A TEST BETWEEN UNEMPLOYMENT THEORIES USING MATCHING DATA

Melvyn G Coles and Barbara Petrongolo

LABOUR ECONOMICS

No. 3241

Available online at: www.cepr.org/pubs/dps/DP3241.asp
A TEST BETWEEN UNEMPLOYMENT THEORIES USING MATCHING DATA

Melvyn G Coles, University of Essex
Barbara Petrongolo, London School of Economics (LSE) and CEPR

Discussion Paper No. 3241
March 2002

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

This Discussion Paper is issued under the auspices of the Centre’s research programme in LABOUR ECONOMICS. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre’s publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Melvyn G Coles and Barbara Petrongolo
ABSTRACT

A Test Between Unemployment Theories Using Matching Data*

A new methodology is described which tests between various equilibrium theories of unemployment using matching data. The Paper shows how to correct econometrically for temporal aggregation effects, where the econometrician's aim is to identify a matching process using data which is recorded monthly, and also shows how to identify different unemployment theories on the data. As implementing this test requires information on the inflow of new vacancies over time, this Paper uses employment agency data for the UK over the period 1985-99.

Although the standard random matching approach provides a reasonably good fit, the empirical evidence provides greater support for 'stock-flow' matching. Estimates find that over this period, around 87% of newly laid-off workers are on the long-side of their markets and so match with the flow of new vacancies as those vacancies come onto the market. In particular, these workers experience average durations of unemployment which exceed six months and their matching rates are highly correlated with the inflow of new vacancies. This job queue interpretation of the matching data has important implications for government policy on long term unemployment and optimal UI. It also suggests that previous estimates of the so-called matching function have been misspecified, which potentially explains the large variation in results obtained in that literature.

JEL Classification: E24, J41, J63 and J64
Keywords: hypothesis testing, matching and unemployment
* We are grateful to Alan Manning for useful comments.

Submitted 11 February 2002
1 Introduction

This paper uses matching data to test between various equilibrium theories of unemployment. Obviously this is important as different unemployment hypotheses typically have different implications for optimal government policy. One unemployment hypothesis considered here is the frictional matching approach, such as the random matching approach as described in Mortensen and Pissarides [1999] and Pissarides [2000]) or the directed search approach (e.g. Montgomery [1991]). In those literatures, the re-employment rate $\lambda$ of an unemployed worker at any point in time depends on the contemporaneous stocks of vacancies $V$ and unemployed job seekers $U$ in the market; i.e. $\lambda = \lambda(V, U)$. In contrast an efficiency wage theory, or the notion of job queues, suggests instead that unemployed job seekers are on the long side of the labour market. If $v$ denotes the inflow of new vacancies, then as described in Shapiro and Stiglitz (1984), frictionless matching implies $\lambda U = v$, where the outflow of the $U$ unemployed workers equals the inflow of new vacancies. This implies an individual re-employment hazard rate of the form $\lambda = v/U$ which, critically, depends on the inflow of new vacancies $v$ rather than on the current vacancy stock $V$. As the stock of vacancies $V$ and the inflow of new vacancies $v$ have different time series properties [see below], the following discriminates between these unemployment hypotheses using matching data.

In implementing this test, this paper also makes two important methodological contributions. First it explicitly takes note that the econometric framework is trying to identify a continuous time matching process while using time series data which record the matching rate as total matches over each month. Typically such temporal aggregation of the data is ignored in the matching literature, though it has been noted that ignoring such aggregation effects introduces a potential bias [see Burdett et al. [1994] and Berman [1997] and also Petrongolo and Pissarides [2001] for a recent survey]. Here we show that correcting this bias requires constructing monthly ‘at risk’ measures for the number of job seekers and vacancies in the market.

The second contribution is that we show the underlying continuous time matching process can be identified econometrically for two particular cases; (a) random matching, where the stock of vacancies matches with the stock of unemployed job seekers, and (b) stock-flow matching, where the stock of agents on one side of the market matches with the flow of entrants on the other side. Indeed, the efficiency wage literature describes an example of stock-flow matching where all unemployed workers are assumed to be on the short side of the market and so match with the flow of new vacancies coming onto the market. But the stock-flow matching literature, as described by Taylor [1995], Coles and Muthoo [1998], Coles and Smith [1998], Coles [1999], Gregg and Petrongolo [2001], allows two sided stock-flow matching. In particular with a segmented labour market where submarkets are differentiated by skill or location, stock flow matching implies an econometric structure where proportion $p$ of newly unemployed workers are on the short-side of their respective submarket and so
match immediately. But proportion $1 - p$ find there is no suitable vacancy currently on the market and must then wait for one to enter. As implied by the efficiency wage approach, these workers then match at rate $\lambda = \lambda(v, U)$ which depends on the inflow of new vacancies, with crowding out by competing unemployed job seekers.

Clearly a crucial aspect of two-sided stock-flow matching is that there is (unobserved) heterogeneity across vacancies and unemployed job seekers. This approach is not unrelated to the hazard function literature which assumes there are two types of workers, those that match quickly and those that match slowly (see Lancaster and Nickell [1980] and Heckman and Singer [1984]). In contrast, stock-flow matching assumes some workers are on the short-side of their particular job specialisation and so match (arbitrarily?) quickly, while others are on the long side and must wait for new vacancies to enter the market. However most importantly, stock-flow matching implies different restrictions on the underlying matching process. In particular, it implies that the stock of unemployed workers (i.e. those that do not find immediate re-employment) matches with the flow of new vacancies (they are job rationed), rather than with the stock (which assumes frictional unemployment).

Controlling for temporal aggregation effects and distinguishing between the above unemployment theories requires vacancy inflow information. Unfortunately such data is not available for the United States where inches of help-wanted advertisements are used to measure vacancies and there is no information on whether a particular job advertisement is new or is a re-advertisement. Fortunately, Job Center data in the United Kingdom provides this flow information for the U.K. labor market. However, even though the results presented are obtained for the U.K. labor market, the insights provide a useful re-interpretation of U.S. matching data.

For example Figure 1, which is taken from Blanchard and Diamond [1989], describes the number of unemployed workers who find work each month and the stocks of unemployed workers and vacancies in the U.S. manufacturing sector. Blanchard and Diamond [1989] estimate the aggregate matching process assuming random matching and do not attempt to identify other theories of unemployment on these data. However note that these data may also be consistent with the efficiency wage approach. To see why, note that the measured number of matches is much more volatile than the measured change in the stock of vacancies. This implies that a large increase in the number of matches is highly correlated with a large increase in the inflow of new vacancies, thereby leaving the stock of vacancies largely intact. (Otherwise, if the inflow of new vacancies were fairly smooth, a large increase in the number of matches would necessarily result in a large fall in the stock of vacancies.) The inflow of new vacancies, rather than the stock of current vacancies, is much more important in explaining short-run fluctuations in matching rates.

[Figure 1 around here]

Identifying the underlying matching process given temporally aggregated data requires constructing measures of the number of vacancies and number of job seekers
which are ‘at risk’ of matching during the month. The typical matching function approach (implicitly) assumes that the stock variables recorded at the start of the month provide suitable ‘at risk’ proxies. This paper establishes that the appropriate ‘at risk’ measure of vacancies in each month $n$ is instead a weighted sum of the initial stock of vacancies $V_n$ and the inflow of new vacancies within the month, denoted $v_n$ (throughout upper case will refer to stock variables, lower case to flow variables). The appropriate weights in turn depend on the matching rate of vacancies. For example, if the matching rate of vacancies were very slow, the appropriate at risk measure for vacancies would be $V_n = V_n + 0.5v_n$ as each new vacancy is, on average, ‘at risk’ for half of the month. However, if the matching rate of vacancies were arbitrarily fast, we show that the appropriate at risk measure would instead be $V_n = V_n + v_n$ where each vacancy that enters at any stage during the month matches immediately. The appropriate weight therefore depends on the matching rate of vacancies, and this ‘at risk’ insight potentially explains why observed monthly matches are highly correlated with the inflow of new vacancies. Obviously the same insights apply for at risk measures of the number unemployed.

