Eddie Gerba

Reconnecting investment to stock markets: the role of corporate net worth evaluation

Working paper

Reconnecting Investment to Stock Markets: The Role of Corporate Net Worth Evaluation

Eddie Gerba*

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Abstract

Following recent studies by the Bank of England that the low financial market confidence and low expectations about private sector profits over the next three years has lead to unusually low price-to-book ratios, we incorporate a stock market mechanism in a general equilibrium framework. More specifically, we introduce an endogenous wedge between market and book value of capital, and make investment a function of it in a standard financial accelerator model. The price wedge is driven by an information set containing expectations about the future state of the economy. The result is that the impulse responses to exogenous disturbances are on average two to three times more volatile than in the benchmark financial accelerator model. Moreover, the model improves the matching of firm variables and financial rates to US data compared to the standard financial accelerator model. We also derive a model based quadratic loss function and measure the extent to which monetary policy can feed a bubble by further loosening the credit market frictions that entrepreneurs face. A policy that explicitly targets stock market developments can be shown to improve welfare in terms of minimizing the consumption losses of consumers, even when we account for incomplete information of central bankers regarding the current state of the economy.

Keywords: Asset price cycles, financial friction model, monetary policy, asset price targeting

JEL: G32, E44, E52

*Gerba: Research Officer, London School of Economics. E-mail: E.E.Gerba@lse.ac.uk. I would like to thank Jagjit Chadha for his advice and support. In addition we wish to thank the participants at the 18th International Conference for Computing in Economics and Finance in Prague 2012 for useful comments and suggestions.

1 Introduction

In the Financial Stability Report of November 2012, Bank of England observed that the price to book ratio of (bank) equity was down to its' historical lowest at 0.5.[1]The ratio is a wedge between the market value and the book value of capital. Book value of capital is calculated with reference to accounting standards, and represents the value left over for shareholders having paid off its liabilities, and excluding the liquidation costs. Market value, on the other hand, is a broader measure of capital since it includes both the market value of net assets and the present value of future investments. The latter is discounted back to the present using a risk-weighted measure, investors' required return, that characterizes the minimum return that investors demand for the level of risk taken by acquiring that capital. Therefore, when the accounting and market values of existing net assets are equal, the capital will be traded at price to book ratios above one (since the market value contains an additional forward-looking component). They note, however, that the sharp fall in the ratio over the past two years has been the result of two problems: investors question the value of net assets reported in the accounts, but more crucially, they seriously doubt whether the banks will be able to generate earnings sufficient to exceed the required return of investors. The doubts follow the 10 percent fall in market expectations of profits over the next three years, combined with the generally higher risk aversion of investors because of the deteriorating conditions on the global financial markets, who currently demand a higher required return in order to compensate for the (perceived) increased hazardousness of holding that capital.[2]

Moreover, pessimistic market sentiment is not only isolated to the banking sector. Corporate markets on both sides of the Atlantic have equally been affected. Following a persistent boom in the US stock market of 87 percent between 2002:II and 2007:II, in just two years (between 2007:II and 2009:III) the value of S&P500 has fallen by 40 percent. One of the key reasons behind the boom has been the prospect of higher productivity growth of US firms (Jermann and Quadrini, 2007), allowing firm earnings to persistently increase between 2002 and 2007. This consolidation in firm finances resulted in two things. First, it brought the default probability on loans down to negligible levels (because market value of firms increased), leading to

[1]See Chart 2 (page. 6). Further discussion on the developments on the bank capital market can also be found in the preceding Financial Stability Report of June 2012.

[2]Financial Stability Report, December 2011.

an easing in firm external financing constraints (Jermann and Quadrini, 2007). Second, the investors' required earnings sharply declined since the growth in earnings and the relaxed financing constraints assured investors that firms would have enough financial capital to continue growing. On the downside, and as a result of doubts regarding future firm profits due to the contractionary conditions, the default risk increased, pushing investors' required earnings sharply up. To illustrate the boom-bust cycle, net corporate dividends, a good rough for firm net earnings, more than doubled from less than 400 to more than 800 billion dollars during the boom period. Conversely, during the latest recession, the value of net dividends has fallen by 38 percent. The deteriorating stock market value combined with the greatly weakened cash-flows of firms has resulted in a fall in the real private investment of more than one-third (36 percent), or 800 billion dollars between 2007:II and 2009:III.[3]

The financial accelerator effectively links the financial condition of firms to their capital purchases. The net asset position of firms determines the quantity of credit they can borrow on the (constrained) credit markets, and the quantity of new investment they can attain. Because of the endogenous default risk priced into the financial contract of firms, the three components have a positive relationship. An increase in investments, due to higher profits leads to an increase in the net asset position of firms. Since a stronger net asset position of firms implies that their probability of default on the loans is reduced, the credit constraint is loosened, and therefore firms are able to borrow a higher quantity. The higher liquidity means in turn that firms can invest more, generating thus stronger future profit streams. As a result, their net assets in the future will increase, relaxing the credit constraint even further, and allowing them to invest even more. Imagine now that capital markets know all this already in the current period, and therefore supply more capital today to firms because they know that the higher amount of investments will generate a considerably higher present value of future investments. This means that expectations about future earnings are brought forward into today's market value of capital, which will deviate from its' present (point estimate) accounting value. Assuming (for the moment) that the value of net assets reported on accounts is the correct one, the forward looking nature of market prices will mean that the market to book ratio will be significantly above one in financial upturns since future earnings are expected to grow, and with certainty to exceed investors' required return. This in turn will put upward pressure on asset prices, which will improve the value

[3]The data was downloaded from Federal Reserve's St. Louis database.

of firm net worth. The result is a reduction in the risk of default on loans of firms, allowing them to gain some margin on leverage that they otherwise would not have if the return on capital was determined by the book value. So, as long as there is optimism (regarding future earnings) on the market, the market to book ratio will be significantly above one, and firms can expand their investment projects by more than proportional.[4]On contrary, with emerging pessimism on the market (or investors start to question the accounting value of capital), the margin constraints start to hit in much sooner, and the required deleveraging of firms will be more rapid compared to the book value case, creating a negative loop on the (stock) market value of capital due to the fire sales. Therefore we expect to see augmented boom-bust cycles in capital prices, credit supply, firm leverage, investment, production, and output which is more in line with the empirical findings of Chapter 1.

For the default risk, a wedge between market and book value implies a more asymmetric risk distribution. During upturns, lenders' confidence concerning the present value of firms' future investments assures them of the borrowers' future solid repayment status, which brings the default risk down, and the total value of loans supplied on the market up. During downturns, when questions regarding the future cash flows from the investment projects start to appear (or there are uncertainties concerning the value of firm net assets published in the accounts), the perceived riskiness of the borrowers will increase (since borrowers are uncertain that they will generate sufficient future earnings to exceed the required, and to pay off the loan), and so lenders required returns rise. Because the present value of future investments will decrease, which will be priced into the value of current stocks, the asset position of firms will deteriorate today and will require firms to either sell off assets, or issue new equity in order to meet higher loan repayment demands. However, if lenders were already risk averse before, they will be even more so now since the collateral that firms give has become riskier, and so their willingness to lend will be even smaller. The lack of liquidity and the tight margin calls exacerbate the repayment condition of firms more rapidly each time, and the default risk will be augmented by an equal proportion. The result is a more rapid deleveraging compared to the case where investments are priced to book value. The intensity of this (market value) mechanism is in line with the boom and bust cycle of 2000's when the increase in leverage or indebtedness, and the subsequent fall was more intense than implied

[4]Since firms will hold a higher amount of internal funds at a higher value, and access a higher quantity of external funds at a lower value in relative terms.

by the standard financial friction models. This is primarily because the principal-agent problem and risks have been much greater over the past decade following the high accumulation of corporate debt in the firm sector, resulting in a much higher vulnerability of firm balance sheet to stock market fluctuations and more intense problems of asymmetric information on the credit markets (see Chapter 1 of this thesis for more details).[5]

We incorporate these observations into our theoretical analysis by introducing two prices of capital into the canonical Bernanke, Gertler and Gilchrist (BGG, 1999) framework: *market* and *book* values. Firm's optimal decisions are contingent on the (stock) market value of capital, and the level of access to (external) credit is dependent on the market value of the firm. A wedge between the market and book value of capital is developed, which varies positively with projections on future firm and macroeconomic activity (i.e. procyclical). Positive projections about firm profitability and macroeconomic outlook will lead to higher investment in the firm, which will drive up its' market value, allowing it to borrow more in the credit market and invest more in its' production technology. This in turn will create positive expectations about future firm growth, which will push its' stock market value even further, allowing it to borrow even more, and increase the production plant. This positive loop, driven by expectations on the stock market, results in an increasing wedge between the market value of equity, and its' fundamental. A higher wedge implies a more volatile firm balance sheet (more positive in upturns, and more negative in downturns due to the higher deleveraging), and more volatile investment cycles, consequently attenuating the financial accelerator mechanism of the BGG model. This method of introducing an asset price wedge differs from existing approaches in the literature, such as Bernanke and Gertler (2001), Smets and Wouters (2007), or Hilberg and Hollmayr (2011) in two ways. First, the wedge (or bubble as they denominate it) in their models is basically determined by a parameter. By calibrating the bubble parameter to a certain value, the wedge is allowed to persist for a certain number of periods before it dies out. In addition, a shock needs to be placed directly on the bubble in order to trigger it. Hence, the endogenous effects of expectations on the wedge are (unlike this paper's model) not captured by their approach. Second, firm investment is a function of the book value of capital, rather than the market value as in our approach. As a result, firm

[5]See Bebczuk (2003) for a comprehensive discussion on the asymmetric information in financial markets.

investment is less volatile and less persistent in their framework.

In the second part of the paper, we proceed by investigating the role of monetary policy in maintaining the asset price wedge under control, in particular since 2002, and more importantly whether the current recession might have been avoided if alternative policies had been employed. Recent empirical studies from the IMF (2009) and Reinhart and Rogoff (2009) note that recoveries after recessions associated with financial crises are typically much more slower than others. Bean (2012) points to the sharp and sustained decline in confidence as the dominant reason for the delayed recovery at present. The over-confidence and underestimation of risks during the boom gives way to extreme caution in busts. Therefore, investments become costly to reverse, resulting in a sharp decline in capital expenditures. Our framework allows us to study the impact of an endogenous wedge between asset prices on the real economy.

Moreover, we are able to establish the role of monetary policy in nourishing this confidence (and so the asset price wedge) by analyzing alternative policy rules in the model. Previous studies had pointed in two opposite directions. On one hand, a set of studies (Bernanke and Gertler (2001), or Bullard and Shaling (2002)) showed that strong inflation targeting was sufficient and preferred for bringing a stock market boom under control without causing additional instabilities. Others, such as Cecchetti et al (2000, 2002), Bordo and Jeanne (2002), and Mussa (2002) argued that an explicit asset price targeting is not only beneficial, but also necessary if a policy maker has a stable economic development as its goal. With new insights from the pre-crises boom and the Great Recession, we return to this pre-crisis debate and investigate the role that neglecting asset price (wedges) play in times of over-confidence on the stock markets (i.e. in feeding a positive wedge), and how much less of an asset price wedge there might have been if alternative policy rules had been employed. We do this by deriving a second-order approximation of consumers' welfare, and conduct a series of welfare experiments based on this loss function derived from the model's first principles.

We find that making investment a function of an endogenous asset price wedge improves the impulse responses and the fit of the financial accelerator model to the post-2000 US data. Not only does the model better match the data on capital and firm balance sheet such as market value of capital, investment, and firm net worth compared to the standard Bernanke, Gertler and Gilchrist (1999) model, but also the financial, such as the policy rate and the rate of return on capital. Moreover, the endogenous responses to exogenous disturbances are on average two to three times more volatile than in the benchmark BGG model. More interestingly, the model generates an endogenous asset price wedge without the necessity to directly employ a shock to the wedge, which had been the *modus operandi* for most DSGE models until now. The role of monetary policy in an asset price boom is crucial. Experiments using (model derived) quadratic approximate welfare measure of consumers show that consumers are, on average 10 percent better off in terms of foregone consumption with a monetary policy that explicitly targets stock market prices compared to a policy that only targets inflation and output, as in Bernanke and Gertler (2001). The superiority of this policy is maintained even when we take into consideration that for central bankers might hold incomplete information regarding the true state of the asset price wedge. We interpret our results as overturning those of BG (2001) since in order for the standard policy rule to be preferred, the *ex ante* probability of an economy without a wedge has to be higher than the probability of a wedge, which is not reasonable to assume.

The remainder of the paper is outlined in the following way. Following a short empirical motivation of the link between the wedge and investment in the US in Section 2.2, and a brief literature review in Section 2.3, we make investment a function of the endogenous asset price wedge, and accommodate it to the financial accelerator framework in Section 2.4. We proceed with a qualitative and quantitative analysis of our model in Section 2.5. In Section 2.6, we conduct a set of welfare experiments and establish the preferred monetary policy for an economy with a positive wedge. Section 2.7 concludes.

2 Market vs book value and investment demand

The market value of firm and its impact on firm's investment demand has been an attractive research field for decades. The literature was initiated by Tobin's (1969) Q theory. The canonical theory states that the ratio of the market valuation of capital to the replacement cost of capital is a sufficient statistic for the optimal amount of investment of a firm facing simple adjustment costs. Since the market valuation of capital depends on the current stock price, the theory implies a positive contemporaneous covariance between stock prices and investment.

We take a slightly different approach in our study of the link between movements in (stock) market value and investment. We wish to explore whether the wedge between market and book values, which is closely related to the market-to-book ratio and a key feature of our model can explain US private investment. Let us first examine what the empirical literature has found before we perform our own analysis.

Using a long series on stock prices and private investment, Barro (1990) finds that changes in US stock prices, unlike the standard Q-variable, substantially explain the US private investment, even when the model is extended to include cash-flow variables.[6]This is because of the forward-looking nature of stock markets. An exogenous disturbance to, for instance, the future return on capital, shows up contemporaneously as an increase in stock prices, but only with a lag of a year or more as an expansion in the accounts (or book value). Comparing the explanatory power of stock prices to the Q-variable, he finds that the former drastically outperforms the latter, since the majority of change over time is in the market value of equity (and not in the reproduction cost of capital stock and net debt). Baker et al (2003) go a level deeper and test the hypothesis that corporate investment is sensitive to movements in the non-fundamental component of (stock) market prices. The results support their hypothesis and they find that firms who are equity-dependent are up to three times more sensitive to the non-fundamental movements than firms who are not. Even in a model where managers have discretion in determining the level of investment (Dow and Gorton, 1997), the empirics show that stock prices indirectly guide managers' investment decisions since traders extract important information regarding prospective investment opportunities from stock prices. In addition, managers are compensated based on the future stock prices, meaning that they hold incentives to make investment decisions based on stock price movements.

Next, let us examine the structural relationship between investment demand and the stock market. However, instead of regressing stock prices directly on investment, we wish to explore whether the wedge between the market and book value contains (on its own) information about prospective investment returns, and therefore determines the investment demand. We expect to find a statistically significant and positive coefficient of the wedge in the investment equation. Moreover, the wedge should explain a considerable share of the fluctuations in investment demand. We also expect the relationship between the wedge and the corporate bond rate to be significantly negative since an increase in the bond rate rises the cost of borrowing for firms, resulting in an increased default probability of firms, and therefore a re-

[6]Such as contemporaneous and lagged values of net profits.

duction in the wedge. To test our hypothesis, we conduct a simple VAR(2) of the wedge, book value, investment demand, and the corporate bond rate according to the general model:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + e_t \tag{1}$$

where y_t is a $4x1$ vector containing the four variables, and A_1 and A_2 are $4x4$ matrices of two lags for the four variables. c is a 4x1 vector of constants, and e_t is a $4x1$ vector of error terms (one for each equation).[7]The variables enter the model in the following order: the wedge, book value, corporate bond rate, and investment demand. The VAR model is expressed in levels, or I(0). In order to separate movements in the wedge caused by movements in the book value, we include book value in our VAR models and position it after the wedge. Similarly, access to external financing is important for firms since it determines how liquidity constrained a firm is, and therefore how much of their investment demand they can realize (or satisfy). We therefore include a measure of the cost of borrowing in the corporate bond market, and position it after the book value.

In order to calculate the wedge, we first need to establish the market value of capital. We define the market value as the *(stock) market value of shareholder equity of non-farm and non-financial corporate business* computed by the following accounting principle:

Assets = Liabilities + Stockholders Equity

For book value and investment demand, we have used the data on *replacement cost of capital* and *private nonresidential fixed investment*, respectively. These are the best representatives of the respective theoretical definitions given in Bernanke, Gertler and Gilchrist (1999). The wedge between the two capital prices is therefore the differences between stockholders equity and the replacement cost of capital. We use Moody's 30-year BAA rate as a measure for the costs in corporate borrowing. The data set is quarterly, covering the period 1953:I to 2012:II and accessible from the Federal System Economic Database (FRED). The data was logged before the estimation.[8]Results are reported in Table I.1. The stars in each entry indicate that the value is significant at 5 percent significance level.

Apart from the autoregressive coefficients in each VAR equation, we find that the only significant coefficients are the two lags of investment in the book value of equity

[7]The error terms are not correlated.

[8]Except for the BAA-rate and the wedge, since they are expressed in percentage terms.

equation, and the two lags of the wedge in the investment demand equation.[9]Hence we learn two things here. Not only does the wedge explain investment demand, but also investment demand explains the book value of equity. Let us examine this further with the impulse responses.

We calculate the impulse responses to innovations in each of the four variables using the Cholesky decomposition. As before, the order of the variables is: the wedge, book value, corporate bond rate, and investment demand. The innovations are normalized to one standard deviation and the relevant responses are reported in Figures I.1. Let us start with investment demand. One percent increase in the wedge leads to a 0.013 percent rise in investment. Only the investment shock causes a greater response of investment, of 0.04 percent (following a one percent innovation to investment). Turning to the other significant variable in our VAR model, the coefficient of investment demand in the book value equation, we find that an increase of one percent in investment demand leads to a 0.008 percent increase in the book value of equity. Switching around the models, a one percent rise in the book value causes (after 4 quarters) a 0.01 percent rise in investment demand. While the impact of the book value on investment is larger than the other way around, looking at table I.1, it is insignificant.

