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Walrasian Foundations for Equilibria in Segmented Markets*

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Abstract

We study a CAPM economy with segmented financial markets and competitive arbitrageurs who link these markets. We show that the equilibrium of the arbitraged economy is Walrasian in the sense that it coincides with the equilibrium of an appropriately defined competitive economy with no arbitrageurs. This characterization serves to clarify the role that arbitrageurs play in integrating markets.

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1 Introduction

The Arrow-Debreu model provides an elegant and parsimonious theoretical foundation for the study of financial markets. It has proved to be not only the bedrock of textbook financial economic theory, but also the benchmark relative to which the role of "frictions", such as taxes, asymmetric information or limits of arbitrage, can be studied. In this paper we focus on one such friction, namely asset market segmentation.

The same or similar assets are often traded in many different locations and at many different prices. For instance, Mifid II in Europe and RegNMS in the US have given rise to significant market fragmentation in equities. As a result, any one stock is traded on many competing exchanges, multilateral trading facilities, electronic communication networks, dark pools, systematic internalizers, and so on.[1] Similarly, redundant or nearly redundant derivatives on a variety of underlying securities are traded on different venues with different price-discovery processes.

Arbitrageurs, often high frequency traders, exploit the surplus gains from trade arising from this segmentation. These activities lead to some price alignment, the extent of which depends on the degree of competition among arbitrageurs. While it is usually acknowledged that trading is segmented and that most traders focus on a small subset of the available spectrum of securities, it is nevertheless typically assumed that mispricings across securities do not occur and that equilibria can be approximated by a frictionless centralized Walrasian auction. This must be true, so the thinking goes, "because if arbitrages were to occur, then unrestricted traders would compete those gains away immediately, and therefore arbitrages in fact cannot occur".

In this paper, we investigate whether this intuition holds and, if so, under what conditions and for which type of Walrasian auction. The main questions that we seek to answer are the following: Is an equilibrium of a segmented markets economy, with a high degree of competition in the arbitraging sector, a variant of a Walrasian equilibrium with no arbitrageurs? To what extent do arbitrageurs ameliorate the segmentation friction?

We study a two-period model of financial markets with multiple market segments or "exchanges". Markets may be incomplete on any given exchange and the set of tradable payoffs may differ across exchanges. Each exchange is populated by investors who can trade only on that exchange, and have preferences that yield a local CAPM. In addition, there are arbitrageurs who can trade across exchanges. All agents behave

[1] While market fragmentation has accelerated in recent years, it has of course always been an important feature of the economic landscape. Allais (1967) argued for a more realistic "economy of markets" in lieu of a "market economy". In his Nobel speech he says: "... I was led to discard the Walrasian general model of the market economy, characterized at any time, whether there be equilibrium or not, by a single price system, the same for all the operators, - a completely unrealistic hypothesis, - and to establish the theory of economic evolution and general equilibrium, of maximum efficiency, and of the foundations of economic calculus, on entirely new bases resting on ... a new model, the model of the economy of markets (in the plural)".

competitively.

We refer to this economy as the *arbitraged economy*. It has a unique equilibrium, which is arbitrage-free. In particular, equilibrium asset valuations on each exchange coincide with the subjective valuations of the arbitrageurs. Our objective is to provide a Walrasian benchmark that relates this equilibrium to the equilibrium of an appropriately defined competitive economy with no arbitrageurs. (Throughout this paper, we reserve the term "Walrasian" for competitive economies or equilibria with no arbitrageurs.)

A natural candidate for such a benchmark is the well-studied concept of Walrasian equilibrium with restricted participation, wherein all agents face the same asset prices but can only trade payoffs that lie in their local asset span. However, except for a narrow class of economies, this equilibrium, which is unique in our model, is distinct from that of the arbitraged economy. While a restricted-participation equilibrium captures some features of competitive arbitrage, in particular a common asset price vector for all agents, there may nevertheless be arbitrage opportunities that agents are unable to exploit because of their participation constraints. An equilibrium of the arbitraged economy, on the other hand, is arbitrage-free.

Instead, we propose a subtly different notion of equilibrium, which we call Walrasian equilibrium with restricted consumption, wherein all agents face the same asset price vector but can only *consume* payoffs (in excess of their endowments) that lie in their local asset span. Thus agents can trade all the assets in the economy, but may have to discard consumption in some states in order to stay within the imposed span. There is a unique restricted-consumption equilibrium in our setting. This equilibrium is arbitrage-free and has the same asset pricing and allocational outcomes (for investors) as the equilibrium of the arbitraged economy. Furthermore, the restricted-participation equilibrium coincides with the restricted-consumption equilibrium if and only if the former is arbitrage-free.

What does this tell us about the role that arbitrageurs play in the arbitraged economy? A restricted-participation equilibrium does capture the fact that arbitrageurs allow investors to trade their local assets with the rest of the world insofar as these overlap with assets traded elsewhere. But the connection that we establish with the restricted-consumption economy shows that arbitrageurs in fact allow investors to trade *all* the assets in the economy. Investors gain as a result, even if future consumption must be curtailed to respect their local asset market constraints.

Market segmentation has been the subject of a recent and growing literature. In classical general equilibrium, segmentation is captured by restricted-participation constraints on agents (see Polemarchakis and Siconolfi (1997) and Cass et al. (2001)). Strategic arbitrage in a general equilibrium setting is the subject of Zigrand (2004, 2006). Rahi and Zigrand (2009) specialize this framework to a CAPM setting to study security design by arbitrageurs. An extended discussion of the segmented markets literature in finance, including empirical work, can also be found in this paper. A broader "limits of arbitrage" literature considers settings in which arbitrageurs fail to eliminate mispricings due to various constraints that they face (see Gromb and

Vayanos (2010) for a recent survey of this literature). For instance, Gromb and Vayanos (2002, 2009) study the dynamics of competitive arbitrage between identical assets traded in two separate markets, when the arbitrageurs are constrained by margin requirements.

The economy with competitive arbitrageurs that we study in the present paper can be thought of as the limit of an economy in which there are frictions in the arbitraging sector, as these frictions tend to zero. In particular, the Cournot-Walras equilibrium in Rahi and Zigrand (2009), wherein strategic arbitrageurs engage in Cournot competition, converges to the equilibrium of the competitively arbitraged economy that we analyze here.

