

Michael Laver, [Kenneth Benoit](#)

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THE BASIC ARITHMETIC OF LEGISLATIVE DECISIONS

Michael Laver
New York University

Kenneth Benoit
London School of Economics
and Trinity College Dublin

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ABSTRACT

Despite the huge number of possible seat distributions following a general election in a multi-party parliamentary democracy, there are far fewer *classes* of seat distribution sharing important strategic features. We define an exclusive and exhaustive partition of the universe of theoretically possible n -party systems into five basic classes, the understanding of which facilitates more fruitful modeling of legislative politics, including government formation. Having defined a partition of legislative party systems and elaborated logical implications of this partition, we classify the population of postwar European legislatures. We show empirically that many of these are close to critical boundary conditions, so that stochastic processes involved in any legislative election could easily flip the resulting legislature from one type to another. This is of more than hypothetical interest, since we also show that important political outcomes differ systematically between the classes of party system – outcomes that include duration of government formation negotiations, type of coalition cabinet that forms, and stability of the resulting government.

INTRODUCTION

Any legislative election in a multiparty system may distribute seats between parties in a huge number of different ways. Ignoring party names, for example, there are 2,977,866 different distributions of 100 seats between up to 10 parties (Laver and Benoit 2003).¹ Considering the politics of building legislative majorities, however, many seat distributions are functionally equivalent, generating the same set of winning coalitions. Take a five-party 100-seat legislature with a majority winning threshold, and three possible distributions of seats between parties: A(48, 13, 13, 13, 13); B(48, 43, 3, 3, 3); C(40, 15, 15, 15, 15). These very different legislatures are equivalent in the sense that the largest party can form a winning coalition with *any other party*, while *all other parties* must combine to form a winning coalition that excludes the largest. The three legislatures do differ in terms of their “fragility,” however. If the largest party in legislature A loses a single seat to one of the others, then it can no longer form a two-party winning coalition with *any* of the others; the set of winning legislative coalitions radically changes. Legislature C is much less fragile in this sense; at least five seats must change hands to affect the set of winning coalitions.

In what follows, we define a set of equivalence classes that capture such similarities and differences between legislative party systems. Since any observed election result is the realization of a random draw from a *distribution* of expected results, different draws from the same distribution may produce legislatures that fall into different classes. Small reallocations of seats between parties can then flip the realized legislature from one class to another, making the effective election result, in terms of downstream legislative politics, something of a dice roll. Following the realization of an actual election result that leaves the legislature close to a boundary condition, furthermore, *non-random* strategic defections from one party to another may flip the legislature from one class to another, offering rent-seeking opportunities for wannabe defectors.

The strategic implications of such critical thresholds have not passed unnoticed. They give rise to notions such as the Shapley value and to power indices such as the Shapley-Shubik and Banzhaf indices (Banzhaf 1965; Shapley and Shubik 1954; Shapley 1952; Felsenthal and Machover 1998; Stole and Zwiebel 1996).² Many different distributions of seats between parties generate the same vector of Shapley or Banzhaf values. For example, the set of theoretically possible five-party 100-seat legislatures referred to above has 38,225

¹ All replication materials for this paper can be accessed at: TBA. (Materials submitted to Dataverse on 11 February 2014).

² Stole and Zwiebel (1996), among others, derive the Shapley value as a prediction from a non-cooperative alternating offers bargaining game.

different distributions of seats between parties, but only 20 different Shapley vectors (Laver and Benoit, 2003). Shifting a single seat from one party to another can change Shapley values dramatically, or not change them at all. Within the traditions of non-cooperative game theory, these thresholds inform a literature on “minimal integer representations” (MIRs) of weighted voting games (Ansolabehere et al. 2005; Laver et al. 2011; Montero 2006; Snyder et al. 2005; Freixas and Molinero 2009).³

Building on this work, we have three core objectives in this paper. First, we specify an exclusive and exhaustive partition of the universe of legislative party systems and derive theoretically relevant implications of this classification. This partition is far more parsimonious than the set of discrete Shapley or Banzhaf vectors⁴, and its implications are “model free”, in the sense they are accounting identities arising from binding arithmetic constraints and hold regardless of utility functions of key agents or local institutional structure. Second, we show that many real legislatures in postwar Europe were close to critical boundary conditions. Third we show this is substantively important. Different classes of legislature are associated with different political outcomes in real parliamentary democracies. First, however, we motivate our argument with a recent example of government formation where our boundary conditions made a big difference.

GREECE 2012

Greek voters went to the polls in May 2012 facing the specter of default on their country’s sovereign debt. The largest party, New Democracy (ND), won 108 of the 300 legislative seats, 43 short of the majority needed to form a government (see Table 1). The only two-party winning coalition was between ND and the second largest party, Syriza. This generated a “top-two” party system in the terms we define below, complicated by the fact the two largest parties fundamentally disagreed on the EU bailout. ND approached every other party except the extreme anti-European Golden Dawn (XA). Each refused to go into government. As mandated by the Greek constitution, the second largest party (Syriza) and third largest (PASOK) attempted in turn to form governments. These attempts also failed. As a last resort,

³ A minimal integer representation is the vector of smallest integers that generates, for a given winning quota, the same set of winning coalitions as the vector of raw seat totals. Consider three very “different” legislative party systems in a setting with a majority decision rule: (49, 17, 17, 17); (27, 25, 24, 24); and (2, 1, 1, 1). All generate the same set of winning coalitions. The largest party can form a winning coalition with any other; all others must combine to exclude the largest party. These legislative party systems share the same vector of Shapley or Banzhaf values (1/2, 1/6, 1/6, 1/6), and the same MIR (2, 1, 1, 1). Despite large superficial differences, in this precise sense these party systems are in an equivalence class.

⁴ Laver and Benoit (2003: p224) show, for an eight-party 100-seat legislature, there are 930,912 different distributions of seats between parties and 49,493 different Shapley vectors. There remain just five legislative types in our sense.

the President himself proposed a government comprising ND, PASOK and a small left wing party, Democratic Left (DIMAR). However DIMAR, from the beginning reluctant to accept conditions of the EU-IMF package, blocked this, knowing ND and PASOK lacked the 151 seats needed to form a government.

May		June	
Name	Seats	Name	Seats
ND	108	ND	129
Syriza	52	Syriza	71
PASOK	41	PASOK	33
ANEL	33	ANEL	20
KKE	26	XA	18
XA	21	DIMAR	17
DIMAR	19	KKE	12
Total	300		300
Threshold	151		151
Legislative type	D		B

Table 1. *Legislative arithmetic in the Greek elections of May and June 2012 and 2010.*
 “Legislative Type” is explained below.

New elections were called for June, and realized a crucial difference in the legislative arithmetic. The first and third largest parties, two seats short after the previous election, now controlled a majority of seats between them. The Greek legislative party system flipped out of a “top two” state and ND was now a “strongly dominant” party. This substantially weakened the second largest party, Syriza, *even though Syriza increased its seat total from 52 to 71*. The key fact arising from the new legislative arithmetic in Greece was that that ND and PASOK could now form a government alone – *even though the ND seat total declined from 41 to 33*. Given the new reality that the anti-bailout Syriza could not form a government even with all other parties, DIMAR accepted the deal they blocked one month before, joining the government with “conditional support”.⁵ Two election results in Greece, one month apart, generated two very different types of legislature.

⁵ The resulting coalition was a “surplus” majority coalition. DIMAR left this in June 2013, leaving a minimum winning coalition in place as the incumbent government.

CLASSES OF LEGISLATIVE PARTY SYSTEM

An exclusive and exhaustive partition of the universe of legislative party systems

Consider a legislature comprising n perfectly disciplined parties, labeled P_1, P_2, \dots, P_n , in descending order of seat share. The number of seats controlled by P_i is S_i . Any legislative party system can be written as $(W: S_1, S_2, \dots, S_n)$ where, according to binding constitutional rules, a successful proposal must be supported by a coalition of legislators whose number equal or exceeds W . The winning quota is decisive: if a coalition, C , of legislators is winning then its complement, C' , is losing. W must therefore be *at least* a simple majority of legislators, though in most of what follows W could be a supermajority.⁶ We label a coalition between P_x and P_y as $P_x P_y$. A “pivotal” party can render a winning coalition losing by leaving it; a “minimal winning” coalition comprises only parties that are pivotal. Define an exclusive and exhaustive partition of the universe of possible legislative party systems into five equivalence classes, which we call “types”, using sizes of the three largest parties, relative to each other and to W . This is set out in Figure 1.

Universe of possible legislative party systems				
<i>Single winning party</i>	<i>No single winning party</i>			
$S_1 \geq W$	$S_1 < W$			
	$S_1 + S_2 \geq W$			$S_1 + S_2 < W$
	$S_1 + S_3 \geq W$		$S_1 + S_3 < W$	
	$S_2 + S_3 < W$	$S_2 + S_3 \geq W$		
A: Single winning party	B: Strongly dominant party	C: Top-three	D: Top-two	E: Open

Figure 1. *Partitioning the universe of legislative party systems.*

⁶ Note immediately that if W is decisive, then $S_1 + S_2 + S_3 < 2W$ and hence $S_2 + S_3 \leq 4W/3$ and $S_3 \leq 2W/3$.

While our partition specifies constitutionally binding arithmetical *constraints* on legislative bargaining, it is no substitute for a *model* that specifies an institutional environment, agent utility functions, preferences, and so on. Knowing the May 2012 election in Greece returned a top-two legislature does not in itself tell us that government formation must be deadlocked. What it does tell us is that the only two-party winning coalition was between the two largest parties. Our explanation of deadlock, knowing the legislative type, derives from an implicit model of policy-based government formation and the knowledge that the two largest parties held fundamentally opposed positions on key issues. Our explanation of the end of the deadlock in June, assuming agent preferences did not change, is that a new election returning a new type of legislature removed a key constraint, so that, *despite declining in size*, the largest party could now find partners in a winning coalition that did not fundamentally disagree with it.

Definitions and properties of classes of legislative party system

Type A: Winning party ($S_1 \geq W$)

A single “winning” party controls all legislative decisions.

