On the limits of communication in multidimensional cheap talk

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1. Introduction

In this paper we extend the analysis of the cheap talk model of Crawford and Sobel (1982) to a multidimensional state and policy space. In this game, a sender who knows the state of the world sends a message to a receiver. The receiver chooses an action given the sender's message and his prior beliefs about the state. The sender's ideal policy is linear in the state of the world, and his utility decreases in the p-norm distance between his ideal policy and the receiver's action. The receiver's ideal policy is the state of the world and we assume that he chooses an action identical to his posterior expectation of the state of the world. A vector b, the difference between the ideal policy of the sender and that of the receiver, denotes the conflict between the two players. We analyze the (weak) perfect Bayesian equilibria of the game.

The generalization of cheap talk to multiple dimensions introduces two new effects. The first arises from the interaction of the multiple dimensions in the players' preferences. In the multidimensional environment, there is always a direction upon which both players agree.[2] The second effect, overlooked in previous literature, involves the interaction between the different dimensions according to the underlying information structure. Generally, these dimensions may be correlated which implies that

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[2] See Austen-Smith (1993), Spector (2000) and Battaglini (2002).

communication on one dimension often reveals information on others. In this paper we show how both these effects combine to determine the level of communication in equilibrium. We focus on environments in which the magnitude of b is large (which allows us to pin down the structure of equilibria). Our main result establishes an upper bound on the level of communication in equilibrium.

We first show, in Proposition 1, that when the conflict becomes large, the preferences of the sender become highly sensitive to a particular dimension, which we term "the dimension of conflict". We can then derive necessary conditions on equilibria. Specifically, we find that in any equilibrium, all actions that the receiver takes must converge to a particular dimension, which we term "the dimension of agreement" (these two dimensions depend on the primitives of the model, namely, the parameters in the sender's utility function).

Intuitively, the conditions imposed by Proposition 1 may constrain communication in equilibrium. Since the receiver's actions depend on the prior distribution, there is no guarantee that these actions will all be on the "dimension of agreement". To capture this, we characterize the environments that we study according to how the state space is distributed along the "dimension of agreement" and the "dimension of conflict".

Theorem 1 states that for any $\varepsilon > 0$, for a large enough conflict, there is a finite and bounded number of actions that the receiver takes in equilibrium with probability of at least $1 - \varepsilon$. A corollary to Theorem 1 is that in an open set of environments, all equilibria approximate the "babbling" equilibrium (for example, this is the case when the dimension of conflict and that of agreement are affiliated according to the prior distribution). We show how the upper bound on the number of actions in equilibrium depends on the relation between the prior distribution and the sender's preferences. Moreover, we illustrate that this upper bound is binding. Finally, in Section 4, we relate our results to those of Crawford and Sobel (1982), Chakraborty and Harbaugh (2005) and Battaglini (2002, 2004).

2. The model

A receiver has to choose an action in a two-dimensional policy space, $\Re^2$ (the results can be generalized for $\Re^d$ for $d > 2$, see the discussion in Section 3). The appropriate choice of action depends on the realization of a state of the world θ, in $\Re^2$. The receiver initially holds a continuous and proper prior distribution on $\Re^2$ denoted by F, with a density function f, and expectations at the origin. Assume that the receiver always chooses an action at the expectation of θ, $E(\theta)$, according to his posterior. This assumption is made for tractability. It is also consistent with an assumption that the receiver's utility function decreases in the quadratic distance between the action and the state of the world.[3]

A sender who is fully informed about θ (henceforth type θ), has an optimal policy $\theta + b$ for some vector $b = (b_x, b_y) \in \Re^2$. His preferences over the actions of the receiver are represented by $V(\Delta_{\alpha,p}(a, b|\theta))$, where $\Delta_{\alpha,p}(a, b|\theta) = \left(\sum_{i\in\{x,y\}} \alpha_i |a_i - (b_i + \theta_i)|^p\right)^{\frac{1}{p}}$, for $1 < p < \infty$, and the function $V(.)$ is strictly decreasing. The parameters $\{\alpha_x, \alpha_y\}$ are all strictly positive and denote the relative importance of the different dimensions

[3]Proposition 1 does not depend on the receiver's behavior. Theorem 1 could be modified to the case in which the strategy of the receiver is responsive (in a monotone way) to his expectation on the state of the world.

in the sender's utility function. We assume that at least for one dimension $i \in \{x, y\}$, b_i is different from zero.

