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Discussion paper

Fiscal Policy in an Unemployment Crisis[*]

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Abstract

This paper shows that large fiscal multipliers arise naturally from equilibrium unemployment dynamics. In response to a shock that brings the economy into a liquidity trap, an expansion in government spending increases output and causes a fall in the unemployment rate. Since movements in unemployment are persistent, the effects of current spending linger into the future, leading to an enduring rise in income. As an enduring rise in income boosts private demand, even a temporary increase in government spending sets in motion a virtuous employment-spending spiral with a large associated multiplier. This transmission mechanism contrasts with the conventional view in which fiscal policy may be efficacious only under a prolonged and committed rise in government spending, which engineers a spiral of increasing inflation.

Keywords: Fiscal multiplier, liquidity trap, zero lower bound, unemployment inertia.

1 Introduction

The aggressive fiscal response to the financial crisis of 2008 sparked a heated debate over the merits of countercyclical government spending. Critics questioned the transmission mechanisms typically invoked to support the effect of fiscal policy, expressing concerns over their theoretical and empirical foundations. These concerns applied to both the traditional view, essentially inspired by the "Keynesian cross" linking current income to current spending, and the new Keynesian view, which stresses the need to sustain demand over time in order to reduce long-term interest rates by engineering a rise in expected inflation. This paper addresses some of those concerns by exploring a novel channel through which large fiscal multipliers can arise from equilibrium unemployment dynamics.

The key mechanism underlying the main results in this paper stems from the interaction between two widely accepted premises. First, at a zero rate of nominal interest, output is largely determined by demand. If households wish to consume more, firms will also produce more. Second, the labor market is frictional. Any change in current unemployment will therefore partly persist into the future. The core contribution of this paper is in showing that the interaction of these two properties implies that even a temporary expansion in government expenditures can have a large and lasting impact on output.

To appreciate this result, consider the effect of a transitory spending hike. Higher spending raises output (premise 1) and lowers the unemployment rate both in the present *and* in the future (premise 2). As forward-looking agents desire to smooth consumption over time, a rise in future output feeds back to a rise in present private spending, and the unemployment rate falls further. This interplay between present and future economic activity sets off a virtuous "employment-spending" cycle which propagates the effect of demand stimulating policies many times over. The fiscal multiplier associated with these dynamics is often around two, with unambiguous improvements in welfare.[1] These results contrast to the existing literature, in which policy efficacy hinges on either the effect of current spending on current income, or the effects of persistent government spending on expected inflation dynamics.

The main idea can more clearly be illustrated by showing how the same mechanism can tilt the economy into a slump in the first place. I model a negative demand shock as the arrival of disappointing news concerning future labor productivity.[2] The sudden rise in

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[1]A Lucas (1987) style welfare calculation suggests gains of approximately 0.8 dollars of private consumption per dollar spent on government consumption.

Note that Ricardian equivalence implies that there is no conflict between a tax- or debt-financed expansion in government spending, nor any dichotomy between the fiscal multiplier and the balanced budget multiplier (see Barro (1974) and Haavelmo (1945)).

[2]The idea that news about future fundamentals may be a key driver of business cycles originates from

pessimism leads households to save rather than to spend, causing a decline in the nominal interest rate. If the news is sufficiently bad, the interest rate is pushed to zero and nominal spending plummets.

In a Walrasian goods market, a fall in demand causes a decline in the price level. Provided that nominal wages are downwardly rigid, falling prices reduce profits, discourage hiring, and provoke a decline in current economic activity. With *persistent* unemployment, a slump in the present makes the future appear even bleaker. Troubled by an outlook of both low productivity *and* elevated joblessness, households take further measures to smooth consumption. But the additional savings only make matters worse, and the economy is set on a downward spiral of self-reinforcing thrift.

Where does this process end? Agents save because the future appears worse than the present. Eventually, however, the rise in savings erodes enough current output to restore intertemporal optimality, and the spiral comes to a halt. I quantitatively gauge this effect by calibrating the model to the US economy at a quarterly frequency. In the baseline scenario, agents unexpectedly learn that labor productivity will temporarily fall in the consecutive period. Even though the shock is short-lived and reverts back within one quarter, output falls by 1.9 percent on impact, and recovers slowly over the course of about 1.5 years (see Figure 1).

Yet the same force that propels the economy downwards can also be turned around. An increase in government spending increases demand and puts upward pressure on prices. The rise in profits spurs economic activity and encourages hiring, and the unemployment rate falls. Moreover, since also a decline in unemployment is persistent, the future appears brighter. Amidst a less troubling outlook households start to spend, and the vicious cycle is soon turned virtuous. A temporary burst in spending equal to one percent of GDP attenuates the fall output in the above scenario from 1.9 to 0.5 percent. Thus, a simple back of the envelope calculation suggests a fiscal multiplier of around 1.4.

These results relate to a long-standing literature in economics, originating of course with Keynes (1936). But though the Keynesian literature indeed suggests a large fiscal multiplier, it is difficult to draw a meaningful comparison due to the profound methodological differences. Just as the Keynesian narrative draws heavily on agent myopia, the current paper draws equally heavily on the idea that agents are forward-looking and rational. The fiscal multiplier is not large in spite of rational expectations, but because of rational expectations.

The theoretical landscape is of course more leveled now. In the context of a standard

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Pigou (1927). Beaudry and Portier (2004) inspired a renewed interest in the topic (e.g. Jaimovich and Rebelo (2009); Den Haan and Kaltenbrunner (2009)); recent empirical studies lend support to this view of macroeconomic fluctuations (e.g. Beaudry and Portier (2006); Barsky and Sims (2012)).

flexible price, real business cycle model, a rise in government spending reduces private wealth and stimulates labor supply (e.g. Barro and King (1984), Aiyagari et al. (1992), and Baxter and King (1993)). Real wages fall in response to clear the labor market, but the net effect on output is unambiguously positive. However, the same wealth effect which instills a rise in output is also responsible for crowding out private consumption, and the multiplier falls short of unity.[3]

In the context of the new Keynesian literature, a rise in public spending again stimulates labor supply, but increases (instead of decreases) real wages, and cushions the aforementioned fall in private consumption. The eventual effect on economic activity, however, depends crucially on the conduct of monetary policy, and the fiscal multiplier remains below one under a wide range of circumstances.[4]

Those circumstances do not extend to the situation of a liquidity trap. In recent seminal work, Eggertsson (2011) and Christiano et al. (2011) study the effect of government spending in the context of a standard new Keynesian model with staggered pricing. In similarity to this paper, they find that the efficacy of demand stimulating policies can be significantly higher when the nominal interest rate is up against zero.[5] But behind these similarities hide several pronounced differences; some are merely narrative, but most are substantive.

Eggertsson (2011) and Christiano et al. (2011) model a negative demand shock as a decline in the rate of time preference. In similarity to a news shock, this decline pushes the nominal interest rate towards zero, after which any further fall instead erodes output. Without a frictional labor market, however, there are no endogenous intertemporal linkages, and a purely temporary shock subdues any internal propagation. The negative effect on output is small and short-lived, and the fiscal multiplier equals one.[6] These results contrast with those in this paper, in which even a transitory demand shock can lead to a deep and prolonged recession, and a large fiscal multiplier.

However, when the economy instead is struck by an extended sequence of shocks, Eggertsson's (2011) and Christiano et al.'s (2011) results take another turn. The fall in demand

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[3]Baxter and King (1993) show that when the rise in government spending is permanent, the long run multiplier may exceed one at the expense of dynamic efficiency.

From an empirical perspective, Fatas and Mihov (2001), Blanchard and Perotti (2002), Galí et al. (2007), Perotti (2008), Fisher and Peters (2010) find support of a positive response in consumption. But studies such as Ramey and Shapiro (1998) and Ramey (2011) do not.

[4]Monetary policy tends to "lean against the wind". See for instance, Galí et al. (2007), Monacelli and Perotti (2008), and Woodford (2011) for a detailed discussion. Hall (2009) provides an excellent survey.

[5]In the baseline analysis, Christiano et al. (2011) find a multiplier of 3.7, and Eggertsson (2011) of 2.3. The difference can be traced to the choice of preferences.

[6]A multiplier of one follows from temporary demand shock with an expected duration of one period: i.e., setting μ to zero in equation (30) of Eggertsson (2011), and p to zero and σ equal to one in equation (32) of Christiano et al. (2011)

leads to a fall in the contemporaneous price level. This fall would be inconsequential if the duration of the crisis was expected to be short. But in a prolonged slump, staggered pricing implies that only a small fraction of firms will manage to reduce their prices in the present, with a larger mass instead doing so in the future. The associated deflationary path raises the real interest rate, stifles spending, and sets the economy on a vicious downward cycle.

These authors show that a rise in government spending can unwind this cycle.[7] But analogously to the situation of a temporary demand shock, a temporary rise in spending is toothless unless accompanied by an expectation of future increases. Consequently, only a long lasting, committed expansion in government spending becomes an efficacious tool in combatting a deep and prolonged recession.

These results expose some important contrasts to this paper. As here, Eggertsson's (2011) and Christiano et al.'s (2011) results hinge on an intertemporal feedback mechanism which propagates the efficacy of policy. In their studies, however, this feedback stems from the inflationary/deflationary spirals associated with the prolonged nature of the recession and with the positive "news" associated with the committed rise in future spending.[8] In this paper the feedback mechanism is instead endogenous, relying on the inherent sluggishness observed in frictional labor markets which tightly interlinks current economic activity with the future and vice versa.

At a deeper level, the mechanisms emphasized by both approaches reflect distinct channels of policy transmissions. At a first pass, a rise in government spending increases output and makes the private sector richer through an increase in income – but also poorer through an increase in present value taxation. The net result is a wash. Then, however, the stories start to diverge. In this paper, the rise in public spending stimulates private consumption because the rise in income is associated with a job, and jobs last. In Eggertsson (2011) and Christiano et al. (2011), private spending takes off because the general equilibrium effect associated with a committed rise in future government spending sets off an inflationary spiral and lowers the real interest rate.

