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Debt Maturity and the Liquidity of Secondary Debt Markets*

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Abstract

We develop an equilibrium model of debt maturity choice of firms, in the presence of fixed issuance costs in primary debt markets, and an over-the-counter secondary debt market with search frictions. Liquidity in this market is related to the ratio of buyers to sellers, which is determined in equilibrium via the free entry of buyers. Short maturities improve the bargaining position of debtholders who sell in the secondary market and hence reduce the interest rate that firms need to offer on debt. Long maturities reduce re-issuance costs. The optimally chosen maturity trades off both considerations. Firms individually do not internalize that choosing a longer maturity increases the expected gains from trade in the secondary market, which attracts more buyers, and hence also facilitates the sale of debt issued by other firms. As a result, the laissez-faire equilibrium exhibits inefficiently short maturity choices. Empirical implications of the model include that issuance yields and bid-ask spreads should be increasing in maturity.

JEL classifications: G12, G32
Keywords: debt maturity, search, liquidity.

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1 Introduction

In the run up to the recent financial crisis, the use of some types of corporate debt securities created substantial maturity mismatch. For example, Schroth, Suárez, and Taylor (forthcoming) estimate that Asset-Backed Commercial Paper (ABCP) with an average maturity of around 37 days was used to finance assets with an average duration of around 5.8 years. With hindsight, it is clear that maturity mismatch exacerbated the problems that the financial system faced during the crisis. As a consequence, there is now a move towards the explicit regulation of maturity transformation.[1] A central question is why issuers chose to finance long term assets by issuing such extremely short-term debt securities. In this paper, we explore a possible answer to this question that relates to the liquidity of the secondary market for debt securities, and describe a novel type of inefficiency which provides a rationale for mandating longer maturities.

The argument is as follows. First, a short maturity increases the price at which a debtholder who needs to sell is able to do so in an over-the-counter (OTC) secondary market: When this debtholder finds a potential buyer, they start bargaining over the price. The bargaining position of the seller depends on his outside option, which is to refuse to trade at the negotiated price and to find another buyer. Importantly, while searching for another buyer, the debt might mature, and the seller would then receive the repayment of face value in cash. The shorter the maturity, the more likely this is to happen, and hence the better the bargaining position of the seller, and the higher the price he will obtain. Investors who buy in the primary market will anticipate that debt with shorter maturity has a higher secondary market price, and require lower yields at issuance. From the perspective of issuers, a shorter maturity therefore implies a lower cost of debt.[2]

At the same time, the number of potential buyers that are attracted to the secondary debt

[1] An example of such regulation would be the Net Stable Funding Ratio of Basel III (Basel Committee on Banking Supervision, 2010).

[2] To the best of our knowledge, this point was first made by He and Milbradt (2011).

market will depend on the prices that they will have to pay in this market. If the bargaining position of sellers is good because of short maturities, then buyers will have to pay high prices. So few buyers would be interested in being active in this market, and the market would be illiquid as a consequence.

In this paper, we describe this mechanism in a model in which all debt securities are traded in a secondary search market, and in which there is free entry of potential buyers into this search market. Our main result is that in equilibrium the maturities chosen by issuers are inefficiently short. An individual issuer takes into account that increasing the maturity worsens the bargaining position of her debtholders when they have to sell, which raises the yield that the issuer has to promise at issuance in the primary market. However, she does not internalize that doing so also increases the profits of buyers in the secondary market. The presence of more profitable deals available to buyers promotes their entry into the secondary market, which makes it easier for holders of debt of other issuers to locate alternative buyers, and hence reduces the yield that other issuers have to promise at issuance. This is a novel externality that is distinct from the standard externalities present in search models with ex-post bargaining and free entry (Hosios, 1990).

A policy conclusion is that mandating longer maturities would improve welfare. For example, financial institutions issuing commercial paper rely on very short maturities, and the secondary market for this paper is so illiquid as to be almost non-existent. The model suggests that if issuers were forced to sell longer-maturity paper, this would attract more buyers to the secondary market for commercial paper, making it easier to sell, and hence decreasing the need to issue such short maturity paper.

In our continuous-time infinite-horizon model, we have two types of agents with different time preferences. There are *entrepreneurs* who each can set up a firm which undertakes one long-term project that generates perpetual constant cash flows. Entrepreneurs are impatient, i.e. have a high discount rate. There are also *investors*, who are born patient with a low discount rate, but are subject to random preference shocks that make them impatient and increase their discount rate.

We assume that there is a constant and large inflow of new, patient investors. In order to take advantage of the differences in time preferences between entrepreneurs and investors, the firms set up by entrepreneurs issue debt to investors. Investors who become debtholders and then become impatient will want to consume, and can attempt to sell to patient investors with funds in the secondary market.

We model this secondary market as a search market in which sellers (who are impatient) and buyers (who are patient) are matched according to a constant returns to scale matching function. The intensities at which buyers and sellers get matched with each other depend only on the ratio of buyers to sellers in the secondary market. In particular, the rate at which sellers find buyers is increasing in the ratio of buyers to sellers. After a match, the price in the secondary market at which trade is realized is determined through Nash bargaining, and will therefore be increasing in the rate at which sellers find other potential buyers (and hence will be increasing in the ratio of buyers to sellers), because it improves the bargaining position of the seller. At the same time, the price in the secondary market will be decreasing in maturity, because it worsens the bargaining position of the seller. Since investors who buy in the primary market anticipate the effect of maturity, firms pay lower interest rates in the primary market when issuing at shorter maturities.

We model the primary debt market in a reduced form manner. We assume that when issuing, or when re-issuing in order to roll over, firms can place their debt to investors via a competitive auction, which generates a fixed cost. Everything else being equal, firms have an incentive to increase maturity in order to decrease the number of times this fixed cost is paid.[3]

[3] Although there are types of debt securities, such as corporate bonds, for which a (small) fixed cost of issuance appears to exist (Altinkiliç and Hansen, 2000), more generally, this assumption can also be interpreted as shorthand for other mechanisms that generate a preference for longer maturities, for instance roll-over risk (see e.g. He and Xiong, 2012a).

The maturity decisions of firms trade off the frictions in the primary and secondary debt markets. When the ratio of buyers to sellers in the secondary market is low, the effect of maturity on price in the secondary market is strong, and hence the effect on interest rates in the primary market is strong. Firms then find it optimal to issue short maturity debt, even if this implies a high cost of re-issuing. Conversely, when the ratio of buyers to sellers is high, the effect of maturity on the price in the secondary market and hence on interest rates in the primary market is weak. Firms then find it optimal to issue long maturity debt, to reduce the cost of re-issuing.

Finally, the ratio of buyers to sellers in the secondary market is determined through free entry of buyers into the market. This entry in turn depends on the gains from trade that stem from the difference in the valuation of debt claims between (impatient) sellers and (patient) buyers. The longer the maturity of debt, the higher the differences in valuations, and hence the more attractive it is for buyers to enter the secondary market.

In equilibrium, agents in our economy correctly anticipate the ratio of buyers to sellers in the secondary market, and all decisions are optimal. Maturities chosen by firms are inefficiently short, because firms do not internalize how their maturity choice affects the ratio of buyers to sellers: When a firm chooses a longer maturity, this increases the gains from trade that can be realized in the secondary market, which attracts more buyers to the market and increases the ratio of buyers to sellers. This in turn reduces the interest rate that all other firms have to pay in the primary market on their debt and increases their profits.

Finally, we extend the model to include marketmakers that can intermediate trade between (final) buyers and sellers and show that the bid-ask spreads of marketmakers should increase in maturity, a result which is consistent with the empirical evidence on estimated corporate bond transaction price spreads of Edwards, Harris, and Piwowar (2007).

The rest of the paper is organized as follows. Section 2 presents the related literature, both empirical and theoretical. In Section 3 we describe the model. Section 4 discusses the determination of equilibrium. In Section 5, we show that equilibrium maturities are inefficiently short, and discuss the underlying assumptions that generate this inefficiency. We also comment how the inefficiency in our model differs from the standard inefficiency present in models of search with ex-post bargaining and free entry. In Section 6, we extend the model to incorporate marketmakers, and then illustrate the extended model with a numerical example. Finally, Section 7 concludes. All proofs are in the appendix.

2 Related Literature

We first discuss some features of the market for corporate debt claims, and findings of the empirical literature which are relevant for the main mechanism in the model, and then discuss how our paper fits into the theoretical literature.

Secondary trading in corporate debt claims takes place mostly OTC.[4] Most corporate debt claims are traded only very infrequently, at least in comparison to equity claims issued by the same entity. This infrequent trading is apparent for corporate bonds,[5] syndicated loans,[6] as well as for commercial paper,[7] and is indicative of low liquidity. There is evidence that liquidity (or lack

[4]Historically, corporate bonds used to be traded on the NYSE. However, there was a migration towards OTC trading starting in the 1940s (Biais and Green, 2007). As of 2002, only about 5% of all US corporate bonds were still listed on the NYSE (Edwards, Harris, and Piwowar, 2007), and the average trade size on the NYSE is quite small compared to OTC trades (Hong and Warga, 2000).

[5]For instance Edwards, Harris, and Piwowar (2007) report that for their fairly representative dataset, which contains about 12.3 million trades in 22,000 US corporate bonds over 2 years, the median number of trades per day per corporate bond is about one.

[6]Although dollar volumes in the secondary market for the syndicated loan market have exceeded dollar volume in the market for corporate bonds (Bessembinder and Maxwell, 2008), data from the Loan Syndications and Trading Association suggests that most syndicated loans also trade relatively infrequently. The 2008 Loan Market Chronicle reports 63,490 trades on 2,278 facilities over the 3rd Quarter of 2007 (p. 30). By a rough calculation, this suggests an average of 28 trades per quarter per facility, or roughly one every three days.

[7]Covitz and Downing (2007) report that the trades in the secondary market make up only 16% of the total transaction volume in their data (the rest being attributable to primary market transactions).

thereof) is priced, in corporate bonds,[8] syndicated loans,[9] and commercial paper.[10]

Importantly, time-to-maturity appears to matter for liquidity. For corporate bonds, Edwards, Harris, and Piwowar (2007) and Bao, Pan, and Wang (2011) find that their preferred measure of illiquidity (the estimated transaction price spread and negative price autocovariance, respectively) increases with time-to-maturity. For commercial paper, Covitz and Downing (2007) find no direct evidence that links a measure of illiquidity to time-to-maturity, but do show that yield spreads increase in time-to-maturity. (All of these papers control for credit quality.)

Other important characteristics of debt claims that are related to liquidity, and which our model will not be able to shed light on, are age (measured as time since issuance), and credit risk.[11]

Within the theoretical literature, the paper most closely related to ours is that of He and Milbradt (2011), who consider endogenous transaction costs in the secondary market for debt that arise due to search and bargaining frictions, which affect a bankruptcy decision of shareholders similar to that in Leland (1994). They study the dynamic feedback between secondary market illiquidity and default risk that arises from the fact that default not only means lower payoffs, but also makes debt claims more illiquid. Our paper shares with theirs the modeling of secondary trading as a search and bargaining process and also the insight on the effect of debt maturity on the bargaining position of sellers. Our model complements theirs by examining the implications of joint maturity choices for the entry of buyers into the secondary market, which allows us to study

[8] Longstaff, Mithal, and Neis (2005) document that the non-default related component of yield spreads can be up to around 50% of the total, and suggest that this non-default component is strongly related to measures of illiquidity. Bao, Pan, and Wang (2011) find that higher illiquidity is strongly related to lower prices.

[9] Gupta, Singh, and Zebedee (2008) find that loans with higher expected liquidity can be issued with lower spreads.

[10] Covitz and Downing (2007) argue that higher illiquidity raises yield spreads.

[11] For bonds, there is ample evidence that illiquidity increases with age (see e.g. Edwards, Harris, and Piwowar, 2007, Bao, Pan, and Wang, 2011), and that illiquidity increases with credit risk. Conversely, for syndicated loans, there is evidence that illiquidity decreases with credit risk, as market participants appear to be interested mostly in trading distressed loans (see e.g. Gupta, Singh, and Zebedee, 2008).

the efficiency properties of the resulting equilibrium and derive normative implications.

Our paper uses a search-and-matching model with ex-post bargaining. The search approach that we use was initially made popular in a labor market context (Diamond, 1982a,b, Mortensen, 1982, Pissarides, 1985). It has been applied to describing OTC markets by Duffie, Garleanu, and Pedersen (2005), Vayanos and Wang (2007), Vayanos and Weill (2008), Afonso (2011) and others. Our model differs slightly from the typical approach in these papers, in two respects. First, we assume a constant-returns-to-scale (CRS) matching function as opposed to the increasing-returns-to-scale (IRS) matching function used in these papers. Loosely speaking, with an IRS matching function, an additional seller entering the market will make it more attractive for buyers to enter, without making it much harder for other sellers to find buyers. This means that strong positive liquidity externalities are assumed as part of the technology. We assume a CRS matching function in order to highlight that our results do not depend on this technological assumption in order to generate our externality.[12] In this sense, our assumptions are closer to those of Weill (2008) and Lagos and Rocheteau (2007), who also consider alternatives to the IRS matching function used in the first set of papers. Second, we consider a situation with free entry of buyers, similar to Lagos and Rocheteau (2007).

Our paper is also related to an emerging literature which considers the effect of aggregate shocks on financial institutions, and finds market failures that generate excessively short maturity structures. Stein (2012) and Segura and Suárez (2012) find that the interaction between pecuniary externalities in the market for funds during liquidity crises and the financial constraints of banks leads to excessive short-term debt issuance. In Farhi and Tirole (2012), the collective expectation of a bailout gives incentives to choose maturities that are too short. These kind of market failures justify potential regulatory intervention that mandates longer maturities. Our model highlights

[12]For a discussion of the use of CRS versus IRS matching functions in the labor literature, see e.g. Petrongolo and Pissarides (2001).

that mandating longer maturities would also increase the liquidity of secondary markets.

We focus on a particular motivation for maturity choice. Others are considered in the literature. For example, in the classical banking literature, it is often argued that (forms of) short-term debt can act as a disciplining device (see e.g. Calomiris and Kahn, 1991), or that short-term debt can be used to signal quality (see e.g. Diamond, 1991). More recently, Dangl and Zechner (2006) show that shorter maturities can serve to commit equityholders to reducing leverage after poor performance, Greenwood, Hanson, and Stein (2010) develop a model in which firms choose maturities in response to the maturity choices of government, given a fixed demand by investors for certain maturities, and Brunnermeier and Oehmke (2013) argue that an inability of issuers to commit to a maturity structure can lead to a choice of inefficiently short maturities, as creditors who lend at longer maturities know that their claim on firm value is likely to be diluted ex-post through subsequent issuance at shorter maturities.

