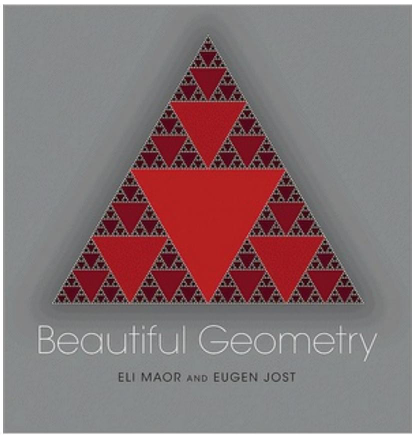
Book Review: Beautiful Geometry by Eli Maor and Eugen Jost

*How do art and mathematics cross over? **Beautiful Geometry** is an attractively designed coffee table book that has 51 short chapters, each consisting of an essay by the mathematical historian **Eli Maor** on a single topic, together with a colour plate by the artist **Eugen Jost**.The book achieves its aim to demonstrate that there is visual beauty in mathematics, finds **Konrad Swanepoel**. It is heartily recommended to mathematics students who want to broaden their horizons and to teachers of mathematics at school level.*

Beautiful Geometry. Eli Maor and Eugen Jost. Princeton University Press. January 2014.



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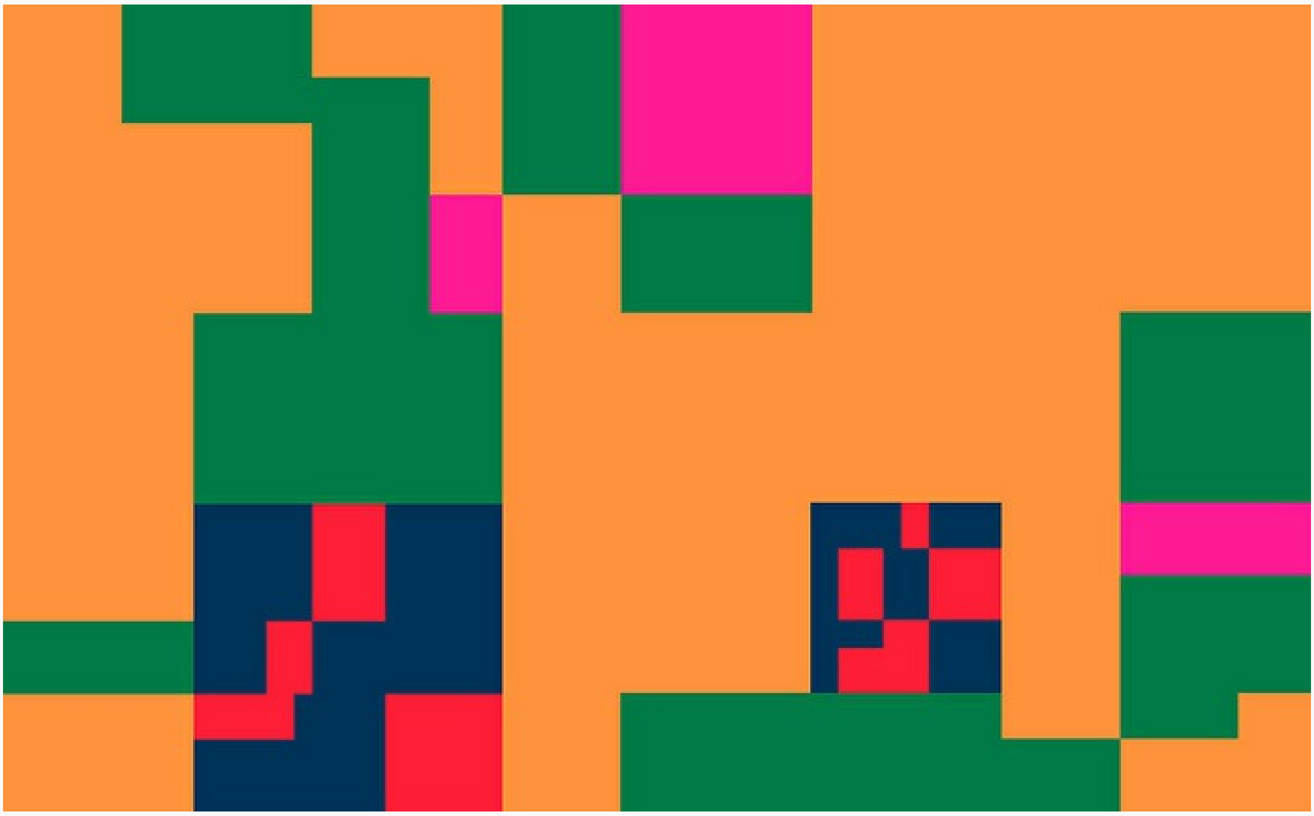
Geometry has a history that goes back at least 3000 years, and under the ancient Greeks achieved a scientific level which even today survives all but the most exacting logical scrutiny. Having always been synonymous with Mathematics, in the 20th century Geometry has simultaneously specialised into a myriad subfields (algebraic geometry, differential geometry, discrete geometry, convex geometry, etc.) while losing its all-encompassing grip on Mathematics. At universities we are eager to teach our students Calculus and Linear Algebra instead of a course on Classical Geometry, now deemed old-fashioned. This book reminds us of what we are missing.