Using standard MLE techniques to identify the appropriate ‘at risk’ measures, the first set of results estimate the identifying equations implied by random (or stock-stock) matching on U.K. matching data for the period 1985-99. Assuming a standard Cobb-Douglas function of the form $\lambda = aU^\alpha V^\beta$, we first establish that the hypothesis of constant returns to matching is accepted. However an important robustness test then includes the flow of new vacancies $v$ as an additional explanatory variable for $\lambda$, where the random matching structure implies $v$ should not have any additional explanatory power. It is found that including $v$ not only much improves the fit, the vacancy stock parameter becomes insignificant and wrong-signed. Although inconsistent with the identifying equations implied by random matching, this implies that temporal aggregation of the data cannot fully explain the high correlation between observed monthly matching figures and the inflow of new vacancies over the month. The underlying implication being that the inflow of new vacancies plays a more direct role on observed matching rates than is allowed by imposing random, or stock-stock, matching.

The second set of results instead estimate those matching equations which are identified by assuming stock-flow matching. Estimation for the period 1985-1999 finds $p \approx 0.1$ and is highly significant. Although unemployment rates for this period were rather high - with an average duration of unemployment of around 6 months - this estimate implies that around 10% of newly unemployed workers had skills which were in relatively short supply and so quickly found work. Estimates also find that the re-employment rates of those who fail to match immediately are driven statistically by the inflow of new vacancies, and are crowded out by the stock of unemployed workers. Indeed, although marginally rejected by the data, the estimated equations are remarkably close to $\lambda = v/U$ as suggested in Shapiro and Stiglitz [1984]. This is strong support for the notion that the longer-term unemployed are job-rationed,
being forced to chase new vacancies as those vacancies come onto the market.

These results have very important policy implications. For example the optimal unemployment insurance (UI) literature, where job search effort is not observed by the government, argues that UI payments should decrease with duration to promote greater job search effort (e.g. Shavell and Weiss [1979] and more recently Hopenhayn and Nicolini [1997] and Fredriksson and Holmlund [2001]). But the above estimated job matching process $\lambda \simeq v/U$ for the longer term unemployed implies this cannot be an optimal policy. In particular with job rationing, reducing the quality of UI coverage to encourage greater job search effort merely generates pure displacement effects. It also suggests that when trying to reduce longer term unemployment, policies aimed at increasing search effort (such as the Job Restart scheme in the United Kingdom) are misplaced. The essential market failure is more likely a wage distortion (so that the longer term unemployed are job rationed) rather than a job search failure.

2 The Empirical Framework

Throughout we suppose that at every date $t$, the re-employment probabilities of unemployed workers are described by a pair $(p(t), \lambda(t))$ where $p(t)$ is the proportion of workers laid off at date $t$ who find immediate re-employment, while $\lambda(t)$ is the hazard rate of re-employment of a worker who has been unemployed for some (strictly positive) period of time. As described in the Introduction, different equilibrium theories of unemployment have different implications for these variables. In particular, each theory $i$ implies functional forms $(p^i, \lambda^i): (U, V, u, v) \rightarrow \mathbb{R}_+^2$ where $U, V$ are the stocks of unemployed workers and vacancies respectively, and $u, v$ refer to the flow of new job seekers and new vacancies into the market.

First consider the efficiency wage approach, theory $i = E$. Involuntary unemployment implies $p^E = 0$; it takes time to find work. Furthermore, in the absence of search frictions, each vacancy is immediately filled as it enters the market. If workers are identical, the re-employment hazard rate of an unemployed worker, denoted $\lambda^E$, is $\lambda^E = v/U$. For econometric purposes we consider a more general specification of the form $\lambda^E = \lambda^E(v, U)$, where the faster the arrival rate of new vacancies $v$, the sooner a worker will escape unemployment, but there is crowding out by the other unemployed job seekers.

The random matching approach, $i = M$, also assumes $p^M = 0$, but unlike the efficiency wage hypothesis assumes a re-employment hazard rate $\lambda^M = \lambda^M(V, U)$, where constant returns to matching imply $\lambda^M = \lambda^M(V/U)$. Note that this specification corresponds to the so-called matching function as estimated in the matching literature.

Stock-flow matching, $i = SF$, assumes a segmented labour market. A job seeker who has just lost his job, samples the stock of vacancies currently on the market. With some probability $p$, the worker is on the short-side of the relevant submarket and so immediately exits unemployment. Coles and Smith [1998] describe a particular case
with pure idiosyncratic match payoffs. Their model suggests that the re-employment probability of a recently laid off worker is an increasing function of the stock of current vacancies \( V \). However, as several workers might be laid off at the same time, stock-flow matching implies there may be crowding out by other recently laid-off workers, which suggests \( p = p^{SF}(V, u) > 0 \).

With probability \( 1 - p^{SF} \) a newly unemployed worker finds there is no suitable vacancy and so has to wait for a suitable vacancy to arrive. In that case, the worker’s re-employment hazard rate is \( \lambda = \lambda^{SF}(v, U) \) where as in the efficiency wage case, this worker has to compete against the other unemployed workers to match with the flow of new vacancies coming onto the market.

Note that these different theories essentially provide different identifying restrictions. Stock-flow matching implies that the stock of unemployed workers cannot match with the current stock of vacancies, the idea being that if such a match exists, the worker should already have exited unemployment. Hence \( p^{SF} \) should only depend on \( (V, u) \) (the stock of vacancies matches with the flow of unemployed workers) and \( \lambda^{SF} \) should only depend on \( (v, U) \) (the stock of unemployed workers matches with the flow of new vacancies). In contrast the random matching approach assumes stock-stock matching, while efficiency wages implies one-sided stock-flow matching, that the stock of unemployed workers matches with the flow of new vacancies and \( p^E = 0 \).

We now show how to identify \( (p^i, \lambda^i) \) using data which is temporally aggregated.

### 3 Temporal Aggregation

As \((U, V, u, v)\) are time varying, let \((p(t), \lambda(t))\) denote the true underlying matching probabilities. If \( M(t) \) denotes the expected flow matching rate at a point in time, it is given by

\[
M(t) = \lambda(t)U(t) + p(t)u(t). \tag{1}
\]

The first term describes the flow out of unemployment by the current stock of job seekers, the second describes the flow of newly unemployed workers \( u(t) \) who immediately become re-employed. The econometric issue is identifying this continuous time matching relationship using data which is recorded as monthly time series.

The data record the stock of unemployed workers \( U_n \) at the beginning of each month \( n \in \mathcal{N} \), and the total inflow of newly-unemployed workers \( u_n \) during each month. Similarly the data record the initial vacancy stock \( V_n \) and the total vacancy inflow \( v_n \). A temporal aggregation bias arises as the data only record the total number of matches during each month which presumably depends on how the stock variables change over the month.\(^2\)

To control econometrically for this bias, consider how the stock of unemployment \( U(t) \) changes over time \( t \) in month \( n \in \mathcal{N} \) where \( t \in [n, n+1) \). Given \( \lambda(t), p(t) \) over this month and entry rate \( u(t) \) of new unemployed workers, the stock of unemployed
job seekers at time $t$ is simply:

$$U(t) = U_n e^{-\int_{n}^{t} \lambda(s)ds} + \int_{n}^{t} u(t')[1 - p(t')]e^{-\int_{n}^{t} \lambda(s)ds}dt'.$$

(2)

The first term describes the number unemployed at the start of the month who remain unemployed by date $t$. The second describes all those laid-off at some date $t' \in [n, t]$ and have also failed to find employment by date $t$.

