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# **Structural Models of the Wage Curve Estimated by Panel Data and Cross-Section Regressions**

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**Structural Models of the Wage Curve Estimated by Panel Data  
and Cross-Section Regressions**

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# **Structural Models of the Wage Curve Estimated by Panel Data and Cross-Section Regressions**

## **Abstract**

Introducing equilibrium unemployment to the solution of the intertemporal allocation of non-leisure time, we derive two wage-setting models which we estimate by panel data and cross-section regressions applied on aggregative data. The results support the empirical relation known as the wage-curve, thus enriching and strengthening the microfoundations of that relation.

*Keywords:* Wage Curve, Intertemporal Allocation, Two-Sector Model

*JEL Codes:* E22, E23, E24

## 1. Introduction

The voluminous empirical evidence supporting an inverse relation between wage rates and unemployment rates at the local level - known as the wage-curve - is well documented in Blanchflower and Oswald (2005). What remains to be firmly established are the microfoundations of this relation. By deriving a wage-curve type relation from first principles and by estimating this relation successfully we help embed the wage-curve literature more firmly in mainstream economics.

Sections 2-4 derive wage-setting with equilibrium unemployment from first principles, sections 5-6 derive two-versions of wage-curve type relations to be estimated empirically by panel data and cross-section regressions, whilst section 7 reports the empirical findings and concludes the paper. Empirical estimates and information on data and its sources appear in the Appendix.

## 2. Production in a Two-Sector Model

### 2.1 The Final-Goods Sector

To describe production in the final-goods sector we shall let  $Y$  denote the output of the final good,  $A$  the technology level,  $h$  human capital per worker,  $K$  the capital stock,  $N$  the number of workers in the final-goods sector, and  $AhN$  the effective human capital in that sector. Assuming  $Y$  exhibits constant returns to scale in  $AhN$  and  $K$  we shall write:

$$(1) \quad Y = (AhN)^{1-\alpha} K^\alpha = (AhN)(K / AhN)^\alpha, \quad 1 > \alpha > 0$$

An alternative way to model  $Y$  is to let  $L$  denote the labor force,  $\tilde{y} \equiv (Y / AhL)$  and  $\tilde{k} \equiv (K / AhL)$  define output and capital per effective labor force, respectively, and let  $v \equiv (N / L)$  define the share of labor force in the final-goods sector, to write:

$$(2) \quad (\tilde{y} / v) \equiv (Y / AhN) = (K / vAhL)^\alpha \equiv (\tilde{k} / v)^\alpha = (K / AhN)^\alpha$$

## 2.2 The Intermediate-Goods Sector

The intermediate-goods sector transforms  $I$  units of  $Y$  into  $I$  units of installed capital employing the fraction of the labor force not employed in the final-goods sector. Letting  $(1/\sigma)$  denote the productivity of labor engaged in this sector we model capital formation as follows:

$$(3) (I/K) = \{(1/\sigma)(1-\nu)\}$$

## 3. Allocating Non-Leisure Time in a Centralized Economy

To describe optimization in a centralized economy we shall let  $C \equiv Y - I$  define aggregate consumption,  $(C/AhL) \equiv \tilde{c}$  define consumption per effective labor force,  $U(\tilde{c})$  denote the instantaneous utility of the representative agent,  $\rho$  her rate of time preference,  $n$  the rate of population growth,  $\delta$  the rate of capital depreciation and  $g$  the equilibrium growth rate and set up the following objective:

$$(4) \text{ Maximize } \int_0^{\infty} U(\tilde{c}) e^{-\rho t} dt$$

Subject to:

$$(\partial \tilde{k} / \partial t) = (\tilde{y} - \tilde{c}) - (n + \delta + g)\tilde{k}$$

$$(1 - \nu) = \sigma[(\tilde{y} - \tilde{c}) / \tilde{k}] = \sigma(I/K)$$

$$\tilde{y} = \nu^{1-\alpha} \tilde{k}^{\alpha}, \quad \tilde{k}_0 : \text{ is given, and: } 0 \leq \tilde{c} \leq \tilde{y}$$

Solving for the command optimum we arrive at:

$$(5) U'(\tilde{c}) = m_1 - m_2(\sigma/\tilde{k}) = m_1 - (\partial\tilde{y}/\partial v)U'(\tilde{c})(\sigma/\tilde{k})$$

In (5),  $m_1$  and  $\{m_1/U'(\tilde{c})\}$  measure the price of investment in utility, and in intensive consumption, respectively. Letting  $q-1$  denote the marginal cost of investment in intensive consumption yields:

$$(6) q = \{1 + (\partial\tilde{y}/\partial v)(\sigma/\tilde{k})\} = \{m_1/U'(\tilde{c})\}$$