Type B: Strongly dominant party

In strongly dominant party systems P_1 has too few seats to control decisions ($S_1 < W$), but can form a winning coalition with either P_2 or P_3 ($S_1 + S_3 \geq W$), while P_2 and P_3 together cannot form a winning coalition ($S_2 + S_3 < W$). This makes P_1 “dominant” in the sense defined by previous authors (Peleg 1981; Einy 1985; van Deemen 1989), whose definition refers to mutually exclusive losing *coalitions* made winning by adding the largest party. The intuition is more striking if we consider losing *parties*, and call party P^* “strongly dominant” if there are two other parties P_i and P_j such that $S^* + S_i \geq W$ and $S^* + S_j \geq W$ but $S_i + S_j < W$. Define a Type B legislative party systems as one containing a strongly dominant party. There are several striking logical implications of having a strongly dominant party.⁷

Implication B1: If P_1 is strongly dominant, both P_2 and P_3 are members of every winning coalition excluding P_1 .⁸

⁷ Additional implications can be found in the supplementary materials for this paper.

⁸ Since the coalition P_1P_2 is winning by definition of strong dominance, its complement $(P_1P_2)'$ is losing. Thus $(P_1P_2)'$ must add either P_1 or P_2 to become winning. If it excludes P_1 it must add P_2 . Thus if P_1 is strongly dominant, any winning coalition excluding P_1 must include P_2 . An identical argument applies to P_3 .

Implication B2: If P_1 is strongly dominant, P_1 and only P_1 is a member of every winning two-party coalition.⁹

The strategic significance of this is that a strongly dominant party holds a privileged bargaining position. If it is excluded from *any* winning coalition, which must then include both P_2 and P_3 , it can tempt *either* P_2 or P_3 , and quite possibly other pivotal parties, with an offer that can be implemented solely by dominant party and tempte, without regard to any other party. Only a strongly dominant party can be in this position. We show below that this is empirically relevant because legislatures with a strongly dominant party are not only common in postwar Europe, but also tend to be associated with minority governments that include the dominant party.

Type B: System-dominant party*

A special case of a strongly dominant party occurs when the largest party P_1 is not winning on its own but can form a winning coalition with *any* other party ($S_1 + S_n \geq W$). Call such a party, P^{**} , “system-dominant”.

Implication B3: Any winning coalition excluding P^{} must include all other parties.** This is a necessary and sufficient condition for system dominance.¹⁰

This implies a strategic setting described by game theorists as an “apex game”. Identifying the sub-type of B* party systems is useful theoretically because, moving beyond three parties, apex games have a structure that is more tractable analytically than many others (Fréchette et al. 2005a; Montero 2002). Such systems are tractable because minimal winning coalitions comprise: the largest party plus any other; or every party except the largest. All parties except the largest are in this sense perfect substitutes for each other. Adding other as yet unmodeled constraints on government formation, arising from personal animosities, policy differences between the small parties or anything else, can make it extremely difficult to exclude a system dominant party from government. This in turn leads us to expect that Type B* party systems may be associated with minority governments comprising the system dominant party. Identifying Type B* systems is important empirically because, as we show

⁹ Since the largest possible two-party coalition excluding P_1 , which is P_2P_3 , is losing, then every possible two-party coalition excluding P_1 is losing.

¹⁰ For example, in a 100-seat legislature with a simple majority rule, this would arise if the partition of seats between 6 parties was (40, 12, 12, 12, 12, 12). Aragonés (2007) offers a similar result, confined to four-party systems.

below, these do indeed tend to be associated with single party minority cabinets, as well as significantly shorter government formation negotiations, and longer cabinet durations.

Type B^k: k-dominant party

We can generalize the notion of a system-dominant party to that of a k -dominant party, defined as a largest party able to form a winning coalition with P_k but not with P_{k+1} . For example, in the legislature (51: 35, 25, 16, 16, 8), the P_1 would be 4-dominant, able to form a winning coalition with P_4 but not with P_5 . A system-dominant party in an n -party system would be n -dominant. While not part our system of legislative types because it is not driven by the sizes of the three largest parties, this refinement may be useful in future work. Valuing parsimony, we do not pursue it here.

Type C: “Top-three” party system

A “top-three” legislative party system arises when *any* pair of the three largest parties can form a winning coalition. $S_2 + S_3 \geq W$ is thus a necessary and sufficient condition for a top-three system. Logically, this implies:

Implication C1: Regardless of the number of parties in a top-three system, only the three largest parties can be pivotal.¹¹

Implication C2: Any coalition excluding any two of the three largest parties in a top-three system is losing.¹²

Implication C3: The three largest parties in a top-three system are perfect substitutes for each other in the set of minimal winning coalitions.¹³

By symmetry, the Shapley values and minimum integer weights (MIWs) of the top three parties must all be equal, and those of all other parties must be zero. In practical terms, this means an analyst looking at a new legislature with no majority party should first check to see whether the second and third largest parties can form a winning coalition. If they can, we are in the very distinctive bargaining environment of a top-three party system, in which any two

¹¹ If P_2P_3 is winning then its complement, $(P_2P_3)'$, the coalition between P_1 and all parties outside the top three, is losing. Similarly, P_1P_3 winning implies $(P_1P_3)'$ losing, and P_1P_2 winning implies $(P_1P_2)'$ losing. No party outside the top three can render winning a coalition *excluding* two of the top three parties, since every such coalition must be losing. Yet, by definition of Type C, every coalition *including* two of the top three parties is winning regardless of the addition or subtraction of another party outside the top three.

¹² By definition S_1S_2 , S_1S_3 , and S_2S_3 are all winning, so their complements are all losing.

¹³ This follows from the definition of a Type C legislature and implications C1 and C2.

of the three largest parties can form a winning coalition and, *no many how many other parties there might be*, none of these is ever pivotal.

The theoretical relevance of top-three party systems arises because of their analytical tractability. Settings with only three legislative parties, where any pair may form a winning coalition, produce a very tractable set of winning coalitions but are almost unheard of in practice, rendering “three-party” results of dubious empirical relevance. Top-three party systems are analogous, on some modeling assumptions, to three-party systems to which a set of “dummy” agents have been added who have no effect on play.¹⁴ The empirical relevance of top-three systems arises, as we show below, because minimal winning coalitions (MWCs) are very much more likely to occur in Type C than in any other type of party system. Indeed, it is only in Type C systems that MWCs are the most likely type of government.

Type D: “Top-two” party system

Top-two legislative party systems arise when the two largest parties can form a winning coalition ($S_1 + S_2 \geq W$) but P_1 and P_3 cannot ($S_1 + S_3 < W$). The only two-party winning coalition is between the two largest parties, since P_1P_3 , the next-largest two-party coalition, is losing. Logically, this implies:

Implication D1: One or other of the two largest parties in a top-two system is a member of every winning coalition.¹⁵

Note there are top-two systems that privilege the largest party¹⁶ and others that do not¹⁷. For example, it may be that $S_1 + S_3 + S_4 \geq W$ while $S_2 + S_3 + S_4 < W$, giving P_1 more options than P_2 . This suggests subdivisions of the top-two legislative type, though these require looking beyond sizes of the three largest parties, so we leave these for future consideration. Nonetheless, P_1 and P_2 are at the “top” of any top-two party system in the sense that one or the other must be part of every winning coalition, while they and only they can form a winning coalition between themselves that excludes all others.

¹⁴ This sets aside the possibility that parties outside the top three may find ways to make binding commitments to vote together in the legislature, in effect combining into a single new legislative party and flipping the legislature into a new equivalence class.

¹⁵ Since P_1P_2 is winning its complement is losing. Note therefore that Result D1 also applies to Type B and Type C systems.

¹⁶ For example (51: 35, 20, 13, 12, 10, 10).

¹⁷ For example (51: 29, 26, 13, 12, 10, 10).

Type E: “Open” systems

The defining inequality, $S_1 + S_2 < W$, of the residual class of “open” party systems implies there is *no winning two-party coalition*. It must also be true that $S_2 < W/2$, a necessary condition for an open system. Logically, this implies a striking result focusing on $W/2$:

Implication E1: $S_1 < W/2$ is a sufficient condition for an open party system.¹⁸

Every legislature in which the largest party has fewer seats than half the winning threshold has an open legislative party system, which immediately suggests another useful practical check for an observer looking at a new multi-party legislature.

Implication E2: An open party system and majority decision rule imply $N \geq 5$.¹⁹

It is therefore necessary to model at least five-party systems to cover the full range of logical possibilities arising from the legislative arithmetic we outline. The theoretical significance of open legislatures arises because it is never possible for a party excluded from a winning coalition to tempt any single pivotal member of that coalition with an offer that can be implemented exclusively by temptor and temptee, since any two-party coalition must be losing. This means even the largest party must deal with *coalitions* of other parties – and with potential collective action problems within such coalitions – in order to put together a winning coalition. In all other types of legislative party system, if the largest party does not win single-handed, it can win by forming a coalition with no more than one other party, at the very least the second-largest party. It can win without having to *coalesce with coalitions*.

The empirical significance of open legislative party systems arises, as we show below, because they are associated with significantly longer government formation negotiations, with significantly shorter cabinet durations, and with surplus majority or minority governments.

Legislative types and politicians’ policy preferences

Our argument in this paper is intended to facilitate conclusions about legislative bargaining in multi-party systems that are model-free implications of constitutionally binding arithmetical constraints. Adding modeling assumptions about agent utilities or institutional structure may well refine our understanding of legislative bargaining, subject to the constraints we specify. In this context, our partition clearly has a bearing on how we think

¹⁸ $S_1 + S_2 < W$ implies $S_1 < W/2$ since $S_1 \geq S_2$

¹⁹ A majority decision rule, $N = 3$ and $S_1 + S_2 < W$ imply $S_3 \geq W$. $N = 4$ and $S_1 + S_2 < W$ imply $S_3 + S_4 \geq W$. Since $S_1 \geq S_2 \geq S_3 \geq S_4$, both implications are contradictions.

about the legislative politics of *policy* decisions. For example, it is easy to see that *a system dominant party must control the median legislator on every policy dimension for which it is not at one of the two extreme positions*, which has a bearing on the likelihood of minority governments. It is also easy to see that *the median legislator on any policy dimension in a top-three system must belong to the most central of the three largest parties*. Our approach thus enhances the modeling of legislative bargaining over policy. To develop this in any explicit way, however, requires assumptions about agent utility functions, from which we refrain here, though further discussion of this can be found in supplementary materials.