In the game, the sender observes the state of the world θ and then chooses a message in $\Re^2$. Following the message, the receiver takes an action a in the set $A = \Re^2$. We analyze (weak) Perfect Bayesian equilibria of this game. Note that an equilibrium always exists, e.g., a 'babbling' equilibrium.

We refer to the vector b as the "conflict" between the sender and the receiver. This terminology may be misleading as other primitives in the model, such as α, p and perhaps even the prior distribution, could be part of a more general definition of a conflict. Indeed, we will show how all these parameters are relevant for determining the level of communication in equilibrium.

Our paper focuses on large levels of conflict. For any vector of conflict b denote by $b^{\rightarrow}$ its direction in $\Re^2$. We refer to the triple $(F, \alpha, b^{\rightarrow})$ as an *environment.* In the analysis we will keep the environment fixed while increasing the magnitude of the vector of conflict. For expositional purpose, we assume that the conflict on the $x-$axis, b_x, is positive whereas there is no conflict on the $y-$axis, i.e., $b_y = 0$. Our analysis focuses therefore on increasing $b_x (\equiv b)$.

3. Equilibria with high levels of conflicts

Our first result characterizes the preferences of the sender over the possible actions of the receiver when the conflict is large. Focusing on large conflicts will allow us to impose a particular structure on the sender's preferences and hence on equilibria. As we will show later on, this structure will constrain the possibility of communication in equilibrium.

When b becomes large, the distance between typical actions that the receiver may take and the sender's ideal policy increases. We show that this implies that the sender's preferences (over actions in some compact set) become highly sensitive to a particular dimension, the $x-$dimension.

PROPOSITION 1 *Fix a compact set $C \subset \Re^2$. For any $\varepsilon > 0$, there exists a $\bar{b}$ such that for all $b > \bar{b}$ and any two distinct actions (a'_x, a'_y) and (a''_x, a''_y) in C : (i) A sender type $\theta' \in C$ is indifferent between the two actions only if $|a'_x - a''_x| \leq \varepsilon$; (ii) If θ',$\theta'' \in C$ are both indifferent between these two actions, then $|\theta'_y - \theta''_y| \leq \varepsilon$.*[4]

PROOF OF PROPOSITION 1: Consider any two distinct actions, a',$a'' \in C$. For any b, any sender type θ' that is indifferent between these two actions satisfies:

$$|a'_x - (\theta'_x + b)|^p - |a''_x - (\theta'_x + b)|^p = \frac{\alpha_y}{\alpha_x}(|a''_y - \theta'_y|^p - |a'_y - \theta'_y|^p) \tag{1}$$

Note that $a'_y \neq a''_y$ (otherwise by (1) the two actions are not distinct). Without loss of generality let $a''_y > a'_y$. For any $b > \max\{a'_x, a''_x\} - \theta_x$, we can re-arrange (1):

$$\frac{\sum_{j=1}^{p}(\theta'_x + b - a'_x)^{p-j}(\theta'_x + b - a''_x)^{j-1}}{\kappa} = \tag{2}$$

$$\begin{cases} \frac{\alpha_y}{\alpha_x}\sum_{j=1}^{p}(\theta'_y - a''_y)^{p-j}(\theta'_y - a'_y)^{j-1} & \text{if } \theta'_y \geq a''_y \\ \frac{\alpha_y}{\alpha_x}\frac{a''_y + a'_y - 2\theta'_y}{a'_y - a''_y}\sum_{j=1}^{p}(a''_y - \theta'_y)^{p-j}(\theta'_y - a'_y)^{j-1} & \text{if } \theta'_y \in [a'_y, a''_y] \\ -\frac{\alpha_y}{\alpha_x}\sum_{j=1}^{p}(a''_y - \theta'_y)^{p-j}(a'_y - \theta'_y)^{j-1} & \text{if } \theta'_y \leq a'_y \end{cases}$$