Setting  $(1-\alpha)(Y/N) = \omega$  to define the wage rate, we arrive, after some algebra, at:

$$(7) \left[ \frac{(q-1)I}{1-v} \right] = \left[ \frac{\omega N}{v} \right]$$

As (7) asserts: *At the optimum, the value added per unit labor equalizes across sectors.*

#### 4. Introducing Equilibrium Unemployment

To quote Greenwood and Hercowitz (1991): "...household activities involve approximately as much capital as business activities." This suggests that one can motivate equilibrium unemployment by defining the intermediate-goods sector broadly to include household investment. To illustrate, imagine an economy of family establishments producing the market good. When market activity is low it becomes profitable to reallocate family time in favor of household investment. In this stylized economy investment in home capital is positively

correlated with unemployment since homework is not included in employment statistics.

To model equilibrium unemployment we revisit (7) to observe: Firstly, the higher is  $q$  the higher the value added in the intermediate-goods sector and the stronger the motive to reallocate labor in favor of this sector. Secondly, the higher is  $q$  the lower is the return on capital and the weaker is the motive to reallocate labor services from the future to the present. Thus, with an intermediate-goods sector defined to include household investment, the motives to substitute labor supply intratemporally and intertemporally coincide and reinforce each other so that  $q$  is positively correlated with equilibrium unemployment. Thus, to introduce equilibrium unemployment, we begin by letting  $s = (I/Y)$  define the saving rate to rearrange (7) as follows:

$$(8) \quad q = ((1-v)/v)((1-\alpha)Y/I) + 1 = ((1-v)/v)(1-\alpha)(1/s) + 1$$

Assuming  $(1-v)$  to be “sufficiently” small and taking logarithms yields:

$$(9) \quad \ln q = \ln[((1-v)/v)(1-\alpha)(1/s) + 1] \approx ((1-v)/v)(1-\alpha)(1/s)$$

Letting  $u$  denote the equilibrium unemployment rate and observing that  $(u/(1-u))$  and  $((1-v)/v)$  are positively correlated, we shall write:

$$(10) \quad ((1-v)/v) = \tilde{\mu} (u/(1-u)), \quad \tilde{\mu} > 0$$



## 5. A Wage-Curve Model Estimated By Panel Data Regressions

In competitive equilibrium the price of labor equals the marginal product of labor and the rental price of capital equals the marginal product of capital. Accordingly, letting  $r$  and  $(r + \delta)q$  denote the interest rate and the rental price of capital, respectively, we shall write:

$$(11) \quad \omega = (1 - \alpha)(Y / N) = (1 - \alpha)(Ah)(K / AhN)^\alpha = (1 - \alpha)(Ah)(Y / K)^{-(\alpha/1-\alpha)}$$

$$(12) \quad \alpha(Y / K) = (r + \delta)q$$

Assuming capital is perfectly mobile so that equilibrium interest rates equalize across countries, we may take  $(r + \delta)$  to be constant. Thus, subscripting the  $i$ th unit at time  $t$  by  $it$  and combining (9)-(12), we shall write:

$$(13) \quad \ln \omega_{it} = B + \ln(Ah)_{it} - (\alpha/(1 - \alpha)) \ln q_{it} = B + \ln(Ah)_{it} - (\alpha/s) \tilde{\mu}(u/(1 - u))_{it}$$

$$B \equiv \ln(1 - \alpha) - (\alpha/(1 - \alpha)) \ln((r + \delta)/\alpha)$$

To model  $A$  we let  $A_0$  to control for the initial, world-wide, stock of ideas in the public domain, and a time trend to control for technical progress. To control for  $h$  we use the logarithm of the schooling years attained by those over the age of 25 reported in Barro and Lee (2001). Accordingly we re-specify (13) as follows:

$$(14) \quad \ln \omega_{it} = (B + A_0) + \beta_1 t + \beta_2 \ln \text{Schooling}_{it} + \beta_3 (u/(1 - u))_{it} \quad , \quad \beta_3 \equiv -(\alpha/s) \tilde{\mu}$$