3 The Model

Time is continuous and indexed by $t \geq 0$. There are two types of infinitely-lived, utility-maximizing, and risk-neutral agents: Entrepreneurs and investors. There are many entrepreneurs. Each entrepreneur has a large endowment of funds, and can set up a firm that can operate one project. The project requires an initial investment of 1 at $t = 0$, and subsequently produces a perpetual cash flow of $x > 0$. Entrepreneurs have discount rate $\rho > 0$.

Each investor is endowed with an equivalent small amount of funds. We normalize this so that a measure 1 of investors has a total endowment of 1. An investor is either *patient* and has a discount rate of 0, or *impatient* and has a discount rate of ρ. Patient investors are subject to (idiosyncratic) liquidity shocks that arrive at Poisson rate θ and are i.i.d. across investors. Once hit by the shock, a patient investor irreversibly becomes impatient. At every time t there is a large inflow of patient investors into the economy. Investors can consume their endowment, can store it at a net rate of return of zero, or can buy the debt issued by firms, as described below. Without loss of generality, we assume that investors only consume their funds when they are impatient.

Since entrepreneurs attach a higher value to present consumption than patient investors, they may prefer to let the firm finance the investment in the project through issuing debt which is placed with investors. Each firm can have a single debt issue outstanding, with an aggregate face value of 1.[13] We assume that maturity is stochastic and arrives at Poisson rate $\delta \geq 0$, chosen by the firm at $t = 0$ and held fixed through time.[14] When a debt issue matures, the repayment of principal is financed via funds raised from re-issuing the maturing debt. We will refer to δ as the *refinancing frequency*. We consider debt without embedded put options.[15]

Debt also pays a continuous interest rate of r, set in an auction as described below. There is a *primary* and *secondary market* for debt.

In the primary market, firms issue debt at $t = 0$ and then refinance it every time it matures. Debt is placed to investors through an auction in which all investors can freely participate. Investors observe the refinancing frequency δ of a debt issue, and then submit bids of interest rates r at which they are willing to buy a unit of the debt issue at par.

We assume that firms incur a cost $\kappa > 0$ each time an auction for debt issuance is set up. Because of the assumption of stochastic maturity, firms would be exposed to the risk of having to pay κ at random times in order to re-issue debt. This risk would not be present in a model with deterministic maturity. To simplify, we assume that firms can insure against this risk and cover these costs by paying a flow of $\delta\kappa$ per unit of time, equal to the expected issuance cost. As in

[13]One can consider a version of the model in which there is a choice as to how much debt to issue. This adds complexity, but does not provide important additional insights.

[14]This is for the purpose of analytical tractability, as in Blanchard (1985), Leland (1998), He and Xiong (2012b).

[15]In Appendix C, we consider embedding put options, and show that this is not in general optimal in our setting.

Dangl and Zechner (2006), debt issuance costs generate a preference for issuing debt with longer maturities that reduce the frequency at which the cost is incurred.

For convenience, we maintain an assumption on parameter values throughout most of the paper.

Assumption 1.

$$\min\left(\frac{x}{\rho}, 1\right) - \kappa > \frac{\theta}{\rho + \theta} \tag{1}$$

This assumption is sufficient to ensure that the utility that can be obtained from a debt-financed project is positive, and exceeds the utility that can be obtained from a project financed with the entrepreneur's own funds.

A debtholder who becomes impatient attaches a lower value to a debt claim than an investor who is still patient. The gains from trade between these two types of agents can be realized in a secondary market, which is subject to search frictions. The debt of all firms trades in the same secondary market. Searching buyers in this market incur a non-pecuniary flow cost of effort $e_B > 0$ per unit of time while they are searching. For simplicity, we assume that sellers incur no such cost.[16]

We let $\mu(\alpha_t^S, \alpha_t^B)$ denote the aggregate flow of matches between sellers and buyers, where α_t^S, α_t^B are the measures of sellers and buyers, respectively, in the secondary market at time t. These measures will be endogenously determined in equilibrium. The matching function satisfies $\mu(0, \alpha^B) = \mu(\alpha^S, 0) = 0$, is increasing in both arguments, and has continuous derivatives. In order to highlight that the results derived in the paper do not rely on the strong "thick market externalities" inherent in an increasing returns to scale matching function, we assume that the

[16]We have explored a version of the model in which sellers also incur a search cost. This complicates the analysis substantially but leaves the main result unchanged. The only additional result is that there can also exist equilibria in which there is no trade in secondary markets.

matching function exhibits constant returns to scale, and let μ be concave and homogeneous of degree one in (α^S, α^B). As long as $\alpha^S > 0, \alpha^B > 0$, we can define $\phi := \frac{\alpha^B}{\alpha^S}$, and then define $\mu_S(\phi) := \mu(\alpha^S, \alpha^B)/\alpha^S = \mu(1, \phi)$ as the rate at which sellers find a counterparty, and $\mu_B(\phi) := \mu(\alpha^S, \alpha^B)/\alpha^B = \mu(\phi^{-1}, 1)$ as the rate at which buyers find a counterparty. We assume that these rates satisfy the following *congestion properties*:

$$\begin{aligned} &\lim_{\phi \to 0} \mu_S(\phi) = 0, \quad \lim_{\phi \to \infty} \mu_S(\phi) = \infty, \\ &\lim_{\phi \to 0} \mu_B(\phi) = \infty, \quad \lim_{\phi \to \infty} \mu_B(\phi) = 0. \end{aligned} \tag{2}$$

These equations simply state that when there are more sellers (buyers) in the market it is more difficult for a seller (buyer) to get matched with a buyer (seller). From the perspective of all agents, the ratio of buyers to sellers ϕ will be sufficient for describing the state of the secondary market.

When sellers and buyers get matched, they engage in Nash bargaining over the trading price with bargaining powers $\beta, 1 - \beta$, respectively, with $\beta \in (0, 1)$.

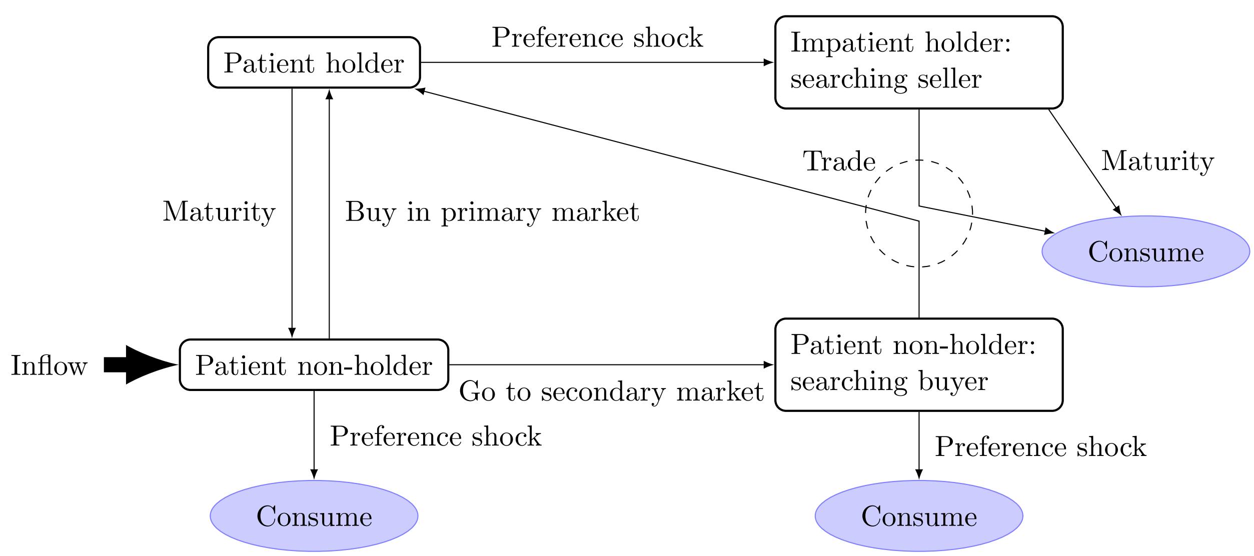

Figure 1: Flow diagram for investors

This flow diagram illustrates the possible states that investors can transition through in the model.

To summarize, decisions are as follows: At $t = 0$, firms decide on the refinancing frequency

δ of their debt. They take this decision based on an expectation of the ratio of buyers to sellers $(\phi_t)_{t\geq 0}$. Then, for every $t \geq 0$ patient investors with funds decide whether to bid in the primary market auctions of any current debt (re-)issue, whether to search to buy in the secondary market, or whether to store their endowment. Impatient investors with funds will consume, and impatient debtholders decide whether to search to sell in the secondary market. These decisions are taken based on the publicly known refinancing frequency choices δ of firms and on an expectation of the ratio of buyers to sellers $(\phi_{t'})_{t'\geq t}$. We illustrate these decisions in Figure 1.

We focus on steady-state equilibria in which all quantities that are determined in equilibrium are constant through time. The equilibrium is characterized by a pair (δ^e, ϕ^e) such that: first, given an expectation of the ratio of buyers to sellers $\phi = \phi^e$, the refinancing frequency choices δ^e of firms are optimal, and second, the free entry decisions of investors into both the primary and secondary market are optimal given (δ^e, ϕ^e), which amounts to the condition that investors obtain no rents in any of these markets.

4 Equilibrium

We find the equilibrium of the economy by following a sequence of steps: We first work out how free entry of investors into the primary market determines the interest rate r that firms have to pay on debt as a function of their choice of refinancing frequency δ, taking the ratio of buyers to sellers ϕ as given. We then analyze the firm's optimal choice of refinancing frequency δ, given ϕ. Finally, we determine the ratio of buyers to sellers ϕ that is compatible with free entry of investors into the secondary market, for a given refinancing frequency δ chosen by all firms. Taken together, equilibrium is characterized by the intersection point of two curves in (ϕ, δ)-space.

4.1 The interest rate in the primary market

In order to compute the interest rate that is determined in the primary market auctions, we first need to consider the utility that investors derive from holding debt that pays an interest rate of r and has a refinancing frequency δ. We use $V_0(r, \delta, \phi)$ and $V_\rho(r, \delta, \phi)$ to denote the utility that a patient and an impatient debtholder obtain, respectively, from holding a unit of the debt issue (r, δ), for a given ratio of buyers to sellers ϕ. Below, we will omit the arguments of V_0 and V_ρ where possible to reduce notational clutter.

Due to the higher discount rate of impatient investors, we have that $V_\rho < V_0$. We normalize the utility that is obtained from consuming an amount of funds equivalent to an investor's endowment to 1.

Patient debtholders do not search to sell in the secondary market, because buyers do not attach a higher utility to holding the debt, and hence there are no potential gains from trade. In contrast, there are gains from trade between impatient debtholder and buyers: Suppose that an impatient debtholder is matched with a patient buyer, and that trade takes place at price P per unit of face value. Then the surplus that the impatient seller obtains is $\Delta_S := P - V_\rho$. The surplus a patient buyer obtains is $\Delta_B := V_0 + 1 - P - V_B$, where we let V_B denote the value for a patient investor who is searching to buy in the secondary market.[17] The total gains from trade are therefore:

$$\Delta_S + \Delta_B = V_0 - V_\rho + 1 - V_B. \tag{3}$$

We will see later that free entry of investors into the (buy side of) the secondary market implies that in equilibrium, $V_B = 1$, and hence total gains from trade in equilibrium are equal to:

$$\Delta_S + \Delta_B = V_0 - V_\rho > 0. \tag{4}$$

[17]Note that this surplus includes the term $1 - P$ to account for the part of the buyer's endowment that is left over after paying the price P.

The gains from trade are positive, which confirms that every match will result in a trade, with the price P splitting the surplus according to Nash bargaining,

$$P = \beta V_0 + (1-\beta)V_\rho, \tag{5}$$

where β and $1-\beta$ are the bargaining power parameters of the seller and buyer, respectively. Since $P \geq V_\rho$ we see that it is optimal for impatient debtholders to search to sell in the secondary market.

We can now write a system of recursive flow-value equations that V_0 and V_ρ satisfy in steady state:

$$r + \delta(1 - V_0) + \theta(V_\rho - V_0) = 0, \tag{6}$$

$$r + \delta(1 - V_\rho) + \mu_S(\phi)(P - V_\rho) = \rho V_\rho. \tag{7}$$

The first equation states that for a patient investor, the utility flow stemming from the continuous interest payments, the possibility of maturity, and the possibility of becoming impatient, just balance the reduction in utility due to discounting at rate 0 (which is zero). The second equation states that for an impatient investor, the utility flow stemming from the continuous interest payments, the possibility of maturity, and the possibility of locating a buyer in the secondary market and selling at price P, just balance the reduction in utility due to discounting at rate ρ.

Obviously, the utility of a patient debtholder $V_0(r, \delta; \phi)$ is increasing in the interest flow r, and the profits of the firm and hence the utility of the entrepreneur are decreasing in r. There is free entry of patient investors into the primary market auctions, who will compete by bidding successively lower interest rates r, until, in equilibrium,

$$V_0(r, \delta; \phi) = 1. \tag{8}$$

Given the expression for $V_0(r, \delta; \phi)$ that can be derived from equations (6) and (7), using (5), the condition (8) determines the interest rate $r(\delta; \phi)$ that firms have to pay when issuing debt. We summarize this discussion in the following lemma:

Lemma 1. *For a given ratio of buyers to sellers ϕ and refinancing frequency choice δ of a firm, the interest rate $r(\delta;\phi)$ that is set in the primary market auctions is given by:*

$$r(\delta;\phi) = \frac{\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta}\theta. \tag{9}$$

The interest rate exceeds 0, the discount rate of patient investors, because bidders require compensation for the utility losses associated with the frictions faced when attempting to sell in the secondary market. They will suffer these losses in case they become impatient before maturity, and need to sell, so that the interest rate can be interpreted as an illiquidity premium. The magnitude of frictions can be indirectly measured via the discount in the secondary market price $1 - P$ that impatient debtholders accept in order to be able to liquidate their position, which can be calculated using equation (6) as

$$1 - P(\delta;\phi) = (1-\beta)(V_0 - V_\rho(\delta;\phi)) = (1-\beta)\frac{r(\delta;\phi)}{\theta}. \tag{10}$$

In equilibrium, the interest rate will just compensate for the expected loss incurred when becoming impatient, which is $\theta(V_0 - V_\rho)$. We can see that the price discount $1-P$ (which benefits the buyer), is just equal to the buyers share $1-\beta$ of the gains from trade $V_0 - V_\rho$. The gains from trade, the interest rate, and the price discount are therefore all tightly linked.

