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**Working paper**

**Original citation:**

Michailat, Pascal (2012) *A theory of countercyclical government multiplier*. Working paper series, Social Science Research Network.

Originally available from [Social Science Research Network](#)

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Available in LSE Research Online: November 2013

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# A Theory of Countercyclical Government Multiplier

By PASCAL MICHAILLAT\*

*I develop a New Keynesian model in which a type of government multiplier doubles when unemployment rises from 5 percent to 8 percent. This multiplier indicates the additional number of workers employed when one worker is hired in the public sector. Graphically, in equilibrium, an upward-sloping quasi-labor supply intersects a downward-sloping labor demand in a (employment, labor market tightness) plane. Increasing public employment stimulates labor demand, which increases tightness and therefore crowds out private employment. Critically, the quasi-labor supply is convex. Hence, when labor demand is depressed and unemployment is high, the increase in tightness and resulting crowding-out are small.*

*JEL: E24, E32, E62*

A recent literature has argued that the effect of government policy can be different across stages of the business cycle if the zero lower bound on nominal interest rates is reached during a recession.<sup>1</sup> In this paper, I present a New Keynesian model in which the effect of government policy varies across stages of the business cycle even when the zero lower bound does not bind.<sup>2</sup> I consider a policy in which the government increases the size of the public-sector workforce. I measure the effect of this policy with the public-employment multiplier, defined as the additional number of workers employed when one more worker is employed in the public sector. I find that this multiplier doubles when the unemployment rate rises from 5 percent to 8 percent.

In the model, the effect of government policy varies over the business cycle because of the structure of the labor market, adapted from the search-and-matching framework of Michaillat (2012). Since government and firms hire from the same pool of jobseekers, increasing public employment crowds out private employment. To increase public employment, the government posts additional vacancies. Furthermore, increasing public employment mechanically reduces the number of jobseekers. Therefore, increasing pub-

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<sup>1</sup>See for instance Christiano, Eichenbaum and Rebelo (2011) and Eggertsson and Krugman (2012).

<sup>2</sup>Canzoneri et al. (2012) propose an alternative model in which the effect of government policy varies over the business cycle. Their model features costly financial intermediation and countercyclical financial frictions.

lic employment raises labor market tightness—the number of vacancies per jobseeker—and makes it more costly for firms to hire workers, thus reducing private employment. The government policy reduces unemployment more effectively in recession than in expansion because crowding-out is weaker then. The extent of crowding-out is determined by the amplitude of this increase in labor market tightness. When unemployment is high, the government needs few vacancies to hire additional workers because the matching process is congested by jobseekers; moreover, the number of jobseekers is so large that the vacancies posted and jobseekers hired by the government have little influence on tightness. Consequently, the increase in tightness is small and crowding-out is weak. The same mechanism leads to strong crowding-out when unemployment is low and the matching process is congested by vacancies.

In Section I, I develop a simple search-and-matching model to highlight the key economic forces that drive the results. I do comparative steady-states exercises because they are transparent: they provide an analytical expression for the public-employment multiplier and can be represented diagrammatically. Indeed, the steady-state equilibrium is the intersection of an upward-sloping, convex quasi-labor supply curve and a downward-sloping aggregate labor demand curve in a (employment, labor market tightness) plane. The quasi-labor supply is the employment rate when labor market flows are balanced, and the aggregate labor demand is firms' labor demand plus public employment. The properties of the curves arise from a standard matching function and a production function with diminishing marginal returns to labor. I first compare a steady state to another steady state with one more public worker. The difference in aggregate employment is the public-employment multiplier. I find that the multiplier is between 0 and 1. In the diagram, the aggregate labor demand curve shifts outward when public employment is higher, leading to higher employment and higher tightness. Next, I compare a steady state to another steady state with a higher wage and thus higher unemployment. I find that the multiplier is higher when wage and unemployment are higher. In the diagram, the aggregate labor demand curve shifts inward when the wage is higher, and the convex quasi-labor supply curve is flatter at the equilibrium point. Thus, increasing public employment leads to a smaller increase in tightness and a larger increase in employment.

In Section II, I embed the search-and-matching model into a New Keynesian model. I simulate the responses to a range of technology shocks. Unemployment rises after negative shocks because the real wage is somewhat rigid. I compare the response of employment when the government hires additional public workers after the shock and when it does not. The resulting public-employment multiplier doubles from 0.24 to 0.49 when the unemployment rate rises from 5 percent to 8 percent.

The public-employment multiplier is a type of government-consumption multiplier in that it measures the macroeconomic effect of an increase in government consumption. It is not the typical government-consumption multiplier, which measures the response of output to an increase in government purchases of goods from the private sector. It is nonetheless relevant for policy.<sup>3</sup> First, on average from 1947 to 2011 in the US, public

<sup>3</sup>Public employment has long been recognized as an important component of government consumption. See for instance Kahn (1931), Finn (1998), Cavallo (2005), and Pappa (2009).

employment represents 63 percent of government consumption, whereas purchases of goods from the private sector represent only 37 percent.<sup>4</sup> Second, public employment has been used to stimulate employment in recession, for instance during the Great Depression by the Roosevelt administration (Neumann, Fishback and Kantor, 2010).

In the model, hiring workers in the public sector is more effective when unemployment is higher; furthermore, hiring public workers always reduces unemployment whereas purchasing goods from the private sector has no effect on unemployment. These results suggest that empirical estimates of multipliers obtained by averaging the effects of government spending over the business cycle may not apply in recession; moreover, estimates obtained by averaging the effects of different types of government spending may not apply to a particular type of spending. Section III discusses such implications of the model, and some to its shortcomings.

## I. Comparative Steady-States Analysis of the Multiplier

This section builds a simple model that adds public employment to the search-and-matching framework of Michaillat (2012).<sup>5</sup> Comparative steady states show that the public-employment multiplier is always between 0 and 1, and that it is closer to 1 when the unemployment rate is higher. Section II complements these analytical results with numerical results obtained by simulating a temporary increase in public employment at different stages of the business cycle with a New Keynesian model.

### A. A Search-and-Matching Model

LABOR MARKET. — A measure 1 of identical workers participate in a labor market composed of two sectors. The government employs  $g_t < 1$  workers in the public sector. A measure 1 of identical firms employ  $l_t$  workers in the private sector. Aggregate employment is  $n_t = l_t + g_t$ . At the end of period  $t - 1$ , a fraction  $s$  of the  $n_{t-1}$  existing worker-job matches is exogenously destroyed. Workers who lose their job start searching for a new job immediately. At the beginning of period  $t$ ,  $u_t = 1 - (1 - s) \cdot n_{t-1}$  unemployed workers search for a job. Jobseekers apply to jobs randomly, without directing their search to the private or public sector. Jobseekers who find a job start working in period  $t$  with the  $(1 - s) \cdot n_{t-1}$  incumbent workers.

Firms and the government post a total of  $v_t$  vacancies to hire workers. The number of matches in period  $t$  is given by a Cobb-Douglas matching function:  $h_t = m \cdot u_t^\eta \cdot v_t^{1-\eta}$ . The parameter  $m > 0$  measures the effectiveness of matching, and  $\eta \in (0, 1)$  is the elasticity of the matching function with respect to unemployment. Let  $\theta_t \equiv v_t/u_t$  be the

<sup>4</sup>In national accounts, government consumption is the cost of producing the services provided by the government, such as education or health care. Table 3.10.5 in the National Income and Product Accounts, titled “Government Consumption Expenditures and General Government Gross Output,” shows that 55 percent of these production costs are compensations of public employees, 33 percent are purchases of intermediate goods and services from the private sector, and 12 percent is consumption of government fixed capital, an imputed rent on capital from which I abstract.

<sup>5</sup>For other search-and-matching models with public employment, see Quadrini and Trigari (2007), Gomes (2010), and Burdett (2012).

labor market tightness. Jobseekers find a job with probability  $f(\theta_t) = h_t/u_t = m \cdot \theta_t^{1-\eta}$ , and vacancies in both sectors are filled at the same rate  $q(\theta_t) = h_t/v_t = m \cdot \theta_t^{-\eta}$ .<sup>6</sup>

**LARGE HOUSEHOLD.** — All workers belong to a large household that consumes a final good and a public good. The household's time discount factor is  $\beta < 1$ . The final good is purchased from firms. The public good is provided free of charge by the government. The household finances its consumption of final good with its income. Employed workers receive a real wage  $w_t$  taxed at rate  $\tau_t$ , unemployed workers receive no income, and firms distribute their real profits  $T_t$  to the household because it owns them. Hence, the household's consumption of final good is  $c_t = (1 - \tau_t) \cdot w_t \cdot n_t + T_t$ .

Given the matching process on the labor market, the household's employment rate is

$$(1) \quad n_t = (1 - s) \cdot n_{t-1} + [1 - (1 - s) \cdot n_{t-1}] \cdot f(\theta_t).$$

In steady state, inflows to unemployment,  $s \cdot n$ , equal outflows from unemployment,  $[1 - (1 - s) \cdot n] \cdot f(\theta)$ , and the employment rate is a function of labor market tightness given by

$$(2) \quad n^s(\theta) = \frac{f(\theta)}{s + (1 - s) \cdot f(\theta)}.$$

I refer to this function as quasi-labor supply. It translates the search decision of workers into the employment rate that prevails when the labor market is in steady state. Of course, in this model there is no active search decision: workers' search effort is exogenously set to 1. But it is easy to endogenize the search decision as a function of the flow values of work and unemployment. In that case, the quasi-labor supply incorporates the optimal search choice and translates it into a steady-state employment rate.<sup>7</sup> The quasi-labor supply is therefore similar to a conventional labor supply in that it gives the quantity of labor arising from workers' optimal choice based on prevailing economic conditions, especially the return of work relative to non-work (leisure or job search). However, there is one difference between the two concepts of labor supply. A conventional labor supply indicates directly workers' optimal employment choice (number of hours, or number of workers with indivisible labor). But in presence of matching frictions, workers cannot directly choose how much they work; they can only choose how much they search for jobs when they are unemployed. Therefore, the quasi-labor supply indicates the steady-state employment rate that prevails when workers' search choice is optimal.