EMPIRICAL DISTRIBUTION OF PARTY SYSTEM TYPES

We now describe the empirical distribution of types of legislative party system in 29 European parliamentary democracies during the period 1945-2010, using a dataset assembled by the European Representative Democracy (ERD) project (Andersson and Ersson 2012).²⁰ Winning coalitions in these *empirical* data are those comprising a simple majority of legislators. We partitioned all 361 European post electoral party systems in the ERD data universe into our six (including B*) basic types. Figure 2 maps out, for minority legislatures, the partition of party systems specified in Figure 1. Left panels show regions defined by seat shares of the three largest parties. Boundaries of these regions are specified by the inequalities set out in Figure 1. For example, a lower region of the upper left hand plot is the exclusive preserve of “open” party systems, given the defining inequality $S_1 + S_2 < W$. A region of the lower left-hand plot is the exclusive preserve of “top-three” party systems given the defining inequality $S_2 + S_3 \geq W$ and our deduction that $S_2 + S_3 \leq 4W/3$. Right panels of Figure 2 map the party systems of postwar Europe into the theoretically possible regions. The key empirical pattern is that *regions close to boundary conditions are densely populated with empirical cases*. Very small changes in the seat distributions of many actual legislatures would have flipped them from one type of party system to another.

²⁰ For scrupulous documentation of coding protocols for this dataset, see <http://www.erdda.se>. Countries from the former Soviet bloc, as well as Spain, Portugal and Greece, were included after their first democratic election.

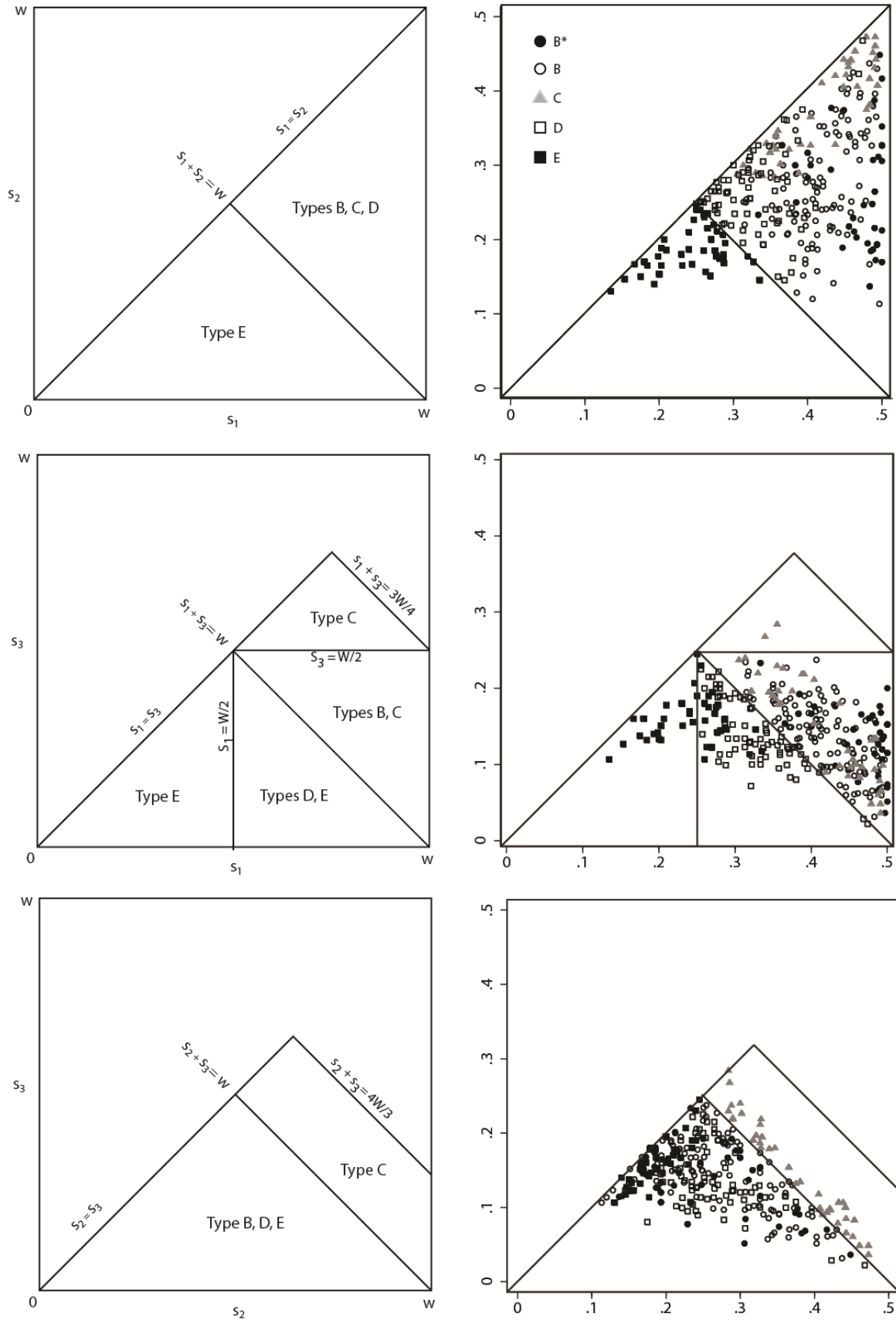


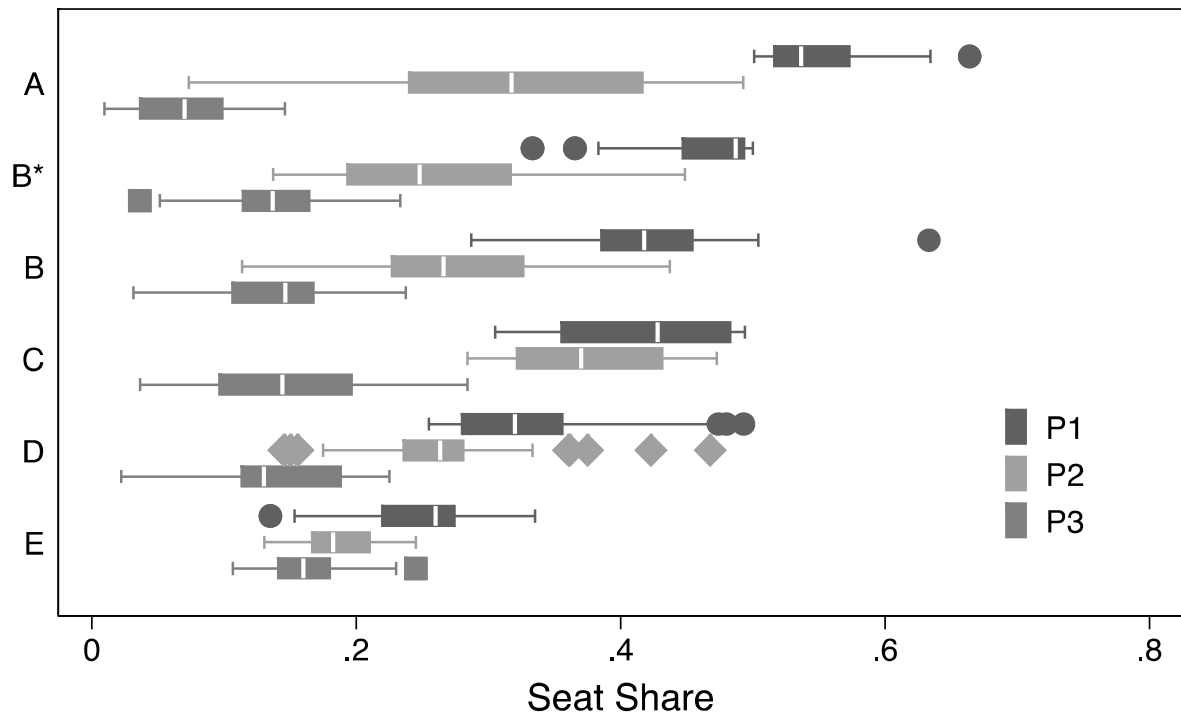
Figure 2. *Partition of party systems in theory (left), and observed in postwar Europe (right).*

Table 2 shows that 90 percent of postwar European legislatures with six parties or fewer fall into the highly constrained types A to C. In contrast, 57 percent of those with seven parties or more fall into the relatively unconstrained types D and E, where the number of arithmetically possible majority coalitions is very much greater and, in this sense, legislative politics is more complicated. We also see that dominant parties are not theoretical curiosities. Notwithstanding the typical PR electoral systems and resulting multi-party politics in postwar Europe, it is common to find legislative party systems dominated by one party able to play off the rest against each other.

	A	B*	B	C	D	E	
<i>Number of legislative parties</i>	<i>Single party winning</i>	<i>System dominant party</i>	<i>Strongly dominant party</i>	<i>Top three</i>	<i>Top two</i>	<i>Open</i>	<i>Total</i>
2-6	47 23%	37 18%	64 32%	35 17%	18 9%	1 0%	202 100%
7-16	19 12%	2 1%	43 27%	4 3%	50 31%	41 26%	159 100%
All	66 18%	39 11%	107 30%	39 11%	68 19%	42 12%	361 100%

Table 2. *Frequencies of legislative types in European legislative elections, 1945-2010.*

Figure 3 plots relative seat shares sizes of the three largest parties in postwar European legislatures. Similar seat shares across especially the second and third largest parties result in different types of party system. More than party seat shares *per se*, it is precise relationships between seat shares of the top three parties, relative to boundary conditions, that determine the type of party system.

Figure 3. *Plots of $S_1 - S_3$ by legislative type: post-election party systems in the ERD Dataset.*

FRAGILITY OF LEGISLATIVE STATES

If the distribution of expected legislative seat shares following an election straddles one of our boundary conditions, then small random shocks to vote shares, amplified in complex ways by electoral formulae, can have big effects. As long as the process generating votes has some residual variance—as does every model from the vast empirical literature in electoral behavior and electoral systems—then the process generating votes will be to some degree stochastic. When these differences are multiplied across numerous constituencies, with multiple parties and candidates, their aggregate effects can easily produce small shifts in seats from one party to another, even if underlying political and contextual factors remain unchanged. We simulate this in a simple and intuitive way by representing election results as random draws from an underlying distribution of expected results, where expected seat proportions remain constant but the prior distribution is assigned a non-zero variance. We draw a new seat allocation for each party from a multinomial distribution where the

proportions p_i are the actual seat share for party i , and n is the total number of seats.²¹ By drawing new “shocked” seat allocations based on observed party seat shares, we generated a set of election results that might plausibly have been realized within a specified range of expected variance.²² To simulate a range of “possible” distributions of legislative seats for every post-war European legislature in the ERD dataset – each consistent with the realized outcome – we drew 100 new elections for each observed seat allocation, and computed the legislative type associated with each possible outcome. The proportions of “shocked” legislative types associated with each observed legislative type are shown in Figure 4.

