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Article (Accepted version)
(Refereed)

Original citation:

Liu, Ting and Schiraldi, Pasquale (2014) *Buying frenzies in durable-goods markets*. [European Economic Review](#), 70. pp. 1-16. ISSN 0014-2921

DOI: [10.1016/j.euroecorev.2014.03.008](https://doi.org/10.1016/j.euroecorev.2014.03.008)

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Buying frenzies in durable-goods markets*

Ting Liu[†] & Pasquale Schiraldi[‡]

March, 2014

Abstract

We explain why a durable-goods monopolist would like to create a shortage during the launch phase of a new product. We argue that this incentive arises from the presence of a second-hand market and uncertainty about consumers' willingness to pay for the good. Consumers are heterogeneous and initially uninformed about their valuations but learn about them over time. Given demand uncertainty, first period sales may result in misallocation and lead to active trading on the secondary market after the uncertainty is resolved. Trading on the second-hand market will generate additional surplus. This surplus can be captured by the monopolist ex-ante because consumers are forward-looking, and the price they are willing to pay incorporates the product's resale value. As a consequence, when selling to uninformed consumers, the monopolist faces the trade-off between more sales today and a lower profit margin. Specifically, because the product's resale value is negatively related to the stock of the good in the second-hand market, selling more units today will result in a lower equilibrium price of the product. Therefore, the monopolist may find it optimal to create a shortage and ration consumers to the second period. We characterize conditions under which the monopolist would like to restrict sales and generate buying frenzies.

Keywords: Buying frenzies, second-hand market, durable goods, consumer uncertainty.

*We thank Martin Pesendorfer and two anonymous referees for their comments and suggestions.

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1 Introduction

Introductions of new goods are often featured by serious shortage and such phenomenon is particularly pronounced in a durable-goods environment where shortage is coupled with active trading on second-hand markets. Examples include video games, game consoles, iPads, iPhones and luxury cars. Although shortages might be driven by limited capacity, shortage of components or demand uncertainty, their repeated occurrence in durable goods markets suggests that firms may use scarcity as a strategic choice. If the firm benefits from scarcity strategies, what is the mechanism behind them? What are the welfare implications of buying frenzies? How does the existence of a second-hand market affect the firm's optimal selling strategy? These are the questions addressed in this paper.

The internet revolution has substantially enhanced active trading on second-hand markets when buying frenzies occur.¹ When iPad 2 was launched, Apple stores across the U.S. sold out of the tablet while the price of it spiked on eBay.² A similar phenomenon was documented for other electronics including Wii and PlayStation 2 (see Stock and Balachander, 2005). Despite the important role played by the second-hand market, it is ignored by the existing literature aiming to explain firms' scarcity strategies. In fact, the predominant theories are not robust against resale. To the best of our knowledge, our paper is the first one to study durable-goods producers' incentives to induce buying frenzies while taking into account active trading on the second-hand market. Contrary to the existing literature, we argue that the existence of a second-hand market can be one of the driving forces for buying frenzies.

We develop a model in which production and sales of a durable good occur in two periods. There is a monopolistic firm in the market and a mass one of two types (optimistic and pessimistic) of consumers heterogeneous in their valuations for the good. A consumer's type is her private information which determines the probability distribution of her valuation for the good. Specifically, optimistic consumers are *affectionados* whose distribution of valuation first-order stochastically dominates the distribution of pessimistic consumers'

¹See, for example, Rapson and Schiraldi, 2013, for an empirical analysis of the internet impact on the trade volume of used cars on the second-hand market.

²"iPad 2 Prices Are Spiking on eBay", the Atlantic Wire, March 14, 2011.

valuation. So, optimistic consumers on average value the good more than pessimistic consumers.

The first period is the launching phase of the new product and consumers are uncertain about their valuations. However, they learn about their valuations in the second period which is the product's mature phase and is characterized by the presence of an active second-hand market. When the monopolist sells to uninformed consumers in the first period, the product may end up with those who turn out to have low valuations. Hence, re-allocation of the good among consumers takes place through the second-hand market when the uncertainty about consumers' valuations is resolved. Trading on the second-hand market will generate an additional surplus. This surplus can be captured by the monopolist *ex-ante* because consumers are forward-looking, and the price they are willing to pay incorporates the product's resale value. As a consequence, when selling to uninformed consumers, the monopolist faces the trade-off between more sales today and a lower profit margin. Specifically, because the product's resale value is negatively related to the stock of the goods in the secondary market, selling more units today will result in a lower equilibrium price of the product. Therefore, the monopolist may find it optimal to create a shortage and ration consumers to the second period.

Buying frenzies arise when the monopolist intentionally undersupplies the product (rationing occurs) and some of the consumers are strictly worse off when being rationed out. In our model, buying frenzies occur when it is optimal for the monopolist to sell to both types of consumers and ration some of them to the second period. Among consumers rationed out, optimistic ones are strictly worse off because they strictly prefer to consume the product in period one. In contrast, pessimistic consumers are indifferent between consuming the product in period one and postponing consumption. Specifically, buying frenzies are more likely to happen when i) there is a large number of pessimistic consumers, and ii) the ex-ante surplus from selling to pessimistic consumers is sufficiently small. Under the former condition, it is optimal for the monopolist to charge pessimistic consumers' maximum willingness to pay and sell to both types. Conditional on selling to both types, the latter condition ensures that the monopolist makes more profit from undersupplying the product than selling to all. To see this, suppose that the monopolist sells to everyone in period one. By undercutting the supply, the monopolist increases the product's resale value and hence consumers' willingness to pay in period one. So, the marginal benefit from restricting supply is

the incremental increase in the product's first period price multiplied by the total mass of consumers. The marginal cost of undercutting the supply is the forgone surplus from selling to the marginal consumer in period one, which is the ex-ante surplus from selling to pessimistic consumers. Thus, when condition ii) holds, the marginal benefit from undersupplying the product outweighs the marginal cost and it is optimal for the monopolist to ration consumers.

We also analyze the monopolist's optimal selling strategy when the secondary market does not exist. In this case, the monopolist never rations consumers. We compare the social welfare in the presence of the secondary market and buying frenzies with that when there is no secondary market. Social welfare is higher without the secondary market when marginal cost is sufficiently low. The existence of the secondary market and consequently buying frenzies may improve social welfare when marginal cost is sufficiently high.

While we assume for simplicity that the monopolist commits to future price and quantity, the driving force for buying frenzies is robust against the monopolist's commitment power. We found that when the monopolist cannot commit to future price and quantity, it may ration consumers more aggressively than it would like to when it has the commitment power. This is because when the monopolist lacks the commitment power, it will make too much sales in the second period with respect to what it would like to do from the first period point of view. Therefore, in order to maintain a high resale value for the goods, the monopolist will try to counterbalance this effect by reducing the first period sales more aggressively which in turn leads to more consumers rationed to the second period.

Our paper is most closely related to DeGraba (1995). DeGraba argues that when a discrete number of consumers learn their valuations over time, the monopolist can extract more consumer surplus by committing to a fixed output short of demand in the first period. When output is short of demand, consumers risk losing the opportunity to buy the good if they strategically delay purchases. As a consequence, consumers all rush to buy the good when they are uninformed. DeGraba's results rely on the following assumptions: no secondary market, no production in the second period and the monopolist cannot commit to future price. The option of purchasing the good in the second period in case of further production or an active secondary market void the risk borne by consumers when they delay consumption. We instead show that the monopolist still has

incentives to induce buying frenzies when we relax these assumptions in an environment with a continuum of consumers. Moreover, we show that the occurrence of buying frenzies does not depend on the monopolist's ability to commit to future price. Finally, we argue that our results hold under any rationing rule except for the efficient rationing rule whereas DeGraba focuses on the class of rationing rule with "last customer rationing monotonicity".

Several other papers including Courty and Nasiry (2013), Denicolo and Garella (1999), Stock and Balachander (2005) and Allen and Faulhaber (1991) have offered alternative theories for monopolist's scarcity strategies. None of these papers allows resale. In Courty and Nasiry (2013) the monopolist cannot commit to second period price, therefore, when it produces more units in period one, consumers' option value of waiting becomes larger due to a lower second period price. As a consequence, their willingness to pay in period one becomes smaller. Hence, it may be optimal for the monopolist to ration consumers in period one. Denicolo and Garella (1999) study a model without demand uncertainty. They argue that rationing reduces the monopolist's incentive to lower future prices and can convince consumers to buy without strategic delay. This may allow the monopolist to increase its discounted profit. In Denicolo and Garella, if consumers can resell, the arbitrage across periods will make the firm's rationing strategy less profitable. Stock and Balachander (2005) and Allen and Faulhaber (1991) show that product scarcity can be used to signal a high quality of the product.

In our model, the monopolist may prefer a smoothly functioning secondary market for a reason different from that in the existing literature (Swan (1980), Rust (1986), Hendel and Lizzeri (1999), Schiraldi and Nava (2012)). When trade is driven by uncertainty in demand, the secondary market can help the monopolist to extract surplus generated by reallocation of the goods. In a similar context, Johnson (2011) studies the implications of uncertainty in demand and the presence of transaction costs on monopoly profit and its choice of durability.

