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# Collective Decision-Making and Supervision in a Communist Society

Moshé Machover

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## Foreword

This article brings together my commitment to communism and my scientific work in the theory of social choice, particularly the measurement of voting power. I have been composing it, on and off, for several years: writing a few paragraphs, then putting it aside and turning to more urgent current tasks.

Originally I used the term ‘socialism’ in the title and the body of the article; but I have now changed this to ‘communism’. Let me explain why. The terms ‘communist’ and ‘communism’ have undergone a significant semantic shift since the beginning of the 21st century. For long these terms had been usurped and virtually monopolized by Stalinized ‘official communism’. For this reason, anti-Stalinist communists tended to avoid these terms as self-description and a description of the kind of society they strive for. Instead, they used terms such as ‘revolutionary socialists’ as self-description, and ‘socialism’ for their aim. (The term ‘socialism’ was also usurped by Stalinists, but they never managed to monopolize it.) However, with the collapse of ‘official communism’, there has been a growing tendency to reclaim the former terms and use them in their older sense as in the *Communist Manifesto* and Marx’s *Critique of the Gotha Programme*. I am happy to go along with this semantic shift and actively promote it.

A brief summary of the article is in Chapter 5. You can read it now, but I placed it at the end because I would like you to re-read it after going through the first four chapters: I hope it will then make better sense.

The Appendix, which contains some mathematical technicalities, is optional. If you are daunted by mathematics, you need not read it; but then you will have to take my word for some assertions I make in the main text.

I am indebted to Dan S Felsenthal for arousing my interest in the mathematical theory of voting; for collaboration in joint technical work, on which I have drawn in this article; as well as for providing the data on Swiss referenda in Chapter 2. He bears no responsibility whatsoever for the political views expressed or implicit in the article. Discussions with my friends and comrades Ehud Ein-Gil and Hannu Reime helped me to crystallize the ideas expressed in this article. I gratefully acknowledge useful comments by other friends and comrades: Majed Chergui, Paul Cockshott, Emmanuel Farjoun, Arie Finkelstein, Simon Pirani, Stephen R Shalom and Dave Zachariah. Needless to say, they may not agree with some of the views I put forward.

# Collective Decision-Making and Supervision in a Communist Society

## 1 Introduction

The importance of this issue cannot be over-stated: it concerns the very essence of communism. If communism means anything at all, it means a radical eruption of democracy. Bursting its present narrow political confines, where it is allowed to hold truncated and partly illusory sway, democracy is to engulf all spheres of social life. This applies in particular to what is, under capitalism, the alienated sphere of economics: major choices that are now made behind the backs of society – imposed by private owners who monopolize wealth, or left to the chaotic play of blind market forces – will be decided consciously and collectively by the community concerned. The enormous extension of the sphere of collective decision-making will necessarily imply a corresponding expansion and deepening of the scope of public supervision, ensuring proper implementation of decisions.

Capitalism depends for its stability and ideological legitimation on a separation between the political and economic spheres: the former ruled by formally democratic public decision-making and supervision, the latter by a combination of the micro-tyranny of private ownership and the macro-anarchy of the market. In both spheres the citizen is reduced to a passive voter or consumer, offered a limited choice between brands of political parties or soap powder whose minor differences are inflated by smooth spin machines and privately owned brain-washing media.

The borderline between the two spheres is however far from fixed: privatization pushes it in one direction, allowing public politically-controlled domains to be hived off, sold on the market and annexed by private owners. Communism demands a far-reaching shift of the borderline in the opposite direction – turning production into a social service – as well as blurring of the borderline itself, until eventually the very division between the two spheres is effaced.

Yet, socialists have devoted far too scant attention to the question as to how communist democracy is to function. There is very little detailed discussion of the institutional framework that a communist commonwealth might use for making collective decisions and supervising their implementation. The relatively few discourses that do exist in the socialist literature are, for the most part, very sketchy; perhaps worse, they depend on extremely utopian assumptions – more often than not unstated – about communist

society.<sup>1</sup>

Among Marxists, in particular, there is a reluctance to engage in what is felt to be speculative drawing of blueprints. This is justified up to a point, but must not be taken too far. We cannot win people over to communism if we remain too vague about what a communist commonwealth might be like. Moreover, given the twentieth-century experience of tyrannies that claimed to be ‘socialist’, and the moral and political bankruptcy of their ‘communist’ apologists, most people are suspicious of the true intentions of communists, and doubtful of the very compatibility of communism with democracy.

Many Marxists – particularly those of the left-wing, councillist variety – tend to be too insouciant about the whole issue.<sup>2</sup> Surely, with the disappearance of classes and the withering away of the state, social conflicts will cease to exist and collective decision-making – reduced to matters of mere administration – will no longer be problematic.

This is a grave error. First, the disappearance of classes and the withering away of the state cannot happen overnight but – following a revolutionary crisis – must extend over decades of changing consciousness. In the meantime, acute social conflicts are inevitable. Second, even in a classless society, many matters coming up for collective decision are bound to be hotly contested. In all probability, conflicting local, sectorial and personal interests, not to mention sharp differences of taste and opinion, will still exist. It is quite naïve to assume that all such conflicts and differences have a class basis. Third, a decision – whether or not hotly contested – needs to be implemented, which usually requires a social mechanism of supervision, to ensure its proper implementation.

## 1.1 The social context

My purpose in this article is to address some of the problems of democratic decision-making and supervision in a prospective communist commonwealth. It is a very incomplete attempt, designed to kick off a discussion. I confine myself to discussing the institutional framework and do not address here other essential aspects of democracy, such as freedom of speech and information.

The society I envisage here is not in turmoil of revolutionary transition,

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<sup>1</sup>An exception is the sober discussion by Daryl Glaser [11]. His article contains useful references to earlier discussions of the topic in the socialist literature. Cockshott and Cottrell’s book [6] and Shalom’s articles [17, 18], with which I take issue below, also contain some valuable ideas.

<sup>2</sup>As a typical example, see Anton Pannekoek’s famous book *Workers’ Councils* [14].

but one in which communism has established itself. I will therefore avoid facile far-reaching extrapolations from the experience of working-class democracy in periods of revolutionary upsurge – all of them tragically brief – since the Paris Commune of 1871. This historical experience is inspiring and instructive; but it would be dangerously naïve to assume that institutional forms that arise during periods of extreme collective political mobilization and social ferment are necessarily appropriate without much modification to a more sedate social environment.

It would be a serious misunderstanding to read this article as a discussion of the ‘political’ structure appropriate to a communist society. As I have pointed out, the separation between the political and other spheres of public social life – primarily the ‘economy’ – is an artefact of capitalism. It is even arguable that the very distinction between the spheres is unique to capitalism. In relation to a social formation in which the distinction no longer exists, use of the terms ‘political’ and ‘economic’ is misleading if not meaningless. For this reason I insist on using the term *collective decision-making and supervision* rather than talking about a ‘political’ structure.<sup>3</sup>

Some of what I shall say may strike the reader as formal and abstract. This is because democratic decision-making has some formal aspects that are universal, and in this sense abstract; this does not make them unimportant – quite the contrary.<sup>4</sup> As I noted above, in a communist commonwealth democracy must be radically extended in scope and intensified. No doubt some aspects of the very mechanism of decision-making must be determined by the substance of the matters being decided, and adapted to the social context in which these decisions are made. Yet some important generic aspects and principles of democratic decision-making are independent of content and context.<sup>5</sup>

## 1.2 Some general principles

I list below the most important constitutive principles of democratic decision-making. In each case, I shall comment briefly on the problems – some quite obvious, some less so – that the principle in question involves.

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<sup>3</sup>In my opinion Glaser [11] is wrong to use a purely political terminology. The same applies to the less critical discussion of decision-making by Shalom [17], which is endorsed by Albert [1].

<sup>4</sup>The formal and largely mathematical aspects of collective decision-making are the subject of a specialized academic discipline: *social choice*.

<sup>5</sup>This is, after all, why we can speak of the *extension* of democracy rather than of its replacement by something else.

**1.2.1 Sovereignty of the people** This principle, often proclaimed by the slogan ‘*Power to the people!*’, is the most fundamental principle of democracy: the power of making decisions concerning the running of a society belongs collectively to the people constituting that society. This principle does not imply that all decisions must be made *directly* by the whole community; as I shall argue below, that would in any case be impossible. But it does imply that whenever decisions are not made directly by the whole community, the direct decision-makers must be expressly mandated by it, and can only derive their authority in this way.

**1.2.2 Empowerment of the people** The slogan ‘*Power to the people!*’ also implies that the ability of citizens to influence the running of their society must be maximized; hence the dilution of people’s power – which some forms of indirect decision-making necessarily involve – must be minimized.

**1.2.3 Universal suffrage** This principle is not as unproblematic as it may first seem. Until not very long ago, it used to be proclaimed by the slogan ‘*One man, one vote*’, a hangover from the time when ‘universal’ suffrage meant universal *male* suffrage.<sup>6</sup> We now say ‘*One person, one vote*’; but who counts as a ‘person’ for this particular purpose? Surely, some demarcation by age is necessary; but where exactly do you draw the line? Other restrictions may also be justified,<sup>7</sup> but their exact determination is bound to be problematic, and to some extent arbitrary.

Another type of demarcation problem is that of determining the constituency, the part of society that is to make a given decision. The democratic ideal of communism implies *subsidiarity*: a fair degree of decentralized decision-making, allowing a great deal of autonomy to collectives determined geographically (local communities) and functionally (occupations,

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<sup>6</sup>On 29th October 1647, the second day of the Putney Debates of the English Parliament’s army, Commissary-General Henry Ireton (Oliver Cromwell’s son-in-law) objected to the idea that ‘every man that is an inhabitant is to be equally considered, and to have an equal voice in the election of those representers, the persons that are for the general Representative...’. To which the Leveller leader Colonel Thomas Rainborough made the memorable reply: ‘[R]eally I think that the poorest he that is in England hath a life to live, as the greatest he; and therefore truly, sir, I think it’s clear, that every man that is to live under a government ought first by his own consent to put himself under that government; and I do think that the poorest man in England is not at all bound in a strict sense to that government that he hath not had a voice to put himself under...’

<sup>7</sup>For example, it would be fair to deny the vote (for a limited period or permanently) to persons guilty of certain anti-social misdeeds, such as electoral fraud. It is naïve to expect that anti-social behaviour will ever be totally eliminated, or that it will not need to be penalized. But punishment should fit the crime. Needless to say, the blanket disfranchisement of all prisoners or convicted felons – as practised, for example, in some US States – is anti-democratic and a violation of human rights.

workplaces or enterprises). Yet a local community cannot be permitted to exercise parochial monopoly over resources that happen to be located in its area but in which wider society has a legitimate interest. The trees of a forest, the oil and other mineral deposits in the ground, must not be left to the people of the vicinity to dispose of as they alone see fit. Similarly, a self-managing production unit or workplace cannot be allowed absolute authority to make decisions that impinge on legitimate concerns of society at large. The number of motor vehicles of each category operating in a region or a country cannot be left to the sole discretion of those engaged in vehicle production. Greenhouse gasses and other pollutants, produced locally, are a global plague.

Control of production by the direct producers is essential to communist democracy. Yet it cannot be exercised properly at an exclusively local level, separately in each separate workplace. Those working in a given workplace cannot be allowed to run it entirely as they alone please, ignoring the wishes and interests of other members of the wider society.

A major weakness of anarchism is its failure to provide satisfactory answers to this kind of problem. The totally decentralized society that it envisages would leave resources entirely under localized control – resulting in gross inequalities. Without a great deal of non-local decision-making, macro planning would be impossible – leaving overall regulation to revert inevitably to blind anarchic market forces.