As the ratio of buyers to sellers increases, it becomes easier for sellers to find a buyer. The bargaining position of sellers therefore improves, and hence the price discount and the interest rate decrease. In the limit as $\phi \to \infty$, sellers can find a buyer instantaneously, and the price discount and the interest rate tend to zero. As refinancing frequency δ increases, searching sellers are more likely to have their debt mature before they find a buyer, which improves their bargaining position, implying a higher secondary market price (and hence a lower price discount), and a lower interest rate, as shown in Figure 2. As liquidity shocks become more frequent (θ increases) the interest rate that investors demand increases because it is more likely that they become impatient before the

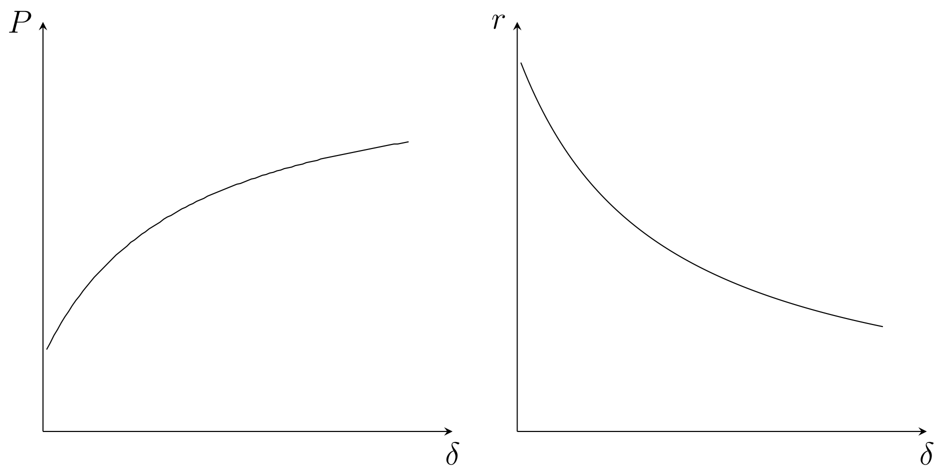

Figure 2: Price in the secondary market, interest rate in the primary market
Interest rate r that the firm has to pay in the primary market and secondary market price P, both as a function of refinancing frequency δ.

debt matures.

4.2 The firm's problem

At $t = 0$, firms choose whether to issue debt and undertake the project. If the project is undertaken and debt is issued, the firm also needs to decide on the refinancing frequency δ. The firm anticipates that in order to issue debt at par, it needs to pay an interest flow of $r(\delta;\phi)$ as given by (9). Debt issuance is feasible as long as the cash flow from the project, x, exceeds the flow cost of debt, $r(\delta;\phi) + \delta\kappa$.

An entrepreneur consumes the residual cash flows and hence her utility when there is investment and the firm issues debt with refinancing frequency δ is:

$$U(\delta, r(\delta;\phi)) = -1 - \kappa + 1 + \int_0^\infty e^{-\rho t}(x - r(\delta;\phi) - \delta\kappa)dt,$$

where the first term is the cost of the investment and the second the cost of the initial debt issuance (which needs to be paid by the entrepreneur). The third accounts for the proceeds from debt issuance. The last term accounts for the discounted value of the net excess cash flows that the firm generates. The expression for U can be rewritten as:

$$U(\delta, r(\delta;\phi)) = \frac{x}{\rho} - \kappa - \frac{r(\delta;\phi) + \delta\kappa}{\rho}. \tag{11}$$

The first term corresponds to the present value that an entrepreneur attaches to the gross cash flows of the project. The second term is the first issuance cost, and the last term represents the discounted cost of servicing the debt for $t > 0$, which includes re-issuance costs as well as interest payments.

The firm's optimal refinancing frequency choice can then be written as:

$$\max_{\delta \geq 0} U(\delta, r(\delta;\phi)) \quad \Leftrightarrow \quad \min_{\delta \geq 0} \frac{r(\delta;\phi) + \delta\kappa}{\rho}. \tag{12}$$

We can see that the firm chooses a refinancing frequency to minimize the cost of debt service, trading off a higher re-issuance cost against lower required interest rates at higher refinancing frequencies (i.e. at shorter maturities).

The optimal descision of the firm is described in the following lemma:

Lemma 2. *It optimal to undertake the project and to issue debt. In addition, for every ϕ, the firm's problem* (12) *has a unique solution $\delta^*(\phi)$ which is given by:*

$$\delta^*(\phi) = \max\left\{\sqrt{\frac{\rho\theta}{\kappa}} - \theta - \rho - \mu_S(\phi)\beta, 0\right\}.$$

We illustrate how the optimal choice of refinancing frequency δ^* varies with the ratio of buyers to sellers ϕ in Figure 3. As buyers become scarce and $\phi \to 0$, the only way in which investors can liquidate their investment is by being repaid at maturity. This makes long maturity debt very expensive for firms and they choose a high refinancing frequency (a short expected maturity). As ϕ increases, the maturity of debt becomes less important to investors, since they can more easily liquidate their investment by selling in secondary markets. Hence firms find it optimal to choose

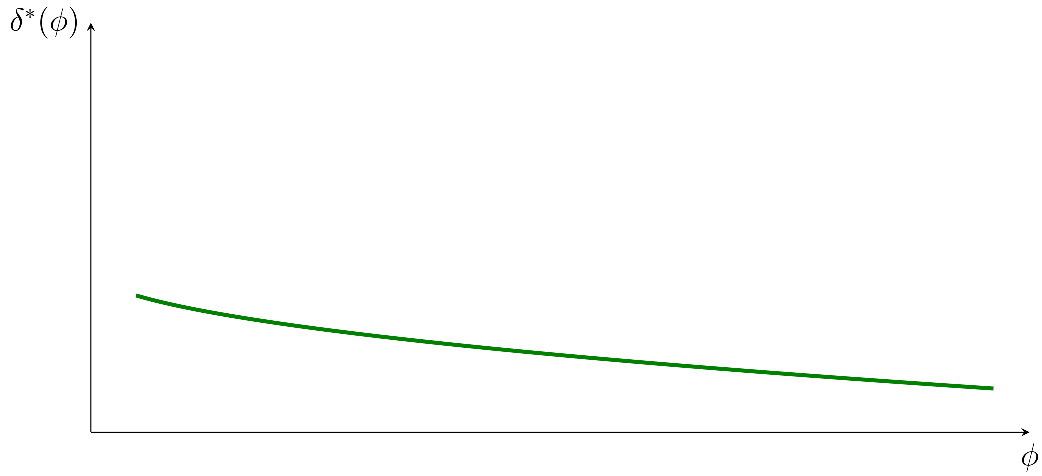

Figure 3: Optimal refinancing frequency as function of ϕ
Optimal refinancing frequency $\delta^*(\phi)$ chosen by firms, as a function of the ratio of buyers to sellers ϕ.

a lower refinancing frequency (that is, to lengthen the expected maturity), in order to reduce the expected issuance costs. When the ratio of buyers to sellers ϕ becomes sufficiently large, the firm eliminates re-issuance costs completely by setting $\delta = 0$, that is, by issuing perpetual debt.

4.3 Entry into the secondary market

We now consider what ratio of buyers to sellers ϕ is consistent with free entry of buyers into the secondary market, given a choice of refinancing frequency δ by firms.

We let $V_B(\delta; \phi)$ denote the utility that a patient investor who enters the secondary market to attempt to buy obtains when firms have chosen a refinancing frequency δ, and the current ratio of buyers to sellers is ϕ.

First, we establish that there must be trade after a match: If the patient investor who enters to attempt to buy is matched with a seller, there is trade if and only if the total gains from trade given in equation (3) are positive. If there was no trade following the match, then since searching is costly the entrant would be better off consuming his endowment and not entering, implying that it should be the case that $V_B(\delta;\phi) < 1$. We can see immediately from equation (3) that total gains from trade would be positive in this case, which is a contradiction. Therefore there has to be trade after a match.

Since there is trade after a match, $V_B(\delta;\phi)$ satisfies the following flow-value equation in steady state:

$$-e_B + \mu_B(\phi)(1-\beta)\left(V_0 - V_\rho + 1 - V_B\right) + \theta(1 - V_B) = 0. \tag{13}$$

The equation states that the (dis-)utility flow from the effort cost of searching, the possibility of meeting a seller which leads to trade at a price that gives a fraction $1-\beta$ of the total surplus to the buyer, and the possibility of becoming impatient and having to consume the endowment must just balance the reduction in utility due to discounting at rate 0.

We now turn to possible equilibrium values for $V_B(\delta;\phi)$. Intuitively, if buyers could obtain positive rents in the secondary market, a very large number would enter. This increased competition would have two effects: First, it would become very difficult for any particular buyer to be matched with a seller. Also, it would drive down the profits buyers can obtain if matched with a seller. Both effects would drive down the expected profits to buyers in the market until $V_B \leq 1$. At the same time, if $V_B < 1$, then no buyers should enter. Since there would still be sellers in the market, any buyer that did enter would be matched instantaneously with a seller, and would obtain positive rents. This discussion can be formalized and leads to the following lemma:

Lemma 3. *In equilibrium, free entry ensures that the utility of a searching buyer satisfies* $V_B(\delta;\phi) = 1$.

After substituting $V_B = 1$ into equation (13) and using equation (6), we obtain a *free entry condition* that describes how buyers enter the secondary market, which is summarized in the following lemma:

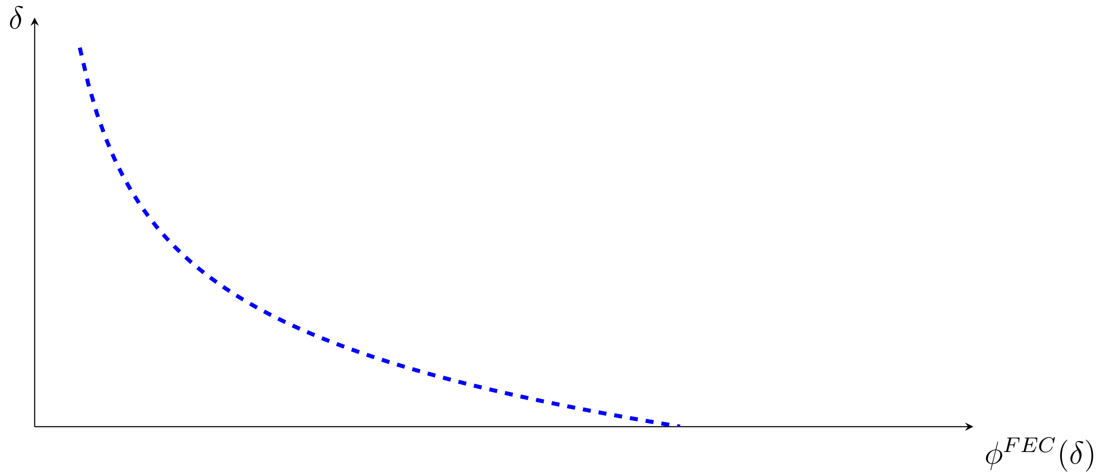

Figure 4: Free entry, refinancing frequency, and the ratio of buyers to sellers

The ratio of buyers to sellers ϕ produced via free entry of buyers as a as a function of refinancing frequency δ chosen by firms, as described by $\phi^{FEC}(\delta)$. Note that to facilitate comparison with Figure 3, we have reversed the order of the axes, that is, we are plotting the inverse of $\phi^{FEC}(\delta)$.

Lemma 4. *Free entry into the secondary market implies the following* free entry condition*:*

$$e_B = \mu_B(\phi)(1-\beta)\frac{r(\delta;\phi)}{\theta}. \tag{FEC}$$

This equation defines a strictly decreasing function $\phi^{FEC}(\delta)$ which describes the ratio of buyers to sellers that results from free entry of buyers for each possible choice of δ by firms. This function is maximized for $\delta = 0$, when it takes a finite value $\hat{\phi}$, and tends to zero as $\delta \to \infty$.

Figure 4 plots $\phi^{FEC}(\delta)$ (with the axes reversed to facilitate comparison with Figure 3). Using equation (6) we can see that the gains from trade in the secondary market are $1 - V_\rho(\delta;\phi) = \frac{r(\delta;\phi)}{\theta}$. At higher refinancing frequencies, the bargaining position of sellers improves, and the gains from trade in the secondary market (as well as the interest rate) decrease. This makes entering the market less attractive for buyers, and reduces the ratio of buyers to sellers ϕ. Such a reduction in ϕ has two effects that lead to the reestablishment of the free entry condition (FEC). First, it increases the matching rate $\mu_B(\phi)$ of buyers. Second, it decreases the matching rate $\mu_S(\phi)$ for sellers, meaning that they are in a worse bargaining position when selling, which increases the interest rate paid on debt and the gains from trade in the market. These two effects offset the impact of the increase in refinancing frequency, with the end result that $\phi^{FEC}(\delta)$ is a decreasing but not very steep function of δ. Conversely, the inverse of this function, call it $\delta^{FEC}(\phi)$, is a decreasing and very steep function of δ.

We note that there is a maximum ratio of buyers to sellers of $\phi = \hat{\phi}$ that can be induced via free entry when firms issue perpetual debt ($\delta = 0$). Also, as firms choose refinancing frequencies that tend to infinity, ϕ tends to zero as the gains from trade in the secondary market vanish and buyers choose not to enter.