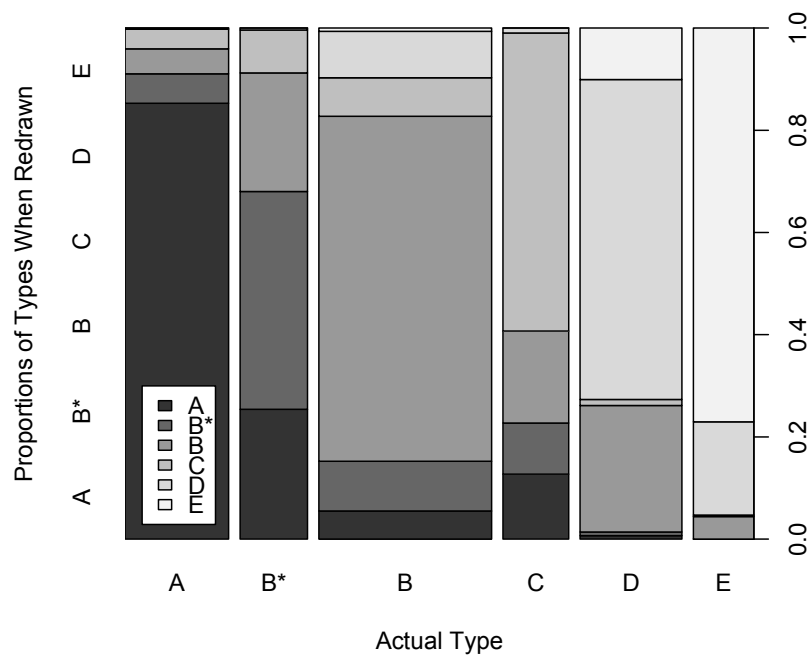


Figure 4. *Transitions from actual post-election governments to other legislative types, following simulated repeats of each election.* Each of 361 post-election governments was redrawn 100 using observed seat proportions from a multinomial draw, and the y-axis reflects the proportions by original type of each of the 36,100 simulated types. The width of the columns is proportional to the relative frequency of observed legislative types from Table 2.

Most shocked Type A party systems, for example, remained in Type A. The most common realization of a shock to a Type B* party system was to remain in Type B*, but about 25% became Type A systems, another 20% became Type B, and just under 10%

²¹ This means that parties who won no seats cannot win seats in any of the simulations, as $p_i=0$ for a party that won no seats. An alternative would be to use Laplace smoothing where we added one seat to each party, but we avoided this because it would change the number of parties in the system and potentially represent a different legislative dynamic.

²² We present stress tests of this assumption about the distribution of possible election results variance at alternative settings, along with supporting empirical evidence, in the supplementary materials for this paper. The full dataset of simulated results is also available with the replication materials for this paper.

became Type C. Similar transition probabilities for the other legislative types are presented in Figure 4.

Moving beyond aggregate patterns reported in Figure 4, we now predict the particular legislative types that result from small shocks to seat shares associated with each election result. To illustrate our core argument most clearly, Table 3 highlights predictions of changes in the odds of flipping to each legislative type, given a change in the seat share of the *smallest* party – a party rarely the focus of attention in opinion polls or discussions of government formation. As control variables, we include differences between seat shares of each of the top three parties and their closest competitor, to hold constant the main effects that determine legislative types. Our estimations in Table 3 report five multinomial logistic regressions, one for each legislative type, except the majority Type A party system.²³ Each exponentiated coefficient represents the relative risk (analogous to an odds ratio) of changing from the type that heads each column to the new type labeled in the row, given a one unit change in the relevant explanatory variable. Each column represents a separate multinomial logistic regression. To illustrate the interpretation of results from Table 3, consider the effect of a change in the seat share of the smallest party on the odds of becoming a Type D system. Look at the gray horizontal band of coefficients near the bottom of the table, associated with transitions to Type D party systems. A one per cent increase in the seat share of the *smallest* party increases the relative risk of a Type B party system becoming a Type D party system (thereby undermining the dominant position of the *largest* party) by about 15%. The same shift in the smallest party seat share increases the probability of Type C party system transitions into Type D (thereby making parties outside the top three pivotal in majority coalitions) by about 40%. Our classification of legislative types shows that small changes in the sizes of even the *smallest* party in the legislature can have big effects on legislative politics when no single party wins a majority.

²³ Each regression uses the original legislative type (before simulating a new seat allocation) as the base outcome, and reports exponentiated coefficients representing relative risk ratios, or the multiplicative change in odds of the stated outcome relative to the base category, for a percentage point change in seat share (or seat share difference).

New Type	Variables	Original Legislative Type				
		(1)	(2)	(3)	(4)	(5)
		B*	B	C	D	E
A	P1 % Lead	1.258 [1.224 - 1.293]	1.325 [1.283 - 1.369]	1.366 [1.263 - 1.479]	1.243 [1.047 - 1.475]	
	P2 % Lead	1.198 [1.174 - 1.223]	1.252 [1.227 - 1.276]	1.149 [1.102 - 1.197]	1.303 [1.201 - 1.414]	
	P3 % Lead	1.176 [1.146 - 1.208]	1.118 [1.086 - 1.150]	0.932 [0.831 - 1.045]	1.321 [0.911 - 1.914]	
	Pn %Δ	0.782 [0.741 - 0.825]	0.749 [0.680 - 0.826]	0.856 [0.743 - 0.986]	0.637 [0.350 - 1.159]	
B*	P1 % Lead		1.022 [1.007 - 1.037]	1.252 [1.193 - 1.314]	1.071 [0.976 - 1.175]	
	P2 % Lead		1.036 [1.027 - 1.046]	0.925 [0.906 - 0.943]	1.151 [1.102 - 1.202]	
	P3 % Lead		0.929 [0.910 - 0.949]	0.666 [0.631 - 0.704]	0.915 [0.709 - 1.180]	
	Pn %Δ		1.025 [0.976 - 1.076]	1.308 [1.238 - 1.382]	0.447 [0.328 - 0.610]	
B	P1 % Lead	1.097 [1.081 - 1.114]		0.849 [0.816 - 0.883]	1.069 [1.054 - 1.085]	1.149 [1.100 - 1.202]
	P2 % Lead	1.031 [1.017 - 1.045]		0.913 [0.897 - 0.928]	1.058 [1.047 - 1.069]	1.13 [1.015 - 1.259]
	P3 % Lead	0.916 [0.888 - 0.945]		0.835 [0.803 - 0.867]	1.18 [1.151 - 1.209]	1.484 [1.398 - 1.576]
	Pn %Δ	0.956 [0.918 - 0.995]		1.217 [1.164 - 1.273]	0.825 [0.783 - 0.869]	0.628 [0.525 - 0.751]
C	P1 % Lead	0.783 [0.756 - 0.811]	0.82 [0.804 - 0.836]		0.827 [0.731 - 0.936]	0.427 [0.427 - 0.427]
	P2 % Lead	0.978 [0.966 - 0.991]	1.034 [1.023 - 1.045]		1.037 [0.989 - 1.087]	0.022 [0.00270 - 0.179]
	P3 % Lead	1.317 [1.259 - 1.378]	1.259 [1.230 - 1.289]		1.216 [1.100 - 1.345]	10.81 [9.383 - 12.46]
	Pn %Δ	0.954 [0.919 - 0.990]	0.783 [0.741 - 0.826]		0.474 [0.396 - 0.569]	0.279 [0.0309 - 2.519]
D	P1 % Lead	0.987 [0.923 - 1.056]	0.924 [0.912 - 0.937]	0.586 [0.503 - 0.684]		1.077 [1.055 - 1.100]
	P2 % Lead	0.961 [0.896 - 1.030]	0.986 [0.978 - 0.995]	0.808 [0.757 - 0.863]		1.456 [1.383 - 1.533]
	P3 % Lead	0.623 [0.417 - 0.932]	0.901 [0.882 - 0.920]	0.695 [0.617 - 0.782]		1.402 [1.352 - 1.454]
	Pn %Δ	0.996 [0.841 - 1.180]	1.155 [1.107 - 1.206]	1.406 [1.278 - 1.546]		0.762 [0.697 - 0.832]
E	P1 % Lead		0.936 [0.887 - 0.988]		0.897 [0.879 - 0.917]	
	P2 % Lead		0.603 [0.518 - 0.703]		0.745 [0.723 - 0.767]	
	P3 % Lead		0.83 [0.756 - 0.911]		0.785 [0.755 - 0.816]	
	Pn %Δ		1.173 [0.958 - 1.437]		1.123 [1.048 - 1.203]	
	Observations	3,900	10,000	2,700	5,900	3,500
	Log-likelihood	-4272.1183	-9665.0572	-2748.0535	-5111.8676	-1878.9035

Table 3. *Multinomial logistic regressions predicting simulated types from original legislative types.* All coefficients are exponentiated to represent risk ratios, relative to the original type as a baseline. 95% confidence intervals are in brackets, with bold coefficients statistically significant at the $p \leq .05$ level. Data are the same as for Figure 4.

TYPES OF LEGISLATIVE PARTY SYSTEM, TYPES OF POLITICAL OUTCOME

Types of legislative party system and the “difficulty” of forming a government

Rational politicians with complete information should negotiate equilibrium cabinets without delay: “... for the environments most interesting in policy-making applications, delay will almost never occur” (Banks and Duggan 2006, 72-73). It is well known, however, that some government formation negotiations drag out much longer than others. If the environment evolves stochastically, and/or if party leaders exploit private information (about personal preferences or which proposals their legislators will accept) bargaining delays may arise in equilibrium (Merlo 1997; Merlo and Wilson 1995). Diermeier and van Roozendaal apply this insight to government formation negotiations, and find a strong empirical relationship between measures of uncertainty and durations of negotiations (Diermeier and Van Roozendaal 1998). Martin and Vanberg, and more recently Golder, confirm these findings in different ways (Golder 2010; Martin and Vanberg 2003). Their strongest conclusion is that negotiations immediately following an election tend to take much longer than those taking place between elections, following defeat or resignation of an incumbent.

Each of these authors treats post-electoral government formation as an indicator of *uncertainty*, on the ground there is less information about preferences of new legislators immediately after an election. We also note that inter-electoral government formations are often *endogenous* to legislative politics; when a majority of legislators vote a government out of office, mid-term, they presumably have some preferred alternative in mind. Inter-electoral formation negotiations may be shorter because they commence with this preferred alternative.

