Eddie Gerba, Emmanuel V. Pikoulakis

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Working paper

Explaining Cross-Country Differences in Productivity: Is it Efficiency or Factor Endowments?

Eddie Gerba

Research Fellow

London School of Economics (LSE)

Emmanuel V. Pikoulakis

Hull University Business School (HUBS)

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In this paper we develop a two-sector growth model of optimizing agents and apply this model to the data for the purpose of addressing the two interrelated questions that preoccupy the literature on development and growth accounting, namely: (1) What determines sustained growth and (2) What explains the vast cross-country differences in labor productivity. Concerning the first questions our findings support the view that to some extend the growth in effective human capital is a by-product of learning –by-doing. On the second question we find that differences in factors of production explain twice as much of the difference in labor productivity between developed and developing countries than differences in efficiency.

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Explaining Cross-Country Differences in Productivity

I. Introduction

The Caselli (2005) suggestion that "Chipping Away" at the Solow residual may prove a promising strategy to explaining away cross-country differences in income proved too irresistible to ignore. Our chipping away strategy has been to create a two-sector growth model of optimizing agents that retains some of the key features of growth and development to be found in the literature and apply this model to the data. As we illustrate, the data is shown to be consistent with three hypotheses. Firstly, that knowledge creation is, to some extent, a byproduct of newly created physical capital. Secondly, that knowledge creation requires part of the labor force to be allocated to sectors that contribute to knowledge. And, thirdly, that the quality of education plays an important role in explaining differences in productivity and growth. Our findings suggest that differences in the endowment of factors of production explain as much as 70% of the difference in labor productivity between rich and poor. Like De Michelis, Estevao, and Wilson (2013), our findings also suggest that there are circumstances where there is a trade-off between TFP and employment. In particular, when unemployment and quality of education rise (fall) *together* labor productivity also rises (falls).

The structure of this paper is as follows: Sections II-VI develop the micro-foundations of a two-sector version of a growth model. In Section 6 we sketch out an adaptation of the model to the study of the evolution of effective human capital and its implications on productivity differences. In Section VII we apply the model to the data and discuss in detail our findings. Section VIII concludes the main body of the paper. The description and sources of the data together with the statistical properties of the estimated relations are shown in Appendix A. Appendix B develops and discusses the dynamics of the theoretical model presented in Sections II-VI.

II. Production in a Two-Sector Growth Model

The model of growth we propose to present resembles a two-sector version of optimizing behavior of the Ramsey-Cass-Koopmans variety. The economy consists of a final-goods sector and an intermediate-goods sector. Every period the final-goods sector produces Y units of the final product, the intermediate -goods sector uses I units of Y to install newly created capital whilst the remainder $Y - I = C$ units are used for consumption.

II.1 Modeling Production in the Final-Goods Sector

To describe production in the final –goods sector we shall let K denote the stock of physical capital, L the labor force, v the fraction of the labor force employed in the final –goods sector, N the number of workers employed in that sector, A the level of labor-augmenting technical progress, and h the level of skills embodied in each member of the labor force. Assuming Y exhibits constant returns to scale in physical capital and in effective human capital, AhN, we shall write:

$$Y = (AhN)^{1-\alpha} K^{a} = (AhN)(K / AhN)^{a} = (AhvL)(K / AhvL)^{\alpha} = (AhvL)^{1-\alpha} K^{\alpha} \tag{1}$$

Letting $\tilde{y} \equiv (Y / AhL)$, and $\tilde{k} \equiv (K / AhL)$ define output and capital in intensive form, the production of output in intensive form shall be described by:

$$(Y / AhN) = (\tilde{y} / v) = (\tilde{k} / v)^{\alpha} = (K / AhN)^{\alpha} \tag{2}$$

II. 2 Modeling Production in the Intermediate-Goods Sector

Mussa (1977) defines the total marginal cost (TMC) of capital accumulation in terms of foregone consumption to include two components: (a) "the internal marginal cost of investment incurred directly by consumption goods producers" and (b) "the marginal cost of diverting labor into capital goods production". He goes on to say that the latter marginal cost “may be thought of as "external" to the individual consumption goods producer, even though it is clearly internal to the economy."

Each period, the intermediate-goods sector transforms I units of the final good into I units of installed capital-this is the "internal" marginal cost of investment. By assumption this sector employs only labor- this is the "external" marginal cost of investment. Observing that $(1-v)$ measures the fraction of the labor force employed in the intermediate-goods sector and letting $(1/\sigma)$ denote a vector of time –invariant variables to describe the productivity of $(1-v)$, we shall model gross capital formation as follows:

$$\{(1-v)/\sigma\} = [(\tilde{y}-\tilde{c})/\tilde{k}] = (I/K) \tag{3}$$

A much broader definition of non-leisure time spent in installing capital would include time spent in all types of homework activities that are akin to installing such capital. An even more encompassing definition of the intermediate-goods sector would include activities akin to creating additional human capital.