The paper is organized as follows. We introduce the arbitraged economy in Section 2 and characterize its (unique) equilibrium in Section 3. In Section 4, we propose the notion of Walrasian equilibrium with restricted consumption as the appropriate benchmark for the equilibrium of the arbitraged economy. We analyze Walrasian equilibrium with restricted participation in Section 5, leading to explicit characterizations of valuation in the restricted-consumption economy in Section 7. In Section 8, we bring together the various preceding results to provide an overall picture of the sense in which the arbitraged economy is Walrasian. Section 9 concludes.

2 The Arbitraged Economy

We consider an economy with two dates, 0 and 1, and a single physical consumption good. Assets are traded at date 0, in several locations or "exchanges", and pay off at date 1. Uncertainty is parametrized by the state space $S := \{1, \ldots, S\}$. Asset payoffs on exchange $k \in K := \{1, \ldots, K\}$ are given by a full column rank payoff matrix R^k of dimension $S \times J^k$. The asset span on exchange k is the column space of R^k, which we denote by $\langle R^k \rangle$. Asset spans may differ across exchanges, and we do not assume that markets are complete on any exchange. Assets are in zero net supply.

Associated with each exchange is a group of competitive investors who can trade only on that exchange. Investor $i \in I^k := \{1, \ldots, I^k\}$ on exchange k has endowments $(\omega_0^{k,i}, \omega^{k,i}) \in \mathbb{R} \times \mathbb{R}^S$, and preferences which allow a quasilinear quadratic representation,

$$U^{k,i}(x_0^{k,i}, x^{k,i}) = x_0^{k,i} + \sum_{s \in S} \pi_s \left[x_s^{k,i} - \frac{1}{2}\beta^{k,i}(x_s^{k,i})^2 \right],$$

where $x_0^{k,i} \in \mathbb{R}$ is consumption at date 0, $x^{k,i} \in \mathbb{R}^S$ is consumption at date 1, and π_s is the probability (common across agents) of state s. The coefficient $\beta^{k,i}$ is positive.

The setting described so far applies to all the economic environments that we study in this paper. For the arbitraged economy there is, in addition, a competitive arbitrageur who can trade both within and across exchanges. He has no endowments and he cares only about date 0 consumption. We can think of this arbitrageur as standing in for a continuum of identical arbitrageurs.

Definition 2.1 *An equilibrium of the arbitraged economy is an array of asset prices, asset demands, and arbitrageur supplies,* $\{q^k \in \mathbb{R}^{J^k},\ \theta^{k,i} \in \mathbb{R}^{J^k},\ y^k \in \mathbb{R}^{J^k}\}_{k\in K,\, i\in I^k}$, *such that*

1. *Investor optimization: For given* q^k, $\theta^{k,i}(q^k)$ *solves*
$$\max_{\theta^{k,i}\in\mathbb{R}^{J^k}} \; x_0^{k,i} + \sum_{s\in S}\pi_s\left[x_s^{k,i} - \frac{\beta^{k,i}}{2}(x_s^{k,i})^2\right]$$
subject to the budget constraints:
$$\begin{aligned} x_0^{k,i} &= \omega_0^{k,i} - q^k\cdot\theta^{k,i},\\ x^{k,i} &= \omega^{k,i} + R^k\theta^{k,i}. \end{aligned}$$

2. *Arbitrageur optimization: For given* $\{q^k\}_{k\in K}$, $\{y^k(\{q^k\})\}_{k\in K}$ *solves*
$$\max_{\{y^k\in\mathbb{R}^{J^k}\}_{k\in K}} \sum_{k\in K} q^k\cdot y^k \qquad s.t.\ \sum_{k\in K} R^k y^k \le 0.$$

3. *Market clearing:* $\{q^k\}_{k\in K}$ *solves*
$$\sum_{i\in I^k}\theta^{k,i}(q^k) = y^k(\{q^k\}), \qquad \forall k\in K.$$

Note that the arbitrageur maximizes date 0 consumption, i.e. profits from his arbitrage trades, subject to a no-default constraint at date 1.

It is convenient to cast our analysis of equilibrium prices in terms of state-price deflators. To this end, we introduce some more notation. Let $\Pi := \text{diag}\,(\pi_1,\dots,\pi_S)$. For $x\in\mathbb{R}^S$, the $L^2(\Pi)$-norm of x is $\|x\|_2 := (x^\top\Pi x)^{\frac{1}{2}}$. Let
$$P^k := R^k({R^k}^\top\Pi R^k)^{-1}{R^k}^\top\Pi.$$
Since P^k is idempotent, it is a projection. Indeed, it is an orthogonal projection in $L^2(\Pi)$ onto the asset span $\langle R^k\rangle$.

A vector $p\in\mathbb{R}^S$ is a state-price deflator[2] for (q^k,R^k) if $q^k = {R^k}^\top\Pi p$. If markets are incomplete on exchange k, there is a multiplicity of state-price deflators p, all of which satisfy $q^k = {R^k}^\top\Pi p$. Hence, it is often useful to identify the valuation functional for exchange k by the projected state-price deflator P^kp. Clearly, if p is a state-price deflator for (q^k,R^k), so is P^kp, since ${R^k}^\top\Pi P^kp = {R^k}^\top\Pi p$. Indeed, P^kp is the unique state-price deflator that is also marketed, i.e. in the span $\langle R^k\rangle$.

We shall also use the term state-price deflator to describe subjective, as opposed to equilibrium, valuations. Thus a state-price deflator p^A for the arbitrageur implies the subjective asset valuation ${R^k}^\top\Pi p^A$ on exchange k. We say that state-price deflators p and p' are *equivalent*, denoted by $p\equiv p'$, if $P^kp = P^kp'$, for all $k\in K$. Equivalent state-price deflators imply the same asset valuation on any given exchange.

[2]We do not restrict state prices to be nonnegative, since we will have occasion to consider economies with no arbitrageurs later in the paper. In such economies there may be unexploited arbitrage opportunities, and hence negative state prices, in equilibrium. See Example 5.1.