Golder (2010) and others also associate longer formation negotiations with more “complex” bargaining environments, measuring complexity in terms of the number and ideological polarization of parliamentary parties. We argued above that different types of legislative party system are associated with different levels of complexity or “difficulty” in coalition formation. Moving from Type A to Type E systems, we move from the simplest setting, with a single majority party, through settings with a dominant party in the catbird seat, through “top-three” systems with only three pivotal parties no matter how many others there are, to the least constrained “open” systems with many pivotal parties and many possible majority coalitions to explore. Our conjecture is that, as complexity of the coalition formation environment increases, so will the “difficulty” and hence duration of government formation negotiations. Table 4 shows mean durations of formation negotiations, by type of

party system. The bottom row replicates previous findings that post-electoral negotiations last much longer (on average 39 days) than those between elections (13 days). The rightmost column supports our conjecture that mean durations of government formation negotiations should increase monotonically as the legislative arithmetic becomes less constrained.

Type of system	Post-election	Inter-election	All formations
A: Single majority party	20.3 <i>3.6</i>	8.1 <i>2.7</i>	15.7 <i>2.5</i>
B*: System dominant party	24.9 <i>5.4</i>	2.9 <i>0.9</i>	17.2 <i>3.8</i>
B': Strongly dominant party	32.6 <i>3.3</i>	16.1 <i>2.1</i>	25.0 <i>2.1</i>
C: Top-three system	48.7 <i>7.7</i>	10.0 <i>4.2</i>	33.4 <i>5.5</i>
D: Top-two system	46.5 <i>4.9</i>	18.5 <i>5.6</i>	34.0 <i>3.9</i>
E: Open system	72.3 <i>7.0</i>	12.7 <i>2.0</i>	36.3 <i>4.2</i>
All formations	38.6 <i>2.2</i>	13.3 <i>1.4</i>	27.1 <i>1.4</i>

Table 4. *Mean durations of government formation negotiations in postwar Europe, by type of legislative party system.* Standard errors in italics. Formation durations data, taken from the ERD dataset, count days between election/government resignation and investiture of new government.

Creating binary variables for legislative types, we use the Cox proportional hazards model specified by Golder (2010) to investigate whether these types predict delays in government formation. We follow Golder in using the number of legislative parties as an indicator of uncertainty, controlling for existence of a single majority party, and distinguishing post- and inter-electoral formations. Rather than using the subjective and potentially endogenous notion of “positive parliamentarianism, we use the objective and binding constitutional constraint of a constructive vote of no confidence. *Inter*-electoral government formations should be much quicker with a constructive vote of no confidence, since the next government must be explicitly identified in the no confidence motion that defeats the incumbent. The constructive vote of no confidence should however have no effect

on post-electoral formations.²⁴ Unlike the dataset used by Golder, which is confined to Western Europe and ends in 1998, the ERD dataset ends in 2010 and includes 10 former communist countries in Central and Eastern Europe (CEE). We therefore include a CEE dummy since we expect greater uncertainty, hence longer bargaining delays, in these new party systems.²⁵ Table 5 shows Cox proportional hazards estimates of the effects of independent variables on durations of government formation negotiations in postwar Europe.²⁶

	<i>Model 1</i>		<i>Model 2</i>		<i>Model 3</i> (country fixed effects)	
	<i>Post-election</i>	<i>Inter-election</i>	<i>Post-election</i>	<i>Inter-election</i>	<i>Post-election</i>	<i>Inter-election</i>
Number of parties	-0.10** (0.02)	-0.14** (0.02)	-0.08** (0.03)	-0.13** (0.03)	-0.01 (0.04)	-0.03 (0.04)
Constructive vote of no-confidence	-0.14 (0.12)	0.85** (0.22)	-0.11 (0.11)	0.94** (0.23)	0.79 (0.63)	1.84** (0.44)
CEE country	-0.10 (0.11)	-0.59** (0.16)	-0.11 (0.14)	-0.60** (0.15)	-1.19 (0.74)	-3.62** (0.79)
Minority parliament	-0.51** (0.21)	-0.55** (0.17)				
B*: System-dominant party			-0.23 (0.32)	0.45 (0.28)	-0.49 (0.30)	0.10 (0.26)
B': Strongly-dominant party ²⁷			-0.31 (0.21)	-0.28 (0.26)	-0.64** (0.25)	-0.26 (0.22)
C: Top-three system			-0.94** (0.27)	-0.24 (0.33)	-0.42 (0.31)	-0.68** (0.25)
D: Top-two system			-0.65** (0.22)	-0.14 (0.27)	-0.70** (0.27)	-0.06 (0.33)
E: Open system			-0.90** (0.23)	0.09 (0.29)	-1.20** (0.32)	0.03 (0.32)
Log likelihood	-1572	-1193	-1562	-1228	-1446	-1172
Observations	331	266	331	272	331	272

Table 5. *Cox proportional hazards models of durations of government formation negotiations in Europe, 1945-2010*²⁸

²⁴ If we include the ERD variable for positive parliamentarianism in models that also include the constructive vote of no confidence, it has no significant effect on bargaining delays. It has the effects observed by Golder if the no-confidence variable is dropped.

²⁵ Golder included a measure of ideological polarization as another indicator of bargaining difficulty. When we included the ERD measure of ideological polarization, however, we found no significant effect, and therefore excluded it from the analysis we report here.

²⁶ Rather than following Golder and using interaction terms to capture effects of key independent variables, conditional on whether negotiations follow an election, we estimate different models for post-electoral and inter-electoral settings, since these differ in many ways relevant to government formation.

²⁷ Systems labeled B' in have a strongly dominant party that is not system dominant.

Model 1 is a stripped-down benchmark. It replicates findings from previous work that increasing the number of parties, which has an exponential effect on the number of winning coalitions and hence the amount of information needed to take every possibility into account, reduces the hazard rate and thereby increases typical durations of government formation negotiations.²⁹ This effect is essentially the same in post- and inter-electoral negotiations. As expected, a constructive vote of no confidence significantly shortens *inter*-electoral formation negotiations, but has no significant effect on *post*-electoral negotiations. Former Communist states do have longer negotiations in inter-electoral settings, but not immediately after elections.

Model 2 replaces the simple distinction between systems with or without a majority party with the different types of legislative party system specified in Figure 1, using single party majority systems as the baseline. Coefficients for other independent variables are essentially unchanged. Types of legislative party system have the predicted effects on durations of *post-electoral* formation negotiations. These do not take significantly longer in systems with dominant parties than in those with majority parties.³⁰ In contrast, there are significantly longer formation delays in Type C, D and E systems. Note in particular that, while our classification of party systems is affected strongly by the number of legislative parties, *effects of party system types on bargaining delays are measured holding the number of parties constant*. In contrast, differences between types of legislative party system have no systematic effect on durations of *inter-electoral* government formation negotiations. This is consistent with Golder's argument that inter-electoral formations are high-information settings, so that the different information requirements posed by different types of party system do not bite. It is also consistent with the view that there may be a particular candidate government in inter-electoral formations, so that the full range of coalition possibilities is less likely to be explored. Either way, our Model 2 estimates show that post- and inter-electoral government formations are completely different. Conventional arguments about government formation apply to negotiations immediately following elections, but not to those taking place mid-term.

²⁸ Classifications of party systems by the authors; all other data from the ERD dataset.

²⁹ Diermeier and van Roozendaal (1998) use the *effective* number of legislative parties in this context, but Golder uses the absolute number. It is this latter number that has a direct effect on the number of winning coalitions. We also agree with Golder that it is not a good idea to use the number of parties *in government*, as do Martin and Vanberg (2003); this is clearly endogenous to government formation negotiations.

³⁰ Non-significant effects are in the "right" direction, with negotiations tending to be longer than in Type A systems.

Model 3 replicates Model 2, but adds a full set of country fixed effects, to eliminate the possibility that different countries tend to have different types of party system, with government formation negotiations tending to last longer in some countries as result of unmodeled differences between countries.³¹ Our classification of legislative party systems should pick up significant variation between different types of party system within the same country. We see that country fixed effects wash out the impact of the number of legislative parties but that the impact of party system types on *post-electoral* negotiations is robust to these. Legislative settings with system dominant parties do not have significantly longer formation negotiations than those with majority parties; Type D and Type E systems do have significantly longer formations. The differences are that Type B systems, with strongly dominant parties, have longer bargaining delays when country fixed effects are added, and top-three systems do not. All coefficients are in the predicted direction. The non-effect of party system types on *inter-electoral* formation durations is also robust to adding country fixed effects. Our legislative types effectively classify post-war European party systems according to the “difficulty”, measured as the duration of negotiations, of forming governments in minority parliaments.

Types of legislative party system and types of government

Different types of legislative party system are also associated with different types of coalition cabinet. Theoretical and empirical accounts of government formation in parliamentary democracies typically distinguish between:

- minimal winning coalitions (MWCs);
- surplus coalitions, which include at least one member whose defection leaves the coalition winning;
- minority cabinets, comprising parties that do not between them control a majority.

Models assuming politicians are motivated only by private benefits of office tend to imply MWCs. Models assuming politicians are motivated by preferences over public policy outcomes may also imply minority or surplus majority cabinets (Laver 1998). There is also an informal folk-wisdom that surplus cabinets provide insurance against defections in times of high uncertainty or low party discipline (Laver and Schofield 1998). Table 6 classifies European postwar governments formed in minority situations into MWCs, minority and

³¹ Luxembourg, close to the overall mean for formation negotiations, is the excluded category.

surplus majority cabinets,³² further classifying minority governments into coalition and single party cabinets. It shows a striking relationship between type of legislative party system and type of government.³³ Recall that top-three systems are the closest real-world analogue to analytically tractable “three-party” systems and that models assuming office-seeking politicians, typically predict MWCs. Table 6 shows that MWCs are the norm for actual top-three party systems, though such systems arise after only 11 percent of postwar European elections. Table 6 restates the well-known empirical pattern that well under half of all governments arising from post-war European minority systems are MWCs, while well over half are minority or surplus majority coalitions (Gallagher et al. 2012). Notwithstanding many theoretical models, MWCs are *not* the norm in real parliamentary settings and our classification of legislative party systems throws light on why this might be.