III. The Command Optimum

III.1 The Objective of the Social Planner

Letting $\tilde{c} \equiv C/AhL$ denote consumption per effective labor force unit, $U(\tilde{c})$ the instantaneous utility of the representative agent who is infinitely lived, and ρ her rate of time preference, we shall describe the objective of the central planner as follows:

Maximize $$\int_0^\infty U(\tilde{c})e^{-\rho t}dt \tag{4}$$

Subject to:

$$(\partial\tilde{k}/\partial t) = (\tilde{y}-\tilde{c}) - (g+n+\delta)\tilde{k}\ , \tag{5a}$$

$$(1-v) = \sigma[(\tilde{y}-\tilde{c})/\tilde{k}] = \sigma(I/K) \quad (5b)$$

Where: $\tilde{y} = v^{1-\alpha}\tilde{k}^{a}$, $\tilde{k}_0$: is given, and: $0 \le \tilde{c} \le \tilde{y}$

III.2 Solving for the Equilibrium Allocation of Non-Leisure Time

Letting Λ_C denote the Current Value Lagrangian and Λ the Langrangian we shall write:

$$\Lambda_c = U(\tilde{c}) + m_1\{(\tilde{y}-\tilde{c}) - (g+n+\delta)\tilde{k}\} + m_2\{(1-v) - \sigma[(\tilde{y}-\tilde{c})/\tilde{k}]\} = \Lambda e^{\rho t} \quad (6)$$

The first-order conditions for an optimum require setting all the partial derivatives of Λ_C equal to zero and solving simultaneously. Accordingly:

Setting: $(\partial\Lambda c/\partial\tilde{c}) = 0$, $(\partial\Lambda c/\partial v) = 0$,

and combining results, we arrive at:

$$U^{/}(\tilde{c}) = m_1 - m_2(\sigma/\tilde{k}) = m_1 - (\partial\tilde{y}/\partial v)U^{/}(\tilde{c})(\sigma/\tilde{k}) \quad (7)$$

Re-arranging (7) yields:

$$U^{/}(\tilde{c})\{1 + (\partial\tilde{y}/\partial v)(\sigma/\tilde{k})\} = m_1 \quad (8)$$

Setting $(\partial\Lambda c/\partial m_2) = 0$, we recover (3). To complete the solution we must observe the equation of motion of the co-state variable given by:

$$(\partial m_1/\partial t) = \rho m_1 - (\partial\Lambda_c/\partial\tilde{k}) \quad (9)$$

In the above relations m_1 is the price of newly created capital measured in utility whereas $\{m_1/U^{/}(\tilde{c})\} = \{1 + (\partial\tilde{y}/\partial v)(\sigma/\tilde{k})\}$ is the price of (gross) investment measured in consumption per effective unit of labor. Letting $q-1$ denote the marginal cost of gross investment in units of consumption per effective unit of labor we have:

$$q = \{1 + (\partial\widetilde{y} / \partial v)(\sigma / \widetilde{k})\} = \{m_1 / U^{/}(\widetilde{c})\} \tag{10}$$

Manipulating (10) we arrive at:

$$\begin{aligned} q-1 &= (\partial\widetilde{y} / \partial v)(\sigma / \widetilde{k}) = ((\partial\widetilde{y} / \partial v)v) / v))(\sigma / \widetilde{k}) = (1-\alpha)(\widetilde{y} / \widetilde{k})(\sigma / v) = \\ &= (1-\alpha)(Y / I)(I / K)(\sigma / v) = [\{(1-v) / v\}][\{(\omega N) / I\}] \end{aligned} \tag{11}$$

Where ω is the real wage rate set equal to the marginal product of N. Re-arranging (11) we arrive at:

$$(q-1)I = (1-v)\omega(N / v) = (1-v)\omega L \tag{12}$$

As (12) makes clear, *in equilibrium, the value added by the intermediate-goods sector must equal the opportunity cost of allocating non-leisure time to that sector.* Using (3) to rearrange (12) we get:

$$\begin{aligned} (1-v) &= \frac{(q-1)I}{\omega L} = \frac{(q-1)I}{\{(\omega N) / v\}} = \frac{v(q-1)I}{(1-\alpha)Y} = \\ &= \frac{v(q-1)(I / K)(K / Y)}{(1-\alpha)} = \frac{v(q-1)(1-v)(1/\sigma)(K / Y)}{(1-\alpha)} \end{aligned} \tag{13}$$

Solving for v we arrive at:

$$v = \frac{\sigma(1-\alpha)(Y / K)}{(q-1)} \tag{14}$$

The solution for v expressed by (14) may be seen as the outcome of behavior that reflects the workings of an *intertemporal* as well as an *intratemporal* substitution of labor supply: A rise (fall) in $[(Y / (q-1)K]$ -taken to correspond to a rise (fall) in the interest rate -will induce some workers to raise (reduce) the non-leisure time they allocate to the market sector as a whole-this is the intertemporal substitution effect. If the intermediate – goods sector were broadly defined to include activities akin to installing capital in the household sector, the rise (fall) in $[(Y/(q-1)K]$ must be associated with a rise (fall) in the share of labor employed in the final -goods sector at the expense of the intermediate- goods sector– this is the intratemporal substitution effect.