3 Equilibrium of the Arbitraged Economy

Letting $\mathbf{1} := (1 \dots 1)^\top$, an $S \times 1$ vector of ones, we can write investor (k,i)'s optimization problem as follows:

$$\max_{\theta^{k,i} \in \mathbb{R}^{J^k}} \omega_0^{k,i} - q^k \cdot \theta^{k,i} + \mathbf{1}^\top \Pi(\omega^{k,i} + R^k\theta^{k,i}) - \frac{\beta^{k,i}}{2}(\omega^{k,i} + R^k\theta^{k,i})^\top \Pi(\omega^{k,i} + R^k\theta^{k,i}).$$

The first-order condition is

$$-q^k + {R^k}^\top \Pi \mathbf{1} - \beta^{k,i} {R^k}^\top \Pi(\omega^{k,i} + R^k\theta^{k,i}) = 0,$$

which gives us the asset demand function,

$$\theta^{k,i}(q^k) = \frac{1}{\beta^{k,i}}({R^k}^\top \Pi R^k)^{-1}[{R^k}^\top \Pi \mathring{p}^{k,i} - q^k], \tag{1}$$

where $\mathring{p}^{k,i} := (\mathbf{1} - \beta^{k,i}\omega^{k,i})$. Notice that $\mathring{p}^{k,i}$ is a no-trade state-price deflator for agent (k,i) since $\theta^{k,i} = 0$ at $q^k = {R^k}^\top \Pi \mathring{p}^{k,i}$. The aggregate demand function for exchange k is, therefore, given by

$$\theta^{k}(q^k) = \frac{1}{\beta^{k}}({R^k}^\top \Pi R^k)^{-1}[{R^k}^\top \Pi \mathring{p}^{k} - q^k], \tag{2}$$

where $\beta^k := [\sum_{i \in I^K} (\beta^{k,i})^{-1}]^{-1}$, $\omega^k := \sum_{i \in I^k} \omega^{k,i}$, and $\mathring{p}^k := \mathbf{1} - \beta^k\omega^k$. The vector $\mathring{p}^k$ is an autarky state-price deflator for exchange k, i.e. $\sum_{i \in I^k} \theta^{k,i} = 0$ at $q^k = {R^k}^\top \Pi \mathring{p}^k$.

Using the market-clearing condition, $\theta^k = y^k$, we get

$$q^k = {R^k}^\top \Pi[\mathring{p}^k - \beta^k R^k y^k].$$

From this expression we can see that the parameter β^k measures the "depth" of exchange k: the state s value of the state-price deflator, $\mathring{p}^k - \beta^k R^k y^k$, falls by β^k for a unit increase in arbitrageur supply of s-contingent consumption. We can interpret equilibrium prices as risk-neutral prices ${R^k}^\top \Pi \mathbf{1}$ from which a risk-aversion discount $\beta^k {R^k}^\top \Pi(\omega^k + R^k y^k)$ is subtracted.

In order to fully characterize an equilibrium, we need a preliminary result. We say that a vector $x \in \mathbb{R}^S$ satisfies condition **C** if

(C1) $x \geq 0$;

(C2) $\sum_{k \in K} \frac{1}{\beta^k} P^k(\mathring{p}^k - x) \leq 0$; and

(C3) $x \cdot \left[\sum_{k \in K} \frac{1}{\beta^k} P^k(\mathring{p}^k - x)\right] = 0$.[3]

[3]Notice that each of the S terms that are summed up in the inner product must be less than or equal to zero, due to **C1** and **C2**. Hence all of these terms must in fact be zero.

Lemma 3.1 *Suppose $x, y \in \mathbb{R}^S$ both satisfy condition* **C**. *Then $x \equiv y$.*

Proof In order to save on notation, we use the following shorthand:

$$A := \sum_{k \in K} \frac{1}{\beta^k} \Pi P^k, \tag{3}$$

$$b := \sum_{k \in K} \frac{1}{\beta^k} \Pi P^k \mathring{p}^k. \tag{4}$$

The vectors x and y satisfy condition **C** if and only if

$$x \geq 0, \qquad Ax - b \geq 0, \qquad x^\top (Ax - b) = 0, \tag{5}$$

$$y \geq 0, \qquad Ay - b \geq 0, \qquad y^\top (Ay - b) = 0. \tag{6}$$

Since ΠP^k is positive semidefinite for all k, A is positive semidefinite as well. Hence,

$$(x - y)^\top A(x - y) \geq 0, \tag{7}$$

or, equivalently,

$$y^\top Ax \leq \frac{1}{2}(x^\top Ax + y^\top Ay). \tag{8}$$

Furthermore, since $y \geq 0$, from (5) and (6) we have $y^\top Ax \geq y^\top b = y^\top Ay$, and similarly $y^\top Ax \geq x^\top b = x^\top Ax$. Therefore, (8) must hold with equality, and hence so must (7), i.e. $\sum_k \frac{1}{\beta^k}(x-y)^\top \Pi P^k (x-y) = 0$. Again using the fact that ΠP^k is positive semidefinite for all k, this implies that $(x - y)^\top \Pi P^k (x - y) = 0$, or $\|P^k(x - y)\|_2^2 = 0$, for all k. Hence, $P^k(x - y) = 0$, for all k. $\square$

In particular, the lemma tells us that all state-price deflators that satisfy condition **C** induce the same asset valuation on any given exchange.

We now present our equilibrium characterization. It turns out that there is a unique[4] equilibrium of the arbitraged economy. The equilibrium valuation on each exchange coincides with the (subjective) valuation of the arbitrageur, given by a state-price deflator p^A, where p^A_s is the arbitrageur's marginal shadow value of consumption in state s.[5]

Let q^k be an equilibrium asset price vector for exchange k, and p^k a corresponding state-price deflator (i.e. p^k satisfies $q^k = {R^k}^\top \Pi p^k$).

Proposition 3.1 *There is a unique equilibrium of the arbitraged economy, with $p^k \equiv p^A$, for all k, where p^A is a state-price deflator for the arbitrageur, a vector satisfying*

[4]By uniqueness we mean that the equilibrium allocation and pricing functional on each exchange are unique. There may, of course, be multiple state-price deflators that induce the same equilibrium pricing functional.