Our last exercise is the variance decomposition. The graphs are reported in Figures I.2, but we will concentrate mainly on the decomposition of investment. Not very surprising, the investment shock explains the majority of variation in investment. However, the wedge is jointly with the book value of equity the second most important factor in explaining the fluctuations in investment, of about 10 percent each. This means that one fifth of the variation in investment is explained by movements in stock market prices (which is the sum of the wedge and the book value of capital). Summarizing our results, we find a structural relation going from the asset price wedge to investment. Both of the lagged wedge coefficients in the investment equation are statistically significant. In addition, we find that investment has a significant (albeit small) impact on the book value of capital. Lastly we find that 10 percent (one fifth) of the movement in investment can be explained by the wedge (stock market prices).

With this structural link between the wedge and investment in mind, we need to establish a set of stylized facts for the wedge, market value of capital, as well as

[9]We also find that the first lag of corporate bond rate is significant in explaining the wedge, but since only one of the two lags in the independent variables is significant, we exclude it from the main discussion.

the book value that we want our model of the asset price wedge to be consistent with. For that purpose, we HP-filter the three variables, and report the relevant correlations and relative standard deviations to output in Table I.2. Moreover, we report the graphs of the series in Figure I.3. Again we use the same sample period as before. The wedge is procyclical. Its correlation to output is 0.30 for the whole sample period. Moreover it has increased over the past two decades. While already in the early 1990's the wedge became much more procyclical than in the past (with a correlation coefficient of 0.64 since 1991:II), the correlation has continued to grow, reaching a value of 0.76 over the latest business cycle (2001:IV-2012:II).[10]

For market versus book value of capital, we observe similar correlations for both series (0.34 and 0.40 for the entire sample period). Similar to the wedge, the correlations have increased over the past two decades, and during the last cycle, the correlations grew to 0.83 and 0.90 for both capital prices. But, on the volatility side, we find that market value is more volatile than the book value (by a factor of 5). Whereas the book value is in the full sample 1.3 times more volatile than output (growing to 2.14 during the past two decades), market value of capital is 7.2 times more volatile than output (or 11.1 times during the past two decades).

These empirical findings motivate this paper. We need to construct a financial accelerator model which accommodates for a highly procyclical asset price wedge, and where the (stock) market value is more volatile than the book value. Once we have captured these stylized facts, we can investigate the role of stock market swings (optimistic and pessimistic phases) for credit supply and cost of borrowing, investment and production, debt and equity stocks of firms, and more generally, for the stability of the economy.

3 Literature review

Incorporating a market value mechanism into a standard macroeconomic model is a relatively young research field.[11]The work of Bernanke and Gertler (2001) is one of

[10]On the volatility side, the wedge is 8.7 times more volatile than output for the entire sample period. However, over the past two decades, the volatility it has almost doubled to 15.7 times that of output.

[11]On the other hand, there is a vast variety of research on the relationship between monetary policy and asset prices. Most researchers explore this topic without defining a separate market price mechanism. Among many, Gilchrist and Leahy (2002), Chadha et al. (2004), Detken and Smets (2004), Gilchrist and Saito (2006), Geromichalos, Licari and Suarez-Lledo (2007) and Nosal and Rocheteau (2012) explore the link between asset prices and the macroeconomy without defining a separate (endogenous) wedge.

the seminal examples of the literature. In that paper, the authors were interested in examining whether monetary policy should react to price movements on the stock market, particularly when asset price bubbles are present. Therefore the model establishes a simple theoretical framework for evaluating market values of firms. In this framework, the price of capital is divided into two pieces: the market value of capital and the fundamental value of capital. A positive deviation of the market value from the fundamental value defines a *stock market bubble* and these deviations are *exogenously driven* by some parameters. As a result, for instance, a productivity shock does not feed a stock market bubble since the variations in stock prices are not endogenously driven by other model variables.[12]

During subsequent years, researchers had developed a variety of theoretical models structuring a market price mechanism. Three common characteristics define these models. First, the expected dividend and real interest rate play a key role in the determination of market prices. With this property, they adapt the neoclassical security valuation approach, which defines the value of an asset as the (discounted) present value of expected dividends (Rubinstein, 1976). Second, particular attention was not devoted to the relationship between investment demand and stock market prices, and third, the spread between market and fundamental values of capital was modeled as an exogenous process governed by some parameters. Examples of these models include Carlstrom and Fuerst (2007), Castelnuovo and Nistico (2010), and Hilberg and Hollmayr (2011), and since their mechanism is very similar to Bernanke and Gertler (2001), the bubble (or asset price wedge) is only triggered when explicitly shocked. Therefore, the general effects of the bubble on the dynamics of the economy are limited.

Our paper is distinct from this literature in many ways. First, it explicitly tries to capture the strong correlation between investment demand and the asset price wedge (market-to-book ratio) as confirmed by the empirical data. Therefore, in our macroeconomic structure investment demand is contingent on the movements in stock market prices which, because of the link to external finances, in turn also determines the borrowing capacity of firms and the debt stock of the production sector. There are plenty of empirical studies supporting this argument, such as Bond and Cummins (2001), Goyal and Yamada (2004), Dupor (2005), and Chaney, Sraer and Thesmar (2010).

[12]In other words, you have to shock explicitly the asset price bubble in order to create a bubble effect.

Second, empirical studies have found that stock market prices fluctuate largely endogenously depending on the existing and expected states of the economy (Nasseh and Strauss (2000), Errunza and Hogan (2002), Campbell (2005), Hanousek et al (2009), Chen (2012), Schwert (2012)). Stock market investors use every information at micro and macro levels to find a *fair* value of the stock price now, and in the future. If there is no trading activity based on expectations, the market value of the shareholders equity value would be equal to the value of the capital as stated on the balance sheet of the firm. Consequently, the stock market value of a firm, or net worth can be literally reduced to the value of net assets on the books plus some additional wedge formed on the basis of expectations regarding future corporate performance, which itself develops endogenously, and is exposed to exogenous shocks.

The contemporaneous macroeconomic research on this specific topic is expanding and far from a consensus on the way to establish a valuation method for a firm's market value. This provides us with some flexibility in the design of it. Here we mostly benefited from the corporate finance literature, in which there is a bulk of studies exploring this subject.

In that literature, the seminal work of Ohlson (1995) provides the most suitable stock market value model, so called *the earnings capitalization model*, since it provides a direct relation between the market value of capital, and its accounting (book) measure, with the difference reduced to contemporaneous and future expected earnings.[13]The model is therefore a convex combination of a pure 'flow' (or profit capitalization) and a pure 'stock' (or balance sheet based) model of value.

The earning capitalization model is attractive for many reasons of which the most important are: first, technically the approach is theoretically constructed from first principles and therefore it is straightforward to adapt it to DSGE models. Second, many studies have empirically validated and confirmed that the model fits well the stock market data.[14]Third, the model is theoretically well recognized within the neoclassical theory and the corporate finance literature. In addition, the model does not require the Modigliani-Miller irrelevance property to hold in order for the valuation to be consistent (even if it satisfies them), which facilitates the pricing

[13]Since the model is based on the clean surplus relation of accounting statements, all changes in assets and/or liabilities unrelated to dividends must pass through the income statement. That is why the model considers earnings as its argument.

[14]See Dechow et al (1999), McCrae and Nilsson (2001), Callen and Segal (2005), Gregory et al (2005).

theory to be easily integrated within the financial friction structure of the DSGE models (Larran and Lopez, 2005).

4 Model

We use a standard New-Keynesian model with financial frictions, real and nominal stickiness and market price mechanism. The backbone of our macroeconomic framework is formed by Bernanke, Gertler and Gilchrist's (1999) (BGG model, thereafter) general equilibrium model. The market price mechanism is adapted from Ohlson (1995)'s earning capitalization model.

Since this paper's model is very similar to the BGG model, for the sake of focus below we only discuss the core equations specific to our framework. The full log-linearized system is outlined in Appendix II, including a list explaining the parameters, and their calibrated values.

4.1 The rate of return on capital

The return on capital occupies an important place in the determination of default risk and risk premia in the BGG model. The wedge between expected and ex-post returns on capital drives this premia and the cost of external borrowing.

We start with the definition of expected return on capital in the economy. Denoting output as Y, the capital stock as K, the share of capital in production by α, depreciation rate of capital as δ, the return on capital as R^k, the value of capital as Q and the mark-up of retail goods over wholesale goods as X, the aggregate expected return on capital in the canonical BGG model takes the following form:

$$E_t[R^k_{t+1}] = E_t[\frac{(\frac{1}{X_{t+1}})(\frac{\alpha Y_{t+1}}{K_{t+1}}) + Q_{t+1}(1-\delta)}{Q_t}] \tag{2}$$

This definition states that the expected return on a unit of capital is the sum of the mark-up over the cost of capital and the net capital gains due to the change in capital price. Our first modification of the canonical BGG model is to incorporate an (endogenous) asset price wedge into the general equilibrium. We therefore need to adopt the above expression to express the gains from holding a unit capital in terms of the market price (which includes the wedge) according to:

$$E_t[R^{ks}_{t+1}] = E_t[\frac{(\frac{1}{X_{t+1}})(\frac{\alpha Y_{t+1}}{K_{t+1}}) + S_{t+1}(1-\delta)}{S_t}] \quad (3)$$

where S_t is the market value of capital at time t, and E_t is the expectations operator at t. The definition provides a reasonable demand curve for capital and it amplifies the accelerator effect on the accumulation of entrepreneurial net worth (since the market return R^{ks}_t has more intense cycles compared to the capital return in the canonical BGG model).

4.2 Net worth

The aggregate net worth of firms at the end of period t, N_{t+1}, is given as:

$$N_{t+1} = \gamma V_t + W^e_t \quad (4)$$

where V is the equity of entrepreneurs in the firm and W^e is the wage of entrepreneurs.[15]The entrepreneurial equity is formed by the following identity:

$$V_t = R^k s_t S_{t-1} K_t - (R_t + \frac{\mu \int_0^{\omega_t} R^k s_t S_{t-1} K_t dF(\omega)}{S_{t-1}K_t - N_{t-1}})(S_{t-1}K_t - N_{t-1}) \quad (5)$$

where $\mu \int_0^{\omega_t} R^k s_t S_{t-1} K_t dF(\omega)$ is the default cost and $S_{t-1}K_t - N_{t-1}$ represents the quantity borrowed. The external finance premium on borrowings is given by the following expression:

$$R_t + \frac{\mu \int_0^{\omega_t} R^k s_t S_{t-1} K_t dF(\omega)}{S_{t-1}K_t - N_{t-1}}$$

Turning to Equation 5, the second term on the right hand side gives the repayment of borrowings. On the other hand, the first term on the right hand side, $R^k s_t S_{t-1} K_t$, gives the gross return on holding a unit of capital from time t to $t+1$. Consequently this statement implies that the entrepreneurial net worth equals gross return on holding a unit of capital minus the repayment of borrowings.

The return on capital is determined by the market price of capital in this paper's model, which implies that we have thus created an explicit link between the market value and net worth. Hence, a rise in the (stock) market value of capital, but not in the book value will lead to an increase in firm's net worth via two channels. The first is due to the fact that higher gross return on capital will inevitably lead

[15]Which we calibrate to play a minor role in this setting.

to higher net worth accumulation. The second is because a higher r^{ks} reduces the probability of default, reducing thus the amount to be repaid on the loan to the financial intermediary.[16]

4.3 Investment

Our second modification of the BGG (1999) regards firm investment demand, which we now make a function of the (expected) market value of capital.[17]Entrepreneurs' appetite for new investment is determined by the price of capital they expect on the market in the next period (inclusive of the wedge), and the increasing marginal adjustment costs in the production of capital, $Theta(.)$[18]according to:

$$E_t[S_{t+1}] = [\Theta'(\frac{I_t}{K_t})]^{-1} \tag{6}$$

where $E_t[S_{t+1}]$ is the expected market price of a unit of capital, I is investment and δ is the parameter that governs the depreciation rate. $\Theta(.)$ can be thought of as a capital production function generating new capital goods and is increasing and concave in investment. New investment (represented as percentage of the existing capital stock) will have a positive impact on the price via the demand channel. Later on in the paper, we will quantify the effects on model dynamics (impulse responses) and data matching from allowing investment to depend on market value of capital as opposed to allowing it to depend on a fundamental or book value, as in the canonical BGG model.

4.4 Financial accelerator

We start with the constraint on the purchases of capital:

$$S_t K_{t+1} = N_{t+1} \tag{7}$$

That is, firm is not allowed to borrow above its net worth. But for any firm, if the expected return is above the riskless rate there will be an incentive to borrow

[16] Nolan and Thoenissen (2009) find that a shock to the default rate in the net worth equation is very powerful in driving the entire model dynamics, and the shock accounts for a large part of the variation in output. It is also strongly negatively correlated with the external finance premium.

[17] We will show later that this is equivalent to making investment contingent on the (expected) asset price wedge since the wedge is the actual value of capital above its book/fundamental value, and it is therefore directly related to the ratio of market-to-book value.

[18] In steady state, the price of capital is unitary, meaning that the adjustment cost function is normalized.

and invest:

$$s_t \equiv E_t(\frac{R^{ks}_{t+1}}{R_{t+1}}) \tag{8}$$

where s_t is the expected discounted return on capital. For entrepreneurs to purchase new capital in the competitive equilibrium it must be the case that $s_t \geq 1$. The investment incentive and the strength of financial accelerator are both underpinned by this ratio. This incentive can be incorporated into the borrowing constraint:

$$S_t K_{t+1} = \psi(s_t) N_{t+1} \tag{9}$$

This definition states that capital expenditure of a firm is proportional to the net worth of entrepreneur with a proportionality factor that is increasing in the expected return on capital, s_t. Putting it in another way, the wedge between R^{ks} and R and the firm's net worth underpin the investment demand to build new capital good. Equation 9 can be equivalently expressed in the following form:

$$E_t[R^{ks}_{t+1}] = s(\frac{N_{t+1}}{S_t K_{t+1}}) R_{t+1}, \quad s < 0 \tag{10}$$

To recall, the firm borrows the amount $S_t K_{t+1} - N_{t+1}$; therefore $(N_{t+1}/S_t K_{t+1})$ gives the financial condition of the firm. And [10] relates the financial condition of the firm to the expected return on capital which is increasing in net worth but decreasing in borrowing. This is the financial accelerator. Since capital return depends on (stock) market prices, the financial accelerator is enhanced in this paper's model. Firms face higher net worth in stock market upturns, which allows them to borrow more and thus invest more in new capital compared to the canonical model. Conversely, in downturns the Kiyotaki-Moore (negative) credit cycle is triggered faster and margin calls hit sooner. As a result, less credit is supplied on the financial market.

Therefore, we should expect to see more volatile investment cycles in this paper since not only are we making investment depend on the (stock) market value of capital, but we are also attenuating the financial accelerator mechanism.

4.5 Market value and the asset price wedge

While we have incorporated the market value of capital S into the financial accelerator framework, we still need to derive its value. Our aim is to describe it in terms of the book value and the present value of future earnings. For this purpose, we follow the Ohlson (1995, 2001) model explained in Appendix II (and section 2.2) in deriving an analytical expression relating the market value to the book value. We start with the neoclassical view of the market value which simply states that the market value of the firm is the present value of expected sum of dividends discounted by the risk-free rate (non-arbitrage condition):

$$S_t = \sum_{\tau=1}^{\infty} R^{-\tau} E_t[D_{t+\tau}] \tag{11}$$

where S_t represents the (stock) market value of capital, R is the risk-free interest rate and $E_t[D_{t+\tau}]$ is the expectation on dividends that the firm is expected to generate in the future. To keep matters simple risk neutrality applies so that the discount factor equals the risk-free rate. This is the same PVED condition we described before. Next, we need to relate dividend payments to the book value and firm earnings.

Let X_t be the earning on equity from period $t-1$ to t. The basic clean surplus condition of financial statements defines the fundamental relation between book value of capital Q_t, dividend payments D_t, and earnings as:

$$X_t = Q_t - Q_{t-1} + D_t \tag{12}$$

Equation 12 states that the *earnings* at the end of the period t is the sum of two components: the change in the book value, and dividend payment. It means that all changes in assets/liabilities unrelated to dividends must be recorded by earnings in the income statement.[19]In addition, we impose the restriction that dividends effect negatively the book value of capital, but not current earnings, i.e.:

$$\frac{\partial Q_t}{\partial D_t} = -1 : \frac{\partial X_t}{\partial D_t} = 0 \tag{13}$$

Together with 12, this represents the clean surplus relation found in many financial models.[20]

One can now use the clean surplus relation above to express the market value in terms of future expected earnings (instead of the sequence of expected dividends)

[19]The reason lies with the dividend policy of a firm. Following a rise in the value of firm stocks, a firm can choose not to pay them out as dividends, but rather use them as retained earnings, which would contribute to larger earnings in the subsequent period.

[20]See Larran and Lopez, 2005 for a review on the application of the clean surplus relation in financial and accounting models.

in 11. However to complete that, we first need to define an additional financial variable. In particular, we follow Ohlson (1995) and define abnormal, or *residual earnings* as:

$$X_t^{re} \equiv X_t - [R-1]Q_{t-1} \tag{14}$$

where residual earnings X_t^{re} are described as firm earnings above the net book value at time $t-1$ (times the interest rate), or the (replacement) cost of using the capital. Hence, during profitable periods, earnings are above the cost of using the capital, or the same as saying positive 'residual earnings'.[21]

We are now in a position to express the market value in terms of the book value and residual earnings, by combining equations 12 and 14:

$$X_t^{re} = Q_t - Q_{t-1} + D_t - RQ_{t-1} + Q_{t-1} \Rightarrow D_t = X_t^{re} - Q_t + RQ_{t-1} \tag{15}$$

and using this last expression to replace d_{t+1}, d_{t+2}, d_{t+3}...in 11 to yield the market value as a function of the book value and the present value of future (expected) residual earnings:

$$S_t = Q_t \sum_{\tau=1}^{\infty} R^{-\tau} E_t[X_{t+i}^{re}] = Q_t E_t[X_{t+\tau}^{re}/R^{\tau}] \tag{16}$$

provided that $\frac{E_t[X_{t+i}^{re}]}{R^{\tau}} \to 1$ as $\tau \to 0$.[22]Hence, the market value in 11 can equivalently be expressed as 16, and our objective at the beginning of this section is accomplished. Relation 16 implies that the fluctuations in market value are the result of two factors: the variations in the book value and the present value of future residual earnings. In other words, the future profitability of capital, as measured by the present value of future (anticipated) residual earnings sequence reconciles the difference between the market and the book value of capital. The next step is to define the properties of residual earnings.