Courty (2003a, 2003b) studies a monopolist's selling strategy in ticket markets when there is demand uncertainty. In Courty, the monopolist sells either in an early market when consumers are uninformed about their valuations for tickets or in a late market where their valuations are revealed. Despite the similar features

shared with Courty, we have different findings. Courty found that the monopolist does not gain by selling tickets in the early market and rationing consumers compared with selling in the late market. In contrast, we show the monopolist may strictly prefer to sell the good in the early market and ration consumers. The driving force for the difference is because we focus on durable goods while Courty studies goods that can only be consumed once.

Finally, our paper is related to the literature on intertemporal pricing. Previous work has studied how the monopolist can use advance-purchase discount (Nocke et al. 2011, Dana 1998) or refund (Courty and Li 2000) to price discriminate between consumers when the uncertainty of consumers' valuations is resolved over time. Our paper differs from the previous work by allowing consumers to resell, and in particular, we focus on how the option of consumer resale affects the monopolist's optimal selling strategy.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium and shows the conditions under which buying frenzies occur. Section 4 analyzes the monopolist's optimal selling strategy when there does not exist a secondary market and investigates the welfare implications of buying frenzies. Section 5 shows that our main results are robust against the monopolist's commitment-power assumption. Section 6 concludes.

2 The model

A risk-neutral monopolist sells an indivisible durable good in two trading periods. There is a continuum of consumers who live for two periods. The mass of consumers is normalized to one. Consumers enter the market in period one, each buying, at most, one unit of the durable good in her life time. A consumer's valuation for the good is indexed by θ , with $\theta \in [0, \bar{\theta}]$. We interpret a consumer's valuation as her taste θ multiplied by the product's quality which is commonly known and normalized to one. If a consumer with valuation θ buys the good at price p in period t , $t = 1, 2$, her utility in period t is $\theta - p$. For expositional simplicity, we assume the seller and consumers do not discount.³

³We could allow the seller and consumers to have different discount factors. Our main results hold as long as the consumer's discount factor is not too low. When the consumer's discount factor is very low, they do not incorporate the product's resale value into their first period willingness to pay. This will destroy the monopolist's incentive to ration consumers in the first period.

In period one, consumers face uncertainty about their valuations and differ in their valuation distributions. There are two types of consumers, type O and P with fractions β and $1 - \beta$, respectively. Throughout the paper, we will call type O optimistic consumers and type P pessimistic consumers. A consumer's type is her private information which determines the probability distribution of her valuation for the good. Type i , $i = \{O, P\}$, consumers' valuations are distributed according to cumulative distribution function $F_i(\theta)$ and density function $f_i(\theta)$ on the interval $[0, \bar{\theta}]$. Optimistic consumers' distribution of valuation first-order stochastically dominates the distribution of pessimistic consumers' valuation. That is, $F_o(\theta) \leq F_p(\theta)$, $\forall \theta \in [0, \bar{\theta}]$. Accordingly, optimistic consumers have a greater mean valuation of the good compared with pessimistic consumers. Throughout the paper, we make the following technical assumption on the distributions of consumers' valuations to ensure the uniqueness of the monopolist's profit maximization problem.

Assumption 1 $\beta f_o(\theta) + (1 - \beta)f_p(\theta)$ is *logconcave*.

Consumers become informed about their actual valuations in period two. We assume a consumer learns her true valuation regardless of her purchase decision in period one. This assumption is reasonable in a number of situations. For example, a gamer can learn how much she likes a game console by playing it at her friend's house or the game store; a car buyer will learn her valuation for a new car by test drive. At the beginning of period 2, a secondary market is opened. The good does not depreciate and consumers can resell it at zero transaction cost.⁴

The seller wishes to maximize the total profit from the two periods. The marginal cost of the good is assumed to be constant at c . We assume that the seller can commit to future price and quantity. We focus on the commitment case to abstract away from the well-studied role of time inconsistency. This assumption simplifies our analysis and the results remain qualitatively the same even when the monopolist lacks commitment power. We discuss the No Commitment case in Section 5.

Throughout the model we assume that consumers are rationed according to the proportional rationing rule. Buying frenzies continue to exist under other rationing rules except for the case of efficient rationing.

⁴See Anderson and Ginsburg (1994); Hendel and Lizzeri, (1999); Hideo and Sandfort, (2002); Schiraldi (2011) and Waldman (1996) among others for a discussion about the role of the second-hand market when quality depreciates and/or transaction costs are present.

With efficient rationing, we still have a rationing equilibrium but consumers are not strictly worse-off when rationed out. When a new product is introduced, it is normally purchased on a first-come-first-served basis. The order of arrival is not only determined by consumers' desire to consume the good, but also by their opportunity costs of shopping. There is no reason to believe that optimistic consumers have uniformly lower opportunity costs of shopping than pessimistic consumers. In addition, ex-post some pessimistic consumers will have higher valuations than optimistic ones. Therefore, proportional rationing seems more natural than efficient rationing in our context. The timing of the model is summarized as follows:

- Period 1. At the beginning of the period, consumers learn their types. The seller announces the prices and supplies in the two periods. Then, consumers decide whether to buy the good after observing (p_1, q_1, p_2, q_2) .
- Period 2. Consumers learn their true valuations. A secondary market is opened. Consumers who have purchased the good in period one decide whether to resell it, and those who haven't bought the good decide whether to buy it and from whom to buy.

3 Equilibrium

In the first period, the monopolist faces a cohort of consumers with two different levels of willingness to pay. If the monopolist charges the pessimistic consumers' maximum willingness to pay, optimistic consumers will also purchase the good at this price because they have a higher expected valuation. In this case, the monopolist can sell the good to both types of consumers. Alternatively, the monopolist can charge the optimistic consumers' maximum willingness to pay and exclude the pessimistic consumers in the first period

In this section, we first discuss the monopolist's maximum profit from selling to both types of consumers in the first period. The case of selling to optimistic consumers exclusively follows. We then characterize the conditions under which buying frenzies happen or rationing occurs.

3.1 Selling to both types of consumers

Consider a type i , $i = \{O, P\}$, consumer's purchase decision in period one. If the consumer buys the good immediately, she enjoys flow utility $E_i(\theta) - p_1$. In the next period, the consumer will keep the good if her valuation turns out to be greater than the resale price p_2 ; the consumer will resell the good in the secondary market at price p_2 , otherwise. Hence, the consumer's expected payoff from purchasing the good in period one is

$$E_i(\theta) - p_1 + [(1 - F_i(p_2))E_i(\theta|\theta > p_2) + F_i(p_2)p_2]. \quad (1)$$

Alternatively, the consumer can delay consumption until her valuation is revealed in period two. Anticipating that she will buy the good only when her valuation is greater than p_2 , the consumer's expected payoff from waiting is

$$(1 - F_i(p_2))[E_i(\theta|\theta > p_2) - p_2]. \quad (2)$$

Comparing (1) and (2), the consumer will buy the good in period one if and only if

$$p_1 \leq E_i(\theta) + p_2.$$

Observe that the consumer's first period maximum willingness to pay is increasing in p_2 . This is because the secondary market provides an insurance to the consumer. When the good maintains a high resale value, the consumer will bear a smaller loss if she turns out to have a low valuation. Hence, the consumer is willing to pay more for the good up front. Notice that the monopolist benefits from having a smoothly functioning second-hand market. If transaction costs were present, they would reduce the maximum willingness to pay of consumers because they would expect a lower resale value for the purchased goods which in turn would reduce the monopolist's profit. A smooth second-hand market increases surplus which is captured by the monopolist.

Since $E_p(\theta) < E_o(\theta)$, when the monopolist charges pessimistic consumers' maximum willingness to pay $E_p(\theta) + p_2$, both types of consumers are willing to buy the good. The monopolist chooses q_1 , p_2 and q_2 to maximize its expected profit subject to the market clear condition in the second period.⁵ Given that

⁵When the monopolist charges $p_1 = E_p(\theta) + p_2$, pessimistic consumers are just indifferent between purchasing

consumers can trade with each other in the secondary market, the equilibrium secondary market price must equal the second period price charged by the monopolist. Otherwise, there will be arbitrage opportunities and the prices will be adjusted. The secondary market will allocate the good to consumers who value it the most. Consequently, the market clear condition requires

$$\beta \int_{p_2}^{\bar{\theta}} f_o(\theta) d\theta + (1 - \beta) \int_{p_2}^{\bar{\theta}} f_p(\theta) d\theta = q_1 + q_2. \quad (3)$$

The right hand side of (3) is the total stock of the good in period two, whereas the left hand side of (3) is the number of consumers with valuations greater than p_2 . To simplify exposition, we define $G(\theta) \equiv \beta F_o(\theta) + (1 - \beta)F_p(\theta)$ and $g(\theta) \equiv \beta f_o(\theta) + (1 - \beta)f_p(\theta)$. The market clear condition is simplified to

$$q_1 + q_2 = 1 - G(p_2). \quad (4)$$

When selling to both types of consumers, the monopolist solves the following program:

$$\text{Max}_{q_1, p_2, q_2} (E_p(\theta) + p_2 - c)q_1 + (p_2 - c)q_2 \quad (5)$$

subject to the market clear condition (4) and the boundary conditions

$$0 \leq q_1 + q_2 \leq 1 \quad (6)$$

$$q_1, q_2 \geq 0. \quad (7)$$

We first show that it is never optimal for the monopolist to shut down the market in period one and only sell in period two. Notice that the following lemma is also valid when the monopolist targets optimistic consumers and sell to them exclusively in period one.