Several studies have explored the empirical support for each channel. Bachmann et al. (forthcoming) study US households' readiness to spend in response to changes in inflation expectations. They find that the effect is statistically insignificant outside a liquidity trap, and significant but negative inside. Dupor and Li (2013) question whether the US economy was characterized by a deflationary spiral before the passage of the American Recovery

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[7]A rise in government spending – the argument goes – increases output, and puts upward pressure on prices. If this rise in prices sets the economy on an inflationary path, the vicious cycle is turned around.

[8]Woodford (2011), p. 24, phrases this as: "Eggertsson (2011) obtains a multiplier of 2.3, 1.0 of this is due to the increase in government purchases during the current quarter, while the other 1.3 is the effect of higher anticipated government purchases in the future". An analogous argument applies to Christiano et al.

and Reinvestment Act of 2009. They argue that while measures of expected inflation had increased by a modest amount by the time of the Act's passing, there is no evidence of a systematic link between an individual forecaster's expectation of government spending and his or hers expectation of inflation. On the contrary, in the context of a structural VAR, Dupor and Li (2013) show that inflation systematically responds negatively to innovations in government spending – even during periods of passive monetary policy. Overall, the empirical support for the inflation-expectation channel is not strong.

Evidence related to the mechanism studied in this paper is produced by Bachmann and Sims (2012). These authors assess the impact of government spending on output through a mechanism of consumer "confidence", which is strongly predictive of future fundamentals (c.f. Barsky and Sims (2012)).[9] Following the methodology developed in Auerbach and Gorodnichenko (2012), Bachmann and Sims (2012) deploy a regime-switching structural VAR and find that fiscal multipliers are small or even negative in expansions, but rise to about two in recessions. More interestingly, about half of the effect of government spending on output in recessions can be attributed to the associated – and causal – rise in confidence.

Monacelli et al. (2010) study the effect of government spending on factors closely related to the functioning of the labor market. Using a structural VAR they find that a rise in spending equal to one percent of GDP not only increases output with about 1.6 percent, but also raises labor market tightness with around 20 percent and employment by 1.6 percent, lowering the unemployment rate by 0.6 percentage points. Recent cross-state studies further corroborate these findings. Suárez Serrato and Wingender (2011), Chodorow-Reich et al. (2012) and Shoag (2013) assess the effect of the Recovery Act on job creation. Suárez Serrato and Wingender (2011) find that each job-year cost around $30,000 in government spending, suggesting around 3.3 job-years created per $100,000 spent. Chodorow-Reich et al. (2012) find that $100,000 in government spending generated 3.8 job-years, of which 3.2 were outside the government, health, and education sectors.[10] Shoag (2013) finds that $100,000 in government spending added around 4.8 jobs, of which 2.5 can be attributed to a reduction in unemployment, with the addition 2.3 stemming from a rise in labor market participation.[11] Lastly, Nakamura and Steinsson (2014) use historical data on military procurement spending

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[9]Bachmann and Sims (2012) use the Index of Consumer Expectations from the Michigan Survey of Consumers as a proxy of confidence.

[10]Chodorow-Reich et al. (2012) assume that jobs end immediately with the Act's expiry, and therefore interpret these numbers as a lower bound. Using an average total compensation of $56,000, Chodorow-Reich et al. (2012) estimate the associated fiscal multiplier to be about two.

[11]Other studies such as Wilson (2012) and Feyrer and Sacerdote (2012) suggests smaller effects with around a cost of $125,000 and $107,000 per job, respectively. The differences across studies appear to lie in the precise data used, the definition of spending, and the source of exogenous variation. All these studies come with the usual "local multiplier" caveat (see Nakamura and Steinsson (2014)).

and find that a rise in government expenditures equal to one percent of GDP increases the employment rate by about 1.5 percentage points.

While these results can be considered tentative at best, they do lend support to the view taken in this paper. The transmission mechanism of fiscal policy appears to be closely intertwined with the labor market. A rise in government spending can have a profound, positive effect on job-creation and confidence, and can jointly raise both employment and output.

2 Two-period intuition

This section presents a simplified version of the model to provide some key intuition. The full model is introduced in Section 3, which is self-contained. To enhance analytical tractability, I here make use of two simplifying assumptions. First, the persistence in unemployment is assumed to be exogenous. As shown below, this assumption comes at a surprisingly small loss in generality. Second, because of price rigidities, the model displays a disequilibrium in which labor demand falls short of supply.[12] Thus, the simplified model presented here will to a large extent follow Krugman (1998), extended with an inertial labor market. Both assumptions will be relaxed in the subsequent section.

2.1 Baseline setting

An optimal intertemporal plan allocates consumption between periods t and $t+1$ according to the Euler equation

$$u'(c_t) = \beta(1 + i_{t+1})\frac{p_t}{p_{t+1}}u'(c_{t+1}), \tag{1}$$

where, as usual, c_t denotes consumption, p_t the price level, and i_{t+1} the nominal interest rate. The utility function, $u(\cdot)$, is of constant relative risk aversion such that $u(c) = c^{1-\gamma}/(1-\gamma)$, with $1/\gamma < 1$. To keep the analysis as simple as possible, I consider a deterministic setting.[13]

A cash-in-advance structure determines the price level. If the cash-in-advance constraint is binding in period $t+1$, which I assume, the price level is given by $p_{t+1} = m_{t+1}/y_{t+1}$, where m_{t+1} denotes the monetary base. The price level in period t is instead rigid and set to one.[14]

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[12]This type of analysis is susceptible to Barro's (1977) criticism. Despite this, disequilibrium analysis has recently experienced a bit of a renaissance due to its usefulness as an illustrative tool. For recent, and related, examples, see for instance Shimer (2012), Schmitt-Grohé and Uribe (2012), or Simsek and Korinek (2014). The model in the subsequent section avoids the objections raised by Barro (1977) in the way proposed by Hall (2005).

[13]Section 4.3.1, page 23, deals with the stochastic case.

[14]An alternative interpretation is that the price level instead is rigid at $p_t = m_t/\hat{y}_t$, where $\hat{y}_t$ denotes

The economy is closed and there are no investments. The resource constraint is therefore given by $y_t = c_t + g_t$, where g_t denotes government spending. Since the main focus of this paper is on the aggregate effects of a contemporaneous rise in government spending, g_{t+1} is equal to zero.[15]

Using this additional structure we can rewrite equation (1) as

$$u'(y_t) = \beta(1 + i_{t+1})\frac{y_{t+1}}{m_{t+1}}u'(y_{t+1}), \tag{2}$$

where g_t, for the time being, is left out. With an elasticity of intertemporal substitution (EIS), $1/\gamma$, less than one, the nominal interest rate is an increasing function in future output. Thus, in similarity to Krugman (1998), "bad news" about the future prompts a saving motive which, in equilibrium, lowers the nominal interest rate.[16] Of course, if this news is sufficiently bad the nominal interest rate falls to zero and the economy is dragged into a liquidity trap.

How bad is bad enough? Let potential output in period t be equal to one. Then the value of y_{t+1} that puts the economy right on the cusp of a liquidity trap, y^*, is given by

$$u'(1) = \beta\frac{y^*}{m_{t+1}}u'(y^*), \tag{3}$$

or simply as $y^* = (m_{t+1}/\beta)^{\frac{1}{1-\gamma}}$. Then for any $y_{t+1} \geq y^*$ the nominal interest rate is positive and satisfies equation (3) with $y_t = 1$. The fiscal multiplier is in this case zero as a rise in government spending raises the nominal (and real) interest rate sufficiently to crowd out private consumption entirely.

For any $y_{t+1} < y^*$, however, the nominal interest rate is instead zero. Period t output satisfies

$$u'(y_t) = \beta\frac{y_{t+1}}{m_{t+1}}u'(y_{t+1}), \tag{4}$$

and several of Krugman's (1998) results follow: Worse news about the future exacerbates the current recession; a committed rise in future money supply (i.e. *forward guidance*) dampens the present fall in output; and an increase in government spending gives rise to a multiplier

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"potential output" which can be normalized to one. Then m_{t+1} can be reinterpreted as the *relative* expansion (if $m_{t+1} > 1$) or contraction (if $m_{t+1} < 1$) in money supply between period t and $t+1$. The next subsection shows that price rigidities can be reinterpreted as nominal wage rigidities.

[15]See Section 4.3.1, page 23, for the case of a committed rise in government spending.

[16]In contrast to Krugman (1998), however, the value of the EIS turns out to be important for this result. The reason is that, here, the price level in $t+1$ is endogenous. Thus a rise in y_{t+1} carries both a wealth effect, by increasing future income, and a substitution effect, by increasing future prices and thereby lowering the real interest rate. An EIS which is less than one ensures that the income effect dominates the substitution effect. In Krugman (1998) the future price level is exogenous, and the substitution effect absent.

of *precisely one.*

Lastly, and before I extend the model with an inertial labor market, it is useful to solve for y_t in terms of the threshold value y^* and the news y_{t+1}

$$y_t = \left(\frac{y_{t+1}}{y^*}\right)^{\frac{\gamma-1}{\gamma}} < 1. \tag{5}$$

Thus, output in period t is given by potential output (which is one), multiplied by a fraction determined by the ratio of news to the threshold, y^*. If γ approaches infinity, the EIS approaches zero and y_t approaches a trough of y_{t+1}/y^*.[17] In other words, if future output is known to fall by 2 percent below the threshold that brings the economy on the cusp of a liquidity trap, current output will fall by 2 percent below its potential.

2.2 An inertial labor market

To capture the idea of frictions in the labor market, I will assume that firms produce output according to $y_t = z_t n_t$, where z_t denotes labor productivity and n_t labor.[18] Inertia is then introduced through the law of motion

$$n_{t+1} = n_t^\alpha, \quad \alpha \in [0, 1], \tag{6}$$

where the parameter α governs the degree of frictions in the labor market. If α is equal to zero, the labor market in period $t + 1$ clears seamlessly, and employment is equal to its potential, which is normalized to one. If α is close to one, however, employment displays hysteresis and a reduction in the present can last into the future.

Setting z_t to one, equation (3) is under the current conditions given by

$$u'(y_t) = \beta(1 + i_{t+1})\frac{z_{t+1}y_t^\alpha}{m_{t+1}}u'(z_{t+1}y_t^\alpha). \tag{7}$$

Following the same steps as in the preceding subsection, we can define a threshold value of z_{t+1} that puts the economy on the cusp of a liquidity trap, z^*. However, since employment in period t does not deviate from its potential in this case, z^* takes on the same value as y^* previously.