4.4 Equilibrium

Summarizing the discussion in the previous subsections, a steady-state equilibrium can be characterized by the pairs (δ^e, ϕ^e) for which refinancing frequencies are optimal, and for which the free entry condition for buyers into the secondary market is satisfied:

$$\delta^e = \delta^*(\phi^e) \text{ and } \phi^e = \phi^{FEC}(\delta^e).$$

Proposition 1. *There exists a unique steady-state equilibrium* (δ^e, ϕ^e) *in the economy.*

The steady-state equilibrium can be described by the intersection of a *refinancing frequency curve*, and a *free entry curve* as illustrated in Figure 5. Since both curves (seen as functions of ϕ) are decreasing, there could exist multiple intersection points: If firms expect a high ratio of buyers to sellers, they could issue debt with low refinancing frequency which generates important gains from trade in the secondary market. This in turn could attract many buyers, and produce the anticipated high ratio of buyers to sellers. Proposition 1, however, states that this kind of self-

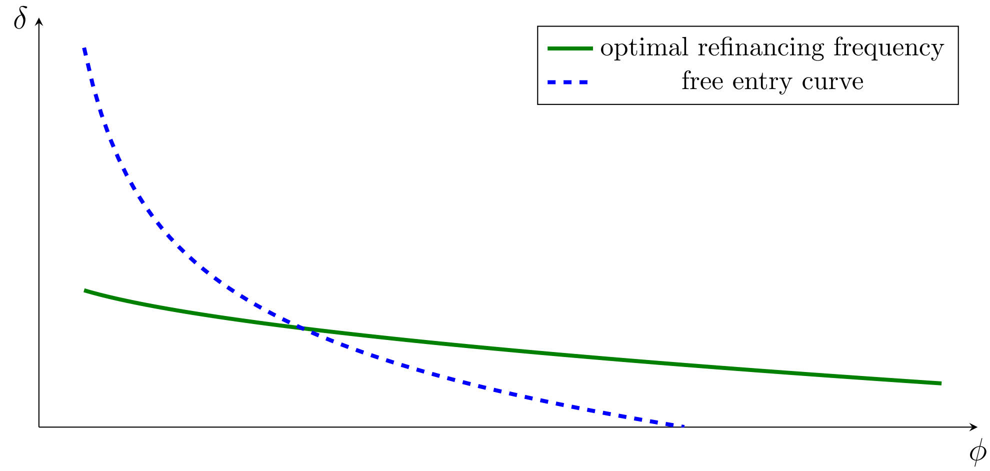

Figure 5: Equilibrium

The optimal refinancing frequency $\delta^*(\phi)$ (green solid line) and the free entry curve $\delta^{FEC}(\phi)$ (blue dashed line). The unique steady-state equilibrium (δ^e, ϕ^e) occurs at the intersection of the two curves. Parameters are as described at the beginning of Section 3.

fulfilling equilibrium does *not* arise in the model. The intuition is that, while the optimal maturity function $\delta^*(\phi)$ depends on the ratio of buyers to sellers ϕ only via the matching intensity of sellers that determines the interest rate set in the primary market, the free entry curve $\delta^{FEC}(\phi)$ depends on ϕ via the matching intensity of sellers as well as that of buyers (as was argued in previous section after Lemma 4). As a consequence, the function $\delta^{FEC}(\phi)$ is more sensitive to changes in ϕ than the function $\delta^*(\phi)$, so that its slope is steeper, and therefore there exists a unique intersection point between the two curves.

Various comparative static results can be derived. When e_B increases, for any refinancing frequency δ chosen by firms, less buyers find it optimal to enter. As a result the free entry curve shifts leftwards and ϕ^e decreases while δ^e increases. When κ increases, for a given ratio of buyers to sellers ϕ in the secondary market, firms find it optimal to reduce their optimal refinancing frequency in order to compensate for the increase in reissuance costs. As a result the refinancing frequency curve shifts downwards and δ^e decreases while ϕ^e increases.

The equilibrium effect of β, θ and ρ is more difficult to determine since changes in these parameters shift the two curves that determine the equilibrium. When β increases, sellers are able to extract more surplus out of a trade in the secondary markets, which decreases the interest rate and leads firms to lower the refinancing frequency, i.e. the refinancing frequency curve shifts downwards. Also, the reduction of the bargaining power of buyers discourages their entry and the free entry curve shifts leftwards. These shifts lead generally to an ambiguous effect but it is possible to show that δ^e is increasing in β if and only if the elasticity of $\mu_B(\phi)$ is lower than β, and decreasing otherwise.[18]

As θ increases, patient investors become impatient at a faster rate. As a consequence, the difference in the utility of being an impatient and a patient debtholder decreases, such that the gains from trade in the market decrease. Hence, the free entry curve shifts leftwards. Because of this, and the fact that patient investors worry more about becoming impatient, firms will have to pay higher interest rates (keeping the refinancing frequency fixed). For low values of θ, the interest rate is close to zero, and the effect of the refinancing frequency on the interest rate is small, so firms choose a low refinancing frequency (to save on the refinancing cost κ). As θ increases, the interest rate becomes more sensitive to the refinancing frequency, so firms choose larger refinancing frequencies in order to reduce interest payments. As θ becomes large, the interest rate is close to ρ, and the effect of the refinancing frequency on the interest again becomes small, so that firms choose a low refinancing frequency. This means that as θ increases, the refinancing curve first shifts up and then shifts down, such that δ^e first increases and then decreases.

Also, it can be shown that for a sufficiently high values of θ, firms will not find it optimal to issue debt, and issuance in the primary market collapses.[19] This is because in this situation, the

[18]The relationship between the elasticity of the matching function and the bargaining power parameter is also closely related to the question of efficiency of entry, as discussed in Section 5.

[19]Note also that for sufficiently high θ, the *sufficient* condition in Assumption 1 for firms to find optimal

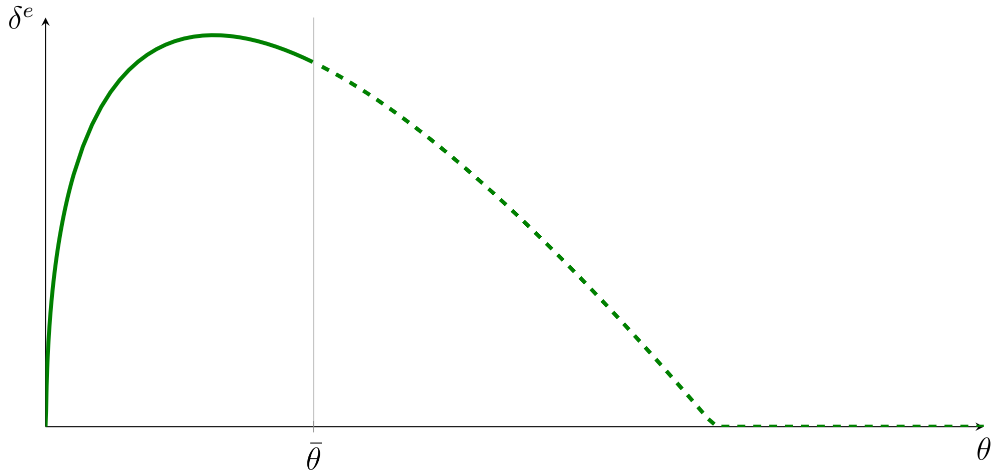

Figure 6: Equilibrium refinancing frequency δ^e as a function of θ
Equilibrium refinancing frequency δ^e as a function of θ, assuming that firms issue debt. Here, $\bar{\theta}$ indicates the critical level of θ beyond which firms would find it optimal not to issue debt.

interest rate firms have to pay (at all refinancing frequencies) is so close to ρ, the discount rate of entrepreneurs, that entrepreneurs prefer either to finance the project out of their own funds or not to invest in it. We illustrate this in Figure 6.

5 Efficiency of equilibrium

It is well known that models of search with ex-post bargaining and free entry exhibit a generic inefficiency related to the level of entry (see e.g. Pissarides, 1990, chapter 7). This type of inefficiency also exists in our model, but it is unrelated to our main result. To see this, consider a situation in which we fix a choice of refinancing frequency. Then it is still the case that investors who enter the secondary market in order to buy impose a negative externality on other buyers, by making it more difficult for them to be matched with a seller. This externality could lead to an inefficiently high level of entry. At the same time, buyers do not appropriate the whole surplus from a match, and thus they do not have enough incentives to incur the cost of searching, which might lead to

to issue debt is not satisfied.

an inefficiently low level of entry. The relative importance of the two opposing forces depends on the bargaining power of buyers in the market: when it is high the first dominates and there is excessive entry, when it is low the second dominates and there is insufficient entry. The amount of entry will therefore only be socially efficient for a particular value of the bargaining power of buyers which exactly balances the two effects (keeping the refinancing frequency fixed). At this level of the bargaining power, the price in the secondary market is such that the marginal rates of substitution of the price versus ϕ are equalized across buyers and sellers.[20]

Since this general inefficiency due to congestion externalities associated with entry decisions is well known in the search literature, we focus in this section on the efficiency properties of our model from a second best perspective. In particular we assume that a Social Planner (SP) can choose the refinancing frequency of debt, but cannot influence the entry decisions of investors. The SP chooses the refinancing frequency in order to maximize surplus in the economy. Our objective is to understand whether there exist differences between the laissez-faire equilibrium and the welfare maximizing allocation chosen by the SP, and if so, to understand the roots of the discrepancy.

In this scenario, there is still free entry into both the primary and secondary debt markets and, as in the previous section, investors just break even and obtain a utility equal to the utility associated with instantaneously consuming their endowment. The only agents who obtain a surplus are entrepreneurs, and therefore the SP will choose δ in order to maximize their utility. The maximization problem of the SP differs, nevertheless, from the firm's problem in how it takes into account the ratio of buyers to sellers in the secondary market: while firms take ϕ as *given*, the SP *internalizes* the effects of maturity choices on the ratio of buyers to sellers.

More formally, the SP internalizes that a refinancing frequency δ induces a ratio of buyers to

[20]Given a fixed refinancing frequency in our model, the first order condition for maximization of welfare with respect to β holds when $\beta = -\phi\mu'_B(\phi)/\mu_B(\phi)$, i.e. when β is equal to the elasticity of $\mu_B(\phi)$ with respect to ϕ. This is a very standard condition in the labor-search literature, sometimes referred to as the "Hosios Condition," see Pissarides (1990, chapter 7), or Hosios (1990).

sellers $\phi^{FEC}(\delta)$ in the secondary market. We can write the SP's optimization problem in terms of the expression for the utility of entrepreneurs in equation (11) as follows:

$$\max_{\delta \geq 0} U^{SP}(\delta) = U(\delta, r(\delta; \phi^{FEC}(\delta))).$$

Now if the competitive equilibrium (δ^e, ϕ^e) has $\delta^e > 0$, then the first order condition for firms implies that at the equilibrium values (δ^e, ϕ^e),

$$\frac{\partial U}{\partial \delta} + \frac{\partial U}{\partial r}\frac{\partial r}{\partial \delta} = 0,$$

whereas the first order condition for the social planner is

$$\frac{dU^{SP}}{d\delta} = \frac{\partial U}{\partial \delta} + \frac{\partial U}{\partial r}\left(\frac{\partial r}{\partial \delta} + \frac{\partial r}{\partial \phi}\frac{d\phi^{FEC}}{d\delta}\right) = 0. \tag{14}$$

We note that $\frac{\partial U}{\partial r} < 0$ or that the utility of entrepreneurs is decreasing in interest rates, that $\frac{\partial r}{\partial \phi} < 0$ or that interest rates are decreasing in the ratio of buyers to sellers, and $\frac{d\phi^{FEC}}{d\delta} < 0$ or that the ratio of buyers to sellers induced by free entry is decreasing in refinancing frequency. Hence the first order condition for firms implies that at the equilibrium values (δ^e, ϕ^e),

$$\frac{dU^{SP}(\delta^e)}{d\delta} < 0, \tag{15}$$

and hence that the equilibrium refinancing frequency chosen by firms, δ^e, is too large from the perspective of the SP, as illustrated in Figure 7.

The SP can therefore increase aggregate welfare by reducing the refinancing frequency. The reason is that any given firm does not internalize that by choosing a smaller refinancing frequency, it will increase the gains from trade in the secondary market, which increases the ratio of buyers to sellers and so makes it easier for sellers, including those who want to sell the debt of other firms, to find a buyer. This reduces the interest rates that *all* firms pay. Since investors always break even,

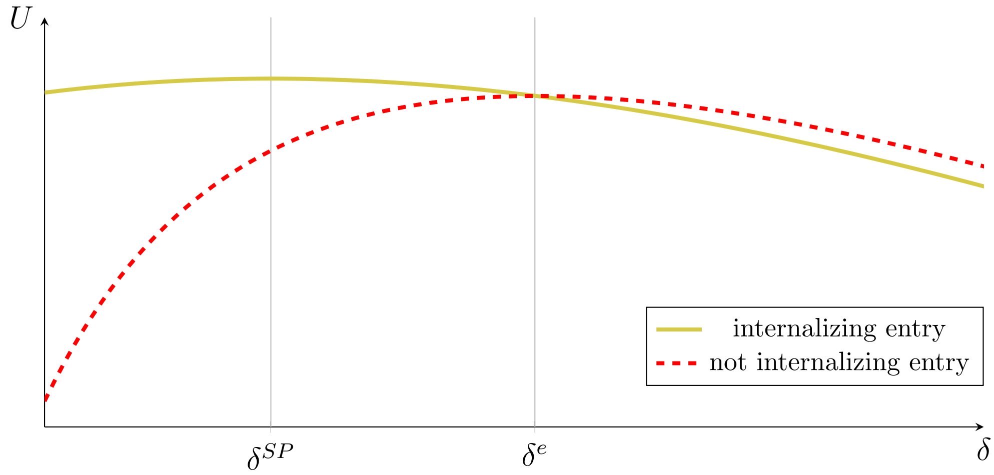

Figure 7: Firm profit U as function of refinancing frequency δ
Firm profit U as a function of refinancing frequency δ, (a) as perceived by the social planner, internalizing the effect of δ on entry and hence $\phi = \phi^{FEC}(\delta)$ (solid yellow line), and (b) as perceived by an individual firm in equilibrium, not internalizing the effect of δ on entry and hence $\phi = \phi^{e}$ (red dashed line).

the increase in the utility of entrepreneurs brought about by a decrease in refinancing frequency is a Pareto improvement.

The local argument above can be extended to a global result which is the main result of the paper:

Proposition 2. *Let (δ^e, ϕ^e) be an equilibrium with $\delta^e > 0$. Then the solution δ^{SP} to the Social Planner's problem satisfies $\delta^{SP} < \delta^e$ and induces $\phi^{SP} > \phi^e$, and it Pareto improves the competitive equilibrium.*

The key implicit assumption that generates the externality is that debt claims with different maturities are all traded in a *single* secondary search market, in the sense that if there are claims with different maturities being sold in the market, buyers cannot search to be matched only with specific maturities. This means that a single firm that deviates from the equilibrium refinancing frequency and chooses $\delta \neq \delta^e$ knows that this deviation will not affect the distribution of maturities available in the market, hence knows that this will not affect entry, and hence will correctly anticipate that this will not affect the ratio of buyers to sellers ϕ^e.

Conversely, consider a situation in which debt claims with different maturities are all traded in different sub-markets, and that buyers can decide in which sub-market they search, and hence can search to be matched only with a specific maturity. In this case, we would have a free entry condition for each sub-market i, and a corresponding ratio of buyers to sellers ϕ_i^e. Suppose that firms who deviate and offer a maturity not yet traded in the market know that this creates a new sub-market. This means that even a single firm which deviates from the equilibrium refinancing frequency knows that the ratio of buyers to sellers will be determined by its maturity choice. In this situation firms would internalize the effect of maturity on entry, and on the ratio of buyers to sellers in sub-markets, ϕ_i^e. As a consequence, the maximization problem of the SP would coincide with the one of firms and the laissez-faire equilibrium would exhibit efficient maturity choice.

We note that in a *competitive search* model (or *directed search* model) (Moen, 1997) neither the general inefficiency described above, nor our inefficiency would exist. In such a model, there exist sub-markets, and prices are competitive in the sense that they equalize marginal rates of substitution across buyers and sellers in each sub-market. They key feature in that type of model that would eliminate our externality is the existence of sub-markets, not the competitive pricing.