**1.2.4 Equal suffrage** The slogan ‘One person, one vote’ has an additional meaning: suffrage ought to be not only universal, but also equal. On the face of it, this is quite unproblematic – and so it is as far as direct decision-making is concerned: simply give each voter a single vote. However, things are much less simple under certain forms of indirect decision-making. Consider a council made up of delegates from several constituencies; each delegate is mandated to vote in the council according to the majority opinion in his or her constituency. In this two-tier setup, the citizens in the various constituencies are the indirect decision-makers, acting through their respective delegates. If the constituencies vary in size – as is bound to happen if they are not arbitrarily demarcated but ‘organic’ entities, such as local communities or functional collectives – then it would clearly be unfair to give all delegates equal votes in the council: it would mean that members of large constituencies are under-represented and have less indirect influence on the final outcome compared to citizens of small constituencies. We can try to remedy this by giving delegates unequal weighted votes. But finding the right weighting is a tricky business: the obvious common-sense solution of making the weights proportional to the respective sizes of the constituencies

turns out to be quite wrong. To take a very simple ‘toy’ example, consider a council of three delegates, one representing a workplace numbering 3,000 persons, and each of the other two representing 1,000 persons. Suppose the delegate of the larger constituency is given three votes and the other two delegates one vote each. The council makes decisions by a majority of votes. Then the ‘heavier’ delegate decides all issues, as s/he is always in the majority (casting three out of a total of five votes), while the other two delegates, and hence all their 2,000 constituents, have no influence at all on the final outcome, and are thus in effect disfranchised.<sup>8</sup>

**1.2.5 Majority rule** According to this principle, decisions are to be made by *simple* (or *absolute*) *majority*: a resolution is passed if the number of those voting for it exceeds that of those voting against it; otherwise, the resolution is defeated.<sup>9</sup> Note that majority rule is associated with individual equality, because it assigns equal weight to all votes.<sup>10</sup> Also, perhaps less obviously, of all possible decision rules the simple majority rule endows voters with the greatest possible power.<sup>11</sup>

However, unrestricted use of this democratic principle is undesirable, for two reasons. First, certain minority interests and preferences ought to be protected against being overridden: otherwise, the result may be an oppressive dictatorship by the majority. Second, some fundamental aspects of the status quo (such as constitutional ground rules) ought to be privileged and protected against being overturned by a slim majority margin: otherwise, the result may be great instability, with far-reaching changes being all too easily made and just as easily reversed.<sup>12</sup>

## 2 Referenda and their limitations

A referendum is a procedure whereby all members of the society who are entitled to have a say in an issue do so by voting on it directly, rather than

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<sup>8</sup>I shall return to this problem in Section 3.3 and address it in detail in the Appendix, Subsection 6.1.7.

<sup>9</sup>Here I am assuming of course that all those whose votes are counted are entitled to vote on the given issue (see Subsection 1.2.3 above). On the other hand, not all those entitled to vote need do so. In a decision made by simple majority, abstentions are ignored.

<sup>10</sup>The term ‘simple majority’ is sometimes used in a more general sense, to apply to cases where voters may have unequal weights. I will avoid this usage in the present article.

<sup>11</sup>See Subsection 6.1.4 in the Appendix.

<sup>12</sup>In this article I will not discuss institutional mechanisms for restricting majority rule. For a discussion of this issue see Shalom [17].

handing the decision over to an elected council or assembly. In an obvious sense, this is the most democratic decision-making system. Any other procedure, whereby decisions are mediated via delegates or representatives,<sup>13</sup> must raise concerns as to the reliability of the mediating mechanism: do the elected representatives vote in accordance with the wishes and interests of their constituents? are they immune to improper pressures and to temptations of self-interest? Moreover, as we shall see later, some indirect decision-making systems – including the one most highly praised and recommended in the anarchist and Marxist literature (insofar as this literature discusses such questions) – dilute the influence of the ground-level (‘grass root’) indirect voters and may violate majority rule, *even if all elected delegates act in the most proper and scrupulously honest manner*. It is therefore clear that in a communist democracy direct unmediated decision by referendum ought to be used as much as possible.

Yet it is also clear that only a small proportion of collective decisions can be made in this way. For one thing, a referendum is suited to deciding relatively brief, clear and simple questions, not complex resolutions. But the main reason is more basic: the limitation of time. In a typical national bourgeois parliament, several hundred votes are taken each year. In a communist commonwealth, the number of decisions that will have to be made at a national level will no doubt be greater, because (as I have argued in the Introduction) many more issues will be subject to collective decision-making. To this must be added decisions at other levels: local, regional and – let us not forget – supra-national. All in all, the collective decisions that affect any given citizen and in which s/he ought therefore to have some say will, at a reasonably conservative estimate, number many hundreds, perhaps a few thousand, per year. If all collective decisions were to be made directly, then every conscientious citizen would have to engage in voting continually: it would become more than a full-time occupation, to the exclusion of any other social or private activity. And there would hardly be time for proper discussion of the issues being decided or for due consideration of the arguments *pro* and *con*, or for exchange of information on the factual and technical background needed for forming a well-founded opinion.<sup>14</sup> If put in such a position, most normal people would react by abstaining altogether, or by voting in a haphazard and capricious way. Thus decisions would actually be made by a self-selecting clique of busybodies and an ill-informed and so easily misled mass. This would make a mockery of democracy. In the context of a complex and technically advanced global commonwealth, the ar-

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<sup>13</sup>For the difference between a delegate and a representative, see Section 3.1 below.

<sup>14</sup>An essentially similar point is made by Shalom [18].

adian utopia in which most social decisions are made directly by referendum transmutes into a recipe for a reactionary dystopia.

Referenda – while playing an important role in a communist commonwealth – must therefore be confined to a relatively small number of major issues such as constitutional questions, and to cases where a sufficient number of citizens actively demand it.

## 2.1 The Swiss experience

In this connection it will be instructive to have a brief look at the role of referenda in Switzerland. Of course, as in all capitalist countries, democracy in Switzerland is confined to the political sphere; but within this relatively narrow domain, decision-making is arguably more democratic than in any other country, largely thanks to the major role of referenda in the Swiss constitution.

I will confine myself here to discussing Federal – that is to say nationwide – referenda. The Swiss constitution provides for three kinds of Federal referendum.

- The government can initiate a referendum on any issue it chooses.
- Citizens can initiate a referendum for repealing a Federal law. The proposers of such a referendum must collect 50,000 signatures or secure a majority in the legislatures of at least eight (out of the 26) cantons.<sup>15</sup>
- Citizens can initiate a referendum for amending the Swiss Constitution. The proposers of such a referendum must collect 100,000 signatures.<sup>16</sup>

The first thing to note is that the number of signatures – 50,000 or 100,000 – required for triggering a referendum by popular initiative is quite small compared to the size of the Swiss electorate: just under 5 million.<sup>17</sup>

During the decade 1999–2008, referenda were held on 31 occasions. The

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<sup>15</sup>Unlike the practice in many other countries (notably the United States), the Swiss constitution does not allow the Supreme Court to annul a law by declaring it unconstitutional. So in Switzerland the electorate at large, acting directly by referendum, is the final arbiter on all legislation.

<sup>16</sup>For a more detailed statement of these constitutional provisions, as well as for data on Swiss referenda since 1848, see Diskin et al. [7]. For the official source of the data I quote below, see <http://tinyurl.com/cyfo9z>.

<sup>17</sup>The official number of eligible voters for the last referendum conducted in 2008 was 4,996,626. At that time the total Swiss population was estimated at 7.7m.

total number of propositions was 95.<sup>18</sup> Of this total, only 14 were put by the government. Of the remaining 81 propositions, which were put to referendum by citizens' initiative, 31 aimed to repeal Federal laws, and 50 to amend the Constitution.<sup>19</sup>

The picture that emerges from these data appears to be one of a lively referendum culture. Indeed, this culture is well entrenched and rooted in a long and cherished tradition. Yet there are indications that the frequency of national referenda, as well as the number of propositions put to referendum, cannot be increased much further without adverse effects. Already, the level of Swiss citizens' participation in referenda is worrying low: in the 10-year period 1999–2008, the average turnout rate was approximately 45%. A most likely reason for the decline in participation – apart from a general loss of interest in politics – is ‘the significant growth over time in the number of referenda conducted per year which causes citizens to become weary or apathetic with this instrument of direct democracy’.<sup>20</sup> One may hope that in an established communist commonwealth popular enthusiasm for voting will be greater; but it would be unwise to stretch it too far.

## 2.2 Supervision and monitoring

From the foregoing it is clear that referenda should not be too frequent; most decision-making (except on matters of exclusive concern to small local or functional collectives) will have to be mediated through elected delegates or representatives.

There is another purpose, closely connected to decision-making, for which elections are necessary. Decisions are worse than worthless if they are not implemented. Implementation is, by definition, an *executive* function, which in most cases is assigned to designated persons, mandated to act on behalf of society.<sup>21</sup> Whether these persons are elected, appointed or volunteer to

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<sup>18</sup>The number of propositions put on a single occasion ranged from one (on 18.4.1999, 21.5.2000, 5.9.2005, 21.5.2006, 11.3.2007 and 17.6.2007) to nine (on 18.5.2003).

<sup>19</sup>The government was successful in 11 of the 14 propositions it put to referendum. Of the 31 citizens' initiatives to repeal laws, 25 succeeded. Of the 50 amendments to the Constitution proposed by citizens, only three passed: Swiss voters are generally reluctant to approve constitutional amendments.

<sup>20</sup>Diskin et al. [7, p. 332].

<sup>21</sup>Note that many functions that under capitalism are regarded as belonging to the private ‘economic’ sphere will be recognized in a communist commonwealth for what they really are: social tasks, performed on behalf of society as services to it; and will be institutionalized as such.

perform their tasks,<sup>22</sup> their performance needs to be monitored and supervised. Society has to be satisfied that its decisions are implemented correctly, efficiently and honestly; and this cannot be left to the say-so of those performing the executive function – however conscientious they are – but must be verified objectively, independently and *systematically*.

Thus we have a third type of function – monitoring and supervision of performance – which is distinct from both decision-making and implementation, but is more naturally associated with the former and must keep a certain distance of objectivity from the latter.

Monitoring and supervision cannot be left to the spontaneous initiative of society at large. That would lead in some cases to excessive meddling by too many self-appointed inspectors, impeding those being supervised in going about their normal business. But in other cases it would leave executive tasks unsupervised: if no-one in particular is assigned the duty of supervision, everyone may assume it is, or should be, done by someone else.

Some monitoring tasks – for example, the detailed auditing of accounts – are best left to specialist comptrollers. But for the most part supervision of implementation should go hand in hand with decision-making, as they are in a sense two successive phases of the same process. In other words, those elected to be direct decision-makers must double also as monitors, checking the implementation of collective decisions.

This dual role of decision-making and monitoring is of course commonplace in bourgeois parliamentarism: legislative assemblies – or, more usually, their sub-committees – engage in monitoring and supervising executive actions. However, in present-day society this public monitoring function, just like its decision-making counterpart, is confined to the narrow sphere of what is admitted as public business; it excludes social activity that – in a perverted market-dominated logic – is defined as ‘strictly private’, beyond the reach of legislation and public scrutiny.

### 3 The council model

As we have seen, collective decision-making in a communist commonwealth, as well as the closely associated task of monitoring and supervising the implementation of decisions, must for the most part be mediated through a structure in which citizens at large act *indirectly*: by entrusting the *direct* performance of these tasks to persons specifically elected for this purpose.

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<sup>22</sup>By the way, I must make it clear that the logically necessary distinction between the *functions* of decision-making and implementation need not imply a separation between two lots of persons performing each function: they may well overlap.

In this chapter I will consider one model of such a structure: a multi-tier structure of councils. This model is the one favoured by anarchists as well as by most Marxists, especially of course by those of the leftist, councillist tendency.<sup>23</sup> I will argue that, despite its great merits, it cannot serve on its own as sole indirect mechanism for democratic decision-making, but ought to operate in tandem with a simple two-tier parliamentary structure. In the next chapter I will discuss a model of the latter kind most suited for a communist commonwealth.

### 3.1 The council pyramid

Grass-root councils have formed in all major working-class revolutionary and pre-revolutionary upheavals since the Paris Commune of 1871, and have proved themselves as an indispensable form of working-class and popular self-activity and mobilization. But there is no historical experience that can be drawn upon directly to assess the council system as a decision-making structure in a post-revolutionary communist democracy. My discussion of this issue must therefore be purely theoretical, and therefore tentative.

Let me first outline the general form of such a structure. Here I borrow from an article by Stephen R Shalom [17],<sup>24</sup> which is also cited approvingly in a recent book by Michael Albert [1, pp. 25f].

There would be primary-level councils that would include every adult in the society. The number of members in these primary-level councils would be somewhere between 25–50. Each primary-level council would choose a delegate to a second-level council. (Each second-level council would be composed of 20–50 delegates, probably the same size as the primary councils, but not necessarily so.) Likewise, each second-level council would choose delegates to third-level councils, and so on, until there was one single top-level council for the entire society.

The number of members on each council would be determined on the basis of a society-wide decision, and perhaps revised on the basis of experience, so as to meet the following criteria: small enough to guarantee that people can be involved in deliberative bodies, where all can participate in face-to-face discussions; but yet big enough so that (1) there is adequate diversity of opinion included; and (2) the number

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<sup>23</sup>Lenin also advocates such a model in his 1917 pamphlet *State and Revolution*, which represents the most leftist phase in his thinking. Like most others who have advocated this model, he does so in very general terms, without going into detail.

<sup>24</sup>Shalom has elaborated on his scheme in subsequent articles; see Shalom [18] and articles cited therein.

of layers of councils needed to accommodate the entire society is minimized. For example, if all councils have 25 members, then, assuming half the population consists of adults, then 5 layers could accommodate a society of 19 million people; with councils of 40 members, 5 layers could accommodate 200 million people; and 50-person councils could accommodate 625 million people by the fifth level. With a sixth level, even 25-person councils could accommodate a society of about half a billion people.