[21]One can link this idea back to the Bank of England 2012 report by viewing the profitable periods as periods of optimism. During periods of high market confidence, the capital is expected to generate a present value of future earnings above the required demanded by investors, or the same as saying, positive residual earnings. On the contrary, during times of distrust, the capital is not expected to generate future earnings above investors' required, either because the capital profitability has fallen, or investors' required earnings have increased, or a mix of both, implying that residual earnings will be negative.

[22]In the Ohlson (1995, 2001) paper, the author expresses the market value in linearized terms. However, because we are interested in studying the non-linear dynamics before we linearize the system, we express the value in non-linear terms

4.6 Residual earnings

To proceed, we need to characterize the process governing residual earnings. Our main purpose is to establish a bridge between the residual earnings and the general state of the economy. Feltham and Ohlson (1995, 1999), and Ohlson (1995, 2001, 2003) assume that X^{re} follows an AR (1) process and we extend it to additionally depend on economic fundamentals in the next period, $F_{t+1}|I_{t+1}$ according to:

$$X_{t+1}^{re} = \rho_x(X_t^{re}) + F_{t+1}|I_{t+1} \tag{17}$$

where ρ is restricted to be positive.[23]

4.6.1 Comparison to the BG (1999, 2001) bubble augmented FA model

Before we go on to simulate the model, let us briefly compare our approach to an asset price wedge to the one used in the BG (2000, 2001) model discussed briefly in section 2.3. While their model also starts off by defining the asset prices using the dividend discount model, they apply it to the fundamental value of capital, instead of the market value, i.e.:

$$Q_t = \frac{\sum_{\tau=1}^{\infty}(1-\delta)^{\tau} E_t(D_{t+\tau})}{R_{\tau+1}^{q}} \tag{18}$$

with Q_t denoting the fundamental value of capital, and $R_{\tau+1}^{q}$ is the discount rate on the fundamental value of capital. While this is not wrong, it is more accurate to apply the dividend discount model to market price, since firms pay out dividends to the shareholders based on the value of firm equity quoted on the stock market. Next, they assume that the market price of capital, S_t may differ from the fundamental value for a fixed period of time. The deviation in the two prices is called a bubble in the model, and is governed by the process:

$$S_{t+1} - Q_{t+1} = \frac{a}{p}(S_t - Q_t)R_{t+1}^{q} \tag{19}$$

where p is the probability the bubble persists in period t, and s is the growth rate of the bubble. However, since BG (2000, 2001) assume that a is smaller than one, the parameter can be interpreted as the rate at which the bubble component converges to zero. If $a = 1$, then we have the rational bubble scenario in the sense of

[23] For a more detailed discussion on the economic fundamentals that are relevant for this model, see Appendix II.

Blanchard and Watson (1982). Moreover, it is assumed that one a bubble bursts, it does not re-emerge. Hence, the wedge between the two prices is basically governed by these two parameters, and the model assumes that the deviation is only temporary, and because $a < 1$, it will disappear in the medium-run. BG (2001) calibrates the two values such that the bubble persists for 5 periods only. The influence of the expectations channel on the market price of capital is not endogenously captured in this model.

The second remark relates to the determinants of firm investment. The BG (2001) model (just as in the canonical FA model of BGG (1999)) assumes that firms make investments based on fundamental valuation of capital, such as net present value, according to:

$$Q_t = [\Theta'(\frac{I_t}{K_t})]^{-1} \tag{20}$$

By assuming this, the model ignores the endogenous effects of stock market bubbles on firm balance sheets and income flows, and thus the cost of capital. Thus, firms are not allowed to issue new shares to finance new capital, or building new capital and selling it at the market price inclusive of the bubble. In our framework, on the other hand, we want to contrast this by allowing investment to depend on market value of capital. Apart from being more realistic, it allows us to quantify the increase in swings in the business cycles (or equivalently, the intensification of the financial accelerator mechanism) from allowing firms to evaluate equity, and invest on the stock market based on the market value of capital.

4.7 Forcing variables

The model has two types of disturbances: productivity and monetary policy shocks. They are the same as in the BGG model, and the following law of motion describes the productivity shock:

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \tag{21}$$

Let ϵ_t^a be mean zero white noise shock. We calibrate ρ_a to 0.99 in our simulations.

Moreover, to examine a full boom-bust cycle in asset prices, we introduce an additional shock to our model. More specifically, we introduce an exogenous disturbance to the residual earnings equation II.54. The shock can be viewed as unexpected news (good or bad) regarding future economic performance that arrives, and influences stock market investments in that period. We label it *wedge shock*.

In our simulations, the standard errors of the monetary policy-and news shocks are calibrated to 1 percent, while for the productivity shock it is calibrated to 0.1 percent.

5 Quantitative analysis

5.1 Calibration

Table II.1 lists the parameter values for our two models. Most of the variables are calibrated following the values given in BGG (1999), and are standard to the literature. This applies both to the canonical BGG as well as the extended model in this paper. There are only a few minor differences. Our consumption-output ratio in the steady state includes both the private and public consumption, hence why the value is slightly larger in our calibration.[24]We calibrate the share of capital in production, α to 0.20. For robustness purposes, we also tried with $\alpha = 0.35$, the other common value in the literature, but no differences were observed. Finally, in order to replicate the stylized facts of the asset price wedge (including the market and book values) that we outlined in section 2.2, we parameterize ν, the elasticity of EFP to leverage to 0.13. It is slightly higher than the 0.05 in the original BGG model, but follows the estimation results for the US of Caglar (2012), and it represents well the post-2000 period, when the leverage of firms increased drastically, and so the sensitivity of financial lending rates to leverage was high.[25] For reasons of comparison between the canonical and extended BGG-Gerba model, we also calibrate ν to 0.13 in the canonical BGG model. In the same wave, we consider an accommodative monetary policy, replicating thus the Fed's stance during most of the past decade, and use the Taylor rule parameters of 0.2 for the feedback coefficient on expected inflation, ζ along with a value of 0.95 for the smoothing parameter.

The new parameter in this paper's framework is the AR(1) process of residual earnings. Borrowing from the insights in the corporate finance literature, and the US estimation results for the residual earnings process of Caglar (2012), we set the value equal to 0.67. Lastly, the weight on expected evolution of the economy is 0.18.

[24]In the canonical BGG (1999) model, the C/Y ratio is calibrated to 0.568. However, if we also include the public consumption in that ratio, which they calibrate to 0.2, the value is almost the same to our, which we calibrate to 0.806.

[25]See Gerba (2012), or the first chapter of this thesis on the balance sheet changes and the financial exposure that firms underwent during the past decade.

5.2 Impulse response analysis

5.2.1 Technology and Monetary shocks

In this section we examine the dynamic properties of the extended model using the impulse response analysis to three shocks: a technology, a nominal interest rate, and a wedge shock. The model responses to these shocks are presented in Figures I.4-I.6.

Generally, the impulse responses of the model in the current paper are qualitatively very similar to the canonical BGG model, albeit quantitatively they differ considerably. Moreover, since the wedge shock is a feature of the extended model, this is a novelty to the BGG (1999) framework. Hence, we will discuss it in further detail below.

We begin our analysis with the impacts to a technology shock, depicted in Figure I.4. A positive (0.1 percent) shock leads to a rise in the marginal product of capital. Since higher productivity implies higher expected return on existing capital stock, there will be an incentive for firms to raise investment demand. As the investment rises, demand for capital goods and asset prices rise, corporate net worth gradually accumulates; and accordingly external finance premium and borrowing cost fall. The reduction in the external finance premium will further create an incentive for firms to increase external funding and to raise investment. The incentive for external borrowing will continue until the marginal return on capital is equal to the cost of external funds used to build the last unit of capital stock. Concentrating further on asset prices, in the extended model we observe how market value rises beyond the book value. The spread between the two values is 0.2 percent from the steady state. Hence, a technology shock does not only lead to a rise in value of the fundamentals of assets, but because of positive expectations regarding future economic growth (as a result of better technology), the expectations generate further rise in the market value of those assets, which is maintained above the book value for 24 quarters.

Quantitatively, we see significant disparities in the two model versions. As a result of the higher expansion on the stock market, investment increases by 1 percent in this paper's model compared to the 0.25 percent in the canonical BGG. We also see a stronger wealth effect, since consumption increases by 0.1 percent and entrepreneurial consumption by 0.6 percent instead of the 0.1 in the canonical. Higher market value of assets means also that the net worth rise is significantly higher in the extended model, 0.6 percent compared to the canonical 0.13 percent.[26]This allows entrepreneurs to borrow significantly more on the external loan market in the current model, resulting in two and a half times higher output expansion compared to the canonical version.

Figure I.5 shows the responses to a simulated monetary policy shock. The unexpected rise in nominal interest rate generates a contractionary economic environment, leading to a fall in output, investment, consumption, inflation and corporate net worth. At this stage of the economy, firms' are reluctant to invest due to the decrease in profitability on existing capital stock. This is because the elevated nominal and real interest rates diminish the discounted cash flow of investment projects making them less profitable. As a consequence, investment, asset prices and net worth fall, whilst external finance premium rises which further suppress the desire to invest. Market value of assets decreases by more compared to the book value since a rise in the interest rate does not only reduce the discounted flow of investment projects (which determine the market value of capital), but also the expected residual earnings (since the general economy contracts). As a result, less will be invested in stock markets in the future, depressing further the market value. As a result, the spread between market and book value falls by 4 percent.

Again we observe significant differences in the magnitudes of the responses in this paper's model compared to the canonical BGG. The contraction is much more significant in the first. To a 1 percent reduction in the policy rate, investment decreases by 15 percent (compared to 7.5 in the canonical), and consumption falls by 1 percent in the extended version compared to the 0.6 percent in the canonical. Because of the much higher decrease in investment in the extended version, the cash flow of entrepreneurs is also reduced by significantly more, causing the net worth to fall by three times more (12 percent in the extended compared to the 4 percent in the canonical). Since entrepreneurs' borrowing conditions are affected more heavily in the extended model, the fall in output is therefore stronger and lasts for longer (since lower future cash flow is maintained for a longer period of time), resulting in a 4 percent fall in output compared to the 1.75 percent in the canonical.

Note that a deviation in dynamics of the extended version (in this paper) to the canonical BGG model is observed without the necessity to introduce an explicit stock market bubble shock to trigger that dynamics. We thus observe an endogenously

[26]This is almost equivalent to saying that the return on capital is 5 times higher in the extended model.

driven asset price wedge without the necessity to explicitly shock that it. This is an advantage compared to the current generation of models where the asset price bubble needs to directly be shocked in order to trigger a wedge in asset prices. Nevertheless, to illustrate the effects of a financial shock on the real economy, we will hereafter directly shock the wedge.

5.2.2 Wedge shock

In addition to the two standard shocks, we wish to explore the dynamics of the model in relation to the updating of beliefs.[27]We consider a positive (1 percent) shock to residual earnings, and the responses are reported in Figure I.6. Note that, unlike the BG (2001) model, we do not need to explicitly shock asset prices in order to generate a wedge between the two prices. However, we are interested in examining the effects that updating of beliefs has on asset prices, and the wider economy.

Overall, the wedge shock generates a strong (boom-bust) cycle in both the asset prices and the general economy. The shock causes optimism on the stock market since the capital is expected to generate a much higher return than the fundamental one, causing market value of capital to rise already today. The market value increases by 0.4 percent, which is 1.2 percent above the book value. There are two effects from this. The immediate effect is that capital is more attractive, and so induces more investment by entrepreneurs. In addition, because the value of net worth increases (since net worth is dependent on the market value of capital), this eases the borrowing constraints that entrepreneurs face. As a result, they can take out more loans, and use the credit to invest further into capital, The total effect is that net worth increases by 1 percent as a result of higher market value, and investment rises by 1 percent. In line with the empirics, there is also a wealth effect on consumption from a higher stock market value. The wealth effect on households is marginal since consumption increases by only 0.015 percent but the larger effect comes from entrepreneurial consumption, which rises by 1 percent. The final effect from the demand-side expansion is that output expands by 0.2 percent.

Nonetheless, as soon as expectations about the future return of assets deteriorate (after the second quarter), a negative spiral starts to hit in. The market return on capital falls by 0.2 percent, which causes the market value of assets to fall, and also investment, since it is now less attractive to invest because of lower expected capital

[27]For instance, Gertler and Karadi (2011) consider a similar shock in their version of the financial accelerator model with explicit banking.

returns. Additionally, falling asset prices mean that the value of internal funds starts to fall, which results in higher restrictions to external financing (since the collateral constraint binds sooner). This will cause a further fall in investment, which will result in lower net worth in the subsequent period, and so on. Hence, 4 quarters after the initial shock, market value of capital drops to below the steady state level, which causes investment to fall below its' steady state level in the subsequent quarter. The total effect on production is immediate, and output starts to contract 4 quarters after the initial shock. Despite the relatively slower fall of market value of capita compared to the book value, the negative economic prospects cause a steady drop in the market value, resulting in output being below its steady state level for almost 10 quarters. Only 4 years (or 15 quarters) after the initial shock does the economy recover from the contraction, and output turns back to its steady state level. We therefore observe the full cycle in our impulse responses. Our output (and investment) cycle is in line with the empirical literature which finds that output, on average takes longer time to recover after a stock market boom than after any other type of expansion. Moreover, the recessionary period is longer than the expansionary as a result of a stock market boom, which is in line with our model simulations.

5.3 Second moment analysis

Let us proceed by comparing the model generated second moments to US data. We have included information on the correlation coefficients of output to the remaining model variables, as well as the relative standard deviations with respect to output and we will compare those to the post-2000 US data moments. Table I.3 reports the correlations, and Table I.4 presents the relative standard deviations.

This paper's model succeeds in replicating all of the data correlations. More important, the high procyclicality of both the wedge and the two asset prices (market and book values) are correctly captured by the model. Compared to the canonical BGG version, there is some improvement in moment matching. The policy rate, while countercyclical in the canonical version, is like in the data, highly procyclical in the extended version.

Turning to volatilities, the current model matches most of the data. Market asset value is more volatile than both output, and the book value. The wedge is also more volatile than output, similar to investment and net worth. Moreover the model correctly matches Inflation and the policy rate, both smoother than output, while capital return, following the data, is approximately as volatile as output. The only shortcoming is in replicating consumption, which contrary to data, is smoother than output in the model. While this is a good replication of consumption before the 1990's, it is not representative of consumption thereafter. Book value volatility is also, to a certain extent, not completely captured by the model. While it is as volatile as output in the model and in the pre-1990's sample, it has turned slightly more volatile thereafter. However, neither the model in this paper nor the canonical BGG versions have been able to capture that.

Compared to the canonical version, we find a somewhat better fit. In particular, investment, net worth and capital return are more volatile in this paper's model compared to the canonical BGG model, which is consistent with the data. Nonetheless, there is still some room for improvement for the current model (as well as the canonical BGG) in matching the book value of capital, the market value of capital, consumption, the policy rate, and inflation which, while having the right sign, are even so more volatile in the data compared to the model.

To sum up, the model in this paper improves slightly the moment matching to the post-2000 US data compared to the canonical BGG. In particular, it captures the correlations and relative standard deviations of capital market variables such as the wedge, market value of assets, investment, net worth, and capital return. In addition, the correlation of the wedge and the policy rate to output are identical to the ones found in the data. The reason is that the effects from the financial accelerator mechanism are intensified in this paper's model. By allowing the market value to deviate from the book value depending on the expectations regarding economic fundamentals, and by making investment a function of it, positive (negative) outlook on firm growth will drive the wedge up (down). This will push firm investment up (down), which will lead to higher (lower) capital return and firm net worth in the subsequent period. Since expectations about economic fundamentals are tightly linked to the general business cycle, the firm and capital market variables listed above will also become more tightly linked to the business-cycle movements at the same time as their volatilities are intensified. Since the economy becomes more cycle-driven, the policy rate will also respond in a procyclical way to these cycles to dampen the expansionary (recessionary) effects on prices from a higher growth (contraction). This is more in line with the empirical findings discussed in the introduction of this chapter and Gerba (2012) where we identify investor confidence and a higher vulnerability of firm balance sheet to stock market fluctuations as two of the main reasons behind the increased procyclicality of firm balance sheet and flows over the past ten to fifteen years.

Notwithstanding, there is still some room for improvement in the matching of the (relative) standard deviations of the two asset prices (including their wedge), consumption, the policy rate and inflation which, while having the right sign, are even so more volatile in the data compared to the model.

6 Welfare analysis

The quantitative analysis in the previous section showed that by allowing investment to be a function of an endogenous asset price wedge in a financial accelerator framework, the economy becomes two to three times more responsive to the same shocks compared to a standard BGG model since investment and external finance premium become more elastic to asset price movements. As a result, the firm balance sheet becomes much more volatile.[28]In such circumstances the role of monetary policy becomes crucial in bringing down these large fluctuations. But bearing in mind that the asset price wedge is at the epicentre of the higher responsiveness of the economy, the key question becomes whether the monetary authority should directly target asset prices, and smoothen their boom-bust cycles? In a standard financial accelerator framework, Bernanke and Gertler (1999), and Bullard and Schaling (2002) showed that a sufficiently aggressive inflation (and output) targeting is both sufficient and optimal.[29]However, having extended the canonical mechanism, we wish to re-assess the Kansas City consensus and test whether they still hold under the modified model, or whether a policy rule that includes stock market developments performs better in stabilizing the economy, as Cecchetti et al (2000, 2003), Bordo and Jeanne (2002), and Mussa (2002) have argued. For that purpose, we will conduct a welfare analysis of alternative policy rules by comparing the losses in consumer welfare. We will contrast policies that include explicit asset price targets, and compare them to a standard policy rule. Lastly we will extend the analysis to include incomplete information regarding the true state of the economy on the part of monetary authorities and assess whether the same results hold when we include such information

[28]See Gerba (2012) for further details on the stock market exposure of firm-and household balance sheets and income flows.

