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Manipulating Market Sentiment*

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Abstract

We analyze a simple model of an asset market, in which a large rational trader interacts with "noise speculators" who seek short-run speculative gains, and become active following a prolonged episode of mispricing relative to the asset's fundamental value. The model gives rise to price patterns such as bubble dynamics, positive short-run correlation and vanishing long-run correlation of price deviations from the fundamental value. We argue that this example model sheds light on the question as to whether rational speculators abet or curb price fluctuations.

KEYWORDS: behavioral finance, price manipulation, bounded rationality, trading rules, speculative trade

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1 Introduction

One of the main themes in the behavioral finance literature has been the effect that boundedly rational traders have on price fluctuations in financial

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markets. In seminal papers such as Shleifer et al. (1990a,b) and Hong and Stein (1999), conventionally rational traders coexist with “noise traders” (agents whose trading behavior follows some exogenous stochastic process), or with agents who follow trading rules, such as “fundamental trading” or “trend seeking”, based on an incomplete understanding of the market.

A maintained assumption in this literature has been that the market is competitive in that rational traders are price takers and have no market power. In many markets, however, some rational traders have genuine market power: a large hedge fund acting in a (relatively thin) derivative market, for example, or a large oil producing country in a market for oil-related securities.

This short paper is a modest attempt to explore the effects of boundedly rational trading on price fluctuations when some rational traders have market power. We analyze a simple example of speculative trade between a large rational trader and boundedly rational speculators who follow a trading rule that conditions on the observed price history. We use this example to show how rich patterns of asset price fluctuations can emerge from very simple boundedly rational trading rules, as a result of their interaction with a large rational trader. Specifically, although the speculators' trading rule neither follows a trend nor responds to price trends, the expected asset price induced by the large trader's optimal strategy displays “bubble dynamics”: during periods of low volume of speculative trade, the expected price strictly increases (decreases) when it is above (below) the fundamental value. This means that during these periods, price discrepancies are positively correlated in the short run and negatively correlated in the long run. The effects are suggestive of phenomena that have been documented in real-life financial markets (e.g., see Daniel et al. (1998)).

2 The Model

An asset is traded in a market in periods $0, 1, 2, ...$ This asset has a constant fundamental value equal to v. At the beginning of each period t, a long-lived large trader chooses a quantity x^t of the asset that he supplies to the market. Let p^t denote the price in period t and $\theta^t = p^t - v$ denote the deviation from the fundamental value. Demand for the asset is generated by two groups of agents:

Arbitrageurs. Their net demand in period t is $y^t = D(\theta^t)$. The function

$D : \mathbb{R} \to \mathbb{R}$ thus represents the arbitrageurs' reactivity to current price discrepancies. We assume that D is continuous, strictly decreasing and odd (i.e., $D(\theta) = -D(-\theta)$). Let $\Theta(y) = D^{-1}(y)$ denote the inverse demand.

Noise speculators. Their net demand at period t, denoted z^t, is a stochastic function of the history of price deviations $(\theta^s)_{s=1}^{t-1}$. We will impose structure on this function below.[1]

The market price p^t is determined by a market clearing condition

$$x^t = D(\theta^t) + z^t.$$

The large trader's information set at period t consists of the entire history of price deviations $h^t = (\theta^1, .., \theta^{t-1})$. His profit in period t is $x^t\theta^t$. He chooses a trading strategy - namely, a function that assigns a supply quantity to every information set - that maximizes his discounted expected profits. The discount factor is $0 < \delta < 1$.

The interpretation of the market structure is as follows. The large trader can access a large external competitive market and buy or sell any quantity at the fundamental price v, with no transaction cost. The implicit assumption behind the arbitrageurs' demand function is that they can also access the external market, albeit with increasing transaction costs that cause their net position to grow in absolute terms with the magnitude of the price discrepancy. Thus, one could view this market as a local marketplace set up by the large trader, who takes advantage of his privileged access to a global competitive market. The large trader may have an incentive to manipulate the local market price in anticipation of the noise speculators' reaction, but is mindful of the (limited) arbitrage activity that exploits deviations of the local market price from the fundamental value.