Lemma 1 *Selling in the second period only is never optimal.*

The intuition behind Lemma 1 is that for a given level of output, it is always more profitable for the monopolist to sell a small fraction of the good in the first period and the remaining units in the second

the good immediately and waiting. However, in equilibrium, the first period demand must be at least the supply q_1 . To see this, suppose the monopolist's optimal profit π^* is achieved at $q_1^* < 1$. If the demand is less than q_1^* , the monopolist can achieve a profit arbitrarily close to π^* by undercutting price slightly below $E_p(\theta) + p_2$ and still manage to sell q_1^* units.

period. This is because the monopolist can charge a strictly higher price for the units sold in the first period without affecting the second period price which is determined by the total stock of the good.

At the price $E_p(\theta) + p_2$, all consumers are willing to buy the good in period one, but the monopolist may not be interested in selling to all of them. When is it optimal for the monopolist to ration consumers? The solution for the monopolist's profit maximization program is summarized by the following lemma:

Lemma 2 *When the monopolist sells to both types of consumers, it only sells in period one, i.e. $q_2^* = 0$. If $E_p(\theta) - c < \frac{1}{g(0)}$, the monopolist chooses $q_1^* < 1$ and the corresponding profit is $(E_p(\theta) + p_2 - c)(1 - G(p_2))$ with $E_p(\theta) + p_2 - c = \frac{1 - G(p_2)}{g(p_2)}$. If $E_p(\theta) - c \geq \frac{1}{g(0)}$, the monopolist chooses $q_1^* = 1$. The corresponding profit is $E_p(\theta) - c$.*

When choosing the first period output q_1 , the seller faces the trade-off between more sales in period one and a lower profit margin. By selling more of the good today, the seller will increase the stock in the secondary market and hence reduce the good's resale value. Anticipating this, a forward looking consumer is willing to pay less in the first period because she expects to receive less payment from resale.

To better understand Lemma 2, imagine that the monopolist sells to all consumers in period one. That is, it chooses $q_1 = 1$. Because no one will buy the good from the secondary market, the good's resale value is reduced to zero. Now, suppose the seller slightly undercuts the first period output. Some consumers will be rationed out and their purchases in the second period will create a positive resale value of the good. This in turn will increase the good's first period price. How much is the monopolist's marginal benefit from undercutting the first period output by one unit? The marginal change in p_2 in response to a marginal change in q_1 at $p_2 = 0$ is

$$\frac{dp_2}{dq_1} \Big|_{p_2=0} = \frac{dG^{-1}(1 - q_1 - q_2)}{dq_1} \Big|_{q_1+q_2=1} = \frac{-1}{g(0)}.$$

Because pessimistic consumers' maximum willingness to pay in period one is $E_p(\theta) + p_2$, the monopolist's first period price will increase by $\frac{1}{g(0)}$. Hence, its marginal benefit from undercutting the first-period output by one unit is $\frac{1}{g(0)}$ multiplied by 1, the total mass of consumers. Now, consider the monopolist's marginal cost of undercutting q_1 . When the monopolist sells to everyone in the first period, the maximum price it can charge is $E_p(\theta)$. So, if the monopolist reduces one unit output in period one, it loses an amount $E_p(\theta) - c$

which is its marginal cost from undercutting q_1 . When the condition $E_p(\theta) - c < \frac{1}{g(0)}$ holds, the monopolist's marginal benefit from undercutting q_1 outweighs its marginal cost. So, it is optimal for the monopolist to ration some consumers to the second period.

Corollary 1 *The monopolist's optimal profit from selling to both types of consumers is (weakly) increasing in β .*

To see the corollary, first consider $E_p(\theta) - c < \frac{1}{g(0)}$. The monopolist rations consumers in period one and makes the optimal profit $(E_p(\theta) + p_2 - c)(1 - G(p_2))$. The derivative of its optimal profit with respect to β is $(E_p(\theta) + p_2 - c)(F_p(p_2) - F_o(p_2)) \geq 0$. When the fraction of optimistic consumers becomes larger, the monopolist can sell more units in period one while retaining the product's resale value. Hence, its profit is increasing in β . By Lemma 2, the monopolist sells to all consumers when $E_p(\theta) - c \geq \frac{1}{g(0)}$ and its profit is constant in β .

When consumers are rationed to the second period, a fraction β of them are optimistic consumers and a fraction $1 - \beta$ of them are pessimistic consumers. Because pessimistic consumers are charged their maximum willingness to pay in the first period, they are indifferent between buying the good in the first period and the second period. By contrast, optimistic consumers strictly prefer to buy the good in period one. So, rationing has different welfare impacts on the two types of consumers.

Corollary 2 *When the monopolist rations consumers, optimistic consumers are strictly worse off when rationed out but pessimistic consumers are not worse off.*

3.2 Selling to optimistic consumers exclusively

Now, we turn to the case where the monopolist targets optimistic consumers and sells to them exclusively. We have shown in the previous analysis that optimistic consumers' maximum willingness to pay in period one is $E_o(\theta) + p_2$. Hence, the monopolist chooses q_1 and p_2, q_2 to maximize

$$(E_o(\theta) + p_2 - c)q_1 + (p_2 - c)q_2$$

subject to the market clear condition (4) and the boundary conditions

$$0 \leq q_1 \leq \beta \tag{8}$$

$$0 \leq q_2 \leq 1 - q_1.$$

First, notice that the same market clear condition (4) holds both when the monopolist targets pessimistic consumers and optimistic consumers. This is because the market clear condition only requires that the second period price equal the valuation of the marginal consumer in the second period. The marginal consumer in the second period is determined by the total stock of the good $q_1 + q_2$ and does not depend on who owns the good in the first period. Second, for a fixed p_2 , pessimistic consumers' maximum willingness to pay is $E_p(\theta) + p_2 < E_o(\theta) + p_2$. Hence, at price $E_o(\theta) + p_2$, the first period demand is at most β .

Lemma 1 has shown that it is never optimal for the monopolist to sell in period two only. So, we focus on the case where $q_1 > 0$ and analyze when the monopolist may benefit from rationing optimistic consumers in the first period. To begin, we first present a lemma which will be used to analyze the monopolist's optimal selling strategy. Define $H(\beta) \equiv G^{-1}(1 - \beta) - c - \frac{\beta}{g(G^{-1}(1 - \beta))}$.

Lemma 3 $H(\beta)$ is strictly decreasing in β .

When selling to optimistic consumers exclusively, the monopolist's optimal selling strategy depends on the value of the function $H(\beta)$ and is characterized in the following lemma.

Lemma 4 As β increases from zero to one, the monopolist's optimal strategy is summarized by the following table

Case	condition	q_1^*	q_2^*	$\pi(q_1^*, q_2^*)$
1	$0 < H(\beta)$	β	> 0	$(E_o(\theta) + p_2 - c)\beta + (p_2 - c)q_2^*$
2	$-E_o(\theta) \leq H(\beta) \leq 0$	β	0	$(E_o(\theta) + G^{-1}(1 - \beta) - c)\beta$
3	$H(\beta) < -E_o(\theta)$	$< \beta$	0	$(E_o(\theta) + G^{-1}(1 - q_1^*) - c)q_1^*$

where in Case 1, q_2^* and p_2 are determined by $G^{-1}(1 - \beta - q_2^*) - c = \frac{\beta + q_2^*}{g(G^{-1}(1 - \beta - q_2^*))}$ and $p_2 = G^{-1}(1 - \beta - q_2^*)$; in Case 3, q_1^* is determined by $G^{-1}(1 - q_1^*) - c - \frac{q_1^*}{g(G^{-1}(1 - q_1^*))} = -E_o(\theta)$.

When choosing optimal first period sales, the monopolist faces the same trade-off between more sales in period one and lower profit margin as in the case of selling to both types of consumers. This trade-off depends on the relative proportion of the two groups. If β is very low, the monopolist will sell to all

optimistic consumers in period one because this will not reduce the good's resale value significantly. In addition, the monopolist will also sell the good in the second period to capture the large demand from pessimistic consumers. As β increases the incentive to sell in the second period decreases because by selling in the second period, the monopolist will drive down the equilibrium secondary market price and consequently reduce the price and profit from selling to optimistic consumers in the first period. When β is large enough, the monopolist will not only stop selling in the second period (Case 2) but eventually find it optimal to restrict sales in the first period below β in order to maintain a high resale value of the good (Case 3). We characterize the monopolist's optimal profit as a function of β in the next corollary.

Corollary 3 *When the monopolist sells to optimistic consumers exclusively, the optimal profit is continuous and increasing in β .*

By charging the optimistic consumers' maximum willingness to pay, the monopolist gives up the pessimistic consumers in the first period. So, when the fraction of optimistic consumers becomes larger, the monopolist has less to lose by selling to optimistic consumers exclusively in the first period and hence will make more profit.

In Case 3, some of the optimistic consumers are rationed to the second period, but since these consumers are charged their maximum willingness to pay in the first period, they are not worse off when rationed out.

Corollary 4 *When the monopolist sells to optimistic consumers exclusively, consumers are not worse off when rationed to the second period.*

3.3 Optimal selling strategy and buying frenzies

Corollaries 1 and 3 show that the monopolist's optimal profit is increasing in β both when the monopolist sells to all types of consumers and optimistic consumers exclusively. Hence, for a given β , the comparison of the profitability between these two selling strategies depends on the shapes of the distribution functions $F_o(\theta)$ and $F_p(\theta)$. Nevertheless, we can rank these two selling strategies when β is sufficiently large or small.