Given this definition of z^*, if $z_{t+1} \geq z^*$ the model repeats the previous framework, with

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[17]It should be noted that y^* is itself a function of γ such that $\lim_{\gamma \to \infty} y^* = 1$. However, this dependence does not alter the results, and is left out for expositional clarity.

[18]Notice that the first order condition of a price taking firm is then given by $p_t z_t = w_t$, where w_t denotes the nominal wage. Nominal wage rigidity therefore implies price rigidity, and vice versa. Thus, the rigidity of prices should be seen more as a rhetorical tool than a statement of some fundamental pricing distortion.

identical properties. However, once z_{t+1} falls below z^*, the economy falls into a liquidity trap and output is instead given by

$$y_t = \left(\frac{z_{t+1}}{z^*}\right)^{\frac{\gamma-1}{\gamma-\alpha(\gamma-1)}} \leq \left(\frac{z_{t+1}}{z^*}\right)^{\frac{\gamma-1}{\gamma}}, \tag{8}$$

where the expression on the right-hand side of the inequality sign rephrases equation (5) but in terms of labor productivity.

Equation (8) captures the consequences of unemployment inertia on output succinctly. Consider the polar cases $\alpha = 0$, and $\alpha = 1$. In the former case the weak inequality in equation (8) turns into an equality and the model collapses to that of the previous subsection. Again, as the EIS approaches zero output approaches a trough of z_{t+1}/z^*. However, when α is instead equal to one this result takes an unexpected turn. With an EIS close to zero, output approaches a trough of zero, and unemployment soars to 100 percent. Thus, in this latter case, even a small reduction in future labor productivity shuts the entire economy down.

The intuition behind this result is relatively straightforward. As previously, a known drop in future labor productivity encourages saving and pushes the nominal interest rate to zero. Any further reduction in future output then translates instead to a contraction in the present. With rising and persistent unemployment, however, the future is no longer only unproductive but also plagued by a higher unemployment rate. The rise in unemployment thus reinforces the saving motive, exacerbates the current recession, and sets off a downward "unemployment-saving" spiral.

So what can the government do to address this vicious cycle? Since prices are rigid, a transitory expansion in government spending raises demand and increases output one-to-one. With rising output the unemployment rate falls. But since a reduction in the unemployment rate is also expected to last, the boost in the present makes the future appear less troubling, and the desire to save is weakened. With more private spending the unemployment rate takes yet another drop, the future appears even less troubling, and the vicious cycle turns into a virtuous. Proposition 1 formalizes this intuition.

Proposition 1. *When $z_{t+1} < z^*$ the fiscal multiplier is given by*

$$\frac{\partial y_t}{\partial g_t} = \frac{1}{1 - \alpha(1 - \frac{1}{\gamma})} \in [1, \gamma].$$

Proof. The Euler equation is in this case given by

$$u'(y_t - g_t) = \beta \frac{z_{t+1} y_t^{\alpha}}{m_{t+1}} u'(z_{t+1} y_t^{\alpha}). \tag{9}$$

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A straightforward application of the implicit function theorem around $g_t = 0$ gives the result. □

Consider again the polar opposites $\alpha = 0$, and $\alpha = 1$. In the former case the fiscal multiplier is precisely equal to one. This should come as no surprise given the results of the previous subsection. In the latter case however, the fiscal multiplier is instead equal to γ; the reciprocal of the EIS. The reason is that a rise in current government spending raises output in the present *and* in the future. Since the rise in future output reduces the future price level, the real interest rate increases and suppresses current demand. When the EIS is very large, the dampening effect from a higher real interest rate is substantial, and the multiplier is therefore quite small. However, when the EIS is instead small, current consumption demand is largely insensitive to intertemporal price changes and the multiplier is instead much larger.

Before I turn to the full model, three remarks are in order. First, and in contrast to Eggertsson (2011) and Christiano et al. (2011), the above mechanism does not rely on the government setting off an inflationary spiral. In fact, a rise in government spending tends to lead to less inflation, not more. If a central bank would not tolerate these variations in the future price level, output in period t and the associated multiplier are instead given by

$$y_t = \left(\frac{z_{t+1}}{z^*}\right)^{\frac{1}{1-\alpha}}, \qquad \frac{\partial y_t}{\partial g_t} = \frac{1}{1-\alpha},$$

with z^* given as $z^* = (\pi/\beta)^{\frac{1}{1-\gamma}}$, and π denoting some inflation target.[19] Thus, the cash-in-advance structure introduces a counteracting, rather than amplifying, force.

Second, since the effect of current spending on output lasts for several periods into the future the possibility of a cumulative multiplier arises.[20] In particular, notice that $y_{t+j} = y_t^{\alpha^j}$. Thus,

$$\frac{\partial y_{t+j}}{\partial g_t} = \alpha^j y_t^{\alpha^j - 1} \frac{\partial y_t}{\partial g_t} = \alpha^j \frac{y_{t+j}}{y_t} \frac{\partial y_t}{\partial g_t} \geq \alpha^j \frac{\partial y_t}{\partial g_t}.$$

A lower bound for the cumulative multiplier is therefore given by

$$\sum_{j=0}^{T} \frac{\partial y_{t+j}}{\partial g_t} \geq \frac{1 - \alpha^{T+1}}{1 - \alpha} \frac{\partial y_t}{\partial g_t},$$

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[19]In fact, the assumption of $1/\gamma < 1$ is redundant in this case.

[20]Although this section uses the narrative of a two-period model, there is nothing inherently two-period about it. In fact, as long as the economy is not in a liquidity trap in period $t+1$, the model can be interpreted as cast in an infinite horizon. See Rendahl (2014) for more details.

which can be many times larger than the impact multiplier itself.

Lastly, while government spending is welfare improving it can never attain the social optimum. Since private consumption is monotonically decreasing in the output gap, the optimal level of spending is determined by the minimum amount necessary to bring the economy back to its full potential. Private consumption is then given by

$$c_t = \left(\frac{z_{t+1}}{z^*}\right)^{\frac{\gamma-1}{\gamma}},$$

which falls short of one. Thus an alternative policy which lowers the real interest rate – such as a temporary cut in sales taxes (Correia et al., 2013), or a commitment from the monetary authority "to act irresponsibly" (Krugman, 1998) – can bring the economy back to its potential at a consumption level possibly closer to the social optimum.[21] The underlying reason is that while government spending can indeed bring the economy back to its potential, it does so at the cost of wastefully consuming a non-negligible share of the additional resources it generates.

The simple framework analyzed above is cast in a deterministic setting. I will have reason to return to this model in Section 4.3.1, and then extend it to include a demand spell of stochastic duration. As we will see, this will provide a useful lens through which some of the richer dynamics in Section 4 can be understood.

3   Model

The economy is populated by a government, a large number of potential firms, and a unit measure of households. The planning horizon is infinite, and time is discrete. As in the previous section there are two types of commodities in the economy. Cash, m_t, which is storable, but not edible. And output, y_t, which is edible, but not storable. Cash assumes the role of the numeraire, and output trades at relative price p_t. In order to abstract from interaction effects with monetary policy, I assume that cash is in fixed supply, such that $m_t = m$ for all time periods, t. The output good, however, is repeatedly produced in each period using labor, n_t, and labor productivity, z_t, according to $y_t = z_t n_t$. The labor market is frictional in the Mortensen-Pissarides tradition. There is no physical capital in the conventional sense, but there are investments.

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[21]I use the word "possibly" here as such policies may come with challenges and trade-offs of their own.

3.1 Households

Households initiate their lives in period zero. They supply labor inelastically and the time-endowment is normalized to one. Employment is denoted n_t, and the unemployment rate is therefore given by the difference in labor supplied and labor demanded, $u_t = 1 - n_t$. In a frictional labor market employment is beyond the control of the households and will, for the time being, be treated as given. The wage rate in the economy is denoted $\tilde{w}_t$.

Each household owns firm equity. I will let q_t^t denote the quantity of shares held in period t (subscript), and purchased in period t (superscript). As a fraction, δ, of firms will exit the market in each period it follows that $q_{t+1}^t = (1 - \delta)q_t^t$.[22] Equity dividends are denoted $\tilde{d}_t$.

Total income, or simply *income*, w_t, constitutes both total labor income, $n_t \times \tilde{w}_t$, and dividends, $q_t^t \times \tilde{d}_t$. There are complete insurance markets across households, so each household earns income w_t irrespective of whether she is employed or not. Following Lucas (1982), income is received at the very end of a period – i.e. after any consumption decisions – and is therefore *de facto* disposable first in the ensuing period.

A representative household enters period t with bonds b_t, and equity q_t^{t-1}. She receives income w_{t-1}, as well as any unspent cash from the preceding period. Bonds are nominally riskless and pay net return i_t. The price of equity in terms of the output good is denoted J_t, so the total nominal equity value is given by $p_t J_t q_t^{t-1}$. Out of these nominal resources, the household pays lump-sum taxes T_t, and may spend the remainder on consumption, $p_t c_t$, or on purchases of new assets. Households are restricted to settle all consumption purchases in cash, such that $M_t \geq p_t c_t$. The sequence of budget constraints is therefore given by

$$b_t(1 + i_t) + p_t J_t(q_t^{t-1} - q_t^t) + (M_{t-1} - p_{t-1}c_{t-1}) + w_{t-1} - T_t = M_t + b_{t+1}, \tag{10}$$

$$M_t \geq p_t c_t, \qquad t = 0, 1, \dots, \tag{11}$$

where the term $(M_{t-1} - p_{t-1}c_{t-1})$ refers to unspent cash in the preceding period.

To keep the analysis as clear as possible, it is useful to make the change of variables $x_{t+1} = M_t - p_t c_t$, and rewrite equations (10)-(11) as

$$b_t(1 + i_t) + p_t J_t(q_t^{t-1} - q_t^t) + x_t + w_{t-1} - T_t = p_t c_t + x_{t+1} + b_{t+1}, \tag{12}$$

$$x_{t+1} \geq 0, \qquad t = 0, 1, \dots \tag{13}$$

The variable x_{t+1} will be referred to as *excess cash* in period t.