The most obvious – and necessary – feature of this kind of structure is that it is *pyramidal*: it consists of several tiers, starting from the bottom tier at the ‘grass roots’, and culminating at the apex in the top-tier council for the entire society. Altogether, about five tiers would be required for national decision-making, and at least six if we take into account supra-national decisions, which will surely be much needed in an increasingly interdependent global humanity.

Note also that a lower-tier council chooses a *delegate* rather than a *representative* to speak for it at a council in the next higher tier. The difference is important.

A delegate is mandated to vote in a specific way: in accordance with the majority opinion in the delegating body that elected him or her. The mandate cannot normally be too rigid: there is little point to having any debate at a council of delegates whose votes are entirely predetermined. Nor can it usually be very explicit and detailed, for the same reason that very frequent referenda are impracticable. But a delegate can be recalled and replaced at any time, so the delegating body retains a great measure of control over how s/he votes.

A representative, on the other hand, is elected for a fixed term and is not subject to recall during that term, except under narrowly prescribed extraordinary circumstances. S/he is free to decide how to vote on any issue that comes up in the body to which s/he has been elected, although s/he is expected to remain faithful to a platform that s/he had published or endorsed before the elections.

The pyramidal council structure has some undeniable inbuilt attractive advantages, the most important of which are the following two.

First, it encourages social cohesion. Individual citizens are not represented as disconnected atoms but organized as members of ‘organic’ units: geographically defined local communities or functionally determined collectives.

Second, it encourages wide *active participation* of citizens at the grass-root level.

### 3.2 Sociological critique of the council pyramid

However, each of these undoubted advantages has an undesirable flip side.

First, categorizing a person *solely* as member of a local or functional group is one-sided. A free human being, citizen of a free commonwealth, is not merely a member of such a group but also, and at the same time, a member of the entire society at large. Let us consider decisions that concern a whole country and so need to be taken at a national level. For some such decisions, local or sectorial interests and viewpoints are relevant, and so it is reasonable for citizens to be represented through a structure that reflects their local or functional affiliation. But there are surely some national-level decisions for which these affiliations are either irrelevant, or should be ignored, even overruled, in order to prevent undue special pleading. As Glaser [11, p. 157] puts it:

Representation based on enterprises alone privileges a uni-situational identity – that of individual as producer; based on a collection of different kinds of communities, it represents individuals through a whole *series* of capacities. Neither, however, represents the individual as a ‘citizen’ – as a composite member of a wider national-territorial unit, endowed with rights, powers and obligations commensurate with those of other ‘citizens’. The concept of citizens is often criticised as an abstract category designating a legal and formal condition of existence but concealing real socio-economic, gender and other inequalities; it is also accused of fostering an atomistic individualism that is uncondusive to solidaristic activity and likely to leave the individual isolated and divided in the face of powerful institutions like the state, private corporations, and so on. These criticisms are well taken: any defensible model of democracy must include self-governing ‘real communities’ and an active, voluntary civil society mediating between the individual and larger, more powerful bodies. But one of the prerequisites of an active civil society is a national ‘public sphere’ in which individuals confront each other as citizens with common rights and commensurate terms of political and legal reference. Not only is this the precondition for activating a national civil society but it alone can ensure that ‘real communities’ do not become mired in an uncommunicative parochialism which could block the free exchange of ideas, make it impossible to establish uniform norms of material distribution between communities, and licence violation within particular communities of rights enjoyed by individuals as citizens. Moreover, there are areas of decision-making which are most appropriately addressed by individuals as citizens, as possessors of what Polan calls ‘trans-situational’ legal and political identities, rather than as members of one or even a

proliferation of distinct, quite often occupationally specialised or culturally discrete, base communities: matters such as citizens' rights, national defence, foreign policy – to name but a few. The nation-state can, in *certain* respects, itself constitute a 'real community'.

Second, while the pyramidal council structure *encourages* active participation of citizens at the grass-root level, it also *depends* on it for proper functioning. Let me quote Glaser [11, p. 159] once again:

The second . . . premise of pyramidal representation is that individuals *within* 'real communities' will be more-or-less continuously politically mobilised and active. There is no point (democratically speaking) in placing a layer of delegates between the electorate and the central assembly unless that intermediate body itself serves as a focal point of active, widespread, sustained popular mobilisation, enabling ordinary people to confront higher tiers of elective officials with their collective power rather than in passive isolation.

Glaser rightly comments that this premise is 'rather precarious' and 'unsustainable'. There is no structural, inbuilt guarantee that widespread grass-root activism will be maintained in the long term at a sufficiently high level. But if a majority of the citizens are not active, positions taken at the bottom tier of cells and transmitted upwards will be biased in favour of the opinions and concerns of a self-selecting activist minority.<sup>25</sup>

### 3.3 Problematics of equal suffrage

Shalom's outline of the council pyramid quoted above (Section 3.1) is extremely schematic. Trying to unpack it raises some serious problems for implementing equal suffrage.<sup>26</sup>

Can all grass-root ('primary level') councils – or, for that matter, all councils at a given tier of the pyramid – be of the same size? Clearly, citizens are not 'naturally' grouped in cells of 25 or 50 persons. Organic units – whether they are geographically defined local communities or functionally determined collectives – are objectively given and vary considerably in size. Cutting across them in an arbitrary way must surely be ruled out. But if councils at a given tier vary in size, then – as I pointed out in Subsection 1.2.4 – the principle of equal suffrage seems to require that their respective delegates to the next-higher tier should be given unequal weighted votes. As noted there, this is trickier than it first appears.

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<sup>25</sup>A more far-reaching critique of the council structure is levelled by Cockshott and Cottrell [6, Ch. 13]; cf. footnote 55 in Section 4.5.

<sup>26</sup>Cf. the discussion of these problems by Glaser [11, pp. 153–156].

An even more difficult problem arises when we ask ourselves whether the grass-root cells ought to be determined geographically or functionally. The only reasonable answer is that *both* kinds are needed: geographically-based councils are essential for dealing with local issues, and functionally-based cells for self-management of production and of other sectorial functions that are not anchored in a person's specific place of residence. So the council pyramid must consist – at least in its lower tiers – of two kinds of council.

A person belonging to cells of both kinds will then have two votes, one in each cell. But a citizen who is not active in production (due to illness or old age), or who works alone or in a very small group, can only have one vote. This is inconsistent with the principle of equal suffrage.

### 3.4 Technical critique of the council pyramid

But in my opinion the greatest shortcomings of the pyramidal council system are technical structural ones, which as far as I know have not received any attention at all in the socialist literature. These shortcomings are necessary consequences of the extremely indirect nature of decision-making in such a system.

As I noted in Chapter 2, direct decision-making by referendum is the most democratic system. Mediated decision-making is a necessary evil – necessary because making every collective decision by referendum is not feasible, and holding too many referenda is counter-productive.

But the pyramidal council system is, in this sense, the very opposite of direct democracy: it is far more indirect than parliamentary democracy. In the latter, the citizens' decisions are mediated by one tier of elected representatives; whereas in the former four, five, six or more mediating tiers are interposed between the citizens and decision at the national or supranational level. The direct decision-makers at the apex are extremely remote from the base.

Glaser [11, p. 156] rightly notes that 'one of the paradoxes of the councilists' aspiration to some approximation of direct democracy is that their proposals for achieving it frequently invite the opposite result: the most indirect kinds of democracy'. He points out that this can easily lead to 'undue concentration of power in particular groups of individuals or in higher tiers of political and bureaucratic authority'. Such unwelcome consequences can indeed follow if the council system malfunctions due to insufficiently widespread mobilization at the grass-root level.

But the point I wish to make here is quite different. Even if the pyramidal council system does not malfunction at all in its own terms, even if it works in its ideal form, with full mass mobilization and absolutely faithful

transmission of resolutions adopted by majorities at each tier to the next higher tier – even in this ideal case, it necessarily clashes with two cardinal principles of democracy.

First, it dilutes the voting power of citizens, their degree of influence over the final decisions made at the apex. In this way it attenuates people’s power (Subsection 1.2.2).

Second, it conflicts with the principle of majority rule (Subsection 1.2.5): a resolution that wins a majority in a council of delegates may go against a majority of the citizens at the base of that council.

Both of these phenomena are amplified with each additional tier in the pyramid.

These matters are analysed mathematically in the scientific literature on voting power. Here I can only give a brief outline of the conclusions of that analysis. Some further details, including a few mathematical technicalities, are presented in the Appendix.

**3.4.1 Dilution of voting power** The scientific theory of voting power was founded by Lionel S Penrose [15, 16]. Penrose’s theory conceptualizes the ‘voting power’ of a member, say  $v$ , of a decision-making body as the degree to which  $v$  is able to affect the outcome of a division of that body on a hypothetical proposed resolution into ‘yes’ and ‘no’ camps. According to this theory, the voting power of  $v$  is equal to the probability that  $v$  will be in a position to cast a ‘decisive vote’; that is, that the remaining voters (all voters other than  $v$ ) will be so divided, that  $v$ ’s vote will tip the balance and so decide the outcome.<sup>27</sup>

Let us consider a two-tier decision-making structure. At the base – the lower tier of the structure – there are a number of cells (grass-root councils) of citizens. For simplicity, let us assume that all these cells are of equal size. The upper tier consists of a council of delegates, one delegate from each lower-tier cell. Decisions are made by simple majority at the upper tier; so the delegates are the direct decision-makers. But each delegate is committed to vote ‘yes’ or ‘no’ according to the majority opinion in his or her grass-root cell; so the citizens are *indirect* decision-makers in this structure, influencing the final decision via the votes of their respective delegates at the upper tier. Let us assume that all the delegates are perfectly faithful to their commitment to reflect the majority view in their respective cells. So in order for a resolution to be approved by this two-tier structure, it must win majorities in a majority of cells. (For example, if there are 75 cells, each comprising 50 citizens, then a resolution will be approved if it is supported by at least 26 citizens in each of at least 38 cells.)

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<sup>27</sup>For technical details see Subsection 6.1.2 in the Appendix.

Now let us compare the (indirect) voting power of a citizen in the two-tier structure with the (direct) voting power that the citizen would have if the decision were taken by simple majority in a referendum among the citizens at large. (In our example, the referendum would be conducted directly among the 3,750 citizens, irrespective of the lower-tier cell to which they belong.)

It turns out that the voting power of a citizen in the two-tier structure is smaller than it would be in the direct referendum. In fact, the former power is approximately 79.79% of the latter – a loss of approximately 20.21%.

This dilution of power is compounded with each additional tier. So the voting power of a citizen in a three-tier pyramid is approximately 79.79% of the voting power in a two-tier system (with the same total number of citizens). The loss of voting power compared to that in a direct referendum is then approximately 36.34%. In a four-tier pyramid, a citizen loses almost half of the voting power s/he would have in a direct referendum. In a five-tier pyramid, the loss is almost 60%.<sup>28</sup>

Let me emphasize: this loss of people’s power is not due to usurpation by evil bureaucrats or to dereliction of duty by disloyal delegates. It involves no erosion of the people’s sovereignty (see Subsection 1.2.1). Rather, it is inherent in a pyramidal decision-making structure: the vote of an individual citizen counts for less, has a smaller chance of affecting the final outcome, than in a referendum.

**3.4.2 Majority deficit** In any pyramidal decision-making structure of the kind considered in this section, it is always possible for the outcome to go against the votes of a majority of the citizens at large. When this occurs, we say that there is a *majority deficit*, whose size is the margin by which the majority of the citizens at large who oppose the outcome exceeds the minority who support it. In cases where the majority of the citizens at large support the outcome, or where the citizens are evenly split, the *majority deficit* is 0.

Here is a simple ‘toy’ example illustrating this phenomenon. Consider a two-tier structure of 3 cells, each consisting of 3 citizens. The upper tier is a council of 3 delegates, one from each cell. Suppose a resolution is supported by a majority of 2 to 1 in each of two of the cells and is opposed by all 3 members of the third cell. In the council – assuming that the delegates vote in accordance with the majority opinion in their respective cells – the resolution is approved by a majority of 2 to 1. In this case a majority of the citizens at large – 5 out of 9 – oppose the outcome, whereas only a minority – 4 out of 9 – support it; so the majority deficit is  $5 - 4 = 1$ . Had the 9

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<sup>28</sup>For details of this calculation see Subsection 6.1.9 of the Appendix.

citizens decided the resolution directly by referendum, it would have been defeated by a majority of 5 to 4.