To capture the effect of unemployment benefits we construct a variable labeled *Replacement Wage* measured by the product of the wage rate and the replacement rate divided by 100 - (See OECD (1994), Martin (1996)) and append in (14) an  $\ln(\text{Replacement Wage})$  term and its interaction with  $\ln(\text{Schooling})$  to write:

$$(15) \quad \ln \omega_{it} = (B + A_0) + \beta_1 t + \beta_2 \ln(\text{Schooling})_{it} + \beta_3 (u/(1-u))_{it} + \beta_4 \ln(\text{Replacement Wage})_{it} \\ + \beta_5 \{ \ln(\text{Schooling}) \times \ln(\text{Replacement Wage}) \}_{it}$$

Once schooling and the replacement wage are controlled separately, their interaction captures a reduction in the number of hours of work due to increased opportunities for part-time employment. Since  $\omega$  measures the hourly wage, we expect  $\beta_5$  to be negative because part-time employment reduces the hourly wage. Estimation results are presented in the Appendix and discussion in 7.1.

## 6. A Wage-Curve Model Estimated by a Cross-Section Regression

An alternative way to derive wage-setting from first principles is to multiply (7) by  $(v/N)$  and taking logarithms arrive at:

$$(16) \quad \ln \omega_{it} = \ln(q-1)_{it} + \ln(K/N)_{it} + \ln(I/K)_{it} + \ln(v/(1-v))_{it}$$

Taking  $(v/(1-v))$  to be positively correlated with  $\{(1-u)/u\}$ , we write:

$$(17) \quad \ln(v/(1-v))_{it} = \phi \ln\{(1-u)/u\}_{it}, \quad \phi > 0$$

Applying a Taylor expansion around  $u = u^*$  we arrive at:

$$(18) \ln(v/(1-v))_{it} = \beta_0 + \phi \ln\{(1-u)/u\} \approx \beta_0 + \beta_1 u_{it}, \quad \text{where: } \beta_1 \approx -\frac{\phi}{(1-u^*)u^*} < 0,$$

To model  $\ln(I/K)$  we revisit (3) to replace the unobservable  $(1/\sigma)$  with a measure of education quality, labeled  $eq$ , and the unobservable  $(1-v)$  with  $u$  since  $u$  and  $(1-v)$  are taken to be positively correlated. Assuming the effect of  $u$  on  $(I/K)$  depends positively on  $eq$ , we re-specify (3) as follows:

$$(19) \ln(I/K)_{it} = \beta_2(eq)_{it} + \beta_3\{(eq)(u)\}_{it} \quad \beta_2 > 0, \beta_3 > 0,$$

Collecting terms we arrive at:

$$(20) \ln(\omega)_{it} = \ln(q-1)_{it} + \ln(K/N)_{it} + \beta_1 u_{it} + \beta_2 eq_{it} + \beta_3\{(eq)(u)\}_{it}$$

## 7. Empirical Findings and Concluding Comments

### 7.1 The Wage-Curve Elasticity in (15)

In our earlier working paper (Pikoulakis and Wisniewski (2009)) we estimate (15) by applying fixed-effects and random-effects panel data regressions on the aggregative data of 20 OECD economies and we conduct endogeneity tests that reject endogeneity bias. For a description of the data set used to estimate (15) consult Pikoulakis and Wisniewski (op.cit.). Table I in the Appendix presents the regression statistics on (15).

Focusing attention on the wage-curve elasticity, we observe that the coefficient estimates on  $(u/(1-u))_t$  and  $(u/(1-u))_{t-1}$  are all significant at the 1% level and their value, averaged over the 6 regressions, is -0.8621. This estimate, together with a sample average for  $u$  equal to 0.062, delivers an average wage-curve elasticity of -0.061. At the 99% confidence interval this elasticity lies between -0.04 and -0.09.

### 7.2 The Wage-Curve Elasticity in (20)

Table II in the appendix presents the statistical findings on (20). These estimates derive from a cross-section regression applied on the aggregative data of 45 economies which differ in economic development. Table III describes the data set used to estimate (20).

With sample averages for  $u$  and  $eq$  equal to 0.0912 and 1.0781, respectively, the estimated parameter values attached to the  $u$ , and the  $(eq)(u)$  terms, imply a wage-curve elasticity of -0.1172.

### **7.3 Concluding Comments**

Tables I - II strongly support the hypothesized relations in (15) and (20) thus confirming that the wage-curve relation is embedded in a structure of wage setting rich enough to address issues such as the return on schooling, the impact of unemployment benefits and the income share of human capital at an aggregative level.