[5]Formally, p^A_s is the Lagrange multiplier associated with the arbitrageur's no-default constraint in state s.

condition **C**. *The equilibrium demands of investors for state-contingent consumption are given by*

$$R^k\theta^{k,i} = \frac{1}{\beta^{k,i}} P^k(\mathring{p}^{k,i} - p^k), \qquad k \in K,\, i \in I^k. \tag{9}$$

Proof From (1), the demands of investors for state-contingent consumption are given by (9). In order to pin down the equilibrium prices, we solve the optimization problem of the arbitrageur. The Lagrangian is

$$\begin{aligned}\mathcal{L} &= \sum_{k\in K} q^k \cdot y^k - {p^A}^\top \Pi \sum_{k\in K} R^k y^k \\ &= \sum_{k\in K} (p^k - p^A)^\top \Pi R^k y^k,\end{aligned}$$

where p^A is the Lagrange multiplier vector attached to the no-default constraints, and can be interpreted as a (shadow) state-price deflator of the arbitrageur. At the optimum, the first-order conditions are satisfied:

$${R^k}^\top \Pi(p^k - p^A) = 0, \qquad k \in K, \tag{10}$$

together with complementary slackness:

$$p^A \geq 0, \qquad \sum_k R^k y^k \leq 0, \qquad \text{and} \qquad p^A_s \cdot \left[\sum_k R^k y^k\right]_s = 0, \quad \forall s. \tag{11}$$

The existence of the multipliers follows as usual from the linearity of the inequalities, as shown in Arrow et al. (1961), for instance.

From (10), it is immediate that $p^k \equiv p^A$, for all k. Due to market clearing,

$$R^k y^k = R^k \theta^k = \frac{1}{\beta^k} P^k(\mathring{p}^k - p^k), \qquad k \in K, \tag{12}$$

where the second equality follows from (2). Equations (11) and (12) together imply that p^A satisfies condition **C** (and, therefore, so does p^k). From Lemma 3.1, any choice of p^A that satisfies condition **C** gives us the same asset valuation, i.e. $P^k p^A = P^k p^k$ is unique. Clearly, the corresponding demands, given by (9), are unique as well. □

Since the arbitrageur takes prices as given, his optimal trade is unbounded if the valuation on any exchange does not agree with his own valuation. Thus we must have $p^k \equiv p^A$ for all k. Given this, his optimal supplies are indeterminate; he simply absorbs the excess demands of investors on each exchange. His profits are zero in equilibrium. Note that the arbitrageur valuation p^A need not be strictly positive in every state. This is due to our assumption that the arbitrageur consumes only at date 0. If his trades result in excess consumption in some state s at date 1, we have $p^A_s = 0$.

Since p^A satisfies condition **C**, it is a sort of average of the autarky state-price deflators $\{\mathring{p}^k\}_k$, with the weights depending on the depths and asset spans of the various exchanges (we will make this precise in Proposition 7.2). Due to arbitrageur-mediated trading, equilibrium prices on any one exchange reflect the valuations and depths of all exchanges in the economy. Such an outcome is what one would expect from a Walrasian auction mechanism. Indeed, we will show that the equilibrium of the arbitraged economy coincides with the equilibrium of an appropriately defined competitive economy with no arbitrageurs. This characterization turns out to be very useful in elucidating the role of arbitrageurs in integrating markets. As mentioned in the Introduction, we reserve the adjective "Walrasian" for competitive economies or equilibria with no arbitrageurs.

4 Walrasian Equilibrium with Restricted Consumption

In this section we analyze a competitive economy with no arbitrageurs which has the following convenient property: an equilibrium state-price deflator of this economy is equivalent to p^A, the arbitrageur's subjective state-price deflator in the arbitraged economy.

Definition 4.1 *A Walrasian equilibrium with restricted consumption (WERC) is a state-price deflator p^{RC}, and portfolios $\{\theta^{k,i}, \varphi^{k,i,\ell}\}_{k\in K,\, i\in I^k,\, \ell\in K}$, such that*

1. *Investor optimization: For given $q^k = R^{k\top}\Pi p^{RC}$, $k \in K$, $\{\theta^{k,i}, \{\varphi^{k,i,\ell}\}_{\ell\in K}\}$ solves*

$$\max_{\theta^{k,i}\in\mathbb{R}^{J^k},\, \varphi^{k,i,\ell}\in\mathbb{R}^{J^\ell}} \quad x_0^{k,i} + \sum_{s\in S}\pi_s\left[x_s^{k,i} - \frac{\beta^{k,i}}{2}(x_s^{k,i})^2\right]$$
$$s.t. \quad x_0^{k,i} = \omega_0^{k,i} - q^k\cdot\theta^{k,i} - \sum_{\ell\in K} q^\ell\cdot\varphi^{k,i,\ell},$$
$$x^{k,i} = \omega^{k,i} + R^k\theta^{k,i},$$
$$\sum_{\ell\in K} R^\ell\varphi^{k,i,\ell} \geq 0.$$

2. *Market clearing:*

$$\sum_{k\in K,\, i\in I^k} R^k\theta^{k,i} + \sum_{k\in K,\, i\in I^k,\, \ell\in K} R^\ell\varphi^{k,i,\ell} = 0.$$

At a WERC, agents can trade any asset in the economy, facing a common state-price deflator p^{RC}, but the date 1 consumption of agents on exchange k, in excess of their endowments, is restricted to lie in $\langle R^k\rangle$. For agent (k,i), the portfolio that leads to future consumption is $\theta^{k,i}$. He can choose, in addition, an auxiliary portfolio $\{\varphi^{k,i,\ell}\}_{\ell\in K}$, provided the payoff of this portfolio is nonnegative. As we shall explain later (in particular, see Example 5.1 and the ensuing discussion), the auxiliary portfolio mimics the role played by the arbitrageur in the arbitraged economy, by allowing investors access to global markets but not to additional consumption outside their local asset span.

Given asset payoffs $\{R^k\}_{k\in K}$, we say that asset prices $\{q^k\}_{k\in K}$ are globally weakly arbitrage-free if an agent with access to all the asset markets in the economy is unable to construct a weak arbitrage, i.e. for any portfolio $\{z^k\}_{k\in K}$ satisfying $\sum_{k\in K} R^k z^k \geq 0$, we have $\sum_{k\in K} q^k \cdot z^k \geq 0$. By the fundamental theorem of asset pricing, this is the case if and only if there exists $\psi \geq 0$ such that $q^k = {R^k}^\top \Pi\psi$, for all k. Clearly, due to the auxiliary portfolio, there cannot be a global weak arbitrage at a WERC. Hence an equilibrium state-price deflator p^{RC} can always be chosen to be nonnegative. Moreover, as the following proposition shows, there is a unique WERC and the asset valuation at this WERC coincides with the asset valuation of the arbitrageur at the equilibrium of the arbitraged economy.