Given there is no other available information, the first identifying assumption is that the entry rates of new unemployed workers and new vacancies are constant within the month. As the data record the total inflows within the month, which we denote as $u_n, v_n$, this identifying restriction implies $u(t') = u_n, v(t') = v_n$ for all $t' \in [n, n + 1)$, and (2) reduces to

$$U(t) = U_n e^{-\int_{n}^{t} \lambda(s)ds} + u_n \int_{n}^{t} [1 - p(t')]e^{-\int_{n}^{t} \lambda(s)ds}dt'.$$  

The second identifying restriction uses a plausible approximation suggested by the data. In both the Blanchard and Diamond [1989] data, and the data used here (see Figures 2 and 3 below), the proportional monthly change in the stock of unemployment and vacancies is small. Assuming the matching elasticities of $\lambda, p$ with respect to the stock variables are not too high (see the estimates reported below) and given the identifying restriction that the flow variables are constant within the month, a reasonable approximation is that $\lambda, p$ do not vary much within the month. In that case, assuming $\lambda(t) = \lambda_n, p(t) = p_n$ for all $t \in [n, n + 1)$, integrating the above now gives

$$U(t) = U_n e^{-\lambda_n(t-n)} + u_n[1 - p_n] \frac{1 - e^{-\lambda_n(t-n)}}{\lambda_n}.$$  

Computing expected total matches over the month, denoted $M_n$, where

$$M_n = \int_{n}^{n+1} M(t)dt,$$

and using the above identifying assumptions implies

$$M_n = \int_{n}^{n+1} [\lambda_n U(t) + p_n u_n]dt.$$  

Substituting out $U(t)$ using the above and integrating implies the following Proposition.

Proposition 1: The Temporally Aggregated Matching Function

Given the identifying assumptions

(i) $u, v$ are constant within the period, and

(ii) $\lambda, p$ are constant within the period,
then expected total matches over the period are
\[ M_n = U_n[1 - e^{-\lambda_n}] + u_n p_n + u_n (1 - p_n) \left[ \frac{e^{-\lambda_n} - 1 + \lambda_n}{\lambda_n} \right]. \]  
(3)

Proof follows directly from the above.

The temporally aggregated matching function described in (3) is composed of three terms. The first describes those in the initial stock of unemployed workers who successfully match within the month, the second describes those laid off who immediately find work, and the third describes those laid off who subsequently match with a new vacancy.

This temporally aggregated matching function was first identified by Gregg and Petrongolo [2001]. The argument is symmetric for vacancies. In particular, if a new vacancy matches immediately with probability \( q \) and if it fails to match immediately subsequently matches at rate \( \mu \), symmetry implies

\[ M_n = V_n[1 - e^{-\mu_n}] + v_n \left[ 1 - (1 - q_n) \frac{1 - e^{-\mu_n}}{\mu_n} \right]. \]  
(4)

Estimating these matching equations (3) and (4) consistently involves the construction of suitable ‘at risk’ measures. However as the different matching theories imply different identifying restrictions, we consider each separately.

4 Identification with temporally aggregated data.

4.1 Random Matching

Random matching implies the identifying restriction \( p_n = 0 \), which says that it takes time for an unemployed job seeker to locate a suitable vacancy. The temporally aggregated matching function described in Proposition 1 is then

\[ M_n = U_n[1 - e^{-\lambda_n}] + u_n \left[ \frac{e^{-\lambda_n} - 1 + \lambda_n}{\lambda_n} \right]. \]  
(5)

Now define \( \overline{U}_n \) as that number where

\[ M_n = \overline{U}_n[1 - e^{-\lambda_n}]. \]

As each unemployed worker matches at rate \( \lambda_n \) over month \( n \), and as \( M_n \) is the expected total number that match over the month, then \( \overline{U}_n \) as defined must describe the average number of unemployed workers ‘at risk’ over that month. (5) implies the relevant measure for \( \overline{U}_n \) is:

\[ \overline{U}_n = U_n + \frac{e^{-\lambda_n} - 1 + \lambda_n}{\lambda_n[1 - e^{-\lambda_n}]} u_n. \]  
(6)
To see that this is the appropriate monthly 'at risk' measure of unemployment with temporally aggregated data, first suppose $\lambda_n \approx 0$, that each unemployed worker matches very slowly. (6) then implies $\overline{U}_n \approx U_n + 0.5 u_n$ which follows as each newly unemployed worker is 'at risk' in the market for on average half of the month. In contrast suppose $\lambda_n \to \infty$ which implies $U_n + u_n$ is the effective total number 'at risk' as each unemployed worker who enters the market matches immediately. (6) therefore computes the consistent 'at risk' measure of unemployment for all possible matching rates $\lambda_n \geq 0$.

The argument is symmetric for vacancies. Random matching implies $q_n = 0$, it takes time to find a job seeker, and so (4) implies

$$M_n = V_n[1 - e^{-\mu_n}] + v_n \left[1 - \frac{1 - e^{-\mu_n}}{\mu_n}\right]. \quad (7)$$

Now define $\overline{V}_n$ as that number where

$$M_n = \overline{V}_n[1 - e^{-\mu_n}],$$

and note that (7) implies

$$\overline{V}_n = V_n + \frac{e^{-\mu_n}}{\mu_n[1 - e^{-\mu_n}]} v_n, \quad (8)$$

where the same 'at risk' interpretation for $\overline{V}_n$ applies.

Given these at risk measures $\overline{U}_n, \overline{V}_n$, (5) and (7) together imply

$$\overline{U}_n[1 - e^{-\lambda_n}] = \overline{V}_n[1 - e^{-\mu_n}], \quad (9)$$

which is an identifying restriction - the number of workers who match must equal the number of vacancies that match.

Given $\lambda(t) = \lambda_n$ within the month, a useful econometric specification for $\lambda^M$ is

$$\lambda_n = \lambda^M(\overline{U}_n, \overline{V}_n; \theta), \quad (10)$$

which says that the average matching rate of an unemployed job seeker in month $n$, depends on the average number of vacancies which are 'at risk' over that month, with crowding out by the average number of job seekers 'at risk' over that month, where $\theta$ are the underlying parameters to be estimated. Most importantly, specification (10) now implies the data are exactly identified. Given $\theta$ and period $n$ data $(u_n, v_n, U_n, V_n)$, the equations (6),(8) for $\overline{U}_n$ and $\overline{V}_n$, the identifying restriction (9) and the functional form (10) provide 4 equations for the four period $n$ unknowns $\lambda_n, \mu_n, \overline{U}_n, \overline{V}_n$ [where $p_n = q_n = 0$]. The predicted number of matches in the month is then

$$M_n(\theta) = \overline{U}_n[1 - e^{-\lambda_n}], \quad (11)$$
where the identifying restriction (9) implies this measure is consistent with both of
the temporally aggregated matching functions defined in (3) and (4).5

The above has essentially assumed a continuum of agents to compute the average
number of agents at risk over the month. The data however is generated by atomistic
agents, where realised total matches $\tilde{M}_n$ are the outcome of a random matching pro-
cess. In each month there is approximately $\tilde{U}_n$ ‘trials’ where, in the United Kingdom,
$\tilde{U}_n$ is of the order of a million. As each worker has a monthly matching probability
of around 1/6 [expected duration of unemployment in the United Kingdom for this
period of time is around 6 months] we assume these large numbers imply realised
matches, denoted $\tilde{M}_n$, are approximately normally distributed with mean $M_n(\theta)$. We
can then estimate $\theta$ using standard MLE techniques.