<i>Cabinet type</i>	<i>B*</i> <i>System</i> <i>dominant</i> <i>party</i>	<i>B'</i> <i>Strongly</i> <i>dominant</i> <i>party</i>	<i>C</i> <i>Top</i> <i>three</i>	<i>D</i> <i>Top</i> <i>two</i>	<i>E</i> <i>Open</i>	<i>Total</i>
MWC	24	68	48	26	28	194
Single party minority	29	62	7	16	5	119
Minority coalition	3	29	3	33	21	89
Surplus	4	32	1	42	38	117
<i>Total</i>	60	191	59	117	92	519

Table 6. *Types of government forming from minority settings in Europe, 1945-2010.*

First, note that Type B* and Type B' party systems are strongly associated with minority governments. Over half of real parliaments with a system-dominant party, and nearly half of those with a strongly-dominant party, generate minority governments, typically comprising the single largest party. Without getting into fine print of any particular model of government formation, this reflects the plain fact that few winning coalitions exclude system-dominant parties in particular, and strongly-dominant parties more generally. As other (modeled or unmodeled) constraints are brought to bear on government formation – squalid personal animosities, lofty policy disagreements, or anything in between – it can quickly

³² This includes all governments, not just those forming immediately after an election.

³³ We have specified type B systems as supersets of type B* systems. In this table and all that follow, however, we create an exclusive and exhaustive partition of systems by dividing type B into types B* and B'. Type B' is a type B legislature that is not B*.

happen that all winning coalitions excluding the dominant party become infeasible for one reason or another. This leaves the dominant party able to form a minority government because no feasible winning coalition agrees on an alternative.

Turning to surplus majority cabinets, Table 6 shows these are strongly associated with the Type D and Type E party systems³⁴ which, as we have seen, tend to sustain many more possible winning coalitions. If we assume that uncertainty about which coalition deals might or might not work increases with the number of winning coalitions, such uncertainty is much higher in the relatively unconstrained Type D and E party systems. The prevalence of surplus majority coalitions in these thus comports with the folk wisdom that surplus majority governments are responses to high levels of uncertainty whereby politicians insure against future intra-coalition disagreements by taking on surplus members.

Overall, the striking patterns in Table 6 are that: Type B' and B* systems dominated by the largest party are associated with minority cabinets; “three-pivotal-party” negotiations in Type C systems are associated with minimal winning coalitions; and the less constrained and arguably more uncertain negotiations found in Type D and E systems are associated with surplus majority cabinets.

Types of party system and typical government durations

Once a government has taken office in a parliamentary democracy, a key question concerns how long it will last, in a setting where any government can at any time resign or be dismissed by a majority vote of no confidence. There is a substantial political science literature on government stability and it is not feasible to review or extend this here (Diermeier and Stevenson 1999, 2000; King et al. 1990; Laver and Shepsle 1998; Lupia and Strom 1995; Warwick 1994; Browne et al. 1986). Our conjecture, in the context of this literature, is that governments should tend to last longer in the most constrained Type A and Type B* systems, and less long in Type E systems where the number of winning alternatives to any incumbent government is highest. Table 7 shows this to be true in aggregate terms for post-war European governments, whether these are formed immediately after elections or during the inter-electoral period following the exit of an incumbent.

³⁴ With 36% and 41% of the relevant cases.

<i>Type of legislative party system</i>	<i>Post- election</i>	<i>Inter- election</i>	<i>All cabinets</i>
A: Single majority party	1082 <i>59</i>	552 <i>61</i>	891 <i>51</i>
B*: System dominant party	942 <i>71</i>	509 <i>74</i>	786 <i>59</i>
B': Strongly dominant party	831 <i>52</i>	451 <i>36</i>	652 <i>35</i>
C: Top-three system	987 <i>91</i>	425 <i>85</i>	775 <i>74</i>
D: Top-two system	929 <i>55</i>	346 <i>41</i>	676 <i>45</i>
E: Open system	695 <i>77</i>	289 <i>31</i>	455 <i>41</i>
All formations	909 <i>27</i>	414 <i>20</i>	688 <i>20</i>
Minimal winning cabinets	1034 <i>43</i>	528 <i>48</i>	875 <i>37</i>
Single-party minority cabinets	735 <i>57</i>	373 <i>42</i>	568 <i>40</i>
Minority coalition cabinets	659 <i>78</i>	315 <i>41</i>	451 <i>43</i>
Surplus majority cabinets	774 <i>58</i>	414 <i>37</i>	587 <i>36</i>
Non-CEE	936 <i>29</i>	431 <i>24</i>	726 <i>23</i>
CEE	761 <i>63</i>	362 <i>31</i>	534 <i>40</i>

Table 7. *Mean government durations, in days, by type of party system and cabinet.*
Standard errors in italics.

Moving beyond a simple table, we deploy the Cox proportional hazards approach used above, taking account of key findings in the government termination literature. First, government durations are treated as “censored” if they are brought to an “artificial” end by a scheduled election. The data show a big spike in durations at about 1400 days, given a typical constitutional inter-election period of four years. Accordingly, government durations over 1350 days are treated as censored. Second, governments forming between elections have lower potential durations than governments forming immediately after elections, while governments formed mid-term are negotiated in settings where a previous equilibrium

cabinet has been destabilized for some unmodeled reason. We therefore consider only governments forming immediately after an election.³⁵ The empirical work cited above shows that the type of coalition cabinet in a minority setting has a significant bearing on its expected duration, as does the “complexity” of the bargaining environment in which it is set. Our types of legislative party system capture the complexity of the bargaining environment, but the stripped down benchmark model uses the number of legislative parties to measure this.³⁶ Table 7 clearly shows that the key distinction in relation to cabinet types is between minimal winning cabinets and others, be they minority or surplus majority administrations. Accordingly, we control for cabinet type using a binary variable for whether the cabinet is minimal winning. Finally, we already assumed more uncertainty in the relatively new party systems of the post-communist CEE, and Table 7 confirms that governments tend to last less long in CEE countries, so we include a binary control for whether the cabinet was in a CEE country.

Table 8 reports Cox proportional hazard estimates for three models of durations of governments formed after elections in postwar Europe. Model 1 is a stripped-down benchmark, using the absolute number of legislative parties to measure the complexity of the bargaining environment, an MWC dummy to control for cabinet type, and a CEE dummy to identify less-established post-communist party systems. Increasing the number of legislative parties, hence the number of possible legislative coalitions, significantly increases the hazard of a government termination, as does the fact that the cabinet is in a CEE country. Minimal winning coalitions are estimated to have lower probabilities of termination, holding other factors constant, though this coefficient is not statistically significant.

Model 2 adds binary variables for type of party system, treating the least stable Type E system as the baseline type in minority settings. Proportional hazards estimates for these are all significant and negative, showing that each party system type is associated with a lower hazard rate (cabinets of longer duration) than those in Type E. As Table 7 suggests, the big difference in cabinet durations is between cabinets forming in Type E, open, systems and the rest. Model 3 adds a full set of country fixed effects, and shows that the lower hazard rates of cabinets in non-Type E systems are robust to this.³⁷

³⁵ Diermeier and Stevenson (1999, 2000) take a different approach to the same, measuring the competing risks of scheduled and unscheduled terminations. Both approaches share the view that it is the unscheduled terminations that convey more information.

³⁶ Previous scholars typically use the effective number of parties in this context but, for reasons noted above, we feel the absolute number of parties, which has a direct and exponential effect on the number of possible coalitions, is a better measure of complexity.

³⁷ Finland, the country with mean durations closest to the overall mean, is the excluded category.

	Model 1	Model 2	Model 3 (country fixed effects)
Number of parties	0.17** (0.06)	0.02 (0.07)	-0.32* (0.16)
CEU country	1.21** (0.29)	0.93** (0.36)	1.55* (0.76)
Minimal winning coalition	-0.44 (0.24)	-0.40 (0.24)	-0.35 (0.39)
B*: System-dominant party		-1.32* (0.67)	-4.03* (1.62)
B: Strongly-dominant party		-1.33** (0.44)	-3.57** (1.32)
C: Top-three system		-2.03** (0.66)	-4.71** (1.50)
D: Top-two system		-0.82* (0.38)	-3.08* (1.28)
Log likelihood	-213	-209	-173
Observations	279	279	279

Table 8. *Cox proportional hazards models of post-electoral cabinet durations in European minority settings, 1945-2010.*

CONCLUSIONS

Despite the vast number of theoretically possible seat distributions that could arise after any legislative election in a multiparty system, legislative party systems fall into a much smaller number of theoretically relevant equivalence classes. We exploit this to generate a mutually exclusive and exhaustive partition of the universe of possible seat distributions into five fundamental “types” of legislative party system (Figure 1). We show that these types differ from one another in theoretically significant ways. For example, in a Type B system with a dominant party, the largest party, and only the largest party, is a member of every two-party winning coalition. In a Type C system, no party outside the largest three is pivotal in any winning coalition. There is no two-party winning coalition in a Type E system, the only type

of party system not subject to the arithmetical constraints we identify, and which must comprise at least five parties.

We classify postwar European party systems and show that regions of the “party system space” close to critical boundary conditions between types are densely populated (Figure 2). Any legislative election is subject to stochastic processes, so that the result is in effect a random draw from a distribution of expected seat distributions. If this distribution straddles a key boundary condition, as Figure 2 implies it often does, different random draws from the same underlying distribution may well flip the resulting real party system from one state to another with theoretically critical effects. For example, as party systems flip stochastically into and out of Type C, a set of parties outside the top three flip into and out of a situation in which they are pivotal in winning coalitions, with substantial consequences for legislative bargaining. We also show that our exclusive and exhaustive partition of legislative party systems is of more than hypothetical interest. Differences between types of party system have substantial effects on: how long it takes to form a government (Tables 4 and 5); the type of government that eventually forms (Table 6); and the typical duration of the government that does form (Tables 7 and 8).

Insights derived from our partitioning of legislative party systems are “model free”, logical implications of the basic arithmetic of legislative voting. They do not depend on utility functions of key agents. They apply whether legislators are motivated by perks of office, by public policy preferences, by spite envy and greed, or by anything else – provided they seek to realize these objectives by forming winning coalitions in the legislature. They apply no matter what detailed institutional structures exist to circumscribe legislation or structure government formation. Such institutions may make a huge difference, but the basic legislative arithmetic imposes its own severe constraints on what can happen, regardless of whether the constitution allows the President to nominate the Prime Minister (as in France) or stipulates (as in Greece) that party leaders lead government formation negotiations in strict order of party size. Notwithstanding such important institutional factors, the basic legislative arithmetic still applies. Proposals must still win legislative votes, and the constraints imposed by our boundary conditions still bite. While particular well-specified models of legislative bargaining and/or government formation may well *further constrain* the set of outcomes implied by the basic legislative arithmetic we set out above, they cannot *transcend* this.