Let us turn to the description of the noise speculators' behavior. Fix $\alpha \geq 0$. For any given finite history $h^t = (\theta^1, .., \theta^{t-1})$, if $\left|\theta^{t-1}\right| > \alpha$, let $B(h^t)$ be the largest integer s for which $sign(\theta^{t-k}) = sign(\theta^{t-1})$ and $\left|\theta^{t-k}\right| > \alpha$ for all $k = 1, ..., s$. That is, $B(h^t)$ is the duration of the most recent episode of persistent mispricing of magnitude greater than α in a given direction relative to the fundamental value. If $\left|\theta^{t-1}\right| \leq \alpha$ or $t = 0$, set $B(h^t) = 0$. We assume that at every period t,

$$z^t = \varepsilon^t + w^t - w^{t-1}$$

[1] Note that while the arbitrageurs' behavior at period t reacts instantaneously to θ^t, all other traders react to the price history up to period $t - 1$.

where ε^t is *i.i.d* according to a density f that is symmetric around zero, $w^t = 0$ when $B(h^t) < L$, and $w^t = a \cdot sign(\theta^{t-1})$ when $B(h^t) \geq L$, where $L > 2$ is an integer and $a \in \{-1, 1\}$. Let F be the *cdf* induced by f, and assume $F(\alpha) < 1$.

The interpretation of the process governing z^t is as follows. Noise speculators consist of one conventional noise trader and one naive speculator. The former agent's net demand at period t is ε^t. The latter agent takes a buy or short position of one unit in each period; this position must be closed one period later. He takes a non-zero position only after a sufficiently long sequence of price discrepancies in the same direction and of sufficient magnitude. The naive speculator's net position at t is thus $w^t - w^{t-1}$. We say that a history h^{t-1} is *inactive* if $w^{t-2} = w^{t-1} = 0$ and $B(h^{t-1}) < L$. At an inactive history, the naive speculator is "waiting" for a critical streak of price discrepancies to form and does not take a non-zero position. Since we allow a to be either positive or negative, we can capture two types of "market sentiment". When $a = -1$, it is apt to refer to the noise speculator as a "fundamental trader", because he acts at period t as if the market is about to correct the mispricing. On the other hand, when $a = 1$, we may refer to the noise speculator as a "momentum trader". Our results can be extended to the case in which a is stochastic. The large trader's activity thus manipulates market sentiment in the sense that it helps activating the perception that the market is about to crash, or that it has gained momentum, etc.

If the large trader only faced arbitrageurs and conventional noise traders - i.e., if $w^t = 0$ for all t - he would be unable to make any speculative gain, and his optimal policy would be to supply a zero quantity in every period. Thanks to the naive speculator, the large trader has an incentive to manipulate the market price, in order to induce the naive speculator to become active, and then lean against him when he does.

3 The Result

Our objective is to provide a qualitative characterization of the price fluctuations that emerge from the large trader's optimal net supply of the asset in each period. We first observe that the large trader faces a Markov decision problem. The naive speculator's net demand at period t following the history h^{t-1} is a deterministic function of the state defined by $q(h^{t-1}) = ((sign(\theta^{t-2}), B(h^{t-2})), ((sign(\theta^{t-1}), B(h^{t-1})))$. Since the behavior of arbitrageurs and the conventional noise trader is entirely stationary, it follows that the large trader's dynamic optimization problem is Markovian w.r.t to the set of states Q defined above. An inactive history h^{t-1} corresponds to a state with $B(h^{t-2}), B(h^{t-1}) < L$.

Let $V(q)$ be the value function given by a solution to this problem. Note that the arbitrageurs' demand function D, the density f and the naive speculators' trading rule are all symmetric w.r.t the sign of price discrepancies. Therefore, V is symmetric in the following sense. Let $q = ((i, B), (j, B'))$ and $q' = ((-i, B), (-j, B'))$. Then, $V(q) = V(q')$.

The following notation will be useful. Consider an inactive history h^{t-1} with $B(h^{t-1}) = B < L$, and $\theta^{t-1} > 0$. We denote the state that corresponds to this history by ρ^B. We use F^q to denote the *cdf* of z^t conditional on a history h^{t-1} that corresponds to the state $q(h^{t-1})$. The expected price deviation at period t given x^t and a history h^{t-1} is thus

$$E\left(\theta^t \mid h^{t-1}, x^t\right) = \int \Theta\left(x^t - z^t\right) dF^{q(h^{t-1})}(z^t)$$

Note that this expression is decreasing in x^t.