4.2 Stock-Flow Matching

The same approach applies to stock flow matching, but the identifying restrictions
are different. Stock flow matching implies that those new vacancies which match
immediately, do so with an unemployed worker who is on the long side of their
particular submarket. Using (3) and (4), this implies

$$q_n v_n = U_n [1 - e^{-\lambda_n}] + u_n (1 - p_n) \left[ 1 - \frac{1 - e^{-\lambda_n}}{\lambda_n} \right],$$

which says that the number of unemployed workers who match with the flow of new
vacancies equals the number of new vacancies that match immediately. Now define $\tilde{U}_n$ where

$$\tilde{U}_n = U_n + \frac{e^{-\lambda_n} - 1 + \lambda_n}{\lambda_n [1 - e^{-\lambda_n}]} (1 - p_n) u_n,$$

and write this matching condition more simply as

$$q_n v_n = (1 - e^{-\lambda_n}) \tilde{U}_n.$$

Symmetry implies the appropriate at risk measure for vacancies, $\tilde{V}_n$, is

$$\tilde{V}_n = V_n + \frac{e^{-\mu_n} - 1 + \mu_n}{\mu_n [1 - e^{-\mu_n}]} (1 - q_n) v_n,$$

and stock-flow matching implies the identifying restriction:

$$p_n u_n = (1 - e^{-\mu_n}) \tilde{V}_n,$$

where the inflow of new unemployed workers potentially match immediately with the
current stock of vacancies.

Given the data is temporally aggregated, we adopt the econometric specification:

$$\lambda_n = \lambda^{SF}(v_n, \tilde{U}_n; \theta),$$

$$p_n = p^{SF}(u_n, \tilde{V}_n; \theta),$$

10
which says that the stock of unemployed workers $U_n$ match at rate $\lambda_n$ with the flow of new vacancies $v_n$, while the flow of newly laid-off workers potentially match immediately, with probability $p_n$, with the stock of vacancies $V_n$.

Again these specifications imply the theory is exactly identified. This time given $\theta$ and period $n$ data, we have 6 equations which jointly determine $(\overline{U}_n, \overline{V}_n, \lambda_n, \mu_n, p_n, q_n)$. Expected matches are then

$$M_n(\theta) = \overline{U}_n(1 - e^{-\lambda_n}) + p_n u_n,$$  \hfill (18)

and we use MLE techniques to estimate $\theta$.

### 4.3 Efficiency Wage

This is a special case of stock flow matching with $p = 0, q = 1$; i.e. there is one-sided stock flow matching where the stock of unemployed workers matches with the flow of vacancies. This implies a directly testable, over-identifying restriction for the stock-flow case.

### 5 Estimation

As explained above, the different matching theories, random or stock-flow, imply different identifying restrictions on the temporally aggregated data. Given those identifying restrictions and the data $(u_n, U_n, v_n, V_n)$, (11) for random matching and (18) for stock-flow matching imply expected matches $M_n(\theta)$ in month $n$, where $\theta$ are the parameters of interest. Given the actual measure $\overline{M}_n$ of matches in month $n$, the set of parameters $\theta$ is estimated by non-linear least squares; i.e.

$$\min_{\theta} \sum_n [\overline{M}_n - M_n(\theta)]^2,$$  \hfill (19)

and we assume the residual error is approximately normal to construct standard errors.

### 5.1 The data

Clearly this approach requires data which distinguish between vacancy flows $v$ and stocks $V$. Using inches of help-wanted advertisements to measure vacancies, as is the general procedure for the United States,\(^4\) is not sufficient as there is no information on whether a particular job advertisement is new or is a re-advertisement. However Job Center data provides this information for the U.K. labor market.

The U.K. Job Center system is a network of government funded employment agencies, where each town/city typically has at least one Job Center. A Job Center’s services are free of charge to all users, both to job seekers and to firms advertising
vакансий. Истинно, чтобы получать пособие, неработающий получатель в Великобритании обязан зарегистрироваться в Бюро по безработице.

Большинство вакансий на Центрах по бирже труда являются мало- и полувыездными. Конечно, профессионально подготовленные люди мало вероятно найдут работу там. Тем не менее, как большая часть безработицы приходится на мало- и полувыездных работников, а не на профессионалов, кажется разумным, что понимание определяющих факторов риска безработицы в этом уровне соответствия оказывает полезную дифференцирующую информацию между конкурирующими теориями равновесной безработицы.

Данные являются месячным временем ряда, начиная с сентября 1985 года и до декабря 1999 года [172 наблюдений]. Данные не только включают число безработных (U_n) и число незанятых вакансий (V_n) за предыдущий месяц в Великобритании, но также число новых зарегистрированных искателей труда (u_n) и новых вакансий (v_n) в каждом месяце n. Данные также включают число вакансий, которые были заполнены (которое предоставляет наш показатель соответствий M_n) и те, которые были отозваны работодателями без заполнения.

Все данные, используемые в настоящем, извлечены из базы данных NOMIS и не сезонно приспособлены. Эти данные представлены на Фигурах 2 и 3. Как также указано в данных Бланчард и Диаманд [1989], Фигура 3 показывает, что число работников, которые соответствуют в течение каждого месяца, тесно коррелирует с инфлюном новых вакансий, в то время как запас вакансий слабо коррелирует с соответствием. Коэффициент корреляции составляет 0.78 и 0.08 соответственно. Для безработных, корреляционный коэффициент между инфлюном и сопутствующим излучением составляет 63%, а между сопутствующим излучением и запасом 50%. Данные также показывают, что турновская скорость по вакансиям гораздо выше, чем по безработным, отношение сопутствующего излучения/запаса вакансий составляя 0.15 для безработных и 1.12 для вакансий.

[Фигуры 2 и 3 здесь]

Есть несколько данных вопросов. Во-первых, вакансии, предложенные в Центрах по бирже труда, являются только частью существующих вакансий - Грегг и Уадsworth [1996] сообщили, что Центры по бирже труда используют лишь около 50% работодателей. Есть два способа получения подходящего масштабирования для нашего показателя занятости M_n, V_n, v_n. Во-первых, если M_n являются общими использованием Великобритании, показателем занятости, тогда \lambda_n = M_n / U_n было бы среднее время ухода с безработицы в месяц n, и, следовательно, 1/\lambda_n = U_n / M_n было бы среднее ожидаемое время безработицы [измеренное в месяцах]. Рассчитывая среднее значение 1/\lambda_n, используя M_n, имелось бы среднее время безработицы около 14 месяцев. В противоположность этому, среднее время безработицы для этого периода составляет около 6.5 месяцев. Соотношение [6.5]:[14] равно 0.46, оно говорит, что показатель Центров по бирже труда занятости, который соответствует числу вакансий, которые были заполнены, M_n, приближается к 46% реального в Великобритании общего числа соответствий. Альтернативный подход состоит в том, чтобы отметить, что общие хиры могут быть предсказаны M_n = u_n + \Delta N_n, где \Delta N_n - это изменение в сопутствующем излучении в общем
employment and $u_n$ is the inflow into unemployment in month $n$. $\gamma_n = M_n/H_n$ is then the share of total U.K. matches recorded in Job Centres. According to our data, $\gamma_n$ ranges between 0.25 and 0.75, with an average of 0.44, and does not display any definite trend over the sample period.

Given both these figures are broadly consistent with Gregg and Wadsworth [1996], we assume that Job Center vacancies report 44% of total vacancies in the United Kingdom, and so rescale all Job Center vacancy measures $M_n$, $V_n$, and $v_n$, by dividing through by 0.44. However it should be noted that this rescaling is largely cosmetic. As we use log linear functional forms [i.e. Cobb-Douglas matching specifications], such rescaling has little qualitative impact on the empirical results - its main effect is to renormalise the intercept term. Nevertheless this rescaling is relevant for two reasons: (i) it ensures that the predicted average duration of unemployment is consistent with the data [around 6.5 months], and (ii) the identifying restriction - that the measured number of vacancies which match is equal to the number of unemployed job seekers that find work - is not unreasonable.\(^7\)

A more fundamental data issue is that the time series for the stocks of unemployment and vacancies are not stationary.\(^8\) Indeed there is quite a literature on so-called shifting ‘Beveridge curves’ (see Petrongolo and Pissarides [2001] for a survey). Of course the matching structure defined above describes short-run variations in matching rates due to short-run variations in labour market conditions. It cannot be used to explain long-run matching trends due to, say, changes in the composition of the workforce [more workers now attend higher education], or changes in job skill requirements, or even medium-term regional migration.