[29]Since it does not increase the range of instability of models caused by the high information uncertainty related to the identification of an asset price bubble, nor depress 'healthy' economic growth.

asymmetries.

6.1 Loss function

In the welfare experiments we consider, the central bank is assumed to minimize a quadratic loss function of consumers (or welfare function as it is also commonly denoted) for each monetary policy rule. The loss function is derived from a second-order approximation of consumers' welfare in the model, and full details on the derivations can be found in Appendix III.

In addition to the minimum losses, the optimal weights on each variable in the monetary policy reaction function are estimated for that specific minimum loss. The following points summarize the steps we will follow in these experiments:

1. Define the loss function in the certainty equivalence policy experiments. It is important to keep the loss function constant throughout the experiments.

2. Two alternative monetary policy reaction functions are specified:

3. The minimum aggregate welfare loss of a particular reaction function is computed. The experiments are repeated for two scenarios: an economy with-and without an asset price wedge.

4. The optimal weights for that particular reaction function that generate the minimum loss are additionally estimated.

5. Alternative reaction functions and their corresponding weights are evaluated using the minimum loss as a general criterion. The monetary policy that generates the smallest welfare loss is preferred.

Following Woodford (2003), Chadha et al (2010), DeFiore and Tristani (2012), we derive a policy loss function by taking the second order approximation to the utility of consumers in this model. The approximation to the objective function takes a form which is relatively standard to the New-Keynesian model (see Woordford, 2003). Appendix II shows that the loss function which the policy maker minimizes is:

$$\arg\min_{\theta \in \ominus} \ell(\theta_j, (\alpha, \beta)) = E_t(\epsilon * \chi_y \sigma_Y^2 + \epsilon * \chi_\pi \sigma_\pi^2) \tag{22}$$

with χ_y denoting the weight on output y_t and χ_π the weight on inflation π_t. θ is the vector of estimated optimal weights in the monetary policy reaction function that gives the minimum loss. The two constituents of the loss function, σ_Y^2 and σ_π^2 are the variances of output and inflation. ϵ is the frequency of the losses we estimate.[30]The corresponding weights of output and inflation in the loss function are, as Appendix III shows, [0.05,1]. Hence, the function depends mainly on the variance of inflation, but also of output to some extent. The two terms are common to the New-Keynesian model. Intuitively, social welfare decreases with variations of inflation around its target, and of the output around its steady state level. The first reason for disliking variations in the output gap is that consumers wish to have a smooth consumption pattern over time. Just as in the benchmark new-Keynesian model, the consumption smoothing motive applies to total output variation. Second, households wish to smooth their labor supply (DeFiore and Tristani, 2012).

It is slightly counterintuitive that the social welfare function 22 is a standard New-Keynesian since one would expect that a model including financial frictions would produce a loss function which includes financial factors, such as asset prices. However, because only firms face financial constraints in this framework, and they are the ones exposed to stock market fluctuations, a second-order approximation of household welfare will not include the financial prices since their welfare does not directly depend on the fluctuations in these variables. Since households are assured a non-state contingent and risk-free return on their deposits (by financial intermediaries), they do not internalize the risks from financial fluctuations, and therefore only variability in the real variables matter for them.

6.2 Monetary policy rules

In order to facilitate the comparison of our policy analysis to Bernanke and Gertler (2001), and to keep the discussion as focused as possible, we will evaluate two types of reaction functions in what follows. The first is a standard inflation-forecast-based (IFB) rule with output. IFB rules are preferred to a current-inflation-based rule for their ability to include a great deal of information in a single object (Coletti (1996)). Levin et al (2003) find that this type of rules is robust provided that the horizon of the inflation lead is short. The second is augmented with a reaction from the market price:

[30]Which we express in annual terms, and hence set ϵ=4.

$$R_t = E_t(\pi_{t+1}) + y_t \tag{23}$$

$$R_t = E_t(\pi_{t+1}) + y_t + s_t \tag{24}$$

where $E_t(\pi_{t+1})$ is the expected level of inflation at t, y_t is the output level, and s_t is the market value of assets. Following the arguments in Bernanke and Gertler (1999) that an asset price wedge is hard to identify (and therefore carries risk if incorrectly identified), we will directly deal with a market price target, which is easily recognizable and available, instead of a wedge target.

We consider three shocks in our experiments: productivity, monetary policy, and wedge shock.[31]We assume the shocks to be uncorrelated, and to have the variance-covariance matrix of a diag[1 1 1].

6.3 Results

Results from the welfare experiments are reported in Tables IV.1 and IV.2. The first one reports the experiment results in an economy with a positive asset price wedge, whereas the second is from an economy without a wedge. The column with initial guesses represent the coefficients/weights given to the different variables in the reaction function before the estimation process, while Optimal weights represents the optimal estimated coefficients/weights on those variables. The optimizer uses the quasi-Newton method with BFGS updates, via the inverse positive definite Hessian in order to find the minimum value of the loss function (or the negative likelihood of the objective loss function) for a set of initial values of the reaction function. Different initial guesses of the monetary policy reaction function parameters/weights did matter for the minimization of the loss function, and for the estimated optimal parameters, since the optimizer searches for the minimum value of the welfare function around the neighborhood of the initial guesses of the reaction function.[32]

In the following discussions, we will be looking at mainly two issues. The first question we want to answer is whether the benefits of responding strongly to inflation (and output) exceed the benefits of responding to stock market developments in terms of welfare maximization. The second issue, which we will outline in more detail below, regards the preferred policy when Type I and II errors are taken

[31]BG(2001) only considered two shocks in their welfare experiments: productivity, and bubble shocks.

[32]For a more detailed description of the routine, please refer to Annex IV.

into account. Let us start with the first issue. Ultimately, we wish to examine whether the BG (2001) conclusions hold when we run similar calculations on the welfare effects of the two monetary policies, but using a different (asset price) wedge mechanism.

6.3.1 Targeting vs. not targeting asset prices

For each policy rule and wedge scenario, we conducted multiple simulations where we varied the initial weights of each policy target within the interval [0,2] for inflation and output and [0.01, 0.5] for market prices. We changed only one initial weight/parameter at a time so that we could appreciate the gradient of the loss function. Beginning with the economy with an asset price wedge, overall we observe a lower loss with the policy that includes a market price target. For the same weights on inflation and output in both types of reaction functions, the loss that the policy including market price reaction generates is, on average, 0.0004 units (or 10 percent) lower. In addition, comparing the global minimum losses (for any standard parameter weight within the interval [0,2]), the function including asset prices generates globally a lower loss (global minimum at 0.0033) than the function excluding it (at 0.0034). Assessing the estimated optimal weights for both reaction functions, we find that the weights on inflation and output are largely the same, but that the additional weight on asset prices (0.24) in the rule including market prices is what reduces the losses by more (i.e. is welfare improving).

Turning to the economy without a wedge, we find that the policy that targets market prices is welfare improving for low weights on inflation and output, while the opposite is true when we increase the weights on those two variables. So for weights on inflation and output under $[1.5, 0.5]$, the reaction function including asset prices is preferred. However, for an aggressive inflation and output policy (weight of 2 on inflation, and 0.5 and above on output), like in Bernanke and Gertler (2001), we find that the policy that does not target asset prices generates less losses, and is therefore favored. Thus, only for an aggressive policy response does Bernanke and Gertler (2001) results hold.

Moreover, we find that the loss function for the policy rule that does not include asset prices (in both economies) is more elastic than the other. In relative terms, this means that small changes in weights on policy targets can change the welfare of consumer by a considerable amount. Therefore, for a monetary authority that does not choose to target asset prices, the choice of weights becomes more critical for the success of their policy, than for an authority that chooses to include stock market developments. This can be interpreted as the asset price target being, apart from the above, a more secure option for stabilizing/controlling the economy. This is in stark contrast to the findings of BG (2001).

6.4 Type I and II errors

Our next question relates to the robustness of our previous findings. Do the same conclusions hold when we accommodate for the possibility that the central bank holds incomplete information regarding the current state of the economy, and therefore can not say with certainty whether the economy is exposed to a positive asset price wedge (stock market boom), or not. In effect, the policy targets will be set conditional on the information that the central bank authority holds at that specific moment, which might generate errors.[33]In other words, the issue is what are the (additional) losses generated by the central bank when it thinks there is an asset price boom and reacts to it despite there not being one (Type I error), and when it thinks that the wedge is zero and does not reacts to it, when there is one (Type II error)? In order to answer this question, we calculate the (minimum) losses that a standard monetary policy rule generates in a canonical BGG framework (where there is no wedge) and in this paper's model (where a positive wedge exists), and compare it to the (minimum) losses that an asset-price augmented policy (conditional on the same weights on inflation and output) generates in a canonical BGG framework and in this paper's framework. The diagram below summarizes the experiment design:

Table 1: The Game

Asset price wedge scenarios /Policy response	Positive wedge	No wedge	State-independent losses
Asset price targeting	Losses	Losses from error type I	Total losses
No asset price targeting	Losses from error type II	Losses	Total losses

We assume in this game that the likelihood of an economy having and not having a wedge to be equal, which means that the probability of a wedge is 50 percent. Therefore following the principles from expected utility theory (which indicates that the losses of each entry in 1 should be premultiplied by the probabilities of each state), we can simply add up the losses in the rows of Table 1 and make a state-independent judgment about the welfare improving and favored policy rule. In other words, what is the end result from subtracting total losses in the first row to the total

[33]See Mishkin (2004).

losses in the second row, or the relative gains from using an asset price augmented policy rule? We wish to compare our findings to the conclusions of BG (2001) that a monetary policy excluding asset price target is optimal.

To conclude our discussion on monetary policy, we will relax the above assumption of equal probability between having and not having a wedge, and ask how much lower the probability of the wedge has to be in comparison to the probability of no wedge in order for the monetary authority to be indifferent between the two policies? Expressed differently, at what probability of the wedge do the two policy rules generate the same losses, conditional on the same weights on inflation and output? In terms of the game above, we need to find the probabilities of each scenario/state at which the difference between the entries in the third column is zero.

6.5 Desirable monetary policy when errors type I and II are included

Table IV.3 reports the results from our simulations when both errors I or II are included in the losses of each policy. To get these values, we have added up the losses generated by each one of the two policies in both the positive wedge-and no wedge scenarios. In this way, we can compare the total losses a specific reaction function produces, independent of the state of the current economy.

For similar weights on output and inflation in both reaction functions, we find that the policy which additionally reacts to market prices generates overall smaller losses. Putting it differently, in order for the standard policy to generate the same losses as the policy reacting to market prices, the weight on output has to be double as large (while that of inflation is maintained the same). For instance, a policy augmented for market prices with optimal weights on inflation, output, and market prices of $[0.96, 0.54, 0.24]$ generates a total loss of 0.008 when error type I is included. In order for the standard policy to generate the same total loss (inclusive of error II) as the previous, the optimal weight on output has to be twice as large, or 1.01 (instead of 0.54 before). Likewise, with the optimal weights on inflation, output, and market prices of $[1.71, 0.79, 0.59]$, the policy which reacts to (stock) price movements generates a total loss of 0.0068. For the standard policy to reach that same level of losses, the weights on inflation and output need to be $[1.74, 1.76]$, or the responsiveness of the policy to movements in output needs to be more than twice as aggressive compared to a market price augmented rule.

To simplify somewhat the interpretation of the losses across the two policies, let us calculate the average losses for each policy when the problem of error I for the market price augmented rule, and error II for the standard rule are taken into account. In other words, what is the average loss for each reaction function in Table IV.3? We find that the average loss for a standard policy rule, $[E_t(\pi_{t+1}), y_t]$ is 0.00753, while for a stock market augmented one, $[E_t(\pi_{t+1}), y_t, s_t]$ it is 0.00687. This confirms our conclusions so far that a monetary policy which explicitly targets (stock) market prices on the whole generates around 10 percent smaller losses, or equivalently, is consumer welfare improving.

Hence, in a financial accelerator framework where investment is a function of an endogenous asset price wedge, and the central bank holds incomplete information regarding the true state of the economy, it is desirable for the policy maker to, apart from expected inflation and output, react to market price movements. The aggressive output and inflation only policy generates, on average, 10 percent higher consumption losses than the policy including an asset price target. Albeit both papers use a similar method to compare the two monetary policy rules, our conclusions are significantly different from the ones of BG (2001). We believe that the difference is due to the fact that investment in our framework is dependent on the wedge (that is closely related to the market-to-book ratio used in practice), which means that stock market booms and busts affect firm balance sheet and income flows, and therefore the intensity of the financial accelerator effect. Since the intensity is higher in our framework compared to theirs, a policy that targets stock market developments will be more stabilizing since it will more effectively control the expectations on the stock markets (via the residual earnings process), and therefore speculative bubbles will be minimized, or at best avoided. In addition, there might be measurement differences. While we measure the preference for one policy over the other by comparing the losses that each policy generates in terms of foregone consumption, a welfare function which we derive from a second-order approximation of consumers' optimization, BG (2001) measure the optimality of a policy in terms of the standard deviations of inflation and output that each policy generates. Hence, their loss function is pre-determined exogenously, and the weights on each variable in the loss function are assumed to be identical and equal to one.

Lastly, we wish to find a probability of the wedge at which the monetary authority is indifferent between the two policies. Calculating from the output tables IV.1 and IV.1, it turns out that the probability of the wedge has to be between 2 and 6 percent lower than the probability of an economy without a wedge in order for the two policies to generate the same losses (see Table IV.4. The slight difference in percentage rates is due to the variation in weights on inflation and output in the monetary policy rule that we impose. For weights on inflation below 2 and weights on output below 1, the probability of a wedge (no wedge) needs to be 47 (53) percent in order to make the central banker indifferent between the policies. On the other hand, for weights on inflation of 2, and output of 1, the probability of the wedge (no wedge) needs to be 49 (51) percent in order to generate this indifference. The latter set of weights on inflation and output is the ones that Bernanke and Gertler (2001) use to show that a standard policy rule is superior to an asset price augmented one in their model. Thus, our results overturn their conclusion, since in order for the standard policy rule to be preferred, the probability of an economy without a wedge has to be at least 3 percent higher, which ex ante is not reasonable to assume.

7 Summary and Conclusions

The recent financial turmoil has revived the debate on whether central banks should respond to asset price movements. Previous studies in this literature had shown that strong inflation targeting was sufficient in bringing down the stock market booms and control the economy, often relying on the premises that the expansionary effects of asset price bubbles are well captured by inflation and output. Moreover, since bubbles are hard to observe, a policy maker that tries to target them might increase the range of instability of models (Bullard and Shaling (2002)). But recent empirical studies from the current financial crisis have pointed out that the unusually low Federal Funds rate during the entire boom period in the 2000s contributed largely to feeding the stock-and property market bubbles. Capitalizing on these new observations, our paper returns to the pre-crisis debate, and in a financial frictions model with endogenous asset price wedge and (stock) market contingent investment of firms, investigates what role neglecting asset prices from monetary policy play in a bubble, and how much less of a bubble might there have been if alternative policy rules had been employed.

We find that making investment a function of an endogenous asset price wedge improves the impulse responses and the fit of the financial accelerator model to the post-2000 US data. Not only does the model better match the data on capital and firm balance sheet such as market value of capital, investment, and firm net worth compared to the standard Bernanke, Gertler and Gilchrist (1999) model, but also the financial, such as the policy rate and the rate of return on capital. Moreover, the endogenous responses to exogenous disturbances are on average two to three times more volatile than in the benchmark BGG model. More interestingly, the model generates an endogenous asset price wedge without the necessity to directly employ a shock to the wedge, which had been the *modus operandi* for most DSGE models until now. The role of monetary policy in an asset price boom is crucial. Experiments using (model derived) quadratic approximate welfare measure of consumers show that consumers are, on average 10 percent better off in terms of foregone consumption with a monetary policy that explicitly targets stock market prices compared to a policy that only targets inflation and output, as in Bernanke and Gertler (2001). The superiority of this policy is maintained even when we take into consideration that for central bankers might hold incomplete information regarding the true state of the asset price wedge. We interpret our results as overturning those of BG (2001) since in order for the standard policy rule to be preferred, the *ex ante* probability of an economy without a wedge has to be higher than the probability of a wedge, which is not reasonable to assume.

Bordo and Jeanne (2002), and Chadha et al (2004) argue that the monetary authority might be employing a non-linear policy rule in asset prices. An interesting extension would therefore be to perform a welfare analysis for non-linear policy rules within our framework. Looking beyond, although the BGG model incorporates financial imperfections, it would be interesting to assess the general impact on the economy of incorporating an endogenous intermediation sector as a driver of market liquidity for magnitudes that we observed on markets during the 2000s. We are therefore also interested in exploring the robustness of our conclusions to these alternative mechanisms that may provide a more realistic characterization of the financial sector.

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I Tables and Figures

Table I.1: VAR(2) model output

Lags	Market-to-book wedge	Book value	Corporate bond rate	Investment demand
Wedge (-1)	0.97*	0.01	0.29	0.013*
Wedge (-2)	-0.05*	0.002	0.04	-0.03*
Book value (-1)	-0.21	0.8*	0.7	0.08
Book value)-2)	0.15	0.19*	-1.57	-0.07
Bond rate (-1)	-0.04*	-0.003	1.06*	-0.003
Bond rate (-2)	0.03	0.003	-0.13*	0.003
Investment (-1)	-0.24	0.12*	1.81	1.59*
Investment (-2)	0.32	-0.10*	-0.95	-0.60*

Note: Stars in each entry indicate that the coefficient is significant at 5 percent significance level. For book value and investment demand, we have used the data on *replacement cost of capital* ((HCVSNNWHCB-SNNCB) and *private nonresidential fixed investment* respectively. For the corporate bond rate, we have used *Moody's 30-year BAA yield* (BAA), and the wedge is the difference between the *market value of equities* ((MVEONWMVBSNNCB) and the book value above. The acronyms in paranthesis denote the labeling of the dataset in the Federal System Economic Database (FRED).