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SUPPLEMENTARY MATERIAL

FURTHER IMPLICATIONS OF INEQUALITIES DEFINING LEGISLATIVE TYPES

Implication B4: The sizes of the three largest parties determine whether P_1 is strongly dominant (the size of any other party has no bearing on this). If two smaller parties, P_i and P_j , render P_1 strongly dominant, then P_2 and P_3 also render P_1 strongly dominant.³⁸ The inequalities $S_1 + S_3 \geq W$ and $S_2 + S_3 < W$ are necessary and sufficient conditions for P_1 to be strongly dominant.

Implication B5: If P_1 is strongly dominant, then $S_3 < W/2$.³⁹

Implication: C4: $S_2 \geq W/2$ is a necessary condition for a top-three legislative party system.⁴⁰ This is the third result focusing our attention on $W/2$.

Implication C5: If $S_1 + S_3 \geq W$ and $S_2 \geq W/2$, then $S_1 + S_2 + S_3 \geq 3W/2$. The top three parties must between them control one and a half times the winning threshold in a top-three system, which can therefore never arise when the winning quota is greater than two-thirds of total seats.

Implication D2: Since $S_1 + S_3 < W$, we know $S_3 < W/2$ and since $S_1 + S_2 \geq W$, we know $S_1 > W/2$ ⁴¹; indeed these are necessary conditions for a top-two party system.

PARTY SYSTEMS AND POLICY DECISIONS

Assume legislators vote on particular issues, and that possible positions on any given issue can be placed on a latent policy dimension. Assume that, for any issue under consideration, legislators have an ideal point on the latent dimension concerned, and a component of their utility function that declines monotonically as the policy agreed by the legislature moves further way from this. Differences between the types of party system set out above bear in striking ways upon policy outcomes that might emerge in such settings, because our boundary conditions impose different constraints on the identity of the party occupying the pivotal position on an arbitrary issue dimension – a dimension for which we are ignorant, *a priori*, of the ordering of party positions. First note that, if a party is pivotal to no legislative majority, it can never be in the pivotal position on any particular issue dimension.⁴² This is why our classification of legislative party systems bears directly on legislative voting on policy issues.

In Type B* systems the system-dominant party, while not winning on its own, can form a winning coalition with any other party. It must therefore occupy the pivotal position on any issue dimension for which there is a party on either side of it. Logically, this implies:

Implication B5*: A system-dominant party must be at the pivotal position on any issue dimension for which it is not at one of the most extreme party positions. If P^{**} is at the extreme of some issue dimension, then the pivotal party must be adjacent to P^{**} .⁴³ The a

³⁸ Since $S_2 \geq S_3 \geq S_i \geq S_j$, if the first two conditions strong dominance hold for S_i and S_j , they hold *a fortiori* for S_2 and S_3 . To see that the third condition also holds, note that if $P_i P_j$ is winning then its complement $(P_i P_j)'$ is losing. For any $j > 3$, $P_2 P_3$ is a subset of $(P_i P_j)'$ and thus $S_2 + S_3 < W$. Thus, if the defining inequalities of strong dominance are fulfilled for any P_1 , P_i and P_j , they are fulfilled for P_1 , P_2 and P_3 .

³⁹ If $S_2 + S_3 < W$ and $S_2 \geq S_3$, then $S_3 < W/2$.

⁴⁰ $S_2 + S_3 \geq W$, implies $S_2 \geq W/2$, since $S_2 \geq S_3$.

⁴¹ Since $S_1 \geq S_2 \geq S_3$

⁴² Note also that, taking at set of issue dimensions together and treating these as a multidimensional issue space, parties may occupy strategically important locations by virtue solely of their issue positions. However, leaving aside the possibility of log-rolling, when legislatures dispose of one issue at a time it remains true that a party pivotal to no legislative majority can never be pivotal on any issue dimension under consideration.

⁴³ Since P_1^{**} can form a winning coalition with any other party

priori probability that a system dominant party in an n -party system is pivotal on some arbitrary issue dimension under consideration by the legislature is therefore $(n-2)/n$.

Even when there is an indefinite number of unknown issue dimensions that might form the basis of legislative decisions, therefore, the pivotal party on any issue is either the system-dominant party or the party adjacent to it, regardless of the positions of all other parties. A system dominant party therefore has substantial control over legislative policy outputs.

In Type B systems a strongly-dominant party, P^* , can form majority coalitions with both P_2 and P_3 , which implies:

Implication B5: If P_2 and P_3 are on opposite sides of P^* on some issue dimension, then P^* is at the pivotal position, regardless of the positions of all other parties.

This gives a P^* a somewhat privileged position in affecting legislative policy outputs, though clearly less than that enjoyed by a P^{**} . In Type C, top-three systems, no party outside the top three can be pivotal, so the pivotal party on any conceivable policy dimension must be to one of the three largest parties. Logically, this implies:

Implication C5: The pivotal party on any issue dimension must be the most central of the top three, regardless of the issue positions of the smaller parties.⁴⁴

In Type D, “top-two”, party systems, it follows logically that:

Implication D2: The pivotal party on any issue dimension must be located on the interval between P_1 and P_2 , regardless of the positions of smaller parties.⁴⁵

This is much less a constraint on the location of the pivotal party on an arbitrary issue dimension than in the previous three settings. Indeed if P_1 and P_2 are at opposite ends of some issue dimension, it is no constraint at all. In Type E “open” systems, the defining inequality, $S_1 + S_2 < W$, implies that that all two-party coalitions are losing. This imposes no constraint of substance on the location of the pivotal party on an arbitrary issue dimension.

The results set out above highlight a stronger relationship than might hitherto have been suspected between constant sum bargaining over a fixed set of perquisites and variable sum bargaining over policy. The reason for this is that *the identity of the pivotal party on an arbitrary policy dimension in a weighted voting game is determined as much if not more by party sizes as by party policy positions*. One consequence of this is that the normalized Shapley value, typically seen as applying to constant sum bargaining over a fixed cake, has a precise interpretation in terms of variable sum legislative bargaining over policy. The normalized Shapley value of party P is the proportion of all orderings of coalition formation in which P is pivotal. This means that it is also the proportion of all orderings on an arbitrary policy dimension in which P is pivotal. The Shapley value of party P , therefore, is precisely the probability that P is pivotal on an arbitrary policy dimension. In this sense, the Shapley value has an intuitively meaningful interpretation in terms of legislative bargaining over public policy.

SIMULATIONS OF SHOCKS TO SEAT DISTRIBUTIONS

Here we provide additional detail about the simulation of seat allocations as realizations of a stochastic process, as described in the section of our paper “Fragility of Legislative States.” We also provide supporting evidence for some of the assumptions behind our approach, especially the size of the unexplainable variance in seat allocations, in case the reader is not convinced that the actual variances would be as large as that specified by the multinomial distribution.

⁴⁴ For any top-three party that is not the most central on some issue dimension, there must be a winning coalition of the two other top-three parties, either to the right or the left of it. Therefore the non-central top-three party cannot be pivotal on this dimension.

⁴⁵ Since P_1P_2 is a winning coalition, the pivotal party on any issue dimension cannot be either to the left or to the right of both P_1 and P_2 .

Simulating outcomes according to the multinomial distribution assumes that the expected proportions of seat shares are fixed, but that each election represents a stochastic draw from this distribution. The fixed component can be thought of as a vector of seat proportions that is determined by a variety of political and contextual factors, some specific to parties, others to electoral system, political climate, and so on. Nonetheless, this set of fixed proportions forms only the expectation of a given seat allocation, from a stochastic (i.e. not deterministic) process. What are the sources of this variation? Random differences will come from a variety of factors that cannot be fully controlled, such as small variations in turnout, the unpredictable decisions of voters making up their minds at the last minute, or even votes cast by mistake. If a computer disaster were to strike following a day of voting, before any results were tabulated, and an embarrassed government were to ask the voters to turnout again the next day, we would not expect an *identical* outcome, although we would expect the party mean vote shares to be the same. The realization of these small differences in outcomes will be magnified by electoral systems that permit differences in votes to affect seat allocations in multiple contests. According to Borman and Golder (2013), of the 1173 electoral systems they surveyed from 1946-2000, the median number of districts in an electoral system was 26, with over 22% also possessing upper tiers for more complex (mixed-level) allocations. Another 11% of these mixed majoritarian and proportional rules. The complexity of these systems means that small differences at a local level can produce shifts that when aggregated to national results, cause the shift of a few seats from one party to another. If an election were repeated under slightly different conditions, perhaps under different weather, or perhaps on the next day, we posit, the results would reflect these slight shifts, even though the systematic factors explaining most of the outcome would remain unchanged.

The multinomial distribution as a model of seat outcomes

Because legislative sizes are fixed, seats (and seat shares) are compositional data. Since our objective is to model the process according to a stochastic draw from vector of expected seat proportions, we employ the multinomial distribution, a distribution that produces expected integer counts given a set of proportions and a total number of “trials” (in our case, seats to be awarded).

A feature of the multinomial distribution is that the variance of each outcome is linked to the proportion, through $\text{Var}(s_i) = p_i(1 - p_i)n$ where p_i refers to the proportion of seats expected for party i , and n refers to the total number of seats in the legislature that will be allocated. This means that as a party’s seat shares approach 0.5, variance is maximized, but also that the variance of the outcome increases as the size of the legislature increases. Both ideas are plausible: parties with greater numbers of expected seats will have higher possible numbers of seats reallocated randomly with each election, and a difference of five seats is much more plausible for a party expecting 100 seats than a party expecting 10. Similarly, the greater the total number of seats, the larger we would expect the effect of random shocks to have. A difference of 2 seats for a party can be treated as noise in the 650-member British parliament, for instance, yet would precipitate a fundamental political crisis if it occurred in the seven-member Swiss federal council.