Lemma 1 establishes a few properties of the quasi-labor supply:

**LEMMA 1:** *The function  $n^s(\theta)$  is strictly increasing, strictly concave,  $\lim_{\theta \rightarrow +\infty} n^s(\theta) = 1$ , and  $\lim_{\theta \rightarrow 0} n^s(\theta) = 0$ .*

<sup>6</sup>Formally, the definition of the matching function includes the restriction that  $h_t \leq u_t$ , which leads to the restriction that  $f(\theta_t) \leq 1$ . I suppress these restrictions for notational convenience.

<sup>7</sup>See Landais, Michailat and Saez (2010).

The proof follows from the properties of  $f(\theta)$ . The lemma says that when labor market flows are balanced and labor market tightness is high, employment is high. The reason is that jobseekers find jobs quickly when tightness is high.

**FIRMS.** — Firms produce a final good and sell it on a perfectly competitive market. A representative firm uses labor  $l_t$  to produce output  $y_t$  according to the production function  $y_t = l_t^\alpha$ , where  $\alpha \in (0, 1)$  measures diminishing marginal returns to labor.

The firm pays a real wage  $w_t$  to its employees. In addition, the firm incurs a cost to hire workers. In period  $t$ , the firm hires  $l_t - (1 - s) \cdot l_{t-1}$  workers. Posting a vacancy for one period costs  $r$  units of final good, where  $r > 0$  measures resources spent recruiting workers. I assume no randomness at the firm level: a firm hires a worker with certainty by opening  $1/q(\theta_t)$  vacancies and spending  $r/q(\theta_t)$ . Hence, the firm's real profits in period  $t$  are

$$l_t^\alpha - w_t \cdot l_t - \frac{r}{q(\theta_t)} \cdot [l_t - (1 - s) \cdot l_{t-1}].$$

Given  $\{\theta_t\}_{t=0}^{+\infty}$  and  $\{w_t\}_{t=0}^{+\infty}$ , the firm chooses  $\{l_t\}_{t=0}^{+\infty}$  to maximize the discounted sum of real profits. In steady state, the optimal employment choice satisfies

$$(3) \quad \alpha \cdot l^{\alpha-1} = w + [1 - \beta \cdot (1 - s)] \cdot \frac{r}{q(\theta)}.$$

The firm hires labor until the marginal product of labor,  $\alpha \cdot l^{\alpha-1}$ , equals the marginal cost of labor, which is the sum of the real wage,  $w$ , plus the amortized hiring cost,  $[1 - \beta \cdot (1 - s)] \cdot r/q(\theta)$ . The firm's labor demand is the employment level that solves (3), expressed as function of labor market tightness and real wage:

$$(4) \quad l^d(\theta, w) = \left[ \frac{1}{\alpha} \cdot \left\{ w + [1 - \beta \cdot (1 - s)] \cdot \frac{r}{q(\theta)} \right\} \right]^{\frac{-1}{1-\alpha}}.$$

The aggregate labor demand is the sum of firms' labor demand and public employment, expressed a function of labor market tightness, real wage, and public employment:

$$(5) \quad n^d(\theta, w, g) = g + l^d(\theta, w).$$

Lemma 2 establishes a few properties of the aggregate labor demand:

**LEMMA 2:** *The function  $n^d(\theta, w, g)$  is strictly decreasing in  $\theta$  and  $w$ ,  $\lim_{\theta \rightarrow +\infty} n^d(\theta, w, g) = g$ , and  $\lim_{\theta \rightarrow 0} n^d(\theta, w, g) = n^*$ , where  $n^* = g + [w/\alpha]^{\frac{-1}{1-\alpha}}$ .*

The proof follows from the properties of  $q(\theta)$ . The lemma says that firms' desired employment is low when the real wage or labor market tightness are high. The reason is

simple: when the real wage is high, the marginal cost of labor is high; and when tightness is high, the hiring cost is high so the marginal cost of labor is high as well. The quantity  $\min\{1, n^*\}$  is the employment rate that prevails if the recruiting cost,  $r$ , is 0. If  $n^* < 1$ , then the labor market does not converge to full employment when the recruiting cost converges to 0: jobs are rationed in the sense of Michaillat (2012). Here, jobs are rationed if  $w > \alpha \cdot (1 - g)^{\alpha-1}$ .

REAL WAGE. — As in Hall (2005), I assume that the real wage  $w_t$  is exogenous. Assuming an exogenous real wage is one way to resolve the indeterminacy of wages that arises in search-and-matching models because worker and firm determine the wage in a situation of bilateral monopoly.<sup>8</sup> This situation arises because worker and firm must share a positive surplus, created by their matching. The positive surplus arises because the firm's marginal product of labor always exceeds the worker's flow value of unemployment when they match. In the steady-state equilibrium, firms' hiring decisions impose that the real wage falls between the marginal product of labor and the flow value of unemployment; therefore, the real wage is necessarily pairwise Pareto efficient.

GOVERNMENT. — The government employs  $g_t$  workers that produce a public good  $z_t$  according to the production function  $z_t = \sigma \cdot g_t^\alpha$ , where  $\sigma > 0$  scales the productivity of the government relative to that of firms. The government balances its budget each period. Government expenditures are the compensations of public workers paid at the private-sector wage,  $g_t \cdot w_t$ , and the cost incurred by hiring public workers,  $[g_t - (1 - s) \cdot g_{t-1}] \cdot r/q(\theta_t)$ . To finance the expenditures, the government levies a labor tax that yields  $\tau_t \cdot w_t \cdot n_t$ .

### B. Steady-State Equilibrium

This section solves for the steady-state equilibrium of the model taking as given the values  $w$  and  $g$  of the real wage and public employment. The equilibrium consists of two endogenous variables, aggregate employment  $n$  and labor market tightness  $\theta$ . Equilibrium labor market tightness equalizes quasi-labor supply to aggregate labor demand:

$$(6) \quad n^s(\theta) = n^d(\theta, w, g).$$

Equilibrium employment is obtained from the aggregate labor demand:

$$(7) \quad n = n^d(\theta, w, g),$$

<sup>8</sup>The indeterminacy of the wage in a situation of bilateral monopoly was first highlighted by Edgeworth (1881) and was discussed by Howitt and McAfee (1987), Pissarides (1989), and Hall (2005) in the context of search-and-matching models. Since the work of Diamond (1982) and Mortensen (1982), the common way to resolve the indeterminacy in search-and-matching models is to set the wage using the Nash bargaining solution.

where  $\theta$  satisfies (6). Lemmas 1 and 2 imply that the equilibrium exists and is unique. Figure 1(a) depicts the equilibrium in a  $(n, \theta)$  plane.<sup>9</sup> The quasi-labor supply curve is upward sloping and convex. The aggregate labor demand curve is downward sloping. The aggregate labor demand curve intersects the x-axis at  $n^*$ . Quasi-labor supply curve and aggregate labor demand curve intersect at the equilibrium point.

Equilibrium is reached through posting of vacancies. Imagine that aggregate labor demand is greater than quasi-labor supply. Then, firms and government post additional vacancies to hire more workers. On the one hand, more jobseekers find a job. On the other hand, the vacancy-filling rate falls such that the hiring cost rises and the number of workers desired by firms falls. As a result, the gap between demand and supply diminishes. Firms and government post more and more vacancies until the gap between demand and supply is completely closed.

### C. Low-Unemployment and High-Unemployment Steady States

This section studies steady-state equilibria that differ by the value  $w$  of the real wage. This comparative steady-states exercise is useful because in the context of business cycles generated by a combination of technology shocks and real wage rigidity (as in Section II), the key difference between expansions and recessions corresponds to a difference in  $w$ . To simplify the derivations, I assume that a change in the value  $w$  of the real wage is mechanically accompanied by a change in the value  $g$  of public employment. More precisely, I assume that across steady-state equilibria parameterized by different  $w$ , the ratio of public employment to private employment is constant:  $g/l = \zeta$ , where  $\zeta > 0$  is a parameter.<sup>10</sup>

Lemma 3 establishes how the labor market changes across steady-state equilibria parameterized by different  $w$ :

**LEMMA 3:** *The labor market variables satisfy  $d\theta/dw < 0$ ,  $dl/dw < 0$ ,  $dn/dw < 0$ , and  $du/dw > 0$ .*

The lemma says that in a steady state in which the real wage is high, labor market tightness, private employment, and aggregate employment are low, and unemployment is high. Appendix A contains the proof of the lemma, but the main idea can be seen graphically by comparing the steady state with high wage depicted in Figure 1(b) to the steady state with low wage depicted in Figure 1(a). In the high-wage steady state, the aggregate labor demand is depressed. Accordingly, the aggregate labor demand curve is located inward. Hence, in equilibrium, labor market tightness and employment are low, and unemployment is high. The high-wage steady state mimics a recession, and conversely, the low-wage steady state mimics an expansion.

<sup>9</sup>Landais, Michaillat and Saez (2010) introduced this representation to study optimal unemployment insurance.

<sup>10</sup>This assumption is perhaps unrealistic, and I will relax it in the simulations in Section II.