This attractively designed coffee table book has 51 short chapters, each consisting of an essay by the mathematical historian Eli Maor on a single topic, together with a colour plate (occasionally two) by the artist Eugen Jost. The colourful artwork is pleasant to look at, but the text requires somewhat more engagement than what is usually required for popular mathematics books. Although many of the topics are not (any more) in the school syllabus, school level mathematics should be sufficient for a detailed reading. While most readers will undoubtedly end up looking only at the colour plates, of which there are more than 60, this is nevertheless an interesting read for anyone interested in elementary geometry.

Although it is impossible for each of the 60+ colour plates to be a masterful work of art (indeed most of them are essentially functional), some are brilliant, and very many display a fine conceptual humour. For someone who not only wants to appreciate the visual beauty of the artwork, but also the conceptual beauty of the geometry itself, this book is best appreciated sequentially. There is a slow and subtle development of ideas. Chapter 6 on Euclid's proof of the theorem of Pythagoras depends on Chapter 2 on the area of triangles. We might at first object to the discussion of prime numbers in Chapters 14 and 15 as having nothing to do with geometry (although present in Euclid)¾however, all is revealed in Chapter 23, where it is shown that prime numbers are essential to the solution of the classical problem of which regular polygons can be constructed by straightedge and compass. To take another example, Chapter 30 explains how to colour in a square that on the one hand seems to miss some part o the square, yet on the other hand, uses up the whole area of the square (if this sounds impossible, this chapter is well worth a look). This foreshadows Chapter 50 on Sierpinski's triangle, where the whole area of a triangle is removed in such a way to leave a fractal shape of zero area. Perhaps it would have helped the reader to indicate these connections. However, Chapters 17 and 24, on the numbers 11 and 50, seem out of place. If there is a connection to any other chapter or to the subject of geometry itself, I can't see it.

The reader who would like to see what geometry involves nowadays would have to look somewhere else. The research mathematician will page through it and wonder why virtually everything that has happened in the 20th century and even the late nineteenth century has been ignored. Some history is discussed, but only of isolated episodes. For instance, one of the most momentous events in the history of geometry, the discovery of hyperbolic geometry in the 19th century, is not mentioned at all (despite there being very beautiful pictures in hyperbolic geometry, cf. M. C. Escher's woodcut Circle Limit III). Felix Klein's influential 1872 manifesto, the so-called *Erlangen Program*, setting out a vision of what geometry should be, is also not mentioned. A natural place would have been the two chapters (44 and 45) on symmetry, where group theory is introduced. At least in the final chapters (46–51) there are some relatively new mathematics: the Reuleaux triangle from convex geometry, Morley's Theorem, a result with a classical flavour yet only discovered in 1899, Pick's Theorem in discrete geometry, some early 20th century mathematics in the form of fractals, and Cantor's groundbreaking work on infinite sets. In fact the book cover displays the intriguing Sierpinski triangle, a fractal discussed in Chapter 50.



The Mathematics is consistently explained in an excellent and engaging way, and I can only make two tiny criticisms that will only be of interest to a mathematician. One is that the example of a geometrical proof by Euclid presented in Chapter 2 is a bit of an anticlimax, as it is taken out of context from Euclid's books and depends on a previous result from Euclid, which is not explained. The second is that in the chapter on Koch's snowflake (Ch. 49), it is implied that the fractal obtained as a limit shape has infinite perimeter because the perimeter at each step in its construction grows without bound. Unfortunately this is not sufficient for a proof as it is possible to find sequences of curves with lengths going to infinity, yet the limit object has finite length. For a rigorous proof that the Koch snowflake has infinite perimeter something more is needed (essentially the fact that its fractal dimension is strictly larger than 1). However, it has to be noted that even Koch made the same omission in his original paper, and I haven't seen this point cleared up in any of the many popular accounts on fractals.

Of course, this book is not intended to be a textbook. It achieves its aim to demonstrate that there is visual beauty in Mathematics. I heartily recommend it for anyone to at least page it through, and for anyone interested in geometry to read, especially mathematics students who want to broaden their horizons and teachers of mathematics at school level.

Konrad Swanepoel is a mathematician at the London School of Economics. His research interests are in discrete and combinatorial geometry, as well as the geometry of Banach spaces and extremal combinatorics.
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