In this toy structure the majority deficit cannot be greater than 1; but if each of the three cells had 25 rather than 3 members, the majority deficit could be as large as 23.<sup>29</sup>

Clearly, if a positive majority deficit occurs, the losing majority camp of citizens have every right to resent the outcome, which violates the principle of majority rule (see Subsection 1.2.5). The larger the majority deficit, the greater and more justified the resentment.<sup>30</sup>

The vulnerability of a given decision-making system to this highly undesirable phenomenon is measured by its *mean majority deficit* (MMD) – the statistical average of the majority deficit that can result in it under all possible divisions of the citizens at large into ‘yes’ and ‘no’ camps. For example, in our toy 9-voter two-tier structure, there are 512 possible ways in which the 9 citizens can divide into ‘yes’ and ‘no’ camps. Of these, 54 divisions yield a majority deficit of 1; in the remaining 458 divisions the majority deficit is 0. So here the MMD equals  $\frac{54}{512}$ , or approximately 0.105. In the example cited in Subsection 3.4.1, with 3,750 citizens in 75 cells of size 50, the MMD is approximately 5.<sup>31</sup>

It can be proved mathematically that the MMD is closely connected with the dilution of voting power discussed above in Subsection 3.4.1. In fact, the MMD is directly proportional to the sum total loss of voting power of all the citizens (compared to their voting power under simple majority in a direct referendum).<sup>32</sup> Therefore, for a given total number of citizens, the majority deficit phenomenon tends to be amplified with each additional tier in the

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<sup>29</sup>This would occur if the majority camp consisted of 49 citizens, of whom 25 constituted the whole of one cell, and the remaining 24 were split evenly between the other two cells. The minority camp of 26 would constitute a majority within these two cells, and thus win the outcome in the top-tier council.

<sup>30</sup>This phenomenon is familiar from real-life politics. The US President is elected by an indirect two-tier procedure, essentially similar to the one discussed in this section. In the 2000 elections, Al Gore won a plurality of the popular vote – even if one accepts at face value the extremely shady result in Florida – but GW Bush had a majority in the Electoral College and won the election. The same kind of thing happened in two previous presidential elections. In 1876 Rutherford B Hayes defeated Samuel J Tilden, who had won a majority of the popular vote. In 1888 Benjamin Harrison defeated Grover Cleveland, who had won a plurality of the popular vote. A masterly account of the 1876 fiasco – which also involved shady shenanigans in Florida – is contained in the historical novel *1876* by Al Gore’s distant relative Gore Vidal.

<sup>31</sup>This can be verified using formula (11) of the Appendix.

<sup>32</sup>For further details, see Subsection 6.1.5 in the Appendix.

pyramidal decision-making structure.<sup>33</sup>

**3.4.3 Collectivist bias** All the shortcomings of the council pyramid that have been discussed above have a common underlying cause: the ‘lumpiness’ of this structure, its collectivist bias, which makes it relatively unresponsive to individuals, and entails loss of individual detail.

This is clearest with regard to majority deficit (Subsection 3.4.2). The position adopted by a council will be transmitted upwards to the next-higher tier, as the corporate consensus of that council, with exactly the same effect whether the council adopted it unanimously or by a slim majority margin. The dissenting view of the minority will be submerged, even if it happens to be the at-large majority view of voters at the same tier.

As for the dilution of citizens’ voting power (Subsection 3.4.1), its connection with the lumpiness of the structure is a bit harder to see.<sup>34</sup> Here is how it works. An individual citizen’s vote can only decide the final outcome if it succeeds in jumping through the collective hoops of a whole hierarchy of groups to which the citizen belongs. For Citizen Alice’s vote to be decisive in the entire pyramidal structure, a conjunction of several independent events has to occur: first, Alice’s vote must tip the balance in the grass-root cell to which Alice belongs; second, the vote of Ben, the delegate of this cell, must be in a position to tip the balance in the second-tier council to which Ben has been delegated by Alice’s cell; third, the vote of Carol, the delegate of this second-tier council, must be in a position to tip the balance in the third-tier council to which she has been delegated by that second-tier council . . . and so on up the pyramid. It turns out that the probability of this chain of events is significantly smaller than the probability of Alice’s vote being decisive in a referendum among the citizens at large; and the longer the chain, the smaller its probability.

The lumpiness of the pyramidal structure is also a cause of its vulnerability to insufficient grass-root mobilization (see above, p. 15). The position taken by a grass-root cell will be transmitted up the pyramid as though it is the consensus of the entire cell even when in fact it may just reflect the view of an activist minority, while a majority of the members are too tired or bored to turn up regularly to the cell’s frequent meetings.

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<sup>33</sup>Shalom [18] addresses the problem of majority deficit, but the solution he proposes is in my opinion inadequate. He says, in effect, that where there is reason to believe that a positive majority deficit exists, the issue should revert to the primary level to be decided by referendum. But in order to be sure whether a positive majority deficit occurs, one would need to conduct a prior shadow referendum on all, or almost all, issues decided at a higher-tier council.

<sup>34</sup>But note that, as I pointed out above, there is a direct mathematical relation between this phenomenon and majority deficit.

Finally, the problems of equal suffrage (Section 3.3) arise because the pyramid structure is based on groups and collectives rather than individuals.

### 3.5 Bicameralism

What conclusion should we draw from the foregoing critical discussion? In my opinion it follows that the council pyramid cannot be used *on its own*, as the *sole* decision-making structure of a communist commonwealth: that would result in an unacceptable democratic deficit, especially at the upper tiers of the pyramid.

On the other hand, I believe that it would be wrong to discard this decision-making system altogether. Its advantages (see p. 13) are real enough and are not nullified by its shortcomings. Indeed, a structure of this sort is indispensable for a non-authoritarian, largely decentralized commonwealth, based on self-management and a reasonable degree of local autonomy (see Subsection 1.2.3).

What is needed at the national and supra-national level, as well as at the highest sub-national level – that of cantons, provinces or member-states of a national federation – is a *bicameral* setup, in which a directly elected assembly of representatives functions as the ‘lower’ chamber, whereas the council of delegates at the corresponding tier of the pyramid serves as the ‘upper’ chamber.

This dual structure is most suitable for resolving the tension between the two complementary aspects of a communist commonwealth: individuality and collectivity. The citizen is a free individual member of the entire commonwealth at large, a universal person, directly equivalent and equipotent with all other citizens; and at the same time a member of one or more ‘real communities’.<sup>35</sup>

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<sup>35</sup>In this connection it may be noted that for Marx ‘individual’ and ‘individuality’ always had a positive connotation. His vision of communism posited full development of free individuality, within a communal social matrix.

‘Relations of personal dependence (entirely spontaneous at the outset) are the first social forms, in which human productive capacity develops only to a slight extent and at isolated points. Personal independence founded on *objective (sachlicher)* dependence is the second great form, in which a system of general social metabolism, of universal relations, of all-round needs and universal capacities is formed for the first time. Free individuality, based on the universal development of individuals and on their subordination of their communal, social productivity as their social wealth, is the third stage. The second stage creates the conditions for the third.’ (*Grundrisse*, Chapter on money)

Most remarkable, in particular, is Marx’s seemingly paradoxical repeated insistence that in a communist society the form of property will be *individual*, based on collective possession of the means of production by the associated producers.

In such a bicameral setup, direct election of representatives to the lower chamber can implement those major principles of democracy that are essentially concerned with individual rights and equality: greater empowerment of the individual citizen, equal suffrage, and majority rule. This compensates for the weaknesses and faults of the council pyramid in these respects – pointed out in Sections 3.2, 3.3 and 3.4 – which are caused by the collectivist bias discussed in Subsection 3.4.3. In fact, this bias itself becomes an advantage in the bicameral setup: the role of a council of delegates as an upper chamber is to reflect the corporate interests and preferences of organic communities and collectives rather than those of individual citizens as such.

Bicameralism is of course by no means a new idea. In medieval Europe, bicameral (or multicameral) setups were designed to give separate representation to different classes or ‘estates’. The rationalist Founding Fathers of the United States of America adapted the bicameral design and transformed it radically in the Great Compromise, so as to reflect the federal structure of the US. In the Federal lower chamber, the House of Representatives, individual citizens are supposed to be represented on an equal basis: Representatives are elected by constituencies of approximately equal population size.<sup>36</sup> In the US

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‘The capitalist mode of appropriation, the result of the capitalist mode of production, produces capitalist private property. This is the first negation of individual private property, as founded on the labour of the proprietor. But capitalist production begets, with the inexorability of a law of Nature, its own negation. It is the negation of negation. This does not re-establish private property for the producer, but gives him individual property based on the acquisition of the capitalist era: i.e., on co-operation and the possession in common of the land and of the means of production.’ (*Capital* vol. I, Ch. 32)

‘Yes, gentlemen, the Commune intended to abolish that class property which makes the labour of the many the wealth of the few. It aimed at the expropriation of the expropriators. It wanted to make individual property a truth by transforming the means of production, land, and capital, now chiefly the means of enslaving and exploiting labour, into mere instruments of free and associated labour. But this is communism, “impossible” communism!’ (*The Civil War in France*)

<sup>36</sup>Until the Civil War, this lofty principle was perverted by slavery. Originally, Article I, Section 2(3) of the US Constitution said:

‘Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons.’

‘Other persons’ was a euphemism for slaves. Although they had no votes (and they and their descendants continued to be denied the vote long after the abolition of slavery), each slave counted as three fifths of a person for the purpose of allocation of seats in the House. This gave a considerable advantage to the slave States.

In present-day USA, the principle of equal and fair representation is subverted by ger-

Senate, States rather than individual citizens are equally represented: each State, irrespective of size, by two members.<sup>37</sup>

Bicameral legislatures are the rule in other federal states – including Australia, Belgium, Canada, India, Malaysia, Brazil, Switzerland, Pakistan, South Africa and Germany. In some federations, member-states or provinces also have bicameral legislatures: this is the case in the US (with the exception of Nebraska) and Australia (with the exception of Queensland).

Traditionally, socialists have been hostile to bicameralism, because historically it often served an anti-democratic purpose in the interest of the privileged classes: the role of the upper chamber was to shackle the potential radicalism of a democratically elected lower chamber. But this is by no means the only use to which a bicameral structure may be put. In essence bicameralism is an institutional framework for a dual structure of decision-making. It can be put to very diverse uses, depending on the social content of the duality.

A suitably adapted bicameral design is highly desirable, indeed necessary, for balancing the individual and collective aspects of citizenship in a communist commonwealth. And in such a design the council pyramid has an indispensable role.

Before leaving the topic of the council pyramid, I must point out that I have only dealt with its ‘macro’ aspects, its overall structure. I have not said anything at all about its ‘micro’ aspects: the mode of operation of the grass-root cells – most importantly, those of the functional kind, which are the vital units of self-management. How are these grass-root councils to conduct their business? What rules and procedures should they use in making decisions? How should persons be chosen for executive roles, and how will their powers and responsibilities be defined and delimited? These and similar questions are of course at the heart of a communist commonwealth, based on self-managing collectives. But they are outside the scope of this article.

## 4 Electing a representative assembly

I have argued in Chapter 3 that a pyramidal multi-tier structure of councils is inadequate on its own, or in combination with referenda, as a decision-

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rymandering and fraud on a staggering scale. See Campbell [5].

<sup>37</sup>According to the original US constitution, senators were chosen by their State’s legislature. But in 1913 the 17th amendment prescribed that they be elected by popular vote.

making system in a communist commonwealth, but must be supplemented by *directly* elected assemblies at the supra-national, national, and the highest sub-national levels.

These assemblies are not designed to reflect the *collective* views and *corporate* interests of organic (geographical or functional) communities: this is the job of the councils. The purpose of an assembly is to reflect the concerns and preferences of the citizens of the commonwealth at large, as *individuals*. Whereas a delegate is mandated by the collective that elects him or her, and is supposed to speak for it as a corporate whole, elected members of the assembly do not speak for any organic collectives that could mandate them; therefore they ought to be representatives rather than delegates.<sup>38</sup>

I now turn to discussing the method of electing such an assembly. This is a crucial issue, because the procedure of electing representatives makes considerably more difference, and is much more problematic, than most people realize.

For the sake of simplicity I will deal here with election of a national assembly: a representative assembly of a whole country. But similar considerations apply at other levels.

I shall assume the existence of *parties*: nationwide or locally-based organizations advocating views and policies on social issues; these may range from single-issue campaigns to parties committed to a platform on a whole range of issues.<sup>39</sup>

## 4.1 Alternative concepts of electing representatives

A great many procedures have been used for electing representative assemblies such as national parliaments and similar large decision-making bodies,<sup>40</sup> and many more have been proposed but never used. Before we can advocate a particular procedure, we must address an underlying conceptual question: what do we mean by ‘representation’? Or, in other words, in what sense is the assembly supposed to ‘represent’ the electorate?

There are in fact two quite different answers to this question, each of which is embodied in a distinct mode of representation and a corresponding type of election procedure.<sup>41</sup>

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<sup>38</sup>See Section 3.1.

<sup>39</sup>For a reason explained in the Introduction, I do not refer to these organization as ‘*political parties*’.

<sup>40</sup>By ‘large’ I mean here: comprising, say, 100 or more members.