Proposition 4.1 (WERC) *There is a unique WERC with $p^{RC} \equiv p^A$, and net trades of state-contingent consumption given by*

$$R^k\theta^{k,i} = \frac{1}{\beta^{k,i}} P^k(\mathring{p}^{k,i} - p^{RC}), \qquad k \in K,\, i \in I^k. \tag{13}$$

Proof The Lagrangian for agent (k,i)'s optimization problem is

$$\begin{aligned}\mathcal{L} = \omega_0^{k,i} - q^k \cdot \theta^{k,i} - \sum_{\ell\in K} q^\ell \cdot \varphi^{k,i,\ell} + \mathbf{1}^\top \Pi(\omega^{k,i} + R^k\theta^{k,i}) \\ - \frac{\beta^{k,i}}{2}(\omega^{k,i} + R^k\theta^{k,i})^\top \Pi(\omega^{k,i} + R^k\theta^{k,i}) + {\psi^{k,i}}^\top \Pi \sum_{\ell\in K} R^\ell \varphi^{k,i,\ell},\end{aligned}$$

where $\psi^{k,i} \in \mathbb{R}^S$ is a vector of Lagrange multipliers. Writing $q^k = {R^k}^\top \Pi p^{RC}$, the first-order conditions are equivalent to:[6]

$$\theta^{k,i} = \frac{1}{\beta^{k,i}}({R^k}^\top \Pi R^k)^{-1} {R^k}^\top \Pi(\mathring{p}^{k,i} - p^{RC}), \tag{14}$$

$${R^\ell}^\top \Pi \psi^{k,i} = q^\ell = {R^\ell}^\top \Pi p^{RC}, \qquad \forall \ell \in K, \tag{15}$$

$$\psi^{k,i} \geq 0, \tag{16}$$

$$\sum_{\ell\in K} R^\ell \varphi^{k,i,\ell} \geq 0, \tag{17}$$

$$\psi^{k,i} \cdot \left(\sum_{\ell\in K} R^\ell \varphi^{k,i,\ell}\right) = 0. \tag{18}$$

[6]In view of (16), equation (18) holds if and only if each of the S terms that are summed up in the inner product is zero.

In addition, we have the market-clearing condition:

$$\sum_{k,i} R^k \theta^{k,i} = -\sum_{k,i,\ell} R^\ell \varphi^{k,i,\ell}. \tag{19}$$

A WERC is completely characterized by equations (14)–(19). Equation (14) gives us the desired allocation (13), which in turn implies that

$$\sum_{k,i} R^k \theta^{k,i} = \sum_k \frac{1}{\beta^k} P^k (\mathring{p}^k - p^{RC}). \tag{20}$$

Equations (15) and (16) are the usual no-arbitrage conditions. In particular, $R^{\ell\top}\Pi\psi^{k,i}$ is independent of (k,i), so we can choose $\psi^{k,i}$ to be the same for all (k,i), and p^{RC} equal to this common value. Thus

$$p^{RC} = \psi^{k,i} \geq 0. \tag{21}$$

Equations (17)–(21) together imply that p^{RC} satisfies condition **C**. Since p^A also satisfies condition **C** (Proposition 3.1), we see from Lemma 3.1 that $p^{RC} \equiv p^A$, and moreover that the implied asset valuation is unique. The equilibrium allocation is then uniquely determined by (13). $\square$

Proposition 4.1 shows that a WERC is the appropriate Walrasian foundation for an equilibrium of the arbitraged economy. Before expanding on this theme, we consider another, more familiar, notion of restricted Walrasian equilibrium.

5 Walrasian Equilibrium with Restricted Participation

Segmented asset markets have been widely studied in the general equilibrium literature in the context of a Walrasian economy with restricted participation. In such an economy, agents face a common state-price deflator p^{RP}, but agents on exchange k can trade claims in $\langle R^k \rangle$ only. In this section, we show that valuation in a restricted-participation economy differs in a subtle way from valuation in the restricted-consumption economy studied above. In general, Walrasian equilibrium with restricted participation is not a suitable benchmark for an equilibrium of the arbitraged economy, as it captures only a subset of arbitrageur-mediated trades.

Definition 5.1 *A Walrasian equilibrium with restricted participation (WERP) is a state-price deflator p^{RP}, and portfolios $\{\theta^{k,i}\}_{k\in K,\, i\in I^k}$, such that*

1. *Investor optimization: For given $q^k = R^{k\top}\Pi p^{RP}$, $\theta^{k,i}$ solves*

$$\begin{aligned} \max_{\theta^{k,i}\in\mathbb{R}^{J^k}} \quad & x_0^{k,i} + \sum_{s\in S} \pi_s \left[x_s^{k,i} - \frac{\beta^{k,i}}{2}(x_s^{k,i})^2 \right] \\ s.t. \quad & x_0^{k,i} = \omega_0^{k,i} - q^k \cdot \theta^{k,i}, \\ & x^{k,i} = \omega^{k,i} + R^k \theta^{k,i}. \end{aligned}$$

2. Market clearing:

$$\sum_{k\in K,\, i\in I^k} R^k\theta^{k,i} = 0.$$

Defining

$$\lambda^k := \frac{\frac{1}{\beta^k}}{\sum_{j=1}^K \frac{1}{\beta^j}},$$

we have the following characterization of a WERP, analogous to Proposition 4.1 for a WERC:

Proposition 5.1 (WERP) *There is a unique WERP, with state-price deflator* p^{RP} *that solves*

$$\sum_{k\in K} \lambda^k P^k(\mathring{p}^k - p^{RP}) = 0, \tag{22}$$

and net trades of state-contingent consumption given by

$$R^k\theta^{k,i} = \frac{1}{\beta^{k,i}} P^k(\mathring{p}^{k,i} - p^{RP}), \qquad k \in K,\, i \in I^k. \tag{23}$$

Proof For a given q^k, agent (k,i) solves the same optimization problem as in the arbitraged economy. Hence his optimal portfolio is given by (1). Writing $q^k = R^{k\top}\Pi p^{RP}$, we obtain (23). The market-clearing condition is

$$\sum_{k,i} R^k\theta^{k,i} = \sum_k \frac{1}{\beta^k} P^k(\mathring{p}^k - p^{RP}) = 0,$$

which is equivalent to (22).