Proposition 1 *The monopolist charges pessimistic consumers' maximum willingness to pay and sells to*

both types of consumers when $\beta < \widehat{\beta}$, where $\widehat{\beta} \in (0, H^{-1}(0))$. It sells to optimistic consumers exclusively when $\beta \geq \widehat{\beta} \equiv H^{-1}(-E_o(\theta))$.

For a fixed second period price p_2 , optimistic consumers have a higher willingness to pay than pessimistic consumers in the first period. Therefore, when choosing the optimal first period price, the monopolist faces the standard trade-off between efficiency and rent seeking and the trade-off hinges on the fraction of optimistic consumers β . When the majority of consumers are pessimistic ($\beta < \widehat{\beta}$), selling to optimistic consumers exclusively will create a large inefficiency which constrains the maximum surplus the monopolist can possibly extract from consumers. By contrast, if the monopolist charges pessimistic consumers' maximum willingness to pay and sells to both types of consumers, the efficiency is significantly improved. Even though the monopolist has to give up some rent to optimistic consumers, the gain in surplus dominates the rent given to optimistic consumers because of their small size. As a result, the monopoly profit is maximized by selling to both types of consumers.

Alternatively, when the majority of consumers are optimistic ($\beta \geq \widehat{\beta}$), the monopolist would have given up too much rent to optimistic consumers if it charges pessimistic consumers' maximum willingness to pay. The monopolist benefits from selling to optimistic consumers exclusively and extracts the maximal possible surplus from them. Proposition 1 characterizes the optimal selling strategy for β sufficiently high or sufficiently low. For intermediate value of β , it is not possible to establish the optimal monopoly strategies (selling to both types vs. optimistic consumers exclusively) because it will depend on the specific shape of the demand.

Based on the monopolist's optimal selling strategy, we summarize in the following proposition the sufficient conditions for rationing and buying frenzies to occur in period one.

Proposition 2 *The monopolist rations consumers in period one when $\beta < \widehat{\beta}$ or $\beta > \widehat{\beta}$. Buying Frenzies occur under the former condition if $E_p(\theta) - c < \frac{1}{g(0)}$.*

If the fraction of pessimist consumers is large ($\beta < \widehat{\beta}$), the optimal first period price will attract both types of consumers. However the monopolist prefers to choose $q_1^* < 1$ and create buying frenzies when

$E_p(\theta) - c < \frac{1}{g(0)}$ (Lemma 2). In this case, pessimistic consumers are indifferent between purchasing the good in the first period and postponing consumption because the first period price is equal to their expected utility. Optimistic consumers, however, are strictly worse-off. If the fraction of optimistic consumers is large enough ($\beta > \widehat{\beta}$), the monopolist will sell to optimistic consumers exclusively and ration some of them to the second period. Nevertheless, optimistic consumers are not worse off when rationed out because they are just indifferent between buying the good in period one and in period two.

Proposition 2 characterizes the sufficient condition for buying frenzies. Notice that buying frenzies arise only when both types of consumers are willing to buy the good in the first period and some optimistic consumers are rationed out. Does the monopolist's incentive to ration consumers depend on the rationing rule applied? Suppose that the monopolist finds it optimal to sell to both types of consumers in period one. The first period optimal output q_1^* is not affected by the nature of the rationing rule. To see this, notice that pessimistic consumers' maximum willingness to pay in period one is invariant to the specification of the rationing rule. This is because the equilibrium secondary market price p_2 is determined solely by $q_1 + q_2$, the total stock of the good, and is not affected by how q_1 is allocated between consumers in period one. Therefore, the monopolist's profit maximization problem (5) remains unchanged when we adopt a different rationing rule.

Although the monopolist's optimal first period output q_1^* does not depend on the nature of the rationing rule, whether buying frenzies will occur hinges on whether the rationing is efficient. Since consumers are only aware of their types in period one, efficient rationing in our context means that optimistic consumers will receive the good before pessimistic consumers. To see how the rationing rule may affect the occurrence of buying frenzies, suppose that the monopolist finds it optimal to sell to both types of consumers and choose $q_1^* \in (\beta, 1)$. Given the efficient rationing rule, only pessimistic consumers are rationed to the second period and they are indifferent between buying the good immediately and postponing consumption. Any other rationing rule which leaves some optimistic consumers without the good will generate buying frenzies. As we discuss in the model section, a non-efficient rationing rule like the proportional rationing rule seems more plausible in the context of a new product launch.

4 No secondary market & Welfare

In this section, we study the monopolist's optimal selling strategy when there is no secondary market and explore the welfare implications of having or banning the secondary market. We find that when the secondary market is banned, the monopolist never rations consumers. This result highlights that the presence of a secondary market is the key driving force for rationing and buying frenzies in our model.

4.1 Optimal selling strategy without a secondary market

As in the previous section, we first analyze the monopolist's optimal strategy when it targets different types of consumers. Then we characterize the monopolist's overall optimal strategy. We adopt the following assumption to rule out the uninteresting case where the monopolist only sells in period two.

Assumption 2 $2E_p(\theta) > \bar{\theta}$

Consider a pessimistic consumer's purchase decision in period one. Because there is no secondary market, the consumer must consume the good in both periods once she purchases it in period one. Hence, the consumer's expected utility from purchasing the good immediately is $2E_p(\theta) - p_1$. By contrast, the consumer's expected utility from postponing consumption is $(1 - F_p(p_2))(E_p[\theta|\theta \geq p_2] - p_2)$. Comparing these two options, the consumer will purchase the good in period one if and only if

$$p_1 \leq 2E_p(\theta) - (1 - F_p(p_2))(E_p[\theta|\theta \geq p_2] - p_2). \quad (9)$$

Given fixed prices p_1 and p_2 , how is the consumer's first period decision different from the case when a secondary market is available? The consumer's utility from waiting is the same regardless of the availability of the secondary market. However, the consumer receives a higher utility from purchasing the good in period one when there is a secondary market. This is because the consumer can resell the good and receive price p_2 if her true valuation turns out to be lower than the secondary market price. By contrast, the consumer has to consume the good and enjoy utility $E_p(\theta|\theta < p_2)$ when the secondary market is unavailable. Since $E_p(\theta|\theta < p_2) < p_2$, the consumer has a lower maximum willingness to pay in the first period when the secondary market is unavailable. Next, we analyze the monopolist's optimal selling strategy when it sells to

both types of consumers.

Lemma 5 *Selling in the second period only is never optimal.*

Suppose that the monopolist only sells in period two. Then at most it will make a profit $\bar{\theta} - c$. By Assumption 2, we have $\bar{\theta} - c \leq 2E_p[\theta] - c$, where $2E_p[\theta] - c$ is the monopoly profit from selling to all the pessimistic consumers in period one while charging the second period price $p_2 = \bar{\theta}$. This contradicts the initial hypothesis. Next, we characterize the monopolist's optimal selling strategy when selling to both types of consumers in period one.

Lemma 6 *If the monopolist charges pessimistic consumers' maximum willingness to pay in period one, its optimal strategy is $q_1^* = 1$, $q_2^* = 0$ and $p_2^* = \bar{\theta}$*

The monopolist extracts the entire surplus from all the pessimistic consumers by choosing $q_1 = 1$ and charging $p_2 = \bar{\theta}$. If instead the monopolist rations out some consumers and sells them in period two, it loses profit due to the reduced sales in period one. Moreover, the monopolist has to reduce the price in period one because consumers anticipate that the monopolist will continue to sell in period two and therefore have an incentive to postpone their purchases to period two when they become informed. The total loss in profit is not compensated by the profit generated in the second period because only a fraction of the consumers rationed out will purchase the good in period two.

Next we show the optimal monopoly strategy if the monopolist sells to optimistic consumers exclusively in period one.

Lemma 7 *If the monopolist targets optimistic consumers and sells to them exclusively in the first period, the optimal strategy is $q_1^* = \beta$, $q_2^* \geq 0$ and $p_2^* \leq \bar{\theta}$*

Similarly to the discussion after Lemma 6, the monopolist finds that it is more profitable to extract as much as possible surplus from consumers when they face uncertainty about their valuations in period one compared with the case when they become fully informed in period two. Notice that p_2^* not only affects the demand from pessimistic consumers in the second period but also constrains the maximum price that can be

charged in the first period. This is because optimistic consumers can delay consumption and buy the good at p_2^* when they become informed in period two. The following proposition characterizes the monopolist's overall optimal selling strategy as a function of β .

Proposition 3 *The monopolist will sell to both types of consumers when $\beta < \tilde{\beta}$, where $\tilde{\beta} \in (0, 1)$, and it will sell to optimistic consumers exclusively when $\tilde{\beta} \leq \beta$.*

Similarly to the previous section with a secondary market, the monopolist finds that it is more profitable to attract both types of consumers when the fraction of optimistic consumers is low enough. Otherwise, it maximizes profit by selling to optimistic consumers exclusively in period one and then selling also to (a fraction of) pessimistic consumers in period two.

4.2 Buying Frenzies and Welfare

We have shown in the previous sections that buying frenzies are driven by trading on the secondary market. In this section, we discuss the welfare implications of banning the secondary market when buying frenzies arise.