This raises the question as to what extent the assumption of a single secondary market for debt claims (i.e. no sub-markets) is empirically plausible. First, it is clear that the externality cannot operate across debt markets that are clearly distinct. For instance, in practice, maturity decisions on corporate bonds are unlikely to affect the ratio of buyers to sellers in the market for commercial paper or syndicated loans, and hence the maturity decisions on commercial paper and syndicated loans. However, we believe that the externality can operate within one of these markets. From interviews with market practitioners we learnt that there is often not much specialization of traders in terms of maturities. For instance, on corporate bond trading desks (or commercial paper trading desks), there will frequently be one trader assigned to a set of issuers, trading in bonds of all maturities issued by these issuers. Investors who contact traders therefore cannot know ex-ante what specific maturities the traders will be interested in trading. (This contrasts with sovereign bonds, where there are typically several traders assigned to a single large sovereign, where each trader specializes in trading bonds in a certain maturity range. Investors who contact a trader will know ex-ante what range of maturities a trader will trade.) We therefore believe that the metaphor of a single search market is plausible for describing e.g. the corporate bond market in isolation, or the commercial paper market in isolation.

6 Marketmakers

In this section, we describe informally how the model can be extended to incorporate a new class of agents, *marketmakers*, who intermediate between buyers and sellers in the secondary market. This extension allows us to derive the empirical prediction that bid-ask spreads of marketmakers should be decreasing in refinancing frequencies (i.e. increasing in expected maturities). We then illustrate the extended model in the context of a numerical example. We defer a formal discussion of the extended model to Appendix B.

As in the model of Duffie, Garleanu, and Pedersen (2005), marketmakers are risk-neutral and utility-maximizing agents, who have no funds. Marketmakers can intermediate as follows: They search in the secondary market to be matched with buyers and sellers, via a special matching technology (described below), and also have access to an inter-dealer market in which they can instantly unload the positions which they enter into with investors in the secondary market (so that they hold no inventory at any time). The bid (B) and ask (A) prices at which marketmakers are willing to buy from and sell to investors, respectively, are determined through Nash bargaining where the bargaining power parameter of marketmakers is γ.[21]

Let α^M denote the measure of marketmakers, and then let $\overline{\mu}(\alpha^i, \alpha^M)$ denote the matching function which describes matches between market makers and searching buyers or searching sellers (where $\alpha^i \in \{\alpha^S, \alpha^B\}$ can describe the measure of sellers or buyers respectively). We deviate from Duffie, Garleanu, and Pedersen (2005) by assuming that this matching function is of the same class as our matching function $\mu(\cdot, \cdot)$. In particular, it also exhibits constant returns to scale, and satisfies the congestion properties (2). This again allows us to define ratios, such as e.g. the ratio of marketmakers to sellers, $\chi_S = \frac{\alpha^M}{\alpha^S}$, and then define e.g. the rate at which sellers are matched with a market maker via a suitably defined function of that ratio, $\overline{\mu}_S(\chi_S)$. Finally, we assume that there is a large measure of marketmakers, who can freely enter the secondary market, and who incur a (non-pecuniary) flow cost of searching in this market.

As in the baseline model, the interest rate that a firm faces is determined by the bidding of investors in primary market auctions. The closed-form expression that can be derived is similar, but does not only depend on the ratio of buyers to sellers ϕ and the refinancing frequency δ, but also on the ratio χ_S of marketmakers to sellers. This is because impatient debt holders who search to sell can now be matched with a marketmaker, and sell at bid price B. Using the expression for the interest rate, it can be shown that e.g. when $\phi > 1$, the bid-ask spread takes the following form:[22]

$$A - B = \gamma(V_0 - V_\rho) = \frac{\gamma\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta + \overline{\mu}_S(\chi_S)(1-\gamma)} \tag{16}$$

As can be seen from this expression, the bid-ask spread is decreasing in the refinancing frequency δ, and hence increasing in maturity. (Also, the bid-ask spread is a constant fraction of the gains

[21]Marketmakers do not need funds to intermediate because they can instantly off-set any position taken with a final investor at the inter-dealer market at a price Q that in equilibrium satisfies $B \leq Q \leq A$.

[22]When $\phi < 1$, the expression is identical except that the term in $\overline{\mu}_S(\chi_S)$ is not present (Lemma B.6, Appendix B). In this case, marketmakers meet more sellers than buyers, and marketmakers respond by setting a low bid price $B = V_\rho$ such that sellers do not gain from meeting a marketmaker, and hence the rate at which they meet is irrelevant for the determination of the interest rate.

from trade, increasing in the bargaining power of marketmakers, and decreasing in the ratio of marketmakers to sellers.)

The equilibrium can be found in the same way as for the baseline model: For given ratios ϕ, χ_S, firms make their optimal choice of refinancing frequency based on the interest rate they have to offer in the primary market. And for every refinancing frequency chosen by firms, free entry of buyers and marketmakers into the secondary market determines the ratios ϕ, χ_S.[23] It can be shown that an equilibrium exists, is unique, and exhibits an inefficiently short maturity, as in the baseline model.

We now illustrate the analytical results via a numerical example, in the extended version of the model with marketmakers. We pick parameters that lead to short maturities, as observed in practice for commercial paper.

We use the following parameters: We measure time in years, and choose a cash flow of the projects of $x = 1\%$, and assume that investors become impatient at rate of $\theta = 1$ (i.e. the expected time until becoming impatient is 1 year). We fix the discount rate of impatient investors at $\rho = 10\%$ p.a., and the cost of refinancing at $\kappa = 2$ basis points.[24] For investors, we pick a Cobb-Douglas matching function, as follows:

$$\mu(\alpha^S, \alpha^B) = 10 \left(\alpha^S\right)^{\frac{1}{2}} \left(\alpha^B\right)^{\frac{1}{2}} .$$

We assume equal bargaining power parameters for sellers and buyers, $\beta = 1 - \beta = \frac{1}{2}$, and a flow cost of searching to buy of $e_B = 5\%$. For marketmakers, we also pick a Cobb-Douglas matching

[23] Because buyers can also be matched and trade with marketmakers, the ratio of marketmakers to buyers, $\chi_B := \frac{\alpha^M}{\alpha^B}$, also affects the entry decision of both buyers and marketmakers. Since $\chi_B = \frac{\chi_S}{\phi}$, it can be seen that the variables ϕ and χ_S are sufficient to characterize matching intensities for all possible pairings of agents in the secondary market.

[24] With these parameters, it can be checked that it is optimal for the entrepreneur to undertake debt-financed projects, even though the sufficient (but not necessary) condition given in Assumption 1 does not hold.

function, with different parameters, as follows:

$$\overline{\mu}(\alpha^i, \alpha^M) = 100 \left(\alpha^i\right)^{\frac{1}{2}} \left(\alpha^M\right)^{\frac{1}{2}}, \tag{17}$$

where $i = \{B, S\}$. We note that this makes the matching technology of marketmakers 10 times more efficient, in the sense that for the same measures, marketmakers would obtain 10 times the match rate. We also assume that their flow cost of searching is much higher than the one of buying investor, $e_M = 50\%$, and that they have high bargaining power, $\gamma = 0.95$. The high cost of searching could be interpreted as justifying both the access to the more efficient matching technology and the high bargaining power after a match as well as the access to the inter-dealer market.

With these parameters, we have an equilibrium refinancing frequency of $\delta^e \approx 11.42$. This implies an expected maturity of debt claims of about $1/\delta^e \approx 32$ days. The ratio of buyers to sellers is $\phi^e \approx 1.25$, implying more sellers than buyers. The ratio of marketmakers to sellers is $\chi_S^e \approx 0.72$. These ratios imply that the expected time for a seller to contact and trade with any counterparty (buyer or marketmaker) is 3.8 days, and that a fraction of about 88% of all sales are to marketmakers and only 12% are direct to buyers.[25] The interest rate / illiquidity premium r that firms have to pay at this maturity of 32 days is equal to about 45bp. Entrepreneur utility is equal to $U \approx$0.032.

To understand prices, first note that in this example, the value that an impatient investor attaches to the security is $V_\rho \approx$0.9955. The value that a patient investor attaches to the security is V_0=1, so that the gains from trade are about 45bp.[26] The price that investors who meet directly agree to when they trade is equal to $P \approx= 0.9978$, indicating that because of the equal bargaining power, they equally divide the gains from trade (slightly more than 22bp per party).

Because there are more buyers than sellers, marketmakers will find more buyers than sellers.

[25]Regarding buyers, the expected time until they contact and trade with any counterparty (seller or marketmaker) is 4.3 days.

[26]As noted in Section 4, the gains from trade can be computed as r/θ. Since, $\theta = 1$, here the gains from trade are equal to r.

As a response to the relative abundance of sellers, marketmakers will set the ask price A to the reservation value $V_0 = 1$ of buyers. The relative abundance of buyers also implies that the inter-dealer market price Q will equal the ask price $A = V_0 = 1$. Due to their high bargaining power, marketmakers will set a low bid price of $B \approx 0.9958$, leaving a seller who meets a marketmaker with a very small gain from the resulting trade of only 3bp, and keeping the bid-ask spread of 42bp as an intermediation profit.

In this situation, a social planner would choose a refinancing frequency of $\delta^{SP} \approx 2.49$, implying an expected maturity of around 147 days. This would induce entry of buyers and of marketmakers, leading to a ratio of buyers to sellers of $\phi = 2.23$, and a ratio of marketmakers to sellers of χ_S =1.29. The expected time for a seller to contact any counterparty is 2.8 days. In comparison to the laissez-faire equilibrium, the time to trade is decreased.

The longer maturity increases the gains from trade from about 45bp to about 60bp. The bid and ask prices are $B \approx$0.9943 and A =1, implying a larger bid-ask spread of 57bp, which sustains the increased numbers of marketmakers. (The price at which investors trade is $P \approx$0.9970)

The interest rate / illiquidity premium r that firms have to pay at this new maturity of 147 days is equal to 60bp. At the same time, an individual firm that considers deviating from the laissez-faire equilibrium would perceive the interest rate required for issuing debt at a maturity of 147 days to be 74bp. The difference (74bp versus 60bp) arises because a coordinated increase in maturity choice increases gains from trade in the secondary market and encourages the entry of buyers and marketmakers into this market.

Finally, even though the interest rate at the socially regulated maturity is higher than in the laissez-faire economy, entrepreneurs benefit because the longer maturity allows them to save on the refinancing cost, leading to a higher entrepreneur utility of $U \approx$0.035. This represents an increase in entrepreneur utility of around 9% over the laissez-faire equilibrium.

7 Conclusion

Debt holders who need to sell in an OTC secondary market are in a worse bargaining position the longer they are locked-in into their contracts, i.e. the longer the time-to-maturity of their debt is. This worse bargaining position implies a larger discount when selling. Firms anticipate that they need to offer higher yields on debt with longer maturities, and especially so if the secondary market is very illiquid, in the sense that the ratio of buyers to sellers is low. But the entry of buyers into the secondary market and hence its liquidity is a function of the profits that buyers can obtain in this market, which decreases in the bargaining position of the sellers. We present a model in which the liquidity of secondary markets for corporate debt, and maturities, are jointly determined in equilibrium, on the basis of this mechanism.

Our main result is that in equilibrium, maturities chosen by firms are inefficiently short. This is because firms do not internalize the effect of their maturity decisions on the gains from trade in the secondary market and hence on the incentives for buyers to enter this market. When an individual firm increases the maturity of its debt, this worsens the bargaining positions of the holders of this debt who need to sell. But this also increases the gains from trade in the secondary market. The latter attracts more buyers into the secondary market in search of more profitable deals, which increases liquidity and reduces the interest rates demanded by investors on the debt of all firms, at all possible maturities.

From a practical perspective, this might e.g. explain why prior to the crisis, financial institutions relied on extremely short term asset-backed commercial paper in order to fund long term assets, while at the same time the secondary market for commercial paper was so illiquid as to be almost non-existent. Our model highlights that if issuers were forced to sell longer maturity paper, this would attract more buyers to the secondary market for commercial paper, making it easier to sell, and hence decreasing the need to issue such short maturity paper.

There are some avenues for future research that could be pursued using the type of model that we describe. For example, one could examine the consequences of a financial transaction tax, as currently being considered by the European Commission.[27] It can be shown that when such a tax is introduced into our model (e.g. modeled as a cash amount to be paid by the buyer every time a transaction occurs), the equilibrium refinancing frequency increases (the equilibrium maturity decreases), indicating that a financial transactions tax might interfere with regulatory objectives such as the reduction in the maturity mismatch produced by financial intermediaries. More generally, it would be interesting to evaluate how this type of tax affects the externality that we have identified (as well as the standard congestion externality present in search models with ex-post bargaining and free entry), and hence its impact on welfare.

[27] See "Proposal for a council directive implementing enhanced cooperation in the area of financial transaction tax," http://ec.europa.eu/taxation_customs/resources/documents/taxation/com_2013_71_en.pdf.

Appendix

A Proofs

Proof of Lemma 1: Substitute (5) into (7), and solve the resulting equation together with (6) for V_0. Apply condition (8) and solve for r to obtain the result. ■

Proof of Lemma 2: Assumption 1 guarantees that undertaking a debt-financed project with $\delta = 0$ produces a positive utility that exceeds the utility of undertaking a project financed with own funds, even when $\phi = 0$. Since r is decreasing in ϕ, it must produce positive utility when $\phi > 0$. Assumption 1 also guarantees that $\frac{x}{\rho} > \frac{x}{\rho} - \kappa > \frac{r(0;0)}{\rho} > \frac{r(0;\phi)}{\rho}$ or $x > r(0;\phi)$, which implies that debt with $\delta = 0$ is feasible (in the sense that the cash flow x is sufficient to cover the cost of debt service).

For an optimal choice of δ, call it δ^*, we furthermore have that $U(\delta^*, r(\delta^*;\phi)) \geq U(0, r(0;\phi))$, which implies that $r(\delta^*;\phi) + \delta^*\kappa < r(0;\phi)$. Since debt with $\delta = 0$ is feasible, it therefore must be the case that debt with $\delta = \delta^*$ is feasible.