[4]Note that since we set $b_y = 0$, α_x and α_y play no role in the result. After presenting Theorem 1, we explain how the results are modified in the general model.

where $\kappa = \frac{a'_y - a''_y}{a''_x - a'_x}$. Note that $\sum_{j=1}^{p} (\theta'_x + b - a'_x)^{p-j} (\theta'_x + b - a''_x)^{j-1} \to \infty$ uniformly and that the right-hand-side of (2) is bounded. Thus, $\kappa \to \infty$ uniformly. This proves (i). Moreover, for any sequence $\{b, a', a''\}_n$, the left-hand-side must have a converging sub-sequence with some limit γ. Note also that the right-hand-side of (2) is monotone and continuous in θ'_y. This implies that θ'_y converges to a finite value. We have to make sure that the convergence is uniform. Note that the derivative of the left-hand-side of (2) with respect to θ_x is of order $\frac{(\theta'_x + b - a'_x)^{p-2}}{\kappa}$. This expression is bounded by a bound that converges to zero (since $\frac{(\theta'_x + b - a'_x)^{p-1}}{\kappa}$ is bounded). We can therefore take the sequence of a', a'' which constitutes the worst case scenario by making κ the smallest possible. For this sequence, $\frac{(\theta'_x + b - a'_x)^{p-2}}{\kappa}$ converges to zero. This implies that the left-hand-side of (2) converges in a uniform way.■

In equilibrium, whenever the receiver takes two distinct actions, then some sender types must be indifferent between them. Thus, Proposition 1 imposes a particular structure on equilibria for large b. First, (i) is a necessary condition on "most" equilibrium actions. It implies that any two equilibrium actions in a compact set will converge to a line parallel to the $y-$axis. Moreover, in equilibrium the receiver also chooses actions according to his posterior beliefs about the state of the world. Thus, these actions must converge to a particular line, the one which passes through the prior expectation (namely, the $y-$axis itself).

Second, (ii) imposes conditions on the shape of the set of sender types who send some particular message in equilibrium. Sender types who are indifferent between two equilibrium actions in a compact set tend to converge to a line parallel to the $x-$axis.

This implies that the receiver's actions will tend to be his conditional expectation over subsets of the state space bounded by lines parallel to the $x-$axis.

One could try and generalize Proposition 1 to other preferences. A necessary condition is that the utility function of the sender is strictly quasi-concave.[5] This implies that as b grows large, in any compact set, the indifference curves are 'stretched' out and converge to lines. An additional requirement is that the slopes of these lines become independent of the type θ of the sender when b increases.

The requirements imposed by Proposition 1, along with the equilibrium condition that the receiver updates his beliefs in a Bayesian manner, suggest that what might be important is the relation between the prior distribution function and the two relevant dimensions, the $x-$dimension, which we term "the dimension of conflict", and the $y-$dimension, which we term "the dimension of agreement". To explore this relation, we define the *reaction curve,* $\gamma(y)$. The reaction curve is the graph of conditional expectation over x for some fixed value of y :

$$\gamma(y) = E[x|y] = \int_{-\infty}^{\infty} x \frac{f(x,y)}{\int_{-\infty}^{\infty} f(z,y)dz} dx.$$

The reaction curve allows us to categorizes different environments according to how the state space is distributed along the "dimension of agreement" and the "dimension of conflict". This is done in the following definition.

Definition 1 *For a finite k, an environment satisfies the k-crossing property if the reaction curve $\gamma(y)$ crosses the $y-$axis exactly k times.*

[5] To see why *strict* quasi-concavity is needed, consider the absolute value preferences ($p = 1$). With these preferences, the first part of the proposition still holds but the second one fails.