Table I.2: Correlations and variances

Correlation (Relative Standard Deviation)	Output 1953:I-2012:II	Output 1991:II-2012:II	Output 2001:IV-2012:II
Wedge	0.30	0.64	0.76
	(8.77)	(15.72)	(12.42)
Market value	0.34	0.74	0.83
	(7.22)	(11.1)	(10.33)
Book value	0.40	0.73	0.90
	(1.31)	(2.14)	(2.02)

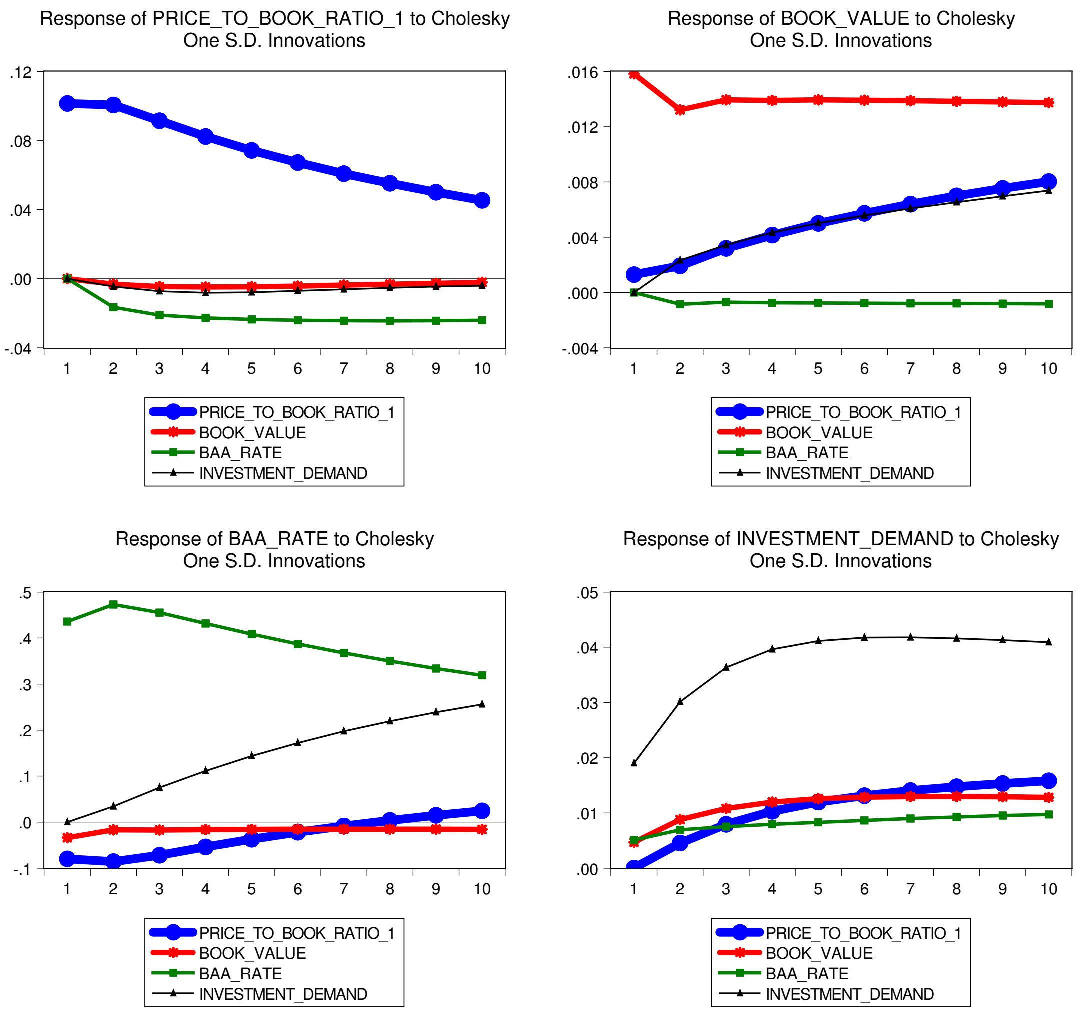


Figure I.1: VAR(2)-Impulse responses

Notes: The impulse responses to the four shocks (market-to-book, book value, corporate bond rate, and investment demand) using Cholesky decomposition are reported. Investment demand is excluding residential investment and the BAA-rate is the corporate bond rate used.

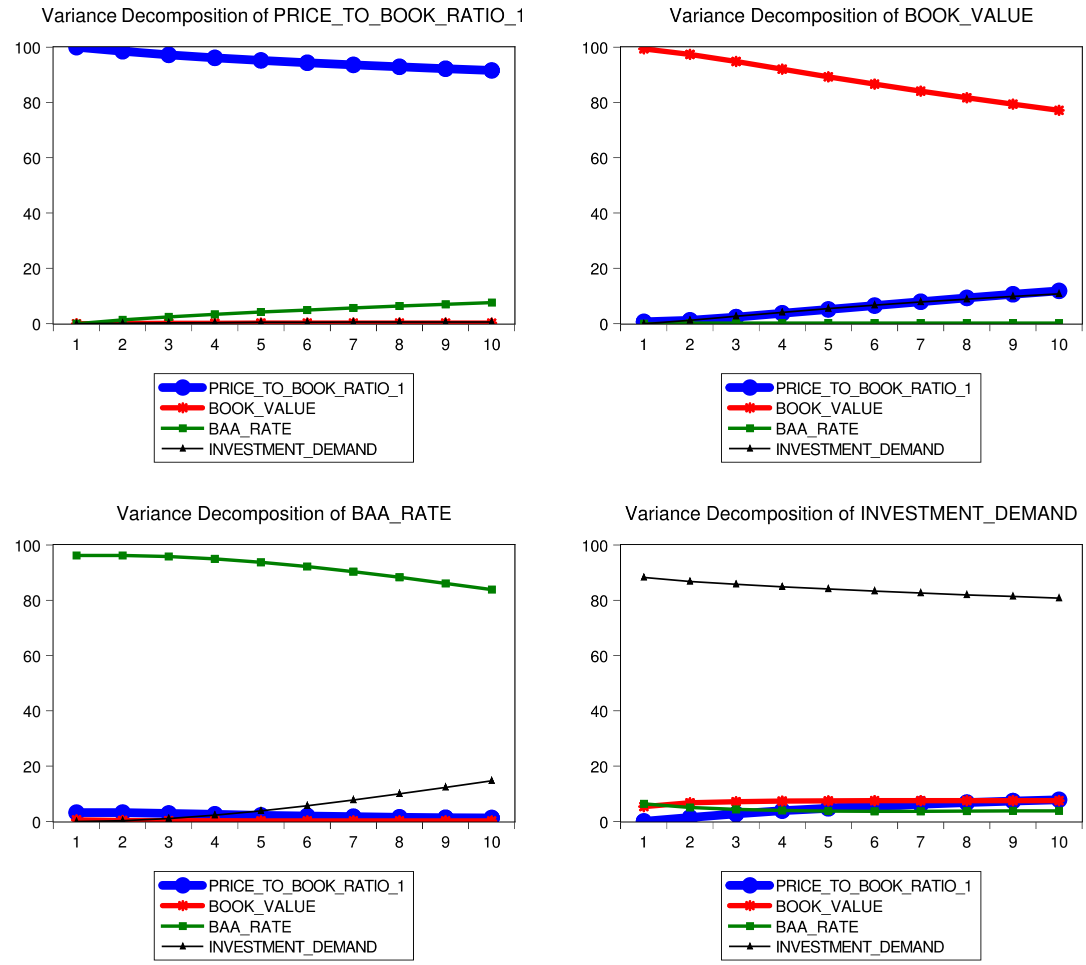


Figure I.2: VAR(2)-Variance decomposition

Notes: The variance decomposition of the four variables (market-to-book, book value, corporate bond rate, and investment demand) are reported. Investment demand is excluding residential investment and the BAA-rate is the corporate bond rate used.

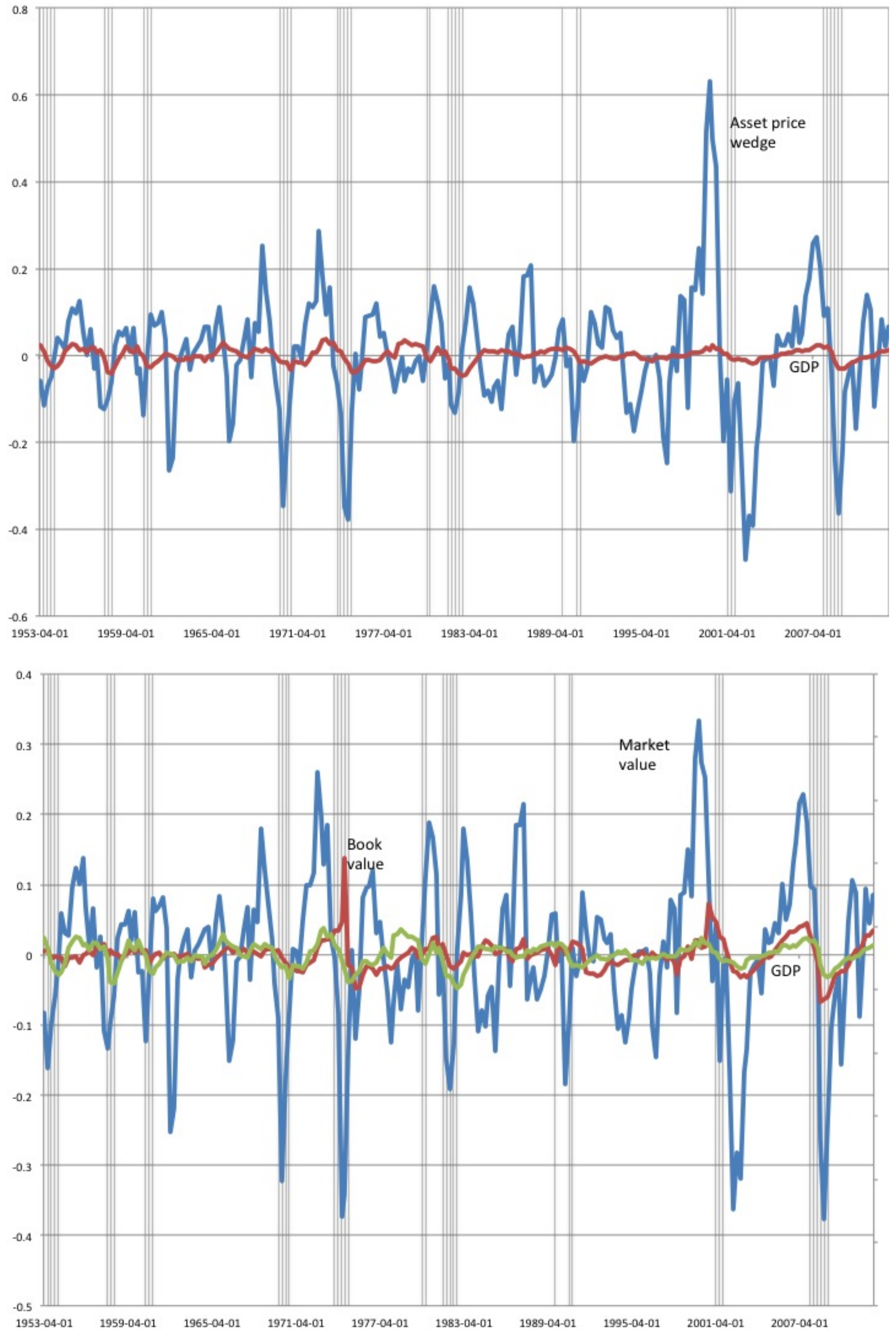
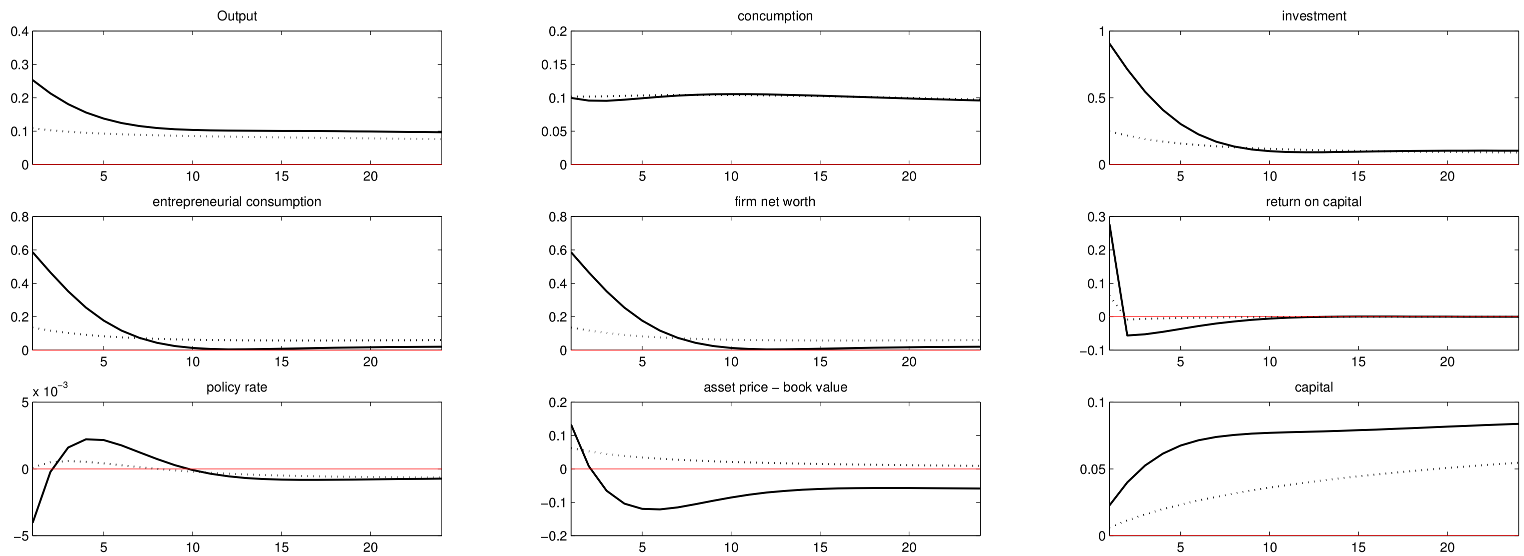


Figure I.3: Capital prices-Stylized facts

Figure I.4: Responses to a productivity shock



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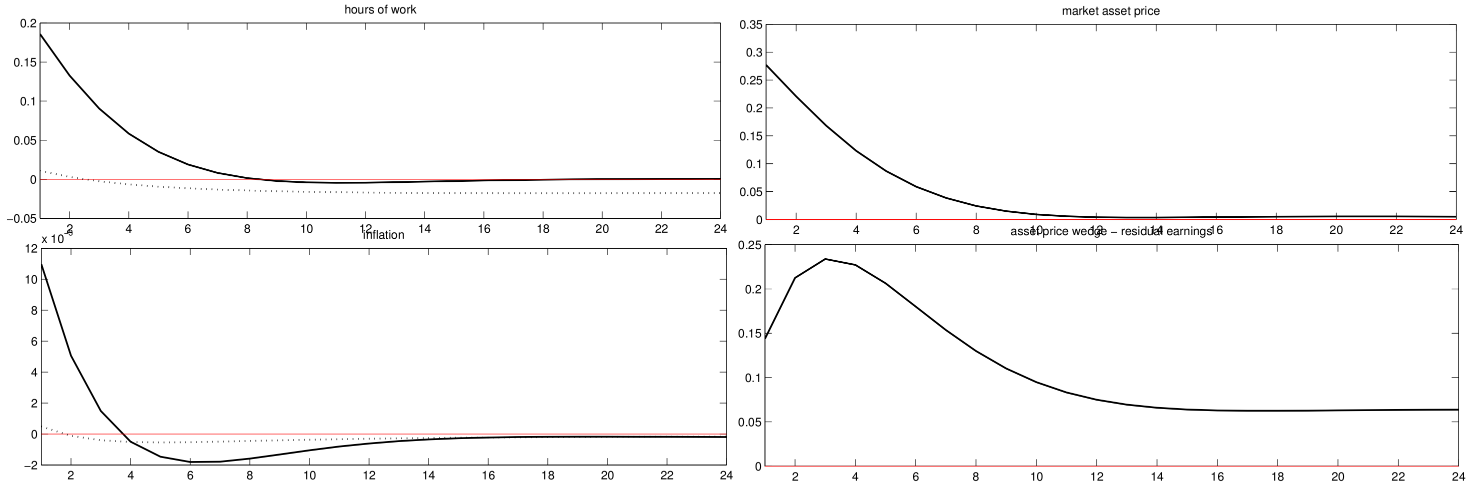
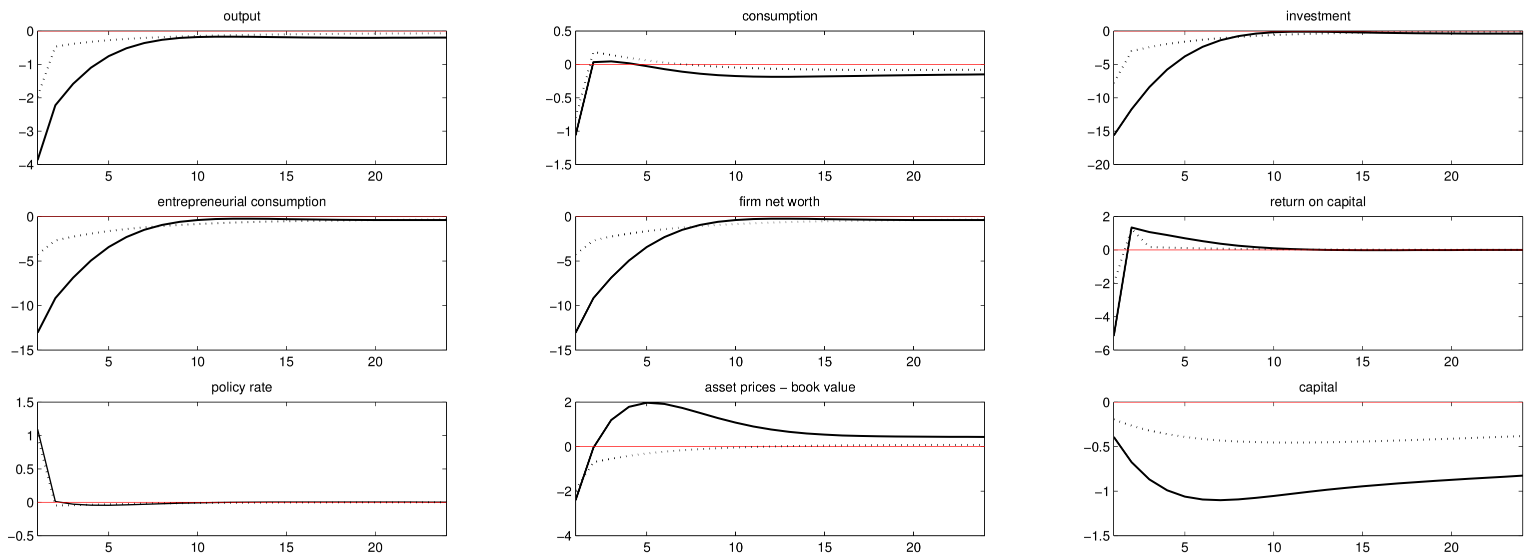


Notes: Impulse responses to a technology shock in both the canonical BGG and Gerba models. The dotted lines are the responses in the BGG, and the full lines in the Gerba model. The responses of asset prices - book value is the ‘q’ in both models, and the wedge is equivalent to residual earnings in the gerba model.