III.3 Equilibrium in the Market for Assets

Applying some algebra to the system expressed by (6)-(9) we arrive at the condition that describes equilibrium in the market for assets as follows:

$$(\dot{q}/q) + (\pi_K/q) = \gamma(\dot{\tilde{c}}/\tilde{c}) + \rho \tag{15}$$

In (15) above, γ is the elasticity of the marginal utility of consumption and π_K is the net profit per unit of capital defined by:

$$\pi_K = \alpha(Y/K) - (n + \delta + g) \tag{16}$$

Where n is the rate of population growth, δ is the depreciation rate of physical capital, and g is the steady-state growth rate of the economy.

Letting r denote the rate of interest we have,

$$r = \gamma(\dot{c}/c) + \rho = \gamma\{(\dot{\tilde{c}}/\tilde{c}) + g\} + \rho = (\dot{q}/q) + (\pi_K/q) + \gamma g \tag{17}$$

At the steady-state path: $(\dot{\tilde{c}}/\tilde{c}) = (\dot{q}/q) = 0$. Therefore, along this path:

$$(\pi_K / q) = \rho\,, \tag{18}$$

and:

$$\{\alpha(Y/K) - (n + \delta + g)\}/\rho = q \tag{19}$$

As (19) makes clear, steady-state earnings per unit capital exceed investment per unit capital and therefore this model economy satisfies the condition for dynamic efficiency.

IV. The Dynamics of the Model

In what follows we shall be making the rather simplifying assumption that agents' expectations follow a perfect- myopic foresight path along which $(\dot{\tilde{c}} / \tilde{c}) = 0$. This rather special assumption will allow for an analytically tractable presentation of the model's dynamics in the two-dimensional space defined by q and $\tilde{k}$. As it turns out the derivation of the equations of motion that describe the evolution of q and $\tilde{k}$ is rather tedious. Therefore, in this section, we shall confine ourselves to simply reporting the relevant specifications and shall delegate the full description of the model's dynamics to Appendix B.

The required equations of motion read as follows:

$$\dot{\tilde{k}} = [(1/\sigma) - \{(1-\alpha)/(q-1)\}\{(q-1)/\sigma(1-\alpha)\}^{-\{(1-\alpha)/\alpha}\tilde{k}^{-\{1-\alpha)/\alpha}\} - (n + \delta + g)]\tilde{k} \tag{20}$$

$$\dot{q} = \{\rho q + n + \delta + g\} - \alpha[\{(q-1)\tilde{k}\}/\{(\sigma(1-\alpha)\}]^{-((1-\alpha)/\alpha)} \tag{21}$$

Given that q is "forward" looking and $\tilde{k}$ is short-run predetermined stability requires that equilibrium is a local saddle- point.

V. Motivating Equilibrium Unemployment

Our purpose in this section is two-fold: Firstly, to further argue that an intermediate-goods sector broadly defined helps motivate an intratemporal substitution of labor effect, and secondly, to illustrate how such an effect can motivate voluntary/equilibrium unemployment.

As Rupert et al. (2000) report, omitting homework activity biases downwards the responsiveness of market hours to the real wage rate. To quote Benhabib *et al.* (1991): "In contrast to the standard model, which relies exclusively on intertemporal substitution, the home production model also includes intratemporal substitution between market work and homework at a given point in time. This makes the labor supply response in the home production economy more similar to that in the data. "

To illustrate how an intratemporal substitution of labor supply can motivate equilibrium unemployment imagine an economy dominated by small family establishments producing a marketable good. When market demand is low some family members may find it profitable to reallocate some of their time in favor of homework activities whilst others may take the opportunity to improve their skills through additional schooling and/or training. In this scenario periods of low market activity will be associated with labor hours being reallocated from the final- goods sector to the intermediate goods sector broadly defined to include the household sector. To the extent that homework is not included in the employment statistics, measured unemployment will rise – a rise that would overstate the degree of resource underutilization since those classified as unemployed spend their non-leisure time in homework activities which increase value added to the economy. These observations suggest the following conclusions: Firstly, when an agent takes time off from her market activities to invest in home capital and/or human capital measured unemployment is likely to rise. Secondly, to the extent that some of the value added by the unemployed is captured by the national accounts statistics, to that extent, variations in unemployment will be positively associated with the Solow residual. Thirdly, to the extent that unemployment above (below) its “natural” rate induces those in employment to exert more (less) effort, to that extent unemployment will be positively correlated with labor productivity, *other things equal*. Fourthly, the national accounts may tend to define the stock of capital too narrowly, as Barro and Sala-i-Martin (1999) seem to suggest. In a nutshell, we shall be assuming that fluctuations in $(1-v)$ are positively correlated with fluctuations in equilibrium unemployment and with labor productivity, *other things equal.* Finally, it is worth noting, that the motive to generate equilibrium unemployment can be further strengthened by introducing heterogeneity between workers.