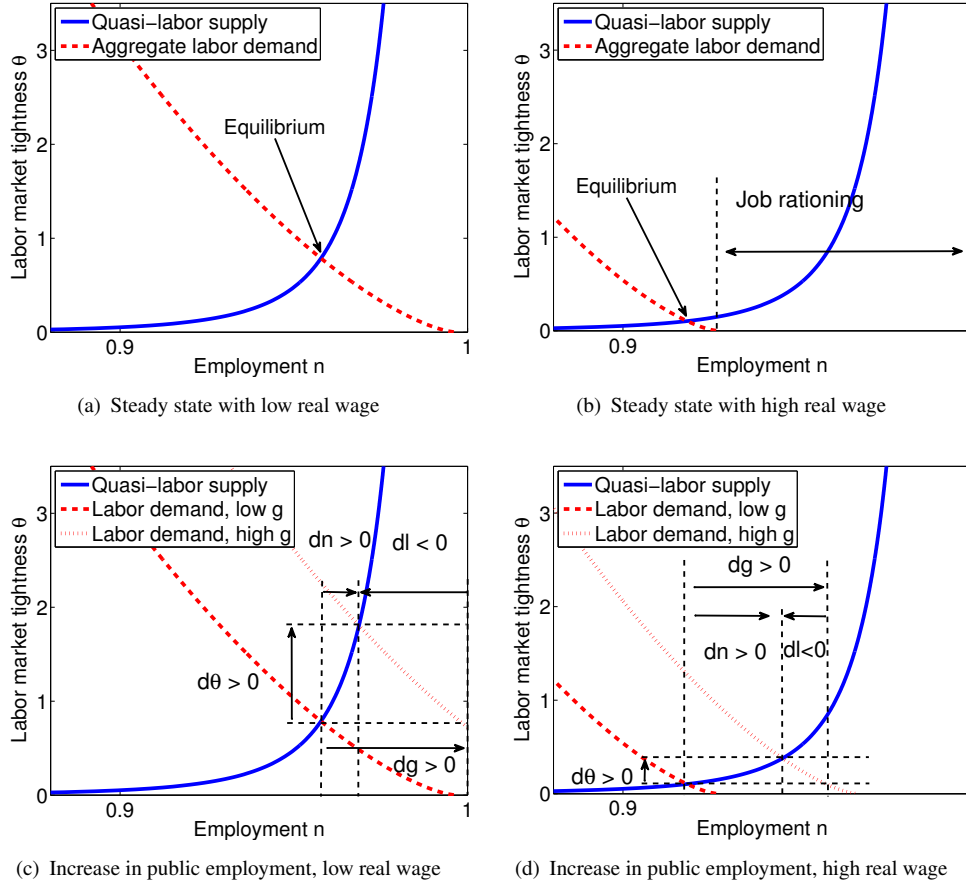


FIGURE 1. STEADY-STATE EQUILIBRIA IN THE SEARCH-AND-MATCHING MODEL

#### D. Public-Employment Multiplier

In this section, I take the economy in a steady state, increase the value  $g$  of public employment, compute the new steady state, and compare employment in the two steady states. Then, I study how the change in employment depends on the value  $w$  of the real wage in the initial steady state.

I measure the difference in employment between the two steady states by a multiplier:

**DEFINITION 1:** *The public-employment multiplier  $\lambda$  is the additional number of workers employed when one additional worker is employed in the public sector:*

$$\lambda \equiv \frac{\partial n}{\partial g}.$$

Proposition 1 establishes some properties of the public-employment multiplier:

**PROPOSITION 1:** *The public-employment multiplier  $\lambda$  satisfies three properties: (i)  $\lambda < 1$ ; (ii)  $\lambda > 0$ ; and (iii)  $d\lambda/dw > 0$ .*

Part (i) shows that the multiplier is less than 1. In other words, increasing public employment necessarily crowds out private employment. This result is illustrated in Figures 1(c) and 1(d). After an increase in public employment, the aggregate labor demand curve shifts outward. At the current tightness, the quasi-labor supply falls short of the aggregate labor demand. To reach a new equilibrium, tightness increases. Thus, the vacancy-filling rate falls and the hiring cost rises. As a consequence, firms reduce employment.

Part (ii) shows that the multiplier is positive. In other words, increasing public employment crowds out private employment strictly less than one-for-one, and it necessarily stimulates aggregate employment. If crowding-out were one-for-one, the new equilibrium would have the same labor market tightness but lower private employment. The marginal cost of labor would be the same, but the marginal product of labor would be higher by diminishing marginal returns to labor; therefore, the firm's optimal employment choice would be violated. To conclude, crowding-out is strictly less than one-for-one.

Part (iii) shows that the multiplier is higher in steady states in which the real wage is higher. In other words, in steady states in which the aggregate labor demand is weaker and unemployment is higher, crowding-out is weaker, and increasing public employment reduces unemployment more effectively. This result is illustrated by comparing the high-wage steady state in Figure 1(d) to the low-wage steady state in Figure 1(c). In the high-wage steady-state, the quasi-labor supply is flat at the equilibrium point. Thus, a shift in the aggregate labor demand curve following an increase in public employment leads to a small increase in tightness and a large increase in employment. That is, crowding-out is weak and the multiplier is large. On the contrary, in the low-wage steady-state, the quasi-labor supply is steep at the equilibrium point. Thus, the shift in the aggregate labor demand curve leads to a large increase in tightness and a small increase in employment. That is, crowding-out is strong and the multiplier is small.

Part (iii) can also be explained by thinking directly about vacancies and the matching process. To increase public employment, the government posts additional vacancies. Furthermore, increasing public employment mechanically reduces the number of jobseekers. Therefore, increasing public employment raises labor market tightness—the number of vacancies per jobseeker. The extent of crowding-out is determined by the amplitude of this increase in labor market tightness. When unemployment is high, the government needs few vacancies to hire additional workers because the matching process is congested by jobseekers; moreover, the number of jobseekers is so large that the vacancies posted and jobseekers hired by the government have little influence on tightness. Consequently, the increase in tightness is small and crowding-out is weak. On the contrary, when unemployment is low, the government needs many vacancies to hire additional workers because the matching process is congested by vacancies; moreover, the number of jobseekers is small such that the vacancies posted and jobseekers hired by the

government have a large influence on tightness. Consequently, the increase in tightness is large and crowding-out is strong.

The complete proof of the proposition is relegated to Appendix A, but I provide a sketch here. Let  $\epsilon^s \equiv (\theta/n^s) \cdot (\partial n^s/\partial\theta) > 0$  and  $\epsilon^d \equiv -(\theta/n^d) \cdot (\partial n^d/\partial\theta) > 0$  be the elasticities of quasi-labor supply and aggregate labor demand with respect to tightness. (The elasticities are normalized to be positive.) Implicit differentiation of equations (6) and (7) yields

$$(8) \quad \lambda = 1 - \frac{1}{1 + (\epsilon^s/\epsilon^d)}.$$

The increase in aggregate employment equals the increase in public employment attenuated by a factor  $1/[1 + (\epsilon^s/\epsilon^d)]$ . This factor measures the crowding-out of private employment. The proof shows that  $\lambda \in (0, 1)$  because both  $\epsilon^s$  and  $\epsilon^d$  are positive and finite. The proof also shows that  $\epsilon^s$  is proportional to unemployment and  $\epsilon^d$  is proportional to the share of the hiring cost in the marginal cost of labor. Since this share decreases with unemployment, both  $\epsilon^s/\epsilon^d$  and  $\lambda$  are larger when unemployment is higher.

Matching frictions and diminishing marginal returns to labor are critical for the results in Proposition 1. Without matching frictions, the multiplier would be 1 in any steady state. For instance, imagine that there is no recruiting cost ( $r = 0$ ). Then the aggregate labor demand does not depend on  $\theta$ . In Figures 1(c) and 1(d), the aggregate labor demand curve is vertical. There is no crowding-out. As a result, the multiplier is 1. Formally, the aggregate labor demand is inelastic so  $\epsilon^d = 0$ , crowding-out is  $1/[1 + (\epsilon^s/\epsilon^d)] = 0$ , and  $\lambda = 1$ .

With constant returns to labor instead of diminishing marginal returns, the multiplier would be 0 in any steady state. With constant returns to labor ( $\alpha = 1$ ), the firm's optimal employment choice solely determines equilibrium tightness. Combined with the quasi-labor supply, equilibrium tightness determines equilibrium employment independent of public employment. Since aggregate employment is independent of public employment, an increase in public employment must be offset by a commensurate decrease in private employment. As a result, the multiplier is 0. Formally, the aggregate labor demand is perfectly elastic so  $\epsilon^d = +\infty$ , crowding-out is  $1/[1 + (\epsilon^s/\epsilon^d)] = 1$ , and  $\lambda = 0$ .

To conclude, I connect the results in Proposition 1 to the results in Michailat (2012). These sets of results are distinct even though they rely on the same ingredients—variations in real wage and diminishing marginal returns to labor. Proposition 4 in Michailat (2012) establishes a property of the model based on its behavior at the limit where matching frictions vanish. It shows that when the real wage is high enough, the labor market does not converge to full employment when the recruiting cost converges to 0. Figure 1(b) illustrates the property. The aggregate labor demand intersects the x-axis below 1; thus, it is not profitable for firms to hire all the workers even when recruiting is costless—jobs are rationed. On the other hand, Proposition 1 in this paper establishes a property of the model based on the behavior of the slope of the quasi-labor supply, measured by  $\epsilon^s$ , relative to the slope of the aggregate labor demand, measured by  $\epsilon^d$ . The behavior of

$\epsilon^s/\epsilon^d$  determines the behavior of the public-employment multiplier.

## II. Multiplier Dynamics

This section uses simulations to explore the effect of a temporary increase in public employment at different stages of the business cycle. To improve realism, the search-and-matching model of the previous section is embedded into a New Keynesian model. Simulations of this model calibrated to US data confirm the comparative steady-states results: the public-employment multiplier is positive and countercyclical.

### A. A New Keynesian Model

OVERVIEW. — This model embeds the search-and-matching model of Section I; therefore, the model presents three departures from the textbook New Keynesian model. First, since government consumption arises not from purchases of goods from the private sector but from compensations of public employees, government consumption appears not in the resource constraint but in the aggregate labor demand. Second, monopolistic firms are subject not to the price-setting friction of Calvo (1983) but to the quadratic price-adjustment cost of Rotemberg (1982); therefore, the Phillips curve admits a different expression. I introduce a quadratic price-adjustment cost because it yields a closed-form expression for the Phillips curve, which simplifies the simulations.<sup>11</sup> Third, the labor market is not perfectly competitive but adopts a search-and-matching structure.<sup>12</sup> This departure introduces four modifications to the model. First, the labor supply is replaced by the quasi-labor supply. Second, firms' labor demand accounts for hiring cost. Third, the model counts one more variable: labor market tightness, determined by the equality of quasi-labor supply and aggregate labor demand. Fourth, the model counts one more equation: a rule that shares the surplus arising from each worker-firm match and thus determines the real wage.

SHOCK. — Business cycles are driven by technology, modeled as a stochastic process  $\{a_t\}_{t=0}^{+\infty}$ .

LABOR MARKET. — The private sector is now composed of a continuum of intermediate-good firms indexed by  $i \in [0, 1]$ . Firm  $i$  employs  $l_t(i)$  workers. Private employment is  $l_t = \int_0^1 l_t(i) di$ .

<sup>11</sup>Braun, Körber and Waki (2012) also take advantage of the simplicity brought by this price-adjustment cost to compute the equilibrium of a nonlinear model of the zero lower bound. For other New Keynesian models using this price-adjustment cost, see Hairault and Portier (1993), Chéron and Langot (2000), and Krause, Lopez-Salido and Lubik (2008).