To focus on explaining the short-run variations on observed matching rates, an obvious approach is to detrend the data series, as already done in the matching literature by Yashiv [2000]. Our first set of results, Tables 1 and 2, uses detrended data obtained by filtering all time series with a Hodrick-Prescott [1997] filter with smoothing parameter equal to 14400. To preserve series means we have added to the detrended series their sample averages.

Our second set of results, which are presented in Tables 3 and 4 in the Appendix, use the original, non-filtered data but include year dummies to capture any long term trends. The advantage of using year dummies instead of HP filtering is that, by estimating structural breaks and matching function parameters simultaneously, it allows for a possible correlation between shift variables and other right-hand side variables. The disadvantage is that it generates discontinuous “jumps” at arbitrary discontinuity points, instead of a smooth long-run trend.

To keep the discussion coherent, the main text shall report and comment only on the results obtained using the HP filtered data. However comparing Table 1 results against those in Table 3, and Table 2 against Table 4 quickly establishes that the results are qualitatively identical. To see the difference between the two approaches, Figures 4 and 5 plot predicted matches for the parameter estimates obtained when estimating the random matching function using the HP filtered data (Figure 4, with
parameters given by Table 1) and using the non-filtered data with year dummies (Figure 5, with parameters given by Table 3 in the Appendix).

As one would expect, the estimates using the filtered data imply predictions which at times drift away from actual matches, but do a good job at reproducing the short-run fluctuations. In contrast, while the estimates using non-filtered data and year dummies do not explain the short-run fluctuations so well, they do not drift so much from the actual series. As it seems less appropriate to identify short-run matching behaviour based on medium to long run trends in the data, we prefer the results obtained using the HP filtered data, but note that the discussion is qualitatively identical for both approaches.

5.2 Results

5.2.1 Random Matching

Given some initial parameters $\theta_0$, then for each observation $n = 1, ..., 172$, we solve numerically (6)-(10) for $U_n, V_n, \lambda_n, \mu_n$. Predicted matches for each $n$ are then $M_n(\theta_0) = U_n[1 - \exp(-\lambda_n)]$. Assuming residual errors are normally distributed, a maximum likelihood estimator is obtained using a standard hill-climbing algorithm.

Given the identifying restrictions for random matching, Table 1 describes the MLE results using various functional forms for $\lambda_n = \lambda^M(\cdot)$. As the data is not seasonally adjusted, all estimated equations include monthly dummies, which turn out to be jointly significant in all specifications.\(^9\)

Column 1 assumes the standard Cobb-Douglas specification - that

$$\lambda_n = \exp[\alpha_0 + \alpha_1 \ln V_n + \alpha_2 \ln U_n].$$

The coefficients on (time-aggregated) vacancies and unemployment have the expected sign and are significantly different from zero. Constant returns to scale in the matching function are not rejected, given a Wald test statistics of 1.536 on the restriction $\alpha_1 = -\alpha_2$. However unlike results typically found in the empirical matching literature, the estimated parameter value on $V$ is remarkably close to unity. This suggests an aggregate matching function $M \approx \alpha_0 V$; i.e. that matching over this period is entirely driven by the availability of vacancies, and there is pure crowding out by the stock of unemployed job seekers.\(^{10}\)

Column 2 imposes constant returns to matching. In columns 1 and 2 the predicted value of $\lambda_n$ is consistent with an expected unemployment duration of 6.8 months (computed as the sample average of $1/\lambda_n$), in line with the actual unemployment duration during the sample period.
Column 3 describes a robustness check. Recall that random matching implies the matching rate of individual workers does not depend directly on the inflow of new vacancies. While still controlling for temporal aggregation effects as described above, Column 3 asks whether including the flow of new vacancies as an added explanatory variable for $\lambda_n$ improves the fit. In fact the fit is not only much improved, the vacancy stock variable becomes insignificant and wrong signed. Column 4 drops the vacancy stock term and the fit is essentially unchanged. In both Columns 3 and 4 constant returns in the matching function are rejected in favour of decreasing returns, although imposing constant returns does not reduce substantially the log-likelihood or the goodness of fit [Column 5].

The results of Table 1 establish that random matching and the temporal aggregation of the data do not explain the high correlation between total monthly matches and the inflow of new vacancies. Of course columns 3-5 are inconsistent with the identifying assumptions for Table 1, that the stock of unemployed job seekers matches with the stock of vacancies. To obtain consistent estimates of the matching process we now turn to the stock-flow identifying restrictions.

### 5.2.2 Stock-Flow Matching

Under stock-flow matching, the at risk measures and matching probabilities $\bar{U}_n, \bar{V}_n, \lambda_n, \mu_n, p_n, q_n$ are obtained by numerically solving (12)-(17) for each $n$, and a maximum likelihood estimator is obtained using a hill climbing algorithm which solves (19). Assuming errors are normally distributed, the results for stock-flow matching are reported in Table 4 under alternative specifications for $\lambda_n = \lambda_{SF}(.)$ and $p_n = p_{SF}(.)$. Recall that in contrast to random matching, stock flow matching implies $\lambda_n$ depends on the vacancy in flow and not on the stock of vacancies, while the efficiency wage hypothesis predicts also $p_n = 0$.

[Table 2 around here]

Column 1 adopts the functional form

$$\lambda_n = \exp \left( \alpha_0 + \alpha_1 \ln \bar{V}_n + \alpha_2 \ln v_n + \alpha_3 \ln \bar{U}_n \right),$$

while $p_n$ is estimated as a constant parameter, and constrained to be non-negative, i.e. $p_n = \exp(\beta_0)$. Note that the stock-flow identifying restrictions are only consistent with $\alpha_1 = 0$, while the efficiency wage approach implies $\beta_0 \ll 0$. The results of column 1 in Table 2 find that the vacancy stock effect is indeed insignificant [and wrong-signed], but the initial matching rate of the newly unemployed, $p_n$, is around 8%, and significantly different from zero, with a standard error of 0.030$^{11}$ In column 2 we drop the vacancy stock from the specification of $\lambda_n$, and re-estimate. The parameter estimates are largely unchanged and strongly support the notion that the exit rates of the longer term unemployed, $\lambda_n$, are driven by the inflow of new vacancies with an estimated elasticity which is very close to unity.
Columns 3-5 consider a more general specification for  
\[ p_n = \exp (\beta_0 + \beta_1 \ln V_n + \beta_2 \ln v_n + \beta_3 \ln u_n) , \]
while leaving the specification of \( \lambda_n \) as in column 2 [which is consistent with the identifying assumptions]. Unfortunately and perhaps due to the complexity of having to solve jointly 6 non-linear identifying equations, we found that the parameter estimates only converged when we imposed constant returns on the estimation routine; i.e. set \( \alpha_2 + \alpha_3 = 0 \) and \( \beta_1 + \beta_2 + \beta_3 = 0 \). We are therefore unable to test for constant returns to matching. However, imposing constant returns generates a remarkable result - the column 3 estimates for \( \lambda_n \) are very close to the Shapiro and Stiglitz [1984] prediction that \( \lambda = v/U \); recall we are estimating a log-linear form and estimates find that the constant term is close to zero and the elasticities are close in value to unity [also see column 5 in Table 1, though note that those estimates are inconsistent with the identifying assumptions]. Unfortunately the specification for \( p_n \) is not well determined.

As including the vacancy inflow \( v \) in the specification of \( p \) is inconsistent with the identifying assumptions, column 4 re-estimates by omitting \( v_n \) from \( p_n \) while imposing constant returns.\(^{12}\) The fit in Column 4 is largely unchanged and the estimated parameters for \( p_n, \lambda_n \) are all significant and correctly signed. The estimates for \( \lambda_n \) again suggest \( \lambda \approx v/U \) (i.e. the longer term unemployed are job rationed) while the probability a newly laid off worker obtains immediate re-employment is an increasing function of the number of vacancies currently on the market, with crowding out by other recently laid off workers. The sample averages of column 4 imply that of the newly unemployed, around 13% on average find immediate re-employment. Of those that enter the stock of longer term unemployment, their average matching rate \( \lambda_n \approx 0.124 \), which implies an expected duration of unemployment of 8.5 months. Conditional on being laid-off, the average duration of unemployment is 7.5 months. Column 4 provides strong support for the notion of stock-flow matching.