We now characterize the optimal refinancing frequency decisions for given ϕ. Substituting the expression for $r(\delta;\phi)$ in equation (9) into the program (12), we obtain

$$\min_{\delta \geq 0} \frac{\rho\theta}{\delta + \theta + \rho + \mu_S(\phi)\beta} + \delta\kappa$$

The expression describes a function with negative curvature on $(0, +\infty)$ which tends to a positive constant as $\delta \to 0^+$ and to $+\infty$ as $\delta \to +\infty$ and hence has at a unique minimum on $(0, +\infty)$, as characterized by the first order condition

$$-\frac{\rho\theta}{(\delta + \theta + \rho + \mu_S(\phi)\beta)^2} + \kappa = 0$$

which has only one solution that can be non-negative:

$$\delta^\dagger(\phi) = \sqrt{\frac{\rho\theta}{\kappa}} - \theta - \rho - \mu_S(\phi)\beta.$$

Since the choice of δ is constrained to be positive, we can see that U is maximized for

$$\delta^*(\phi) = \max\{\delta^\dagger(\phi), 0\},$$

which concludes the proof. ■

Proof of Lemma 3: Consider equation (13). If $V_B(\delta;\phi) > 1$, patient investors would strictly prefer searching to buy over consuming their endowment. Due to the assumption of a large inflow of such investors, this would imply $\alpha^B \to \infty$ and hence $\phi \to \infty$. But since $\lim_{\phi\to\infty} \mu_B(\phi) = 0$, and $\lim_{\phi\to\infty} V_0 - V_\rho = 0$, equation (13) would then imply that $0 = -e_B + \theta(1 - V_B) < 0$, which is a contradiction. (Note that investors who contemplate searching to buy in the secondary market take the interest rate of claims in the market as fixed, and hence the limit of $V_0 - V_\rho$ should be taken for V_0, V_ρ expressed in terms of a fixed r.) On the other hand, if $V_B(\delta;\phi) < 1$ patient investors would strictly prefer consuming their endowment over searching to buy. Therefore it should be the case that $\alpha^B = 0$ and hence that $\phi = 0$. But since $\lim_{\phi\to 0} \mu_B(\phi) = \infty$, while $\lim_{\phi\to 0} V_0(\delta;\phi) - V_\rho(\delta;\phi) = C$ for a positive constant C, equation (13) would imply that $\infty = 0$ which is obviously a contradiction.

Proof of Lemma 4: In order to prove that (FEC) defines a function $\phi^{FEC}(\delta)$, we substitute the expression for $r(\delta;\phi)$ in (9) into the free entry condition in (FEC) and obtain

$$\mu_S(\phi)\beta + \rho + \theta + \delta = \mu_B(\phi)\frac{1-\beta}{e_B}\rho. \tag{18}$$

We note that as $\phi \downarrow 0$ the left hand side tends to a positive constant which is a function of δ, whereas the right hand side tends to ∞. As $\phi \uparrow \infty$, the left hand side tends to infinity, whereas the right hand side tends to 0. Furthermore, from the properties of the matching function, we know that $\mu_S(\phi)$ is continuous and strictly increasing in ϕ (and hence so is the left hand side), and that $\mu_B(\phi)$ is continuous and strictly decreasing in ϕ (and hence so is the right hand side). It therefore follows that for each $\delta \in [0, \infty)$, there exists a unique ϕ that satisfies (18). We denote the function that describes this mapping as $\phi^{FEC}(\delta)$. Its domain is $[0, \infty)$. Using the implicit function theorem, it can be seen that the function is strictly decreasing, implying that it is maximized at $\hat{\phi} := \phi^{FEC}(0)$. We note that since $\lim_{\delta \to \infty} \phi^{FEC}(\delta) = 0$, the function $\phi^{FEC}(\delta)$ has as its image the interval $(0, \hat{\phi}]$. Since $\phi^{FEC}(\delta)$ is strictly decreasing its inverse function $\delta^{FEC}(\phi)$ is well defined, its domain is the interval $(0, \hat{\phi}]$, its image is the interval $[0, \infty)$ and it is strictly decreasing. ■

Proof of Proposition 1: We first consider existence, and distinguish between two cases.

First, let us suppose that $\delta^*(\hat{\phi}) = 0$. By definition $\delta^{FEC}(\hat{\phi}) = 0$. Then trivially $\delta^e = 0, \phi^e = \hat{\phi}$ is an equilibrium. Second, suppose the converse, that $\delta^*(\hat{\phi}) > 0$. By definition, $\delta^{FEC}(\hat{\phi}) = 0$, and hence $\delta^{FEC}(\hat{\phi}) < \delta^*(\hat{\phi})$. At the same time, $\lim_{\phi \to 0} \delta^{FEC}(\phi) = \infty$, while $\delta^*(0)$ is finite, implying that $\delta^{FEC}(\phi) > \delta^*(\phi)$ for ϕ sufficiently close to zero. By continuity of the two functions $\delta^{FEC}(\phi), \delta^*(\phi)$, there must then exist a pair (δ^e, ϕ^e) such that $\delta^{FEC}(\phi^e) = \delta^*(\phi^e) = \delta^e$. This pair is an equilibrium.

We now prove uniqueness. In order to do so it suffices to prove that

$$\frac{d\delta^{FEC}(\phi)}{d\phi} < \frac{d\delta^*(\phi)}{d\phi} \text{ for all } \phi \in (0, \hat{\phi}]. \tag{19}$$

From the expression for $\delta^*(\phi)$ in Lemma 2, it can be seen that

$$\frac{d\delta^*(\phi)}{d\phi} \geq -\beta \frac{d\mu_S(\phi)}{d\phi}, \quad \forall \phi. \tag{20}$$

From (18), we obtain

$$\frac{d\delta^{FEC}(\phi)}{d\phi} = -\beta \frac{d\mu_S(\phi)}{d\phi} + \frac{d\mu_B(\phi)}{d\phi} \frac{1-\beta}{e_B} \rho, \quad \forall \phi \in (0, \hat{\phi}]. \tag{21}$$

Since $d\mu_S(\phi)/d\phi > 0$ and $\mu_B(\phi)/d\phi < 0$, a direct comparison between equations (20) and (21) leads to the inequality (19).

■

Proof of Proposition 2: For all $\delta > \delta^e$ we have $\phi^{FEC}(\delta) < \phi^{FEC}(\delta^e)$. It follows that for $\delta > \delta^e$,

$$U^{SP}(\delta) = U(\delta, r(\delta; \phi^{FEC}(\delta))) < U(\delta, r(\delta; \phi^{FEC}(\delta^e))) \leq U(\delta^e, r(\delta^e; \phi^{FEC}(\delta^e))) = U^{SP}(\delta^e)$$

where in the first inequality we have used that $U(\delta, r)$ is decreasing in r, which in turn is decreasing in ϕ, and in the second that by the definition of equilibrium, δ^e maximizes firms' objective function for liquidity $\phi^{FEC}(\delta^e)$. Using the inequality $\frac{dU^{SP}(\delta^e)}{d\delta} < 0$ which has been proven in the main text we conclude that:

$$\arg\max_{\delta \geq 0} U^{SP}(\delta) < \delta^e.$$

$\blacksquare$

B Marketmakers

In this appendix, we formalize the discussion in Section 6 that considers the introduction of marketmakers.

There exist risk-neutral and utility-maximizing marketmakers, who are not endowed with funds, and have discount rate ρ. While marketmakers search in the secondary market, they incur a flow utility cost of $e_M > 0$.

α^M denotes the measure of marketmakers searching in the secondary market. We define $\chi_S = \frac{\alpha^M}{\alpha^S}, \chi_B = \frac{\alpha^M}{\alpha^B}$ as the ratios of marketmakers to sellers and buyers, respectively, and use $\overline{\mu}_S(\chi_S)$ ($\overline{\mu}_M(\chi_S)$) to denote the rate at which a seller (marketmaker) is matched with a marketmaker (seller) and, analogously, use $\overline{\mu}_B(\chi_B)$ ($\overline{\mu}_M(\chi_B)$) to denote the rate at which a buyer (marketmaker) is matched with a marketmaker (buyer). We assume that $\overline{\mu}(\cdot,\cdot)$ satisfies the same properties as $\mu(\cdot,\cdot)$, such that the matching rates $\overline{\mu}_S(\cdot), \overline{\mu}_B(\cdot)$ satisfy the standard congestion properties (cf. equation (2)). Note that once we know the ratio of buyers to sellers and the ratio of marketmakers to buyers, we can compute the ratio of marketmakers to sellers ($\chi_S = \phi\chi_B$), such that the variables ϕ and χ_B will be sufficient to determine all the marginal matching intensities. The arrival of matches is independent and thus in a short time interval dt a marketmaker cannot be matched with both a seller and a buyer.

After a match involving a marketmaker, there is Nash bargaining, where the bargaining power parameter of marketmakers is γ, for both matches with buyers and matches with sellers. Taking into account the bargaining powers of the different agents and the equilibrium surpluses to be shared after a match, we can write the following equations that the prices must satisfy, as a function of the values that investors attach to holding the assets:

$$
\begin{aligned}
P - V_\rho &= \beta(V_0 - V_\rho), && (22)\\
Q - B &= \gamma(Q - V_\rho), && (23)\\
A - Q &= \gamma(V_0 - Q). && (24)
\end{aligned}
$$

These three equations together with the condition that the inter-dealer market must clear will determine the four prices A, B, Q, P.

In order for the inter-dealer market to clear, the equilibrium inter-dealer price Q must lie in the interval $[V_\rho, V_0]$.[28] To describe where in the relevant interval Q is located, it will be useful to define a variable λ that we call the *inter-dealer price index*, as follows:

$$\lambda := \lambda \in [0,1] \text{ such that } Q = \lambda V_0 + (1-\lambda)V_\rho.$$

To consider inter-dealer market clearing and the determination of λ (or equivalently Q) in more detail, it will be useful to sequentially consider the cases where $\phi < 1, \phi > 1$ or $\phi = 1$.

If $\phi < 1$, there are more sellers than buyers and thus there are more matches between marketmakers and sellers than between marketmakers and buyers. For the inter-dealer market to clear, it must be the case that some of the matches between marketmakers and sellers will not lead to trade, which can happen only if there are no gains from trade associated with these matches, i.e. $\lambda = 0$ and $Q = V_\rho$.

If $\phi > 1$, there are more buyers than sellers and thus there are more matches between marketmakers and buyers than between marketmakers and sellers. For the inter-dealer market to clear, it must be the case that some of the matches between marketmakers and buyers will not lead to

[28] If Q lies outside this range, one type of match produces a negative surplus and hence no trade, while the other match produces a positive surplus, and hence trade. Hence marketmakers are only buying from or only selling to investors, which means that the inter-dealer market is not clearing.

trade, which can happen only if there are no gains from trade associated to these matches, i.e. $\lambda = 1$ and $Q = V_0$.

If $\phi = 1$, there are as many buyers as sellers and thus there are as many matches between marketmakers and buyers as between marketmakers and sellers. In this case inter-dealer market clearing on its own is insufficient to pin down λ (or equivalently Q), but the fact that ϕ needs to be equal to 1 provides the necessary additional condition.

In all three cases, the system of recursive flow-value equations that V_0, V_ρ satisfy in steady state is as follows:

$$r + \delta(1 - V_0) + \theta(V_\rho - V_0) = 0 \tag{25}$$

$$r + \delta(1 - V_\rho) + \mu_S(\phi)(P - V_\rho) + \overline{\mu}_S(\chi_S)(B - V_\rho) = \rho V_\rho \tag{26}$$

The first equation is the same as equation (6). The second equation corresponds to equation (7), with the new term $\overline{\mu}_S(\chi_S)(B - V_\rho)$ that accounts for the possibility of locating a marketmaker and selling to him at the bid price B.

Using the conditions on the equilibrium prices P and B, the fact that free entry into the primary market auctions implies $V_0(r, \delta; \phi, \chi_B, \lambda) = 1$ (cf. equation (8)), and that $\chi_S = \phi\chi_B$, we can work out the interest rate as in Section 4.1, and obtain the following result (which is the analogue to Lemma 1):

Lemma B.1. *In the model with marketmakers, for a given ratio of buyers to sellers ϕ, a given ratio of marketmakers to buyers χ_B, inter-dealer price index λ, and refinancing frequency choice δ of a firm, the interest rate $r(\delta; \phi, \chi_B, \lambda^e)$ that is set in the primary market auctions is given by:*

$$r(\delta; \phi, \chi_B, \lambda) = \frac{\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta + \overline{\mu}_S(\phi\chi_B)(1 - \gamma)\lambda}\theta. \tag{27}$$

Proof. Solve (25) and (26) for V_0, then find the r that satisfies $V_0(r, \delta; \phi, \chi_B, \lambda) = 1$ to obtain the stated result. ■

We can then follow the same procedure as in the baseline model to work out the optimal refinancing frequency choice, and obtain the following result (which is the analogue to Lemma 2):

Lemma B.2. *In the model with marketmakers, under assumptions 1, it is feasible and optimal to issue debt and undertake the project. In addition, for every ϕ, χ_B, λ, the firm's problem* (12) *has a unique solution $\delta^*(\phi, \chi_B, \lambda)$ which is given by:*

$$\delta^*(\phi, \chi_B, \lambda) = \max \left\{ \sqrt{\frac{\theta\rho}{\kappa}} - \theta - \rho - \mu_S(\phi)\beta - \overline{\mu}_S(\phi\chi_B)(1-\gamma)\lambda, 0 \right\}. \tag{28}$$

Proof. In the model with marketmakers, the interest rate is equal to or lower than the interest rate in the model without market makers. Hence it follows from the fact that under Assumption 1, undertaking debt-financed projects in the model without marketmakers is optimal and feasible (Lemma 2) that it is also optimal and feasible to do so here. Finally, with some simple algebra paralleling that in the proof of Lemma 2 (omitted here), the stated expression for $\delta^*(\phi, \chi_B, \lambda)$ can be obtained. ■

Similarly, a free entry condition for buyers corresponding to that in Lemma 4 can be derived:

Lemma B.4. *Free entry into the secondary market implies the following* free entry condition *for buyers:*

$$e_B = [\mu_B(\phi)(1-\beta) + \overline{\mu}_B(\chi_B)(1-\gamma)(1-\lambda)] \frac{r(\delta; \phi, \chi_B, \lambda)}{\theta}. \tag{29}$$

Proof. Following the same procedure as for Lemma 4, we note that $V_B = 1$ and hence that

$$\begin{aligned} e_B &= \mu_B(\phi)(V_0 - P) + \overline{\mu}_B(\chi_B)(V_0 - A) \\ &= \mu_B(\phi)(1-\beta)(V_0 - V_\rho) + \overline{\mu}_B(\chi_B)(1-\gamma)(1-\lambda)(V_0 - V_\rho) \\ &= [\mu_B(\phi)(1-\beta) + \overline{\mu}_B(\chi_B)(1-\gamma)(1-\lambda)] \frac{r(\delta; \phi, \chi_B, \lambda)}{\theta}. \end{aligned} \tag{30}$$

In the first equation we now take into account that buyers can now also be matched with marketmakers, in the second, we use the conditions on P and A, and in the third equation we use the fact that $V_0 = 1$ as well as the definition of r. ■

Finally, a free entry condition for marketmakers can be derived, as in the following lemma:

Lemma B.5. *Free entry into marketmaking implies the following* free entry condition *for marketmakers:*

$$e_M = (\lambda\overline{\mu}_M(\phi\chi_B) + (1-\lambda)\overline{\mu}_M(\chi_B))\,\gamma\frac{r(\delta;\phi,\chi_B,\lambda)}{\theta}. \tag{31}$$

Proof. The utility flow to a marketmaker is

$$\overline{\mu}_M(\chi_S)(Q-B) + \overline{\mu}_M(\chi_B)(A-Q) - e_M,$$

reflecting that marketmakers can be matched either with sellers and buyers, and incur the (non-pecuniary) flow cost e_M.

