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Policy Design in a Model with Swings in Risk Appetite

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Abstract
This paper studies the policy implications of habits and cyclical changes in agents' appetite for risk-taking. To do so, it analyses the non-linear solution of a New Keynesian (NK) model, in which slow-moving habits help match the cyclical properties of risk-premia. Our findings suggest that the presence of habits and swings in risk appetite can materially affect policy prescriptions. As in Ljungqvist and Uhlig (2000), a counter-cyclical fiscal instrument can eliminate habit-related externalities. Alternatively, monetary policy can partially curb the associated overconsumption by responding to risk premia. Specifically, periods in which risk premia are elevated (compressed) merit a looser (tighter) policy stance. However, the associated welfare gains appear quantitatively small.

Keywords: Policy design, cyclical risk aversion, New Keynesian model, habit formation
JEL Classifications: E32, G12

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1. Introduction

Over the past decade, New Keynesian (NK) models have become one of the most popular tools used for policy analysis. They were seen to epitomise the consensus – broadly shared by policy makers and academics – that central banks should set nominal interest rates in order to control aggregate demand and achieve stable prices.[1] The recent financial crisis, however, has reopened the debate on the optimal conduct of monetary policy, and has led many economists to revisit the key assumptions on which the "inflation targeting" paradigm was founded.[2]

Disruptions in asset markets during the crisis motivated many modifications of the benchmark model used for policy analysis. The approach in a number of studies was to increment the original New Keynesian model to explicitly include financial frictions (see, for example, Curdia and Woodford (2010), Christiano, Ilut, Motto, and Rostagno (2010), Rabanal, Kannan, and Scott (2009), Sgherri and Gruss (2009), Gertler and Karadi (2011), and Gertler and Kiyotaki (2010)). While these analyses are undoubtedly of interest, and arguably of crucial importance, we believe that the recent events were heavily influenced by swings in investors' risk appetite (e.g. the high, pre-crisis levels of risk taking and the subsequent hike in risk aversion and risk premia). Accordingly, this is the feature that we give prominence to in our analysis.[3]

To generate swings in risk aversion, we deviate from the benchmark New Key-

[1] Woodford (2003) is an established reference.

[2] See also Rabanal (2011) for a concise discussion.

[3] We should note that our model does not feature financial crises or disruptions in asset markets, but only incorporates booms and busts (or cyclical swings) in agent's appetite for risk. Arguably, recently proposed models of the crisis (e.g. the leverage cycle model of Geanakoplos (2009)) can endogenously generate similar swings in aggregate risk aversion (e.g. via exit / bankruptcy of optimistic investors in bust periods). If this line of research was to be developed further, one would be able to make a more direct link between general economic developments and asset market features.

nesian model by allowing for *persistent* external habits in consumption. Different types of habits (internal, external, in ratios or differences) have long been present in the monetary and macro literatures (e.g. Muellbauer (1988), Abel (1990), Constantinides (1990), Gali (1994), Heaton (1995), Alessie and Lusardi (1997)) with the seminal contributions of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) helping solidify their place in applied macroeconomic modelling.[4,5] Ljungqvist and Uhlig (2000) discussed the implications of habits for fiscal policy, while Fuhrer (2000a,b) and Amato and Laubach (2004) studied monetary-policy implications of internal habits. Dennis (2003), however, in the context of a model in which the monetary authority follows optimal discretionary policy, documented that the dynamics implied by internal and external habits significantly differ and argued in favour of the latter specification. Other important contributions include Levine, Pearlman, and Pierse (2008), who derived optimal monetary rules in the case of external, non-persistent habits, and Leith, Moldovan, and Rossi (2009) who went on to study the case of "deep" habits.[6]

The asset pricing literature incorporating persistent habits preceded and partially motivated some of the macro and monetary studies. There, Campbell and Cochrane (1999) (CC hereafter) have shown that slow-moving external habits can generate time-varying risk aversion and can play a key role in accounting for the dynamic properties of the equity risk premium. Related setups – in which the entire history of aggregate consumption determines current habit levels – proved

[4]Historically, first references to habits can be traced as far back as Aristotle's Nicomachean Ethics (350BC) and in the economics literature to Smith (1776) and Pigou (1903).

[5]Boldrin, Christiano, and Fisher (2001) and Uhlig (2007) demonstrate, however, that simply assuming habits in a production economy does little to generate plausible asset price dynamics as agents have many opportunities of smoothing consumption risks.

[6]The term "deep habits" stands for the case in which consumers form habits at the level of individual goods rather than at the level of an aggregate consumption basket.

instrumental in generating a high equity premium or matching expected stock return volatility (Constantinides, 1990; Campbell and Cochrane, 1999; Tallarini and Zhang, 2005; Abel, 2006) and were used in the foreign exchange literature (Verdelhan, 2006; De Paoli and Sondergaard, 2009) as well as yield-curve studies (Wachter, 2006; Gallmeyer, Hollifield, Palomino, and Zin, 2009). In those contributions habits introduced swings in risk appetite into agents' behaviour and allowed their strength to affect risk premia (and thus asset prices).

One general conclusion from the macro and monetary studies referred to above is that external habits lead to consumption which is in excess of socially optimal levels. This occurs because individual agents fail to internalise the negative implications of their spending decisions on the welfare others, and it implies that optimal policy should aim to mitigate overconsumption. The broad question that we seek to address in this paper is how can this be practically achieved? More specifically, given of the success of persistent habit specifications in generating realistic asset price dynamics, and given the on-going policy debate, we analyse policies that directly respond to asset prices. Our analysis builds on De Paoli and Zabczyk (2012a) (DPZ hereafter) who showed that slow-moving, external consumption habits justify tighter monetary policy following productivity shocks and may, under sufficient degrees of habit persistence, flip the optimal response from a policy loosening to a tightening. In the present paper we study whether one can formulate policies – both monetary and fiscal – which, by responding to asset prices, curb overconsumption and increase social welfare.[7]

To shed light on the issue, we proceed in several steps. Firstly, we derive a micro-founded quadratic loss function. We do so in order to obtain the "traditional" representation of welfare losses. Crucially, the effect of habits on risk

[7] Contrary to De Paoli and Zabczyk (2012a) we do not study monetary policy based on the unobservable natural rate.

aversion implies that one cannot write that loss function solely in terms of the output gap and inflation. We show that social losses also depend on the volatility of the stochastic discount factor, which induces agents to save for precautionary reasons and determines the size of risk premia.

As an alternative to monetary policy, we propose a fiscal instrument that can offset the externality introduced by habits.[8] We show that it is optimal to tax consumption in periods of low risk aversion and high asset prices. For the special case in which habits are not persistent, we also show that the efficient allocation can be achieved with the use of three policy tools: the aforementioned state-contingent consumption tax, monetary policy that stabilises inflation and a steady state labour subsidy/tax that offsets firms' monopoly power.