Using the shorthand notation defined in (3) and (4), a solution p^{RP} to (22) exists if and only if b is in the column space of A, or equivalently b is orthogonal to the orthogonal complement of the column space of A, i.e. $b^\top v = 0$ for every v satisfying $v^\top A = 0$. Consider such a vector v. Then $v^\top Av = \sum_k \frac{1}{\beta^k} v^\top \Pi P^k v = 0$. Since ΠP^k is positive semidefinite for each k, this implies that $v^\top \Pi P^k v = 0$, or $\|P^k v\|_2^2 = 0$, for all k. Hence $P^k v = 0$, for all k. It follows that

$$b^\top v = \sum_k \frac{1}{\beta^k} v^\top \Pi P^k \mathring{p}^k = \sum_k \frac{1}{\beta^k} (P^k v)^\top \Pi \mathring{p}^k = 0,$$

where we have used the symmetry of ΠP^k. This establishes existence.

Finally, we show uniqueness. Let x and y be two values of p^{RP} that satisfy equation (22). Then we have $Ax-b=0$ and $Ay-b=0$, so that $(x-y)^\top A(x-y) = 0$, i.e. $\sum_k \frac{1}{\beta^k}(x-y)^\top \Pi P^k (x-y) = 0$. Now we use the same argument as in the previous paragraph, exploiting the fact that ΠP^k is positive semidefinite for each k, to infer that $P^k(x-y) = 0$, for all k. Thus x and y are equivalent state-price deflators which give us the same asset valuation on each exchange. Portfolios are then uniquely pinned down by (23). $\square$

While there is always a nonnegative p^{RC}, by Proposition 4.1, this is not the case for p^{RP}. The following example illustrates:

Example 5.1 (WERP vs WERC) Consider an economy with two states of the world, two exchanges, and a single agent on each exchange. We refer to the agent on exchange k as agent k, $k = 1, 2$. The payoff matrices are

$$R^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad R^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

The two exchanges are equally deep, with β^1 and β^2 both equal to $\bar{\beta}$, which satisfies

$$0 < \bar{\beta} < \frac{\pi_1}{1 + \pi_1}. \tag{24}$$

Date 1 endowments are as follows: $\omega^1 = \mathbf{1}$ and $\omega^2 = (1/\bar{\beta} - 1)\mathbf{1}$. Autarky state-price deflators are, therefore, $p^1 = (1 - \bar{\beta})\mathbf{1}$ and $p^2 = \bar{\beta}\mathbf{1}$, respectively. Agent 1 values date 1 consumption more than agent 2. In autarky, $q^1 = (1 - \bar{\beta})\pi_1$, and $q^2 = \bar{\beta}$. The restriction (24) implies that $q^1 > q^2$. Thus there exist profit opportunities for arbitrageurs, buying on exchange 2 and delivering to exchange 1.

Now consider a WERP of this economy: agents face a common state-price deflator p^{RP}, but can only trade claims that lie in their local asset span. Since $\langle R^1 \rangle \cap \langle R^2 \rangle = \{0\}$, however, the two agents cannot trade with each other. Equilibrium asset prices are the same as in autarky. Since these prices allow for an arbitrage, albeit for a hypothetical agent with access to all markets, at least one of the state prices must be negative. The state-price deflator p^{RP} (which is unique since markets are complete in the integrated economy) solves $q^k = {R^k}^\top \Pi p^{RP}$, $k = 1, 2$:

$$p^{RP} = \begin{bmatrix} 1 - \bar{\beta} \\ (1/\pi_2)[(1 + \pi_1)\bar{\beta} - \pi_1] \end{bmatrix}.$$

It follows from (24) that $p_2^{RP} < 0$. Notice that this is not due to the non-monotonicity of quadratic utility. Equilibrium consumption at date 1 (which is just the initial endowment for both agents) is below the bliss point $1/\bar{\beta}$.

At the WERP, agents are unable to exploit the arbitrage opportunity, because doing so would take them outside their local asset span. In particular, if agent 1 were to buy the riskfree asset (which is underpriced from his perspective) from agent 2, he would end up with excess consumption in state 2. At the WERC, on the other hand, agents can arbitrage away the mispricing. Agent 1 simply disposes of the state 2 consumption good that he acquires from agent 2. Consequently $p_2^{RC} = 0$ (implying that $q^1 = q^2$). The equilibrium net trade of state-contingent consumption is given by $R^k\theta^k = \frac{1}{\beta^k} P^k(\mathring{p}^k - p^{RC})$, $k = 1, 2$. The projections P^1 and P^2 are:

$$P^1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad P^2 = \begin{bmatrix} \pi_1 & \pi_2 \\ \pi_1 & \pi_2 \end{bmatrix}.$$

Therefore, noting that $p_2^{RC} = 0$,

$$R^1\theta^1 = \frac{1 - \bar{\beta} - p_1^{RC}}{\bar{\beta}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad R^2\theta^2 = \frac{\bar{\beta} - \pi_1 p_1^{RC}}{\bar{\beta}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Market clearing for state 1 (in which there is no excess consumption) gives us $p_1^{RC} = \frac{1}{1+\pi_1}$, so that

$$R^1\theta^1 = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}, \qquad R^2\theta^2 = \begin{bmatrix} -\alpha \\ -\alpha \end{bmatrix},$$

where

$$\alpha := \frac{\pi_1}{\bar{\beta}(1+\pi_1)} - 1.$$

From (24) it follows that $\alpha > 0$. Agent 2 effectively sells α units of the riskfree asset to agent 1. Through this trade, agent 2 reduces his date 1 consumption by α in both states (and increases his date 0 consumption). Agent 1, on the other hand, is constrained by his local asset span to augment his date 1 consumption only in state 1. He increases his consumption in state 1 by α, while disposing of α units of state 2 consumption. It is easy to check that date 1 consumption at the WERC is below the bliss point $1/\bar{\beta}$ for both agents. Indeed, this would be the case even if agent 1 were allowed to consume the "excess" consumption in state 2; in other words, this consumption is in excess because it lies outside the permissible span, not because it takes the agent past his bliss point. $\quad\|$

The two notions of restricted Walrasian equilibrium differ in two key respects (both of which are captured by the auxiliary portfolios $\{\varphi^{k,i,\ell}\}$). First, at a WERP agents cannot trade claims outside their local asset span, while they can at a WERC. Second, the market-clearing condition at a WERC is weaker: at a WERC, we have $\sum_k R^k\theta^k \leq 0$, while at a WERP, $\sum_k R^k\theta^k = 0$. There may be arbitrage opportunities at a WERP that investors are unable to exploit due to their restricted-participation constraints. This is not the case at a WERC.