<sup>41</sup>Surprisingly, the difference between these two rival concepts has received very little explicit attention in the social-choice literature. However, the concept underlying a given

The first mode may be called *district representation* (DR): every member of the assembly is personally elected for representing a particular constituency, which is defined geographically. Accordingly, there is a large number of constituencies, each of which elects a single representative or a small number – at most a handful – of representatives. Naturally, such a constituency may be, and normally is in fact, quite heterogeneous: its citizen-voters may differ considerably from one another in their interests, preferences, tastes and opinions.

The second mode is *proportional representation* (PR): the assembly is supposed to be a microcosm of the society at large, like a statistical sample, reflecting in *true proportion* (or as near to it as possible) the various shades of opinion that exist in the society as a whole. Thus it can stand as proxy for a market-place meeting of the entire citizenry; and a vote taken in the assembly may be regarded as a close approximation to a referendum. Here a member of the assembly represents not a geographically defined constituency (which may well be very heterogeneous in the above sense) but a like-minded section of the electorate at large, which may well be geographically dispersed.

Each of these two modes of representation suffers from serious defects, some of which are obvious and well known, some less so.

## 4.2 District representation

An obvious defect of DR is that it is not proportional: it may, and very often does, produce an assembly in which the number of seats won by a party or a current of opinion is grossly disproportional to the number of votes cast for its candidates nationally.<sup>42</sup>

However, the fundamental defect of DR is that on close analysis the concept itself turns out to be deeply problematic. Let me explain briefly why this is so.<sup>43</sup>

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election procedure can readily be inferred from it by implication.

<sup>42</sup>The *single transferable vote* (STV) election procedure – used, for example, in the Republic of Ireland for electing the lower legislative assembly, the Dáil Éireann – is often claimed by its advocates to be proportional. This is not strictly true: STV is essentially a DR procedure that provides no guarantee of proportionality, although in practice it usually does tend to produce results that are less badly disproportional than the very crude ‘first past the post’ election procedure used in many other countries. Besides, STV suffers from the more fundamental defect of DR discussed below, and is consequently afflicted by some paradoxical pathologies.

<sup>43</sup>For further discussion, See the special literature on this subject; for example, Brams [4], Dummett [8] and Nurmi [13]. By the way, the same kind of incoherence applies also to a choice by a collective of the ‘best’ option out of several, whether these options are candidates or, say, courses of action.

The question that a DR election procedure needs to resolve is which of several candidates ‘best’ represents a constituency, in the sense of providing the ‘best’ reflection of the preferences of its voters. If there are just two candidates,  $A$  and  $B$ , the answer is fairly straightforward: clearly, if the number of voters who prefer  $A$  to  $B$  is greater than that preferring  $B$  to  $A$ , then  $A$  is the better representative and ought to be elected.<sup>44</sup>

But when there are more than two candidates, things become much less clear-cut. Suppose there are three candidates,  $A$ ,  $B$  and  $C$ . Now the electorate can be divided into six distinct camps according to their preference among the candidates:  $(A, B, C)$ ,<sup>45</sup>  $(A, C, B)$ ,  $(B, A, C)$ ,  $(B, C, A)$ ,  $(C, A, B)$  and  $(C, B, A)$ .<sup>46</sup> A *profile* of the electorate is a list of six numbers, giving the numerical size of each of the camps. If a single candidate is to be elected, then we need a procedure that selects, for each possible profile, the ‘best’ candidate. It turns out that any such procedure must produce in some cases a result that is in some sense unreasonable or unsatisfactory. The same applies *a fortiori* when there are more than three candidates, one or several of whom are to be elected.<sup>47</sup>

But quite apart from the defects of DR, this mode is inappropriate for the kind of assembly we are seeking: as I pointed out at the beginning of this section (p. 24), the assembly should represent the citizens as individuals, rather than corporate bodies or collectives of citizens; whereas under DR a

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<sup>44</sup>What if there is an equal number preferring  $A$  to  $B$  as  $B$  to  $A$ ? Such a dead heat is extremely unlikely if the number of voters is very large. However, it cannot be ruled out; and the most reasonable way to resolve it is by lot. Indeed, this is what the existing electoral laws in many countries prescribe in such cases. For example, the British *Representation of the People Act 1983*, Chapter 2 (49), says: ‘Where, after the counting of the votes (including any re-count) is completed, an equality of votes is found to exist between any candidates and the addition of a vote would entitle any of those candidates to be declared elected, the returning officer shall forthwith decide between those candidates by lot, and proceed as if the candidate on whom the lot falls had received an additional vote.’

<sup>45</sup>This stands for:  $A$  is preferred to  $B$  and  $B$  to  $C$ .

<sup>46</sup>For the sake of simplicity I have excluded orders of preference that are indifferent between two or all three of the candidates. If indifferences are admitted, there are not six but thirteen possible camps!

<sup>47</sup>For details, see the literature cited in footnote 43. By the way, this incoherence of collective choice of the ‘best’ out of more than two options is of much less concern with regard to electing a delegate of a council to speak for it in a higher-tier council. This is because the delegate is mandated to transmit to the higher tier the majority view of the council that delegated him or her; and this normally boils down to a choice between just two options: for or against a given proposed resolution. Moreover, a delegate can be replaced at any time; so that when faced with an issue of some importance the delegating council can make sure that its delegate supports the majority view, whether for or against the proposed resolution.

representative is supposed to represent an entire constituency as a whole.

### 4.3 Proportional representation

The most consistent PR procedure is the party-list system used in many countries for parliamentary elections and in most EU countries for elections to the Europarliament. The country is divided into a small number of large constituencies. (In some small countries, such as The Netherlands and Israel, the whole country is a single constituency.) In each constituency a party can present a list of candidates, and a voter casts a vote for one of these lists. The assembly seats are allocated to each party in proportion to the number of votes cast for it.<sup>48</sup> In the ‘closed list’ variant of the list system, the seats are allocated to a party’s candidates in the order in which they appear on its list. In the ‘open list’ variant, voters may indicate preference for a particular candidate in the list of their choice, and seats are allocated accordingly.

The arguments often voiced against this consistent PR system are for the most part specious, if not outright reactionary. However, it has one obvious fault: a representative can only be elected through a party. This tends to give undue power to party leaderships and bureaucracies. Although this can be overcome to some extent by using primary election within the party for choosing its list of candidates, it still requires a candidate to be nominated by a party. It is virtually impossible for a candidate to stand as an independent.

Another fault of this system is that the voter is presented with long lists of candidates’ names, most of whom are likely to be just that: names. The average voter is unlikely to have more than a vague idea – if that – who most of these candidates are. Although the rationale of the party-list system is that the voter should choose primarily between party platforms rather than between individuals, the personal qualities of the candidates do and should matter. Ideally, a voter should have a good idea not only *what* but also *whom* s/he is voting for.

There are some *hybrid* election procedures that combine DR with PR, aiming to obtain the advantages and mitigate the faults of both systems. Such is the *additional member* procedure, variants of which are used for electing the German *Bundestag* (Federal lower house), the Scottish *Parliament* and the London Assembly (elected council of Greater London). Under this procedure, each voter has two votes: one for a representative under a DR system, and one for a party list. The assembly thus consists of two kinds of member, elected

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<sup>48</sup>In practice, a deviation from strict proportionality may be introduced whereby a list obtaining less than a certain threshold – typically set at between 2% and 5% of the total vote – is not allocated any seats, and the votes cast for it are disregarded.

by DR and PR systems respectively; and the number of the latter can be adjusted so as to achieve – or at least approach – overall proportionality.

This procedure does mitigate the worst faults of both pure DR and pure PR systems, but does not eliminate them altogether.

#### 4.4 Election by lottery

There is however a radical hybrid procedure that eliminates all the main faults of both DR and PR procedures and has all their main advantages. In form it closely resembles a common (and very faulty) DR procedure, the plurality (‘first past the post’) system – with one crucial difference: the winner in each constituency is chosen by weighted lottery, as explained below. In its overall outcome it produces a proportional assembly.

As far as I know, this procedure was first seriously proposed and discussed in 1984 by the progressive American jurist and political scientist Akhil Reed Amar,<sup>49</sup> who argued persuasively that it would allow fair representation to diverse minorities. The lottery procedure has never been implemented in modern political elections,<sup>50</sup> and as we shall see there is an important reason why it is very unlikely to be adopted by any bourgeois state but also why it is highly suited to electing a representative assembly of a communist commonwealth.

This is how it works in detail. The whole country is divided into geographically defined constituencies of approximately equal population size, each of which elects a single representative. A candidate can be nominated by a party, or stand as an independent. Each voter is allowed to vote for one candidate.

Up to this point, the procedure is exactly the same as the plurality system. But now comes the crucial difference. Under the plurality system, the votes cast in each constituency are counted, and the candidate who obtains the greatest number of votes is the winner. Under the lottery system, a lottery is conducted among the candidates, such that the probability of any candidate winning is proportional to the number of votes cast for that candidate. To visualize this, imagine that all the ballot papers cast in the constituency are put in a large container and thoroughly stirred; and then a single ballot paper is blindly drawn out. The winner is the candidate marked on this ballot paper. Of course, this manual lottery can be mimicked by more efficient me-

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<sup>49</sup>See Amar [2].

<sup>50</sup>Note however that in ancient Athens most civilian political office holders were chosen by lot (see <http://tinyurl.com/mewhtc>). In modern times, trial juries in many countries are also chosen by lot.

chanical or electronic means. The crucial point is that the lottery is weighted by the numbers of votes cast for the candidates. For example, if a candidate obtained 25% of the votes, her probability of winning (getting elected) is  $\frac{1}{4}$ . Or, in betting terms: the odds in favour of her winning are 1 : 3, because for each vote cast for her there are three cast for other candidates.

Let me now highlight the main features of this lottery voting procedure (briefly: LVP).

**4.4.1 Proportionality** According to the laws of probability – specifically, *Kolmogorov’s Strong Law of Large Numbers* – the overall outcome under the lottery procedure is almost certain to be extremely close to proportionality. More precisely, if the number of constituencies is fairly large (say 100 or more) then the total number of seats won by candidates representing a given party or informal trend of opinion is very highly likely to be closely proportional to the total number of votes cast at large (throughout the country) for such candidates. For example, suppose that the number of constituencies (which is also the number of seats to be filled) is 200. If 25% of the votes in the whole country were cast for the candidates of a given party, then the number of seats won by that party is very likely to be in the close proximity of 50. Deviations from 50 are possible, but the larger the deviation the less likely it is; and really large deviations can be ruled out with near certainty.

The proof of proportionality<sup>51</sup> assumes only that the constituencies are of equal population size; this renders gerrymandering – manipulating constituency boundaries – nugatory.

Due to its proportionality, decision-making by an assembly elected by LVP is immune to the main technical faults of a pyramidal council structure: dilution of citizens’ voting power (Subsection 3.4.1) and majority deficit (Subsection 3.4.2). The assembly is a statistical sample of the entire electorate, so that its decisions approximate quite closely the outcomes of putative referenda.

Note however that LVP is unsuitable for electing a small committee, or a single person to an individual post: the Law of Large Numbers does not apply to such cases, so the risk of a perverse outcome is too great.

**4.4.2 Insecurity of tenure** Under LVP the more popular the candidates, the greater their chance of getting elected. In particular, incumbent representatives who attain great popularity by their work in the outgoing assembly thereby increase their chance of being re-elected. But there is no such thing as a *secure* seat: to be certain of winning, a candidate must get 100% of the constituency’s votes – a virtual impossibility.

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<sup>51</sup>For technical details see Section 6.2 of the Appendix.

Moreover, the probability of getting re-elected several times in succession is small even for very popular candidates. For example, the odds against a candidate who consistently gets 50% of the votes winning three times in succession are 7 : 1; and a candidate who consistently gets a massive 75% of the votes has considerably less than even chance of winning three times in succession.<sup>52</sup>

So LVP has an inbuilt mechanism for turnover of representatives. This is why it is unlikely to be adopted for electing a legislature of a bourgeois state. Members of these legislatures normally regard politics as a career; and incumbents have a proprietary attitude to ‘their’ seats. The last thing these politicians would welcome is a procedure that puts their security of tenure at greater risk.

On the other hand, precisely this feature of LVP makes it suitable for electing representatives in a communist commonwealth, in which decision-making must not become a profession or a career but a temporary service to society. Turnover is a positive feature; and in order to maximize it elections to representative assemblies ought to be held fairly frequently, say annually or every other year.

Note however that LVP is suited only for electing representatives for a fixed term, not for electing delegates who can be recalled at any time. Otherwise, supporters of a candidate who has lost an LVP election would be tempted to get the winner recalled and force a re-election, hoping to get lucky next time.

**4.4.3 Localization** Since candidates stand for election in a localized constituency, they would tend to campaign locally and make themselves reasonably well known to the constituency’s voters.