VI. An Application to the Study of Growth and Development

VI.1 An Introduction to the Modeling of Effective Human Capital, Ah

Our primary purpose in this paper is to apply the model presented above to the study of some key issues that preoccupy the literature on growth and development. To do so we must model the evolution of effective human capital, Ah, and apply this model to the data. As we shall illustrate in the sections to follow, the evolution of Ah in the data is shown to be consistent with the following three hypotheses. Firstly, that the creation of knowledge is, to some extent, a byproduct of newly created physical capital. Secondly, that the creation of knowledge requires part of the labor force to be allocated to sectors that contribute to knowledge creation. Thirdly, that the quality of education plays an important role in explaining differences in productivity and growth.

To sketch out, step by step, how we arrived at our preferred specification for Ah we shall begin with a partial adjustment model that reads as follows:

$$\ln(Ah)_t = \beta \ln k_{t-1} + \gamma \ln(Ah)_{t-1} \quad , \beta > 0, \qquad 1 > \gamma > 0, \tag{22a}$$

Where $k \equiv (K/N)$ defines output per worker.

Moving terms one period forward and applying the Δ operator we shall write:

$$\Delta \ln(Ah)_{t+1} = \beta \ln k_t + (\gamma - 1)\ln(Ah)_t \tag{22b}$$

Solving for $\ln(Ah)_t$:

$$\ln(Ah)_t = (\beta / 1 - \gamma)\ln k_t - (1/1-\gamma)\Delta \ln(Ah)_{t+1} \tag{22c}$$

Dropping time subscripts, a parsimonious expression of the model we applied to the data can read as follows:

$$\ln Ah = \delta \ln k - \theta \Delta \ln Ah \quad \delta = (\beta / 1 - \gamma) \ , \ \theta = (1/1-\gamma) \tag{23}$$

Letting $\ln y \equiv \ln(Y/N)$ define the logarithm of output per worker, we shall embed (23) into (1) to write:

$$\ln y = (1-\alpha)\ln(Ah) + \alpha \ln k = \{(1-\alpha)\delta + \alpha\}\ln k - \theta(1-\alpha)\Delta \ln Ah \tag{24}$$

If it turns out that (24) provides a good empirical description of $\ln y$ we would be able to say that capital plays a much bigger role in explaining differences in labor productivity than conventional, development accounting models would suggest since $(1-\alpha)\delta + \alpha > \alpha$. A primary purpose of our empirical investigation is, therefore, to obtain parameter values for the relations described by (23)-(24).

VI.2 Modeling the Growth Rate of Ah

Sustained growth in output per worker requires sustained growth in Ah.This, in turn, begs the question: What are the forces that generate growth in Ah?

In models of the "learning by doing" variety, first developed in Arrow (1962), sustained growth can be a byproduct of physical capital accumulation. In models of the Uzawa-Lukas variety (see, Uzawa (1962) and Lukas (1988)) as well as in some R&D –based growth models, such growth comes from allocating a share of the labor force to a sector dedicated to knowledge creation/innovation. In our model the source of sustained growth is the intermediate-goods sector. If we were to confine the activities of the intermediate-goods sector to the installation of physical capital then growth in our model economy would be purely a by-product of capital accumulation. If, on the other hand, we were to embrace a very broad definition of the intermediate-goods sector and its activities, sustained growth can also arise as the deliberate outcome of allocating labor to knowledge creation activities.

To fix ideas about the model we propose for $\Delta \ln Ah$, let $\Delta \ln k$ define the growth rate of capital per worker and observe that long-run equilibrium requires that $\Delta \ln k = \Delta \ln A.h$. Observe, also, that $(I/K) \equiv \Delta \ln k + \delta + n$ and, that by (3): $(I/K) = \{(1-v)/\sigma\}$. *Accordingly, in modeling the growth rate of* Ah *we need to consider the modeling of* $\{(1-v)/\sigma\}$.

As we have already observed $(1/\sigma)$ can be taken to be a short-hand expression for a vector of variables measuring efficiency, $(1-v)$ can be taken to be positively correlated with variations in unemployment whilst variations in unemployment can be taken to be positively correlated with "effort" according to the tenets of the "efficient-wage-hypothesis". In addition we need to account for a possible interaction between unemployment and efficiency. To model efficiency we propose two variables: a variable to control for the effects of geography/climate on growth, and a variable to control for the quality of education. We shall elaborate on the choice of these variables immediately below.

The evidence provided by Hanushek and Woessmann (2009) and Schoellman (2012) clearly supports that view that a country's quality of education impacts significantly on labor productivity and its rate of growth. To this effect we shall let education quality be one of the determinants of $\Delta \ln Ah$. Regarding the view that differences in climate/geography can be a contributing factor to explaining differences in productivity/efficiency Minoiu and Pikoulakis (2008) report: "In developing countries particularly, geography affects health and, thus, the quality of effort per hour and the number of hours for which a worker is employed (see Bloom, Sachs, Collier, and Udry, 1998; Sachs, 2000, 2003, Hausmann,2001; Carstensen and Gundlach, 2006 on the relation between geography, disease vulnerability and output per worker)".