Both notions of equilibrium capture the idea that arbitrageurs allow investors to trade their own claims abroad. The weaker restrictions implicit in a WERC mimic the allocational role that arbitrageurs play over and above their mediation of this obvious category of trades. Indeed, arbitrageurs allow investors to trade *any* claim available in the economy. Investors can thereby exploit good deals in the global markets, which relaxes their date 0 budget constraint. They are better off as a result, even if they have to discard consumption in some states at date 1 to remain within their local asset span.

Notice that the state-price deflator p^{RC} need not be strictly positive. This is due to the fact that investors by construction behave as if they are satiated in those directions of the consumption space that lie outside the imposed span. The states in which investors dispose of consumption at a WERC are precisely those in which the arbitrageur disposes of consumption at the equilibrium of the arbitraged economy. In these states, $p_s^{RC} = p_s^A = 0$.

6 Walrasian Equilibrium with Complete Markets

A third notion of Walrasian equilibrium, indeed the most natural one, that we will have occasion to consider is Walrasian equilibrium with complete markets (or WECM). Formally, a WECM is just a special case of a WERP, with $R^k = P^k = I$, for all k. We denote a WECM state-price deflator by p^{CM}. Due to market completeness, the state-price deflator associated with a given WECM is unique. In fact, there is a unique WECM:

Proposition 6.1 (WECM) *There is a unique WECM, with*

$$p^{CM} = \sum_{k \in K} \lambda^k \mathring{p}^k,$$

and

$$\theta^{k,i} = \frac{1}{\beta^{k,i}}(\mathring{p}^{k,i} - p^{CM}), \qquad k \in K,\, i \in I^k.$$

This result is immediate from Proposition 5.1. The state-price deflator p^{CM} can be interpreted as the investors' economy-wide average willingness to pay, with the willingness to pay on each exchange weighted by its relative depth.

7 Explicit Characterizations

We have argued above that while a WERC serves as a suitable Walrasian benchmark for an equilibrium of the arbitraged economy, in general a WERP does not. This is because at a WERP there may be unexploited arbitrage opportunities, a situation that clearly cannot arise in the arbitraged economy. However, what if a WERP happens to be arbitrage-free?

Proposition 7.1 *Consider an economy with WERP and WERC state-price deflators given by p^{RP} and p^{RC}, respectively. There is no global weak arbitrage at the WERP if and only if $p^{RP} \equiv p^{RC}$.*

Proof By the FTAP, there is no global weak arbitrage at the WERP if and only if there exists a nonnegative p^{RP}. If $p^{RP} \geq 0$, the equations characterizing the WERC, (14)–(19), are satisfied at $\psi^{k,i} = p^{RC} = p^{RP}$, and $\varphi^{k,i,\ell} = 0$, for all k, i, ℓ. Hence $p^{RP} \equiv p^{RC}$. Conversely, if $p^{RP} \equiv p^{RC}$, then p^{RP} can be chosen to be nonnegative. $\square$

Thus, if there is an arbitrage-free WERP, it does serve as a suitable Walrasian benchmark for the equilibrium of the corresponding arbitraged economy. While this is applicable in an admittedly narrow class of economies, it is nevertheless of interest. As we shall see shortly, the assumptions commonly made in the literature limit us to this set of economies. Moreover, under these assumptions, there is a simple closed-form solution for p^{RP}.

Arbitrage opportunities can arise at a WERP because of the participation constraints that investors face, as in Example 5.1, or because investors who could potentially exploit these opportunities are satiated. A sufficient condition for a WERP to be arbitrage-free is that there is an investor who has access to all asset markets in the economy, and that this investor is nonsatiated at the equilibrium. The market access condition in our setting is simply the following:

(S1) $\langle R^1 \rangle$ contains $\langle R^k \rangle$, for all $k \in K$.

It says that there is an exchange (which we take to be exchange 1 without loss of generality) that has maximal asset span, in that this span contains the spans of all other exchanges. Thus investors on exchange 1 can trade all the assets in the economy.

In order to state the nonsatiation condition in terms of the primitives of the economy, it is convenient to introduce some additional notation. Let

$$\beta := \left[\sum_k (\beta^k)^{-1} \right]^{-1}$$

$$Q^1 := \left[\lambda^1 I + \sum_{k \neq 1} \lambda^k P^k \right]^{-1},$$

$$Q^k := \left[\lambda^1 I + \sum_{k \neq 1} \lambda^k P^k \right]^{-1} P^k, \qquad k \neq 1.$$

The inverse in the definition of Q^k exists since the matrix

$$\lambda^1 \Pi^{-1} + \sum_{k \neq 1} \lambda^k R^k ({R^k}^\top \Pi R^k)^{-1} {R^k}^\top$$

is positive definite, hence invertible. We will employ the following nonsatiation condition:

(N1) $\mathbf{1} - \beta \cdot \sum_{k \in K} Q^k \omega^k \geq 0.$

It says that the representative agent with aggregate preference parameter β is nonsatiated at the weighted aggregate endowment, $\sum_{k \in K} Q^k \omega^k$.

Proposition 7.2 *Under* **S1**, $p^{RP} \equiv \bar{p}^{RP}$, *where*

$$\bar{p}^{RP} := \sum_{k \in K} \lambda^k Q^k \mathring{p}^k. \qquad (25)$$

Furthermore, $\bar{p}^{RP} \geq 0$ *if and only if* **N1** *holds so that, under* **S1** *and* **N1**, $p^{RC} \equiv p^{RP} \equiv \bar{p}^{RP}$.