<sup>12</sup>Several New Keynesian models add matching frictions to the labor market. See Galí (2010) for an overview.

LARGE HOUSEHOLD. — The large household has expected utility

$$(9) \quad \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t \cdot [\ln(c_t) + \chi \cdot \ln(z_t)],$$

where  $\mathbb{E}_0$  is the expectation conditional on period-0 information, and  $\chi$  measures the taste for public good. Workers pool their income before choosing consumption and saving.<sup>13</sup> The household's budget constraint becomes

$$(10) \quad p_t \cdot c_t + b_t = p_t \cdot n_t \cdot (1 - \tau_t) \cdot w_t + R_{t-1} \cdot b_{t-1} + p_t \cdot T_t,$$

where  $p_t$  is the price level,  $b_t$  is the quantity of one-period bonds purchased at time  $t$ , and  $R_{t-1}$  is the one-period gross nominal interest rate that pays off in period  $t$ . The household chooses consumption  $\{c_t\}_{t=0}^{+\infty}$  to maximize (9) subject to (10) and the no-Ponzi-game constraint

$$\mathbb{E}_0 \left[ \lim_{t \rightarrow +\infty} \frac{b_t}{\prod_{i=0}^t R_{i-1}} \right] \geq 0.$$

Let  $\pi_t \equiv (p_t/p_{t-1}) - 1$  be the inflation rate at time  $t$ . The household's optimal consumption path is governed by the Euler equation

$$(11) \quad 1 = \beta \cdot \mathbb{E}_t \left[ \frac{R_t}{1 + \pi_{t+1}} \cdot \frac{c_t}{c_{t+1}} \right].$$

FINAL-GOOD FIRMS. — A measure 1 of identical firms produce the final good and sell it on a perfectly competitive market. The representative final-good firm uses  $y_t(i)$  units of each intermediate good  $i \in [0, 1]$  to produce  $y_t$  units of final good using the production function

$$y_t = \left[ \int_0^1 y_t(i)^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)},$$

where  $\epsilon > 1$  is the elasticity of substitution across intermediate goods.

The final-good firm takes as given the nominal price  $p_t(i)$  of each intermediate good  $i \in [0, 1]$  and the nominal price  $p_t$  of the final good. The firm chooses  $y_t(i)$  for all  $i \in [0, 1]$  to maximize its profits

$$p_t \cdot \left[ \int_0^1 y_t(i)^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)} - \int_0^1 p_t(i) \cdot y_t(i) di.$$

<sup>13</sup>This formulation is standard since Merz (1995). It avoids the complications that would arise if workers had heterogeneous wealth levels that depended on their employment histories.

The first-order condition with respect to  $y_t(i)$  is

$$(12) \quad y_t(i) = y_t \cdot \left( \frac{p_t(i)}{p_t} \right)^{-\epsilon}.$$

This equation describes the demand for intermediate good  $i$  as a function of the relative price  $p_t(i)/p_t$ .

Perfect competition in the final-good market requires that the price of the final good equals its marginal cost of production:

$$p_t = \left( \int_0^1 p_t(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)}.$$

INTERMEDIATE-GOOD FIRMS. — There is no entry or exit into the production of intermediate goods. Intermediate good  $i \in [0, 1]$  is produced by a monopolist. The monopolist uses  $l_t(i)$  units of labor to produce  $y_t(i)$  units of intermediate good  $i$  according to the production function

$$(13) \quad y_t(i) = a_t \cdot l_t(i)^\alpha,$$

where  $a_t$  is the state of technology and  $\alpha \in (0, 1)$  measures diminishing marginal returns to labor.

As in Rotemberg (1982), the monopolist incurs a cost to adjust its nominal price given by

$$\frac{\phi}{2} \cdot \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 \cdot c_t,$$

where  $\phi > 0$  captures resources devoted to adjusting prices. The price-adjustment cost is measured in units of final good, and it increases proportionally with the size of the economy, measured by consumption  $c_t$ . The monopolist also incurs a cost  $r \cdot a_t$  to post a vacancy for one period; therefore, it incurs a total cost  $[l_t(i) - (1-s) \cdot l_{t-1}(i)] \cdot r \cdot a_t/q(\theta_t)$  to hire new workers in period  $t$ . The hiring cost is measured in units of final good, and it increases proportionally with the state of technology  $a_t$ .

The monopolist chooses  $\{l_t(i)\}_{t=0}^{+\infty}$  and  $\{p_t(i)\}_{t=0}^{+\infty}$  to maximize the expected sum of discounted real profits

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \frac{\beta^t}{c_t} \cdot \left\{ \frac{p_t(i)}{p_t} \cdot y_t(i) - w_t \cdot l_t(i) - \frac{\phi}{2} \cdot \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 \cdot c_t - \frac{r \cdot a_t}{q(\theta_t)} \cdot [l_t(i) - (1-s) \cdot l_{t-1}(i)] \right\}$$

subject to (12) and (13). The discount factor  $\beta^t/c_t$  indicates the value of one unit of final good in period  $t$  from the perspective of the household in time 0. The Lagrangian is

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \frac{\beta^t}{c_t} \cdot \left\{ \left( \frac{p_t(i)}{p_t} \right)^{1-\epsilon} \cdot y_t - w_t \cdot l_t(i) - \frac{\phi}{2} \cdot \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 \cdot c_t \right. \\ \left. - \frac{r \cdot a_t}{q(\theta_t)} \cdot [l_t(i) - (1-s) \cdot l_{t-1}(i)] + \Lambda_t(i) \cdot \left[ a_t \cdot l_t(i)^\alpha - \left( \frac{p_t(i)}{p_t} \right)^{-\epsilon} \cdot y_t \right] \right\},$$

where  $\Lambda_t(i)$  is the Lagrange multiplier on constraint (13) in period  $t$ . The multiplier  $\Lambda_t(i)$  is the real marginal revenue of producing one unit of intermediate good  $i$  in period  $t$ . The first-order condition with respect to  $l_t(i)$  is

$$(14) \quad \Lambda_t(i) \cdot \alpha \cdot l_t(i)^{\alpha-1} = \frac{w_t}{a_t} + \frac{r}{q(\theta_t)} - \beta \cdot (1-s) \cdot \mathbb{E}_t \left[ \frac{c_t}{c_{t+1}} \cdot \frac{a_{t+1}}{a_t} \cdot \frac{r}{q(\theta_{t+1})} \right].$$

The first-order condition with respect to  $p_t(i)$  is

$$(15) \quad \frac{p_t(i)}{p_t} = \frac{\epsilon}{\epsilon-1} \cdot \Lambda_t(i) + \frac{\phi}{\epsilon-1} \cdot \frac{c_t}{y_t} \cdot \left( \frac{p_t(i)}{p_t} \right)^\epsilon \\ \cdot \left[ \beta \cdot \mathbb{E}_t \left[ \left( \frac{p_{t+1}(i)}{p_t(i)} - 1 \right) \cdot \frac{p_{t+1}(i)}{p_t(i)} \right] - \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right) \cdot \frac{p_t(i)}{p_{t-1}(i)} \right].$$

REAL WAGE. — As in Blanchard and Galí (2010), the real wage is a simple function of technology:

$$(16) \quad w_t = \omega \cdot a_t^\gamma,$$

where  $\omega$  governs the level of the real wage and  $\gamma$  governs its response to technology. Below, I discuss microeconomic estimates of  $\gamma$  obtained in US data. They indicate that  $\gamma < 1$ . In other words, the real wage is somewhat rigid in that it does not respond one-for-one to technology. This rigidity may be explained by the existence of several barriers that slow down the adjustment of wages to changes in productivity. A first barrier is the organization of firms around internal labor markets that tie wages to job descriptions.<sup>14</sup> Another barrier is the common practice of not cutting wages. Managers are reluctant to cut wages because they think that wage cuts antagonize workers and thus reduce profitability.<sup>15</sup>

<sup>14</sup>For an historical account of the evolution of firms' organization, see Jacoby (1984). For descriptions of internal labor markets, see Doeringer and Piore (1971) and Baker, Gibbs and Holmstrom (1994).

<sup>15</sup>For surveys of managers about wage-setting practices, see Campbell and Kamlani (1997) and especially Bewley (1999). For empirical evidence that wage cuts reduce productivity, see Mas (2006).

MONETARY POLICY. — Monetary policy sets the gross nominal interest rate to

$$(17) \quad R_t = \frac{1}{\beta} \cdot (1 + \pi_t)^{\mu_\pi \cdot (1 - \mu_R)} \cdot (\beta \cdot R_{t-1})^{\mu_R},$$

where  $\pi_t$  is the inflation rate at time  $t$ ,  $\mu_R \in [0, 1)$  measures interest-rate smoothing, and  $\mu_\pi > 1$  measures the response of monetary policy to inflation. I assume that steady-state inflation is zero. The steady-state gross nominal interest rate is  $1/\beta$ .

GOVERNMENT'S BUDGET CONSTRAINT AND RESOURCE CONSTRAINT. — Each period, the government services the debt inherited from the previous period, which costs  $R_{t-1} \cdot b_{t-1}$ , and it issues new debt, which brings  $b_t$ . Therefore, the budget constraint becomes

$$n_t \cdot \tau_t \cdot w_t + \frac{b_t}{p_t} = g_t \cdot w_t + \frac{r \cdot a_t}{q(\theta_t)} \cdot [g_t - (1 - s) \cdot g_{t-1}] + \frac{R_{t-1}}{p_t} \cdot b_{t-1}.$$

Using the household's budget constraint and the definition of profits, I rewrite the government's budget constraint as the resource constraint:

$$(18) \quad y_t = c_t \cdot \left(1 + \frac{\phi}{2} \cdot \pi_t^2\right) + \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1 - s) \cdot n_{t-1}].$$

The resource constraint says that the final good is consumed or allocated to changing prices or allocated to hiring workers.