As a final step before presenting Theorem 1, we define our measure of communication. As usual in cheap talk games, our measure of communication focuses on the responsiveness of the receiver. It differs from standard definitions as we disregard actions that are taken with small probabilities (even though these actions may be taken on the basis of very precise information). Consider some equilibrium and denote by $S(a)$ the set of types who send with a strictly positive density a message upon which the receiver takes the action a (we sometimes call this the *support set* of a). Also, let D be a measurable set and denote the measure of D by $m(D) = \int_D dF$.

Definition 2 *An equilibrium has k actions up to ε, if $k = \inf_{C \in 2^A}\{|C|$ such that $m(\cup_{a \in C} S(a)) > 1 - \varepsilon\}$.*

If an equilibrium has k actions up to ε for any $\varepsilon > 0$, then there are at most k actions that are taken by the receiver with a strictly positive probability.

We now present our main result (the proof follows after a brief discussion).

Theorem 1 *Suppose that the environment satisfies the k-crossing property. Then for any $\varepsilon > 0$ there exists a $\bar{b} < \infty$ such that for all $b > \bar{b}$, any equilibrium has at most k actions up to ε.*[6]

An immediate corollary of Theorem 1 is that for environments which satisfy the one-crossing property, there is only one action that the receiver is likely to take. Thus, all equilibria approximate the "babbling" equilibrium when the conflict is large. The one-crossing property holds, for example, when $F(\theta_x, \theta_y)$ satisfies affiliation.

[6] In other environments the reaction curve may cross the $y-$axis infinitely often. From the proof of the Theorem it follows that full information transmission is impossible as long as the reaction curve is not identical to the $y-$axis.

In the simple model the "dimension of agreement" and the "dimension of conflict", were fully aligned with the dimensions of the state space, the x and y axes. Suppose however that b_x, b_y, α_x and α_y are all different from zero. All the results follow with the modification that the space is now spanned by the following two dimensions: The "dimension of agreement" has a slope of $(\frac{b_y}{b_x})^{p-1}$ and the "dimension of conflict" has a slope of $-\frac{\alpha_x}{\alpha_y}(\frac{b_x}{b_y})^{p-1}$.[7] The reaction curve is now defined as the graph of the conditional expectation on the dimension of conflict. The environment satisfies the $k-$crossing property if the (modified) reaction curve crosses exactly k times the line with a slope $(\frac{b_y}{b_x})^{p-1}$ that goes through the prior expectation. Given these modifications, the statement of Theorem 1 holds in the general model as it is stated above.[8]

Finally, our analysis leaves open the question of whether equilibria with l actions up to ε for $1 < l \leq k$ actually exist when the conflict is large and the environment satisfies the k-crossing property for $k > 1$. In Section 4.1, we construct environments in which there exist equilibria with k actions up to ε for all $\varepsilon > 0$. Thus, the upper bound that we identify is binding.

Proof of Theorem 1: We first start with some helpful definitions and notations. For any $a', a'' \in \Re^2$ let $d(a', a'')$ denote the Euclidean distance between a' and a''. For any $a \in \Re^2$ and any set $A' \subset \Re^2$, define the distance between a and A', $d(a, A') = \inf_{a' \in A'} d(a, a')$.

[7] The proof of Proposition 1 for the general case as well as for more than two dimensions, is conceptually similar to the one presented here and is available upon request.

[8] When there are more than two dimensions, the "dimension of agreement" and the "dimension of conflict" are hyperplanes. To accommodate this, the definitions of the reaction curve and the $k-$crossing property have to be modified. With the appropriate modifications, Theorem 1 holds.

Denote the set of sender types that weakly prefer an action a to an action a' by $R(a, a')$. For any equilibrium with a set of actions A' taken by the receiver, and any action $a \in A'$, let $\bar{S}(a) = \bigcap_{a' \in A'} R(a, a')$. We say that a sender type θ *is in the border of* $S(a)$ if: (i) $\theta \in \bar{S}(a)$; (ii) for all $r > 0$, there exists a type θ' such that $|\theta' - \theta| \leq r$ and $\theta' \notin S(a)$.