However, note that the *primary* allegiance of a representative is to the platform on which s/he stood for election, not to the geographical constituency. The latter serves as a sort of statistical sample of the population at large, used for conducting a lottery as a means of achieving overall proportional representation.

Of course, it would be both natural and commendable for representatives to pay some heed to the concerns, interests and opinions of their constituents.

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<sup>52</sup>The probability of winning three times is equal to the third power of the probability of winning once. So for a candidate who gets  $\frac{1}{2}$  of the votes the probability of winning three times is  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$  – or, in betting terms, odds of 7 : 1 against. For a candidate who gets  $\frac{3}{4}$  of the votes the probability of winning three times is  $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64} = 0.422$  – or odds of 37 : 27 against.

**4.4.4 Variety and choice** Clearly, LVP does not exclude or in any way disadvantage independent candidates.<sup>53</sup> On the other hand, a party can put up several candidates in the same constituency (although of course at most one of them would win). In fact, it is in a party's interest to attract voters by giving them a choice among several candidates of different gender, ethnicity, age or shade of opinion (within the same party).<sup>54</sup> Under the plurality procedure, such a gambit is of course stupidly counter-productive, as it would split the party's support. But under LVP things are very different, because the probability of the party winning the seat (i.e., of the winner being among its candidates) is proportional to the *total number* of votes cast for *all* its candidates in the constituency. By putting up a variety of candidates and offering greater choice, the party tends to increase this total, and hence its chance of winning the seat.

**4.4.5 Sincerity** Unlike all DR election procedures (but like other PR procedures), LVP encourages sincere voting. Every vote cast for a given candidate counts, and no vote is 'wasted' or improves the chances of some other candidate. Therefore honesty is a voter's best policy: vote for the candidate you prefer most.

## 4.5 Selection by at-large lottery

Some of the main attractive features of an assembly elected by LVP – mainly proportionality and insecurity of tenure – would also be possessed by an assembly of representatives selected by (unweighted) lottery from the entire citizenry, in much the same way as juries are selected at present in many countries.<sup>55</sup>

An additional merit of such a system is that it can activate and draw into direct decision-making at a high level persons who would otherwise shy away from standing as candidates for election.

A counter-argument is that it would be undesirable to impose the job of representative for an extended term (say a year or two) on a person who may be quite reluctant to perform it, and who is not called upon to be scrutinized

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<sup>53</sup>A slight problem with LVP is that it may attract practical jokers and loonies, who cannot gain more than a handful of votes, to stand for election on the off-chance of achieving a fluke win. Such time-wasters can be barred by various devices; for example, by requiring each candidate to be endorsed by a reasonable number of sponsors.

<sup>54</sup>It was this feature of LVP that Akhil R Amar [2] considered to be its most attractive feature.

<sup>55</sup>This is the decision-making system advocated by Cockshott and Cottrell [6, Ch. 13], who maintain that all forms of election are undemocratic.

or present any platform to an electorate. Arguably, such an imposition may also be oppressive.

Nevertheless, a jury-type selection may perhaps be used for certain auxiliary purposes. For example, a special assembly selected in this way may be convoked for a *short* period, say a few weeks, to discuss and advise on some special issues. Also, a small number of seats in a representative assembly elected for the most part by LVP may be reserved for members selected by at-large lottery for a *short* term.

## 5 Concluding remarks

The foregoing discussion is not offered as an attempt to draw up a definite constitution for a future society of which we have no actual experience but only a hopeful projection. Nevertheless, I believe that discussing such matters now, albeit tentatively, is by no means a waste of time. Historical experience shows that many key ideas were originated and discussed theoretically well before they could possibly be implemented.

Also, discussion of various aspects of a future better society is at the same time, at least implicitly, a critique of the present social order: whose exploitative basic process is disguised as the operation of impersonal inexorable forces, and whose repressive and authoritarian mechanisms are barely cloaked under a thin mantle of curtailed democracy and confined freedom.

Yet, in order for a discussion of an imagined society to be of real future and present use, it must not indulge in naïve illusion and insouciant utopia. So, while no detailed decision-making arrangement proposed above should be taken as more than a purely tentative suggestion, I do insist on stressing some sober considerations.

First, a future communist commonwealth will not be a state of nirvana, nor will it run mostly on automatic pilot, with just the occasional light touch on the tiller. Its material and intellectual processes will be both complex and dynamic, requiring a very great number of intelligent social decisions at various levels of society, from the micro-local to the global. Most of these social decisions are likely to be about matters that today would be considered ‘managerial’ or ‘economic’ and are either made privately or not made consciously but left to impersonal ‘forces’.

Second, no human society can run like an ant colony, where each individual is almost an automaton, without an independent mind, but where sophisticated social decisions are made by a collective intelligence that has evolved over many millions of years. Humans are individuals with independent (albeit socially conditioned) intelligence; they are often contrary and

sometimes bloody-minded. Moreover, not all human conflicts and interests are class based, and even in a classless society there will be conflicting interests, opinions and tastes. These will have to be resolved by decision-making whose structures and processes are efficient, transparent and fair. And the implementation of decisions will have to be supervised systematically and methodically.

Third, direct decision-making must play an important but restricted role. Due to the very large number of decisions that will have to be made at various levels, only decisions at the lowest (local) level and a small proportion of decisions at higher levels can be made directly by a meeting or referendum of all concerned. The vast majority of decisions at higher levels will have to be made by elected councils of delegates or assemblies of representatives.

Fourth, a pyramidal structure of councils *on its own* is inadequate, because it is too indirect and fails to satisfy some important principles of democracy, namely those concerned with individuals' rights, equality and empowerment. This does not mean that the council structure must be abandoned; but that it would need to be combined with a countervailing structure in order to resolve the tension between collectivism and individualism.

Fifth, a communist commonwealth will allow and promote methods of decision-making and election that are not feasible in today's anti-egalitarian society but only under conditions of material equality.

Communism does not mean abandoning liberal democracy but superseding it by transcending its limitations.

## 6 Appendix: mathematical technicalities

This Appendix contains explanations of mathematical concepts and statements of the mathematical results used in the preceding chapters. It presupposes some knowledge of probability theory. For further details, such as lengthy or difficult proofs, the reader is referred to the technical literature.

### 6.1 Voting power

**6.1.1 Decision rules** Before addressing the problem of measuring voting power, I must first explain what I mean here by *decision rule*. Such a rule applies to some body of *voters*. I will refer here to this body as *assembly*, although it can be a committee, a council or any other such body of voters, including an entire electorate. A voter may be an individual person or a bloc of persons voting regularly as one.

When the assembly is called upon to make a decision on a proposed bill or resolution, each of its members votes either ‘yes’ or ‘no’.<sup>56</sup> This creates a *division* of the assembly into two sides: the ‘yes’ voters and the ‘no’ voters. For an assembly consisting of two voters, there are four possible divisions (both voters say ‘yes’, both say ‘no’, voter *a* says ‘yes’ and *b* says ‘no’, or *a* says ‘no’ and *b* ‘yes’); for an assembly of three voters, the number of possible divisions is eight; an assembly of four gives rise to 16 divisions; and so on. Putting it generally: an assembly of size  $n$  has  $2^n$  possible divisions.

Any division must have one of two possible outcomes: *positive*, in which case the bill is passed; or *negative*, in which case the bill is blocked. The function of a decision rule for a given assembly is to specify the outcome – positive or negative – of each possible division of the assembly.

The most common kind of decision rule are *weighted* rules: each voter is assigned a non-negative number as *weight*; and a positive number is specified as *quota*. What determines the outcome of a division is the total weight of the ‘yes’ voters in it: the outcome is positive just in case the total weight of the ‘yes’ side exceeds the quota. Among weighted rules, the most familiar is the *simple* (or *absolute*) *majority* rule: all voters are assigned equal weight, and the quota is set at half the total weight.

More generally, and put rather abstractly, a decision rule for a given assembly is *any* classification of all its possible divisions into those with positive or negative outcome, subject only to the following three conditions:

- (1) The division in which all the voters unanimously vote ‘yes’ must have positive outcome.

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<sup>56</sup>For the sake of simplicity I will ignore abstentions.

- (2) The division in which all the voters unanimously vote ‘no’ must have negative outcome.
- (3) If a division has positive outcome, and a voter crosses from its ‘no’ side over to the ‘yes’ side, then the resulting division must likewise have positive outcome.

This definition of *decision rule* is really more general, because not every decision rule as defined here can be cast as a weighted rule.<sup>57</sup>

**6.1.2 Penrose’s measure of voting power** By the *voting power* of a voter under a given decision rule we mean, roughly speaking, the amount of influence over the outcome of a division, which the voter possesses by virtue of the rule. In this subsection I will make this somewhat vague notion more precise by quantifying the *amount of influence*.

At first sight it may seem that, at least in the case of a weighted rule, there is no difficulty: surely, the voting powers of the voters are proportional to their weights. But on closer examination this common-sense view is easily seen to be fallacious.

For example, consider an assembly of three voters,  $a$ ,  $b$  and  $c$  with weights 5, 2 and 1 respectively. If the quota is 4 (half the total weight) then  $a$  is in effect a *dictator*: whatever she says goes; so she has all the voting power. And  $b$  and  $c$  are *dummies*: their votes make no difference; so they have no voting power at all.

The scientific study of voting power was initiated in 1946 by Lionel S Penrose [15, 16]. His first key idea was simple: *the more powerful a voter is, the more often will the outcome go the way s/he votes*. In other words, a more powerful voter is a more successful one, one that is more often on the winning side of a division.

Let us denote by  $r$  the proportion of all divisions of the assembly in which a given voter  $v$  is on the winning side:  $v$  votes ‘yes’ and the outcome is positive, or votes ‘no’ and the outcome is negative. This can also be expressed in terms of probabilities: assuming that all the voters vote independently and at random (for example, each voter flips a true coin and votes ‘yes’ if it shows heads and ‘no’ if it shows tails)<sup>58</sup> then  $r$  is the probability that our voter  $v$  will be on the winning side.

Actually,  $r$  itself is not a very convenient measure of voting power, because it runs together luck and genuine influence. Indeed, even for a dummy  $r = \frac{1}{2}$ ,

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<sup>57</sup>For counter-examples see Felsenthal and Machover [9, pp. 31–2].

<sup>58</sup>Regarding the significance of this assumption, see Subsection 6.1.3 below.

because even a dummy finds himself, by sheer luck, on the winning side in half of all divisions. So Penrose proposed

$$\psi := 2r - 1 \tag{1}$$

(‘ $\psi$ ’ is the Greek letter *psi*) as measure of voting power.

This definition of voting power takes the element of luck out of it: for a dummy  $\psi = 0$ , whereas for a ‘dictator’  $\psi = 1$ .

Penrose also observed that  $\psi$  can be characterized directly in the following striking way:

*$\psi$  is the probability that voter  $v$  is critical; in other words, the probability that the other voters are divided in such a way that voter  $v$  is in a position to determine the outcome of the division.*<sup>59</sup>

This characterization is often used, instead of (1), as a definition of the Penrose measure  $\psi$ .

Note that  $\psi$  is in fact a *function*, whose value depends on the decision rule and the voter under consideration. When we need to emphasize this, we denote by  $\psi_v[\mathcal{W}]$  the value of  $\psi$  for voter  $v$  under rule  $\mathcal{W}$ .

Penrose’s pioneering work lay for many years unnoticed or forgotten by mainstream writers on social choice (the science of collective decision-making). But his ideas on measuring voting power are so natural, so compelling, that they forced themselves on several other scholars who tackled the problem of measuring voting power: without knowing of Penrose’s – or of one another’s – work, they re-invented some of his ideas.<sup>60</sup>

**6.1.3 Aprioricity of Penrose’s measure** The voting power that the Penrose measure is intended to quantify is the power that a voter has *solely by virtue of the decision rule itself*. Voting power in this restricted sense is said to be *a priori*, in contradistinction to *a posteriori* or *actual* voting power. The latter may take into account – apart from the decision rule – additional real-life (and transient) factors such as the particular kind of issue to be decided; as well as voters’ actual interests, preferences and temperaments; their persuasive skills; and their mutual affinities and disaffinities.

However, when it comes to designing or discussing the constitution of a future decision-making body, there is a near-consensus that the a priori

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<sup>59</sup>Another term used instead of ‘critical’ is *decisive*. For a proof of the equivalence of the two ways of characterizing  $\psi$ , see [9, pp. 45–46].

<sup>60</sup>The first among them was the American jurist John F Banzhaf, who began to address this problem in 1965 [3]. For a historical survey of the subject, see Felsenthal and Machover [10].

approach is the only right one, and indeed often the only possible one, for two reasons.

First, because it would be unfair to tailor the decision rule to the specific interests, preferences and affinities of the various voters. Even if such information is available, we must go ‘behind a veil of ignorance’ and act impartially, considering only the voting power that the voters will derive from the decision rule itself.