The previous sections have established that the monopolist prefers selling to both types of consumers when β is below a certain threshold and to optimistic consumers exclusively when β is large enough. We restrict attention to the former case because buying frenzies may occur in this parameter range in the presence of secondary market. To simplify exposition, let $E(\theta) \equiv \beta E_o(\theta) + (1 - \beta)E_p(\theta)$ and $E(\theta|\theta < p) = \beta E_o(\theta|\theta < p) + (1 - \beta)E_p(\theta|\theta < p)$.

When $\beta < \min\{\hat{\beta}, \tilde{\beta}\}$, the monopolist will target both types of consumers either with or without a secondary market. Without secondary market, the monopolist will sell to all consumers in period one (see Lemma 6). The total surplus is

$$W^{NS} = 2E(\theta) - c. \tag{10}$$

Since consumers cannot resell the good, they will consume the good for two periods. The first item in W^{NS} is consumer valuation from consumption and the second item is the cost of production.

When there is a secondary market, the monopolist will choose $q_1^* \in (0, 1)$ and $q_2^* = 0$ (see Lemma 2).

The corresponding total surplus is

$$W^S = q_1^*[E(\theta) - c] + q_1^*E(\theta|\theta \geq p_2^*), \quad (11)$$

where p_2^* is determined by $E_p(\theta) + p_2 - c = \frac{1-G(p_2)}{g(p_2)}$ and q_1^* is pinned down by the market clear condition $q_1^* = 1 - G(p_2^*)$. The first item of W^S is the social surplus generated by q_1^* units of the good in the first period. In period two, the q_1^* units are allocated to consumers with valuations greater than p_2^* through trading on the secondary market. The second item of W^S captures consumer valuation from consumption in period two. The difference between the social surplus in the two scenarios is

$$W^S - W^{NS} = q_1^*[E(\theta|\theta \geq p_2^*) - E(\theta)] - (1 - q_1^*)[2E(\theta) - c]. \quad (12)$$

In both scenarios, the good is randomly allocated to consumers in period one. However, when the secondary market exists, the good will be allocated to consumers with the highest valuations in period two. By contrast, the good will continue to be consumed by consumers randomly in period two when there is no secondary market. Hence, secondary market generates a gain in allocation efficiency, which is captured by the first item in (12). The second item in (12) captures the welfare loss due to under production in the presence of secondary market. The monopolist will produce $1 - q_1^*$ additional units when there is no secondary market and the associated total surplus per unit is $2E(\theta) - c$ which is positive (otherwise the monopolist will not sell to all consumers when there is no secondary market) under the assumptions $2E_p(\theta) > \bar{\theta} > c$.

The welfare comparison depends on the trade-off between the gain in allocation efficiency and the loss due to under production. The following proposition characterizes the condition under which buying frenzies reduce welfare.

Proposition 4 *When buying frenzies occur, banning the second-hand market improves welfare if*

$$c < E(\theta) + E(\theta|\theta < p_2^*) \quad (13)$$

and it reduces welfare otherwise.

Notice that the right hand side of (13) is a function of c . Because the difference $W^S - W^{NS}$ is not monotone in c , we cannot tell in general at what level of c the condition (13) is satisfied. To see that

$W^S - W^{NS}$ is non-monotonic in c , recall p_2^* is increasing in c . So the per unit gain in allocation efficiency is increasing in c . However, q_1^* is decreasing in c . As a consequence, the total gain from allocation efficiency (the first item in (12)) is not monotone in c . How about the loss from under production? As c goes up, the social surplus from each unit of the good decreases. However, under production becomes more severe; that is, $1 - q_1^*$ increases in c . This implies that the total loss due to under production is also non-monotonic in c . So, we cannot tell how $W^S - W^{NS}$ changes in c without imposing additional assumptions on the shape of the distribution functions $F_o(\theta)$ and $F_p(\theta)$.

Nevertheless, it is clear that when buying frenzies occur, banning the secondary market improves welfare for sufficiently small c . The condition (13) is satisfied at $c = 0$. By continuity, it is satisfied when c is sufficiently small. In fact, when $c = 0$, the equilibrium without secondary market is socially efficient. This is because the monopolist sells the good to all consumers in period one and therefore consumption is efficient.

It is not clear though whether allowing the secondary market in the presence of buying frenzies could enhance welfare. That is, whether

$$c > E(\theta) + E(\theta|\theta < p_2^*) \tag{14}$$

is satisfied for some c . When c goes up, p_2^* goes up and consequently $E(\theta|\theta < p_2^*)$ becomes larger. Buying frenzies are likely to enhance welfare when demand curve is relatively elastic. In this case, p_2^* goes up slowly when c increases and (14) is likely to be satisfied for large c .⁶

5 No Commitment

In this section, we discuss the role of commitment and how it affects the buying-frenzies strategies. It is well known that when a durable good monopolist cannot commit to future price and quantity, it will make too much sales in the second period with respect to what the monopolist would like to do from the first period point of view. Therefore, in order to maintain a high resale value for the good, the monopolist will try to counterbalance this effect by reducing even more the quantity sold in the first period which in turn leads to a larger fraction of consumers rationed to the second period. Specifically, the next proposition shows that

⁶A similar trade-off arises also when the monopolist target optimistic consumers only.

the lack of commitment power does not reduce the monopolist’s incentive to ration consumers. In fact, when buying frenzies occur, the monopolist will ration at least the same number of consumers to the second period as it would like to do when it has commitment power. Let q_{1c}^r denote the number of consumers rationed to the second period in buying frenzies when the monopolist can commit to p_2 and q_2 .

Proposition 5 *When the monopolist targets both types of consumers, buying frenzies occur if $E_p(\theta) - c < \frac{1}{g(0)}$ and at least q_{1c}^r consumers are rationed to the second period.*

The proof involves several steps and is relegated to an online Appendix. The proposition shows that when it is optimal for the monopolist to attract both types of consumers, the condition for buying frenzies to occur remains unchanged even when the monopolist lacks commitment power. In fact, the monopolist may ration more consumers to the second period in buying frenzies when it lacks commitment power.

6 Conclusion

This paper explains why a durable-goods monopolist would like to restrict supply and induce buying frenzies in the presence of an active secondary market and demand uncertainty. While the existing literature ignores the important role played by the secondary market, we argue that the option of reselling the good on the secondary market can be one of the driving forces for the firm’s scarcity strategy. We show that when consumers are heterogenous in their distribution of valuations, optimistic consumers are strictly worse off when rationed out in buying frenzies. By contrast, pessimistic consumers are indifferent between buying the good in period one and being rationed to period two. We also find that banning secondary market could be welfare enhancing when buying frenzies occur and the marginal cost of production is sufficiently low.

Finally, we emphasize that our explanation does not exclude other explanations for good scarcity. In particular, the scarcity of fashion products can also be driven by consumers’ need for exclusivity. Moreover, similar behavior could be explained in a context where firms can influence social learning among consumers by manipulating the launch sequence of a new good. It is noted that it can be profitable for a firm to restrict the access of a new good to consumers in order to induce a purchasing herd (Liu and Schiraldi 2011).

Appendix

Proof for Lemma 1: We prove Lemma 1 by contradiction. Assume that the monopolist's optimal strategy is to sell only in period two, i.e. $q_1^* = 0$, then the monopolist's profit maximization problem is

$$\text{Max}_{p_2} (p_2 - c)(1 - G(p_2)), \quad (15)$$

given that all consumers become informed about their valuations in the second period. Let p_2^m denote the monopolist's optimal second period price and $(p_2^m - c)$ is the per unit profit obtained by the monopolist. Suppose the monopolist sells one unit of the good in period 1 instead of period 2 and continues to sell the rest of the units in period two. Since the total stock of the good in period two remains unchanged, the market clear condition (4) implies that p_2 remains unchanged under this alternative selling strategy. However, the profit from selling one unit in the first period is $(E_p(\theta) + p_2^m - c)$ which is strictly larger than $(p_2^m - c)$ and contradicts the initial hypothesis. Q.E.D.

Proof for Lemma 2: The proof has two steps. Step 1 shows given $p_1 = E_p(\theta) + p_2$, the monopolist will optimally choose $q_2^* = 0$. Step 2 shows $q_1^* < 1$ if and only if $E_p(\theta) - c - \frac{1}{g(0)} < 0$.