According to a convincing argument presented in Jones (2002, pp.127) the nearer (further) is a country to (from) the technology frontier the slower (faster) is its rate of skill accumulation. This argument would suggest that countries which are nearer to (further from) the tropics may have a faster (slower) rate of skill accumulation since these countries are usually thought to be further from (nearer to) the technology frontier. This argument supports further our decision to use climate/geography as a control variable because it implies that the role of climate/geography as a control variable is not confined only to issues concerning health. This would also suggest that in modeling $\Delta \ln Ah$ one should introduce an interaction term between the quality of education index and the geography/climate index to allow for an accurate contribution of these indexes to a country's ability to benefit from technology innovations.

As we already alluded to above, an important tenet of the efficient wage hypothesis is that workers' effort is positively correlated with the rate of unemployment, *all other things equal*. However all other things may not be equal if the effect of effort on productivity depends on education quality. Therefore, in modeling $\Delta \ln Ah$ we need to introduce an interaction term between the unemployment rate and the index measuring the quality of education.

To summarize, a potentially fruitful model for $\Delta \ln Ah$ may include variables such as: (1) the quality of education, (2) geography/climate, (3) equilibrium unemployment (4) an interaction between geography and education and (5) an interaction between unemployment and education. Such a model may be expressed as follows:

$$(\Delta \ln Ah)^* = \beta_1 eq + \beta_2 geo + \beta_3 \{(eq)(geo)\} + \beta_4 ura + \beta_5 \{(ura)(eq)\} \qquad (25)$$

Where $(\Delta \ln Ah)^*$ denotes the steady-state path for $(\Delta \ln Ah)$, where (eq) is an index that measures a country's quality of education, (geo) is an index that measures a country's distance from the equator, and (ura) is a country's average rate of unemployment.

Since, by (23), $\ln Ah = \delta \ln k - \theta \Delta \ln Ah$, to model the path for Ah empirically one needs to take account of (25) and estimate the following relation :

$$\ln Ah = \delta_0 + \delta_1 \ln k + \delta_2 eq + \delta_3 geo + \delta_4 \{(eq)(geo)\} + \delta_5 ura + \delta_6 \{(ura)(eq)\} \qquad (26)$$

VII. Effective Human Capital Investigated Empirically

VII.1 Introduction

Our empirical investigation comprises a set of cross-section OLS regressions applied to the sample of the 53 countries considered in Casellli and Freyer (2007) over the period from 1996 through 2006 inclusive. During this period, GDP per worker in the five wealthier countries in the sample averaged 75694 PPP dollars per annum measured in 2005 prices. In comparison, the annual average of the GDP per worker in the poorest five countries was 3605 PPP dollars. On the assumption that $\alpha = (1/3)$, one can show that differences in the capital-output ratios can account no more than a 1.35 -fold of the 21-fold difference in labor productivity between these two sets of countries leaving a 15.5-fold gap to be explained by differences in Ah.Put another way, on the assumption that $\alpha = (1/3)$, 74% of the difference in labor productivity between these two sets of countries would seem to be attributed to differences in Ah . The question to answer is then this: Do differences in Ah reflect mainly differences in efficiency or do they mainly reflect differences in factor endowments? To quote Prescott (1998), "Needed: A Theory of Total Factor Productivity".

Caselli (2005) explores a variety of ways of looking afresh at the issue of the importance of efficiency relative to factors of production in explaining income differences between rich and poor countries. As he states, “The current consensus is that efficiency is at least as important as capital in explaining income differences”. However, he goes on to say that his explorations of several extensions to the basic methods and the data suggest that “some of these extensions may lead to a reconsideration of the evidence”

VII.2 The Logarithm of Effective Human Capital, $\ln Ah$, Estimated Empirically

In what follows we shall first report the parameter values obtained by estimating (26) empirically followed by a brief commentary on our findings. The coefficient estimates obtained by applying a cross-section OLS regression to (26) read as follows:

$\hat{\delta}_0$ =0.8218, $\hat{\delta}_1$=0.5986, $\hat{\delta}_2 = 2.7765$, $\hat{\delta}_3$=7.7153, $\hat{\delta}_4$ = -7.3079, $\hat{\delta}_5$ = -64.4331, $\hat{\delta}_6$ =57.7230

Whilst $\hat{\delta}_2$ was found to be rather insignificant in statistical terms, the interaction terms involving the quality of education tell a different story, a story suggesting that education quality has a rather powerful impact on the stock of effective human capital. For instance, a country whose unemployment rate equals the sample average of 0.1009487, whose geography index measures the sample average of 0.4137736, and whose quality of education index equals the sample average of 1.074699, enjoys an elasticity of effective human capital with respect to the quality of its education equal to 3.0124. This statistic is arrived at by calculating [(57.723)(0.1009487) -7.308(0.4137736](1.074699)

To explain in some detail the mechanisms via which the quality of education impacts on growth consider, first, the interaction term between unemployment and the quality of education. As $\hat{\delta}_6$ suggests, the impact of the quality of education on effective human capital depends positively on unemployment. Since, as mentioned above, a rise in unemployment above its natural rate induces those in employment to exert more effort, the impact of education quality on effective human capital is higher when combined with effort. Put differently, when unemployment and education quality *both* rise (fall) the impact on effective human capital is positive (negative).