Second, in general there is very little reliable information about the nature of the *future* issues to be voted on; and the *future* interests, preferences and affinities of the voters.

Because I am taking in the present section an aprioristic stance, when I speak in probabilistic terms I always assume that the voters act independently and vote at random ‘yes’ or ‘no’ with equal probability. This is tantamount to assuming that in an assembly of  $n$  voters all  $2^n$  possible divisions are equally probable. Of course, this is not how voters *actually* behave: they don’t decide how to vote by tossing a coin. Rather, our random-voting assumption is the most neutral one we can make a priori, ‘behind a veil of ignorance’.

**6.1.4 Voting power under majority rule** Intuitively, decision by simple majority seems to be the most democratic way of making decisions. This gut feeling can be made precise and validated mathematically.

Obviously, the simple majority decision rule is egalitarian: since all voters have equal weight and are counted equally, all voters have the same voting power. But it is far from being unique in equalizing voters’ powers: there are many other decision rules (both weighted and unweighted) that do this.

However, I shall now show that there is another property of the simple majority rule that virtually singles it out as in some sense the most democratic of all possible rules.

Consider a given assembly  $N$  of voters. Suppose that this assembly operates according to some decision rule  $\mathcal{W}$ . I shall hold  $N$  fixed throughout the present discussion, but allow  $\mathcal{W}$  to vary over all possible rules.

Recall that if  $v \in N$  (i.e.,  $v$  is a member of  $N$ ) then  $\psi_v[\mathcal{W}]$  denotes the voting power of  $v$  under the rule  $\mathcal{W}$  as quantified by the Penrose measure. Now I put:

$$\Psi[\mathcal{W}] := \sum_{x \in N} \psi_x[\mathcal{W}]. \quad (2)$$

(‘ $\Psi$ ’ is the upper-case form of the Greek letter *psi*.) Thus,  $\Psi[\mathcal{W}]$  is the sum-total of the voting powers of all the voters of  $N$ . This is not a fixed quantity, but depends on the decision rule  $\mathcal{W}$ .

$\Psi[\mathcal{W}]$  can be regarded as a measure of *total voter empowerment* yielded by  $\mathcal{W}$ . Note that the average value of  $\psi_v[\mathcal{W}]$  for voters  $v$  of  $N$  is equal to  $\Psi[\mathcal{W}]$  divided by the number of voters; so the greater the value of  $\Psi[\mathcal{W}]$ , the greater also the voting power of the ‘average voter’.

It is therefore important to find out which rule is most empowering, i.e., maximizes the value of  $\Psi[\mathcal{W}]$ .

The answer is given by the following theorem.<sup>61</sup>

$\Psi[\mathcal{W}]$  attains its maximal value if and only if  $\mathcal{W}$  is such that there is no division of  $N$  in which the winning side comprises a minority of the voters. In other words:

- (i) Any division of  $N$  in which the number of ‘yes’ voters exceeds that of ‘no’ voters has a positive outcome according to  $\mathcal{W}$ ;
- (ii) Any division of  $N$  in which the number of ‘no’ voters exceeds that of ‘yes’ voters has a negative outcome according to  $\mathcal{W}$ .

If the number of voters (members of the assembly  $N$ ) is odd, then clearly the only decision rule that satisfies these two conditions is the simple majority rule. If the number of voters is even, then there are also some other rules, differing very slightly from the simple majority rule, that satisfy the two conditions, because these conditions do not prescribe the outcome in cases where the voters are exactly evenly split between the ‘yes’ and ‘no’ sides.

I shall denote by  $\Sigma_n$  the maximal value of  $\Psi[\mathcal{W}]$  in an assembly of size  $n$ . Thus  $\Sigma_n$  is the value of  $\Psi[\mathcal{W}]$  for the simple majority rule. Further, I shall denote by  $\sigma_n$  the voting power of each voter under the simple majority rule in an assembly of size  $n$ . (‘ $\Sigma$ ’ and ‘ $\sigma$ ’ are respectively the upper-case and lower-case forms of the Greek letter *sigma*.) Clearly,  $\sigma_n = \Sigma_n/n$ .

By what we have just seen,  $\sigma_n$  is the greatest possible voting power that a voter can have in an assembly of  $n$  voters under an egalitarian decision rule.

It is not difficult to obtain exact mathematical expressions for  $\Sigma_n$  and  $\sigma_n$ , but these are somewhat cumbersome for actual calculation, and it is much more convenient to use the following approximations:<sup>62</sup>

$$\Sigma_n \approx K\sqrt{n} \quad \text{and} \quad \sigma_n \approx \frac{K}{\sqrt{n}}, \quad \text{where} \quad K := \sqrt{\frac{2}{\pi}}. \quad (3)$$

<sup>61</sup>For a proof of this theorem see Subsection 6.1.5 below.

<sup>62</sup>For the exact expressions and an indication how the approximations are derived, see Felsenthal and Machover [9, pp. 55–6].

(The symbol ‘ $\approx$ ’ means *approximately equal*.) These are excellent approximations: their relative error is roughly in inverse proportion to  $n$ . For  $n = 10$ , the approximation is well within 3% of the true value (actually, more like 2.6%). For  $n = 100$ , the relative error is well under 0.3%, and so on.

The constant  $K$  will play a significant role in the sequel. Its numerical value is given by

$$K \approx 0.7979. \quad (4)$$

It follows that  $\sigma_n$  is of the same order of magnitude as  $1/\sqrt{n}$ ; thus,  $\sigma_{100} \approx 0.0798$  and  $\sigma_{400} \approx 0.0399$ . On the other hand,  $\Sigma_n$  is of the same order of magnitude as  $\sqrt{n}$ ; thus,  $\Sigma_{100} \approx 7.9788$  and  $\Sigma_{400} \approx 15.9577$ .

**6.1.5 Majority deficit** As before, let  $N$  be an assembly of size  $n$  operating under some decision rule  $\mathcal{W}$ . Consider a given division of  $N$ . If the winning side according to  $\mathcal{W}$  happens to comprise a minority of the voters, we define the *majority deficit* of this division to be the margin by which the losing majority exceeds the winning minority. (For example, if  $n = 10$  and the winning side comprises 4 voters, then in this division the majority deficit is  $6 - 4 = 2$ .) However, if the winning side comprises a majority of the voters, or if the voters are evenly split between the two sides, then we define the *majority deficit* of this division to be 0.

Further, the mean (average) value of the majority deficits of all  $2^n$  possible divisions of  $N$  is called the *mean majority deficit* – briefly, MMD – of  $\mathcal{W}$  and denoted by  $\Delta[\mathcal{W}]$ . (‘ $\Delta$ ’ is the upper-case form of the Greek letter *delta*.)

The MMD  $\Delta[\mathcal{W}]$  is important because it is a measure of the extent to which the decision rule  $\mathcal{W}$  deviates from the simple majority rule.

There is a remarkable and very simple connection between  $\Delta[\mathcal{W}]$  and  $\Psi[\mathcal{W}]$ , namely:<sup>63</sup>

$$\Delta[\mathcal{W}] = \frac{\Sigma_n - \Psi[\mathcal{W}]}{2}. \quad (5)$$

Thus  $\Delta[\mathcal{W}]$  is a linear decreasing function of  $\Psi[\mathcal{W}]$ . The more empowering the decision rule  $\mathcal{W}$ , the smaller is its MMD. Maximizing  $\Psi[\mathcal{W}]$  is tantamount to minimizing the MMD  $\Delta[\mathcal{W}]$ .

The maximization theorem stated in Subsection 6.1.4 follows from this as an easy corollary.

Indeed, since the majority deficit of any division is by definition always non-negative, the same applies to its mean value,  $\Delta[\mathcal{W}]$ . But  $\Delta[\mathcal{W}]$  is equal to 0 – which is its least possible value – precisely in case the majority deficits of *all* divisions are 0. This, in turn, is the case if and only if  $\mathcal{W}$  is such that there is no division of  $N$  in which the winning side comprises a minority of

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<sup>63</sup>For a proof of (5), see Felsenthal and Machover [9, pp. 60–61].

the voters. In other words,  $\Delta[\mathcal{W}]$  attains its minimal value – and hence  $\Psi[\mathcal{W}]$  attains its maximal value – precisely under conditions (i) and (ii) as stated in the theorem.

**6.1.6 Two-tier structures** Consider a *two-tier decision-making structure* having the following architecture.

The *bottom tier* consists of  $m$  assemblies of citizens,  $N_1, N_2, \dots, N_m$ . I shall refer to these grass-root assemblies briefly as *cells*. The *upper tier* is a council of  $m$  delegates, one for each cell. I shall label the delegates by numbers  $1, 2, \dots, m$  corresponding to their respective cells.

The cells are not necessarily of equal size; I shall denote the size of the  $i$ -th cell  $N_i$  by  $n_i$ . I shall assume that the numbers  $n_i$  are sufficiently large for the error of approximations (3) to be negligible.

The cells are *disjoint*; this means that no two of them have any member in common.

The decision-making procedure has two stages. First, when a bill is proposed, the cells divide on it independently of one another, each of them operating under the simple majority rule. Thus we may assume that a division of each cell has an equal a priori probability of coming up with positive or negative outcome.<sup>64</sup>

The second stage is a division of the council. Each delegate (i.e., council member) is assumed to vote ‘yes’ or ‘no’ according to the outcome of the division in his or her cell. I shall denote by  $\mathcal{V}$  the decision rule under which the council operates. For the time being I shall not specify this rule but leave it open, to be determined later, subject to considerations of equitability (see Subsection 6.1.7) and empowerment (see Subsection 6.1.8).

The final outcome of the two-stage procedure is, by definition, the outcome of the division of the council.

Note that while the final outcome is decided directly at the upper tier by the delegates in the council, the citizens (i.e., cell members) at the bottom tier are indirectly the decision makers, whose votes are transmitted upwards via their respective delegates.

The whole of this two-stage procedure can be regarded as a single *composite* decision rule, which I shall denote by  $\mathcal{W}$ . The ‘grand assembly’ on which  $\mathcal{W}$  operates is the citizenry at large: the set-theoretical union of all the cells:

$$N := N_1 \cup N_2 \cup \dots \cup N_m. \quad (6)$$

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<sup>64</sup>This is slightly inaccurate for those  $N_i$  whose size  $n_i$  is an even number, because all divisions in which the members of  $N_i$  are evenly split have negative outcome. However, for sufficiently large numbers  $n_i$ , as assumed here, the probability of even split is sufficiently small for the error of this inaccuracy to be negligible.

Now let  $v$  be a citizen belonging to the  $i$ -th cell  $N_i$ . I wish to calculate the (indirect) voting power of  $v$  under the composite decision rule  $\mathcal{W}$ .

As explained in Subsection 6.1.2, this voting power, denoted by  $\psi_v[\mathcal{W}]$ , is equal to the probability of the event that  $v$  is critical under the composite rule  $\mathcal{W}$ . This event is the conjunction of two events, namely,

$A$ : the event that  $v$  is critical in the division of his or her own cell,  $N_i$ ;

$B$ : the event that delegate  $i$  is critical in the division of the council.

Moreover, events  $A$  and  $B$  are mutually independent, because  $A$  concerns only what happens inside  $N_i$ ; whereas  $B$  depends only on how the delegates other than  $i$  vote in the council, which in turn depends only on the outcomes of the votes in *their* cells.

Therefore, by a well-known law of probability theory,  $\psi_v[\mathcal{W}]$  is the product of the probabilities of  $A$  and  $B$ .

The probability of  $A$  is equal to the (direct) voting power of  $v$  in the cell  $N_i$ , which by assumption operates under the majority rule. Thus by (3) it is given, to an excellent approximation, by  $K/\sqrt{n_i}$ .

The probability of  $B$  is  $\psi_i[\mathcal{V}]$ , the voting power of delegate  $i$  under the decision rule  $\mathcal{V}$  (which I have not specified as yet). So we have the approximation:

$$\psi_v[\mathcal{W}] \approx \frac{K}{\sqrt{n_i}} \psi_i[\mathcal{V}] \quad \text{for any } v \in N_i. \quad (7)$$

**6.1.7 Penrose's square-root rule** A direct consequence of (7) – in view of the fact that  $K$  is a constant, independent of  $i$  – is *Penrose's square-root rule* (briefly, *PSQRR*):

*The (indirect) voting power of the citizens across all cells are equalized if and only if the decision rule  $\mathcal{V}$  of the council is such that the voting power  $\psi_i[\mathcal{V}]$  of the  $i$ -th delegate is directly proportional to the square root  $\sqrt{n_i}$  of the size of the  $i$ -th cell.*

This rule prescribes what we must do to obtain a two-tier structure that is equitable in the sense of treating all citizens in an egalitarian way. Implementing it is a rather intricate issue, which I can only outline here.<sup>65</sup>

In what follows I shall take the number  $m$  of cells and their respective sizes  $n_i$  as given, and will address the problem of choosing the council's decision rule  $\mathcal{V}$  so as to fit the prescription of PSQRR exactly or at least approximately.