Let L denote the set of lines which are parallel to the $x-$axis, such that the $x-$coordinate of the conditional expectation above each such a line is zero. We denote an element of L by l. Finally, an $\eta-$*square* is a square parallel and symmetric to the axes, that embodies $1 - \eta$ of the mass according to the prior distribution.

The proof of the Theorem follows three main steps. Step 1 allows us to translate results on compact subsets to the non-compact state space. Step 2 focuses on large compact subsets and invokes Proposition 1 to show that each border of the support sets of equilibrium actions must converge to some $l \in L$. In Step 3 we use the reaction curve to show that whenever the $k-$crossing property is satisfied, there are at most $k - 1$ lines in L. Finally, we show how these three steps imply that for high b, there will be at most k actions up to ε.

Step 1 For any $\delta > 0$ there exists an $\eta' > 0$, such that for any $\eta < \eta'$, in any equilibrium, there are at most a measure δ of types in the $\eta-$square that are in support sets of actions outside the $\eta-$square.

Proof of Step 1 In what follows, we set "north" to be the upward direction on the $y-$axis. Consider the strip to the east of the $\eta-$square which is bounded above and below by the extensions of the north and south sides of the square. Denote it by E. Any expectations over actions in E must be in E. Note also that the expectation over these actions, where each is weighted by the measure of its support set, is equal to the conditional expectation over the union of their support sets.

Given δ, assume that those in the $\eta-$square who support actions in E constitute a strip of measure δ in the $\eta-$square which is closest to the east boundary. Denote this strip D. Assume that those outside the square who support actions in E are to the east of the east side of the $\eta-$square. Denote this set of supporters by C. Denote by $f(x)$ the marginal distribution on the $x-$axis. Note that since the prior distribution is proper, $\int_E xf(x)dx \underset{\eta\to 0}{\to} 0$ and $\int_C xf(x)dx \underset{\eta\to 0}{\to} 0$.

The conditional expectation over the union of the support sets of actions in E is therefore $\frac{\int_D xf(x)dx}{F(D\cup C)} + \frac{\int_C xf(x)dx}{F(D\cup C)}$. But $\frac{\int_C xf(x)dx}{F(D\cup C)} \underset{\eta\to 0}{\to} 0$, and, when $\eta \to 0$, D converges to the set of states of mass δ that is to the east of a line parallel to the y-axis. Thus, since the distribution is proper, $\frac{\int_D xf(x)dx}{F(D\cup C)} \underset{\eta\to 0}{\to} k$ for some finite k. This implies that there exists an η' such that for all $\eta < \eta'$, the conditional expectation over the support sets of actions in E is actually inside the $\eta-$square, a contradiction. Note that our assumptions about which types support actions in E constitute the worst case scenario and thus a contradiction will be reached for any other assumption. Finally, the same exercise can be applied to other parts outside the square.$\square$

In the next step we use the following Lemma.

LEMMA 1 *Consider any set C with a strictly positive measure, γ. Let l^C be any line separating $\Re^2$ into two regions, one containing C. The distance between the expectation over C, $E(c)$, to l^C is bounded from below by a strictly positive number.*

Proof: For any set C, define the width of C to be the infimum of the shortest side of any rectangle containing C. When there is no such rectangle, let the width of C be infinite. As F is atomless, the width of any C with a measure of $\gamma > 0$ is bounded below by some $\lambda(\gamma) > 0$.

Fix a set C and divide it into two equal measured subsets by a line with the same slope of l^C. Denote the two subsets by U and D, and note that $E[C] = \frac{1}{2}E[U] + \frac{1}{2}E[D]$. Let p be the closest point on l^C to $E(C)$. Either $E(U)$ or $E(D)$ are distanced by at least $\lambda(\frac{\gamma}{2})$ from p. Thus, $E[C]$ must be distanced from p by at least $\frac{\lambda(\frac{\gamma}{2})}{2} > 0.\square$

Step 2 For any $\zeta > 0$ there exists $\bar{\eta} > 0$ and $\bar{b}$, such that for all $\eta < \bar{\eta}$ and $b > \bar{b}$, for each θ in the border of $S(a)$ for a in the $\eta-$square, $d(\theta, l) < \zeta$ for some $l \in L$.

