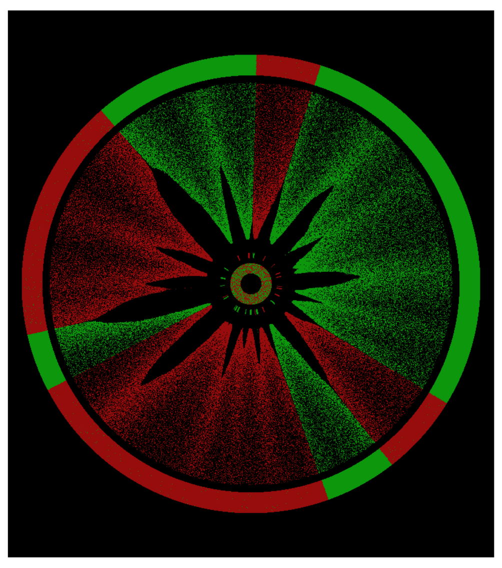
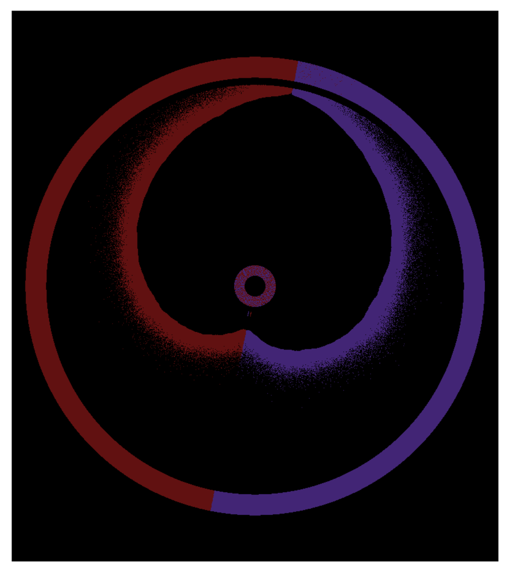
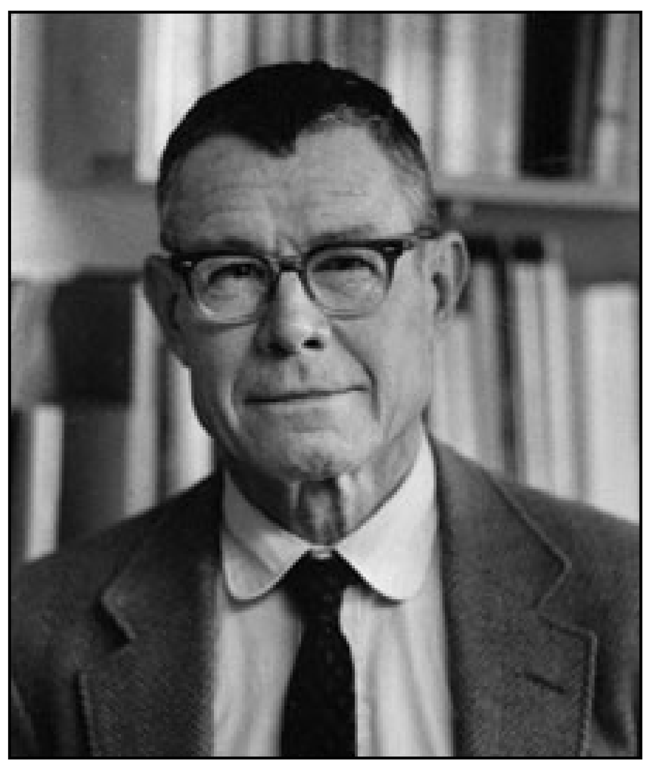