Given a process of taxes, prices, and income, $\{T_t, p_t, J_t, i_t, w_{t-1}\}_{t=0}^{\infty}$, the household decides on feasible consumption, excess cash, and equity plans $\{c_t, x_{t+1}, b_{t+1}, q_t^t\}_{t=0}^{\infty}$, to maximize her

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[22]Households therefore hold a diversified portfolio of otherwise identical firms/assets.

expected net present value utility

$$V(\{c_t\}_{t=0}^{\infty}) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{14}$$

subject to constraints (12)-(13). As previously, the instantaneous utility function $u(\cdot)$ displays constant relative risk aversion, such that $u(c) = c^{1-\gamma}/(1-\gamma)$, with $1/\gamma < 1$. The expectations operator denotes the mathematical expectation with respect to future processes, conditional on information available in period zero.

The first order conditions associated with the problem in (14) subject to (12)-(13) are given by the Euler equation for bond holdings, b_{t+1},

$$u'(c_t) = \beta E_t[(1+i_{t+1})\frac{p_t}{p_{t+1}}u'(c_{t+1})], \tag{15}$$

the Euler equation for excess cash holdings, x_{t+1},

$$u'(c_t) - \lambda_t p_t = \beta E_t[\frac{p_t}{p_{t+1}}u'(c_{t+1})], \tag{16}$$

and the asset pricing equation for equity, q_t^t,

$$J_t = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(\frac{\tilde{d}_t}{p_{t+1}} + (1-\delta)J_{t+1} \right) \right]. \tag{17}$$

The variable λ_t denotes the Lagrange multiplier associated with the cash in advance constraint (13). Thus, λ_t and x_{t+1} satisfy the complementary slackness conditions

$$\lambda_t \geq 0, \quad x_{t+1} \geq 0, \quad \text{and} \quad \lambda_t \times x_{t+1} = 0.$$

As a consequence, excess cash holdings are strictly positive only if the nominal interest rate is zero – i.e. when bonds and cash act as perfect substitutes. I will think of a liquidity trap as a situation in which x_{t+1} strictly exceeds zero.

3.2   Government

Apart from lump-sum taxes, the government has access to two additional policy tools; government spending, G_t, and public debt, d_t. For ease of exposition, they are all denominated in terms of the numeraire. Because of Ricardian equivalence, it is possible to remain agnostic with respect to the timing of taxes. Thus, a *fiscal plan* is a process of taxes, spending, and

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debt $\{T_t, G_t, d_t\}_{t=0}^{\infty}$, which satisfies the sequence of budget constraints

$$T_t + d_{t+1} = G_t + (1 + i_t)d_t, \quad t = 0, 1, \ldots, \tag{18}$$

as well as the no-Ponzi condition

$$\lim_{n \to \infty} \frac{d_{n+1}/p_{n+1}}{\Pi_{s=t}^{n}(1 + i_{s+1})p_s/p_{s+1}} \leq 0. \tag{19}$$

3.3   Firms

A potential firm opens up a vacancy at cost $\kappa \geq 0$. The vacancy cost is denominated in terms of the output good. Conditional on having posted a vacancy, the firm will instantaneously meet a worker with probability h_t. If not, the vacancy is void and the vacancy cost, κ, is sunk. A successfully matched firm-worker pair becomes immediately productive and produces z_t units of the output good in each period.[23] The employment relation may last for perpetuity, but workers and firms separate at rate δ. Equity therefore pays nominal dividends $\tilde{d}_t = p_t z_t - \tilde{w}_t$.

A representative entrepreneur seeks to maximize the share value of the firm, J_t. As a consequence, a vacancy is posted in period t if and only if the expected benefits, $h_t J_t$, (weakly) exceed the associated cost, κ. Free entry ensures that $\kappa = h_t J_t$, for $t = 0, 1, \ldots$

3.4   Matching Market

The labor market is frictional. Let $\hat{u}_t$ denote the *beginning of period* unemployment rate. That is, $\hat{u}_t = u_{t-1} + \delta n_{t-1}$. And let $\hat{v}_t$ denote the measure of vacancies in period t. Following the ideas underlying the Mortensen-Pissarides model (e.g. Diamond (1982); Mortensen and Pissarides (1994)), the measure of successful matches is given by

$$H_t = H(\hat{v}_t, \hat{u}_t).$$

The function $H(\cdot, \cdot)$ exhibits constant returns to scale, and a firm posting a vacancy will therefore find a worker with probability

$$h_t = \frac{H_t}{\hat{v}_t} = h(\theta_t), \quad \text{with} \quad \theta_t = \frac{\hat{v}_t}{\hat{u}_t}.$$

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[23]To ensure that employment, and therefore also output, is elastic with respect to contemporaneous changes in demand, I follow Blanchard and Galí (2010) and assume that a vacancy posted in period t can be filled with a positive probability *within the same period*. This contrasts with, for instance, Hall (2005) in which it is assumed that a vacancy posted in period t can only be filled with a positive probability in period $t + 1$.

As usual, θ_t denotes the labor market tightness in period t. Analogously, a worker unemployed in the beginning of period t will find a job with probability

$$f_t = \frac{H_t}{\hat{u}_t} = f(\theta_t), \quad \text{with} \quad f_t = \theta_t h_t.$$

The law of motion for employment is then given by

$$n_t = \hat{u}_t f_t + (1 - \delta) n_{t-1}.$$

3.4.1. Wage bargaining

Nominal wages, $\tilde{w}_t$, are determined by Nash bargaining. Nash bargaining seeks to maximize the Nash product of each party's surplus associated with a match. As there are complete insurance markets across households, however, unemployment has no financial meaning in the context of a household's surplus, which is therefore always zero. To circumvent this problem, I will follow Den Haan et al. (2000) and Gertler and Trigari (2009), and view workers as separate risk-neutral entities which are owned and traded by households like an asset. As a result there is a price tag attached to each worker. The market price for an employed worker, V_t, and an unemployed worker, U_t, are then given by

$$V_t = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(\frac{\tilde{w}_t}{p_{t+1}} + (1 - \delta(1 - f_{t+1}))V_{t+1} + \delta(1 - f_{t+1})U_{t+1} \right) \right],$$

and

$$U_t = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(\frac{\tilde{b}}{p_{t+1}} + f_{t+1}V_{t+1} + (1 - f_{t+1})U_{t+1} \right) \right],$$

respectively.[24] Here $\tilde{b}$ denotes unemployment benefits. The cost of the unemployment insurance program is financed through lump-sum taxes levied on all households, and is therefore non-distortionary.

Wages are determined as

$$\tilde{w}_t = \text{argmax} \left\{ J_t^{1-\omega} (V_t - U_t)^{\omega} \right\},$$

where the parameter $\omega \in (0, 1)$ governs the worker's relative bargaining power over the firm.

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[24]The asset value V_t also corresponds to the *marginal* value of an additional employed worker to the household. A similar interpretation applies for U_t.

3.5 Equilibrium

We can now state the definition of a competitive equilibrium.

Definition 1. *Given a fiscal plan, a* **competitive equilibrium** *is a process of prices,* $\{p_t, i_{t+1}, \tilde{w}_t, J_t\}_{t=0}^{\infty}$, *and quantities,* $\{c_t, q_t^t, b_{t+1}, x_{t+1}, \theta_t, y_t, n_t, I_t\}_{t=0}^{\infty}$, *such that,*

(i) *Given prices,* $\{c_t, q_t^t, b_{t+1}, x_{t+1}\}_{t=0}^{\infty}$ *solves the households' problem.*

(ii) *Labor market tightness,* θ_t, *satisfies the free-entry condition,* $\kappa = h(\theta_t)J_t$.

(iii) *Employment,* n_t, *satisfies the law of motion*

$$n_t = (\delta n_{t-1} + (1 - n_{t-1}))f(\theta_t) + (1 - \delta)n_{t-1}$$

(iv) *Output,* y_t, *is given by* $y_t = z_t n_t$.

(v) *Investment,* I_t, *is given by* $I_t = p_t \hat{v}_t k$.

(vi) *Wages satisfy* $\tilde{w}_t = \text{argmax}\{J_t^{1-\omega}(V_t - U_t)^{\omega}\}$.

(vii) *Bond markets clear;* $b_t = d_t$.

(viii) *Equity markets clear;* $q_t^t = n_t$.

(ix) *Goods markets clear;* $p_t y_t = p_t c_t + G_t + I_t$.

The definition above omits money market clearing, which follows from Walras law. In particular, consolidating the households' and the government's budget constraints, and using the above equilibrium relations, gives

$$x_t + w_{t-1} = p_t y_t + x_{t+1}. \tag{20}$$

If the left hand side of (20) is equal to m in period t, it must still equal m in period $t+1$, since nominal output, $p_t y_t$, is equal to nominal income, w_t. Thus, as an initial condition I set $x_0 + w_{-1} = m$, which implies that

$$m = p_t y_t, +x_{t+1}, \quad t = 0, 1, \dots, \tag{21}$$

That is, money supply, m, is equal to money demand, $p_t y_t + x_{t+1}$, for $t = 0, 1, \dots$ Money is therefore either used for the purchases of output and/or as cash hoardings in a liquidity trap.[25]

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[25]Notice that the money market equilibrium can be written as $mv_t = p_t y_t$, where v_t is the "velocity of money", $v_t = (m - x_{t+1})/m$. The velocity of money is therefore capped at one, and falls below one only in a liquidity trap.

One of the main reasons for introducing money using a cash-in-advance model is to avoid the multiplicity arising in the cashless limit.[26] The following proposition is therefore particularly pertinent.

Proposition 2. *There exists a unique steady state competitive equilibrium.*

Proof. In Appendix A □

4 Results

This section provides a numerical analysis of the above model. The objective is to understand the mechanisms involved in the richer framework of Section 3, and to gauge the effect of fiscal policy in an equilibrium setting with endogenous labor market frictions. After laying out the calibration, I will describe the main experiments, and graphically illustrate the results. In the simplest case – which is largely similar to that of Section 2 – the impact multiplier is around two, and cumulates to about five over the course of eight quarters. The associated welfare implications are unambiguously positive, and a dollar spent by the government raises welfare by the equivalent of 4 dollars of private consumption. In more complicated experiments the impact multiplier can be smaller, and the welfare effects may even turn negative. When the economy is not in a liquidity trap the fiscal multiplier is always negative, as the associated rise in the real interest rate crowds out investments. The effect of government spending outside a liquidity trap is provided in Appendix B.