In the general case of persistent habits, we obtain results in terms of a fiscal rule that responds to asset prices and find that it is optimal to increase consumption taxes in periods of high equity prices. We then focus specifically on the design of simple interest rate rules, with the aim of coming up with policy prescriptions that can be easily implemented by central banks. We use this exercise to examine whether policy should counteract falls in risk premia and find that indeed the optimal simple interest rule has a negative coefficient on the risk premium. But our findings show that when compared with a monetary policy rule that responds to risk premia, a fiscal policy rule which responds to equity prices is a more effective tool in reducing welfare losses coming from the habit externality.

[8]The idea that a tax instrument should curb overconsumption in models with external habits is in line with the findings Ljungqvist and Uhlig (2000), who characterise an optimal income tax instrument.

2. Model

As noted above, our baseline model is a version of the one proposed in De Paoli and Zabczyk (2012a) and entails a New Keynesian core augmented with slow-moving consumption habits. In the remainder of this section we present the derivations of the dynamic model equations, referring to the appendix for steady-state computations.

2.1. Households

The economy is inhabited by a large number of households, indexed by a. All of them have identical preferences defined over the consumption of a composite good C, and leisure L

$$E_t \sum_{i=0}^{\infty} \beta^i U\left(C_{t+i}\left(a\right), L_{t+i}\left(a\right)\right)$$

where $\beta \in (0,1)$ is the subjective discount factor measuring impatience. Time available for work (denoted by N) and leisure is normalised to one, so that

$$L_{t+i}\left(a\right) = 1 - N_{t+i}\left(a\right).$$

The period utility function is given by

$$U\left(C, L\right) \equiv \frac{\left(C - hX\right)^{1-\rho} - 1}{1-\rho} - \frac{\left(N\right)^{1+\eta} - 1}{1+\eta}$$

where X represents an external consumption habit which is assumed to depend on aggregate consumption as

$$X_t = \phi X_{t-1} + (1-\phi) C_{t-1}. \tag{1}$$

Households' period-by-period budget constraint is given by

$$\begin{aligned} C_t\left(a\right) + \frac{V_t}{P_t} B_t^n\left(a\right) + V_t^r B_t^r\left(a\right) + V_t^{eq} S_t\left(a\right) \\ = \frac{W_t}{P_t} N_t\left(a\right) + \frac{1}{P_t} B_{t-1}^n\left(a\right) + B_{t-1}^r\left(a\right) + \left(V_t^{eq} + D_t\right) S_{t-1}\left(a\right). \end{aligned} \tag{2}$$

On the right-hand side, we have labour income and current values of financial assets held over from the previous period. During the discrete period, households supply N units of labour, which is remunerated at the nominal market wage W. We shall posit complete markets but include in the budget constraint those assets, whose prices will help us define the returns referred to subsequently. Those assets include a one-period, zero-coupon nominal bond B^n with a face value of a unit of money, a one-period, zero-coupon real bond B^r which pays a unit of consumption at maturity, and a share in a real equity index, which is a claim on a portion of all firms' profits, S. We shall adopt the convention that prices of real assets are real (i.e. denominated in units of the consumption good), while prices of nominal assets will be denominated in units of money. We denote the prices of real and nominal bonds by V^r and V^n respectively, while the (real) price of the equity share paying (real) dividends D is denoted by V^{eq}. The left-hand side of the budget constraint (2) captures expenditures on consumption C, and on a new portfolio of assets.

Household a's choice variables are consumption $C(a)$; labour supply $N(a)$; as well as bond and equity holdings $B^r(a), B^n(a)$ and $S(a)$ respectively. The assumption of complete asset markets and focus on a symmetric equilibrium eliminate all aggregation related issues and allow us to replace individual choice variables (a) with economy wide averages (or aggregates, as the mass of households equals 1). In this symmetric equilibrium all bonds are in zero net supply $B^r = B^n = 0$ and equity prices have to be such that households choose to own the entire stock of equity $S = 1$. Defining the gross inflation rate as $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and the Lagrange multiplier on the budget constraint Λ_t, the aggregate first order conditions can be written as

$$(C_t - hX_t)^{-\rho} = \Lambda_t \tag{3}$$

$$N_t^{\eta} = \Lambda_t \frac{W_t}{P_t} \tag{4}$$

$$\Lambda_t V_t^n = E_t \left[\beta \frac{\Lambda_{t+1}}{\Pi_{t+1}} \right] \tag{5}$$

$$\Lambda_t V_t^r = E_t \left[\beta \Lambda_{t+1} \right] \tag{6}$$

$$\Lambda_t V_t^{eq} = E_t \left[\beta \Lambda_{t+1} \left(V_{t+1}^{eq} + D_{t+1} \right) \right] \tag{7}$$

$$C_t = \frac{W_t}{P_t} N_t + D_t \tag{8}$$

where the expressions above are, respectively, those for marginal utility (3), labour supply (4), asset prices (5) - (7) and the budget constraint (8). In what follows we shall also refer to the nominal value of firms' profits Q_t defined as $Q_t \equiv D_t P_t$.

2.1.1. Asset pricing

For future reference, we can also define real and nominal bond returns as

$$R_{t+1}^n = (V_t^n)^{-1} \qquad\qquad R_{t+1}^r = (V_t^r)^{-1}$$

with the one-period real holding returns on equity, R^{eq} given by

$$R_{t+1}^{eq} = \frac{V_{t+1}^{eq} + D_{t+1}}{V_t^{eq}}.$$

The equity risk premium (or simply "the risk premium" in the remainder), can be then be defined as

$$rp_t = E_t(r_{t+1}^{eq} - r_t^r)$$

where lower case variables denote log-deviations from steady state. Note that defining the stochastic discount factor M, as $M_{t+1} \equiv \beta \Lambda_{t+1}/\Lambda_t$ allows us to write the following log-normal approximation to the bond asset pricing equation

(6)

$$r^r_{t+1} = -E_t\left[m_{t+1}\right] - \frac{1}{2} var_t\left(m_{t+1}\right). \tag{9}$$

As elaborated in Campbell and Cochrane (1999) or Verdelhan (2006), the variance term on the right-hand side can naturally be interpreted as capturing the precautionary savings motive. An increase in the volatility of marginal utility that increases agents' willingness to engage in precautionary savings will therefore reduce the mean of the real interest rate.

2.2. Firms

We assume a continuum of monopolistically competitive intermediate good firms and a perfectly competitive final good sector, where the intermediate varieties are "repackaged" into a single consumption bundle. Monopolistic competition in the intermediary sector allows us to have firms that are price-setters, and this in turn facilitates the introduction of nominal rigidities a'la Calvo (1983).

2.2.1. The final goods sector

The final good Y_{t+i} is produced by bundling together a range of intermediate goods $Y_{t+i}(z)$ using the following Dixit-Stiglitz technology

$$Y_{t+i} = \left[\int_0^1 \left(Y_{t+i}(z)\right)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}$$

where σ is the elasticity of substitution between the intermediate inputs. Cost minimisation implies the following demand for each individual variety

$$Y_{t+i}(z) = \left(\frac{P_{t+i}(z)}{P_{t+i}}\right)^{-\sigma} Y_{t+i}, \tag{10}$$

as well as an aggregate price index (equal to the price of the composite bundle) given by

$$P_t = \left[\int_0^1 P_t(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}$$

where $P_t(z)$ is the price of intermediate good z.