SYMMETRIC EQUILIBRIUM. — In a symmetric equilibrium, all intermediate-good firms are identical. For all  $i \in [0, 1]$ ,  $l_t(i) = l_t$ ,  $y_t(i) = y_t$ , and  $p_t(i) = p_t$ . Let  $\Lambda_t$  be the real marginal revenue of producing one unit of intermediate good. Given initial employment  $n_{-1}$ , initial bond holding  $b_{-1}$ , and stochastic processes  $\{a_t, g_t\}_{t=0}^{+\infty}$ , a symmetric equilibrium is a collection of ten stochastic processes  $\{w_t, \theta_t, n_t, l_t, \Lambda_t, \pi_t, c_t, y_t, R_t, b_t\}_{t=0}^{+\infty}$  that satisfy ten relationships: the wage schedule, given by (16); the quasi-labor supply, given by (1); the aggregate labor demand,  $n_t = l_t + g_t$ ; firms' labor demand, deriving from (14) and given by

$$(19) \quad \Lambda_t \cdot \alpha \cdot l_t^{\alpha-1} = \frac{w_t}{a_t} + \frac{r}{q(\theta_t)} - \beta \cdot (1 - s) \cdot \mathbb{E}_t \left[ \frac{c_t}{c_{t+1}} \cdot \frac{a_{t+1}}{a_t} \cdot \frac{r}{q(\theta_{t+1})} \right];$$

the Phillips curve, deriving from (15) and given by

$$\pi_t \cdot (\pi_t + 1) = \frac{1}{\phi} \cdot \frac{y_t}{c_t} \cdot [\epsilon \cdot \Lambda_t - (\epsilon - 1)] + \beta \cdot \mathbb{E}_t [\pi_{t+1} \cdot (\pi_{t+1} + 1)];$$

the household's budget constraint, given by (10); the Euler equation, given by (11); the monetary policy rule, given by (17); the production function,  $y_t = a_t \cdot l_t^\alpha$ ; and the resource



constraint, given by (18).

The zero-inflation steady state of the New Keynesian model is isomorphic to the steady state studied in Section I. The steady state of the New Keynesian model is indeed given by the intersection of a quasi-labor supply and an aggregate labor demand, the sum of firms' labor demand and public employment. The quasi-labor supply remains given by (2). With zero inflation, the Phillips curve implies that  $\Lambda = (\epsilon - 1)/\epsilon$ . Hence, firms' labor demand satisfies (4) except for two changes: the marginal cost of labor is multiplied by a markup  $1/\Lambda = \epsilon/(\epsilon - 1) > 1$  because intermediate-good firms have some monopoly power; and the real wage is replaced by the ratio  $w/a$  because marginal product of labor and hiring cost are proportional to technology  $a$ . With some wage rigidity ( $\gamma < 1$ ),  $w/a = \omega \cdot a^{\gamma-1}$  increases when technology falls; therefore, a steady state with low technology corresponds to a steady state with high real wage from Section I.

### B. Calibration

I calibrate the New Keynesian model to US data. I take one period to be one week. Table 1 summarizes the calibration of all the parameters.

I calibrate a few parameters using conventional values. I set the production function parameter to  $\alpha = 0.66$ ; the elasticity of substitution across intermediate goods to  $\epsilon = 11$ , which yields a monopolistic markup of 1.1; and the discount factor to  $\beta = 0.999$ , which yields an annual interest rate of 5 percent.

Next, I calibrate the labor market parameters. I set the elasticity of the matching function with respect to unemployment at  $\eta = 0.7$ , in line with empirical evidence (Petrongolo and Pissarides, 2001). As Michailat (2012), I set the recruiting cost to  $r = 0.32 \cdot \omega$ , where  $\omega$  is the steady-state real wage. This estimate is constructed from microevidence collected by Barron, Berger and Black (1997) and Silva and Toledo (2009). I estimate the job-destruction rate from the average of the seasonally-adjusted monthly total separation rate in all nonfarm industries constructed by the Bureau of Labor Statistics (BLS) from the Job Openings and Labor Turnover Survey (JOLTS). All nonfarm industries include the nonfarm private sector and the government sector (federal, state, and local government). The monthly separation rate averages 0.036 from 2001 to 2011, so I set the weekly job-destruction rate to  $s = 0.036/4 = 0.009$ . I calibrate the elasticity of the real wage with respect to technology from microeconomic estimates of the elasticity for wages in newly created jobs—the elasticity that matters for job creations (Pissarides, 2009). In panel data following production and supervisory workers from 1984 to 2006, Haefke, Sonntag and Van Rens (2008) find an elasticity of total earnings of job movers with respect to productivity of 0.7.<sup>16</sup> If the composition of the jobs accepted by workers improves in expansion, 0.7 is an upper bound on the elasticity of wages in newly created jobs (Gertler and Trigari, 2009). A lower bound is the elasticity of wages in existing jobs, estimated between 0.1 and 0.45 (Pissarides, 2009). Hence I set  $\gamma = 0.5$ , in the range of plausible values. Since  $\gamma < 1$ , the real wage is somewhat rigid. In the simulations, I

<sup>16</sup>See Table 6, Panel A, column 4 in Haefke, Sonntag and Van Rens (2008).

TABLE 1—CALIBRATION OF THE NEW KEYNESIAN MODEL (WEEKLY FREQUENCY)

Steady-state target		Value	Source
$\bar{a}$	Technology	1	Normalization
$\bar{u}$	Unemployment	0.064	JOLTS, 2001–2011
$\bar{\theta}$	Labor market tightness	0.43	JOLTS and CPS, 2001–2011
$\bar{g}/\bar{n}$	Share of public employment in total employment	0.167	CES, 2001–2011
Parameter		Value	Source
$\eta$	Elasticity of matching function to unemployment	0.7	Petrongolo and Pissarides (2001)
$r$	Recruiting cost	0.21	Barron, Berger and Black (1997), Silva and Toledo (2009)
$s$	Job-destruction rate	0.009	JOLTS, 2001–2011
$\gamma$	Elasticity of real wage to technology	0.5	Pissarides (2009), Haefke, Sonntag and Van Rens (2008)
$\mu_\pi$	Elasticity of monetary rule to inflation	1.5	Convention
$\mu_R$	Elasticity of monetary rule to lag interest rate	0.962	Yields a quarterly elasticity of 0.6
$\phi$	Price-adjustment cost	61	Zbaracki et al. (2004)
$\rho$	Autocorrelation of technology	0.992	MSPC, 1947–2011
$\alpha$	Marginal returns to labor	0.66	Convention
$\beta$	Discount factor	0.999	Yields an annual interest rate of 5 percent
$\epsilon$	Elasticity of substitution across goods	11	Yields a monopolistic markup of 1.1
$m$	Matching effectiveness	0.17	Matches steady-state targets
$\omega$	Real-wage level	0.64	Matches steady-state targets

ensure that wage rigidity never cause the destruction of existing worker-firm matches.<sup>17</sup>

I then calibrate the monetary parameters. I set the parameters of monetary policy to  $\mu_\pi = 1.5$  and  $\mu_R = 0.962$ , corresponding to 0.6 at quarterly frequency. These values are standard. I calibrate the price-adjustment cost from microevidence collected by Zbaracki et al. (2004). Using time-and-motion methods, they study the pricing process of a large industrial firm. They find that the physical, managerial, and customer costs of changing prices amount to 1.22 percent of the firm's revenue in a given year. The firm changed the price of 25 percent of its products that year, and most prices changed by about 4 percent, so I set  $\phi = 61$ .<sup>18</sup>

I assume that log technology follows an AR(1) process:  $\log(a_{t+1}) = \rho \cdot \log(a_t) + \nu_{t+1}$ , where the error term  $\nu_{t+1}$  is a centered normal random variable. I construct log technology as a residual  $\log(a_t) = \log(y_t) - \alpha \cdot \log(l_t)$ , where  $y_t$  and  $l_t$  are seasonally-adjusted quarterly real output and employment in the nonfarm business sector for the 1947–2011 period, constructed by the BLS Major Sector Productivity and Costs (MSPC) program. To isolate fluctuations at business-cycle frequency, I take the difference between log technology and a low-frequency trend—a Hodrick-Prescott filter with smoothing parameter

<sup>17</sup>Thus, wages are always pairwise Pareto efficient, and this wage rigidity is immune to the critique of Barro (1977).

<sup>18</sup>To obtain  $\phi$ , I solve  $\phi/2 \cdot (0.04)^2 \cdot 0.25 = 0.0122$ . The value  $\phi = 61$  is similar to the maximum-likelihood estimate obtained by Ireland (2001) with a New Keynesian model.

$10^5$ . I estimate a quarterly autocorrelation of 0.897, so I set the weekly autocorrelation to  $\rho = 0.992$ .

To calibrate the remaining parameters, I set the steady-state value of some variables to the average of their empirical counterpart. Let  $\bar{x}$  be the steady-state value of  $x$ . I normalize steady-state technology to  $\bar{a} = 1$ . I compute labor market tightness as the ratio of the seasonally-adjusted monthly vacancy level constructed by the BLS from the JOLTS to the seasonally-adjusted monthly unemployment level constructed by the BLS from the Current Population Survey (CPS). Taking the average from 2001 to 2011, I set  $\bar{\theta} = 0.43$ . Similarly, I set  $\bar{u} = 0.064$ , which implies  $\bar{n} = (1 - \bar{u})/(1 - s) = 0.945$ . I compute the share of public employment in total employment using seasonally-adjusted monthly data from the BLS Current Employment Survey (CES). Public employment is employment in the government supersector, including federal, state, and local government. Total employment is the sum of public employment and employment in the total private supersector. The share of public employment averaged 16.7 percent from 2001 to 2011, which implies that  $\bar{g} = 0.167 \cdot \bar{n} = 0.157$  and  $\bar{l} = \bar{n} - \bar{g} = 0.788$ . I calibrate the matching effectiveness from the equality of unemployment inflows and outflows in steady state:  $m = s \cdot \bar{n} \cdot \bar{\theta}^{\eta-1} / \bar{u} = 0.17$ . I calibrate the real-wage level from the optimal employment choice (19):  $\omega = [(\epsilon - 1)/\epsilon] \cdot \alpha \cdot \bar{l}^{\alpha-1} / \{1 + [1 - \beta \cdot (1 - s)] \cdot 0.32/q(\bar{\theta})\} = 0.64$ . I recover  $r = 0.32 \cdot \omega = 0.21$ .

### C. Simulations

Using a shooting algorithm, I simulate an approximation of the calibrated New Keynesian model in which firms and workers have perfect foresight. Since the aim of the simulations is to quantify the nonlinearity of the model once the economy departs from its steady state, I cannot follow the standard procedure of simulating the log-linear approximation of the model.