Step 1. Substitute $p_2 = G^{-1}(1 - q_1 - q_2)$ into the the objective function (5). We first show the profit function

$$\pi(q_1, q_2) = (E_p(\theta) + G^{-1}(1 - q_1 - q_2) - c)q_1 + (G^{-1}(1 - q_1 - q_2) - c)q_2 \quad (16)$$

is concave. Take the derivative of $\pi(q_1, q_2)$ with respect to q_1 and q_2 , respectively.

$$\frac{\partial \pi(q_1, q_2)}{\partial q_1} = E_p(\theta) + G^{-1}(1 - q_1 - q_2) - c - \frac{q_1 + q_2}{g(G^{-1}(1 - q_1 - q_2))} \quad (17)$$

$$\frac{\partial \pi(q_1, q_2)}{\partial q_2} = G^{-1}(1 - q_1 - q_2) - c - \frac{q_1 + q_2}{g(G^{-1}(1 - q_1 - q_2))}. \quad (18)$$

Because $\frac{\partial \pi(q_1, q_2)}{\partial q_2}$ and $\frac{\partial \pi(q_1, q_2)}{\partial q_1}$ are identical except for the constant term $E_p(\theta)$ and q_1 and q_2 enter $\frac{\partial \pi(q_1, q_2)}{\partial q_1}$ and $\frac{\partial \pi(q_1, q_2)}{\partial q_2}$ in the form of $q_1 + q_2$, the second derivatives are

$$\begin{aligned} \frac{\partial^2 \pi(q_1, q_2)}{\partial q_1^2} &= \frac{\partial^2 \pi(q_1, q_2)}{\partial q_2^2} = \frac{\partial^2 \pi(q_1, q_2)}{\partial q_1 \partial q_2} = \frac{\partial^2 \pi(q_1, q_2)}{\partial q_2 \partial q_1} = \\ &= \frac{-1}{g(G^{-1}(1 - q_1 - q_2))} \left[2 + \frac{(q_1 + q_2)g'(G^{-1}(1 - q_1 - q_2))}{g^2(G^{-1}(1 - q_1 - q_2))} \right]. \end{aligned}$$

By Assumption 1, $2 + \frac{(q_1+q_2)g'(G^{-1}(1-q_1-q_2))}{g^2(G^{-1}(1-q_1-q_2))} > 0$. To see this, by Bagnoli and Bergstrom (2005, Economic Theory) Theorem 3, log concave $g(p_2)$ implies log concave survival function $1 - G(p_2)$. So, we have

$$\frac{d^2 \ln(1 - G(p_2))}{dp_2^2} = \frac{-g'(p_2)(1 - G(p_2)) - g^2(p_2)}{(1 - G(p_2))^2} \leq 0 \quad (19)$$

Inequality (19) holds if and only if

$$\begin{aligned} -g'(p_2)(1 - G(p_2)) - g^2(p_2) &\leq 0 \\ \frac{g'(p_2)(1 - G(p_2))}{g^2(p_2)} &\geq -1. \end{aligned} \quad (20)$$

Substitute $p_2 = G^{-1}(1 - q_1 - q_2)$ into (20), it follows that

$$\frac{(q_1 + q_2)g'(G^{-1}(1 - q_1 - q_2))}{g^2(G^{-1}(1 - q_1 - q_2))} \geq -1 > -2.$$

Hence, $2 + \frac{(q_1+q_2)g'(G^{-1}(1-q_1-q_2))}{g^2(G^{-1}(1-q_1-q_2))} > 0$ and $\frac{\partial^2 \pi(q_1, q_2)}{\partial q_1^2} < 0$. So, the Hessian matrix is

$$\left| \frac{\partial^2 \pi(q_1, q_2)}{\partial q_1^2} \right| \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}, \quad (21)$$

which is negative definite. Hence, $\pi(q_1, q_2)$ is global concave.

Now, we show $q_2^* = 0$. First, ignore the boundary conditions. Let q_1^* and q_2^* denote the solution for the optimization problem. Suppose $q_2^* > 0$. Then, (18) ≥ 0 at (q_1^*, q_2^*) . This implies (17) > 0 at (q_1^*, q_2^*) . As a consequence, it must be that $q_1^* = 1$. This contradicts (6).

Step 2. Given that $\pi(q_1, q_2)$ is globally concave and $q_2^* = 0$, the solution $q_1^* < 1$ if and only if

$$\frac{\partial \pi(q_1, q_2)}{\partial q_1} \Big|_{q_1=1, q_2=0} = E_p(\theta) - c - \frac{1}{g(0)} < 0 \quad (22)$$

So, when (22) holds, the monopoly profit is

$$\pi = (E_p(\theta) + p_2 - c)(1 - G(p_2)),$$

where p_2 is determined by

$$E_p(\theta) + p_2 - c = \frac{1 - G(p_2)}{g(p_2)}, \quad (23)$$

and the corresponding optimal first period output is $q_1^* = 1 - G(p_2)$. When (22) is violated, $q_1^* = 1$ and the monopoly profit is $(E_p(\theta) - c)$. Q.E.D.

Proof for Lemma 3: The derivative of $H(\beta)$ is

$$H'(\beta) = -G^{-1}'(1-\beta) - \frac{g(G^{-1}(1-\beta)) + \beta g'(G^{-1}(1-\beta))G^{-1}'(1-\beta)}{g^2(G^{-1}(1-\beta))}.$$

Substituting

$$G^{-1}'(1-\beta) = \frac{1}{g(G^{-1}(1-\beta))}$$

into $H'(\beta)$, we have

$$H'(\beta) = \frac{-1}{g(G^{-1}(1-\beta))} \left[2 + \frac{\beta g'(G^{-1}(1-\beta))}{g^2(G^{-1}(1-\beta))} \right].$$

We have shown in step 1 of the proof for Lemma 2 that $2 + \frac{(q_1+q_2)g'(G^{-1}(1-q_1-q_2))}{g^2(G^{-1}(1-q_1-q_2))} > 0$, $\forall q_1, q_2$, with $0 \leq q_1 + q_2 \leq 1$. Let $q_1 + q_2 = \beta \leq 1$. It follows that $2 + \frac{\beta g'(G^{-1}(1-\beta))}{g^2(G^{-1}(1-\beta))} > 0$ and hence $H'(\beta) < 0$. Q.E.D.

Proof for Corollary 1: The monopolist's optimal profit from selling to both types of consumers is

$$\max\{(E_p(\theta) + p_2 - c)(1 - G(p_2)), E_p(\theta) - c\}.$$

Take the derivative of $(E_p(\theta) + p_2 - c)(1 - G(p_2))$ with respect to β and apply the Envelope theorem, we have

$$(E_p(\theta) + p_2 - c)(F_p(p_2) - F_o(p_2)) \geq 0.$$

The term $E_p(\theta) - c$ is constant in β . As a result, $\max\{(E_p(\theta) + p_2 - c)(1 - G(p_2)), E_p(\theta) - c\}$ is increasing in β . Q.E.D.

Proof for Lemma 4: First notice that Lemma 3 establishes the monotonicity of $H(\beta)$ which characterized the conditions for the different strategies as functions of β . The proof has three steps. Step 1 shows that the monopolist's profit function is globally concave in q_1 and q_2 . Step 2 shows $q_1^* < \beta$ and $q_2^* > 0$ cannot be the solution. Step 3 proves the results in the table.

Step 1. Substituting $p_2 = G^{-1}(1 - q_1 - q_2)$ from (4) into the monopolist's profit function, it follows that

$$\pi(q_1, q_2) = (E_o(\theta) + G^{-1}(1 - q_1 - q_2) - c)q_1 + (G^{-1}(1 - q_1 - q_2) - c)q_2. \quad (24)$$

Note that (24) is identical to (16) except for the constant term in the first parentheses. Because this constant term does not enter the second derivatives, the Hessian matrix is the same as (21) which is shown to be negative definite.

Step 2. We can rewrite (24) as

$$E_o(\theta)q_1 + (G^{-1}(1 - q_1 - q_2) - c)(q_1 + q_2). \quad (25)$$

Suppose $q_1^* < \beta$ and $q_2^* > 0$. Then, the monopolist can make more profit by substituting one unit of q_1 for q_2 . The substitution will not change the total stock $q_1 + q_2$ and hence the second item in (25) remains unchanged. But the first item in (25) becomes larger.

Step 3. Due to step 2, the optimal solution must be $q_1^* < \beta$ and $q_2^* = 0$ or $q_1^* = \beta$ and $q_2^* \geq 0$. Since (24) is globally concave, Case 1 happens if

$$\frac{\partial \pi(q_1, q_2)}{\partial q_1} \Big|_{q_1=\beta, q_2=0} \geq 0 \quad (26)$$

$$\frac{\partial \pi(q_1, q_2)}{\partial q_2} \Big|_{q_1=\beta, q_2=0} > 0. \quad (27)$$

The derivatives

$$\begin{aligned} \frac{\partial \pi(q_1, q_2)}{\partial q_1} &= E_o(\theta) + G^{-1}(1 - q_1 - q_2) - c - \frac{q_1 + q_2}{g(G^{-1}(1 - q_1 - q_2))} \\ \frac{\partial \pi(q_1, q_2)}{\partial q_2} &= G^{-1}(1 - q_1 - q_2) - c - \frac{q_1 + q_2}{g(G^{-1}(1 - q_1 - q_2))}. \end{aligned}$$

Because $\frac{\partial \pi(q_1, q_2)}{\partial q_1} > \frac{\partial \pi(q_1, q_2)}{\partial q_2}$, $\forall q_1, q_2$, (27) implies (26). The condition (27) is satisfied when

$$G^{-1}(1 - \beta) - c - \frac{\beta}{g(G^{-1}(1 - \beta))} > 0.$$

In Case 1, q_2^* is interior and is determined by

$$G^{-1}(1 - \beta - q_2^*) - c - \frac{\beta + q_2^*}{g(G^{-1}(1 - \beta - q_2^*))} = 0. \quad (28)$$

By (4), $p_2 = G^{-1}(1 - \beta - q_2^*)$.