Other things equal, the closer is a country to the equator the smaller is the impact of the quality of its education on its stock of human capital as $\hat{\delta}_4$ suggests. This finding is not surprising: Other things equal countries closer to the equator are less well equipped to benefit from people with higher skills either because of the prevalence of tropical diseases or because of institutional characteristics.

As $\hat{\delta}_3$ suggests, being close to the equator benefits the effectiveness of human capital *all other things equal.* Three possible explanations spring to mind in defense of this rather counter-intuitive finding: Firstly, as mentioned above, other things equal a country which is far from the technology frontier can accumulate skills at a faster rate than a country nearer the technology frontier. Secondly, a country far from the technology frontier has a better chance to receive foreign aid. Thirdly, if the country is a recipient of foreign aid it must be the case that foreign aid is well used.

Finally, $\hat{\delta}_3$ and $\hat{\delta}_4$ taken together, suggest that any *net* benefits to be enjoyed by being closer to the equator are confined to countries whose index of education quality is well below the average. This may well suggest that foreign aid targets mainly those countries with limited education quality: Somehow if there are *net* benefits to be had by being far from the cutting edge of technology, those benefits are targeted for the least able.

As $\hat{\delta}_1$ shows the stock of physical capital is crucial in determining the level of effective human capital: The point elasticity of Ah with respect to k is 0.6, almost twice the size of the elasticity of y with respect to k.

VII.3 The Logarithm of Output per Worker, $\ln y$, Estimated Empirically

Since, $\ln y = (1-\alpha)\ln Ah + \alpha \ln k$, it must be the case that the parameter values of a regression on $\ln y$ are linked to the parameter values of the relation described by (26). To fix ideas suppose we were to describe the path of $\ln y$ as follows:

$$\ln y = \xi_0 + \xi_1 \ln k + \xi_2 eq + \xi_3 geo + \xi_4 \{(eq)(geo)\} + \xi_5 ura + \xi_6 \{(ura)(eq)\} \qquad (27)$$

Then one should expect the coefficients estimates from a regression applied to (27) to be linked to the coefficients estimates of (26) as follows:

$$\hat{\xi}_1 = (1-\hat{\alpha})\hat{\delta}_1 + \hat{\alpha} \qquad \hat{\xi}_2 = (1-\hat{\alpha})\hat{\delta}_2 \qquad \hat{\xi}_3 = (1-\hat{\alpha})\hat{\delta}_3$$

$$\hat{\xi}_4 = (1-\hat{\alpha})\hat{\delta}_4 \qquad \hat{\xi}_5 = (1-\hat{\alpha})\hat{\delta}_5 \qquad \hat{\xi}_6 = (1-\hat{\alpha})\hat{\delta}_5$$

Setting $\hat{\alpha} = (1/3)$ - the value used to derive the Ah series- we should be able to verify that the $\hat{\xi}_i$ hypothesized above match the $\hat{\xi}_i$ obtained by estimating (27) empirically. To confirm that this is indeed the case we report the parameter values obtained by applying a cross-section OLS to (27). These values read as follows:

$\hat{\xi}_1 =$ 0.7324, $\hat{\xi}_2 =$ 1.8513 $\hat{\xi}_3 =$ 5.1444 $\hat{\xi}_4 =$ -4.8727 $\hat{\xi}_5 =$ -42.9525 $\hat{\xi}_6 =$ 38.4796

In conclusion the coefficient estimates labeled $\hat{\xi}_i$ $((i = 1-6)$ do confirm and validate the coefficient estimates obtained by estimating (26) empirically.

VII.4 The Growth Rate of Effective Human Capital, $\Delta \ln Ah$, Estimated Empirically

A further way to validate the empirical estimates obtained by applying (26) to the data would be to estimate a model of the conditional convergence of effective human capital. To do so we shall follow Barro and Sala-i-Martin (1999) and specify the path of Ah as follows:

$$\Delta \ln Ah \equiv \ln(Ah)_{2006} - \ln(Ah)_{1996} = -(1-e^{-\lambda \Delta t})[\ln(Ah)_{1996} - \ln(Ah)^*] \qquad (28)$$

In the above λ measures the speed at which Ah approaches its steady-state path,

$(Ah)^*$denotes the steady path of Ah, and Δt measures the time periods between the end of the initial period and the final period of observation.

Assuming that (26) adequately describes the steady-state path of Ah, and setting $\Delta t = 10$,The results obtained by applying a cross-section OLS the data read as follows:

$$\Delta \ln Ah = -0.5330(\ln Ah)_{1996} + 0.4048 \ln k + 1.8020(eq) + 7.5615(geo) - 7.0059\{(eq)(geo)\}$$
$$- 56.2843(ura) + 50.8461\{(ura)(eq)\} \tag{29}$$

With a coefficient estimate on $\ln(Ah)_{1996}$ equal to -0.5330 , $\lambda = 0.076 = -[\{\ln 1 - 0.5330\}/(10)]$