Proof From (25),

$$\left[\lambda^1 I + \sum_{k\neq 1} \lambda^k P^k\right] \bar{p}^{RP} = \lambda^1 p^1 + \sum_{j\neq 1} \lambda^j P^j p^j.$$

Premultiplying both sides by P^1, and noting that **S1** implies that $P^1 P^k = P^k$:

$$\left[\lambda^1 P^1 + \sum_{k\neq 1} \lambda^k P^k\right] \bar{p}^{RP} = \lambda^1 P^1 p^1 + \sum_{j\neq 1} \lambda^j P^j p^j,$$

i.e.

$$\sum_k \lambda^k P^k (\mathring{p}^k - \bar{p}^{RP}) = 0.$$

Therefore, $p^{RP} = \bar{p}^{RP}$ solves (22). Recalling that $\mathring{p}^k = \mathbf{1} - \beta^k \omega^k$, it is easy to verify that $\bar{p}^{RP} = \mathbf{1} - \beta \cdot \sum_{k\in K} Q^k \omega^k$, so that $\bar{p}^{RP} \geq 0$ if and only if **N1** holds.[7] □

A sharper characterization of p^{RP} can be obtained under the following alternative set of conditions (we define $\omega := \sum_k \omega^k$):

(S2) Either (a) $\langle R^k \rangle = \langle R \rangle$, $k \in K$, or (b) $\mathring{p}^k - p^{CM} \in \langle R^k \rangle$, $k \in K$.

(N2) $\mathbf{1} - \beta\omega \geq 0$.

Condition **S2**(a) specializes **S1** to the case in which all exchanges have the same asset span. **S2**(b) is the condition that characterizes equilibrium security design in Rahi and Zigrand (2009), in a setting in which strategic arbitrageurs play a two-stage game, designing security payoffs in the first stage and carrying out their arbitrage trades in the second. Notice that **S1** and **S2** are not nested. Condition **N2** says that the representative investor for the whole economy with aggregate preference parameter β is weakly nonsatiated at the aggregate endowment ω.

Proposition 7.3 *Under* **S2**, $p^{RP} \equiv p^{CM}$. *Furthermore,* $p^{CM} \geq 0$ *if and only if* **N2** *holds so that, under* **S2** *and* **N2**, *we have* $p^{RC} \equiv p^{RP} \equiv p^{CM}$.

Proof If **S2**(a) holds, $P^k = P$, for all k. Then it is easy to see that $p^{RP} = p^{CM}$ solves (22).[8] Under **S2**(b), $P^k(\mathring{p}^k - p^{CM}) = \mathring{p}^k - p^{CM}$, so p^{CM} solves (22) in this case as well. Finally, note that $p^{CM} = \mathbf{1} - \beta\omega$, so that $p^{CM} \geq 0$ if and only if **N2** holds. □

While condition **S2** is quite restrictive, it is nevertheless more general than the assumption that the same assets are traded on every exchange, an assumption that is commonly made in the literature on arbitrage in asset markets.

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[7] Exactly the same proof goes through if we assume from the start that $P^1 = I$. Thus $\bar{p}^{RP}$ is the (unique) WERP state-price deflator of the economy in which asset payoffs are the same as in the original economy except that markets are complete on exchange 1.

[8] If markets are complete, p^{CM} is the unique solution to (22). If markets are incomplete, so that the common span $\langle R \rangle$ is a strict subset of $\mathbb{R}^S$, p^{CM} is still a solution, but it is not the only one. All solutions are of course equivalent to p^{CM}.

8 Walrasian Foundations

In this section, we give an overview of the Walrasian foundations we have provided in this paper for arbitrage in a segmented markets economy.

Recall that, in the arbitraged economy, the equilibrium prices of assets on each exchange are equal to the arbitrageur valuation of these assets. We have shown that the latter is just the asset valuation at the WERC of the corresponding Walrasian economy with no arbitrageurs (Proposition 4.1). Comparing (9) and (13), we also see that the equilibrium allocation (for investors) in the arbitraged economy is the same as the WERC allocation. Closed-form solutions for the WERC valuation can be derived under restrictions on preferences, endowments and the asset structure that ensure that the WERC and WERP coincide (Propositions 7.2 and 7.3). We summarize these observations in the following proposition, which makes precise the sense in which the arbitraged economy is Walrasian:

Proposition 8.1

1. *For every $k \in K$, the equilibrium valuation on exchange k in the arbitraged economy is equal to the WERC valuation, i.e.* $q^k = {R^k}^\top \Pi p^{RC}$. *Under* **S1** *and* **N1**, *this is also the WERP valuation,* ${R^k}^\top \Pi \bar{p}^{RP}$. *Under* **S2** *and* **N2**, *it coincides with the WECM valuation,* ${R^k}^\top \Pi p^{CM}$.

2. *The equilibrium allocation in the arbitraged economy coincides with the WERC allocation. Under either* **S1** *and* **N1**, *or under* **S2** *and* **N2**, *this is also the WERP allocation.*

As stated in the proposition, under **S2** and **N2**, the WECM valuation obtains. However, we do not get the WECM allocation unless it coincides with the WERP allocation. A sufficient condition for the latter is complete markets on each exchange ($\langle R^k \rangle = \mathbb{R}^S$, for all k), in addition to **N2**. For an economy in which investors on any given exchange have the same no-trade valuations, i.e. $\mathring{p}^{k,i} = \mathring{p}^k$, for all $i \in I^k$, **S2**(b) and **N2** suffice as well.

9 Conclusion

Given an economy with an arbitrary asset structure, if we view Walrasian equilibria with no arbitrageurs as approximations to more complex equilibria with segmented markets connected by arbitrageurs, the concept of Walrasian equilibrium with restricted consumption introduced in this paper is the appropriate benchmark, rather than the well-studied and intuitive notion of Walrasian equilibrium with restricted participation. The subtle difference between these two kinds of Walrasian equilibrium clarifies the sense in which arbitrageurs serve to integrate markets.

We impose strong preference assumptions in order to obtain a tractable framework. While the intuitions garnered from analysis do not specifically depend on these assumptions, a suitable Walrasian foundation for arbitraged economies with more general preferences remains an open question.

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