Free entry will occur until in equilibrium this utility flow is zero, implying

$$\begin{aligned} e_M &= \overline{\mu}_M(\chi_S)(Q-B) + \overline{\mu}_M(\chi_B)(A-Q) = \\ e_M &= \overline{\mu}_M(\phi\chi_B)\lambda\gamma(V_0 - V_\rho) + \overline{\mu}_M(\chi_B)\gamma(1-\lambda)(V_0 - V_\rho) = \\ &= (\lambda\overline{\mu}_M(\phi\chi_B) + (1-\lambda)\overline{\mu}_M(\chi_B))\,\gamma\frac{r(\delta;\phi,\chi_B,\lambda)}{\theta}, \end{aligned} \tag{32}$$

where we have used the conditions on prices and the fact that $\chi_S = \phi\chi_B$, and that $V_0 = 1$, as well as the relationship between V_0, V_ρ and r. ■

An equilibrium of the economy in the extended model can be described by a tuple $(\delta^e, \phi^e, \chi_B^e, \lambda^e)$ of refinancing frequency choice of firms δ^e, and ratios of buyers to sellers and marketmakers to buyers (ϕ^e, χ_B^e) together with an equilibrium inter-dealer price index $\lambda^e \in [0,1]$, such that the refinancing frequency choice of firms is optimal, the entry decisions of buyers and marketmakers are optimal, and the inter-dealer market clears (which amounts to $\lambda^e = 1$ if $\phi^e > 1$ and $\lambda^e = 0$ if $\phi^e < 1$).

In the model with marketmakers, an equilibrium exists, is unique, and is also inefficient, as summarized in the following two propositions (the analogues to Propositions 1 and 2 in the baseline model).

Proposition B.1. *Under Assumption 1, in the model with marketmakers, there exists a unique steady-state equilibrium.*

Proof. The liquidity conditions in the secondary market are described by the variables ϕ, χ_B and λ. The idea behind the proof is to reduce this set of variables to a single variable y, and then, as in the baseline model, to characterize equilibria as the intersection in the (y, δ)-space of a curve describing the solution to the firm's problem, and a curve describing free entry.

In order to construct the variable y we have first to make some definitions and manipulate equations (28), (30), and (31).

We define

$$\chi := \begin{cases} \chi_B \text{ if } \phi \leq 1 \\ \chi_S \text{ if } \phi > 1 \end{cases} .$$

Using the variable χ, and the fact that $\phi < 1 \implies \lambda = 0$, $\phi = 1 \implies \lambda \in (0,1)$, and $\phi > 1 \implies \lambda = 1$, we can re-write the free entry condition for marketmakers (31) as

$$e_M = \overline{\mu}_M(\chi)\gamma \frac{r(\delta; \phi, \chi_B, \lambda)}{\theta}, \tag{33}$$

as well as the free entry condition for buyers. Dividing the free entry condition for buyers by that for marketmakers, we now obtain:

$$\frac{e_B}{e_M} = \frac{\mu_B(\phi)(1-\beta) + \overline{\mu}_B(\chi)(1-\gamma)(1-\lambda)}{\overline{\mu}_M(\chi)\gamma}. \tag{34}$$

It is easy to see that for every ϕ, λ there exists a unique χ such that the equation above is satisfied. Let us denote the function this equation defines by $\chi^{FEC}(\phi, \lambda)$. $\chi^{FEC}(\phi, \lambda)$ is continuous for $\phi \in (0, 1)$ and $\phi \in (1, \infty)$ and for $\lambda \in [0, 1]$, it is increasing in ϕ and is also increasing in λ. In addition:

$$\begin{aligned}
\lim_{\phi \to 1^-} \chi^{FEC}(\phi, \lambda) &= \chi^{FEC}(1, 0) = \lim_{\lambda \to 0^+} \chi^{FEC}(1, \lambda), \\
\lim_{\phi \to 1^+} \chi^{FEC}(\phi, \lambda) &= \chi^{FEC}(1, 1) = \lim_{\lambda \to 1^-} \chi^{FEC}(1, \lambda), \\
\lim_{\phi \to 0^+} \chi^{FEC}(\phi, \lambda) &= 0, \\
\lim_{\phi \to \infty} \chi^{FEC}(\phi, \lambda) &= \infty.
\end{aligned}$$

We now define:

$$F(y) = \begin{cases} (y, 0) \text{ if } y \in (0, 1) \\ (1, y-1) \text{ if } y \in [1, 2] \\ (y-1, 1) \text{ if } y > 2 \end{cases}$$

The function $\chi^{FEC}(F(y))$ is continuous and strictly increasing in $y \in (0, \infty)$. It also satisfies:

$$\begin{aligned}
\lim_{y \to 0^+} \chi^{FEC}(F(y)) &= 0, \\
\lim_{y \to \infty} \chi^{FEC}(F(y)) &= \infty.
\end{aligned}$$

Let us also define the function:

$$G(y) = \begin{cases} y \text{ if } y \in (0, 1) \\ 1 \text{ if } y \in [1, 2] \\ y-1 \text{ if } y > 2 \end{cases}$$

$G(y)$ is a continuous function.

We can use these definitions to rewrite the optimal refinancing frequency choice in equation (28) as

$$\delta^*(y) = \max \left\{ \sqrt{\frac{\theta\rho}{\kappa}} - \theta - \rho - \mu_S(G(y))\beta - \left(\mathbf{1}_{y>2} + \mathbf{1}_{y \in [1,2]}(y-1)\right) \overline{\mu}_S(\chi^{FEC}(F(y)))(1-\gamma), 0 \right\},$$

and to rewrite the free entry condition for buyers in equation (30) as

$$\begin{aligned}\delta^{FEC}(y) &= \frac{\rho}{e_B}\left[\mu_B(G(y))(1-\beta) + \left(\mathbf{1}_{y<1} + \mathbf{1}_{y\in[1,2]}(2-y)\right)\overline{\mu}_B(\chi^{FEC}(F(y)))(1-\gamma)\right] - \\ &\quad -\theta - \rho - \mu_S(G(y))\beta - \left(\mathbf{1}_{y>2} + \mathbf{1}_{y\in[1,2]}(y-1)\right)\overline{\mu}_S(\chi^{FEC}(F(y)))(1-\gamma).\end{aligned} \tag{35}$$

Both functions are continuous.

It is easy to convince oneself that the definitions of $G(y)$, $\chi^{FEC}(F(y))$, $\delta^*(y)$ and $\delta^{FEC}(y)$ are such that if (y^e, δ^e) satisfy:

$$\delta^e = \delta^*(y^e) = \delta^{FEC}(y^e), \tag{36}$$

then, if $y^e < 1$ the tuple $(\delta^e, \phi^e = G(y^e), \chi_B^e = \chi^{FEC}(F(y^e), \lambda^e = 0)$ is an equilibrium, if $y^e > 2$ the tuple $(\delta^e, \phi^e = G(y^e), \chi_B^e = G(y^e)\chi^{FEC}(F(y^e), \lambda^e = 1)$ is an equilibrium and finally if $y^e \in [1,2]$ the tuple $(\delta^e, \phi^e = 1, \chi_B^e = \chi^{FEC}(F(y^e), \lambda^e = y^e - 1)$ is an equilibrium. Conversely, for every equilibrium $(\delta^e, \phi^e, \chi_B^e, \lambda^e)$, we define $y^e = \phi^e$ if $\phi^e < 1$, define $y^e = 1 + \lambda^e$ if $\phi^e = 1$, and $y^e = \phi^e + 1$ if $\phi^e > 1$. Then (δ^e, y^e) satisfy equation (36).

Therefore, pairs (δ^e, y^e) satisfying equation (36) characterize the equilibria of the model. In order to prove the proposition it suffices to prove that the functions $\delta^*(y), \delta^{FEC}(y)$ have a unique intersection point in the interval $(0, \infty)$. From this point onwards the proof is analogous to the one of Proposition 1.

First, existence is consequence of the continuity of both functions and their behaviour at the limits of the interval $(0, \infty)$:

$$\begin{aligned}\lim_{y\to 0^+} \delta^*(y) &< \lim_{y\to 0^+} \delta^{FEC}(y) = \infty, \\ \lim_{y\to\infty} \delta^*(y) &= 0 > \lim_{y\to\infty} \delta^{FEC}(y) = -\infty.\end{aligned}$$

Second, in order to prove uniqueness it suffices to prove that the inequality

$$\frac{d\delta^{FEC}(y)}{dy} < \frac{d\delta^*(y)}{dy},$$

is satisfied almost everywhere.[29] Comparing the analytical expressions for $\delta^*(y)$ and $\delta^{FEC}(y)$ it suffices to prove that

$$\frac{d}{dy}\left[\mu_B(G(y))(1-\beta) + \left(\mathbf{1}_{y<1} + \mathbf{1}_{y\in[1,2]}(2-y)\right)\overline{\mu}_B(\chi^{FEC}(F(y)))(1-\gamma)\right] < 0. \tag{37}$$

Since $\mu_B(\cdot)$ is decreasing in its argument but $\overline{\mu}_B(\cdot)$ is increasing the sign of the expression above is ambiguous. However, we can rewrite equation (34) as:

$$\mu_B(G(y))(1-\beta) + \left(\mathbf{1}_{y<1} + \mathbf{1}_{y\in[1,2]}(2-y)\right)\overline{\mu}_B(\chi^{FEC}(F(y)))(1-\gamma) = \overline{\mu}_M\left(\chi^{FEC}(F(y))\right)\gamma\frac{e_B}{e_M}$$

and insert the result into the inequality. Since $\chi^{FEC}(F(y))$ is strictly increasing in y and $\overline{\mu}_M(\cdot)$ is strictly decreasing in its argument, it is clear that the inequality holds.

■

Proposition B.2. *In the model with marketmakers, let* $(\delta^e, \phi^e, \chi_B^e, \lambda^e)$ *describe an equilibrium with* $\delta^e > 0$. *Then the solution* δ^{SP} *to the Social Planner's problem satisfies* $\delta^{SP} < \delta^e$.

Proof. Let us work in the (y, δ)-space defined in the proof of the previous proposition. The equilibrium can be described by a pair (y^e, δ^e). The function $\delta^{FEC}(y)$ defined in equation (35) is strictly decreasing. Let $y^{FEC}(\delta)$ be its inverse function, which is defined for $\delta \geq 0$, strictly decreasing and differentiable except at $\delta = \delta^{FEC}(1)$ and $\delta = \delta^{FEC}(2)$. The SP maximizes:

$$U^{SP}(\delta) = U(\delta; y^{FEC}(\delta)) = \frac{x}{\rho} + (1-\kappa) - \frac{r(\delta; y^{FEC}(\delta)) + \delta\kappa}{\rho},$$

where $r(\delta; y)$ has the following expression:

$$r(\delta; y) = \frac{\theta}{\delta + \theta + \rho + \mu_S(G(y))\beta + \left(\mathbf{1}_{y>2} + \mathbf{1}_{y\in[1,2]}(y-1)\right)\overline{\mu}_S(\chi^{FEC}(F(y)))(1-\gamma)}\rho.$$

Taking into account that $G(y)$ is increasing in y and that $\chi^{FEC}(F(y))$ is strictly increasing in y we have that:

$$\frac{\partial r(\delta; y)}{\partial y} < 0 \Leftrightarrow \frac{\partial U(\delta; y)}{\partial y} > 0.$$

[29] The functions are not differentiable at $y = 1, 2$ and at the smallest y for which $\delta^*(y) = 0$.

From here, we have as in the baseline model that if $\delta^e > 0$ then:

$$\left.\frac{dU^{SP}(\delta)}{d\delta}\right|_{\delta=\delta^e} < 0.$$

The same arguments as in the proof of Proposition 2 where $\phi, \phi^{FEC}(\delta)$ are replaced by $y, y^{FEC}(\delta)$ also lead to:

$$U^{SP}(\delta) < U^{SP}(\delta^e) \text{ for all } \delta > \delta^e.$$

We conclude from the two previous inequalities that:

$$\arg\max_{\delta \geq 0} U^{SP}(\delta) < \delta^e.$$

■

Finally, for any given equilibrium, the bid-ask spread $A - B$ of marketmakers can be calculated, as stated in the following lemma:

Lemma B.6. *In the model with marketmakers, the bid-ask spread is given by*

$$A - B = \gamma(V_0 - V_\rho) = \frac{\gamma\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta + \overline{\mu}_S(\phi\chi_B)(1-\gamma)\lambda}. \tag{38}$$

Proof. Calculate $A - B$ directly from (23) and (24), and use $\theta(V_0 - V_\rho) = r$ and equation (27) to obtain the stated expression. ■

C Puttable debt

In the main text, we have considered a debt security that does not embed a put option.[30] In this appendix, we consider a debt security that embeds a put option which allows the investor to redeem the debt security for face value at any time before maturity.[31] The firm needs to be able to pay for these redemptions. In order to afford the firm some flexibility, we assume that it has access to a cash storage technology that provides a net rate of return of 0. This will allow the firm to choose the fraction of the (re-)issuance proceeds to be stored in a cash buffer that it can use together with project cash flows to pay for redemptions.

Here, the advantage to an entrepreneur of issuing puttable debt is that the interest rate on such debt can be set to zero, which is lower than the interest rate required for non-puttable debt. The disadvantage is that it is inefficient to store cash at a net rate of return of zero because the discount rate of the entrepreneur exceeds zero. Below, we work out an expression for the utility of the entrepreneur when issuing a general combination of puttable debt and liquidity buffer, characterize the optimal puttable debt structure, and show that for the parameters used in the numerical illustration in Section 6, puttable debt is in fact dominated by the non-puttable debt that we consider in our baseline model.

We maintain the assumption of a stationary debt structure. We assume that puttable debt has deterministic maturity T, meaning that the firm initially issues debt with a face value of 1 as in our baseline model, and re-issues debt with face value of 1 every T units of time. At the time of every issuance, an amount L_0 is taken from the issuance proceeds and stored as a cash buffer. We let $t \in (0, T]$ denote the time since the last issuance, and let $(y_t)_{t\in(0,T]}$ denote the dividends paid out to the entrepreneur between re-issuing. We can then describe a stationary puttable debt structure

[30] Note, though, that as $\delta \to \infty$ in the baseline model, debtholders implicitly have the right to redeem the debt security for face value at any time from the issuer.