Four observations, specific to (29), are worth mentioning: (1) Apart from the coefficient estimate on (eq) all other coefficient estimates are highly significant, (2) Notwithstanding the insignificance of this coefficient, the effect of education quality on the growth rate of human capital is highly significant when one takes account of the two interaction terms in which (eq) appears. Specifically, the elasticity of $\Delta \ln Ah$ with respect to (eq) is equal to 2.4009 which is obtained by evaluating: [-(7.0059)(0.41377)+(50.8461)(0.10095)](1.074699), (3) The speed of conditional convergence is much higher than the speed recorded by Barro and Sala –i-Martin (op.cit) and rather closer to estimates reported by Caselli, Esquivel and Lefort (1996) (4) The coefficient estimates obtained by applying (28) to the data are consistent with the estimates obtained by estimating (26). Put another way, knowledge of the parameter values of (26) and of λ would suffice to predict the 95% confidence interval of the parameters for (28)

VIII. Summary and Policy Conclusions

The question as to whether it is efficiency, or factor endowments that explains most of the difference in labor productivity between rich and poor can best be answered by formulating output per worker and effective human capital as follows:

$$(Y/N) = Ah(K/Y)^{(1/2)} \tag{30}$$

$$Ah = (K/N)^{0.6}(Efficiency)^{x} \tag{31}$$

Since $(K/Y) \equiv (K/N)(N/Y)$, a substitution (31) into (30) gives:

$$(Y/N) = (K/N)^{0.73}(efficiency)^{x/1.5} \tag{32}$$

If we were to assume that output per worker is homogenous of degree one in capital per worker and in efficiency, then $x = 0.4$ and variations in efficiency explain less than 30 per cent of variations in output per worker.

Concerning the evolution of the growth rate in this paper we find, like Michelis, Estevao, and Wilson (2013), that there are circumstances where there may appear to be trade-off between TFP and employment. In particular, when unemployment and quality of education rise (fall) *together* labor productivity also rises (falls). Policy makers should avoid the temptation to use unemployment as a tool to promoting growth partly because this will be counterproductive in the long-term and partly because, as Michelis, Estevao, and Wilson (op.cit) observe workers' happiness and employment are linked.

Appendix A: The Data, its Sources, and the Statistical Properties of The Estimated Relations

A.1 Data Definitions and Data Sources

$y \equiv (Y/N)$ = PPP Converted GDP per Worker at 2005 Constant Prices: Penn World Tables 7.0: rgdpwok

$(Y/K) \equiv$ Output- Capital Ratio: Extended Penn World Tables 4.0, Productivity of Capital: rho

$k \equiv (K/N) = (K/Y)(Y/N)$: Capital per Worker

$Ah \equiv (Y/N)(Y/K)^{(1/2)}$ = Effective Human Capital per Worker

$\Delta \ln Ah \equiv (\ln Ah)_{2006} - (\ln Ah)_{1996}$

Geo: Per Cent of The Country's Land Within the Geographical Tropics (*tropicar),* Gallup, Sachs, and Mellinger (1999)

Eq: Education Quality: Measured by: e^{ERSI} : Where *ERSI= Estimated Returns to Schooling of Immigrants*: Source: Schoellman, T.,(2012) (and Author's Estimates)

Ura: The Average Rate of Unemployment: With the Exception of Burundi, Cote' D'Ivoire and Congo, the Data Source is: UNECE (United Nations Economic Commission for Europe)

For Burundi, Cote' D'Ivoire and Congo, the Data Sources Are:

BURUNDI URATE: WWW.tradingeconomics.com

COTE D'IVOIREWWW.tradingeconomics.com CONGO : CIA FACT BOOK ESTIMATE FOR 2012

A2. The Estimated Relations: Cross –Section OLS Regressions with Robust Standard Errors

$$\ln Ah = 0.821 + 0.599(\ln k) + 2.776(eq) + 7.715(geo) - 7.308\{(geo)(eq)\} - 64.433(ura) + 57.723\{(ura)(eq)\}$$

(2.747) (0.073) (2.272) (4.019) (3.764) (19.971)

(17.93)

F(6,46)=87.33, Prob>F=0.000, R-squared =0.8815, Root MSE =0.3533,

Number of Observations 53

$$\ln y = 0.548 + 0.732(\ln k) + 1.851(eq) + 5.144(geo) - 4.873\{(geo)(eq)\} - 42.952(ura) + 38.480\{(ura)(eq)\}$$

(1.831) (0.0485) (1.515) (2.679) (2.509) (13.314) (11.954)

F(6,46)=233.96 Prob>F=0.000, R-squared =0.9537, Root MSE =0.23554

Number of Observations 53

$$\ln(Ah)_{2006} - \ln(Ah)_{1996} = -0.708 + 0.405(\ln k) + 1.802(eq) + 7.561(geo) - 7.006\{(geo)(eq)\}$$

(2.235) (0.0821) (1.765) (3.396) (3.192)

$$- 56.284(ura) + 50.846\{(ura)(eq)\} - 0.5330(\ln Ah)_{1996}$$

(17.747) (15.985) (0.184)