2.2.2. Intermediate goods sector

There is a continuum of intermediate goods firms (indexed by z) that maximise profits, which are paid out as dividends to households holding shares. Following Calvo (1983), we assume that each period a fraction $1-\alpha$ of randomly-selected firms can adjust their price $P(z)$, while the remaining fraction α can not. Firms maximise profits

$$\max E_t \sum_{i=0}^{\infty} \alpha^i \beta^i \frac{\Psi_{t+i}(z)}{\Psi_t(z)} \left(P_t(z) Y_{t+i}(z) - W_{t+i} N_{t+i}(z)\right) \tag{11}$$

where $\beta^i \Psi_{t+i}(z) / \Psi_t(z)$ is the zth firm's stochastic discount factor. Nominal profits are the difference between revenue and expenditures on labour and expression (11) accounts for the fact that the price $P_t(z)$ chosen in period t will still be in effect in period $t+i$ with probability α^i. Firms face a downward-sloping demand curve (10) and produce intermediate variety $Y(z)$, using hired labour according to the following technology

$$Y_{t+i}(z) = A_{t+i}^{\frac{\eta}{\eta+1}} N_{t+i}(z),$$ [9]

where total factor productivity A_{t+i} is stochastic and follows an AR(1) process of the form

$$\log(A_t) = \left(1-\rho^A\right) \log\left(\bar{A}\right) + \rho^A \log(A_{t-1}) + \varepsilon_t^A, \ \varepsilon_t^A \sim i.i.d.N\left(0, \sigma_{\varepsilon^A}^2\right). \tag{12}$$

Firms optimise over labour input $N(z)$ and the price of their good $P(z)$. As described in Walsh (2003), all firms adjusting prices in period t face the same problem and will thus be choosing the same price. Letting P_t^* denote the optimal price chosen by all firms which can reset, and exploiting the fact that firms are owned by households (i.e. $\Psi_t = \Lambda_t$), the first order condition with respect to P^*

[9] The production function has power $\frac{\eta}{\eta+1}$ on productivity A in order to be consistent with a Yeoman-farmer version of the model.

can be written as

$$\frac{P_t^*}{P_t} = \left(\frac{\sigma}{\sigma - 1}\right)\frac{PB_t}{PA_t} \tag{13}$$

where the auxiliary variables PA_t and PB_t satisfy

$$PA_t = E_t \sum_{i=0}^{\infty} \alpha^i \beta^i \Psi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\sigma-1} Y_{t+i} = \Psi_t Y_t + \alpha\beta E_t \left[PA_{t+1} \left(\Pi_{t+1}\right)^{\sigma-1}\right]$$

$$PB_t = E_t \sum_{i=0}^{\infty} \alpha^i \beta^i \Psi_{t+i} \Upsilon_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\sigma} Y_{t+i} = \Psi_t \Upsilon_t Y_t + \alpha\beta E_t \left[PB_{t+1} \left(\Pi_{t+1}\right)^{\sigma}\right]$$

and where $\Upsilon(z)$ denotes the Lagrange multiplier on the market clearing condition.

Because the adjusting firms were selected randomly from the whole population, therefore the average price used by firms that were not able to adjust will equal the average price level in period $t-1$. This implies that average prices satisfy

$$P_t^{1-\sigma} = (1-\alpha) P_t^{*^{1-\sigma}} + \alpha P_{t-1}^{1-\sigma}. \tag{14}$$

Finally, the first order condition with respect to L_t yields the following labour demand equation

$$-\frac{W_t}{P_t} + \Upsilon_t A_t^{\frac{\eta}{\eta+1}} = 0. \tag{15}$$

2.2.3. Aggregation

We start by combining firms' production function $Y_t(z) = A_{t+i}^{\frac{\eta}{\eta+1}} N_{t+i}(z)$ with the demand curve $Y_t(z) = \left(P_t(z)/P_t\right)^{-\sigma} Y_t$. Integrating these conditions and defining the domestic price dispersion term as $PD_t \equiv \int_0^1 \left(P_t(z)/P_t\right)^{-\sigma} dz$ and aggregate labour supply as $N_t \equiv \int_0^1 N_t(z)\, dz$, we can write

$$Y_t = \frac{A_t^{\frac{\eta}{\eta+1}} N_t}{PD_t}$$

with the Calvo pricing rule implying that price dispersion evolves according to

$$PD_t = (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{-\sigma} + \alpha \left(\Pi_t\right)^{\sigma} PD_{t-1}. \tag{16}$$

Using $N_t = \int_0^1 N_t(z)\,dz$ we can also write

$$\frac{W_t}{P_t} = \Upsilon_t N_t^{-1} Y_t PD_t \tag{17}$$

with aggregation of profits leading to

$$D_t = \frac{Q_t}{P_t} = Y_t - \frac{W_t}{P_t} N_t. \tag{18}$$

2.3. Monetary policy

In order to close the model we need to make assumptions about central bank behaviour. We will consider several alternatives:

- Under strict inflation targeting, the central bank keeps inflation π_t equal to zero at all times, thus replicating the flexible price allocation;
- As is standard, under money market equilibrium, the central bank can also be assumed to set the nominal interest rate $r^{cb} = r^n$. In section 5.1 we therefore analyse a variety of Taylor-type interest rate rules for r^{cb}, the coefficients of which will be chosen optimally.

2.4. Risk premium and precautionary savings: a special case

As discussed in Li (2001) and De Paoli and Zabczyk (2012b), the equity risk premium can be written as

$$cov_t(c^e_{t+1}, r^r_{t+1}) = cov_t(c_{t+1}, r^r_{t+1})\, \mathbf{E}_t \frac{C_{t+1}}{C^e_{t+1}} \tag{19}$$

where excess consumption is defined as $C^e_t = C_t - hX_t$. Moreover, as studied in De Paoli and Zabczyk (2012b), in the special case of flexible prices and inelastic labour – i.e. $\eta \to \infty$ (i.e. in an endowment model where $c_t = \rho c_{t-1} + \varepsilon^A_t$) the derivative of the risk premium rp_t with respect to the current shock realisation can be expressed as

$$\frac{\partial rp_t}{\partial \varepsilon_t} \approx E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi)\Big[C_t - \rho \sum_{s=0}^{t} \phi^s C_{t-s}\Big].^{10} \tag{20}$$