However, there are some caveats. Rather than drop the vacancy flow term in \( p_n \), column 5 instead drops the vacancy stock term. Though inconsistent with the identifying assumptions of Table 2, column 5 provides us with a simple robustness check. Somewhat reassuringly the estimates for \( \lambda_n \) are largely unchanged, but as the overall fit is no worse than column 4, it suggests that \( p_n \) is not well determined. Perhaps this finding is not overly unsurprising. Assuming no search frictions is convenient - it implies a very clean ‘stock-flow’ identifying structure - but is also somewhat extreme. One might expect that even those workers whose job skills are in great demand might still take a week or two to identify a suitable new employer. However, although \( p_n \) is not very well determined, the empirical results are not inconsistent with the identifying stock-flow assumptions. More importantly, the estimates for \( \lambda_n \) are robust and have an obvious economic interpretation - the longer term unemployed are job rationed.
An additional useful consistency check is to note that the parameter estimates for $(p, \lambda)$ and the identifying assumptions imply equivalent information for the probability a vacancy is immediately filled $(q)$, and the matching rate of a vacancy that does not fill immediately $(\mu)$. The results of column 4 in Table 2 imply average $q = 0.556$; i.e. 56% of all vacancies posted fill immediately. Of those that fail to match immediately, average $\mu = 0.075$. Of course, the high initial matching probability $q$ reflects the job rationing interpretation of the data, where $\lambda = v/U$ across the longer term unemployed.

In fact these numbers are not wholly inconsistent with observed vacancy hazard rates. Figure 6, taken from Coles and Smith [1998], describes the empirical hazard rate at which a vacancy posted in a U.K. Job Center is filled by duration. The proportion of new vacancies filled on their first day of being posted in a U.K. Job Center is indeed very high (around 30%), the vacancy hazard rate being much lower thereafter. Our interpretation is that there are many unemployed workers chasing new vacancies as they come onto the market. In fact U.K. Job Centers have a clear weekly cycle. Mondays are very busy - this is when most new vacancies are posted and is the day most active job seekers choose to check the boards - while Fridays are relatively quiet. Indeed given such advertising behaviour, it is perhaps not surprising that the random matching approach does not describe the data well. Of course it might be argued that firms often have an employee in mind when a vacancy is posted, and this ‘cliff’ reflects a legal requirement that all vacancies must be advertised. Nevertheless, the above maximum likelihood estimates do not use this hazard information, and yet obtain results - corrected for temporal aggregation bias - which strongly support this notion - that the stock of unemployed workers chase new vacancies as they enter Job Centers [on Mondays].

However the average duration of a vacancy predicted by column 4 is $(1 - q)/\mu = 6$ months which is much greater than the actual average duration which is around 3 weeks. This reflects that the predicted exit rate $\mu$ is very low. This can be partially explained by noting that over 30% of vacancies that are not filled are actually withdrawn from the Job Center. By ignoring this alternative exit process for vacancies [which would require a more complicated competing risk model] we systematically underpredict the exit rate of unfilled vacancies and so overestimate their average durations. However, it is noticeable that the random matching estimates [see Table 1] do much better in this dimension, giving an estimated average vacancy duration of 7 weeks. This result again suggests that $p_n$ - which describes the matching experience of workers on the other side of the market - is not so well determined as $\lambda_n$. 
# Conclusions

This paper has described a new methodology for estimating matching functions. Most of the previous literature has largely ignored the temporal aggregation problem and simply regressed total matches on the stock variables at the beginning of the month. This paper has shown that correcting for temporal aggregation of the data requires constructing ‘at risk’ variables, which depend not only on the initial stocks but also on the inflow of new participants during the month. The main problem is obtaining data on inflows. However when such data exists, we have also shown that an alternative identifying structure exists, stock-flow matching, which allows the econometrician to discriminate between different matching theories.

Using U.K. Job Center data for the period 1985-99, our main finding is that the inflow of new vacancies plays a much more direct role in matching than is consistent with random matching. The results strongly suggest that most unemployment experience in the United Kingdom over this period is due to job rationing. Around 87% of newly unemployed workers are on the long side of the market, where finding a job relies on waiting for new suitable vacancies to come onto the market, and where the workers’ resulting average duration of unemployment exceeds 6 months. This job queue interpretation of the data has important implications for government policy. The underlying market failure is more likely a wage distortion [leading to job queue formation] than a job search failure and so a Shavell-Weiss type argument, that UI payments should decrease with duration to encourage greater search effort, may not be empirically relevant.

These results also provide an important direction for future research. Although the ‘stock-flow’ identifying structure seems to capture well the matching experience of the longer term unemployed, it does not seem to capture so well the unemployment experience of the very short term unemployed. An alternative is to assume that search frictions bind for those workers who are on the short side of their markets (it takes time to plough through the set of all possible employment opportunities) but do not bind for those workers on the long side of their markets (they have to wait for suitable new vacancies to arrive in the market). As in the hazard function literature [e.g. Lancaster and Nickell (1980)], this implies unobserved heterogeneity across workers, with two types who match at different rates. However in the previous hazard function literature, it has been assumed that matching rates are captured by the vacancy/unemployment ratio. Perhaps the critical contribution here is that this may not be capturing the essential matching experience of most unemployed job seekers. Those workers who match slowly do so because they are on the long-side of the market, and the inflow of new vacancies is the relevant variable when describing their matching rates.
7 Appendix

The following Tables repeat the above estimation procedures, using the original [untransformed] data but including year dummies.

[Tables 3 and 4 around here]

University of Essex
London School of Economics
References


Notes
1. But see also Cahuc and Lehman [2000] for a challenge of this view.
2. Using the end-of-month stock $U_{n+1}$ as a regressor for $M_n$ would not offer a solution to this problem. In this case end-of-month stocks would be depleted by matches, thus generating a simultaneity (downward) bias in the estimated effect of unemployment on matches (see Berman [1997] for a discussion on this).
3. In contrast, Gregg and Petrongolo [2001] do not compute these at risk measures and instead estimate (3) assuming $\lambda = \lambda^M(U_n, V_n; \theta)$ which ignores the matching effects due to the inflow of new vacancies.
5. Gregg and Wadsworth [1996] report that Job Centres are used by roughly 80 percent of the claimant unemployed, 30 percent of employed job seekers and 50 percent of employers.
6. The employment series is also extracted from the NOMIS databank.
7. We have re-estimated the Tables that follow without rescaling the vacancy measures [results not reported here]. The results are qualitatively identical, the main difference being much smaller estimated intercept terms and so higher predicted average durations of unemployment.
8. $ADF$ statistics (with 4 lags) are $-1.181$ and $-0.806$, respectively, against a 5% critical value of $-3.12$.
9. The exact specification used for predicted matches is $M_n(\theta) = U_n[1-\exp(-\lambda_n)] +$ dummies.
10. This property is also confirmed by estimating a log-linear matching function à la Blanchard and Diamond [1989] on detrended data (results not reported here). Similar findings are obtained by Burgess and Profitt [2001]. They use Job Center data disaggregated at the regional level, but do not control for temporal aggregation effects.
11. Using the delta method: $var(p_n) = \exp(2 \cdot \beta_0)var(\beta_0) = 0.0009$.
12. Note, only Columns 2 and 4 in Table 2 are consistent with the stock-flow identifying assumptions. The other specifications describe robustness checks.
Table 1: Estimation results under random matching