Next, the Case 2 happens when

$$\begin{aligned} \frac{\partial \pi(q_1, q_2)}{\partial q_1} \Big|_{q_1=\beta, q_2=0} &\geq 0 \\ \frac{\partial \pi(q_1, q_2)}{\partial q_2} \Big|_{q_1=\beta, q_2=0} &\leq 0, \end{aligned}$$

which are satisfied when $-E_o(\theta) \leq G^{-1}(1 - \beta) - c - \frac{\beta}{g(G^{-1}(1 - \beta))} \leq 0$. By (4), $p_2 = G^{-1}(1 - \beta)$ and hence the monopoly profit is $(E_o(\theta) + G^{-1}(1 - \beta) - c)\beta$.

Finally, Case 3 happens when

$$\begin{aligned}\frac{\partial \pi(q_1, q_2)}{\partial q_1} \Big|_{q_1=\beta, q_2=0} &< 0 \\ \frac{\partial \pi(q_1, q_2)}{\partial q_2} \Big|_{q_1=\beta, q_2=0} &\leq 0,\end{aligned}$$

which are satisfied when $G^{-1}(1 - \beta) - c - \frac{\beta}{g(G^{-1}(1 - \beta))} < -E_o(\theta)$.

In Case 3, q_1^* is the interior solution and is determined by

$$\begin{aligned}\frac{\partial \pi(q_1^*, 0)}{\partial q_1} &= 0 \\ E_o(\theta) + G^{-1}(1 - q_1^*) - c - \frac{q_1^*}{g(G^{-1}(1 - q_1^*))} &= 0.\end{aligned}\tag{29}$$

The corresponding profit is therefore $(E_o(\theta) + G^{-1}(1 - q_1^*) - c)q_1^*$. Q.E.D.

Proof for Corollary 3: We first show that the optimal profit is continuous in β . First, consider $H(\beta) = 0$. By Lemma 3 the function $H(\cdot)$ is strictly decreasing. Given that $H(\beta) = 0$ and q_2^* is determined by $H(\beta + q_2^*) = 0$, it must follow that $q_2^* = 0$ at $H(\beta) = 0$. Hence, the monopolist's optimal profit in Case 1 equals to that in Case 2 at $H(\beta) = 0$. Next, consider $-E_o(\theta) = H(\beta)$. Because q_1^* is determined by $-E_o(\theta) = H(q_1^*)$, it must follow that $q_1^* = \beta$ at $-E_o(\theta) = H(\beta)$. Hence, the monopolist's profit in Case 2 equals that in Case 3 at $-E_o(\theta) = H(\beta)$.

Next, we show that the optimal profit is increasing in β . First, consider Case 1. Let π^1 denote the optimal profit in Case 1. Substitute $q_2^* = 1 - \beta - G(p_2)$ into π^1 and take the derivative of π^1 with respect to β , we have

$$\begin{aligned}\frac{d\pi^1}{d\beta} &= \frac{\partial \pi^1}{\partial \beta} + \frac{\partial \pi^1}{\partial p_2} \frac{\partial p_2}{\partial \beta} \\ &= \frac{\partial \pi^1}{\partial \beta} \\ &= E_o(\theta) + (p_2 - c)(F_p(p_2) - F_o(p_2))\end{aligned}\tag{30}$$

The second equality follows from the Envelope theorem. Since $F_o(\theta)$ first order stochastically dominates $F_p(\theta)$, $F_p(p_2) \geq F_o(p_2)$ and hence $\frac{\partial \pi^1}{\partial \beta} > 0$.

Consider Case 2. Let π^2 denote the optimal profit in Case 2. Take the derivative

$$\frac{d\pi^2}{d\beta} = E_o(\theta) + p_2 - c + \frac{dp_2}{d\beta} \beta.\tag{31}$$

By the market clear condition $G(p_2) \equiv 1 - \beta$, we derive

$$\frac{dp_2}{d\beta} = \frac{F_p(p_2) - F_o(p_2) - 1}{g(p_2)}.$$

Substitute $\frac{dp_2}{d\beta}$ into (31),

$$\frac{d\pi^2}{d\beta} = E_o(\theta) + p_2 - c + \frac{(F_p(p_2) - F_o(p_2) - 1)}{g(p_2)}\beta.$$

Since in Case 2, $-E_o(\theta) \leq H(\beta)$,

$$\frac{d\pi^2}{d\beta} \geq \frac{F_p(p_2) - F_o(p_2)}{g(p_2)}\beta \geq 0.$$

Finally, consider Case 3. Let π^3 denote the optimal profit in Case 3. Take the derivative and apply the Envelope theorem, we have

$$\begin{aligned} \frac{d\pi^3}{d\beta} &= \frac{\partial \pi^3}{\partial \beta} \\ &= (E_o(\theta) + p_2 - c)(F_p(p_2) - F_o(p_2)) \geq 0. \end{aligned} \tag{32}$$

Q.E.D.

Proof for Proposition 1. The proof has two steps. Step 1 shows that the monopolist will sell to both types of consumers in period one when $\beta < \hat{\beta}$. Step 2 shows that the monopolist will sell to optimistic consumers in period one exclusively when $\beta \geq \hat{\beta}$.

Step 1. By Lemma 4, when $\beta < H^{-1}(0)$, the monopolist's maximum profit from selling to optimistic consumers is characterized in Case 1. The monopolist sells $\beta + q^*$ units in the two periods. We can rewrite the monopolist's maximal profit as

$$E_o(\theta)\beta + (p_2 - c)(\beta + q_2^*). \tag{33}$$

Now, suppose that the monopolist charges pessimistic consumers' maximum willingness to pay and sells $\beta + q_2^*$ units in period one and zero unit in period two. By the market clear condition (4), p_2 is determined by the total stock of the good in the two periods and hence remains unchanged in this alternative selling strategy. The monopolist's corresponding profit is

$$\begin{aligned} &(E_p(\theta) + p_2 - c)(\beta + q_2^*) \\ &= (E_p(\theta))(\beta + q_2^*) + (p_2 - c)(\beta + q_2^*). \end{aligned} \tag{34}$$

The profit (34) is greater than (33) if

$$q_2^* > \frac{E_o(\theta) - E_p(\theta)}{E_p(\theta)} \beta, \quad (35)$$

where q_2^* is determined by $G^{-1}(1 - \beta - q_2^*) - c = \frac{\beta + q_2^*}{g(G^{-1}(1 - \beta - q_2^*))}$ (see Lemma 4). By the Implicit Function Theorem, $\frac{dq_2^*}{d\beta} = -1 < 0$. When $\beta = 0$, q_2^* is determined by $F_p^{-1}(1 - q_2^*) - c = \frac{q_2^*}{f_p(G^{-1}(1 - q_2^*))}$. Given that $F_p^{-1}(1 - q_2^*) = p_2$, q_2^* is the optimal static monopoly output when the monopolist faces pessimistic consumers only. Since $c < \bar{\theta}$, $q_2^* > 0$. As a result, (35) is satisfied at $\beta = 0$. Now, consider $\beta = H^{-1}(0) > 0$. At this value of β , Case 2 happens, and $q_2^* = 0$ and hence (35) is violated. Since q_2^* decreases in β , there exists a cutoff value $\hat{\beta} \in (0, H^{-1}(0))$ such that (35) is satisfied for all $\beta < \hat{\beta}$.

Step 2. Consider $\beta \geq H^{-1}(-E_o(\theta))$. We first show that it is more profitable to sell to optimistic consumers exclusively than to sell to all consumers in period one. Given $\beta \geq H^{-1}(-E_o(\theta))$, the optimal profit from selling to optimistic consumers exclusively is summarized in Case 3 of Lemma 4, which is greater than

$$\pi(\beta) = (E_o(\theta) + G^{-1}(1 - \beta) - c)\beta, \quad (36)$$

the profit from selling to all of the optimistic consumers in period one and zero unit in period two. So, at $\beta = 1$, the monopoly profit in Case 3 is at least $E_o(\theta) - c$, which is derived by evaluating (36) at $\beta = 1$. Clearly, $E_o(\theta) - c$ is greater than $E_p(\theta) - c$, the profit from charging pessimistic consumers' maximum willingness to pay and selling to all consumers in period one.

Next, we show selling to optimistic consumers is more profitable than selling to both types of consumers and rationing. Let $p_{2k}^* q_{1k}^*$, $k = o, p$, denote the optimal second period price and first period quantity when the monopolist charges type k consumers' maximum willingness to pay in period one. By Lemma 2, p_{2p}^* is determined by

$$E_p(\theta) + p_{2p}^* - c = \frac{1 - G(p_{2p}^*)}{g(p_{2p}^*)}. \quad (37)$$

Using $G(p_{2o}^*) = 1 - q_{1o}^*$, the market clear condition in Case 3 of Lemma 4, we can rewrite (29) as

$$E_o(\theta) + p_{2o}^* - c = \frac{1 - G(p_{2o}^*)}{g(p_{2o}^*)}, \quad (38)$$

which determines p_{2o}^* . Assumption 1 implies that $\frac{1 - G(\theta)}{g(\theta)}$ is decreasing in θ . Since $E_o(\theta) > E_p(\theta)$, (37) and (38) implies $p_{2o}^* < p_{2p}^*$. In both Lemma 2 and Case 3 of Lemma 4, the second period output is zero. By

the market clear condition $G(p_{2k}^*) = 1 - q_{1k}^*$, $p_{2o}^* < p_{2p}^*$ is equivalent to

$$\begin{aligned} G^{-1}(1 - q_{1o}^*) &< G^{-1}(1 - q_{1p}^*) \\ q_{1p}^* &< q_{1o}^*. \end{aligned}$$

Since $q_{1o}^* < \beta$, it follows that $q_{1p}^* < \beta$. Now, given $q_{1p}^* < \beta$, the monopoly can charge optimistic consumers' maximum willingness to pay and sell the same amount q_{1p}^* in period one and zero unit in period two. Because the total stock of the good doesn't change, the second period price remains unchanged. This alternative selling strategy yields profit

$$(E_o(\theta) + p_{2p}^* - c)(1 - G^{-1}(p_{2p}^*)) > (E_p(\theta) + p_{2p}^* - c)(1 - G^{-1}(p_{2p}^*)).$$

Therefore, when $\beta \geq H^{-1}(-E_o(\theta))$, the optimal profit from charging pessimistic consumers' maximum willingness to pay and rationing is lower than the profit in Case 3 of Lemma 4. Q.E.D.