Figure I.5: Responses to a nominal interest rate shock



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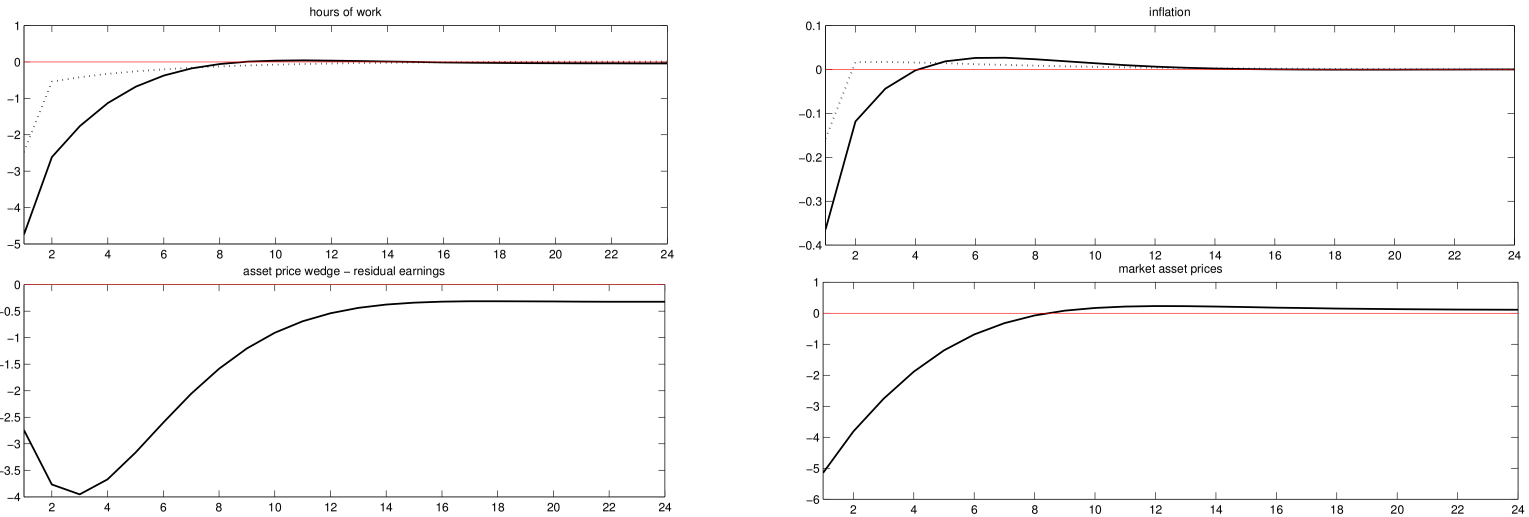
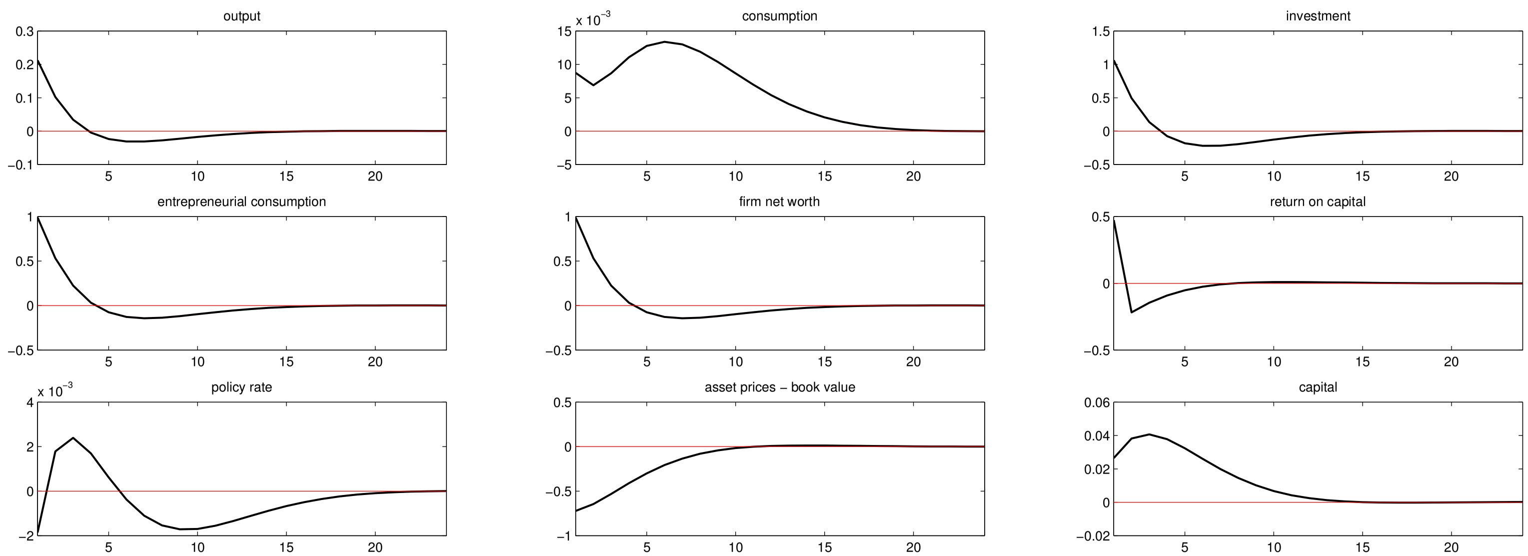


Notes: Impulse responses to a technology shock in both the canonical BGG and Gerba models. The dotted lines are the responses in the BGG, and the full lines in the Gerba model. The responses of asset prices - book value is the ‘q’ in both models, and the wedge is equivalent to residual earnings in the Gerba model.

Figure I.6: Responses to a wedge shock I



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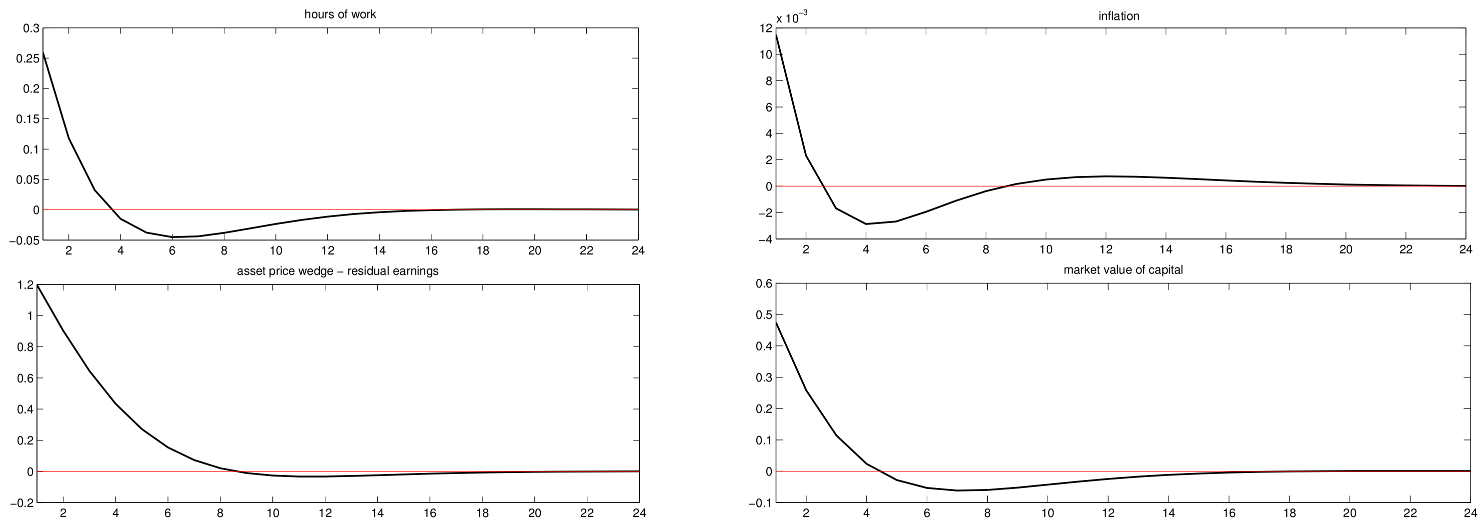

Notes: Impulse responses to a wedge shock in the Gerba model only. The responses of asset prices - book value is the ‘q’ in both models, and the wedge is equivalent to residual earnings in the Gerba model.

Table I.3: Correlations

Variables	Correlation Extended model (Gerba)	Correlation Canonical model (BGG)	Correlation US data
(consumption, output)	0.70	0.83	0.89
(book value, output)	0.50	0.98	0.90
(market value, output)	0.98	-	0.83
(asset price wedge, output)	0.69	-	0.76
(investment, output)	0.99	0.98	0.94
(capital, output)	0.21	0.20	-
(net worth, output)	0.99	0.89	0.76
(capital return, output)	0.59	0.72	0.77/0.38/0.15/0.76
(policy rate, output)	0.79	-0.93	0.76
(inflation, output)	0.92	0.90	0.46/0.52/0.76

Notes: Correlations generated by the two versions of the financial accelerator model are compared to US data. The data correlations were obtained from 2000:I to 2011:II.

Table I.4: Relative Standard Deviations

Variables	Rel.std.dev. Extended model (Gerba)	Rel.std.dev. Canonical model (BGG)	Rel.std.dev. US data
(consumption, output)	0.25	0.46	1.88
(book value, output)	1.0	1.04	2.01
(market value, output)	1.52	-	10.33
(asset price wedge, output)	1.45	-	12.42
(investment, output)	4.53	4.12	5.87
(capital, output)	0.29	0.23	-
(net worth, output)	3.67	2.47	4.52
(capital return, output)	1.36	1.29	0.94/0.25/0.38/0.91
(policy rate, output)	0.26	0.53	0.85
(inflation, output)	0.09	0.09	0.36/0.71/2.67

Notes: Relative standard deviations generated by the two versions of the financial accelerator model are compared to US data. The data standard deviations were obtained from a post-2000 sample ranging from 2000:I to 2011:II. We report four US data variables as an analogue to the capital return variable in the model: 3-month Prime rate, Long-term AAA-rate, Long-term BAA-rate, 3-month Corporate paper rate. Likewise for inflation, we use GDP deflator/CPIU/PPIU.

II Models

II.1 The Ohlson (1995) model

The model uses two standard characteristics of the accounting models, and one behavioral assumption in order to characterize the wedge between market and book values within the neoclassical framework. Therefore, as Rubinstein (1976) shows, the value of an asset can be expressed as the present value of expected dividends (PVED):

$$P_t = \Sigma_{i=1}^{\infty} \frac{E_t \left[d_{t+i}\right]}{(1+r)^i} \tag{II.1}$$

where P_t is the market value of capital, D_t are dividends, and r is the risk-free rate. A two-step procedure derives a particularly parsimonious expression for residual earnings, or goodwill, which collects the difference between the market and book value of assets. First, the clean surplus relation:

$$Q_t - Q_{t-1} = e_t - d_t \tag{II.2}$$

implies the restriction that dividends reduce current book value, but not the current earnings (but negatively the future), i.e.:

$$\frac{\partial Q_t}{\partial d_t} = -1 \tag{II.3}$$

$$\frac{\partial e_t}{\partial d_t} = 0 \tag{II.4}$$

$$\frac{\partial E_t\left[e_{t+1}\right]}{\partial d_t} = -(r-1) \tag{II.5}$$

Peasnell (1981,1982) shows that this condition is sufficient to express market valued in terms of future expected earnings and book value (instead of the sequence of expected dividends). To do so, let us first define residual earnings as:

$$re_t \equiv \frac{e_t}{(r-1)q_{t-1}} \tag{II.6}$$

Combined with the clean surplus condition above (expression II.2, we can express dividends in terms of:

$$d_t = re_t - Q_t + RQ_{t-1} \tag{II.7}$$

Iterating the last expression forward for d_{t+1}, d_{t+2}, etc. and re-inserting it into PVED, we get:

$$P_t = Q_t \sum_{i=1}^{\infty} \frac{E_t\left[re_{t+i}\right]}{r^i} \tag{II.8}$$

provided that $\frac{E_t[y_{t+i}]}{r^i} \to 1$ as $i \to \infty$. Residual earnings is motivated by the concept that 'normal earnings' are return on the capital invested at the beginning of the period, which are equal to the (replacement) cost of using the capital, i.e. $r * Q_{t-1}$ (book value at time $t-1$ multiplied by the (risk-free) interest rate).[34]Hence,

[34]The model assumes risk neutrality and homogeneous beliefs, even though Ohlson (1995) has extended the model to include other risk preferences.

during profitable periods, earnings are above the cost of using the capital, or the same as saying positive 'residual earnings'. One can link this idea back to the Bank of England 2012 report by conceptualizing the profitable periods as periods of optimism. During periods of high market confidence, the capital is expected to generate a present value of future earnings above the required earnings demanded by investors, or the same as saying, positive residual earnings. In other words, the future profitability of capital, as measured by the present value of future (anticipated) residual earnings sequence reconciles the difference between the market and the book value of capital.

Second, to complete the model the time-series behavior of residual earnings need to be specified. Ohlson (1995) assumes an autoregressive process

$$re_{t+1} = \alpha re_t + v_t \tag{II.9}$$

where α is restricted to be positive, and v_t is a scalar variable that represents information regarding future expected (residual) earnings other than the accounting data and dividends. Ohlson (1995) motivates it by the idea that some value-relevant events may affect future expected earnings as opposed to current earnings which means that accounting measures incorporate these value-relevant events only after some time. The scalar information variable is independent of past residual earnings since the value relevant events have yet to have an impact on the financial statements. This is the same as saying:

$$\frac{\partial v_t}{\partial re_{t-1}} = 0 \tag{II.10}$$

since it captures all non-accounting information used in the prediction of future residual earnings. On the other hand, the variable may depend on past realizations of the same scalar (even if that is not necessary), since they can feed expectations about future earnings via past beliefs.

Given the assumption of the stochastic process of residual earnings, one can evaluate $\Sigma_{i=1}^{\infty}\frac{}{r^i}$, and reduce expression II.8 to:

$$P_t = Q_t * \alpha re_t * v_t \tag{II.11}$$

Market value can now be reduced to a composite of book value, residual earnings measuring current profitability and other information that modifies the prediction of future profitability. Rearranging this expression and using the definition of residual earnings in II.6, one can also express next period's expected (total) earnings as:

$$E_t\left[e_{t+1}\right] = (r-1)Q_t * \alpha re_t * v_t \tag{II.12}$$

Note that future earnings only partially depend on the current book value.

Since next period (expected) earnings are formed using information set available up to period t for all the three components (book value, residual earnings, and information), the expression poses no problem. However, for earning forecasts two periods ahead, the model yields no prediction since information from period $t+1$ is necessary in order to forecast this variable.

To conclude, though the process $[P_t - Q_t]$ allows for serial correlations over sufficiently long periods of time, the average realization approximates zero. This means that in the very long-run, book value will become the unbiased estimator of market value.

II.2 The financial accelerator model and the optimization problems

II.2.1 Households

The representative risk-averse household maximizes its lifetime utility, which depends on consumption C_{t+k}, real money balances M/P_{t+k} and labor hours (fraction of hours dedicated to work=H_{t+k}):

$$maxE_t \sum_{k=0}^{\infty} \beta^k [ln(C_{t+k}) + \varsigma ln \frac{M_{t+k}}{P_{t+k}} + \theta ln(1 - H_{t+k})] \tag{II.13}$$

constrained by lump-sun taxes he pays in each period T_t, wage income W_t, and dividends he earns in each period from owning the representative retail firm $\prod_t$, and real savings he deposits in the intermediary,D_t according to the budget constraint:

$$C_t = W_t H_t - T_t + \prod_t + R_t D_t - D_{t+1} + \left[\frac{M_{t-1} - M_t}{P_t}\right] \tag{II.14}$$

The household takes W_t, T_t, and R_t as given and chooses C_t, D_{t+1}, H_t and M/P_t to maximize its utility function subject to the budget constraint.

II.2.2 Entrepreneur

The other key agent in this model, the representative entrepreneur, chooses capital K_t, labor input L_t and level of borrowings B_{t+1}, which he determines at the beginning of each period and before the stock market return has been determined, and pays it back at the end of each period as stated by $R_{t+1}[Q_t K_{t+1} - N_{t+1}]$ (where R_{t+1} is the risk-free real rate that borrowers promise lenders to pay back on their loans, R^{ks}_{t+1} is the return on market value of assets, K_{t+1} is the quantity of capital purchased at 't+1' and N_{t+1} is entrepreneurial net wealth/internal funds at 't+1') to maximize his profits according to:

$$V = \max E_t \sum_{k=0}^{\infty} [(1-\mu) \int_0^{\varpi} \omega dF \omega U^{rk}_{t+1}] E_t(R^{ks}_{t+1}) S_t K_{t+1} - R_{t+1}[S_t K_{t+1} - N_{t+1}] \tag{II.15}$$

with μ representing the proportion of the realized gross payoff to entrepreneurs' capital going to monitoring, ω is an idiosyncratic disturbance to entrepreneurs' return (and ϖ is hence the threshold value of the shock), $E_t R^{ks}_{t+1}$ is the expected stochastic return to stocks, and U^{rk}_{t+1} is the ratio of the realized returns to stocks to the expected return ($\equiv R^{ks}_{t+1}/E_t R^{ks}_{t+1}$). The entrepreneur uses household labor and purchased capital at the beginning of each period to produce output on the intermediate market according to the standard Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{II.16}$$

where Y_t is the output produced in period 't', A_t is an exogenous technology parameter, K_t^{α} is the share of capital used in the production of output, and $L_t^{1-\alpha}$ is the labor share. The physical capital accumulates according to the law of motion:

$$K_{t+1} = \Phi \frac{I_t}{K_t} K_t + (1-\delta) K_t \tag{II.17}$$

with $\Phi \frac{I_t}{K_t} K_t$ denoting the gross output of new capital goods obtained from investment I_t, under the assumption of increasing marginal adjustment costs, which we capture by the increasing and concave function Φ. δ is the depreciation rate of capital.