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln[λₙ] constant</td>
<td>-3.297</td>
<td>-0.572</td>
<td>4.126</td>
<td>2.552</td>
<td>-0.195</td>
</tr>
<tr>
<td></td>
<td>(2.194)</td>
<td>(0.097)</td>
<td>(1.301)</td>
<td>(0.967)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>ln Vₙ</td>
<td>1.231</td>
<td>1.133</td>
<td>-0.170</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.122)</td>
<td>(0.086)</td>
<td>(0.113)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln vₙ</td>
<td>-</td>
<td>-</td>
<td>1.141</td>
<td>1.063</td>
<td>1.112</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.070)</td>
<td>(0.042)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>ln Uₙ</td>
<td>-1.038</td>
<td>-1.133a</td>
<td>-1.276</td>
<td>-1.255</td>
<td>-1.112a</td>
</tr>
<tr>
<td></td>
<td>(0.111)</td>
<td>(0.053)</td>
<td>(0.054)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-0.03003</td>
<td>-0.03024</td>
<td>-0.00747</td>
<td>-0.00762</td>
<td>-0.00795</td>
</tr>
<tr>
<td>R²</td>
<td>0.811</td>
<td>0.810</td>
<td>0.953</td>
<td>0.952</td>
<td>0.950</td>
</tr>
<tr>
<td>CRSᵇ</td>
<td>1.536</td>
<td>-</td>
<td>10.429</td>
<td>7.528</td>
<td>-</td>
</tr>
<tr>
<td>monthly dummies = 0ᶜ</td>
<td>61.111</td>
<td>69.400</td>
<td>231.345</td>
<td>255.691</td>
<td>278.549</td>
</tr>
</tbody>
</table>

Sample averages:

<table>
<thead>
<tr>
<th></th>
<th>λₙ</th>
<th>1/λₙ</th>
<th>μₙ</th>
<th>1/μₙ</th>
</tr>
</thead>
<tbody>
<tr>
<td>λₙ</td>
<td>0.151</td>
<td>0.151</td>
<td>0.143</td>
<td>0.143</td>
</tr>
<tr>
<td>1/λₙ</td>
<td>6.8</td>
<td>6.8</td>
<td>7.3</td>
<td>7.3</td>
</tr>
<tr>
<td>μₙ</td>
<td>0.591</td>
<td>0.597</td>
<td>0.560</td>
<td>0.560</td>
</tr>
<tr>
<td>1/μₙ</td>
<td>1.7</td>
<td>1.7</td>
<td>1.8</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Notes. Monthly data not seasonally adjusted. All series are detrended with a HP filter with smoothing a parameter equal to 14400. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly dummies. Estimation method: non-linear least squares. Heteroskedastic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of 1/λₙ and 1/μₙ, respectively. No. Observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.

b. Wald test, distributed as χ²(1), of the hypothesis that the sum of the coefficients on ln Vₙ, ln vₙ and ln Uₙ is zero. Critical value at 5% significance level: χ²(1) = 3.841.

c. Wald test, distributed as χ²(11), of the hypothesis that monthly dummies are jointly zero. Critical value at 5% significance level: χ²(11) = 19.675.

d. ADF statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: -2.23.
Table 2: Estimation results under stock-flow matching

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln \lambda_n )</td>
<td>constant</td>
<td>4.587</td>
<td>2.771</td>
<td>-0.153</td>
<td>-0.238</td>
</tr>
<tr>
<td></td>
<td>(1.251)</td>
<td>(1.064)</td>
<td>(0.055)</td>
<td>(0.059)</td>
<td>(0.403)</td>
</tr>
<tr>
<td>( \ln V_n )</td>
<td>-0.106</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.089)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \ln v_n )</td>
<td>1.110</td>
<td>1.090</td>
<td>1.167</td>
<td>1.181</td>
<td>1.138</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.046)</td>
<td>(0.058)</td>
<td>(0.064)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>( \ln U_n )</td>
<td>-1.347</td>
<td>-1.300</td>
<td>-1.167a</td>
<td>-1.181a</td>
<td>-1.138a</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.068)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \ln p_n )</td>
<td>constant</td>
<td>-2.535</td>
<td>-2.636</td>
<td>-3.022</td>
<td>-2.310</td>
</tr>
<tr>
<td></td>
<td>(0.374)</td>
<td>(0.398)</td>
<td>(0.628)</td>
<td>(0.531)</td>
<td>(3.177)</td>
</tr>
<tr>
<td>( \ln V_n )</td>
<td>-</td>
<td>-</td>
<td>0.845</td>
<td>0.459</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.644)</td>
<td>(0.234)</td>
<td></td>
</tr>
<tr>
<td>( \ln v_n )</td>
<td>-</td>
<td>-</td>
<td>-1.642a</td>
<td></td>
<td>0.198</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1.125)</td>
<td></td>
<td>(3.479)</td>
</tr>
<tr>
<td>( \ln u_n )</td>
<td></td>
<td></td>
<td></td>
<td>0.797a</td>
<td>-0.459a</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log-likelihood</td>
<td>-0.00729</td>
<td>-0.00739</td>
<td>-0.00754</td>
<td>-0.00776</td>
<td>-0.00785</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.954</td>
<td>0.954</td>
<td>0.953</td>
<td>0.951</td>
<td>0.951</td>
</tr>
<tr>
<td>( CRS^b )</td>
<td>12.295</td>
<td>10.041</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>monthly dummies = 0c</td>
<td>215.251</td>
<td>243.556</td>
<td>207.491</td>
<td>200.159</td>
<td>258.835</td>
</tr>
<tr>
<td>( ADF^d )</td>
<td>-4.645</td>
<td>-4.698</td>
<td>-4.462</td>
<td>-4.632</td>
<td>-5.764</td>
</tr>
</tbody>
</table>

Sample averages:

| \( \lambda_n \) | 0.132   | 0.133   | 0.137   | 0.124   | 0.132   |
| \( p_n \)        | 0.079   | 0.072   | 0.042   | 0.131   | 0.075   |
| \( (1 - p_n) / \lambda_n \) | 7.4   | 7.4   | 7.4   | 7.5   | 7.4   |
| \( \mu_n \)      | 0.046   | 0.042   | 0.025   | 0.075   | 0.042   |
| \( q_n \)        | 0.593   | 0.598   | 0.615   | 0.556   | 0.593   |
| \( (1 - q_n) / \mu_n \) | 9.1   | 9.9   | 17.7   | 6.0    | 9.9    |

Notes. Monthly data not seasonally adjusted. All series are detrended with a HP filter with smoothing a parameter equal to 14400. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly dummies. Estimation method: nonlinear least squares. Heteroskedastic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of \((1 - p_n) / \lambda_n\) and \((1 - q_n) / \mu_n\), respectively. No. observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.
b. Wald test, distributed as \(\chi^2(1)\), of the hypothesis that the sum of the coefficients on \(\ln V_n\) and \(\ln U_n\) is zero. Critical value at 5% significance level: \(\chi^2(1) = 3.841\).
c. Wald test, distributed as \(\chi^2(11)\), of the hypothesis that monthly dummies are jointly zero. Critical value at 5% significance level: \(\chi^2(11) = 19.675\).
d. ADF statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: \(−2.23\).
Table 3: Estimation results under random matching

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_n$</td>
<td>constant</td>
<td>$-13.853$</td>
<td>$-0.903$</td>
<td>$-1.057$</td>
<td>$-3.869$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($0.4099$)</td>
<td>($0.092$)</td>
<td>($2.402$)</td>
<td>($1.999$)</td>
</tr>
<tr>
<td>$\ln V_n$</td>
<td>$1.243$</td>
<td>$0.747$</td>
<td>$-0.233$</td>
<td>$-1.057$</td>
<td>$-3.869$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($0.151$)</td>
<td>($0.056$)</td>
<td>($0.146$)</td>
<td>($0.099$)</td>
</tr>
<tr>
<td>$\ln v_n$</td>
<td>-</td>
<td>-</td>
<td>$1.131$</td>
<td>$1.016$</td>
<td>$0.896$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>($0.87$)</td>
<td>($0.064$)</td>
<td>($0.035$)</td>
</tr>
<tr>
<td>$\ln U_n$</td>
<td>$-0.330$</td>
<td>$-0.747$</td>
<td>$-0.858$</td>
<td>$-0.778$</td>
<td>$-0.896$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($0.151$)</td>
<td>($0.056$)</td>
<td>($0.087$)</td>
<td>($0.068$)</td>
</tr>
</tbody>
</table>