Empirical calibration of simulation variances

A valid question is whether the multinomial variance assumption is an accurate way of model the random, unexplained variance that can result in different seat allocations without the fundamental political circumstances being changed. We test the robustness of this assumption by looking at some empirical evidence on predicted seats shares in the post-war period. To test this we draw on seats and votes data from the Manifesto Project, supplemented with electoral system data from Bormann and Golder (2013). As a test of the relationship of the percentage of the vote to the size of the expected random component, we fit a model predicting seat shares as a function of the interaction of the percentage of the vote, the total seats in the legislature, and the log of average district magnitude, a model that predicts nearly 94% of the variance in absolute seat allocations.

Variable	Estimate	Std. Error
Intercept	1.6600	1.33700
% Vote	**-.0.1409	0.05352
Total Seats in the Legislature	**-.0.02793	0.00415
log(District Magnitude)	-0.3988	0.48210
% Votes x Total Seats	**0.01252	0.00019
% Votes x log(District Magn.)	0.0204	0.02278
Total Seats x log(District Magn.)	**0.006388	0.00172
Tot. Seats x % Votes x log(District Magn.)	**-.0.0003747	0.00009
R^2		0.9357
sigma-hat		13.14
N		2674

Table A1. *Predictive Model of Seat Allocations given electoral size, total seats, and the log of average district magnitude.* Data from Borman and Golder (2013) for electoral system data and the Manifesto Project (2012) for seats and votes data.

Ideally, we would use the post-war electoral data to “calibrate” a model of unexplained variance for each election. This relies heavily on assumptions, however, since we never observe repetitions of an election under identical political conditions, but rather observe different election outcomes—usually separated by years—under different conditions. To justify our simulations using the multinomial variance, however, we need simply to provide evidence that the multinomial variance is conservative: that the real variances are likely to be larger. From Table A1, we computed a predictive confidence interval for SYRIZA, the second-largest party from the May 2012 Greek election, which from the data had 16.79% of the vote, had a log district magnitude of $\log(5.134)$, and a total size of 300 seats. Compared to actual seats won of 52 seats, our model predicted 53.96 seats—very close—with a 95% confidence interval of 28.2 to 79.7. Compared to this predictive interval, the multinomial variance seems quite conservative. It is also easy to show that generally, the estimated σ^2 of $13.14^2 = 172.66$ is greater than the maximum multinomial variances of $.5(1-.5)300 = 75$.

However, the linear model in Table A1 is only an approximation, and part of our argument for using the multinomial variance is that a greater range of deviations will occur when legislative sizes and party vote shares are larger. The relationship of the predicted vote to the residuals to the party’s predicted share of seats is plotted in Figure A1. Not only does the variance in the prediction of seats expand greatly as the party’s expected share of seats increases, but so does the size of the absolute residuals; differences of 10-20 seats are not at all uncommon, even for small seat shares. This suggests that even in a model which can predict 94% of the seat allocations in the post-war period given a fixed vector of expected seat shares and some basic variables on the size of the legislature and rules governing their allocation, there is an unexplained component that will cause results to vary even when these fundamental conditions are controlled for.

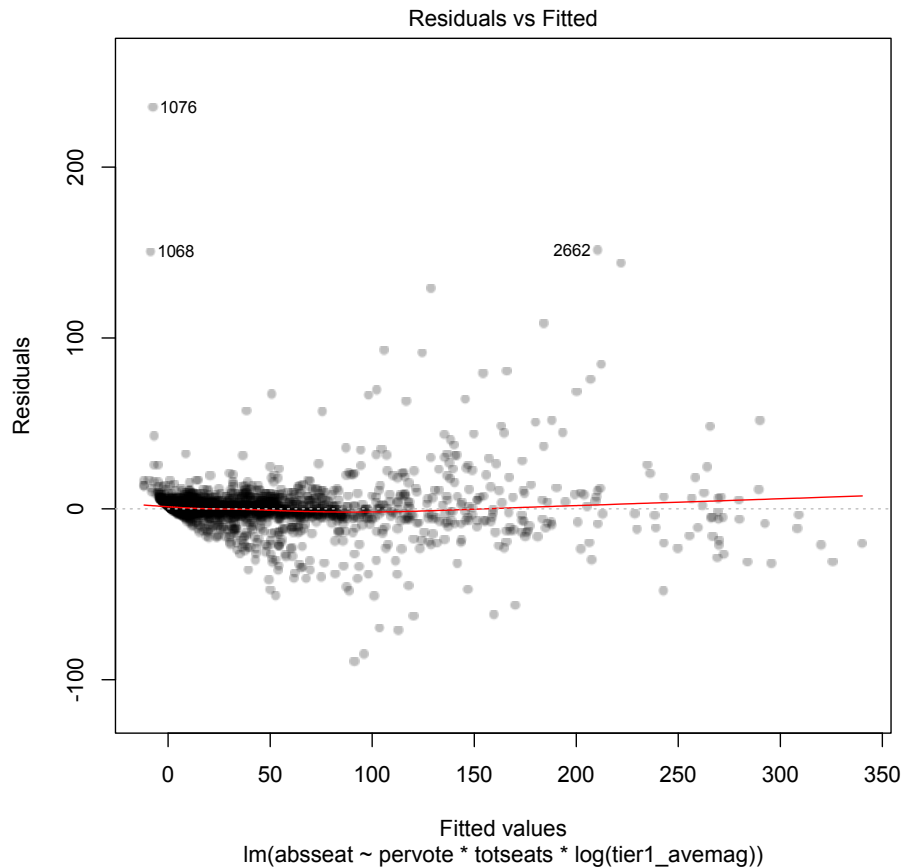


Figure A1. *Residual v. Fitted Plot, in terms of expected versus actual seats from Table A1.*

Specific illustrations using recent elections

To take the first of two concrete examples, consider the Greek elections of May 2012. These resulted in a Type D party system with 108, 52, 41, 33, 26, 21, and 19 seats held by seven parties. From our simulations given this distribution of legislative seat shares, this could have been realized as set of slightly different outcomes, resulting in different legislative types – for example, among many others:

D	104	56	43	43	20	13	21
B	112	52	51	29	29	13	14
B	109	48	48	32	25	21	17
B	113	50	39	33	37	16	12
E	102	44	46	36	34	20	18

The same election could plausibly have realized a Type D, B, or even a Type E party system – each with very different downstream political implications. In our simulations of the uncertainty around this particular observed outcome in Greece, a Type D party system was realized in only about 39% of simulated cases, with a Type B system being the most likely (52%) outcome. We estimated a very low probability that a Type B* or E system would have been realized. Our simulations of the “fragility” of the May 2012 realized outcome in Greece outcome are consistent with the argument that

the June 2012 rerun of this election was in effect another random draw from the same stochastic process of seat distribution.

In a second example, consider the Italian elections which took place in February 2013. Despite an electoral law designed to manufacture majorities by automatically awarding 55% of the seats to the largest party, this election failed to result in a majority party in the Senate, where this bonus mechanism operates only within each of 18 constituencies rather than on the national level. The two main coalitions—a centre-left coalition headed by Bersani and a centre-right coalition led by Berlusconi—polled 31.6 and 30.7 percent of the nationwide Senate vote respectively. The median difference in these two coalitions' vote shares across the Senate constituencies was 0.5%, often just a few percentage points, or even fractions of a point, separating them. In the large constituency of Piedmont, with 22 seats to be allocated, the Bersani coalition took 13 seats because it polled first place in votes, although only by 0.5% of the vote ahead of its rival (which took only four seats). It would not be implausible to model the differences as normally distributed, even unconditionally, as shown in Figure A2.

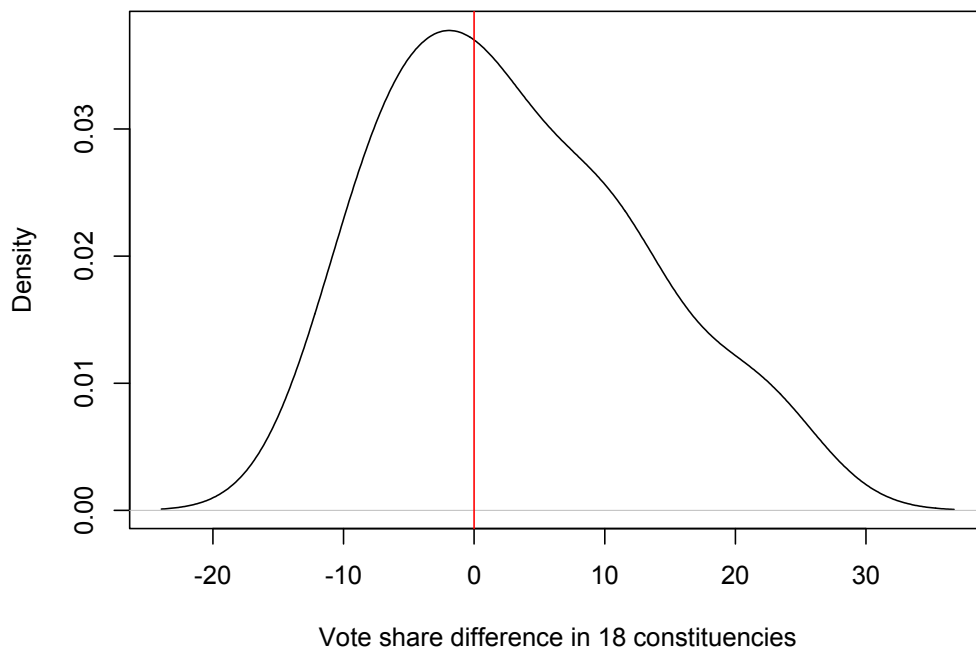


Figure A2. *Density plot of vote share differences in vote shares of the two largest coalitions in 18 multimember Senate constituencies from the Italian elections of February 2013.*

Even if we account for systematic factors, which we have not done here but would be entirely consistent with other election models that predict or forecast vote constituency-level shares (e.g. Linzer 2013), we would still expect a residual with a stochastic distribution with a non-zero variance.

Examining the seat allocation at the national level, Italy's 2013 election to the Senate produced a Type C system⁴⁶. Had an additional 13 seats (of 215) changed hands, however, Italy would have had a Type B system, possibly avoiding the political paralysis that resulted from its result, one that left Italy without a government for two months. While the Italy's electoral rules make the differences in small shocks exaggerated, Italy is hardly alone in having electoral systems where small, effectively random differences at the constituency level can be expected to shift a small yet highly consequential number of seats between parties in hypothetical repetitions of an election.

⁴⁶ The exact seat totals were: Italy: Common Good (Bersani): 123; Centre-right (Berlusconi): 117; Five-Star Movement (Grillo): 54; With Monti for Italy (Monti): 19; and two seats held by regional parties.

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