Proof for Lemma 6: The proof is divided in two steps. Step 1 shows that if the monopolist chooses $q_1 = 1$, the maximum profit it can achieve is $2E_p[\theta] - c$. Step 2 shows that any interior first period output $0 < q_1 < 1$ is dominated by $q_1 = 1$.

Step 1. When the monopolist sells to pessimistic consumers, the maximum first period price it can charge is

$$p_1 = 2E_p[\theta] - (1 - F_p(p_2))(E_p[\theta|\theta \geq p_2] - p_2).$$

The monopolist's profit from selling to all consumers in period one is therefore

$$2E_p[\theta] - (1 - F_p(p_2))(E_p[\theta|\theta \geq p_2] - p_2) - c,$$

which is maximized at $p_2 = \bar{\theta}$. So, the maximum profit the monopolist can make from selling to all consumers in period one is

$$\pi_p(q_1 = 1, p_2 = \bar{\theta}) = 2E_p[\theta] - c.$$

Step 2. The monopolist solves the following problem

$$\max_{q_1, p_2} (p_1 - c)q_1 + (p_2 - c)(1 - G(p_2))(1 - q_1)$$

s.t.

$$p_1 = 2E_p[\theta] - (1 - F_p(p_2))(E_p[\theta|\theta \geq p_2] - p_2). \quad (39)$$

Suppose the optimal first period quantity is interior. That is, $0 < q_1^* < 1$. Let $p_2^* \leq \bar{\theta}$ denote the associated optimal second period price. The monopolist's profit is

$$\begin{aligned} \pi_p(q_1^*, p_2^*) &= (2E_p[\theta] - (1 - F_p(p_2^*))(E_p[\theta|\theta \geq p_2^*] - p_2^*) - c)q_1^* \\ &\quad + (p_2^* - c)(1 - G(p_2^*))(1 - q_1^*). \end{aligned}$$

If q_1^* is interior, it must satisfy the first order condition:

$$2E_p[\theta] - (1 - F_p(p_2^*))(E_p[\theta|\theta \geq p_2^*] - p_2^*) - c = (p_2^* - c)(1 - G(p_2^*)). \quad (40)$$

Now, we take the difference between $\pi_p(q_1 = 1, p_2 = \bar{\theta})$ and that from choosing $q_1^* < 1$ and p_2^* :

$$\begin{aligned} \Delta\pi &= \pi_p(q_1 = 1, p_2 = \bar{\theta}) - \pi_p(q_1^*, p_2^*) \\ &= (2E_p[\theta] - c)(1 - q_1^*) + (1 - F_p(p_2^*))(E_p[\theta|\theta \geq p_2^*] - p_2^*)q_1^* - \\ &\quad (p_2^* - c)(1 - G(p_2^*))(1 - q_1^*). \end{aligned}$$

Using (40), we can simplify $\Delta\pi$ to

$$(2E_p[\theta] - c) - (p_2^* - c)(1 - G(p_2^*)) > 0. \quad (41)$$

The inequality follows from the assumption $2E_p[\theta] \geq \bar{\theta}$ because

$$(p_2^* - c)(1 - G(p_2^*)) < \bar{\theta} - c \leq 2E_p[\theta] - c.$$

In summary, (41) contradicts the initial hypothesis that it is optimal to sell in both periods. Q.E.D.

Proof for Lemma 7: When targeting optimistic consumers, the monopolist chooses q_1 and p_2 to maximize

$$\begin{aligned} \pi_o(q_1, p_2) &= (2E_o[\theta] - (1 - F_o(p_2))(E_o[\theta|\theta \geq p_2] - p_2) - c)q_1 \\ &\quad + (p_2 - c)\{(\beta - q_1)(1 - F_o(p_2)) + (1 - \beta)(1 - F_p(p_2))\} \end{aligned}$$

subject to $0 \leq q_1 \leq \beta$. The first order condition with respect to q_1 is

$$2E_o[\theta] - c - (1 - F_o(p_2))(E_o[\theta|\theta \geq p_2] - c). \quad (42)$$

The assumption $2E_p[\theta] \geq \bar{\theta}$ implies $2E_o[\theta] > \bar{\theta}$. Since $E_o[\theta|\theta \geq p_2] \leq \bar{\theta}$, (42) is greater than

$$\begin{aligned} & \bar{\theta} - c - (1 - F_o(p_2))(\bar{\theta} - c) \\ &= (\bar{\theta} - c)F_o(p_2) > 0. \end{aligned}$$

Hence, the optimal first period quantity must be the corner solution $q_1^* = \beta$. p_2^* is pinned down by $\frac{\partial \pi_o(q_1, p_2)}{\partial p_2} \geq 0$. Q.E.D.

Proof for Proposition 3: The monopolist's maximum profit from targeting the optimistic consumers is

$$\begin{aligned} \pi_o(\beta, p_2^*) &= (2E_o[\theta] - (1 - F_o(p_2^*))(E_o[\theta|\theta \geq p_2^*] - p_2^*) - c)\beta \\ &\quad + (p_2^* - c)(1 - \beta)(1 - F_p(p_2^*)). \end{aligned}$$

By the Envelope theorem,

$$\begin{aligned} \frac{\partial \pi_o(\beta, p_2^*)}{\partial \beta} &= (2E_o[\theta] - (1 - F_o(p_2^*))(E_o[\theta|\theta \geq p_2^*] - p_2^*) - c) - \\ &\quad (p_2^* - c)(1 - F_p(p_2^*)). \end{aligned}$$

Because $F_p(p_2^*) \geq F_o(p_2^*)$, it follows that

$$\begin{aligned} \frac{\partial \pi_o(\beta, p_2^*)}{\partial \beta} &\geq (2E_o[\theta] - (1 - F_o(p_2^*))(E_o[\theta|\theta \geq p_2^*] - p_2^*) - c) - \\ &\quad (p_2^* - c)(1 - F_p(p_2^*)) \\ &= 2E_o[\theta] - c - (1 - F_o(p_2^*))(E_o[\theta|\theta \geq p_2^*] - c) \\ &> (\bar{\theta} - c)F_o(p_2) > 0. \end{aligned}$$

The second to the last inequality follows from $2E_o[\theta] > \bar{\theta}$ and $E_o[\theta|\theta \geq p_2^*] \leq \bar{\theta}$.

Now, we compare the monopolist's optimal profit from targeting the pessimistic consumers $2E_p[\theta] - c$ with $\pi_o(\beta, p_2^*)$. When $\beta = 0$,

$$\pi_o(\beta, p_2^*) = (p_2^* - c)(1 - F_p(p_2^*)).$$

Because $2E_p[\theta] > \bar{\theta}$, $2E_p[\theta] - c > \pi_o(\beta, p_2^*)$. When $\beta = 1$,

$$\pi_o(\beta, p_2^*) = 2E_o[\theta] - c.$$

This is because when $\beta = 1$, $p_2^* = \bar{\theta}$. So, $2E_p[\theta] - c < \pi_o(\beta, p_2^*)$ at $\beta = 1$. Since $\pi_o(\beta, p_2^*)$ is continuous and increasing in β , there exists $\tilde{\beta} \in (0, 1)$ such that

$$\begin{aligned} 2E_p[\theta] - c &> \pi_o(\beta, p_2^*) && \text{for } \beta > \tilde{\beta} \\ 2E_p[\theta] - c &= \pi_o(\beta, p_2^*) && \text{for } \beta = \tilde{\beta} \\ 2E_p[\theta] - c &< \pi_o(\beta, p_2^*) && \text{for } \beta < \tilde{\beta} \end{aligned} .$$

Q.E.D.

Proof for Proposition 4: The difference (12) is negative if and only if

$$\begin{aligned} q_1^* [E(\theta|\theta \geq p_2^*) - E(\theta)] - (1 - q_1^*) [2E(\theta) - c] &< 0 \\ c &< 2E(\theta) - \frac{q_1^* [E(\theta|\theta \geq p_2^*) - E(\theta)]}{(1 - q_1^*)}. \end{aligned} \quad (43)$$

Substituting $q_1^* = 1 - G(p_2^*)$, the right hand side of (43) becomes

$$\begin{aligned} &2E(\theta) - \frac{(1 - G(p_2^*)) (E(\theta|\theta \geq p_2^*) - E(\theta))}{G(p_2^*)} \\ = &E(\theta) + \frac{E(\theta) - (1 - G(p_2^*))E(\theta|\theta \geq p_2^*)}{G(p_2^*)} \\ = &E(\theta) + E(\theta|\theta < p_2^*). \end{aligned}$$

Q.E.D.

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