F(7,45)=11.39 Prob>F=0.000, R-squared =0.4691, Root MSE =0.30793

Number of Observations 53

Appendix B : The Dynamics of the Model

B1. Deriving the Evolution of $\tilde{k}$ and q in $(q,\tilde{k})$ Space

By (2): $(\tilde{y}/v) = (\tilde{k}/v)^{\alpha} = (Y/K)^{-(\alpha/1-\alpha)}$. Substituting this into (11) we arrive at:

$$(q-1)\tilde{k} = \sigma(1-\alpha)(\tilde{y}/v) = \sigma(1-\alpha)(\tilde{k}/v)^{\alpha} = \sigma(1-\alpha)(Y/K)^{-(\alpha/1-\alpha)} \quad \text{(B1)}$$

Solving for (Y/K) we arrive at:

$$(Y/K) = [\{(q-1)\tilde{k}\}/\{\sigma(1-\alpha)\}]^{-\{(1-\alpha)/\alpha\}} \quad \text{(B2)}$$

Using (3) to solve for v we get:

$$v = 1-\sigma(I/K) \quad \text{(B3)}$$

Substituting the expression for v in (A3) into (14) and solving for (I/K) we arrive at:

$$(I/K) = (1/\sigma) - \{(1-\alpha)(Y/K)/(q-1)\} \quad \text{(B4)}$$

Accordingly, the evolution of capital in intensive form can be described by:

$$\dot{\tilde{k}} = [\{(1/\sigma) - (1-\alpha)\{(Y/K)/(q-1)\} - (n+\delta+g)]\tilde{k} \quad \text{(B5)}$$

Using (A2) to substitute (Y/K) out we arrive at:

$$\dot{\tilde{k}} = [(1/\sigma) - \{(1-\alpha)/(q-1)\}\{(q-1)/\sigma(1-\alpha)\}^{-\{(1-\alpha)/\alpha}\tilde{k}^{-\{1-\alpha)/\alpha}\} - (n+\delta+g)]\tilde{k} \quad \text{(B6)}$$

To allow for an analytically tractable analysis of the model's dynamics in $(q, \widetilde{k})$ space it would be convenient to assume that the perfect-myopic foresight path of $(\dot{\widetilde{c}} / \widetilde{c})$ is zero, and proceed to use (15) to arrive at :

$$\dot{q} = \rho q - \pi_K = \rho q - \{\alpha(Y / K) - (n + \delta + g)\} \tag{B7}$$

Using (A2) to substitute (Y / K) out we arrive at:

$$\dot{q} = \{\rho q + n + \delta + g\} - \alpha[\{(q-1)\widetilde{k}\} / \{(\sigma(1-\alpha)\}]^{-((1-\alpha)/\alpha)} \tag{B8}$$

B2. The Stability Conditions, the Speed of Adjustment, and the Saddle Path Slope

Given that q is "forward" looking and $\widetilde{k}$ is short-run predetermined stability requires that equilibrium is a local saddle- point. To establish the stability of equilibrium and to arrive at an expression for the speed of adjustment to the balanced-growth path and for the slope of the saddle-path, I shall take a first -order Taylor approximation to (27) and (30) around $q = q^*$ and $\widetilde{k} = \widetilde{k}^*$, to write,

$$\dot{q} = (\partial \dot{q} / \partial q)(q - q^*) + (\partial \dot{q} / \partial \widetilde{k})(\widetilde{k} - \widetilde{k}^*) \tag{B9}$$

$$\dot{\widetilde{k}} = (\partial \dot{\widetilde{k}} / \partial q)(q - q^*) + (\partial \dot{\widetilde{k}} / \partial \widetilde{k})(\widetilde{k} - \widetilde{k}^*) \tag{B10}$$

where the partial derivatives in (A9) and (A10) are evaluated at: $q = q^*$, and, $\tilde{k} = \tilde{k}^*$. After some algebra one can arrive at the following expressions:

$$a_{11} \equiv \partial \dot{q} / \partial q = \rho + (1-\alpha)\{(Y/K)^*/(q^*-1)\},$$

$$a_{12} \equiv \partial \dot{q} / \partial \tilde{k} = (1-\alpha)\{(Y/K)^*\}^{\{(2-\alpha)/1-\alpha\}}\{1-\sigma(n+\delta+g)\}^{-1}$$

$$a_{21} \equiv \partial \dot{\tilde{k}} / \partial q = [\tilde{k}^*/\{(q^*-1)][\{1-\sigma(n+\delta+g)\}/\sigma\alpha\}]$$

$$a_{22} \equiv \partial \dot{\tilde{k}} / \partial \tilde{k} = (1-\alpha)[\{1-\sigma(n+\delta+g)\}/\sigma\alpha]$$

For the steady-state to be locally a saddle-point the two characteristic roots, call them : μ_S, μ_U, must have opposite signs which, in turn, requires that:

$$\mu_S \mu_U = a_{11}a_{22} - a_{12}a_{21} < 0$$

To measure the speed of adjustment to the balanced - growth path one needs to calculate the stable root, μ_S, by solving the following expression:

$$\mu_S = \frac{(a_{11} + a_{22}) - \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2}$$

Finally, the slope of the saddle-path in $(q, \tilde{k})$ space is given by

$$\frac{\mu_S - a_{22}}{a_{21}} = \frac{a_{12}}{\mu_S - a_{11}}$$

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