APPENDIX

Table I

Empirical Determinants of Ln( $\omega$ )

	Fixed Effect Panel		Random Effect Panel		Fixed Effect Panel with Time Dummies	
	(1)	(2)	(3)	(4)	(5)	(6)
Constant			1.3117 <sup>***</sup> (0.2015)	1.3115 <sup>***</sup> (0.2000)		
Ln(Schooling)	0.2271 <sup>**</sup> (0.1090)	0.2262 <sup>**</sup> (0.1079)	0.2987 <sup>***</sup> (0.1009)	0.2963 <sup>***</sup> (0.1000)	0.3292 <sup>***</sup> (0.0915)	0.3185 <sup>***</sup> (0.0913)
Ln(Rep. Wage)	0.2166 <sup>***</sup> (0.0535)	0.2212 <sup>***</sup> (0.0531)	0.2000 <sup>***</sup> (0.0528)	0.2049 <sup>***</sup> (0.0524)	0.1886 <sup>***</sup> (0.0442)	0.1923 <sup>***</sup> (0.0441)
Ln(Schooling)×Ln(Rep. Wage)	-0.0972 <sup>***</sup> (0.0266)	-0.1009 <sup>***</sup> (0.0264)	-0.0890 <sup>***</sup> (0.0263)	-0.0929 <sup>***</sup> (0.0261)	-0.1064 <sup>***</sup> (0.0220)	-0.1082 <sup>***</sup> (0.0219)
$w/(1-u)$	-0.7580 <sup>***</sup> (0.0266)		-0.7679 <sup>***</sup> (0.1723)		-0.8645 <sup>***</sup> (0.1531)	
$w/(1-u)$ _Lag		-0.9459 <sup>***</sup> (0.1753)		-0.9505 <sup>***</sup> (0.1744)		-0.8856 <sup>***</sup> (0.1548)
Trend	0.0238 <sup>***</sup> (0.0013)	0.0245 <sup>***</sup> (0.0014)	0.0230 <sup>***</sup> (0.0013)	0.0238 <sup>***</sup> (0.0013)		
R-square	90.99%	91.13%	90.70%	90.85%	94.35%	94.36%

Note: Standard errors are given in parentheses. To conserve space the fixed and random effects, as well as the coefficient on time dummies are not reported. For the description of sample and data sources please see Pikoulakis and Wisniewski (2009). \*\*\*, \*\* denote statistical significance at 1% and 5% level, respectively.

**Table II****Empirical Determinants of  $\ln(\omega)$  in a Cross-Section Regression**

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Constant	-0.4430 (2.2757)
$eq$	0.0908 (2.4819)
$(eq)(u)$	57.9024** (25.4645)
$u$	-63.7102** (27.8876)
$\ln(q)$	1.4116*** (0.2499)
$\ln(K / N)$	1.0829*** (0.0513)

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$R^2$	0.9665
Number of observations	45
F(5,39)	287.74
Prob(F-statistic)	0.0000
Root MSE	0.20382

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*Note: Robust standard errors are reported in parentheses. \*\* , \*\* denote statistical significance at 1% and 5% level, respectively.*

### Table III

#### The Countries Included in the Regression of Table II

(1) Algeria (2) Australia (3) Austria (4) Belgium (5) Bolivia (6) Botswana (7) Canada (8) Chile (9) Colombia (10) Costa Rica (11) Cote D'Ivoire (12) Denmark (13) Egypt (14) Finland (15) France (16) Greece (17) Hong Kong (18) Ireland (19) Israel (20) Italy (21) Jamaica (22) Japan (23) Jordan (24) Korea Republic (25) Mauritius (26) Mexico (27) Morocco (28) Netherlands (29) New Zealand (30) Norway (31) Panama (32) Peru (33) Philippines (34) Portugal (35) South Africa (36) Spain (37) Sri Lanka (38) Sweden (39) Switzerland (40) UK (41) Trinidad & Tobago (42) Tunisia (43) Uruguay (44) USA (45) Venezuela

#### The Data Utilized in the Regression of Table II

- $\omega$ : The (average) wage rate  $\equiv$  [the wage share in (GDP)]x[(GDP) per worker]. The source for the wage share is the EPWT 4.0 and for GDP per worker is the PWT 7.0.
- eq: The education quality  $\equiv e^{ERSI}$ : where ERSI= Estimated Returns to Schooling of Immigrants in: Schoellman (2012)
- $u$ : The average rate of unemployment: Source: UNECE (United Nations Economic Commission for Europe)
- $q$ : The price of capital  $\equiv$  {(price level of investment)/(price level of GDP)}: Source: PWT 7.0.
- $(K/N)$ : Capital per worker  $\equiv$  (K/Y)/(Y/N): The source for (K/Y) is EPWT 4.0 and for (Y/N) is the PWT 7.0.



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