4.1 Calibration

The model is calibrated to target the US economy at a quarterly frequency. The discount factor, β, is set to $1.03^{-1/4}$ which corresponds to a 3 percent annual real interest rate. The steady state level of labor productivity, z, is normalized to unity, and cash, m, is set equal to the steady state employment rate, n. As a consequence, the steady state price level, p, equals one. The EIS is set to $1/2$.

The matching function is of a standard Cobb-Douglas type, and given by $H(\hat{v}_t, \hat{u}_t) = \varphi \hat{v}_t^\eta \hat{u}_t^{1-\eta}$. Following Hall (2005), the elasticity of job finding with respect to labor market tightness, η, is set to 0.765.[27] The efficiency of the matching function, φ, is such that the

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[26]See Braun et al. (2012), Christiano and Eichenbaum (2012), Mertens and Ravn (forthcoming) and Cochrane (2013).

[27]An elasticity of 0.765 is in the upper range of empirical estimates (see Petrongolo and Pissarides (2001) for a survey). Hagedorn and Manovskii (2008) use an elasticity of around 0.55, and Shimer (2005) of 0.28. The free entry condition implies that the elasticity of the job finding probability with respect to asset prices is equal to $\eta/(1-\eta)$. Thus, when η is relatively high, even small variations in the asset price translate to

steady state unemployment rate is 6 percent. As a consequence, the associated job finding probability is equal to 62 percent, implying an expected unemployment duration of around 7.5 weeks.[28] The separation rate, δ, is equal 4×0.034 which is in line with the average found in the data.[29]

Unemployment benefits are set to 0.5, which approximates a 50 percent replacement rate (Chetty, 2008). The workers' bargaining power, ω, is set to 0.7. This is in line with Shimer's (2005) value of of 0.72, but higher than both Hall's (2005) value of 0.5 and Hagedorn and Manovskii's (2008) of 0.052. However, it should be noted that neither the bargaining power nor the replacement rate are, in isolation, important factors for the model properties. Rather it is their combined effect on firms' profit margins that matter. Here, the profit margin is around 3.3 percent, which can be compared to 3.4 percent in Hall (2005), 2.25 percent in Hagedorn and Manovskii (2008), 2 percent in Pissarides (2009), and 0.7 percent in Shimer (2005).[30] In this dimension the calibration therefore lies on the conservative side.

Given a labor market tightness normalized to one, the cost of posting a vacancy κ is set to $h(1)J$.[31] Real non-discretionary government spending equals 20 percent of steady state output.

The calibrated parameter values are summarized in Table 1.

4.2 Experiments

The economy is in the steady state at time $t - 1$. In period t agents are notified that labor productivity in $t + 1$ will decline by 1 percent with probability q. The fall in productivity is temporary and reverts back to its steady state value from period $t + 2$ onwards. However, with the complementary probability, $(1 - q)$, there is no decline in labor productivity. Rather the threat continues, and with probability q productivity will instead fall by 1 percent in period $t + 2$, and so on. Under this scenario, the economy falls into a liquidity trap in period t with expected duration of $1/q$ quarters. To keep the analysis as clear as possible, the productivity fall will never materialize. Thus, in the language of Lorenzoni (2009), the shock is more noise than news.

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large variations in job finding, and therefore also in employment. Setting η to 0.5 scales down most results, including the efficacy of policy, to about half. The qualitative story, however, is left intact.

[28] According to the Bureau of Labor Statistics, the median unemployment duration in the United States averages to 8.5 weeks over the years 1967 to 2014, and 7.1 weeks excluding the financial crisis.

[29] The actual monthly average according to JOLTS 2001-2014 is around 0.035. Hall (2005) and Shimer (2005) use a value of 0.034. The results are entirely insensitive to the precise number used.

[30] Setting ω to 0.5 would imply a steady state wage of 0.929, and therefore to a profit margin of 7.1 percent.

[31] Notice that it is possible to multiply the steady state value of θ with a factor of 2 and φ with a factor of 2^ω without altering the steady state values of f and n. The steady state value of h and κ are then halved. Thus, as in Shimer (2005), the steady state value of θ is intrinsically meaningless and can be normalized.

Table 1: Calibrated parameters

Parameter	Interpretation	Value	Source/steady state target
γ	Inverse of EIS	2	Convention
β	Discount factor	0.993	Annual real interest rate of 3%
φ	Efficiency of matching	0.615	Unemployment rate of 6%
δ	Separation rate	0.136	Literature/JOLTS
ω	Workers bargaining power	0.7	Steady state profit margin of 3.3%
η	Elasticity of $f(\theta)$	0.765	Hall (2005)
κ	Vacancy posting cost	0.19	Steady state θ normalized to one
$\tilde{b}$	Unemployment benefits	0.5	Chetty (2008)
$\bar{g}$	Steady state fiscal spending	0.188	20% of GDP

Notes. This table lists the parameter values of the model. The calculations and targets are described in the main text. One period in the model corresponds to one quarter.

Nominal wages are downwardly rigid. In particular, I assume that wages are negotiated before the arrival of the first news shock, and are not renegotiated until the crisis is over. Wage stickiness has received mixed empirical support. Bewley (1999) and Barattieri et al. (2010), for instance, argue that downward nominal wage rigidity is prevalent. Others argue that wages for *new hires* show much more flexibility (e.g. Pissarides (2009) and Haefke et al. (2013)).[32] However, wage stickiness appears important for the transmission of monetary policy (Olivei and Tenreyro, 2007, 2010), and there are reasons to believe that the extent and empirical implications of nominal wage rigidity are not fully understood.[33] Thus, following, Bordo et al. (2000), Gertler and Trigari (2009), Schmitt-Grohé and Uribe (2013a,b), and many others, I will proceed under the assumption that nominal wages are downwardly rigid – at least for a short period of time.

I will consider two different strategies for government spending. The first is a one-shot burst in spending in period t followed by an immediate reversal to its steady state value in period $t + 1$. This is a standard Keynesian experiment. The second follows Christiano et al. (2011) and Eggertsson (2011) and considers the effect of a committed rise in government spending that lasts throughout the entire crisis, but not thereafter. Of course, when q is equal to one, these two policies coincide. The *fiscal multiplier* in the first policy experiment

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[32]Notice that wages of new hires dictate the dynamics of unemployment in most search-theoretic frameworks, including this study.

[33]Diamond (2011), pp. 1062-63, argues that a selection bias arises in observational data as only firms with flexible wage-setting may find it profitable to hire workers in a recession. What matters for job creation is "reservation wages at the marginal vacancy", which is not generally observed.

is given by

$$\chi_T = \frac{\sum_{j=0}^{T} E_t[\Delta y_{t+j}]}{\Delta g_t}, \tag{22}$$

with χ_0 denoting the *impact multiplier*, and χ_n, $n = 1, \ldots, T$, the *cumulative multiplier*. The fiscal multiplier in the second policy experiment is calculated identically to χ_T above, but multiplied by $q/(1 - (1 - q)^{T+1})$.[34] The same terminology is used with respect to the effect on impact versus the cumulative response.

The systems of first order conditions inside and outside a liquidity trap are linearized around the steady state.[35] This has some important implications. The resulting linear policy functions imply that there are no interaction effects across variables. The effect of a deeper crisis, for instance, is therefore only a scaled version of the 1 percent benchmark. Similarly, conditional on the economy being in a liquidity trap, the effectiveness of fiscal policy is unrelated to both the depth of the crisis, the size of the stimulus package and the precise timing of spending. As a consequence, the economy can be fully analyzed through the impulse response functions with respect to shocks at various durations and with respect to the two different spending strategies. Lastly, a non-materialized news shock enters the linearized first order conditions identically to discount factor shock. Thus, the shock driving the economy is directly comparable to that of Christiano et al. (2011) and Eggertsson (2011), although perhaps slightly easier to calibrate.[36]

The impulse response of a variable x_t conditional on some shock s_t is, as usual, defined as $E_t[x_{t+j}|s_t] - E_t[x_{t+j}|\neg s_t]$, $j = 0, 1, \ldots, T$. The computational details are described in Appendix C.

4.3   Results

The solid line in Figure 1 illustrates the response of the economy to a *known* 1 percent decline in period $t + 1$ labor productivity (i.e. $q = 1$). Time is given on the x-axis, and the y-axis illustrates the percent deviation of a variable from its steady state value. Government spending is, however, given as a percent of steady state output. The associated dynamics display what is colloquially referred to as a Pigouvian cycle, in which output, consumption,

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[34]As Δg is assumed to be constant throughout the crisis, we have $\sum_{j=0}^{T} E_t[\Delta g_{t+j}] = \Delta g_t \times (1 - (1 - q)^{T+1})/q$.

[35]In an earlier version of this paper I solve the model using nonlinear methods (Rendahl, 2014). The main message is very similar.