From equation (19) it is clear that if habits are not persistent ($\phi = 0$) then the risk premium is counterfactually pro-cyclical. The study then proceeds to analyse how persistence in habits and shocks is crucial to generate the observed counter-cyclical movements in risk premia. Subsequently, DPZ demonstrate that similar conditions will drive the cyclicality in precautionary savings. In particular, the paper shows that in an endowment economy the variance of the stochastic discount factor can be written as

$$var_t(m_{t+1}) = \kappa_0 + \kappa_1[(1-\phi-\rho)c_t + \phi x_t]$$

where $\kappa_0 = \frac{\rho^2\sigma_\varepsilon^2}{(1-h)^2}$, $\kappa_1 = \frac{2h\rho^2\sigma_\varepsilon^2}{(1-h)^3}$ and σ_ε^2 is the exogenous shock volatility. Given that x_t is a predetermined variable, the equation above proves that $var_t(m_{t+1})$ is countercyclical (i.e. $\frac{\partial var_t(m_{t+1})}{\partial c_t} > 0$) as long as $\phi + \rho > 1$.[11] Expressed alternatively, investors will demand relatively higher compensation for holding risky assets if they expect future economic conditions to remain poor (consumption to persistently undershoot its steady state). If, on the other hand, the expectation is for an improvement in economic prospects, then negative shocks might (counterfactually) not translate into higher risk premia and precautionary behaviour. This occurs when habits are fast moving and consumption reverts back to mean quickly, as investors faced with the bad shock quickly "adapt" to lower levels of

[10] This conditon holds if the conditional variance of returns $\text{var}_t(r_{t+1}^r)$ and their conditional covariance with consumption $\text{cov}_t(r_{t+1}^r, c_{t+1})$ are constant. Furthermore, it holds exactly under the additional assumption that excess consumption and risky returns are jointly conditionally log-normal and that consumption is also conditionally log-normal.

[11] This expression was derived using a second order approximation of the stochastic discount factor.

consumption while at the same time, the expectation is for the latter to recover. This means that investors actually expect consumption to be far above its habit level in the future and might therefore be more inclined to take on risk.

3. Welfare

Welfare losses in the model described in the previous section are driven by different economic inefficiencies. The presence of nominal rigidities introduces relative price distortions. Monopolistic competition implies that firms exert market power and underproduce relative to the social optimum. And finally, external habits lead to overconsumption, as households fail to internalise the effect of their consumption choices on the economy-wide habit level.

We shall now derive a simple characterisation of social losses by computing a second order approximation to the utility function. We note that while the analytical formulae presented below rely on the steady state being efficient, our numerical solution (presented in subsequent sections) does not require such assumptions. Also, our numerical evaluation of welfare is not based on a second order approximation. In fact, in order to take into account how cyclical swings in risk aversion affect agents' precautionary behaviour and asset prices, we approximate welfare and the model dynamics up to fourth order.[12]

As shown in the appendix, which follows Leith, Moldovan, and Rossi (2009), the conditional social loss derived from the utility U_t of the representative agent can be written as

$$L_0 = \frac{1}{2}\overline{\varkappa} E_0 \sum_0^{\infty} \beta^t \left[\frac{1-h}{1-k} \rho (c_t^e)^2 + \eta (y_t^{'})^2 + \frac{\sigma}{\kappa} \pi_t^2 \right] + tip + o[2]^{13}$$

[12]As discussed subsequently, this is required to correctly account for time-varying risk premia and precautionary savings motives.

[13]As shown in the appendix, we define $\overline{\varkappa} = \frac{1-k}{1-h} \left(\overline{C}^e\right)^{1-\rho} = \overline{N}^{1+\eta}$.

where $y_t' = y_t - a_t$, tip stands for terms independent of policies and $o[2]$ denotes terms of order higher than two.

Moreover, the *unconditional* social loss can be written as

$$L \equiv -E(U_t) = \frac{\overline{\varkappa}}{2(1-\beta)}\left[\frac{1-h}{\rho(1-k)}\sigma_m^2 + \eta\sigma_{y'}^2 + \frac{\sigma}{\kappa}\sigma_\pi^2\right] + tip + o[2].$$

Accordingly, social losses depend on the variance of inflation $\sigma_\pi^2 \equiv var(\pi)$, the variance of an output gap measure $\sigma_{y'}^2 \equiv var(y')$ and the variance of the stochastic discount factor $\sigma_m^2 \equiv var(m)$. As shown in De Paoli and Zabczyk (2012a) the variance of the stochastic discount factor drives agents' buffer stock savings and by extension also risk premia. In other words, in a world with endogenous swings in risk taking, economic uncertainty that instigates precautionary behaviour has a direct effect on welfare.

We note that, in the absence of consumption habits, the unconditional loss would be given by

$$\frac{1}{2}\overline{\varkappa}\sum_0^\infty\left[(\eta+\rho)\sigma_{ygap}^2 + \frac{\sigma}{\kappa}\sigma_\pi^2\right]$$

where $y_t^{gap} = y_t - \frac{\eta}{\rho+\eta}a_t = y_t - y_t^{flex}$ and y_t^{flex} is the flexible price output allocation, which coincides with efficient output. So, under the efficient steady state assumption, stabilising inflation would automatically close the welfare-relevant output gap and first-best could be achieved. But in a model with habits, the consumption externality breaks this so-called "divine coincidence".[14]

4. Economic efficiency: an auxiliary fiscal instrument

In this section, similarly to Ljungqvist and Uhlig (2000), we show how economic efficiency can be restored – with the three aforementioned types of market

[14]The term "divine coincidence" was coined by Blanchard and Gali (2007) and corresponds to the situation, in which stabilising inflation also closes the welfare-relevant output gap.

imperfections still in place – using an appropriately designed mix of fiscal and monetary policies (in subsequent sections we analyse how monetary policy alone should be adjusted to tackle such dynamic inefficiencies). The misallocation coming from external habits can be perhaps best understood by contrasting the first order condition with respect to consumption under the competitive equilibrium – equation (3) – with the one that arises in the planner's problem when additionally accounting for habit dynamics (equation (1)). As discussed in DPZ, and covered in more detail in the appendix, if a benevolent social planner were to include the economy-wide habit level (X_t) within her choice variables, then the first order condition with respect to consumption would be given by

$$(C_t^* - hX_t^*)^{-\rho} - \Lambda_t^* - \beta(1-\phi)E_t\Lambda_{t+1}^x = 0 \tag{21}$$

where Λ_{t+1}^x is the Lagrange multiplier on the equation specifying the evolution of habits (1). Moreover, the first order condition with respect to X_t would be

$$-h\left(C_t^* - hX_t^*\right)^{-\rho} + \Lambda_t^x - \beta\phi E_t\Lambda_{t+1}^x = 0. \tag{22}$$

If we further assumed that prices are perfectly flexible and firms have no monopoly power (or that polices are in place that result in such an allocation), then the labour-leisure decision would simplify to

$$(C_t^*)^\eta = \Lambda_t^* A_t^{\frac{\eta}{\eta+1}}. \tag{23}$$

Because in equilibrium $C_t = X_t = Y_t$, therefore the equations above can be used to characterise the efficient allocation of output (with "efficient" variables subsequently denoted using an asterisk). In particular, if we compare the competitive equilibrium with the efficient allocation in steady state, we have

$$\frac{\overline{C}^*}{\overline{C}} = (1-k)^{\frac{1}{\rho+\eta}} \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{\rho+\eta}}$$

where the term $k = \beta(1-\phi)h/(1-\beta\phi)$ is due to the presence of the habit externality. Thus, whether the steady state levels of consumption and output are inefficiently high or low depends on whether overconsumption induced by habits outweighs the underproduction caused by monopolistic competition.