Proof of step 2: For any $\eta-$square, take any θ (in the $\eta-$square) which is in a border of $S(\tilde{a})$ for some action $\tilde{a}$ in the $\eta-$square. Denote by l^θ a line parallel to the $x-$axis that passes through θ. We will show that by choosing a small enough η and a large enough b, the union of the support sets of actions in the $\eta-$square and above l^θ (or below l^θ) coincides with the set of all sender types above l^θ (or below l^θ), up to a small measure.

First, we show that for any $\delta > 0$, there is a η' and a b' such that for all $b > b'$ and $\eta < \eta'$, the measure of all those above (below) l^θ, and in the $\eta-$square, that are in support sets of actions below (above) l^θ, and in the $\eta-$square, is bounded by δ. Take a type θ' above l^θ that supports an action $a' \neq \tilde{a}$ that is below l^θ. The curve of types who are indifferent between $\tilde{a}$ and a' passes in between θ and θ', or in other words, it must pass through or above θ (where only sender types above this curve support a'). As $\eta \to 0$ and $b \to \infty$, such a curve converges to a line in $\Re^2$ parallel to the $x-$axis. As F is proper and continuous and by Lemma 1, the measure of types above l^θ who support a' must converge to zero. Moreover, this convergence is uniform (since the bounds in Lemma 1 and Proposition 1 do not depend on the specifics of equilibria).

Consider now types that support the action $\tilde{a}$. By the definition of the border there exists a type θ'' very near θ that supports some other action a''. Thus the curve of types who are indifferent between $\tilde{a}$ and a'' passes between θ and θ''. By Proposition 1, for a large enough b this curve must be very close to l^{θ}, and hence a small measure of types can be locked in between l^{θ} and this curve. Thus, l^{θ} divides the $\eta-$square so that there are at most a measure δ of types above (below) l^{θ} that support actions below (above) l^{θ}.

Second, by Step 1, we can choose $\bar{\eta} < \eta'$ such that there is at most a measure δ of types in the $\eta-$square that support actions outside the square. Finally, we can choose $\bar{\eta} < \delta$ so that there at most a measure δ of types outside the $\eta-$square (and hence supporting actions inside the $\eta-$square).

We can now prove the statement of Step 2. Take all the actions in the $\eta-$square and above (or below) l^{θ}. The expectation over these actions converges to the y-axis by Proposition 1. The union of the support sets of these actions coincides with the set of types above l^{θ} in $\Re^2$ up to a measure of 4δ (who are either those from above l^{θ} that support actions below l^{θ} and vice versa, or those above l^{θ} supporting actions outside the $\eta-$square and vice versa). As F is proper and continuous, by making δ small enough, l^{θ} converges (uniformly) to some $l \in L$. This implies that for a large enough b, $d(\theta, l) < \zeta$.$\square$

Step 3 If $\gamma(y)$ satisfies the $k-$crossing property then $|L| \leq k - 1$.

Proof of Step 3: Note that there cannot be two lines l and l' that are between two neighboring crossings. If there exist two such lines, then the x-coordinate of the conditional expectation over the subset of the state space bounded by these two lines is either positive or negative. However, by construction, the x-coordinate of the conditional expectation both above l and above l' is zero, implying that it has to be so also for the state space bounded between them, a contradiction. A similar argument applies to show that there cannot be any line (weakly) below the first crossing or above the $k-$th crossing.□