[36]The standard deviation of detrended labor productivity in post-war US data is around 0.02 log points at a quarterly frequency (Shimer, 2005). Using an AR(1) the standard deviation of the forecast error is 0.01 log points. This standard deviation is robust to including k lags.
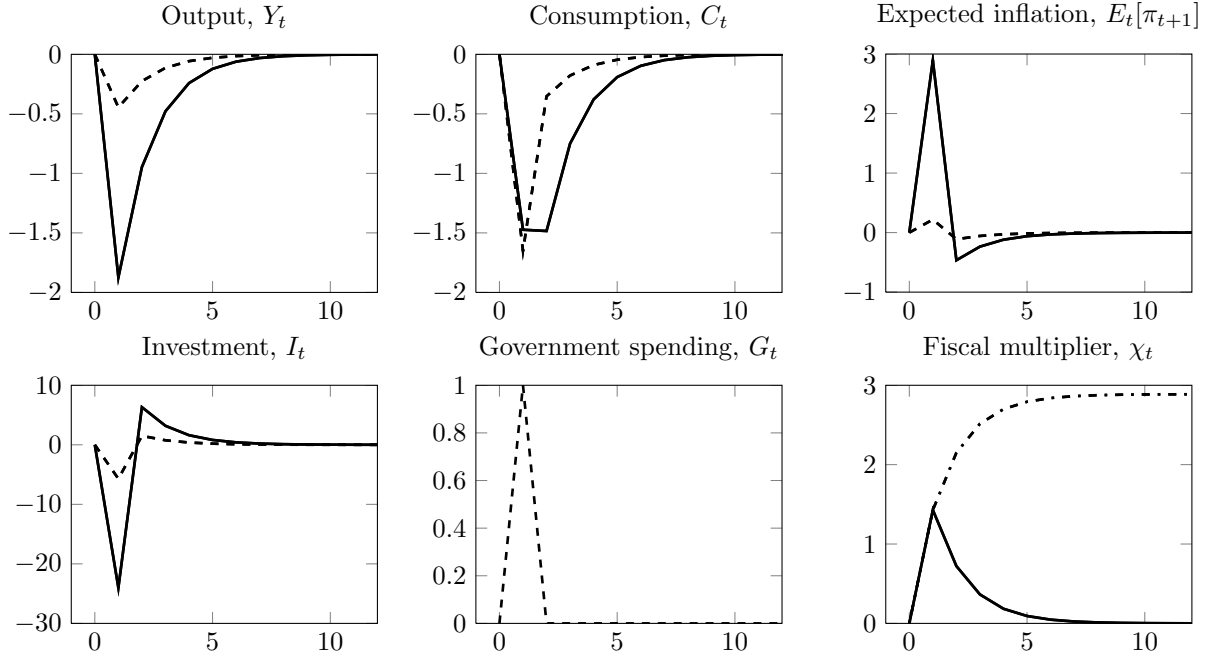
and investment all fall together.[37]

Figure 1: Impulse responses of macroeconomic aggregates to a news shock ($q = 1$), with (solid line), and without (dashed line), a temporary rise in government spending.

The causal mechanism runs as follows. When news arrives, asset prices fall and the vacancy to unemployment ratio declines. Money dominates equity in return, and agents favor excess cash holdings above investments. The ensuing fall in prices raises real wages and provokes a further drop in the share value of a firm. With rising and persistent unemployment, the future looks even bleaker. Excess cash holdings take yet another leap, the price level takes another fall, and there is a further rise in the level of unemployment, and so on.

Where does this process end? As can be seen in Figure 1, investments fall with around 25 percent, which leads to a 1.9 percent decline in output (and roughly a 1.75 percentage point rise in unemployment). Consumption falls slightly less than output, as the reduction in investments helps to buffer the shortfall in income. Lastly there is a sharp rise in expected inflation because the price level in period t declines and is simultaneously expected to rise in period $t + 1$ due to the anticipated negative supply shock.

The dashed line in Figure 1 illustrates the response to an identical shock, but now accompanied by a one-shot expansion in government spending equal to one percent of steady state output. Government spending soaks up excess cash holdings and bolsters demand. The

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[37]Den Haan and Kaltenbrunner (2009) show that standard models with labor market frictions can display Pigouvian cycles even in a non-monetary framework.

associated rise in prices stalls the rise in real wages, and profits improve. More vacancies are posted, the unemployment rate declines, and output increases. An increase in government spending of 1 percent of steady state GDP staves off a 1.4 percent decline in output.

But more importantly, the effect on impact lingers. Output in period $t+1$ is 0.9 percentage points higher than it would be if there was no fiscal expansion in period t. And output in period $t+4$ is still 0.12 percentage points higher than in the absence of fiscal policy. Thus, an expansion in government spending does not only boost output and employment in the present, but also in the future. And as previously argued, it is precisely this interplay between the present and the future that has the capacity to substantially increase the efficacy of fiscal policy.

How powerful is this mechanism? The solid line in the lower right subplot of Figure 1 illustrates the the fiscal multiplier, $\partial y_{t+j}/\partial g_t, j = 0, 1, 2, \ldots$ That is, the change in output in any given period divided by the change in government spending in period t. The fiscal multiplier is slightly above 1.4 on impact, and tapers off over time. The dash-dot line in the same subplot illustrates the cumulative multiplier, defined as the sum of the impact multiplier over time. The cumulative multiplier reaches about 2.9.

4.3.1. Prolonged crises

Figure 2 illustrates the response of the economy to shocks with differing durations. The dotted line illustrates the effect of a liquidity trap expected to last for 8 quarters, and the dash-dot line the effect when the duration reaches 16 quarters. The solid line replicates the results in Figure 1 for ease of comparison

Unsurprisingly, macroeconomic aggregates follow a predictable pattern. With a longer expected duration, the recession is both deeper and longer, with severely depressed consumption and investment levels. What is surprising, however, is the economy's response to policy.

The two pairs of solid lines in the lower right subplot of Figure 2 show the impact and cumulative multiplier in the case of $q = 1$. That is, they repeat the equivalent subplot in Figure 1. The pairs of dotted and dash-dot lines illustrate instead the impact and the cumulative multiplier in the case of $q = 8$ and $q = 16$, respectively. The lowest dotted and dash-dot line show the impact effect of a one-shot burst in government spending, while the highest show the cumulative effect of a committed rise in spending, lasting throughout the crisis.[38]

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[38]The cumulative effect of a one-shot burst in spending is not graphed to keep the figure readable. However, as the effect of a temporary rise in spending is so short-lived, the cumulative effect is indistinguishable from a straight line starting at the peak of the impact multiplier. In the committed case, the impact effect always coincides with the cumulative effect at $t = 1$.
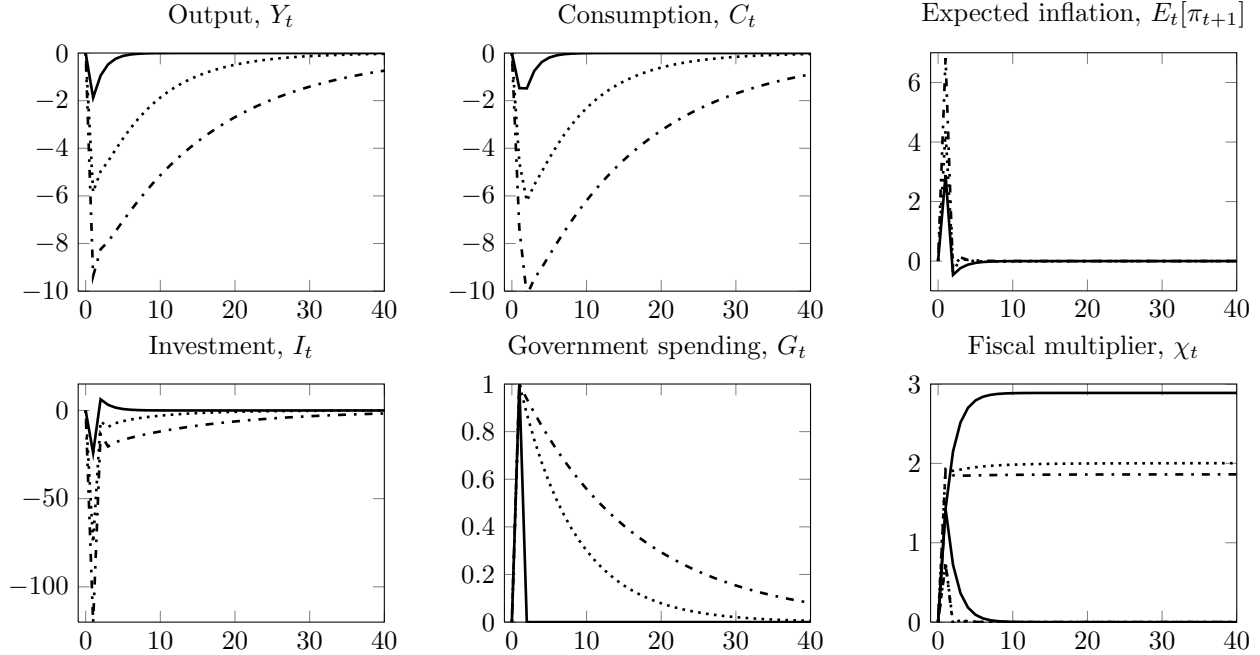
Figure 2: Impulse responses of macroeconomic aggregates to a news shock with duration of 1 (solid line), 8 (dotted line) and 16 (dash-dot line) quarters.

In the Keynesian experiment with a temporary burst in spending, the impact multiplier is relatively small – around 0.75 – and the effect dies out almost immediately. In contrast, the efficacy of policy is significantly larger in the situation in which the government embarks on a committed, long lasting, expansion. To put these numbers in perspective, it is illuminating to compare them to the benchmark experiment. When q is equal to one, the multiplier is 1.4 on impact and accumulates to 2.9. In contrast, at a duration of 8 quarters a committed rise in spending yields an impact multiplier of 1.92, which cumulates to 2.0. And when the duration reaches 16 quarters, the impact and cumulative multiplier coincide at about 1.9.

It is surprisingly easy to make sense of these results. Returning to the simple model of Section 2 – but extended to accommodate the uncertain structure analyzed here – the Euler equation in (7) can be rewritten as

$$u'(\hat{y}) = \beta(qz_{t+1}\hat{y}^\alpha u'(z_{t+1}\hat{y}^\alpha) + (1-q)u'(\hat{y})), \tag{23}$$

where i_{t+1} is zero, m_{t+1} is for simplicity set to one, and $\hat{y}$ denotes the level of output that pertains throughout a liquidity trap absent government intervention. Thus $\hat{y}$ appears on both the right- and the left-hand side of equation (23).[39] In this setting, a committed rise

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[39] As output is entirely demand driven it remains constant in a liquidity trap, and the same level of output appears on both the right- and the left-hand side of the Euler equation. The price level in a liquidity trap

in government spending would manifest itself as $u'(\hat{y})$ being replaced by $u'(y - g)$ in *both* the left- *and* the right-hand side of (23). A one-shot burst would materialize as $u'(\hat{y})$ being replaced by $u'(y - g)$ in the left-hand side *only*.

Consider the case of commitment. By again applying the implicit function theorem, the impact multiplier is in fact identical to the one found in Proposition 1. The associated (expected) cumulative effect, however, is trickier to pin down precisely. Conditional on remaining in a liquidity trap – which appears a justifiable approximation when q is very small – the cumulative multiplier must coincide with the impact multiplier, and therefore appear as a straight line. Figure 2 largely confirms the logic behind both these two results.