Proposition 1 (Bubble dynamics) *Let x^* be a trading strategy that solves the large trader's problem. Consider two inactive histories h^t and h^s for which $B(h^s) < B(h^t) < L - 1$. Then,*

$$\begin{aligned} E\left(\theta^{t+1} \mid h^t, x^*(h^t)\right) > E\left(\theta^{s+1} \mid h^s, x^*(h^s)\right) > 0 \quad if \quad \theta^t, \theta^s > 0 \\ E\left(\theta^{t+1} \mid h^t, x^*(h^t)\right) < E\left(\theta^{s+1} \mid h^s, x^*(h^s)\right) < 0 \quad if \quad \theta^t, \theta^s < 0 \end{aligned}$$

Proof. We will only prove the first row of inequalities, as the remaining set is symmetric. Given an inactive history h, define

$$\pi(x, h) = x \int \Theta(x - z)\, dF^{q(h)}(z)$$

Note that, if $E(z \mid h) = 0$, then $\pi(0, h) = 0$ and $\pi(x, h) < 0$ whenever $x \neq 0$.

Consider the state ρ^B corresponding to an inactive history. Recall that by definition, $B < L$. Note that $V(\rho^B) > 0$, since the myopic maximization of $\pi\left(x, \rho^B\right)$ leads with positive probability to a future history in which the expectation of z is non-zero. Since the value function is symmetric w.r.t the sign of the history of price discrepancies, the Bellman equation for $0 \leq B \leq L-1$ is

$$\begin{aligned} V(\rho^B) &= \max_x \{\pi\left(x, \rho^B\right) \\ &+\delta V(\rho^{B+1})\left(1 - F^{\rho^B}\left(x - D\left(\alpha\right)\right)\right) \\ &+\delta V(\rho^1) F^{\rho^B}\left(x - D\left(-\alpha\right)\right) \\ &+\delta V(\rho^0)\left(F^{\rho^B}\left(x - D\left(\alpha\right)\right) - F^{\rho^B}\left(x - D\left(-\alpha\right)\right)\right)\} \end{aligned}$$

Now,

$$\begin{aligned} V(\rho^0) &= \pi\left(x^*(\rho^0), \rho^0\right) \\ &+\delta V(\rho^1)\left(1 - F^{\rho^0}\left(x^*(\rho^0) - D\left(\alpha\right)\right) + F^{\rho^0}\left(x^*(\rho^0) - D\left(-\alpha\right)\right)\right) \\ &+\delta V(\rho^0)\left(F^{\rho^0}\left(x^*(\rho^0) - D\left(\alpha\right)\right) - F^{\rho^0}\left(x^*(\rho^0) - D\left(-\alpha\right)\right)\right) \end{aligned}$$

Since $\pi\left(\rho^1, x^*(\rho^1)\right) \leq 0$, if $V(\rho^1) \leq V(\rho^0)$, by simple substitution it is easy to verify that $V(\rho^0) \leq 0$, a contradiction. Thus, $V(\rho^1) > V(\rho^0)$. Now let $\hat{B}$ be the smallest $B \leq L-1$ such that $V(\rho^{B+1}) \leq V(\rho^B)$. Then,

$$V(\rho^{\hat{B}}) \leq \pi\left(x^*(\rho^{\hat{B}}), \rho^{\hat{B}}\right) + \delta V(\rho^{\hat{B}})$$

which implies $V(\rho^{\hat{B}}) \leq 0$, a contradiction. Thus, $V(\rho^{B+1}) > V(\rho^B)$ for any $0 \leq B \leq L-1$.
Since $\pi\left(\cdot, \rho^1\right)$ and $F^{\rho^1}\left(z\right)$ are symmetric around zero and $V(\rho^2) > V(\rho^1)$, we have that $x^*(\rho^1) \leq 0$. Otherwise, choosing $-x^*(\rho^1)$ would yield a higher payoff. By definition, for $0 < B < L-1$,

$$\begin{aligned} V(\rho^B) &\geq \pi\left(x^*(\rho^{B+1}), \rho^B\right) + \delta V(\rho^{B+1})\left(1 - F^{\rho^B}\left(x^*(\rho^{B+1}) - D\left(\alpha\right)\right)\right) \\ &+\delta V(\rho^1) F^{\rho^B}\left(x^*(\rho^{B+1}) - D\left(-\alpha\right)\right) \\ &+\delta V(\rho^0)\left(F^{\rho^B}\left(x^*(\rho^{B+1}) - D\left(\alpha\right)\right) - F^{\rho^B}\left(x^*(\rho^{B+1}) - D\left(-\alpha\right)\right)\right) \end{aligned}$$

and

$$\begin{aligned} V(\rho^{B+1}) &\geq \pi\left(x^*(\rho^B), \rho^{B+1}\right) + \delta V(\rho^{B+2})\left(1 - F^{\rho^{B+1}}\left(x^*(\rho^B) - D\left(\alpha\right)\right)\right) \\ &+\delta V(\rho^1) F^{\rho^{B+1}}\left(x^*(\rho^B) - D\left(-\alpha\right)\right) \\ &+\delta V(\rho^0)\left(F^{\rho^{B+1}}\left(x^*(\rho^B) - D\left(\alpha\right)\right) - F^{\rho^{B+1}}\left(x^*(\rho^B) - D\left(-\alpha\right)\right)\right) \end{aligned}$$