The entrepreneur borrows funds from the financial intermediary, as a complement to its internal funds, in order to finance its purchase of new capital, which is described by the following collateral constraint:

$$B_{t+1} \leq N_{t+1} \tag{II.18}$$

and the cost of external funding, which is represented by the external finance premium (EFP) condition:

$$\frac{E_t(R^{ks}_{t+1})}{R_{t+1}} = s\left[\frac{N_{t+1}}{S_t K_{t+1}}\right] \tag{II.19}$$

By assuming a fixed survival rate of entrepreneurs in each period, the model assures that entrepreneurs will always depend on external finances for their capital purchases, and are further assumed to borrow the maximum amount, subject to the value of their collateral (which means that the collateral constraint will bind with equality).

To complete the model, let us look at the maximization problem of the remaining agents in the model: financial intermediaries, retailers and government.

II.2.3 Financial intermediary

The role of the financial intermediary in this model is to collect the deposits of savers, and to lend these funds out to borrowers through 1-period lending contracts against a risk-free return R_t that households demand on their deposits. Because of information asymmetries between lenders and borrowers, the intermediary needs to invest some costly monitoring of the borrowers in order to assure that borrowers survive to the next period and pay back the return on deposits. Therefore the wedge between the rate they charge entrepreneurs for their borrowings, R^{ks}_t, and the one that they pay out to households for their deposits R_t reflects this monitoring cost. Most importantly, the intermediary can not lend out more than the deposits they have (incentive constraint), and it operates in a perfectly competitive market. This means that in each period, intermediaries choose a level of borrowings B_t in order to maximize:

$$maxF = E_t \sum_{t=0}^{\infty} (R^{ks}_{t-1} B_t - B_{t+1}) - (R_{t-1} D_t - D_{t+1}) - \mu(\omega R_{t+1} Q_t K_{t+1}) = \pi_t \tag{II.20}$$

where $\mu\omega R_{t+1}Q_tK_{t+1}$ is the monitoring cost of borrowers. The amount of lending is constrained by the incentive constraint:

$$B_{t+1} \leq D_{t+1} \tag{II.21}$$

and the intermediary makes zero profits in each period:

$$\prod_{t=0}^{\infty} \pi_t = 0 \tag{II.22}$$

II.2.4 Retailer

To incorporate the nominal rigidities, a standard feature of New-Keynesian models, we incorporate a retail sector into this model. Let us look at retailers' problem. They set their price of the final good according to the standard Calvo process (1983), where P_t^* is the price set by retailers who are able to change prices in period 't', and let $Y_t^*(z)$ denote the demand given this price. Then, retailer 'z' chooses P_t^* in order to maximize:

$$maxR = \theta^k E_{t-1} \sum_{k=0}^{\infty} \left[\Lambda_{t,k} \frac{P_t^* - P_t^{\omega}}{P_t} Y_{t+k}^*(z) \right] \tag{II.23}$$

with θ^k being the probability that a retailer does not change his price in a given period, $\Lambda_{t,k} \equiv \beta C_t / C_{t+1}$ denoting the household intertemporal marginal rate of substitution (since households are the shareholders of the retail firms), which they take as given, and $P_t^{\omega} \equiv P_t / X_t$ denoting the nominal price of goods produced by a retailer (X_t is the gross markup of retail goods over wholesale goods). They face a demand curve equal to:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t^f \tag{II.24}$$

where $P_t(z)/P_t^{-\epsilon}$ is the nominal price ratio of wholesale goods for retailer 'z', $\epsilon > 1$ is a parameter on retail goods, Y_t^f is the total final output in the economy which is composed by a continuum of individual retail goods, and P_t is the composite nominal price index of a continuum of individual prices set by retailers.

II.2.5 Government

Finally, a government plans spending, and finances it by either lump-sum taxes, or money creation (Central Bank division). In each period, it chooses spending G_t, and a combination of taxes T_t and money creation M_t so to fulfill the balanced budget condition:

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t \tag{II.25}$$

It chooses money creation for budget financing according to a standard Taylor rule:

$$\frac{R_t^n}{R_{t-1}^n} = \rho + \xi E_t \left[\pi_{t+1}\right] \tag{II.26}$$

where R_t^n is the policy rate in period 't', ρ is the coefficient of interest rate growth, and ξ is the coefficient on expected inflation in the Taylor rule.

II.3 Solutions

Solving the households optimization problem yields standard first order conditions for consumption,

$$\frac{1}{C_t} = E_t \left(\beta \frac{1}{C_{t+1}} \right) R_{t+1} \tag{II.27}$$

labor supply,

$$W_t \frac{1}{C_t} = \theta \frac{1}{1 - H_t} \tag{II.28}$$

and real money holdings,

$$\frac{M_t}{P_t} = \varsigma C_t \left[\frac{R_{t+1}^n - 1}{R_{t+1}^n} \right]^{-1} \tag{II.29}$$

The last equation implies that real money balances are positively related to consumption, and negatively related to the nominal/policy interest rate. Turning to the entrepreneur, his choice of labor demand, capital, and level of borrowings yields the following optimization conditions:

$$(1\alpha) \frac{Y_t}{L_t} = X_t W_t \tag{II.30}$$

$$\tag{II.31}$$

$$\tag{II.32}$$

As is common in the literature, marginal product of labor equals their wage. Labor input in the production of wholesale goods can either come from household or entrepreneurs own labor supply (ie. they can devote a small fraction of their own time to the production activity). Therefore entrepreneurs receive income from supplying labor based on the wage rate above. Since that income stream is assumed to be marginal in this model however, we can assume that the proportion of entrepreneurial labor used for production of wholesale goods is so low that it can be ignored, so that all of labor supply is provided by the household sector.

Continuing our analysis with the retailers, and differentiating their objective functions with respect to P_t^* gives us the following optimal price rule:

$$\theta^k E_{t-1} \sum_{k=0}^{\infty} \left[\Lambda_{t,k} \frac{P_t^*}{P_{t+k}^*}^{-\epsilon} Y_{t+k}^*(z) \left(P_t^* - \frac{\epsilon}{\epsilon - 1} P_t^\omega \right) \right] = 0 \tag{II.33}$$

Retailer's price will be set such that in expectation terms discounted marginal revenue equals discounted marginal cost, subject to the Calvo price setting probability θ. Since only a fraction of retailers will set their price in each period, the aggregate price evolves according to:

$$P_t = [\theta P_{t-1}^{1\epsilon} + (1\theta) P_t^*(1\epsilon)] \frac{1}{1\epsilon} \tag{II.34}$$

To conclude the optimizations, we turn to the representative intermediary. He will set the maximum level of lending such that the incentive constraint is satisfied, and subject to the competitive market condition. Differentiating his value function with respect to B_{t+1} will mean that the level of credit given to the entrepreneurial sector will be:

$$B_{t+1} = D_{t+1} \tag{II.35}$$

This condition will hold in each period, which means that the intermediarys balance sheet expansion is limited to its deposit holdings in each period.

II.4 General equilibrium

The competitive equilibrium is characterized by a set of prices and quantities, such that:

Given $W_t, R_t, \xi, \theta, D_t$ the household optimizes C_t, H_t, D_{t+1}

Given $W_t, R_t, R_t^{ks}, \gamma, \delta, z_t, \mu, N_t, K_t$ the entrepreneur optimizes $I_t, H_t, K_{t+1}, B_{t+1}, N_{t+1}$

Given $R_t, R_t^{ks}, D_{t+1}, B_t$ the financial intermediary optimizes B_{t+1}

Given $\Lambda_{t,k}, \theta, Y_t(z), P_t^{\omega}$ the retailer 'z' optimizes P_t^*

Labor, capital and financial markets clear: $H_t^s = H_t^d, K_t^s = K_t^d, D_t = B_t$

and

Aggregate demand holds: $Y_t = C_t + I_t + C_t^e + G_t$

The complete log-linearized BGG framework is presented below by the Equations II.36 through II.52. In all the equations, lower case letters denote percentage deviations from steady state, and capital letters denote steady state values:

II.5 Log-linearized model

Aggregate Demand:
Resource constraintsource constraint

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{Ce}{Y}c_t^e \tag{II.36}$$

Consumption Euler equation

$$c_t = -r_t + E_t(c_{t+1}) \tag{II.37}$$

Entrepreneurial consumption

$$c_t^e = n_t \tag{II.38}$$

Financial accelerator

$$r_{t+1}^k - r_t = -\nu(n_t - (q_t + k_t)) \tag{II.39}$$

External Finance Premium

$$efp_t = r_t^k - r_t \tag{II.40}$$

Return on capital

$$r_t^k = (1-\epsilon)(y_t - k_t - x_t) + \epsilon s_t - s_{t-1} \tag{II.41}$$

Investment accelerator

$$s_t = \psi(i_t - k_t) \tag{II.42}$$

(Stock) Market value of capital

$$s_t = q_t + re_t \tag{II.43}$$

Residual earnings and formation of stock market expectations

$$re_t = \rho_{re} re_{t-1} + (\chi)(E_t[y_{t+1}] + n_t - E_t[r_{t+1}]) + e_i \tag{II.44}$$

Aggregate Supply:

Cobb-Douglas production function

$$y_t = a + \alpha k_t + (1-\alpha)\omega h_t \tag{II.45}$$

Marginal cost function

$$y_t - h_t - x_t - c_t = \frac{1}{\eta} h_t \tag{II.46}$$

Approximated Philips curve

$$\pi_t = \kappa(-x_t) + \beta \pi_{t+1} \tag{II.47}$$

Evolution of State Variables:

Capital accumulation

$$k_t = \delta i_t + (1-\delta) k_{t-1} \tag{II.48}$$

Net worth accumulation

$$n_t = \gamma R \frac{K}{N}(r_t^k - r_t) + r_{t-1} + n_{t-1} \tag{II.49}$$

Monetary Policy Rule and Shock Processes

Monetary policy

$$r_t^n = \rho r_{t-1}^n + \zeta E_t[\pi_{t+1}] \tag{II.50}$$

Technology shock

$$a_t = \rho_a a_{t-1} + e_a \tag{II.51}$$

Real interest rate (Fisher relation)

$$r_t^n = r_t - E_t(\pi_{t+1}) - e_{r^n} \tag{II.52}$$

Table II.1: Parameters and descriptions

Parameter	Description	Value
Calibrated		Calibration
C/Y	Share of consumption in resource constraint	0.806
I/Y	Share of investment in resource constraint	0.184
C^e/Y	Share of entrepreneunal consumption in resource constraint	0.01
ϵ	Marginal product in investment demand	0.99
X	Gross markup over wholesale goods	1.10
α	Share of capital in production	0.20
Ω	Share of household labour in production	0.99
η	Labour supply elasticity	5.00
κ	Share of marginal cost in Phillips Curve	0.086
θ	Calvo pricing	0.75
β	Quarterly discount factor	0.99
δ	Depreciation rate	0.025
γ	Survival rate of entrepreneurs	0.973
R	Steady state quarterly riskless rate	1.010
K/N	Steady state leverage	2.082
ν	Elast. of EFP to leverage	0.092
ϕ	Elast of inv. demand to asset prices	0.25
ρ_{re}	AR parameter on residual earnings	0.67
χ	Parameter on the expected state of the economy in the residual earnings equation	0.18
ρ	AR parameter in monetary policy rule	0.95
ζ_f	MP response to expected inflation	0.20
ρ_a	AR parameter of productivity shock	0.99
ε_a	Std. parameter of technology shock	0.10
ε_{rn}	Std. parameter of nom. Interest rate shock	1.00
ε_i	Std. parameter of information shock	1.00

Notes: The calibrated values are standard in the literature. Following Caglar (2012), the new AR parameter in the extended model is calibrated to 0.67, in line with the corporate finance literature. Elasticity of external finance premium to leverage, we calibrate to 0.13.

Table II.2: Model variables and descriptions

Variable	Definition
y	Output
c	Household consumption
c^e	Entrepreneurial consumption
i	Investment
g	Government spending
r^n	Nominal interest rate
r	Real interest rate (also the (net) deposit rate of households)
r^k	Rate of return on capital
q	Book value of capital
s	Market value of capital
re	Residual/Abnormal earnings
efp	External finance premium
k	Capital stock
n	Entrepreneurial net worth
x	Mark-up of final good producers
h	Hours of labour input in production
π	Inflation
a	Technological progress
e_i	Information shock

II.6 Residual earnings

Let us characterize the process governing residual earnings. Our main purpose is to establish a bridge between the residual earnings and the general state of the economy. We assume that X^{re} follows an AR (1) process and make it in addition contingent on economic fundamentals in the next period, $F_{t+1}|I_{t+1}$ according to:

$$X_{t+1}^{re} = \rho_x(X_t^{re}) + F_{t+1}|I_{t+1} \tag{II.53}$$

where ρ is restricted to be positive. Following a vast number of empirical studies (outlined in section 2.3) who find a strong link between stock prices and economic fundamentals, this definition gives an important role to the evolution of the economy in determining residual earnings. Since residuals earnings are related to a firm's growth perspectives and their future earnings, the economic fundamentals that are relevant in this case are entrepreneurial output or industrial production Y_{t+1} (from II.16 in the firm's optimization problem), firm equity N_{t+1} (from II.15 and II.18 in the firm's optimization), and he nominal interest rate, R_{t+1} (from the minimization of the borrowing cost in II.19). Thus, we have given the monetary authority an additional channel through which it can influence the stock markets, by altering the prospects for residual earnings of firms. The full residual earnings process can therefore be expressed as (in log-linearized format):

$$re_{t+1} = \rho_{re}re_t + (\chi)E_t[Y_{t+1} + N_t - R_{t+1}] \tag{II.54}$$

The parameters ρ_{re} and χ determine the importance of each factor (the autoregressive process and the (expected) economic fundamentals in the next period) in determining the value of residual earnings.

III Welfare function

III.1 Deriving the second-order approximate function

Our monetary policy objective is derived as the second order approximation to utilities of the household and of the entrepreneur according to:

$$E_0 \sum_{t=0}^{\infty} \left[\frac{C}{Y} U_t^c + \frac{C^e}{Y} U_t^e \right] \tag{III.1}$$

where $\frac{C}{Y}$ and $\frac{C^e}{Y}$ represent the weight coefficients given by the steady state value of consumption over output for the household and the entrepreneur respectively. We follow the methods proposed by Woodford (2003), Gali and Monacelli (2004), Chadha et al (2010), and DeFiore and Tristani (2012). In words of Woodford (2003), our aims of this exercise are to derive an explicit expression for the stabilization loss with which we can evaluate alternative monetary policies, and identify those policies that make this quantity as small as possible. This method is more convenient than other proposed in the literature, such as the optimal simple policy rule of Levine (1991) in that it is time consistent, and hence the choice of optimal rule will not depend on the initial level of interest rate.

We will denote the second order approximation of percent deviations in terms of log deviations by the following generic expression:

$$\widetilde{X} = \frac{X_t - X}{X} \approx X(\widehat{X}_t + \frac{1}{2}\widehat{X}_t^2 + O^3) \tag{III.2}$$

with $\widetilde{X}$ denoting the second-order approximation of X_t in terms of log deviations, $\widehat{X}_t$ denoting the log-deviations of X_t from the steady state, and the term O^3 collects all terms of order third and higher on the amplitude of the relevant shocks.

Since households utility is separable between consumption, labor and real money holdings, and entrepreneurs is separable between investment and consumption. we can consider the second-order approximation to each term separately.

The second-order approximation to the saver's consumption is given by:

$$\ln(c_t) = U_c \breve{c}_t + U_{cc} \frac{\breve{c_t^2}}{2} + O^3 = \frac{1}{c} c(\hat{c}_t + \frac{1}{2}\hat{c_t^2}) - \frac{1}{c^2} \frac{c^2 c_t^2}{2} + O^3 = \hat{c}_t + O^3 \tag{III.3}$$

The second-order approximation to the saver's money holdings is given by:

$$\xi \ln(\frac{M_t}{P_t}) = \xi U_{M/P}(\breve{M/P})_t + U_{(M/P)(M/P)}\frac{\breve{M/P_t^2}}{2} + O^3 = \xi\frac{1}{(M/P)}\frac{M}{P}(\hat{M_t/P_t}\frac{1}{2}(\hat{M_t/P_t}^2)) \\ -\xi\frac{1}{(M/P)}\frac{(M/P)^2(M_t/P_t)^2}{2} + O^3 = \xi \hat{M_t/P_t} + O^3 \tag{III.4}$$

The second-order approximation to the saver's labor is given by:

$$\varsigma \ln(1-H_t) = \varsigma U_{1-H_t} \breve{1-H_t} + U_{(1-H_t)(1-H_t)}\frac{\breve{1-H_t^2}}{2} + 0^3 = \varsigma\frac{1}{1-H}1-H(\hat{1-H_t} + \frac{1}{2}(\hat{1-H_t})^2) \\ -\frac{1}{(1-H)^2}\frac{(1-H)^2(1-H_t)^2}{2} + O^3 = \varsigma \hat{1-H_t} + O^3 \tag{III.5}$$

The second-order approximation to the borrowers investment is expressed as:

$$\ln(i_t) = U_I\breve{i}_t + U_{II}\frac{\breve{i}_t^2}{2} + O^3 = \frac{1}{i}i(\hat{i}_t + \frac{1}{2}\hat{i}_t^2) - \frac{1}{i^2}\frac{i^2 i_t^2}{2} + O^3 = \hat{i}_t + O^3 \tag{III.6}$$

Finally, the second-order approximation to the borrowers consumption is expressed as:

$$\ln(c_t^e) = U_{C^e}\breve{c}_t^e + U_{C^eC^e}\frac{\breve{c}_t^{e\,2}}{2} + O^3 = \frac{1}{c^e c^{e2} c_t^{e2}}2 + O^3 = \hat{c}_t^e + O^3 \tag{III.7}$$

III.2 Simplifying the loss function

Adding the above partial approximations, we get a second-order approximation to the welfare function:

$$U_t - U \approx \frac{C}{Y}\left(\hat{C}_t + \xi(\hat{M_t/P_t}) + \varsigma(\hat{1-H_t})\right) + \frac{C^e}{Y}(\hat{C}_t^e + \hat{I}_t) + O^3 \tag{III.8}$$

We proceed with a number of simplifications. When the economy remains in the neighborhood of an efficient steady state, we get that the inflation is constant, which means that money grows at a constant rate, hence: $\Delta\bar{M}_t = M$, which implies zero inflation, and hence $\Delta P_t = P$.