With a temporary burst in spending, however, the multiplier falls short of that in Proposition 1, and is equal to

$$\frac{\partial y_t}{\partial g_t} = \frac{1}{1 - (1 - \beta(1 - q))\alpha(1 - \frac{1}{\gamma})} \leq \frac{1}{1 - \alpha(1 - \frac{1}{\gamma})}, \tag{24}$$

where the expression on the right-hand side of the inequality repeats Proposition 1 for convenience. Using the result on the right-hand side of the inequality we can match an impact multiplier of 1.4 from the baseline experiment by setting α equal to 0.57. With $1/q$ set to 8 and 16 respectively, the left-hand side of (24) then suggests a multiplier of 1.04 and 1.02. Thus, the potency of a one-shot burst in policy declines quite significantly with the duration of the crisis – again confirming the results in Figure 2.

What is the intuition underlying these results? A *transitory* expansion in government spending gains traction in a liquidity trap as the rise in present output echoes into the future, and vice versa. The future, however, refers to a period in which the economy has left the liquidity trap, and Say's law lets supply – and not demand – dictate economic activity. As the duration of the crisis lengthens, the "future" becomes ever more distant, and its interplay with the present is weakened; so too is the multiplier.

When the government commits to an *extended* rise in spending, however, there will, in contrast, always be some "present" in the instant right before the "future". Thus, in this period the efficacy of spending is once again high. But as this "present" is, itself, some earlier period's "future", policy remains effective even further back in time, and so on. Thus, in anticipation of future policy efficacy, policy efficacy increases already today.

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is assumed to be constant and normalized to one.

4.3.2. Welfare

While the previous section showed that fiscal policy can stimulate overall demand and raise output inside a liquidity trap, it is less obvious whether it may also increase overall welfare. This section explores this possibility, and shows that it often can. However, at a longer duration the effect of a temporary rise in spending weakens, and it becomes increasingly important for the government to commit to future policy actions, which, for time consistency reasons, becomes increasingly difficult.

Let ψ_1 refer to the policy with a one-shot increase in spending, and ψ_2 to the policy with commitment. To assess the implications on welfare, the value function associated with the crisis (ex-post news), conditional on a spending level g, and policy strategy $\psi \in \{\psi_1, \psi_2\}$ is given by

$$v(n, z; g, \psi, q) = u(c(n, z; g, \psi, q)) + \beta E[v(n', z'; g', \psi, q)], \qquad (25)$$

subject to the law of motion for n and z, and the law of motion for g dictated by ψ. The function $c(n, z; g, \psi, q)$ denotes optimal consumption ex-post news, at state (n, z), under government policy (g, ψ), and crisis-duration $1/q$. As usual, $v(\cdot)$ denotes the value function.

With n and z set equal to their steady state values, we can conduct a Lucas (1987) style welfare calculation according to

$$c(g; \psi, q) = [(1 - \beta)(1 - \gamma)v(g, \psi, q)]^{\frac{1}{1-\gamma}},$$

where the dependency on (n, z) is suppressed as both are set at their steady state values. The consumption function $c(g; \psi, q)$ can be interpreted as the level of consumption that leaves an agent indifferent between consuming $c(\cdot)$ for perpetuity and experiencing a crisis with duration $1/q$ under policy (g, ψ). Thus, an easily interpretable measure of welfare under policies ψ_1 and ψ_2 is given by

$$W(q; \psi_1) = \frac{\partial c(g; \psi_1, q)}{\partial g} \times \frac{1}{1 - \beta}, \qquad W(q; \psi_2) = q\frac{\partial c(g; \psi_2, q)}{\partial g} \times \frac{1}{1 - \beta}, \qquad (26)$$

evaluated around $g = 0$. The partial derivative in $W(q; \psi_1)$ can be interpreted as the marginal, perpetual, consumption equivalent change in utility stemming from a one-shot rise in contemporaneous government spending. To convert this perpetual stream of consumption into a present value, the derivative is divided by $(1 - \beta)$. Thus $W(q; \psi_i)$, $i = 1, 2$, captures the welfare associated with policy ψ_i measured as the dollar gained in consumption equivalents per dollar spent by the government. Notice that $W(q; \psi_2)$ contains the factor q,

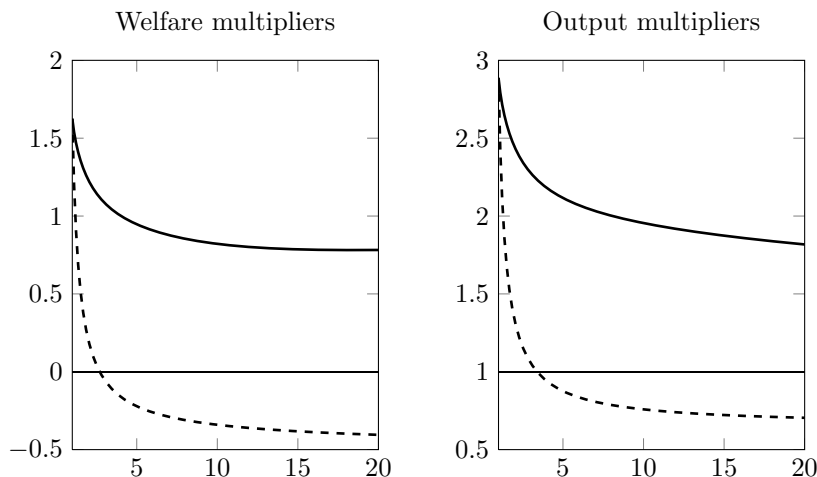since $\sum_{j=0}^{\infty} E_t[\Delta g_{t+j}] = \Delta g/q$.

Figure 3: Welfare and output multipliers with respect to the duration of the crisis, $1/q$. The solid line illustrates the committed strategy, and the dashed line the temporary strategy.

The left graph in Figure 3 illustrates the welfare measures $W(q; \psi_1)$ (dashed line) and $W(q; \psi_2)$ (solid line) with respect to the duration of the crisis, $1/q$. The right graph shows the associated maximal fiscal multipliers. Three results stand out.

First, welfare effects can be sizable. For a short duration, a dollar spent by the government raises welfare by the equivalent of 1.6 dollars of private consumption. For longer durations the welfare effects are smaller and, in the case of commitment, plateaus at around 0.8 dollars of private consumption per dollar of public consumption. With a one-shot burst in spending, however, the welfare effect eventually turns negative.

Second, commitment may be very important. Once the marginal benefit of an additional temporary burst in spending turns negative, policy ψ_2 is time inconsistent. Thus, when the economy is hit by a shock of expected duration longer than 2.7 quarters, commitment is vital from a welfare perspective.

Lastly, the results in Figure 3 lend support to the traditional view that fiscal multipliers in excess of one are welfare improving. Once the one-shot multiplier slightly exceeds unity, the welfare measure reaches its break-even value of zero.[40]

What is the intuition underlying these implications on welfare? It would be tempting, but erroneous, to conclude that theses results stem from the absence of any disutility from work. While working – or searching – does indeed not cause any disutility per se, creating jobs demands resources; and fewer resources means, ceteris paribus, less consumption. This is not a ceteris paribus experiment, however. When the economy is in a sufficiently dire liquidity

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[40]Welfare is zero at an expected duration of 2.7 quarters, at which the one-shot multiplier is equal to 1.1.

trap more resources can be produced using less, and a rise in investments can deliver enough yield to cover its own cost, and more. Thus an increase in government spending increases investment sufficiently to finance both itself and the rise government consumption, and to leave some additional spare resources to be consumed by the private sector. As a consequence, the welfare implications of spending can be large.

5 Concluding Remarks

This paper has shown that large fiscal multipliers can emerge naturally from equilibrium unemployment dynamics. With nominal interest rates stuck at zero, output is largely determined by demand. Forward-looking agents may therefore pass their expectations of future consumption onto current demand, and thus affect contemporaneous economic activity. But with persistent unemployment, changes in current economic activity can similarly affect expectations about future consumption.

This intertemporal propagation mechanism, which is at the core of this paper, amplifies the efficacy of demand stimulating policies many times over. In a stylized framework displaying unemployment hysteresis, the fiscal multiplier is equal to the reciprocal of the elasticity of intertemporal substitution. In a more realistic setting, the effect is somewhat dampened and varies with both the duration of the crisis and the government's commitment to future spending. Yet in most cases the marginal impact of fiscal spending on output is around two, with significant improvements in welfare.

However, the very mechanisms that cause the multiplier to be large also bear on the conduct of fiscal policy. Firstly, government spending should not create jobs for the sake of paying out income to workers. Letting idle workers dig a hole only to fill it up again is not an effective strategy since it is unlikely to deliver a *persistent* decline in the unemployment rate. On the contrary, spending must take the form of purchases of goods and services that would be provided in the economy under normal circumstances if the crisis had not interfered with the macroeconomic equilibrium.

Secondly, government policies must be directed towards economic slack, where the price elasticity of demand is high. Investing in infrastructure during a housing crisis may, for instance, pay a big dividend. However, while government purchases should be directed to sectors or time periods where private demand is temporarily low, public goods must not substitute for private consumption. If the private enjoyment of publicly purchased goods substitute for that of privately purchased goods, the stimulative properties of government spending vanish (cf. Eggertsson (2011)).

Lastly, it should not be forgotten that expansionary government spending may give rise

to further trade-offs that are excluded from this study. High indebtedness can, in some cases, lead to rising risk premia, which would increase real interest rates and possibly dampen the efficacy of policy. Similarly, the anticipation of a future rise in distortive taxation could also cause a drag on the economy that may partly offset some of the beneficial effects of policy explored here.

Nevertheless, the main point of this paper still remains. The joint combination of low rates of nominal interest and persistent unemployment may provide a fertile ground in which accurately targeted fiscal policy can be a potent tool in combatting a deep, demand driven, recession.