I begin by simulating an expansion. At time 0, the economy is in steady state. At time 1, an unexpected positive technology shock  $\nu_1 = +0.054$  occurs. After that, no other shock occurs and technology converges back to its steady-state value. Let  $\hat{x}_t$  be the value of variable  $x$  at time  $t$ . For all  $t \geq 1$ ,  $\log(\hat{a}_t) = \rho^{t-1} \cdot \nu_1$ . Public employment remains constant over time:  $\hat{g}_t = \bar{g}$  for all  $t \geq 1$ , where  $\bar{g}$  is steady-state public employment. The government maintains public employment constant by hiring  $s \cdot \bar{g}$  workers each period. Under perfect foresight, workers and firms do not face any uncertainty: they perfectly anticipate the time path of all relevant variables after time 1.

The solid lines in Figure 2 are the responses to the positive technology shock. At time 1, technology increases, but the real wage increases only partially because of wage rigidity. Relative to technology, the marginal cost of labor falls. In response, firms post more vacancies to hire more workers. Thus, labor market tightness increases, and private employment builds up and peaks after 20 weeks. Consequently, unemployment drops and bottoms at 5.0 percent after 20 weeks. I also plot the response of the gross domestic product (GDP). As in national accounts, I define GDP as output of final good plus government consumption: GDP is  $y_t + w_t \cdot g_t + [g_t - (1 - s) \cdot g_{t-1}] \cdot r \cdot a_t/q(\theta_t)$ .

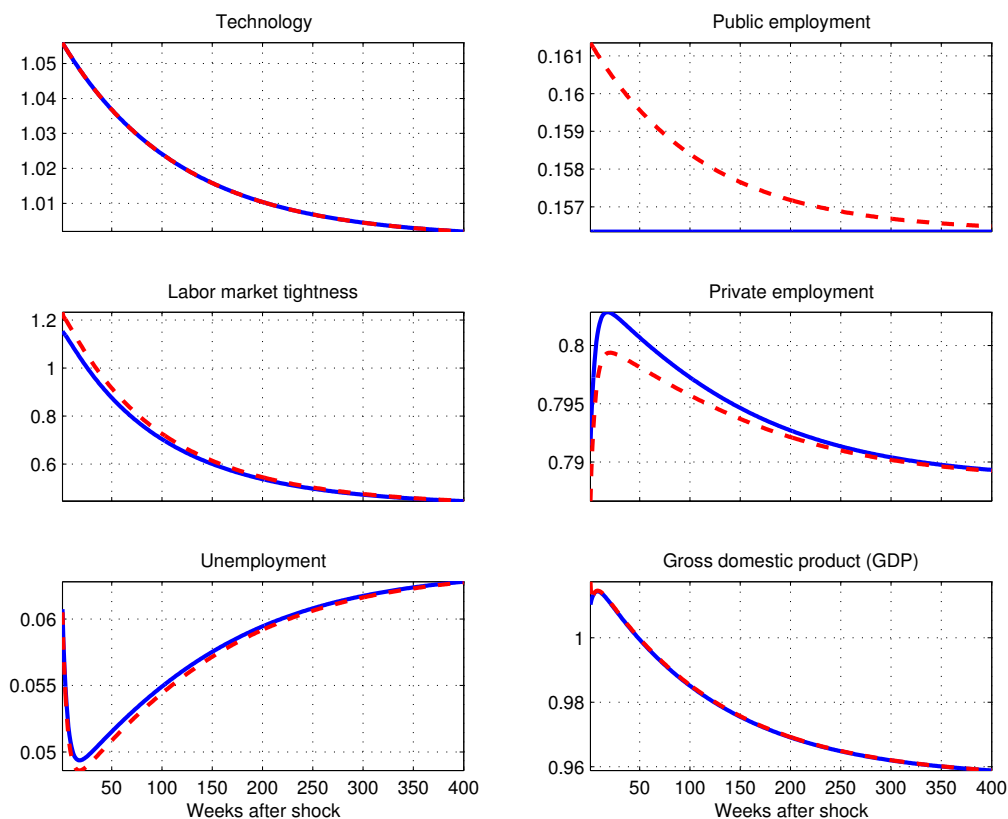


FIGURE 2. RESPONSES TO A POSITIVE TECHNOLOGY SHOCK

*Note:* The solid lines are the responses to an unexpected positive technology shock of +5.4 percent. The dashed lines are the responses to an unexpected positive technology shock of +5.4 percent accompanied by the unexpected hiring of 0.5 percent of the labor force in the public sector. The responses are obtained by simulating an approximation of the calibrated New Keynesian model in which firms and workers have perfect foresight.

At time 1, GDP mechanically increases because technology and thus output increase. GDP further increases in the next 10 weeks because private employment and thus output increase.

To quantify the effect of an increase in public employment during an expansion, I simulate the model when at time 1, the unexpected positive technology shock is accompanied by the unexpected hiring of 0.5 percent of the labor force in the public sector. Let  $x_t^*$  be the value of variable  $x$  at time  $t$ . At time 1,  $g_1^* = \hat{g}_1 + 0.005$ . After that, the government hires as many workers as in the previous simulation:  $g_t^* - (1 - s) \cdot g_{t-1}^* = s \cdot \bar{g}$  for all  $t \geq 2$ . Under perfect foresight, workers and firms perfectly anticipate the time path of public employment after time 1.

The dashed lines in Figure 2 are the responses to the positive technology shock accompanied by the increase in public employment. At time 1, the government posts more

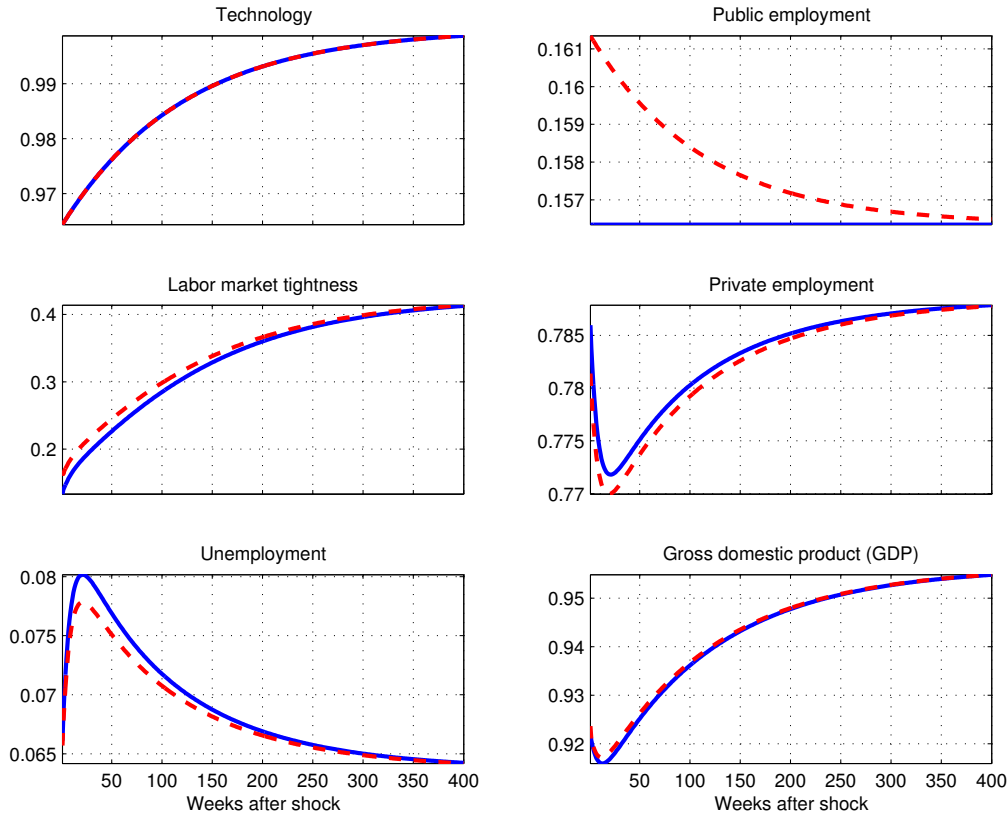


FIGURE 3. RESPONSES TO A NEGATIVE TECHNOLOGY SHOCK

*Note:* The solid lines are the responses to an unexpected negative technology shock of -3.6 percent. The dashed lines are the responses to an unexpected negative technology shock of -3.6 percent accompanied by the unexpected hiring of 0.5 percent of the labor force in the public sector. The responses are obtained by simulating an approximation of the calibrated New Keynesian model in which firms and workers have perfect foresight.

vacancies to hire additional workers in the public sector. As a result, public employment and labor market tightness rise above their previous level. Because labor market tightness increases, it is more costly for intermediate-good firms to recruit workers. Thus, the marginal cost of labor rises, firms reduce hiring, and private employment falls below its previous level. In other words, public employment crowds out private employment. Nonetheless, the net effect of an increase in public employment is positive because unemployment falls below its previous level.

To quantify the effect of an increase in public employment during a recession, I repeat the two previous simulations but replace the unexpected positive technology shock by an unexpected negative technology shock  $\nu_1 = -0.036$ . The results are displayed in Figure 3. At time 1, technology decreases, but the real wage decreases only partially because of wage rigidity. Relative to technology, the marginal cost of labor increases. In

response, firms post fewer vacancies. Thus, labor market tightness, private employment, and GDP fall whereas unemployment increases. Qualitatively, an increase in public employment has the same effect in expansion and recession; but quantitatively, the effects are different. In the expansion of Figure 2, tightness increases by 0.07 from 1.15 to 1.22 after the increase in public employment; in the recession of Figure 3, it only increases by 0.02 from 0.14 to 0.16. Hence, the resulting increase in hiring cost is much larger in expansion than in recession: in the expansion, the expected number of vacancies required to hire a worker increases by 0.27 from 6.41 to 6.68; in the recession, it only increases by 0.16 from 1.47 to 1.64. Thus, crowding-out is larger in expansion than in recession: in the expansion, private employment falls by 0.35 percentage points from 80.29 percent to 79.94 percent at its extremum; in the recession, it only falls by 0.18 percentage points from 77.18 percent to 77.00 percent. To conclude, increasing public employment reduces unemployment more effectively in recession than in expansion: in the expansion, unemployment only falls by 0.08 percentage points from 4.94 percent to 4.86 percent at its extremum; in the recession, it falls by 0.24 percentage points from 8.02 percent to 7.78 percent.