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<sup>65</sup>For further details see Machover [12] and literature cited therein.

For small values of  $m$  there is no guarantee that this can be done, because the choice of decision rules for  $m$  voters is rather limited, and it may well happen that none of them provide a distribution of voting powers for the  $m$  delegates (members of the council) so as to satisfy PSQRR precisely or even to a reasonable approximation.

However, for sufficiently large values of  $m$  – say  $m > 15$  – the position is much better. It is almost always possible to find a decision rule  $\mathcal{V}$  that satisfies PSQRR to a good approximation.<sup>66</sup> It is even possible to find suitable  $\mathcal{V}$  that are weighted, with weights proportional to the square root  $\sqrt{n_i}$  of the respective cell size.

As noted at the beginning of Subsection 6.1.2, the voting powers of voters under a weighted decision rule are not in general proportional to their respective weights. However, it turns out that if the number of voters – in the present case the number  $m$  of delegates – is sufficiently large, then approximate proportionality does hold *provided the quota  $q$  is chosen in an appropriate way*.

For weights proportional to the respective  $\sqrt{n_i}$ , the value of  $q$  that produces the best fit with PSQRR is obtained as follows.

If the  $n_i$  are not all equal, the value of  $q$  that yields the closest fit is somewhat greater than 50% of the total weight (i.e., the sum of the  $m$  weights). A formula for this optimal value of  $q$  was devised by Wojciech Słomczyński and Karol Życzkowski.<sup>67</sup> However, even taking  $q$  to be just 50% of the total weight produces a tolerably good fit with PSQRR. Moreover, as  $m$  increases, the performance of the 50% quota tends to improve.

In the special case where all the  $n_i$  are equal, any value of  $q$  from 50% to just short of 100% of the total weight will clearly provide a perfect fit.

So, to sum up: for sufficiently large values of  $m$  we get a reasonably – albeit not always perfectly – egalitarian two-tier structure by choosing the decision rule  $\mathcal{V}$  of the council to be a weighted rule, in which delegates are assigned weights proportional to the square root  $\sqrt{n_i}$  of their respective cell size; and the quota is chosen as half the total weight.

**6.1.8 Empowerment of citizens** Consider again a two-tier structure as described in Subsection 6.1.6. Again, I take the number  $m$  of cells and their respective sizes  $n_i$  as given. Recall that the (indirect) voting power of a citizen  $v$  belonging to the  $i$ -th cell is given by (7).

I now pose the problem of choosing the decision rule  $\mathcal{V}$  of the council so as to maximize the total voting power  $\Psi[\mathcal{W}]$  of the citizens. This quantity

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<sup>66</sup>The main exception are cases where there is an extreme discrepancy of size among the cells, with some very large  $n_i$  and the rest very small.

<sup>67</sup>See Machover [12].

was defined in (2); but the  $N$  that features in that definition is now taken to be the citizenry at large, as defined in (6).

The answer is provided by the following *second square-root rule* (briefly, *SSQRR*):

*To maximize  $\Psi[\mathcal{W}]$ , choose  $\mathcal{V}$  to be a weighted decision rule, with the weights of delegates proportional to the square root  $\sqrt{n_i}$  of their respective cell size; and the quota is chosen as half the total weight.<sup>68</sup>*

We have just seen at the end of Subsection 6.1.7 that the same prescription produces a reasonably egalitarian two-tier structure, provided  $m$  is sufficiently large. In contrast, the SSQRR works for small  $m$  as well, and is highly accurate, provided the numbers  $n_i$  are sufficiently large for the error of approximations (3) to be negligible.

Recall also that, as explained in Subsection 6.1.5, maximizing  $\Psi[\mathcal{W}]$  is tantamount to minimizing the MMD  $\Delta[\mathcal{W}]$ . Thus the SSQRR tells us how to make a two-tier structure as ‘majoritarian’ as possible, in other words: as close as possible to a referendum of the entire citizenry operating under the simple majority rule.

**6.1.9 Dilution of citizens’ voting power** Even the best two-tier structure falls short of a referendum of the citizenry at large with regard to voter empowerment and majority deficit.

I shall illustrate this by means of a simplified version of the two-tier structure considered above: one in which all the cells  $N_i$  are of equal size, say  $n$ . In other words, I put  $n_i = n$  for all  $i$ .

I will assume that the decision rule  $\mathcal{V}$  of the council is the simple majority rule: all delegates have equal weight, and the quota is half the total weight. (For simplicity, we can take the weight of each delegate to be 1 and the quota to be  $\frac{m}{2}$ .) This clearly satisfies PSQRR, and is perfectly egalitarian. It also satisfies the SSQRR, and hence maximizes the total voting power of the citizens (and minimizes the MMD) as far as possible for the given two-tier structure.

The voting power  $\psi_i[\mathcal{V}]$  of each council member is now equal to  $\sigma_m$ . Since  $n_i = n$  for all  $i$ , I can rewrite (7) in the following simplified form:

$$\psi_v[\mathcal{W}] \approx \frac{K}{\sqrt{n}} \sigma_m \quad \text{for any } v \in N. \quad (8)$$

I shall now make the additional assumption that the number  $m$  of cells (which is also the number of council members) is large enough for approximation

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<sup>68</sup>For proof and discussion of the SSQRR, see Felsenthal and Machover [9, pp. 72-8]

(3) to be applicable to  $\sigma_m$  with negligible error. Hence I can further simplify (8) as follows:

$$\psi_v[\mathcal{W}] \approx \frac{K}{\sqrt{n}} \frac{K}{\sqrt{m}} = \frac{K^2}{\sqrt{nm}} \quad \text{for any } v \in N. \quad (9)$$

On the other hand, if the citizens were to make a decision by direct referendum under the simple majority rule, then we would have an assembly of size  $nm$  and hence the voting power of each citizen would be given by

$$\psi_v[\text{Referendum}] = \sigma_{nm} \approx \frac{K}{\sqrt{nm}} \quad \text{for any } v \in N. \quad (10)$$

Therefore

$$\psi_v[\mathcal{W}] \approx K\psi_v[\text{Referendum}].$$

By (4) this means that the (indirect) voting power of a citizen in the two-tier system is about 79.79% of the power that s/he would have in a direct referendum – a loss of approximately 20.21%.

As for the MMD of the two-tier structure, a similar simple argument – using (5) with  $nm$  instead of  $n$  – shows that

$$\Delta[\mathcal{W}] = \frac{\Sigma_{nm} - \Psi[\mathcal{W}]}{2} \approx \frac{K(1-K)\sqrt{nm}}{2}. \quad (11)$$

In a multi-tier structure, the dilution of voting power (and corresponding increase of the MMD) is compounded with each additional tier. With each additional tier,  $\psi_v[\text{Referendum}]$  is multiplied by an additional factor  $K$ .

Thus in a three-tier structure (assuming that all cells/councils at each level are of equal size, and decisions are made by simple majority), the voting power of each citizen will be approximately  $K^2\psi_v[\text{Referendum}]$ . Since  $K^2 = 2/\pi \approx 0.6366$ , this means a loss of about 36.34% of voting power. A simple calculation shows that in a four-tier pyramid citizens lose almost half of the voting power they would have in a direct referendum; and in a five-tier pyramid, the loss is almost 60%.

## 6.2 Proportionality of LVP

Let us consider elections conducted under LVP, as described in Section 4.4. Suppose there are  $m$  constituencies, which I label by the numbers  $1, 2, \dots, m$ . I shall assume that the number of voters in each constituency is the same, say  $n$ . Then the total number of voters is  $mn$ .

Let us look at a particular party, say the Reds, that puts up candidates for election. Suppose that the number of voters for the Reds in the  $i$ -th constituency is  $r_i$ . I put:

$$p_i := \frac{r_i}{n}. \quad (12)$$

Thus  $p_i$  is the proportion of Red voters in the  $i$ -th constituency.

Now for each  $i$ , let  $X_i$  be the random variable equal to the number of Red candidates elected in the  $i$ -th constituency. Under LVP  $X_i$  has two possible values: 1 and 0, which it takes with probabilities  $p_i$  and  $1 - p_i$ , respectively. The mean value and variance of  $X_i$  are easily seen to be

$$EX_i = p_i \quad (13)$$

and

$$VX_i = p_i(1 - p_i). \quad (14)$$

The  $X_i$  are of course mutually independent.<sup>69</sup>

The total (at-large) number of Red voters is  $\sum_{i=1}^m r_i$ , and hence by (12) the overall proportion  $p$  of Red voters in the entire electorate is given by

$$p = \frac{1}{mn} \sum_{i=1}^m r_i = \frac{1}{m} \sum_{i=1}^m p_i. \quad (15)$$

On the other hand, the total number of seats won by Reds is the random variable  $\sum_{i=1}^m X_i$ , and hence the proportion of Reds in the assembly is given by the random variable

$$\bar{X}_m := \frac{1}{m} \sum_{i=1}^m X_i.$$

Using (13) and (15), we find that the mean value of  $\bar{X}_m$  is

$$E\bar{X}_m = \frac{1}{m} \sum_{i=1}^m EX_i = \frac{1}{m} \sum_{i=1}^m p_i = p.$$

Thus the mean proportion of Reds in the assembly is equal to the proportion of Red voters in the entire electorate.

I claim that if  $m$  increases without bound then  $\bar{X}_m - p$  converges to 0 almost surely (i.e., with probability 1).

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<sup>69</sup>Thus the  $X_i$  constitute a *sequence of  $m$  Bernoulli trials with variable probabilities*.

To prove this, I invoke the theorem known as *Kolmogorov's Strong Law of Large Numbers*.<sup>70</sup> From (14) it follows that  $0 \leq VX_i \leq \frac{1}{4}$ ; hence the sum

$$\sum_{i=1}^m \frac{VX_i}{i^2}$$

remains bounded as  $m$  increases. Thus Kolmogorov's criterion is satisfied, and therefore according to the theorem the probability that  $\bar{X}_m - p$  converges to 0 equals 1, as claimed.

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<sup>70</sup>See the Wikipedia entry <http://tinyurl.com/mgvcpn> for the statement of this theorem. For a proof, consult any good book on probability theory.

## References

- [1] Albert, M 2006: *Realizing Hope: Life Beyond Capitalism*; London & New York: Zed Books (also: Nova Scotia: Fernwood Publishing).
- [2] Amar A R 1984: 'Choosing representatives by lottery voting', *Yale Law Journal* **93**:1283–1308.
- [3] Banzhaf J F 1965: 'Weighted voting doesn't work: a mathematical analysis', *Rutgers Law Review* **19**:317–343.
- [4] Brams S J 1976: *Paradoxes in Politics: An Introduction to the Non-Obvious in Political Science*; New York: Free Press.
- [5] Campbell T 2005: *Deliver the Vote: A History of Election Fraud, an American Political Tradition—1742–2004*; New York: Carrol & Graf Publishers
- [6] Cockshott W P and Cottrell A 1993: *Towards a New Socialism*; Nottingham: Spokesman Books (Bertrand Russell Peace Foundation). Also downloadable from <http://tinyurl.com/ldr9zj>
- [7] Diskin A, Eschet-Schwarz A and Felsenthal D S 2007: 'Homogeneity, Heterogeneity and Direct Democracy: The Case of Swiss Referenda', *Canadian Journal of Political Science* **40**:317–342.
- [8] Dummett M 1997: *Principles of Electoral Reform*; Oxford: Oxford University Press.
- [9] Felsenthal D S and Machover M 1998: *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*; Cheltenham: Edward Elgar.
- [10] ——— 2005: 'Voting power measurement: A story of misreinvention', *Social Choice and Welfare* **25**:485–506.
- [11] Glaser D 1994: 'Paradoxes of the Council State', *Studies in Marxism* **1**:143–177.
- [12] Machover M 2007: *Penrose's square-root rule and the EU Council of Ministers: significance of the quota* <http://eprints.lse.ac.uk/2857/>
- [13] Nurmi H 1999: *Voting Paradoxes and How to Deal with Them*; Berlin: Springer-Verlag.

- [14] Pannekoek A P 1950: *Workers' Councils*. Reprinted 2003, AK Press.  
Also available for downloading from  
<http://www.geocities.com/johngray/wcontit1.htm>
- [15] Penrose L S 1946: 'The elementary statistics of majority voting', *Journal of the Royal Statistical Society* **109**:53–57.
- [16] ——— 1952: *On the Objective Study of Crowd Behaviour*; London: H. K. Lewis & Co.
- [17] Shalom, S R 2005: 'ParPolity: Political Vision for a Good Society',  
<http://www.zmag.org/znet/viewArticle/4957>
- [18] ——— 2009: 'Parpolity and Indirect Elections',  
<http://www.zcommunications.org/znet/viewArticle/22017>