Thus, since $F^{\rho^{B+1}}(x) = F^{\rho^B}(x)$ for any $x \in \mathbb{R}$,

$$\begin{aligned}&\left(1 - F^{\rho^B}\left(x^*(\rho^B) - D(\alpha)\right)\right)\left(V(\rho^{B+1}) - V(\rho^{B+2})\right) \geq \\ &\left(1 - F^{\rho^B}\left(x^*(\rho^{B+1}) - D(\alpha)\right)\right)\left(V(\rho^{B+1}) - V(\rho^{B+2})\right)\end{aligned}$$

which implies that $x^*(\rho^B) \geq x^*(\rho^{B+1})$ as $V(\rho^{B+1}) < V(\rho^{B+2})$. To establish the strictness of the inequality, note that since $V(\rho^B) < V(\rho^{B+1})$, the first-order necessary conditions fail when $x^*(\rho^B) = x^*(\rho^{B+1})$. ■

Proposition 1 describes the evolution of expected prices after consecutive periods of mispricing (above a minimal magnitude) in a given direction. Expected prices deviate further from the fundamental value as the duration of mispricing becomes longer. The intuition for this result is simple. The large trader benefits from speculative trade only because he can anticipate when the naive speculator will submit non-zero demand. As a streak of price deviations in a given direction is formed, the large trader knows that the moment in which the naive speculator becomes active gets nearer. If the noise trader's random demand shocks terminate this episode before the critical date arrives, the opportunity for a speculative gain is lost and the large trader must wait for the next streak to form. Therefore, the large trader has a stronger incentive to prolong the streak, and consequently the expected price deviation becomes larger.

An apt analogy for this predicament is the Greek myth of King Sisyphus rolling a boulder up a hill. Should the boulder fall before Sisyphus reaches the hilltop, he will have to go down and start all over again. Introducing economic reasoning into the myth, Sisyphus's incentive to prevent the boulder from falling is greater as he approaches the hilltop, because the opportunity cost of dropping it becomes larger. Similarly, the large trader has a greater incentive to manipulate the price as the duration of mispricing increases, because the opportunity cost of terminating a streak rises. This translates into an increasingly larger bias relative to the fundamental. Thus, even though the traders' behavior during this phase neither obeys a trend nor responds to one, their interaction with large trader causes the emergence of patterns of increasing or decreasing prices.

A numerical illustration

Suppose that the arbitrageurs' demand is given by $D(\theta^t) = -\theta^t$, and that ε^t is uniformly distributed over $[-k, k]$. As to the naive speculator, let $\alpha = 0$

and $a = -1$ (i.e., the naive speculator is a "fundamental trader"). The following graphs illustrate bubble dynamics by showing $|E(\theta \mid h, x^*(h)|$ as a function of $B(h)$, under various values of k and L, for $\delta = 0.99$.

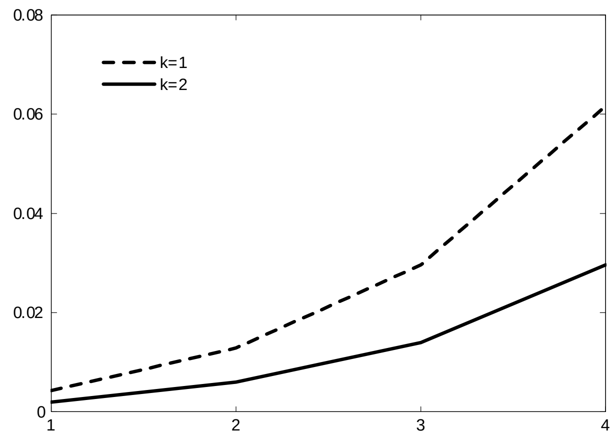

Figure 1: $|E(\theta \mid h, x^*(h)|$ as a function of $B(h)$ for $L = 5$.