To measure the period-by-period effect of an increase in public employment, I compute the *instantaneous multiplier at time  $t$* . This multiplier is defined as  $(n_t^* - \hat{n}_t) / (g_t^* - \hat{g}_t)$ , where  $\hat{n}_t$  and  $\hat{g}_t$  are employment levels before government intervention, and  $n_t^*$  and  $g_t^*$  are levels after intervention. To isolate the marginal effect of an increase in public employment,  $\{n_t^*\}$  and  $\{g_t^*\}$  are obtained by hiring only 0.01 percent of the labor force in the public sector at time 1. The multiplier gives the number of jobs created in period  $t$  for each job added to the public sector in that period. Figure 4(a) shows the response of the instantaneous multiplier to a positive technology shock  $\nu_1 = +0.054$ , and Figure 4(b) shows the response to a negative technology shock  $\nu_1 = -0.036$ . At time 1, the instantaneous multiplier is small in both cases. After the positive shock, the multiplier grows slowly to reach a steady-state value of 0.38. The increase of the multiplier concurs with the increase in unemployment when unemployment reverts to its steady-state level. After the negative shock, the multiplier grows quickly and peaks at 0.58 after 30 weeks. The peak of the multiplier broadly concurs with the peak of unemployment. After the peak, unemployment and multiplier revert to their steady-state levels.

To summarize the effect of an increase in public employment after a given technology shock, I compute the *cumulative multiplier*. This multiplier is defined as

$$(20) \quad \frac{\sum_{t=0}^T (n_t^* - \hat{n}_t)}{\sum_{t=0}^T (g_t^* - \hat{g}_t)},$$

where  $T = 15,000$  is the horizon in the shooting algorithm, long enough for the model to converge back to steady state. The multiplier measures the total number of job  $\times$  weeks created by hiring workers in the public sector divided by the number of job  $\times$  weeks added to the public sector. This multiplier accounts for the persistence of public employment, arising because public workers cannot be dismissed such that public jobs are closed only at a rate  $s$ . I repeat the simulations described in Figure 4 for a collection of 16

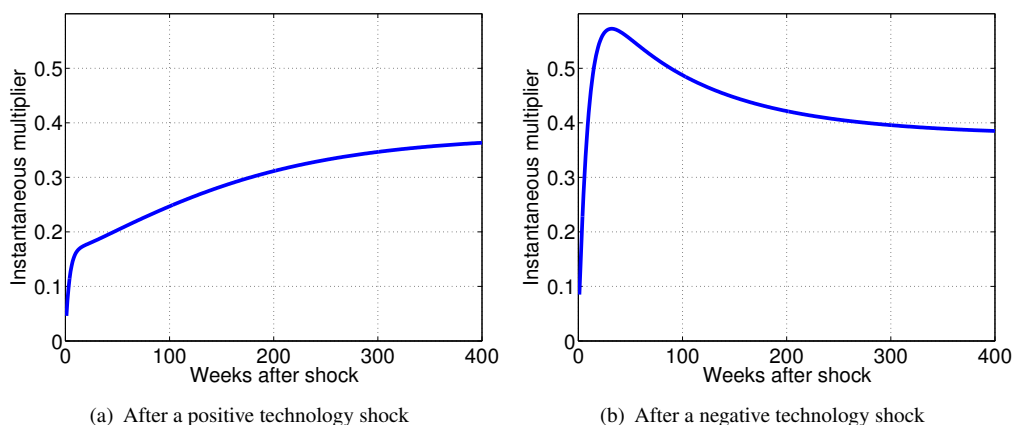


FIGURE 4. MULTIPLIER DYNAMICS IN EXPANSION AND RECESSION

*Note:* The positive shock is an unexpected increase in technology by +5.4 percent. The negative shock is an unexpected decrease in technology by -3.6 percent. The instantaneous multiplier at time  $t$  is  $(n_t^* - \hat{n}_t) / (g_t^* - \hat{g}_t)$ , where  $\hat{n}_t$  and  $\hat{g}_t$  are aggregate and public employment without government intervention and  $n_t^*$  and  $g_t^*$  are aggregate and public employment with government intervention. The intervention is the unexpected hiring of 0.01 percent of the labor force in the public sector. The multiplier dynamics are obtained by simulating the calibrated New Keynesian model.

technology shocks ranging from  $\nu_1 = -0.036$  to  $\nu_1 = +0.054$ . For each technology shock, I compute the cumulative multiplier given by (20), and I measure the extremum of the unemployment response without government intervention. I link each cumulative multiplier to the associated unemployment rate and plot the 16 multiplier-unemployment pairs in Figure 5(a). The cumulative multiplier is countercyclical: it doubles from 0.24 to 0.49 when unemployment increases from 5 percent to 8 percent. The interpretation is that hiring 1 percent of the labor force in the public sector reduces unemployment by 0.49 percentage points in recession and by only 0.24 percentage points in expansion.<sup>19</sup>

The multiplier in Figure 5(a) has not been estimated empirically. To facilitate comparison with empirical evidence, Figure 5(b) displays another cumulative multiplier that measures the percentage-point reduction in unemployment obtained by spending 1 percent of GDP on public employment. This multiplier is given by an expression that differs from (20) on two counts. First, public employment  $g_t$  is replaced by its cost,  $g_t \cdot w_t + [g_t - (1 - s) \cdot g_{t-1}] \cdot r \cdot a_t / q(\theta_t)$ . Second, the expression is multiplied by steady-state GDP,  $\bar{y} + \bar{w} \cdot \bar{g} + s \cdot \bar{g} \cdot r / q(\bar{\theta})$ , to measure the cost of public employment as a fraction of GDP. This alternative multiplier is also countercyclical: it increases from 0.34 to 0.71 when unemployment increases from 5 percent to 8 percent. The interpretation is that spending 1 percent of GDP on public employment reduces unemployment by 0.71 percentage points in recession and by only 0.34 percentage points in expansion. Unem-

<sup>19</sup>During the Great Depression, the Roosevelt administration was concerned that the public jobs created by the New Deal might take away job applicants from firms, thus making it difficult to hire workers in the private sector (Neumann, Fishback and Kantor, 2010). The numerical results address this concern by showing that crowding-out of private employment is weak in recession.

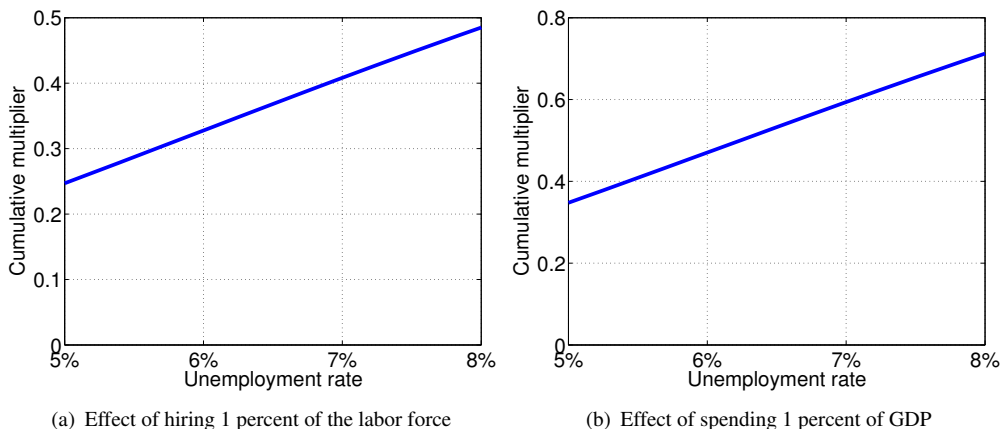


FIGURE 5. CUMULATIVE MULTIPLIERS OVER THE BUSINESS CYCLE

*Note:* The multipliers in Panel (a) are given by (20). They give the percentage-point increase in aggregate employment obtained by hiring 1 percent of the labor force in the public sector. The multipliers in Panel (b) are defined in the text. They give the percentage-point increase in aggregate employment obtained by spending 1 percent of GDP on public employment. Each multiplier is computed by hiring 0.01 percent of the labor force in the public sector in response to 1 of 16 technology shocks ranging from -3.6 percent to +5.4 percent. The unemployment rate on the x-axis is the extremum of the unemployment response after the technology shock, without government intervention. The cumulative multipliers are obtained by simulating the calibrated New Keynesian model.

ployment averaged 5.8 percent in the US from 1954 to 2006; at that rate, the multiplier is 0.45. This value is aligned with the results of Monacelli, Perotti and Trigari (2010). Using US data for the 1954–2006 period, they estimate a structural vector autoregression and compute a cumulative multiplier defined as that in Figure 5(b). At a two-year horizon, they find a multiplier of 0.43.<sup>20</sup>

The level and cyclicity of the multipliers are robust to changes in public-employment policy. Appendix B describes the multipliers obtained when public employment is a constant fraction of private employment in absence of government intervention: for all  $t \geq 0$ ,  $\hat{g}_t = \zeta \cdot \hat{l}_t$ , where  $\zeta = \bar{g}/\bar{l} = 0.20$  is the steady-state ratio of public employment to private employment. The results are almost identical to those displayed in Figure 5.<sup>21</sup>

### III. Conclusion

In this paper, I have developed a theory in which the public-employment multiplier varies across stages of the business cycle. My analysis has two important implications for work that estimates government multipliers. A first implication is that work that estimates average multipliers over all stages of the business cycle may not be informative

<sup>20</sup>See Table 1, column 4 in Monacelli, Perotti and Trigari (2010).

<sup>21</sup>The level and cyclicity of the multipliers are also robust to changes in monetary policy. I redid the simulations with a monetary policy rule that includes an output gap:  $R_t = (1/\beta) \cdot (1 + \pi_t)^{\mu_\pi} \cdot (u_t/\bar{u})^{\mu_u}$ , where  $\bar{u}$  is steady-state unemployment,  $\mu_\pi = 5$ , and  $\mu_u = -0.8$ . The values of  $\mu_\pi$  and  $\mu_u$  are borrowed from Blanchard and Galí (2010). The results were almost identical to those displayed in Figure 5.



for the design of government policy in recessions.<sup>22</sup> The reason is that multipliers may be very different in recessions, compared to other stages of the business cycle. For instance, in this model the public-employment multiplier is much higher in recessions: in Figure 5(b), the multiplier is 0.45 at a normal unemployment rate of 5.8 percent but it reaches 0.71 when the unemployment rate reaches 8 percent.