[31] This puttable debt security could also be described as a form of demand deposit.

via the tuple $(T, L_0, (y_t)_{t\in(0,])})$. We maintain the assumption that a cost κ is incurred each time debt is (re-)issued.

Let $D_t \leq 1$ be the aggregate face value of remaining (unredeemed) debt at time t, and L_t the amount of cash stored at t. For $t \in (0, T)$, these satisfy the laws of motion

$$\frac{dD_t}{dt} = -\theta D_t, \tag{39}$$
$$\frac{dL_t}{dt} = x - \theta D_t - y_t. \tag{40}$$

The first equation states that the face value of remaining debt shrinks at a rate θ, as debtholders who become impatient redeem. The second equation is an accounting identity that states that all cash inflows and outflows must add up to the change in the cash buffer: x represents the cash inflows generated by the project, $-\theta D_t$ represents the cash outflows associated with redemptions, and $-y_t$ is the dividend outflow. Integrating from 0 to $t < T$ we obtain:

$$D_t = e^{-\theta t}, \tag{41}$$
$$L_t = L_0 + \int_0^t \left(x - \theta e^{-\theta s} - y_s\right) ds. \tag{42}$$

In order for a debt structure to be feasible, the following constraints must be satisfied:

$$L_t \geq 0, \tag{C1}$$

$$y_t \geq 0, \tag{C2}$$

$$1 \geq D_T + \kappa + (L_0 - L_T). \tag{C3}$$

(C1) states that the cash buffer cannot be negative, so that the firm can only borrow by issuing debt. (C2) states that dividends cannot be negative. Negative dividends would imply cash injections by the entrepreneur, which would have to come from cash stored outside the buffer. To ensure that we account fully for the opportunity cost of storing cash, we require non-negative dividends. (C3) states that at maturity $t = T$, the re-issuance proceeds, 1, have to be sufficient to repay face value on the remaining debt D_T as it matures, finance the issuance cost κ, and to pay an amount $L_0 - L_T$ into the cash buffer, such that it again contains an amount L_0.

The utility for the entrepreneur when financing the project with puttable debt $(T, L_0, (y_t))$ is

$$\begin{aligned} U^{PD}(T, L_0, (y_t)) = & -1 - \kappa + 1 - L_0 + \frac{1}{1 - e^{-\rho T}} \int_0^T e^{-\rho t} y_t dt \\ & + \frac{1}{1 - e^{-\rho T}} e^{-\rho T} \left(1 - D_T - \kappa - (L_0 - L_T)\right). \end{aligned}$$

The first term and second term represent the cost of the investment and the issuance cost at $t = 0$, respectively. The third and fourth terms account for the funds obtained by issuing puttable debt at $t = 0$ and the fraction of them that are kept in the cash buffer. The net sum of the first four terms $-\kappa - L_0$ is the net amount that needs to be paid by the entrepreneur at $t = 0$. The fifth term is the expected value of the continuous flow of dividends paid by the firm. The last term is the expected value of the left over cash after re-issuing, which is paid as a discrete dividend.

Using equations (41) and (42) we can write

$$\begin{aligned} U^{PD}(T, L_0, (y_t)) &= C(T) - L_0 + \frac{1}{1 - e^{-\rho T}} \int_0^T (e^{-\rho t} - e^{-\rho T}) y_t dt, \\ \text{where } C(T) &:= -\kappa + \frac{e^{-\rho T}}{1 - e^{-\rho T}} \left(1 - e^{-\theta T} - \kappa + \int_0^T (x - \theta e^{-\theta t}) dt\right). \end{aligned}$$

The firms' problem is:

$$\max_{T, L_0, (y_t)} U^{PD}(T, L_0, (y_t)) = C(T) - L_0 + \frac{1}{1 - e^{-\rho T}} \int_0^T (e^{-\rho t} - e^{-\rho T}) y_t dt \tag{43}$$

$$\begin{aligned} \text{s.t.} \quad L_0 + \textstyle\int_0^t \left(x - \theta e^{-\theta s} - y_s\right) ds \geq 0 \quad & \text{(C1)} \\ y_t \geq 0 \quad & \text{(C2)} \\ xT - \kappa \geq \textstyle\int_0^T y_s ds \quad & \text{(C3)} \end{aligned}$$

where we have simplified (C3) using (41) and (42).

Before stating the solution to this program, it will be useful to define $\overline{T} = -\frac{1}{\theta} \ln\left(\frac{x}{\theta}\right)$. Since $x - \theta e^{-\theta t} \leq 0$ if and only if $t \leq \overline{T}$, we can see that $\overline{T}$ describes the first time at which the remaining face value of puttable debt has decreased to the point where the cash inflows from the project, x, are sufficient to cover the cash outflows due to redemptions, $-\theta D_t = -\theta e^{-\theta t}$.

We can characterize the optimal puttable debt structure as follows:

Proposition C.3. *If $\overline{T} > \frac{\kappa}{x}$, the solution $T, L_0, (y_t)$ to the firms' problem with puttable debt satisfies*

$$\begin{aligned} T &\geq \frac{\kappa}{x}. \\ L_0 &= \int_0^{\min(\overline{T},T)} \left(\theta e^{-\theta s} - x\right) ds, \\ y_t &= \begin{cases} 0 \ \textit{if } t \leq \overline{T} \\ x - \theta e^{-\theta t} \ \textit{if } t > \overline{T} \end{cases}, \ \textit{for } t \in [0, T]. \end{aligned}$$

In particular we have $L_{\min(\overline{T},T)} = L_T = 0$.

For the proof, see the end of this appendix. The proposition states that for a given (feasible) choice of maturity T, the optimal puttable debt structure uses the minimum cash buffer necessary to be able to pay for redemptions. This means that if $T > \overline{T}$, the optimal initial size of the cash buffer is such that the buffer is depleted just at $\overline{T}$, at which point the remaining face value of debt has fallen so far that the project cash flows x are sufficient to cover the remaining cash outflows due to redemptions. Only then a continuous dividend equal to the project cash flow net of redemptions is paid out. If $T < \overline{T}$, the optimal initial size of the cash buffer is such that the buffer is depleted just at maturity T, and no continuous dividends are paid. If any funds are left over after reissuing and reestablishing the cash buffer, these are paid out as a discrete dividend. (Also, the debt maturity T has to exceed $\frac{\kappa}{x}$, such that the accumulated cash flows from the project are sufficient to at least cover the reissuance cost that is incurred at maturity.) The optimal puttable debt structure $T, L_0, (y_t)$ given in the proposition can now be substituted into the problem of the firm (43). This reduces the problem to finding the optimal maturity T, which can be done numerically.

For example, for the parameters used for the numerical illustration in Section 6, we have $\overline{T} \approx 4.6$, so that a firm that issues puttable debt would have negative net cash flows for the 4.6

years following every debt issuance. This does not mean that the firm will wait this long until it reissues debt. Indeed, if we numerically solve the problem of the firm we find that the optimal maturity is $T \approx 0.046$, i.e. 17 days approximately. Out of the first issuance the firm keeps a cash buffer of $L_0 \approx 0.0444$ at $t = 0$, so that the funds the entrepreneur has to use out of her wealth at $t = 0$ are $L_0 + \kappa \approx 0.0446$. The initial cash buffer is completely consumed at $t = T$ and since $T < \overline{T}$ the net cash flows are negative in the interval $(0, T)$ and the firm does not pay dividends. At $t = T$ the firm re-issues puttable debt with aggregate face value of 1, and uses the proceeds to repay face value on the remaining debt as it matures. This leaves an amount $1 - e^{-\theta T} \approx 0.0448$ of cash. It pays the refinancing cost κ, reestablishes the initial cash buffer and pays a dividend of about 0.00026. This dividend is paid every 17 days and the entrepreneur utility that this generates is $U^{PD}(T^*) \approx 0.012$, which is below the entrepreneur utility obtained using our optimal non-puttable debt ($U(\delta^*) \approx 0.032$).

Proof of Proposition C.3: Let $T, L_0, (y_t)$ be an optimal solution to the problem above. We must trivially have $T \geq \frac{\kappa}{x}$ in order for (C3) to be satisfied. In addition, $L_0, (y_t)$ has to be optimal for a given T. Consider the restricted problem in which we look for the optimal $L_0, (y_t)$ for fixed T. This is a non-standard optimal control problem, since it involves a continuous control variable y_t, as well as the discrete control variable L_0, which both have an effect on the state variable L_t. The strategy for the proof is to reduce this to a standard optimal control problem. We follow a sequence of steps, which all rely on the fact that it is optimal to minimize the storage of cash:

i) For $t \in (0, \min(\overline{T}, T)]$ we have $y_t = 0$ (a.s.):

Let us suppose that this is not true in a subset of $(0, \min(\overline{T}, T)]$ with positive measure. Then we would have:

$$\int_0^{\min(\overline{T},T)} y_s ds > 0.$$

Let us define

$$\begin{aligned}\widetilde{L}_0 &= L_0 - \int_0^{\min(\overline{T},T)} y_s ds, \\ \widetilde{y}_t &= \left\{ \begin{array}{c} 0 \text{ if } t \le \min(\overline{T},T) \\ y_t \text{ if } \overline{T} < t \le T \end{array} \right. .\end{aligned}$$

Then we have:

$$U^{PD}(T,\widetilde{L}_0,(\widetilde{y}_t)) - U^{PD}(T,L_0,(y_t)) = \int_0^{\min(\overline{T},T)} y_s ds - \int_0^{\min(\overline{T},T)} \frac{e^{-\rho s} - e^{-\rho T}}{1-e^{-\rho T}} y_s ds > 0. \tag{44}$$

Also, since by construction $\widetilde{L}_t = L_t \ge 0$ for $t \ge \min(\overline{T},T)$ and $\widetilde{L}_t$ is decreasing over the interval $(0,\min(\overline{T},T)]$ we have that $\widetilde{L}_t \ge 0$ for all t and (C1) is satisfied. Since $\widetilde{y}_t \le y_t$ for all t (C3) is also satisfied. And (C2) is trivially satisfied. But then inequality (44) contradicts that $T, L_0, (y_t)$ is a solution to the firms' problem.

ii) There exists a $t \le T$ such that $L_t = 0$:

If this is not the case, then L_0 could be lowered, which increases $U^{PD}(T,L_0,(y_t))$, while all three constraints are still satisfied.

iii) Define $I := \{t : L_t = 0\}$ and $t_1 := \inf(I)$. Then $t_1 = \min(\overline{T},T)$ and thus $L_{\min(\overline{T},T)} = 0$:

Since $dL_t/dt < 0$ for $t < \min(\overline{T},T)$ we necessarily have that $t_1 \ge \min(\overline{T},T)$. Let us suppose that $t_1 > \min(\overline{T},T)$. Note that this implies that $\overline{T} = \min(\overline{T},T)$ because of **ii)**.

Let us define the tuple

$$\begin{aligned}\widetilde{L}_0 &= L_0 - L_{\overline{T}}, \\ \widetilde{y}_t &= \left\{ \begin{array}{c} 0 \text{ if } t \le \overline{T} \\ x - \theta e^{-\theta t} \text{ if } \overline{T} < t \le t_1 \\ y_t \text{ if } t_1 < t \le T \end{array} \right.\end{aligned}$$

Then using that L_t satisfies **i)** we have by construction

$$\begin{aligned}U_T^{PD}(\widetilde{L}_0,(\widetilde{y}_t)) - U_T^{PD}(L_0,(y_t)) &= L_{\overline{T}} + \int_{\overline{T}}^{t_1} \frac{e^{-\rho s} - e^{-\rho T}}{1-e^{-\rho T}} (x - \theta e^{-\theta s} - y_s) ds \\ &= \left(1 - \frac{e^{-\rho \overline{T}} - e^{-\rho T}}{1-e^{-\rho T}}\right) L_{\overline{T}} + \int_{\overline{T}}^{t_1} \frac{\rho e^{-\rho s}}{1-e^{-\rho T}} L_s ds > 0, \end{aligned} \tag{45}$$

where in the second equality we have used that $L_t' = x - \theta e^{-\theta t} - y_t$, that by construction $L_{t_1} = 0$, $L_t > 0$ for $t \in (\overline{T}, t_1)$, and we have integrated by parts. It is straightforward to see that the tuple $\widetilde{L}_0, (\widetilde{y}_t)$ satisfies conditions (C1) and (C2). Furthermore, it is also the case that condition (C3) is satisfied: since $L_{\overline{T}} > L_{t_1} = 0$ and $\widetilde{L}_{\overline{T}} = \widetilde{L}_{t_1} = 0$, we have $\int_{\overline{T}}^{t_1} y_s ds > \int_{\overline{T}}^{t_1} \widetilde{y}_s ds$. Hence $\int_0^T y_s ds > \int_0^T \widetilde{y}_s ds$ and since $L_0, (y_t)$ satisfies condition (C3) $\widetilde{L}_0, (\widetilde{y}_t)$ does as well. But then inequality (45) contradicts that $T, L_0, (y_t)$ is a solution to the firms' problem.

Therefore $t_1 = \min(\overline{T}, T)$ and by continuity $L_{\min(\overline{T},T)} = 0$. And using **i)** we recover the expression for L_0 in the proposition.

iv) If $T > \overline{T}$, for $t \in (\overline{T}, T]$ we have $y_t = x - \theta e^{-\theta t}$:

We know that $L_{\overline{T}} = 0$. Then $(y_t)_{t \in (\overline{T},T]}$ has to be a solution to the following restricted problem

$$
\begin{array}{cll}
\max\limits_{(y_t) \text{ for } t \in (\overline{T},T]} & \frac{1}{1-e^{-\rho T}} \int_{\overline{T}}^{T} (e^{-\rho t} - e^{-\rho T}) y_t dt & \\
\text{s.t.} & \int_{\overline{T}}^{t} \left(x - \theta e^{-\theta s} - y_s \right) ds \geq 0 & \text{(C1)} \\
 & y_t \geq 0 & \text{(C2)} \\
 & xT - \kappa \geq \int_{\overline{T}}^{T} y_s ds & \text{(C3)}
\end{array}
$$

This problem is a standard optimal control problem with control variable y_t, state variable L_t and initial condition $L_{\overline{T}} = 0$. Using Pontryagin's maximum principle it is easy to prove that the solution to this problem is $y_t = x - \theta e^{-\theta t}$ (Assumption $\overline{T} > \frac{\kappa}{x}$ ensures that (C3) is satisfied). ■

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