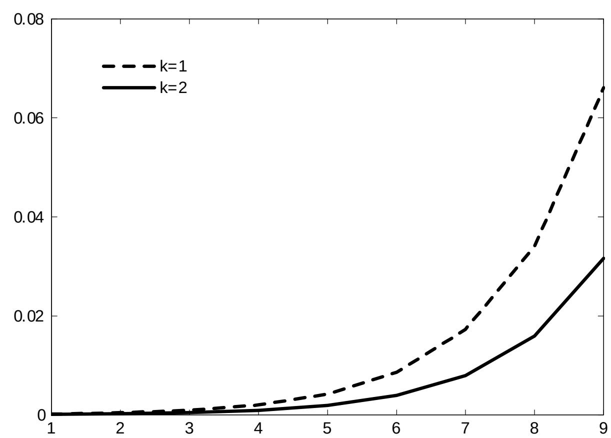

Figure 2: $|E(\theta \mid h, x^*(h)|$ as a function of $B(h)$ for $L = 10$.

Price deviations increase at an accelerated rate as the bubble is prolonged. Note that when k increases, the effect becomes muted. The intuition is that as conventional noise trader's behavior becomes more unpredictable, it becomes less profitable for the large trader to lean against the naive speculator's expected attack of the bubble.

Short-run and long-run correlation of price discrepancies
Our last observation applies Proposition 1 to characterize short-run and long-run correlations of price discrepancies. Specifically, when $w^t = 0$, $\theta^t > \alpha$ and $B(h^t) < L$, the price deviation is likely to persist at $t+1$, in the sense that $\theta^{t+1} > 0$ with probability greater than $\frac{1}{2}$. Put differently, our model implies positive short-run correlation of price discrepancies during periods of low speculative trade volume.

The picture is different in the long run. The process governing z^t is a finite-state Markov process which treats positive and negative price discrepancies symmetrically. Therefore, the invariant measure over the state space is ergodic, and the long-run average price deviation induced by the optimal policy is zero. In other words, the large trader's interventions only affect temporary deviations of the asset price from its fundamental value, but they do not affect its long-run average. Any correlation of price discrepancies vanishes in the long run as deviations in any direction eventually dissipate.

4 Concluding Remarks

Our objective in this short paper was merely to illustrate that the combination of rational traders with market power and price-taking, boundedly rational speculators can generate price fluctuation patterns of interest. However, we believe it also sheds some light on an ancient debate regarding the role of rational speculators in financial markets. Some argue that speculators sow instability and create excess volatility, whereas others argue that speculators have a stabilizing role, as they bring prices back to fundamentals by spotting arbitrage opportunities. Our model synthesizes both views. Even if the large rational trader did not exist, episodes of persistent mispricing would occur spontaneously from time to time, and this would lead to large price fluctuations due to the activity of the naive speculator. The rational trader's strategic, forward-looking behavior in our model can make these episodes more likely to happen, thereby raising the *frequency* of large price movements. Nevertheless, the *amplitude* of the large price movements due to the naive speculator's activity is reduced, because the rational trader leans against him. Although the large rational trader precipitates episodes of large volatility, he curbs their amplitude when they occur. Thus, if we view price volatility as a "problem", then rational speculators are both part of the problem and part of its solution.

Related literature
There is a huge literature in behavioral finance that addresses price fluctuations due to the presence of noise/naive traders. Our paper adds a dimension to this literature by introducing rational, forward-looking traders with market power. As a result, our modeling style is different from the competitive equilibrium methodology that characterizes this literature.

Our paper is also related to a smaller literature that asks whether a large rational speculator can make profits in a market for a financial asset or a storable commodity by manipulating prices (see Hart (1977), Hart and Kreps (1986), Jarrow (1992)). This literature has not addressed effects on price dynamics, and has treated the behavior of traders that the large speculator faces as a black box without deriving it from behavioral rules. The only exceptions we are aware of are Mei, Wu and Zhou (2004) and Rubinstein and Spiegler (2008). The former paper analyzes a finite-horizon model with one large manipulator, a population of rational arbitrageurs and a set of traders prone to the disposition effect (a tendency to avoid selling losing assets), and derive some asset price anomalies. The latter paper analyzes the interaction between a large rational trader and boundedly rational speculators whose trading rule responds to the *ergodic price distribution*. As a result, the Rubinstein-Spiegler model cannot be reduced to a Markov decision problem. Finally, there is a large literature on information-based manipulation of market prices, where a large informed trader exploits informational asymmetries and the presence of noise traders to make speculative profits at the expense of rational, uninformed traders (Kyle (1985), Allen and Gale (1992) are key references in this literature).

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