We can now complete the proof. By Step 1, for any ε, there exists $\tilde{\eta}$ such that for all $\eta \leq \tilde{\eta}$ the measure of types who support actions outside the $\eta-$square is at most $\varepsilon/3$. By Step 3, the set L is finite and thus, by Step 2, for any ε, there exists an $\bar{\eta} < \varepsilon$ and a $\bar{b}(\varepsilon)$ such that for all $\eta < \bar{\eta}$ and $b > \bar{b}(\varepsilon)$, the total measure of types who are locked between some border and some line $l \in L$ to which this border converges to, is less than $\varepsilon/3$. Moreover, there are at most k actions whose measure of support sets does not vanish to zero. Therefore, for any ε, we can take an $\eta-$square for $\eta < \min\{\tilde{\eta}, \bar{\eta}, \varepsilon/3\}$, and hence there exists $\bar{b}(\varepsilon)$ such that for all $b > \bar{b}(\varepsilon)$, in any equilibrium, the measure of those who support the k actions in the $\eta-$square (if k of them exist) is at least $1 - \eta - 2(\varepsilon/3) > 1 - \varepsilon$.■

4. Discussion

4.1. The existence and characteristics of equilibria with communication

Restricting attention to large conflicts allows us to derive a necessary condition on equilibria. We derive this condition by the requirement that the receiver will not learn any information on the "dimension of conflict". For finite b, the conditional expectations of the receiver on other dimensions are also important. In this section we provide sufficient conditions for the existence of informative equilibria.

To do so, we consider an environment with symmetric preferences, i.e., let $b_x = b_y$ and $\alpha_x = \alpha_y$. In this environment, the slope of agreement is -1 and that of the conflict is 1. An important property of this environment is that the set of sender types who are indifferent between any two actions on the dimension of agreement is actually a line (for all p). This line has the slope of the conflict and moreover, all sender types on this line are equidistant to the two actions.

We can now characterize a sufficient condition for equilibrium with communication. Divide the state space by a finite collection of lines with a slope 1. Suppose that we find a prior distribution F such that the conditional expectations on each subset of the state space bounded by any two neighboring lines satisfy the following two requirements. First, they are all on the line with a slope -1 that passes through the prior expectation. Second, any two neighboring conditional expectations are equidistant to the line passing between them.

The above collection of lines represents an equilibrium for any level of conflict; each subset of the state space bounded by any two neighboring lines is a set of sender types who send some particular message. The first condition implies that the first necessary condition of Proposition 1 is never violated. The second condition implies that the sender types on each line are indeed indifferent between any two neighboring actions.

In a paper which is complementary to ours, Chakraborty and Harbaugh (2005, henceforth CH), among other results, have shown that for some environments equilibria with meaningful communication exist for all levels of conflict.[9] There exists a

[9] For this result, CH assume that the sender's preferences and the prior distribution are symmetric

family of environments that is analyzed in both papers.[10] Translated to our framework, their assumptions about the symmetry of preferences and the prior distribution imply that $b_x = b_y$, $\alpha_x = \alpha_y$ and that F is symmetric with respect to the x and y dimensions.

The equilibria that CH analyze, "Comparative Cheap Talk" equilibria, are replicated in our framework by the above construction where the collection of lines is a singleton. The symmetry of F implies that if one chooses the 45 degree line that passes through the prior expectation, the two conditions above will be satisfied.[11] The above discussion shows that the existence of such equilibria may be extended to other environments not covered in CH. In particular one can easily construct examples in which F is not symmetric or in which there are more actions in equilibrium (as above, one can construct environments that would yield as many actions in equilibrium as there are crossings).

But can one construct similar informative equilibria more generally? Falling short of answering this question we can use Theorem 1 to gain some insights about the level of communication in other equilibria. First, if the environment satisfies the one-crossing property, these types of equilibria cannot be constructed (for all levels of conflict). Moreover, when the conflict is large, the receiver will take almost surely only one action in equilibrium. Second, if the environment satisfies the two-crossing property, Theorem 1 implies that for large conflicts, the "Comparative Cheap Talk" equilibria achieve the upper bound on the number of actions in equilibrium. Finally,

across the different dimensions.

[10] Given symmetry of preferences, the set of preferences analyzed in CH is richer than ours.

[11] CH also show that these equilibria are robust to small asymmetries.

if there are more than two crossings, there could potentially be equilibria with more than two meaningful actions.