To offset the distortions caused by habits and monopolistic competition (both the static ones above, as well as dynamic inefficiencies), we posit a consumption tax T_t, which would enter households' budget constraints as

$$T_t P_t C_t(a) + V_t B_t(a) - W_t N_t(a) - B_{t-1}(a) - Q_t = 0.$$

This tax would appear in the first order condition with respect to consumption as

$$(C_t - hX_t)^{-\rho} = T_t \Lambda_t. \tag{24}$$

The appendix derives the optimal tax rule for the special case, in which habits are not persistent ($\phi = 0$). This allows us to derive analytical expressions that help highlight the economic intuition.[15] As shown in the appendix, the tax rule that restores efficiency can be written as

$$T_t = \frac{1}{1 - h\beta\tilde{M}_t}\left(\frac{\sigma}{\sigma-1}\right)^{-1} \tag{25}$$

where $\tilde{M}_t \equiv \frac{E_t(C_{t+1}-hX_{t+1})^{-\rho}}{(C_t-hX_{t+1})^{-\rho}}$ represents agents' expectation of the grown rate of their marginal utility. Under this rule, the competitive equilibrium would coincide with the social planners one – i.e. the tax would eliminate the overconsumption attributable to external habits. Analysing expression (25), we see that the fiscal rule is forward looking and advocates higher taxes in periods, in which agents expect conditions to improve.

Notably, and as discussed in Section 2.1.1, it is exactly this expectation of the growth rate of marginal utility that drives movements in the risk premium and

[15]The corresponding intuition will subsequently be tested in the numerical part of our analysis, where we relax the assumption of $\phi = 0$.

hence also equity prices. In particular, when agents expect conditions to improve the risk premium is compressed and asset prices are higher. So, even though the optimal tax given by equation (25) may not appear implementable – since movements in marginal utilities are unobservable – responding to changes in asset prices is arguably much more feasible. This will be examined more systematically in the next section.

We also note that, in steady state, the optimal tax is larger than one (i.e. the optimal fiscal tool is a tax and not a subsidy) if overconsumption generated by habits (represented by the term $1/(1-h\beta)$ in equation (25)) exceeds the steady-state underproduction induced by monopolistic competition (represented by the term $\sigma/(\sigma-1)$ in equation (25)).

Our finding suggests that fiscal policy should be utilised in periods of exuberant expectations.[16] The general idea that a tax instrument should curb overconsumption in models with external habits is in line with the findings of Ljungqvist and Uhlig (2000), who characterise an optimal income tax instrument (as opposed to consumption taxes analysed above). The insights coming from such results may be useful for the growing literature on macroprudential policy, and the discussion of whether such policy should react to booms in asset prices or compressions in risk premia. As discussed in Rabanal (2011), recent research has suggested that it is possible to improve welfare by including asset price fluctuations or indicators of financial vulnerability in the monetary policy rule or, alternatively, that this should be the target of a macroprudential rule (e.g. Christiano, Ilut, Motto, and Rostagno (2010), Rabanal, Kannan, and Scott (2009), Curdia and Woodford (2010), Sgherri and Gruss (2009)).

Figure 1 shows how the introduction of an efficient consumption tax affects

[16] The term "exuberant" here does not stand for mistaken expectation but only periods in which economic conditions are expected to improve.

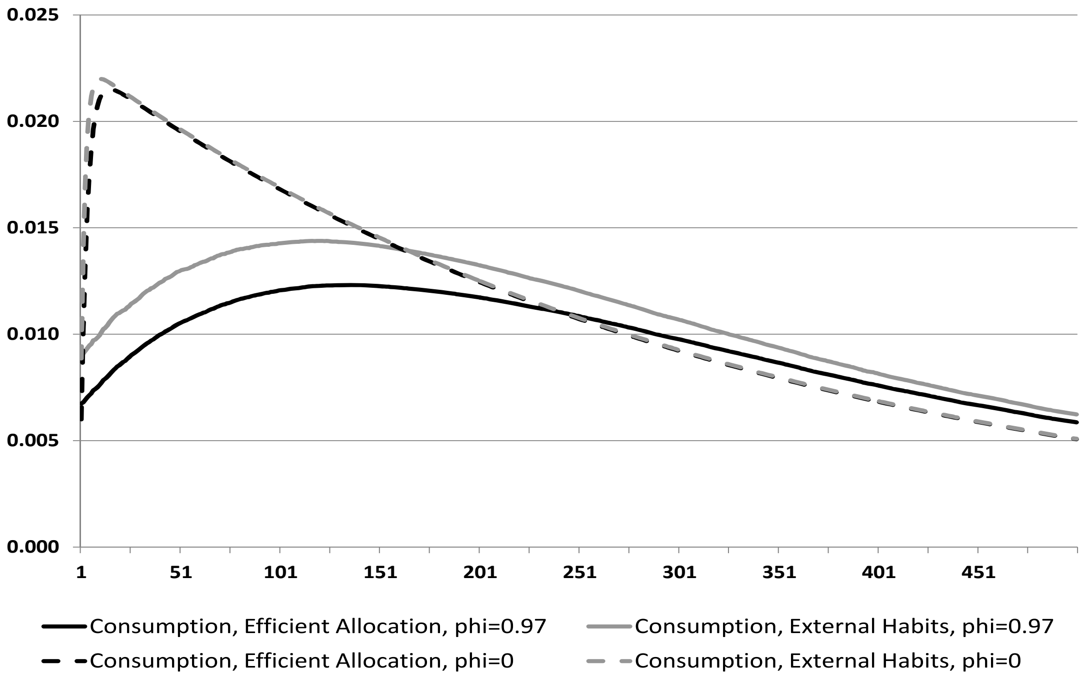

Figure 1: Efficient and flexible price consumption profile following a one standard deviation positive productivity shock (annualised, in percentage points)

the dynamics of consumption after a positive productivity shock.[17] It demonstrates that the optimal fiscal instrument partially offsets the effect of a boom in productivity on consumption, arguably in order to restrain the effect of such consumption increases on the average household's habits. Note that the difference between the efficient allocation and the flexible price equilibrium is more marked when we relax the assumption of $\phi = 0$ (the calibration used in this exercise follows closely DPZ, with the parameter values listed in Table 1).

[17]In this exercise we maintain the assumptions that monetary policy stabilises prices, and that there is a labour subsidy guaranteeing steady state efficiency. We also use a third order perturbation approximation to the model's solution, which is implemented in Dynare++.

Table 1: Parameter Values

Parameter	Value
β	0.99
η	6
ρ	2.37
α	0.66
σ	10
h	0.85
ϕ	0.97 (unless stated otherwise)
γ_{prod}	0.997
sub	$\frac{(1-k)\sigma}{\sigma-1}$ (unless stated otherwise)
$\sigma_{\Delta c}$	0.75%

5. Optimal Simple Rules

Motivated by the previous discussion, which highlighted links between optimal fiscal policy and asset price dynamics, we now explicitly focus on simple rules – both fiscal and monetary – that attempt to move the equilibrium closer to first-best by responding to asset prices.