Log-likelihood | $-0.03355$ | $-0.03737$ | $-0.00955$ | $-0.00984$ | $-0.01039$ |
$R^2$ | $0.927$ | $0.918$ | $0.979$ | $0.978$ | $0.977$ |
$CRS^b$ | $10.131$ | $-0.035$ | $4.558$ | - | - |
monthly dummies $= 0^c$ | $63.497$ | $72.597$ | $227.000$ | $228.913$ | $217.294$ |
yearly dummies $= 0^d$ | $291.810$ | $260.306$ | $351.451$ | $521.222$ | $530.837$ |
$ADF^e$ | $-6.921$ | $-4.485$ | $-5.685$ | $-5.945$ | $-5.861$ |

Sample averages:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_n$</td>
<td>$0.153$</td>
<td>$0.181$</td>
<td>$0.146$</td>
<td>$0.148$</td>
<td>$0.161$</td>
</tr>
<tr>
<td>$1/\lambda_n$</td>
<td>$7.6$</td>
<td>$6.2$</td>
<td>$7.6$</td>
<td>$7.6$</td>
<td>$7.0$</td>
</tr>
<tr>
<td>$\mu_n$</td>
<td>$0.576$</td>
<td>$0.712$</td>
<td>$0.577$</td>
<td>$0.574$</td>
<td>$0.630$</td>
</tr>
<tr>
<td>$1/\mu_n$</td>
<td>$1.8$</td>
<td>$1.5$</td>
<td>$1.9$</td>
<td>$1.9$</td>
<td>$1.7$</td>
</tr>
</tbody>
</table>

Notes. Monthly data not seasonally adjusted. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly and yearly dummies. Estimation method: non-linear least squares. Heteroskedastic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of $1/\lambda_n$ and $1/\mu_n$, respectively. No. Observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.
b. Wald test, distributed as $\chi^2(1)$, of the hypothesis that the sum of the coefficients on $\ln V_n$, $\ln v_n$ and $\ln U_n$ is zero. Critical value at 5% significance level: $\chi^2(1) = 3.841$.
c. Wald test, distributed as $\chi^2(11)$, of the hypothesis that monthly dummies are jointly zero. Critical value at 5% significance level: $\chi^2(11) = 19.675$.
d. Wald test, distributed as $\chi^2(14)$, of the hypothesis that yearly dummies are jointly zero. Critical value at 5% significance level: $\chi^2(14) = 23.685$.
e. $ADF$ statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: $-2.23$. 

25
### Table 4: Estimation results under stock-flow matching

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_n$</td>
<td>-0.161</td>
<td>-3.644</td>
<td>-0.718</td>
<td>-0.567</td>
<td>-0.643</td>
</tr>
<tr>
<td></td>
<td>(2.414)</td>
<td>(1.745)</td>
<td>(0.188)</td>
<td>(0.087)</td>
<td>(0.315)</td>
</tr>
<tr>
<td>$\ln \overline{V}_n$</td>
<td>-0.200</td>
<td>-0.161</td>
<td>-3.644</td>
<td>-0.567</td>
<td>-0.643</td>
</tr>
<tr>
<td></td>
<td>(0.099)</td>
<td></td>
<td>(2.414)</td>
<td>(0.188)</td>
<td>(0.087)</td>
</tr>
<tr>
<td>$\ln \sigma_n$</td>
<td>1.098</td>
<td>1.034</td>
<td>0.875</td>
<td>0.930</td>
<td>0.923</td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td>(0.073)</td>
<td>(0.057)</td>
<td>(0.041)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>$\ln \overline{U}_n$</td>
<td>-0.930</td>
<td>-0.817</td>
<td>-0.875</td>
<td>-0.930</td>
<td>-0.923</td>
</tr>
<tr>
<td></td>
<td>(0.096)</td>
<td>(0.058)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_n$ constant</td>
<td>-2.521</td>
<td>-2.269</td>
<td>-2.476</td>
<td>-2.181</td>
<td>-1.903</td>
</tr>
<tr>
<td></td>
<td>(0.487)</td>
<td>(0.416)</td>
<td>(0.693)</td>
<td>(0.513)</td>
<td>(0.884)</td>
</tr>
<tr>
<td>$\ln \overline{V}_n$</td>
<td>-0.161</td>
<td>-0.161</td>
<td>-1.778</td>
<td>-0.187</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1.057)</td>
<td>(0.511)</td>
<td></td>
</tr>
<tr>
<td>$\ln \sigma_n$</td>
<td></td>
<td>2.713</td>
<td></td>
<td></td>
<td>0.193</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.056)</td>
<td></td>
<td></td>
<td>(0.877)</td>
</tr>
<tr>
<td>$\ln \mu_n$</td>
<td></td>
<td>-0.935</td>
<td>0.187</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.099)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| log-likelihood | -0.00921 | -0.00956 | -0.00894 | -0.00982 | -0.00983 |
| $R^2$          | 0.980    | 0.979    | 0.980    | 0.978    | 0.978    |
| CRSb           | 12.295   | 3.122    | -       | -        | -        |
| monthly dummies = 0 $c$ | 216.745 | 211.398 | 235.091 | 184.676 | 210.050 |
| yearly dummies = 0 $d$ | 316.169 | 536.041 | 292.333 | 444.064 | 548.361 |
| $ADF^e$        | -6.508   | -6.774   | -6.459   | -6.787   | -6.059   |
| Sample averages: |       |       |       |       |       |
| $\lambda_n$   | 0.135    | 0.132   | 0.132   | 0.142    | 0.134    |
| $p_n$         | 0.080    | 0.103   | 0.104   | 0.102    | 0.167    |
| $(1 - p_n) / \lambda_n$ | 7.7     | 7.7     | 7.7     | 7.2      | 7.2      |
| $\mu_n$       | 0.053    | 0.067   | 0.070   | 0.070    | 0.105    |
| $q_n$         | 0.570    | 0.556   | 0.557   | 0.590    | 0.556    |
| $(1 - q_n) / \mu_n$ | 10.6   | 8.3     | 10.8    | 8.0      | 5.1      |

Notes. Monthly data not seasonally adjusted. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly and yearly dummies. Estimation method: non-linear least squares. Heteroskedastic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of $(1 - p_n) / \lambda_n$ and $(1 - q_n) / \mu_n$, respectively. No. observations: 171. Source: NOMIS.

---

$a$. Coefficient constrained to equal the value reported.

$b$. Wald test, distributed as $\chi^2(1)$, of the hypothesis that the sum of the coefficients on $\ln V_n$ and $\ln U_n$ is zero. Critical value at 5% significance level: $\chi^2(1) = 3.841$.

$c$. Wald test, distributed as $\chi^2(11)$, of the hypothesis that monthly dummies are jointly zero. Critical value at 5% significance level: $\chi^2(11) = 19.675$.

$d$. Wald test, distributed as $\chi^2(14)$, of the hypothesis that yearly dummies are jointly zero. Critical value at 5% significance level: $\chi^2(14) = 23.685$.

$e$. $ADF$ statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: $-2.23$. 

26
Figure 2: Unemployment stock, inflow and outflow in Britain: September 1985-December 1999. Source: NOMIS.
Figure 3: Vacancy stock, inflow and outflow in Britain: September 1985-December 1999. Source: NOMIS.
Figure 4: Actual and predicted matches on HP filtered data.
Figure 5: Actual and predicted matches on non-filtered data.
Figure 6: Mean vacancy hazards in Britain by duration (weeks), 1987-1995. Source: Coles and Smith (1998).