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A   Proof of Proposition 2

In a steady state all (real) quantities are constant. As a consequence an equilibrium allocation of prices and quantities must satisfy the following collection of equations for all time periods, t

$$\beta(1+i_{t+1})\frac{p_t}{p_{t+1}} = 1, \tag{A.1}$$

$$p_t y + x_{t+1} = m, \tag{A.2}$$

$$i_{t+1} \geq 0, \tag{A.3}$$

$$x_{t+1} \geq 0, \tag{A.4}$$

$$x_{t+1} i_{t+1} = 0, \tag{A.5}$$

where equations (A.3)-(A.5) capture the complementary slackness conditions in terms of the nominal interest rate. Suppose that $x_{t+1} > 0$ for some t. Then $i_{t+1} = 0$, and equation (A.1) suggests that $p_{t+n} = \beta^n p_t$ for $n = 1, 2, \ldots$ The transversality condition with respect to x_{t+1} is given by

$$\lim_{n\to\infty} \beta^n u'(c)\frac{x_{t+n+1}}{p_{t+n}} \leq 0. \tag{A.6}$$

Using the fact that $x_{t+n+1} = m - p_{t+n}y$, and that $p_{t+n} = \beta^n p_t$, the transversality condition in (A.6) can be rewritten as

$$\lim_{n\to\infty} u'(c)\frac{m - \beta^n p_t y}{p_t} \leq 0. \tag{A.7}$$

Thus, there exists an N such that $\frac{m - \beta^n p_t y}{p_t} > 0$ for all $n \geq N$. Since this violates the transversality condition, x_{t+1} must equal zero for all t. Prices are thus constant and given as $p_t = m/y$, for all t.

Given constant prices, the equilibrium conditions are summarized by

$$J = \beta(1-\eta)(z - \tilde{b}) + \beta J(1-\delta)(1 - \eta f(\theta)), \tag{A.8}$$

$$\kappa = h(\theta)J, \tag{A.9}$$

where the first equation describes the equilibrium steady state asset price at Nash bargained wages, and the second equation is the free-entry condition. Combining equations (A.8) with (A.9) we can define the function $g(\theta)$ as

$$g(\theta) = \frac{1 - \beta(1-\delta)(1 - \eta f(\theta))}{\beta(1-\eta)(z - \tilde{b})} - \frac{h(\theta)}{\kappa}, \tag{A.10}$$

which, in equilibrium, should equal zero. Under standard conditions imposed on the matching function, $h(\cdot)$ is monotone, continuous, and satisfies $\lim_{\theta\to 0} h(\theta) = -\infty$ and $\lim_{\theta\to\infty} h(\theta) = \infty$. Thus there exists a unique steady state equilibrium. $\qquad\square$

B The effect of government spending outside of a liquidity trap

Figure B.1 illustrates the effect of a temporary rise in government spending when the economy is not in a liquidity trap. As can be seen from the graph, a rise in government spending depresses
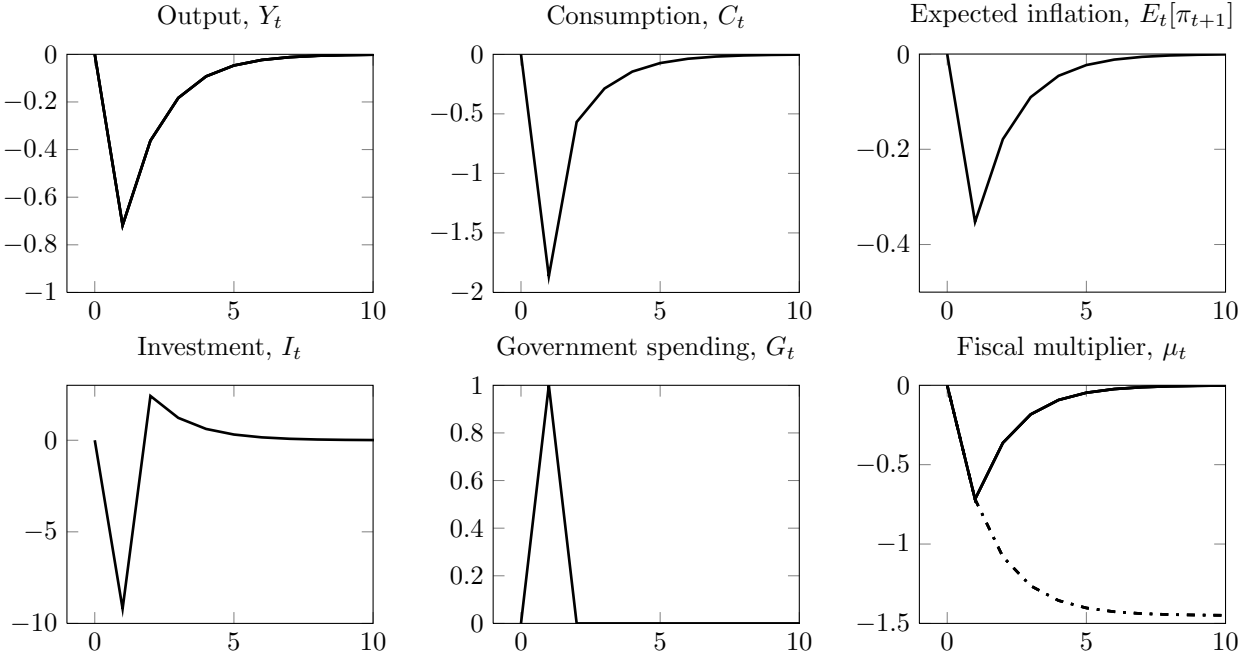

Figure B.1: Impulse responses of the economy to an unanticipated temporary rise in government spending outside of a liquidity trap. The welfare multiplier is -1.4.

both output, consumption and investment, and puts downward pressure on expected inflation. The underlying reason behind these results is that increasing spending raises the real interest rate and crowds out investment. With a higher unemployment rate, consumption takes yet another fall, and the real rate increases further, and so on. An increase in government spending of one dollar decreases welfare by the equivalent of 1.4 dollars of private consumption.

C Computational Details

The economy is characterized by six equations, of which the first four are given by

$$J_t = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(\frac{p_t z_t - \tilde{w}_t}{p_{t+1}} + (1 - \delta) J_{t+1} \right) \right], \tag{C.1}$$

$$n_t = ((1 - n_{t-1}) + \delta n_t) f(\theta_t) + (1 - \delta) n_{t-1}, \tag{C.2}$$

$$\kappa = h(\theta_t) J_t, \tag{C.3}$$

$$z_t n_t = c_t + \kappa \theta_t ((1 - n_{t-1}) + \delta n_t) + g_t. \tag{C.4}$$

Outside a liquidity trap, prices and wages are determined as

$$p_t = \frac{m}{z_t n_t},$$ (C.5)

$$\tilde{w}_t = E_t[\eta p_t z_t + (1 - \eta)\tilde{b} + \eta p_{t+1} J_{t+1}(1 - \delta)f(\theta_{t+1})],$$ (C.6)

and the nominal interest rate satisfies the bond Euler equation. Inside a liquidity trap, however, the nominal interest rate is zero and prices are determined by the Euler equation

$$p_t = E_t\left[\frac{u'(c_t)}{\beta u'(c_{t+1})}p_{t+1}\right].$$ (C.7)

Nominal wages are sticky and equal to their steady state value, $\tilde{w}_t = w_{ss}$. To solve the model I linearize systems (C.1)-(C.6) and (C.1)-(C.4) with (C.7) around the steady state. This gives policy functions, $x_t = A(s) + B(s)x_{t-1}$, where $s \in S = \{0, 1\}$, with a zero denoting that the economy is in a liquidity trap, and a one that it is not. For each $s \in S$, $A(s)$ is a 5×1 vector, $B(s)$ is a 5×5 matrix, and $x_t = (J_t, \tilde{w}_t, n_t, \theta_t, c_t, p_t)'$.

To calculate the impulse responses, consider a generic policy function of the type $x' = g(x, s)$, where x refers to the endogenous state and s is the exogenous state. The probability mass function (pmf) of states in period $t + n + 1$, ψ_{t+n+1}, is given by

$$\psi_{t+n+1}(x', s') = \int_{\{(x,s)\in\Gamma(x')\}} \psi_{t+n}(x, s)P(s'|s),$$

where the set Γ is defined as $\Gamma(x') = \{X \times S : x' = g(x, s)\}$, and $P(s'|s)$ denotes the probability of transitioning to state s' given that the current state is s. Given some initial pmf, ψ_t, it is then quite straightforward, although sometimes computationally tedious, to calculate the sequence $\{\psi_{t+1}, \psi_{t+2}, \ldots, \psi_{t+T}\}$. The associated expected values, $E_t[x_{t+n}|\psi_t]$, can then easily be calculated in order to construct the impulse responses.

However, given the partly linear nature of the policy function this can be done in a computationally very efficient way. In particular, notice that

$$E_t[x_{t+n+1}|s_{t+n+1} = s']P_{t+n+1}(s'|\psi_t) = \int_{x'\in X} x'\psi_{t+n+1}(x', s')dx',$$

where $P_{t+n+1}(s'|\psi_t)$ denotes the probability of state s' occurring in period $t + n + 1$, given some

initial distribution ψ_t . Thus,

$$
\begin{aligned}
E_t[x_{t+n+1}|s_{t+n+1} = s']P_{t+n+1}(s'|\psi_t) &= \int_{x' \in X} x' \left(\int_{\{(x,s) \in \Gamma(x')\}} \psi_{t+n}(x,s)P(s'|s) \right) dx' \\
&= \int_{x \in X} (A(s) + B(s)x) \left(\int_{\{(x,s) \in \Gamma(A(s)+B(s)x)\}} \psi_{t+n}(x,s)P(s'|s) \right) dx \\
&= \int_{(x,s) \in X \times S} (A(s) + B(s)x)\, \psi_{t+n}(x,s)P(s'|s)dx \\
&= \int_{s \in S} (A(s) + B(s)E_t[x_{t+n}|s_{t+n} = s])P_{t+n}(s|\psi_t)P(s',s).
\end{aligned}
$$

As a consequence, the expected value of x_{t+n+1}, conditional on some initial distribution ψ_t, is given by

$$
E_t[x_{t+n+1}|\psi_t] = \int_{s' \in S} E_t[x_{t+n+1}|s_{t+n+1} = s']P_{t+n+1}(s'|\psi_t).
$$