Estimating multipliers that account for the stage of the business cycle in which the government increases spending is therefore essential for policy applications. Two studies offer a promising start on this agenda. Both find countercyclical multipliers.<sup>23</sup> Auerbach and Gorodnichenko (2011) use a direct projection method that allows the multiplier to vary smoothly with the stage of the business cycle. In data for a large number of OECD countries, they find that the multiplier is quite large in recessions but not significantly different from zero in expansions.<sup>24</sup> Nakamura and Steinsson (2011) measure how an increase in government consumption in a given US state during a military build-up affects employment in this state.<sup>25</sup> Their estimate of the multiplier is much larger when unemployment is high than when unemployment is low.<sup>26</sup>

A second implication is that work that estimates average multipliers over all types of government spending may not be useful to assess the effectiveness of specific types of spending. The reason is that multipliers may be very different for different types of spending. For instance, in this model increasing public employment reduces unemployment, but increasing government purchases of goods from the private sector has no effect on unemployment.<sup>27</sup> Hence, estimating separate multipliers by types of government spending is essential for policy applications.<sup>28</sup>

Several restrictions limit, however, the degree to which the theory moves us toward a complete understanding of the role for government spending over the business cycle. A first restriction is that in the model, public-sector jobs are identical to private-sector jobs. But in practice, these jobs may differ. A first difference concerns wages: during the New Deal, hourly wages were substantially lower in relief jobs than in private jobs (Neumann, Fishback and Kantor, 2010); and on average, public-sector wages are higher and more rigid than private-sector wages (Gregory and Borland, 1999; Quadrini and Trigari, 2007). A second difference concerns separation rates: in the US, public-sector

<sup>22</sup>Parker (2011) made this point, and this paper offers a theoretical support for his argument.

<sup>23</sup>Other researchers who estimate such multipliers include Auerbach and Gorodnichenko (2012), Canova and Pappa (2011), Bachmann and Sims (2012), and Holden and Sparrman (2011).

<sup>24</sup>The results are reported in Panel B of Table 3 in Auerbach and Gorodnichenko (2011). Column 1 and 2 show that the multiplier is 0.50 (standard deviation: 0.22) in recessions and -0.11 (standard deviation: 0.15) in expansions. A recession is defined as a period when the detrended unemployment rate is especially high, and an expansion as a period when the detrended unemployment rate is especially low.

<sup>25</sup>The effect of government consumption is isolated from a monetary policy response because US states are part of a monetary union.

<sup>26</sup>Column 3 in Table IV in Nakamura and Steinsson (2011) shows that the multiplier is 1.85 when the US unemployment rate is above the median and 1.10 when the US unemployment rate is below the median. The estimates are not very precise because the number of business cycles in the sample is limited.

<sup>27</sup>Consider the simple model of Section I. Suppose that the government purchases  $G$  units of final good from the private sector.  $G$  only appears in the resource constraint:  $y = c + G + s \cdot n \cdot r/q(\theta)$ . In particular,  $G$  affects neither quasi-labor supply nor aggregate labor demand. Hence,  $G$  has no effect on unemployment. However,  $G$  crowds out one-for-one the household's consumption of final good,  $c$ .

<sup>28</sup>Some researchers who have done so include Auerbach and Gorodnichenko (2012) and Pappa (2010).

jobs last much longer than private-sector jobs because the separation rate in the private sector is almost 3 times higher than in the public sector (Gomes, 2010). If jobs differ across sectors, jobseekers will direct their search toward a specific sector. Studying the effect of public employment in this context may be difficult, but it would offer a more accurate characterization of the cyclical behavior of the public-employment multiplier.

A second restriction is that in the model, firms are always able to sell their entire production at the going price such that the concept of deficient aggregate demand is absent. This assumption explains why government purchases of goods from the private sector have no effect on unemployment. This assumption is also unrealistic, and it is important to relax it to analyze the effect of government purchases on unemployment. Michaillat and Saez (2013) take a first step in this direction. They propose a new kind of business-cycle model with trade frictions in both labor market and product market. Unemployment and unsold production arise in equilibrium. Recessions may be caused by technology shocks or aggregate demand shocks. They study how consumption and unemployment respond when the government purchases goods from the private sector.

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## APPENDIX A: PROOFS

## A1. Proof of Lemma 3

Since  $g/l = \zeta$  in equilibrium, the equilibrium condition for a steady state parameterized by  $w$  can be written as  $n^d(\theta, w) = n^s(\theta)$ , where

$$(A1) \quad n^d(\theta, w) = (1 + \zeta) \cdot \left[ \frac{1}{\alpha} \cdot \left\{ w + [1 - \beta \cdot (1 - s)] \cdot \frac{r}{q(\theta)} \right\} \right]^{\frac{-1}{1-\alpha}}.$$

Implicit differentiation of the equilibrium condition yields

$$\frac{d\theta}{dw} = \underbrace{\frac{\partial n^d}{\partial w}}_{-} \cdot \left[ \underbrace{\frac{\partial n^s}{\partial \theta}}_{+} - \underbrace{\frac{\partial n^d}{\partial \theta}}_{-} \right]^{-1}.$$

The signs of the derivatives come from Lemma 1 and Lemma 2, which also applies to the aggregate labor demand defined by (A1). Thus  $d\theta/dw < 0$ . The other results follow since  $n = n^s(\theta)$ ,  $u = 1 - (1 - s) \cdot n$ , and  $l = n/(1 + \zeta)$ .

## A2. Proof of Proposition 1

I first prove Parts (i) and (ii). Implicit differentiation of equation (6) yields

$$(A2) \quad \frac{\partial n^d}{\partial \theta} \cdot \frac{d\theta}{dg} = -\frac{1}{1 + (\epsilon^s/\epsilon^d)},$$

where  $\epsilon^s \equiv (\theta/n^s) \cdot (\partial n^s/\partial \theta) > 0$ ,  $\epsilon^d \equiv -(\theta/n^d) \cdot (\partial n^d/\partial \theta) > 0$ . Next, implicit differentiation of equation (7) yields

$$\frac{dn}{dg} = 1 + \frac{\partial n^d}{\partial \theta} \cdot \frac{d\theta}{dg}.$$

Combining this result with (A2) yields an expression for the public-employment multiplier:

$$(A3) \quad \lambda \equiv \frac{dn}{dg} = 1 - \frac{1}{1 + (\epsilon^s/\epsilon^d)}.$$

Since  $\epsilon^s \in (0, +\infty)$  and  $\epsilon^d \in (0, +\infty)$ , then  $\lambda \in (0, 1)$ .

Next, I prove Part (iii). The first step is to express  $\epsilon^s$  and  $\epsilon^d$  as functions of endogenous

variables. The definition of  $n^s(\theta)$  implies

$$(A4) \quad \epsilon^s = (1 - \eta) \cdot u.$$

The definition of  $l^d(\theta, w)$  implies  $(\theta/l) \cdot (\partial l^d / \partial \theta) = -[\eta / (1 - \alpha)] \cdot \Omega$ , where

$$\Omega \equiv \frac{[1 - \beta \cdot (1 - s)] \cdot r / q(\theta)}{[1 - \beta \cdot (1 - s)] \cdot r / q(\theta) + w}.$$

Since  $n^d(\theta, w, g) = g + l^d(\theta, w)$  and  $n/l = 1 + \zeta$  in equilibrium, I can relate  $\epsilon^d$  to  $\Omega$ :

$$(A5) \quad \epsilon^d = -\frac{1}{1 + \zeta} \cdot \frac{\theta}{l} \cdot \frac{\partial l^d}{\partial \theta} = \frac{\eta}{(1 + \zeta) \cdot (1 - \alpha)} \cdot \Omega.$$

Lemma 3 and the fact that  $q$  is decreasing imply  $d\Omega/dw < 0$ . Hence, (A5) implies  $d\epsilon^d/dw < 0$ . Lemma 3 and equation (A4) imply  $d\epsilon^s/dw > 0$ . Thus, (A3) implies  $d\lambda/dw > 0$ .

#### APPENDIX B: ROBUSTNESS

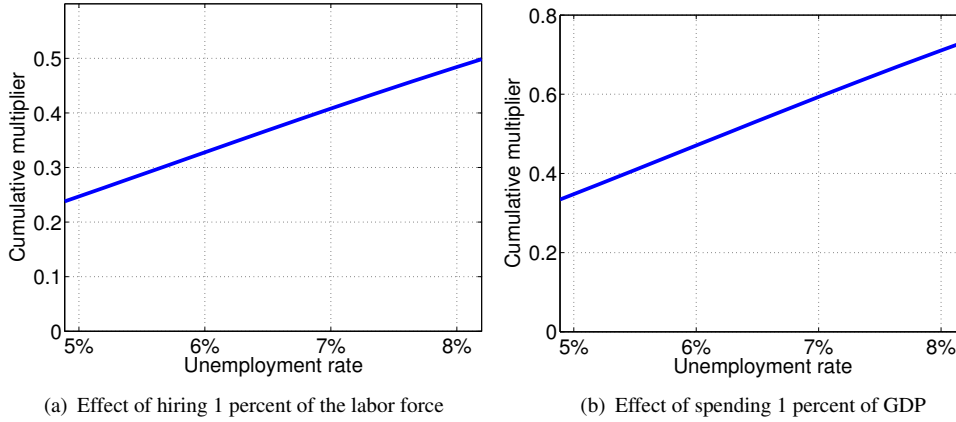


FIGURE B1. CUMULATIVE MULTIPLIERS WITH A CONSTANT RATIO OF PUBLIC TO PRIVATE EMPLOYMENT

*Note:* This figure is obtained as Figure 5, except that public employment remains a constant fraction of private employment over time in absence of government intervention:  $\hat{g}_t = \zeta \cdot \hat{l}_t$ , where  $\zeta = \bar{g}/\bar{l} = 0.20$  is the steady-state ratio of public employment to private employment. The multipliers in Panel (a) are given by (20). They give the percentage-point increase in aggregate employment obtained by hiring 1 percent of the labor force in the public sector. The multipliers in Panel (b) are defined in the text. They give the percentage-point increase in aggregate employment obtained by spending 1 percent of GDP on public employment. The cumulative multipliers are obtained by simulating the calibrated New Keynesian model.