4.2 Many senders Battaglini (2002) analyzes a multidimensional cheap talk game with many senders. In this model, two (or more) senders are perfectly informed about a multidimensional state of the world. He constructs a fully informative equilibrium which does not depend on the magnitude of the conflict between the receiver and each sender. In this equilibrium, each sender reveals information on the dimension upon which she has agreement with the receiver. Together, these two dimensions may span the whole state space and hence all information is aggregated.

As we have shown, an obstacle to information revelation is the lack of commitment on behalf of the receiver to choose actions only on the "dimension of agreement". It is now easy to see that in the model with multiple senders, such commitment may arise endogenously; when both senders know the state of the world, one sender allows the receiver to commit vis-à-vis the second sender to take actions on this particular dimension of agreement.

This interpretation of the equilibrium highlights the importance of the assumptions about the information structure. We now illustrate that it is important that the two senders have perfect information about the state of the world.[12] Suppose that the two senders receive the same imperfect signal s about θ. Suppose that sender 1 has a conflict with the receiver only on the $x-$axis and that sender 2 has a conflict with the receiver only on the $y-$axis. In the equilibrium proposed in Battaglini (2002), each sender reveals one dimension of s, about which he has an agreement with the

[12] It can also be shown that it is important that the senders have the *same* information.

receiver. Thus, sender 1 reveals s_y and sender 2 reveals s_x. However, the existence of this equilibrium depends on the assumptions about the prior and the structure of signals.[13] Assume for example that the random variables θ_x, θ_y, s_x and s_y are affiliated. Recall that by Proposition 1, when the conflict is large, all s types of sender 1 prefer the equilibrium action with the highest $x-$coordinate. However, by affiliation, the receiver's action on the x dimension is monotone in the message of sender 1 about s_y. As a result, sender 1 will not transmit information truthfully and the equilibrium cannot hold.

4.3 Multidimensional versus one dimensional cheap talk Our result echoes the one in Crawford and Sobel (1982), in which large conflicts impose limits on communication. However, the reason why there is an upper bound on information transmission is different. One manifestation of this, is that commitment is useful in the multidimensional policy space. For example, in environments which satisfy the one-crossing property, all equilibria converge to the babbling equilibrium for high levels of conflict. On the other hand, when the receiver can commit, some information can be transmitted in equilibrium.[14] This is in contrast to the unidimensional model. In that model, when the conflict is large, the receiver cannot increase his utility by exercising commitment.[15]

[13]Battaglini (2004) analyzes the case of imperfect information with an assumption that the prior distribution is the non-proper uniform distribution.

[14]In our simplified model, all information would be transmitted on any line parallel to the $y-$axis if the receiver can commit to take actions on such a line, for all values of b.

[15]When conflicts are small, Dessein (2000) shows that commitment is useful in the unidimensional policy space.

Our analysis can shed light on whether there is more scope for information transmission in a multidimensional setting compared with a unidimensional setting.[16]. Specifically, Theorem 1 has unveiled that multidimensional environments may yield less information in equilibrium relative to separate unidimensional models. We show that full information revelation on the $y-$dimension cannot arise when the $k-$crossing property is satisfied. In a one dimensional model, on the other hand, all information can be transmitted on the $y-$dimension, about which there is no conflict between the sender and the receiver.

Another simple example does not depend on the assumption of large conflicts. When the two dimensions are perfectly correlated, and the component of the conflict vector on the $x-$axis is larger than that on the $y-$axis, the multidimensional setting may yield less information as well. In this situation, communication on the two dimensions will be constrained by the larger component of the conflict. On the other hand, separating the two dimensions will allow for more communication.

More research is needed to establish a more general theory of the comparison between the levels of communication in a unidimensional versus a multidimensional setting. Our analysis indicates that for all levels of conflict, what is important in such a theory is the relation between players' prior beliefs and preferences.

[16] Bolton, Freixas and Shapiro (2004) discuss this question in the context of the Glass-Steagall act.

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