Digital morphogenesis via Schelling segregation

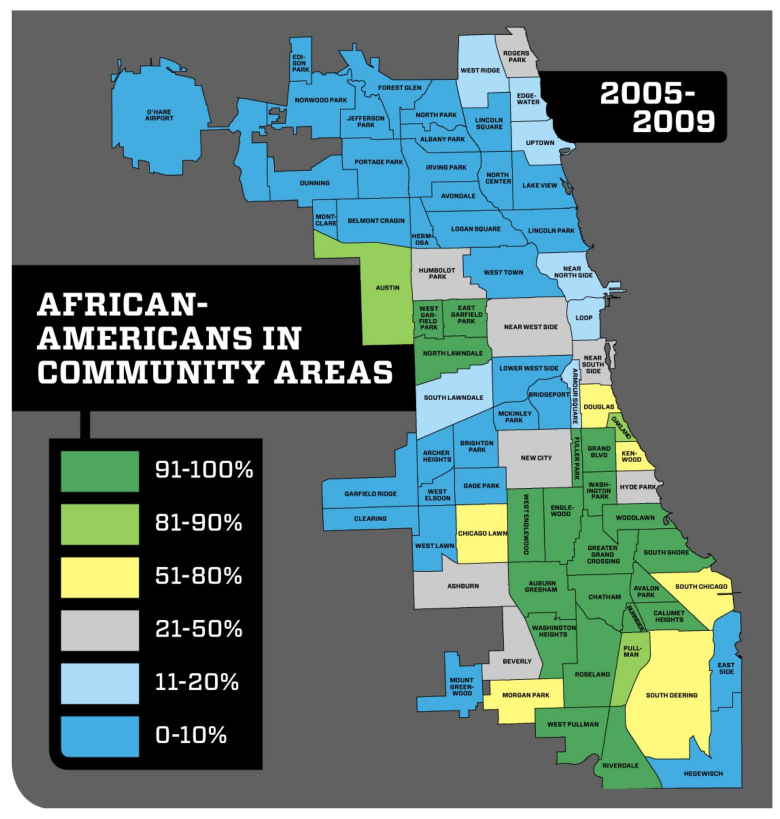
G. Barmpalias, R. Elwes, A. Lewis-Pye



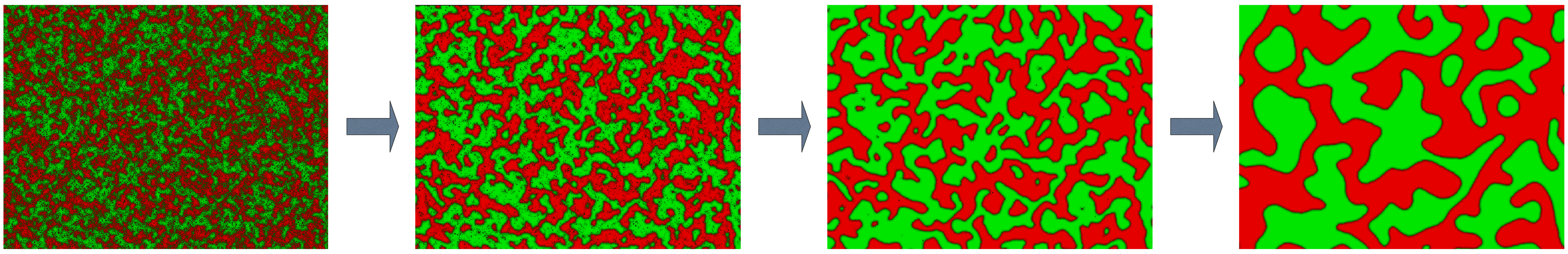
One of the major contributions of the economist and game theorist Thomas Schelling was an elegant model of segregation [3], first described in 1969. The model looks to describe how individuals or particles of different types come to organise themselves spatially into segregated regions, each of largely one type. Today it has become perhaps the best known model of self-organising behaviour, and was one of the reasons cited by the Nobel prize committee upon awarding Schelling his prize in 2005. While the explicit aim was initially to model the kind of racial segregation observed in large American cities, the model turns out to be very widely applicable. There are direct links to areas at the interface between computer science and statistical mechanics, such as the Ising model (used to model physical phase transitions) and the study of contagion and cascading phenomena in networks.