To eliminate consumption, entrepreneurial consumption, and investment terms, we consider a second-order approximation of the aggregate resource constraint:

$$Y(\hat{Y}_t+\frac{1}{2}\hat{Y}_t^2) = C(\hat{C}_t+\frac{1}{2}\hat{C}_t^2)+C^e(\hat{C}_t^e+\frac{1}{2}(\hat{C}_t^e)^2)+G(\hat{G}_t+\frac{1}{2}\hat{G}_t^2)+I(\hat{I}_t+\frac{1}{2}\hat{I}_t^2) \quad \text{(III.9)}$$

which re-arranging, gives:

$$C(\hat{C}_t+\frac{1}{2}\hat{C}_t^2)+C^e(\hat{C}_t^e+\frac{1}{2}(\hat{C}_t^e)^2)+I(\hat{I}_t+\frac{1}{2}\hat{I}_t^2) = Y(\hat{Y}_t+\frac{1}{2}\hat{Y}_t^2)-G(\hat{G}_t+\frac{1}{2}\hat{G}_t^2) \quad \text{(III.10)}$$

To eliminate the first and second order terms $\hat{G}_t$ and $\hat{G}_t^2$, we consider that in a stationary equilibrium, money grows at a constant rate, which implies zero inflation. Since the government runs a balanced budget, and the growth in real money creation is zero, the taxes recollected must be constant, $T = \bar{T}$, since we only consider lump-sum taxes in this model. Therefore, the second-order approximation to the government budget constraint is:

$$G(G^t + \frac{1}{2}G_t^2) = 0 + 0^3 + t.i.p \quad \text{(III.11)}$$

where $t.i.p$ denotes terms independent of policy. For the household, his time that he can spend on work and leisure has an upper bound. In a continuous interval of [0,1], it means that the time he spends on work is inversely related to the time he spends on leisure, and so utility of leisure can be expressed as disutility of work. Therefore in the above utility function of a household, the last term can be expressed as $ln(H_t)$ instead of $+ln(1H_t)$, and so we use that notation hence after. To eliminate $\hat{H}_t$ which allows us to express the welfare function in terms of output, we make use of both the production function and the aggregate intra-termporal demand function for retail goods to get:

$$H_t^{1-\alpha} = \frac{Y_t}{A_t K_t}^{\frac{1}{\alpha}} \int_0^1 (\frac{P_t(z)}{P_t})^{\frac{-\epsilon}{1-\alpha}} \quad \text{(III.12)}$$

which in log-linear form is:

$$(1-\alpha)\alpha\hat{H}_t = \hat{Y}_t - \hat{A}_t + \hat{K}_t + \hat{dp}_t \quad \text{(III.13)}$$

with $\hat{dp}_t$ denoting the final goods price dispersion in the intermediate sector: $\equiv (1-\alpha)\ln[\int_0^1 (\frac{P_t(z)}{P_t})^{\frac{-\epsilon}{1-\alpha}} dz]$, where $P(z)$ is the price of the final good z.

With the previous simplifications, we can now express our welfare function as:

$$U_t - U \approx Y(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2) - [((\hat{Y}_t - \hat{A}_t - \hat{K}_t) + \hat{dp}_t) + \frac{1}{2}\left(\frac{\varsigma}{(1-\alpha)*\alpha}\right)((\hat{Y}_t - \hat{A}_t - \hat{K}_t)^2 + \hat{dp_t}^2)] + O^3 \tag{III.14}$$

We now use two lemmas to simplify our expression for labor.

Lemma 1: $\hat{dp}_t = \frac{1}{2}\frac{\epsilon}{\nu}P(z)_t + O^3$, where $\nu \equiv \frac{1-\alpha}{1+\alpha(\epsilon 1)}$

Proof: See Chadha *et al* (2010), Gali and Monacelli (2005).

Lemma 2: $\Sigma_{t=0}\beta^t P(z)_t = \frac{1}{\lambda}\Sigma_{t=0}\beta^t \pi_t^2$, where $\lambda \equiv \frac{(1\theta)(1\beta\theta)}{\theta}$

Proof: See Gali and Monacelli (2005), and Woodford (2003), Chapter 6.

Under lemmas 1 and 2, we can short-write the compounded parameter as $\eta \equiv (1-\theta)(1-\theta\beta)\alpha(1-\alpha)$, and use it to re-write the welfare function as:

$$U_t - U \approx Y(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2 - [(\hat{Y}_t + \frac{1}{2}\frac{\epsilon}{\eta}\hat{\pi_t^2}) + \frac{1}{2}\left(\frac{\varsigma}{(1-\alpha)*\alpha}\right)(\hat{Y_t}^2)] + t.i.p. + O^3 \tag{III.15}$$

and simplifying, it gives us:

$$U_t - U \approx \frac{1}{2}\hat{Y}_t^2 - \frac{1}{2}\left(\frac{\varsigma}{(1-\alpha)*\alpha}\right)(\hat{Y_t}^2) - \frac{1}{2}\frac{\epsilon}{\eta}\hat{\pi_t^2} + t.i.p. + O^3 \tag{III.16}$$

Collecting second-order terms, we can rewrite the above expression as:

$$U_t - U \approx \frac{1}{2}\left[\left(1 - \frac{\varsigma}{(1-\alpha)*\alpha}\right)\sigma_Y^2 - \frac{\epsilon}{\eta}\sigma_\pi^2\right] + t.i.p. + O^3 \tag{III.17}$$

where σ^2 denotes the variance terms.

We can rewrite the above welfare function in terms of aggregate welfare losses using the following purely quadratic loss function:

$$E_0 \sum_{t=0}^{\infty} \beta^t (U_t U) = \frac{1}{2} E_0 \Sigma_{t=0}^{\infty} \beta^t L_t + t.i.p + O^3 \tag{III.18}$$

with $L_t = \chi_y \sigma_y^2 + \chi_\pi \sigma_\pi^2$

where $\chi_y \equiv \frac{\varsigma}{(1-\alpha)\alpha} - 1)$, $\chi_\pi \equiv \frac{\epsilon}{\nu}$, and $\nu \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{(1-\alpha)\alpha}{1+\alpha(\epsilon-1)}$.

Keeping our benchmark calibration from the IRF and Second moment analyzes, and calibrating the parameter ς according to Christiano *et al* (2010) calibration for the US data, we get the following optimal weights for each of the variables in the loss function:

$$\chi_y \equiv \frac{0.75}{(1-0.20)0.20} - 1 = 3.69 \tag{III.19}$$

$$\chi_\pi \equiv \frac{0.99}{\frac{(1-0.75)(1-0.99*0.75)}{0.75}\frac{(1-0.20)0.20}{1+0.20(0.99-1)}} = 71.95 \tag{III.20}$$

Normalizing to 1 for the inflation, the respective weights are:

$$\chi_y \equiv \frac{3.69}{71.95} = 0.05 \tag{III.21}$$

$$\chi_\pi \equiv \frac{71.95}{71.95} = 1 \tag{III.22}$$

This is exactly the loss function that we defined in section and that we have used for our welfare experiments.

IV Welfare experiments

IV.1 Welfare estimation algorithm

To estimate the loss function for the different policy options, we make use of Sims codes, which represent a robust computational method for rational expectations models based on the QZ matrix decomposition. The program solves nonlinear equations, and minimizes them using quasi-Newton methods with BFGS updates.[35]The particular strength of these codes lies in the ability of the minimizer to negotiate hyperplane discontinuities without getting stuck, allowing us to estimate our loss function more smoothly and efficiently. The only drawback that has been noted until now is that, despite it being more robust against some specifications of the likelihood function, in some cases it is not very clear whether it succeeds in jumping the cliffs in a reliable way, and hence requires a broader depiction of the loss function around the estimated minimum.[36]

[35]This method derives from Newton's method.

[36]Zha (2000), Sims and Zha(2006).

The key file that we call on in our estimations is the **csminwel.m** file that draws on our loss function defined in **lsscby.m**, and estimates it using the inverse Hessian, that has to be positive definite in order for a solution to exist. We therefore need to provide some initial values of the inverse Hessian in **csminwel.m** which will ensure that it is positive definite. Alongside we need to provide the initial (non-estimated) values of the parameter vector in our monetary policy reaction function that we want to estimate and a convergence criterion.[37]One can also specify the maximum number of iterations for one simulation, but if the critical threshold above was specified, then that is not necessary. Finally, one is free to either specify a string of the function that calculates the gradient of the loss function, or if left in blank, the program will calculate a numerical gradient (which is our case). Since we are also estimating the optimal vector of parameters that minimizes the loss function, we need to provide an explicit policy reaction function in **lsscby.m**, which we define based on the simulated DSGE model. With these inputs, the **csminwel.m** routine calls on **csminit.m** that searches for a loss function minimum around the neighborhood of the initial vector of parameters input in **csminwel.m**, and estimates the corresponding vector of parameters in the reaction function that produces that minimum loss. Hence, different initial values of the vector of reaction function parameters will give different minimum losses, since the program searches in the immediate neighborhood ϵ of the initials. Keeping that in mind, in a second step we try to depict the loss function from a broader perspective, and understand whether the minimum loss that was estimated for a reasonable set of parameters in the reaction function represented a local or a global minimum, and whether the estimated vector of parameters for that global minimum represented a reasonable set of weights that a policymaker can make use of in his reaction function.

More condensed, the above algorithm minimizes $f(x)$ in an open subset $U \subset \Re^n$, and can be reduced to the following five steps (Heer and Maussner (2009)):

1. Initialize: Choose a vector of initial values x_0, stopping criteria $\epsilon_1 \in \Re^{++}$, and $\epsilon_1 \in \Re^{++}$, where $\epsilon_1 >> \epsilon_2$, and the Hessian $H_0 = I_n$. Put $k = 0$.

2. Compute the gradient of $\nabla f(x_k)$, the column vector of first partial derivatives of f with respect to x_i, and solve for w_k from $H_s w_k = \nabla f(x_s)$. $w_k = x_{k+1} - x_k$.

[37]Which specifies the threshold value at which it proves impossible to improve the function value by more than the criterion, and hence the iterations will cease. In our simulations, the threshold value was $1 * 10^7$.

3. Use line search algorithm (a step size algorithm that achieves a sufficient decrease in the value of a function to be minimized) with ϵ_2 to find the step length s, and put $x_{k+1} = x_k + sw_k$. This is good to use in order to enhace the global convergence.

4. Check for convergence to see whether the algorithm is close to the minimizer. If so, stop. If not, and if the line search was successful, proceed to the next step. Otherwise stop and report convergence to a non-optimal point.

5. Use $H_{k+1} = H_k + \frac{z_k z_k'}{z_k' w_k} - \frac{H_k w_k w_k' H_k'}{w_k' H_k w_k}$, where $z_k = \nabla f(x_{k+1}) - \nabla f(x_k)$ to get H_{k+1}. This defines the BFGS update formula for the secant approximation of the Hesse matrix. Increase k by a unit and return to Step 2.

The algorithms are relatively simple to implement, and because the programs are more robust than many other standard Matlab codes that do approximately the same thing, they have become increasingly popular during the past few years, and are now a standard tool in optimization/welfare loss analysis in monetary policy theory.

IV.2 Results

Table IV.1: Loss function analysis I - Economy with a positive asset price wedge

Reaction function	Initial weights	Loss value	Optimal weights
$[E_t(\pi_{t+1}), y_t]$	(1, 0.5)	0.0045	(0.77,0.73)
$[E_t(\pi_{t+1}), y_t]$	(1.5, 0.5)	0.0041	(0.99,1.01)
$[E_t(\pi_{t+1}), y_t]$	(2, 0.01)	0.0041	(0.996, 1.01)
$[E_t(\pi_{t+1}), y_t]$	(2, 0.1)	0.0041	(1.04, 1.06)
$[E_t(\pi_{t+1}), y_t]$	(2, 0.5)	0.0039	(1.23, 1.27)
$[E_t(\pi_{t+1}), y_t]$	(2, 1)	0.0037	(1.48, 1.52)
$[E_t(\pi_{t+1}), y_t]$	(2, 2)	0.0034	(2.01, 1.99)
$[E_t(\pi_{t+1}), y_t, s_t]$	(1, 0.5, 0.2)	0.0041	(0.96,0.54,0.24)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.2)	0.0035	(1.65,0.85,0.55)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.01, 0.2)	0.0037	(1.58,0.43,0.62)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.1, 0.2)	0.0037	(1.59,0.51,0.61)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 1, 0.2)	0.0034	(1.75,1.26,0.46)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 2, 0.2)	0.0033	(1.96,2.04,0.24)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.01)	0.0035	(1.55,0.95,0.46)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.1)	0.0035	(1.60,0.90,0.50)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.2)	0.0035	(1.65,0.85,0.55)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.3)	0.0035	(1.71,0.79,0.59)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.4)	0.0035	(1.77,0.74,0.64)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.5)	0.0035	(1.82,0.68,0.68)

Note: The first column in the table states whether the simulations are based on a standard policy, or a market price augmented one. The second column reports the initial weights given to each and every variable in the reaction function prior to the estimation, while the (fourth) last column reports the optimal weights of each variable that was estimated for that specific (minimum) loss.

Table IV.2: Loss function analysis II - Economy without a wedge

Reaction function	Initial weights	Loss value	Optimal weights
$[E_t(\pi_{t+1}), y_t]$	(1, 0.5)	0.0043	(0.77,0.73)
$[E_t(\pi_{t+1}), y_t]$	(1.5, 0.5)	0.0039	(0.99,1.01)
$[E_t(\pi_{t+1}), y_t]$	(2, 0.01)	0.0039	(0.997, 1.03)
$[E_t(\pi_{t+1}), y_t]$	(2, 0.1)	0.0038	(1.04, 1.06)
$[E_t(\pi_{t+1}), y_t]$	(2, 0.5)	0.0033	(1.23, 1.27)
$[E_t(\pi_{t+1}), y_t]$	(2, 1)	0.0031	(1.48, 1.52)
$[E_t(\pi_{t+1}), y_t]$	(2, 2)	0.0030	(2.01, 1.99)
$[E_t(\pi_{t+1}), y_t, s_t]$	(1, 0.5, 0.2)	0.0039	(0.96,0.54,0.24)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.2)	0.0033	(1.65,0.85,0.55)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.01, 0.2)	0.0035	(1.58,0.43,0.62)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.1, 0.2)	0.0035	(1.59,0.51,0.61)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 1, 0.2)	0.0032	(1.75,1.26,0.46)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 2, 0.2)	0.0031	(1.96,2.04,0.24)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.01)	0.0033	(1.55,0.95,0.46)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.1)	0.0033	(1.60,0.90,0.50)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.3)	0.0033	(1.71,0.79,0.59)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.4)	0.0033	(1.77,0.74,0.64)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.5)	0.0033	(1.82,0.68,0.68)

Table IV.3: Loss function analysis III - Total losses including errors I and II

Reaction function	Initial wights	Total loss value	Optimal weights
$[E_t(\pi_{t+1}), y_t]$	(1,0.5)	0.0088	(0.77,0.73)
$[E_t(\pi_{t+1}), y_t]$	(1.5,0.5)	0.0080	(0.99,1.01)
$[E_t(\pi_{t+1}), y_t]$	(2, 0.01)	0.0080	(0.996, 1.01)
$[E_t(\pi_{t+1}), y_t]$	(2, 0.1)	0.0079	(1.04, 1.06)
$[E_t(\pi_{t+1}), y_t]$	(2, 0.5)	0.0075	(1.23, 1.27)
$[E_t(\pi_{t+1}), y_t]$	(2, 1)	0.0068	(1.48, 1.52)
$[E_t(\pi_{t+1}), y_t]$	(3, 0.5)	0.0068	(1.74,1.76)
$[E_t(\pi_{t+1}), y_t]$	(2, 2)	0.0064	(2.01, 1.99)
$[E_t(\pi_{t+1}), y_t, s_t]$	(1, 0.5, 0.2)	0.008	(0.96,0.54,0.24)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.01, 0.2)	0.0072	(1.58,0.43,0.62)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.1, 0.2)	0.0072	(1.59,0.51,0.61)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.2)	0.0068	(1.65,0.85,0.55)
$[E_t(\pi_{t+1}), y_t, s_t])$	(2, 0.5, 0.01)	0.0068	(1.55,0.95,0.46)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.1)	0.0068	(1.60,0.90,0.50)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.3)	0.0068	(1.71,0.79,0.59)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.4)	0.0068	(1.77,0.74,0.64)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 0.5, 0.5)	0.0068	(1.82,0.68,0.68)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 1, 0.2)	0.0066	(1.75,1.26,0.46)
$[E_t(\pi_{t+1}), y_t, s_t]$	(2, 2, 0.2)	0.0064	(1.96,2.04,0.24)
$[E_t(\pi_{t+1}), y_t, s_t]$	(3, 0.5, 0.2)	0.0062	(2.4,1.1,0.8)

Table IV.4: Loss function analysis 4- Indifference between the two policy rules

Reaction function weights on inflation and output	Probability of the wedge	Probability of no wedge	Total losses at those probabilities
(1,0.5)	47 %	53 %	0.0042
(2, 0.01)	47 %	53 %	0.0038
(2, 0.1)	47 %	53 %	0.0038
(2, 0.5)	48 %	52 %	0.0036
(2, 1)	49 %	51 %	0.0033
(3, 0.5)	48 %	52 %	0.0032

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