5.1. Fiscal Rule

We begin by assuming that it is the fiscal authority that responds to conditions in asset markets. More specifically, we consider the case in which consumption taxes respond to equity prices according to

$$T_t = \bar{T} + \theta^e (V_t^{eq} - \overline{V}^{eq}). \tag{26}$$

As before, we initially assume that the monetary authority follows a strict inflation targeting rule, which replicates the flexible price allocation. We then focus on finding values of the policy coefficient θ^e which maximise welfare. In the context of our model, the natural measure of welfare $\mathcal{W}$ is given by the expected discounted value of utility and satisfies the recursive definition

$$\mathcal{W}_t \equiv \mathbf{E}_t \left\{ \frac{\left(C_t - hX_t\right)^{1-\rho} - 1}{1-\rho} - \frac{(N_t)^{1+\eta} - 1}{1+\eta} + \beta \mathcal{W}_{t+1} \right\}. \tag{27}$$

There are several ways in which welfare can be approximated. The simplest option would be to use the steady state level of welfare. Unfortunately, that concept will not depend on policy and is thus unsuitable for our exercise. The alternative, advocated in Schmitt-Grohe and Uribe (2007) (who briefly discuss related issues), is to compute the expected value of welfare *conditional* on being in the steady state. Letting y_t denote the state of the economy, our k-th order perturbation approximation to the model solution $g^k(\cdot,\cdot)$ satisfies

$$y_t \approx g^k(y_{t-1}, \epsilon_t) + o[k+1]$$

and so, assuming welfare is the i-th coordinate of the state, the k-th order conditional welfare measure $\mathcal{W}^k$ could be defined as

$$\mathcal{W}^k \equiv g_i^k(\bar{y}, 0).$$

While Schmitt-Grohe and Uribe (2007) claim that this measure accounts for transitional dynamics, which are important when evaluating welfare, it is not immediately obvious why one should condition the transitional dynamics on the deterministic steady state (which the authors note in passing in their analysis).[18] Since, arguably, and as suggested by our numerical results, this choice of initial condition is likely to underplay the role of uncertainty and associated risk corrections, we also consider two additional welfare measures. The first is the

[18]Notably, however, Villaverde et at (2010) argue that the concept may be well-suited for capturing *changes* in welfare due to uncertainty.

"stochastic fixed point" measure of welfare and the second is the unconditional mean of welfare. The "stochastic fixed point" $\widetilde{y}$ satisfies

$$\widetilde{y} = g^k(\widetilde{y}, 0)$$

i.e. it is the point in which the economy would remain if agents accounted for risk in their decision rules, but if that risk never materialised (hence $\epsilon = 0$ in the formula above). The $k-th$ order stochastic fixed point measure of welfare would thus be given by[19]

$$\mathcal{W}^{k,fp} \equiv g_i^k(\widetilde{y}, 0).$$

And of course the unconditional mean $\mathbf{E}y$ would satisfy

$$\mathbf{E}y = \mathbf{E}g^k(\mathbf{E}y, \epsilon)$$

with the corresponding $k-th$ order unconditional welfare measure given by[20]

$$\mathcal{W}^{k,unc.} \equiv \mathbf{E}g_i^k(\mathbf{E}y, \epsilon).$$

As made clear in the definitions above, all those measures depend on the order or approximation to the solution (i.e. k in the notation above). As argued in DPZ, many of the risk-related / precautionary effects that we are interested in capturing will only manifest themselves at third order. Accordingly, we evaluate the unconditional welfare measures based on simulations conducted using a third order approximation. However, for the conditional welfare measure and the

[19]See also Kamenik (2007) for a discussion of the concept.

[20]In practice, we compute unconditional welfare using Monte Carlo methods and the model solution generated by *Dynare++*. Specifically, for a given set of policy parameters we average over 10K random draws taken from 2000 different simulations (we discard 85% of the sample to eliminate the dependance on initial conditions). We have experimented by sampling over 500K draws but since this appeared not to have any significant impact on the results, we settled on the smaller sample.

stochastic fixed point welfare measure (both of which can be computed directly from the approximate solution, without resorting to simulation) it is possible to show, in line with Schmitt-Grohe and Uribe (2004), that welfare estimates (both conditional and based on the stochastic fixed point) do not change when moving from second to third order.[21] Accordingly, to capture the effects we are interested in, we use the stochastic steady state based on a fourth order approximation to the model's solution.

The results of the exercise described above are presented in Table 2 (which, for reference, also presents results based on a second order approximation).[22] We note that they have been obtained under the assumption that the optimal taxes ensure that the steady-state is efficient. As alluded to previously, this leaves two dynamic distortions in place. Since the central bank offsets that driven by nominal rigidities, therefore the fiscal rule will be left to tackle the one associated with consumption habits. It follows that the optimal coefficient θ^e should be positive, implying counter-cyclical fiscal policy via pro-cyclical consumption taxes. This is also what our numerical simulations imply. In particular, when using the various welfare measures we find values of θ^{e*} ranging from 0.4.to 2.0.[23]

[21] This is because all σ-corrections of an odd order are equal to zero for a symmetric shock distribution like the Gaussian one that we use.

[22] In line with the discussion above, our results based on second order welfare approximations do not take fully into account the effect of uncertainty on equilibrium dynamics – e.g. in this case risk premia and agents' precautionary motives are constant. So such analysis may represent times of low uncertainty – or cases in which the linear approximation of the model (and a linear-quadratic welfare measure) are not a bad approximation of reality. But clearly, even in this case, habits will result in a time-varying aversion to risk and induce overconsumption.

[23] To find the maximum, we evaluate welfare on a grid (in line with SGU). We started by using a grid of [-10,10] with a step of 0.05. Given that the maximum obtained using the conditional welfare measure was always smaller than 2 in absolute value, we then restricted the grid to [-2,2] retaining the step size. In previous versions of the paper we also directly maximised the welfare measures, using a variety of optimisation routines. The conclusions from those exercises were

To give an idea of the size of the welfare gains, Table 2 compares the value of welfare under the optimised rule to that obtained when fiscal policy is passive and only the static efficiency subsidies are in place. We find improvements in social welfare ranging from 0.4% to 15%. Notably, and in line with the previous discussion, the welfare gains obtained using measures not conditioned on the deterministic steady state (i.e. arguably those in which risk plays an additional role), point to greater benefits of active fiscal policies. Accordingly, to avoid appearing to choose welfare measures which artificially inflate the importance of our findings, and also to make it easier to compare our results to Schmitt-Grohe and Uribe (2007), in the remainder we focus on the conditional welfare measure based on a fourth order approximation (i.e. on $\mathcal{W}^4$).