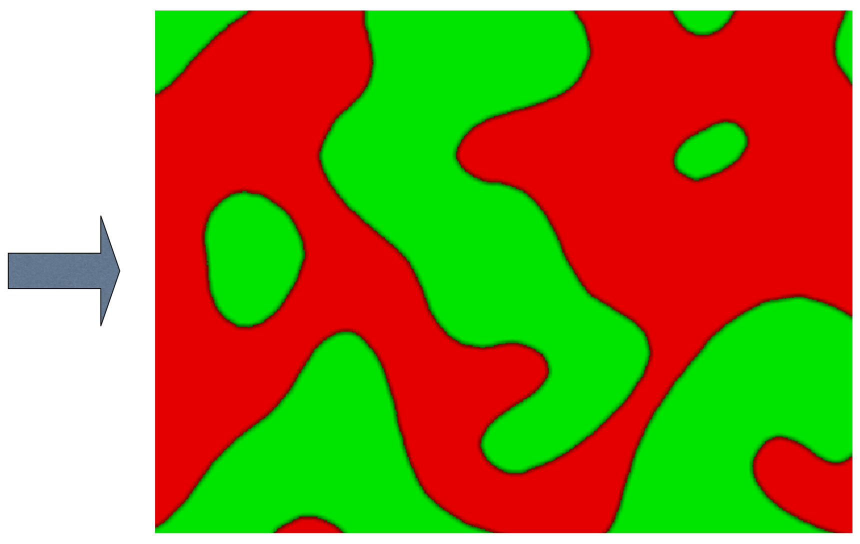


Thomas Schelling



Segregation in Chicago

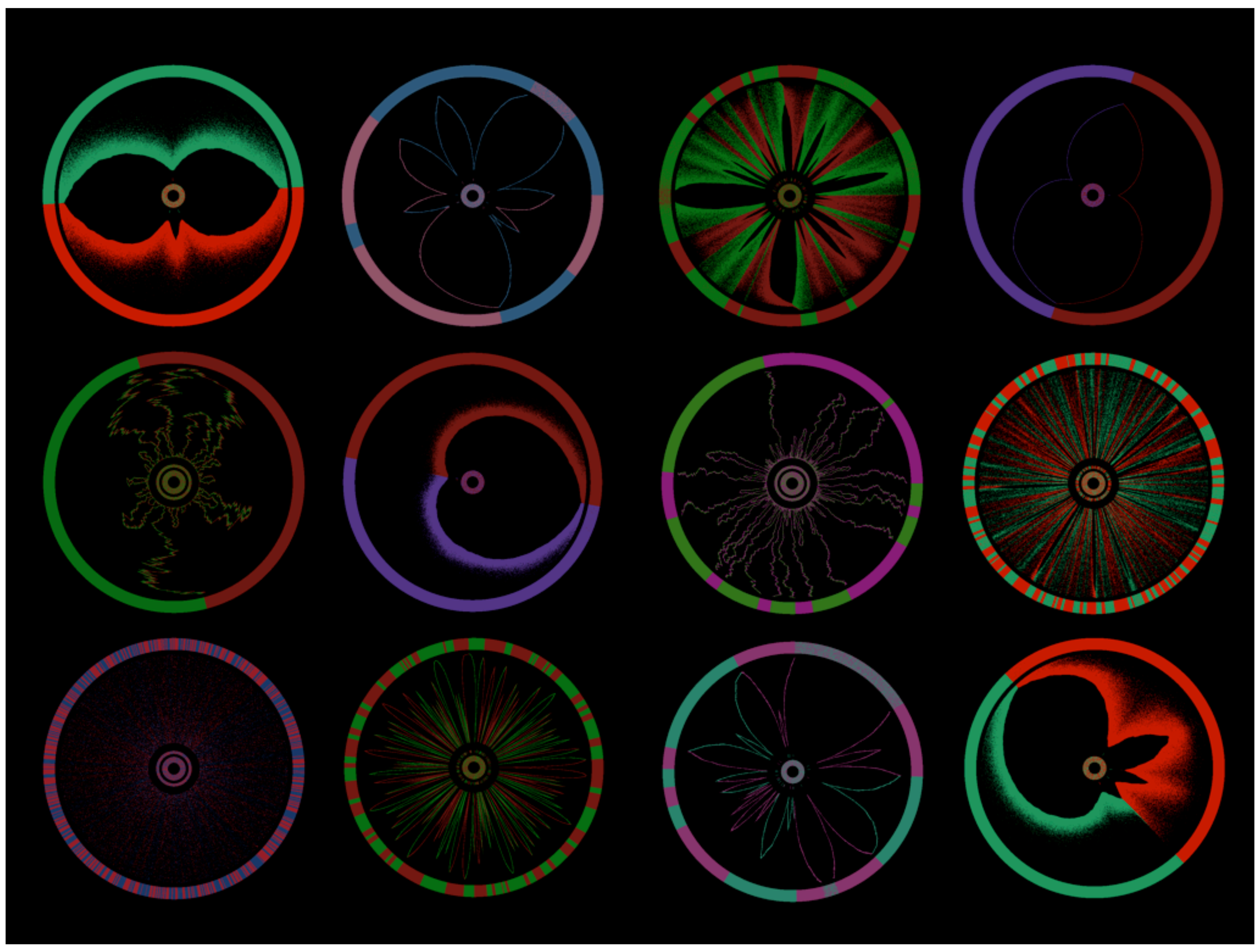




One may consider both 1-dimensional versions of the model, in which individuals are arranged in a circle, and also 2-dimensional versions in which individuals are arranged on a grid. One of the principal parameters is a level of 'intolerance' that individuals have for members of other types. While the model has been extensively studied it has largely resisted rigorous analysis, prior results from the literature generally pertaining to variants of the model which are tweaked so as to be amenable to standard techniques from statistical mechanics or stochastic evolutionary game theory.

In [2], Brandt, Immorlica, Kamath and Kleinberg provided the first rigorous analysis of the unperturbed model, for a specific set of input parameters.

We have now provided a rigorous analysis of the model's behaviour much more generally and have established some surprising forms of threshold behaviour, notably the existence of situations where an *increased* level of intolerance for neighbouring individuals of opposite type leads almost certainly to *decreased* segregation.



While looking to produce our formal proofs, simulations aid in building understanding. The outcomes of some simulations for the 1-dimensional model are illustrated to the left. In the twelve processes depicted here the number of individuals is one hundred thousand. In each case, the inner ring displays an initial mixed configuration, and a stage by stage process then unfolds, in which some individuals are unhappy with their present location causing them to swap with others. The outer ring displays the final, much more segregated, configuration. The process by which the final configuration is reached is indicated in the space between the inner ring and the outer ring in the following way: when an individual changes location this is indicated with a mark, at a distance from the inner rings which is proportional to the time at which the change of type takes place. One may easily observe a number of qualitatively different process taking place as we change the parameter inputs of the model.

Literatur

[1] G. Barmpalias, R. Elwes, A. Lewis-Pye, Digital morphogenesis via Schelling segregation, to appear (available at www.aemlewis.co.uk).

[2] C. Brandt, N. Immorlica, G. Kamath, R. Kleinberg, An Analysis of One-Dimensional Schelling Segregation, *Proc. 44th Annual ACM Symposium on Theory of Computing (STOC 2012)*.

[3] T. Schelling, Models of segregation, *The American Economic Review*, pages 488-493, 1969.