Table 2: Optimal Simple Fiscal Rules

Welfare approximation	θ^{e*}	Welfare gain (relative to $\theta^e = 0$)
2nd order conditional welfare measure $\mathcal{W}^2$	0.4	0.4%
4th order conditional welfare measure $\mathcal{W}^4$	0.4	0.6%
2nd order stoch. fixed point welfare $\mathcal{W}^{fp,2}$	2.0	9.2%
4th order stoch. fixed point welfare $\mathcal{W}^{fp,4}$	1.05	5.7%
2nd order simulated welfare measure $\mathcal{W}^{unc,2}$	1.9	7.2%
3rd order simulated welfare measure $\mathcal{W}^{unc,3}$	2.0	14.2%

Table 3 presents some robustness analysis and examines how the results differ under alternative specifications of the model (it is based on $\mathcal{W}^4$). Firstly, we decrease the habit parameter (and set $h = 0.7$). The smaller habit parameter reduces the strength of the habit externality. As a result, the optimal response to asset prices decreases and so do the associated welfare gains. Similar results arise when we decrease the persistence of habits. The table also shows that allowing for an inefficient steady state does not change the optimal response to equity

in line with the ones we present here.

prices but it does increase the welfare benefits from such responses.

Table 3: Optimal fiscal rule coefficients

	θ^{e*} (4th order conditional welfare gain)
Benchmark	0.4 (0.6%)
Inefficient steady state	0.5 (1.0%)
$h = 0.7$	0.25 (0.2%)
$\phi = 0.7$	0.35 (0.2%)

5.2. Monetary Rule

In this section we assume that the central bank follows a simple Taylor-type rule that responds to inflation as well as asset prices. In particular, we consider the case in which fiscal policy is passive and only ensures steady state efficiency while monetary policy follows

$$r_t^n = \theta^\pi \pi_t + \theta^{rp} rp_t. \tag{28}$$

We first observe that equation (28) fails to include a familiar output gap term. But, if one believes that monetary policy should not deal with steady state inefficiencies – i.e. under an optimal steady state tax, when all economic distortions come from habits and price dispersion – then the presence of inflation and a measure of risk taking might be sufficient. Aside from that, our exercise here has the objective of considering rules which can be easily implemented, and so depend only on "observables" – and it is debatable, whether the welfare-relevant output gap falls into this variable category.

Based on a fourth order unconditional welfare ranking, we find that the optimal value of θ^π and θ^{rp} are 20 and -0.4 respectively (in line with the intuition developed in the analytical part of the paper). The result highlights that the welfare cost associated with nominal rigidities tends to dominate the ones associated with the habit externality. This is consistent with the results of Amato and Laubach (2004). Also, the welfare gains from a non-zero response to equity prices appear very small.

Table 4: Optimal monetary rule coefficients (based on a 4th order conditional welfare measure)

	$\theta^{\pi *}$	θ^{rp*}	Welfare gain
$r_t^n = \theta^{\pi}\pi_t$	20	NA	NA
$r_t^n = \theta^{\pi}\pi_t + \theta^{rp} \cdot rp_t$	20	-0.4	$< 0.1\%$
$r_t^n = \theta^{\pi}\pi_t + \theta^{rp} \cdot rp_t$ (ineff. SS)	20	-0.2	$< 0.1\%$

In summary, one may conclude that periods of high productivity, low risk aversion and compressed risk premia would justify a contractionary bias in policy. Nevertheless, given that fluctuations in risk premia are only a "third order phenomenon" – i.e. they only arise when approximating the model to third-order – the inclusion of a policy response term plays a relatively minor role for welfare.[24]

6. Conclusion

The finance literature has taught us many things about factors driving asset prices and risk premia. The recent financial crisis, in turn, has spurred monetary economists to reinvestigate whether central banks should respond to these financial variables or, more broadly, to asset market conditions. In this paper, we tackled those policy questions using a model inspired by the asset-pricing literature. We showed how habits and the consequent swings in risk appetite affect welfare and highlighted several ways in which movements in asset prices could be incorporated in monetary and fiscal policy analysis.

Our framework was stylised and, accordingly, so was the characterisation of policy. We also needed to resort to higher order approximations to even account

[24]As a check, we have also analysed the case of active monetary and fiscal policies. While for parsimony we no longer report the results, those suggested small gains from accounting for asset prices in a Taylor type rule. And, in line with the results from the previous section, they pointed to significant gains from introducing another instrument to deal with distortions coming from overconsumption.

for fluctuations in risk taking behaviour. As such, using a model in which these shifts in risk preferences are of first order importance may be a fruitful avenue for future research (and may already by under way in a share of the literature studying the role of "financial frictions"). In this paper, we also proposed a fiscal instrument that could deal with so-called "overconsumption" externalities. Extending these results and relating them to the lively debate on macroprudential instruments would thus seem like another clear avenue for future research. Arguably, we also only considered the stylised case in which the fiscal and monetary authorities fully cooperate when trying to optimally set their policies. Thus, another interesting extension would be to assess an equilibrium in which the authorities have different goals or simply set policy independently.

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APPENDIX - DERIVATIONS

Appendix A.1. The social loss function: A second order approximation

The utility function of the household is given by

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - hX_{t+i})^{1-\rho} - 1}{1-\rho} - \frac{(N_{t+i})^{1+\eta} - 1}{1+\eta} \right]$$

and we can approximate each term in turn. Following Leith, Moldovan, and Rossi (2009) and noting our more general specification of habits[25], we have

$$\frac{(C_{t+i} - hX_{t+i})^{1-\rho} - 1}{1-\rho} =$$
$$\left(\overline{C}^e\right)^{1-\rho} \left\{ \frac{1}{1-h} \left[c_t + \frac{1}{2} c_t^2 \right] - h \frac{1}{1-h} \left[x_t + \frac{1}{2} x_t^2 \right] - \frac{1}{2} \rho (c_t^e)^2 \right\} + tip + o[2].$$

But, given the definition of habits,

$$\begin{aligned} x_t + \frac{1}{2} x_t^2 &= \phi \left[x_{t-1} + \frac{1}{2} x_{t-1}^2 \right] + (1-\phi) \left[c_{t-1} + \frac{1}{2} c_{t-1}^2 \right] \\ &= (1-\phi) \sum_{1}^{t} \phi^{s-1} \left[c_{t-s} + \frac{1}{2} c_{t-s}^2 \right]. \end{aligned}$$

Summing up to the future, we can write the utility of consumption as

$$E_t \sum_{t=0}^{\infty} \beta^t \frac{(C_t - hX_t)^{1-\rho} - 1}{1-\rho}$$
$$= \left(\overline{C}^e\right)^{1-\rho} \sum_{0}^{\infty} \beta^t \left\{ \frac{1-k}{1-h} \left[c_t + \frac{1}{2} c_t^2 \right] - \frac{1}{2} \rho (c_t^e)^2 \right\} + tip + o[2]$$

where $k = \beta(1-\phi)h/(1-\beta\phi)$.

Our approximation of the disutility of labour is also based on Leith, Moldovan, and Rossi (2009). However, noting that our specification of the production function implies that $Y_t = A_t^{\frac{\eta}{\eta+1}} N_t / PD_t$, we have

$$\frac{(N_{t+i})^{1+\eta} - 1}{1+\eta} = \overline{N}^{1+\eta} \left[y_t + \frac{1}{2}(1+\eta)(y_t)^2 - \eta y_t a_t + \frac{\sigma}{2} var(p_i) \right] + tip + o[2]$$

[25] The habit specification of Leith, Moldovan, and Rossi (2009) is a special case of the one proposed in this paper in which $\phi = 0$.

and, thus, overall welfare can be written as

$$
\begin{aligned}
& E_t \sum_{t=0}^{\infty} \beta^t \frac{(C_t - hX_t)^{1-\rho} - 1}{1-\rho} - \frac{(N_{t+i})^{1+\eta} - 1}{1+\eta} \\
= \quad & \left(\overline{C}^e\right)^{1-\rho} \sum_0^{\infty} \beta^t \left\{ \frac{1-k}{1-h} \left[c_t + \frac{1}{2} c_t^2 \right] - \frac{1}{2} \rho (c_t^e)^2 \right\} \\
& -\overline{N}^{1+\eta} \sum_0^{\infty} \beta^t \left[y_t + \frac{1}{2}(1+\eta)(y_t)^2 - \eta y_t a_t + \frac{\sigma}{2} var(p_i) \right] + tip + o[2]
\end{aligned}
$$

From the steady state derivation we know that, if we assume an efficient subsidy, we have

$$
\frac{1-k}{1-h} \left(\overline{C}^e\right)^{1-\rho} = \overline{N}^{1+\eta} = (1-h)^{\frac{-\rho(\eta+1)}{\rho+\eta}} (1-k)^{\frac{1+\eta}{\rho+\eta}}.
$$

Defining $\overline{\varkappa} = \frac{1-k}{1-h}\left(\overline{C}^e\right)^{1-\rho} = \overline{N}^{1+\eta}$and noting that $c_t = y_t$, we can write welfare W_0 as

$$
\begin{aligned}
W_0 = & E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - hX_t)^{1-\rho} - 1}{1-\rho} - \frac{(N_{t+i})^{1+\eta} - 1}{1+\eta} \\
= \quad & -\frac{1}{2} \overline{\varkappa} E_0 \sum_0^{\infty} \beta^t \left[\frac{1-h}{1-k} \rho (c_t^e)^2 + \eta (y_t - a_t)^2 + \sigma var(p_i) \right] + tip + o[2].
\end{aligned}
$$

Or following Woodford (2003)

$$
L_0 = -W_0 = \frac{1}{2} \overline{\varkappa} \sum_0^{\infty} \beta^t \left[\frac{1-h}{1-k} \rho (c_t^e)^2 + \eta (y_t - a_t)^2 + \frac{\sigma}{\kappa} \pi_t^2 \right] + tip + o[2]
$$

where $\kappa = (1-\alpha)(1-\alpha\beta)/\alpha$ is the slope of the Phillips curve. One can write the unconditional period loss as

$$
E(L) = \left[\frac{1-h}{1-k} \rho var(c^e) + \eta var(y')^2 + \frac{\sigma}{\kappa} var(\pi) \right]
$$

where $y_t' = y_t - a_t$, with the final two terms identical to the ones that would arise in model without habits.

We finally observe, that if we define the stochastic discount factor as

$$
\mathcal{M}_{t+1} = \beta \frac{(C_{t+1} - hX_{t+1})^{-\rho}}{(C_t - hX_t)^{-\rho}} = \beta \left(\frac{C_{t+1}^e}{C_t^e} \right)^{-\rho}
$$

and, hence, $var_t(m_{t+1}) = var_t(\rho(c^e_{t+1})) = \rho^2 var_t(c^e_{t+1})$, then the unconditional loss function $-E(W_0)$ can be written as

$$\frac{1}{2}\varkappa\sum_{0}^{\infty}\left[\frac{1-h}{1-k}\rho^{-1}\sigma^2_m + \eta\sigma^2_{y'} + \frac{\sigma}{\kappa}\sigma^2_\pi\right].$$

Appendix A.2. Economic Efficiency

As shown in DPZ, if a benevolent social planner were to include the economy-wide habit level (X_t) within her choice variables, then the first order condition with respect to consumption would be given by

$$(C^*_t - hX^*_t)^{-\rho} - \Lambda^*_t - \beta(1-\phi)E_t\Lambda^x_{t+1} = 0 \tag{A.1}$$

where Λ^x_{t+1} is the Lagrange multiplier on the equation specifying the evolution of habits (1). Moreover, the first order condition with respect to X_t would be

$$-h\,(C^*_t - hX^*_t)^{-\rho} + \Lambda^x_t - \beta\phi E_t\Lambda^x_{t+1} = 0. \tag{A.2}$$

If we further assumed perfectly flexible prices and no monopoly power, then the labour-leisure decision would simplify to

$$(C^*_t)^\eta = \Lambda^*_t A_t^{\frac{\eta}{\eta+1}}. \tag{A.3}$$

Because in equilibrium $C_t = X_t = Y_t$, therefore the equations above can be used to characterise the efficient allocation of output. Specifically, in the special case of $\phi = 0$, efficiency implies

$$C^{*\eta}_t = A_t^{\eta/(\eta+1)}\left[(C^*_t - hX^*_t)^{-\rho} - \beta h E_t\left(C^*_{t+1} - hX^*_{t+1}\right)^{-\rho}\right]. \tag{A.4}$$

We now investigate if and how this allocation can be achieved. Let's assume that the central bank targets inflation and fully stabilises prices. In this case the competitive equilibrium, would imply

$$\frac{\sigma\theta^N}{\sigma-1}C^\eta_t = A_t^{\eta/(\eta+1)}\left(C_t - hX_t\right)^{-\rho}. \tag{A.5}$$

But if we additionally posited a consumption tax T_t, which would enter households' budget constraints as

$$T_t P_t C_t(a) + V_t B_t(a) - W_t N_t(a) - B_{t-1}(a) - Q_t = 0$$

then competitive equilibrium would imply the following first order condition with respect to consumption

$$(C_t - hX_t)^{-\rho} = T_t \Lambda_t \tag{A.6}$$

and a labour-leisure indifference condition of the form

$$\frac{\sigma\theta^N}{\sigma-1} C_t^{\eta} = A_t^{\eta/(\eta+1)} \frac{(C_t - hX_t)^{-\rho}}{T_t}. \tag{A.7}$$

If the monetary authority stabilised the price level (ensuring $\pi = 0$) then the optimal level of the fiscal instrument T_t would equal

$$\begin{aligned} T_t &= \frac{\sigma-1}{\sigma} \frac{(C_t - hX_t)^{-\rho}}{(C_t - hX_t)^{-\rho} - \beta h E_t (C_{t+1} - hX_{t+1})^{-\rho}} && \text{(A.8)} \\ &= \frac{\sigma-1}{\sigma} \frac{1}{1 - h\beta \tilde{M}_t}. && \text{(A.9)} \end{aligned}$$

Under this tax rule, the competitive equilibrium would coincide with the social planners one. It follows directly, that in the steady state we would have

$$T = \left(\frac{1}{1-h\beta}\right) / \left(\frac{\sigma}{\sigma-1}\right) \tag{A.10}$$

so the decision on either taxing or subsidising consumption would depend on whether the habit externality (numerator) exceeds the monopolistic distortion (denominator).

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