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**Working paper**

**Original citation:**

Corsi, Fulvio, Hosni, Hykel and Marmi, Stefano (2013) *Risk allocation: the double face of financial derivatives*. The London School of Economics and Political Science. (Unpublished).

This version available at: <http://eprints.lse.ac.uk/50971/>

Available in LSE Research Online: July 2013

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# Risk Allocation: The Double Face of Financial Derivatives

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5 June 2013

## Abstract

For the past two decades, derivatives provided the core financial innovation for risk-management and risk-sharing activities. However, in the aftermath of the 2007-2008 crisis, derivatives have started to receive, partly for good reason, an increasingly bad press. The main purpose of this paper is to lay the foundations for a theoretical framework in which systemic risk is centrally involved in the assessment of derivative usage. We define an allocation to be efficient if it maximizes the Aggregate Sharpe ratio of the economy, i.e. if it allows to finance the maximum amount of productive investments while minimizing the overall systemic risk of the economy. We then say that a derivative is *socially efficient* if it leads to an allocation having higher Aggregate Sharpe ratio. We illustrate the applicability of our model by means of a qualitative analysis of three types of derivatives, namely Plain vanilla, Asset backed securities and Credit default-swaps.

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# 1 Introduction and Motivation

An efficient allocation of the unavoidable risks generated by the business activity of individual agents is vital for the economic system. Insurance contracting and financial markets attain to this crucial goal. Motivated by this objective, which often takes the form of an attempt to achieve the simultaneous reduction of risk for all participants, financial risk-bearing entities may enter into various kinds of formal risk-sharing agreements. Indeed, the idea that enhanced risk allocation should result in real economic benefits for society as a whole is deeply entrenched in political economic thought. To make just one notable example, Irving Fisher remarked, back in 1906, that

*[R]isk is one of the direst economic evils, and all of the devices which aid in overcoming it – whether increased guarantees, safeguards, foresight, insurance or legitimate speculation – represent a great boon to humanity. [25]*

Driven essentially by sheer increase in financial innovation, the practice of risk allocation performed by the financial system has rapidly evolved over the past few decades. Very substantial efforts have been devoted to the design of institutions and products enabling efficient risk sharing among individuals<sup>1</sup>. It is important to stress that innovation in the field of derivative products has significantly increased both the efficiency and the complexity of the allocation of risk. Today, market-based allocation of risk is primarily accomplished through the use of derivative products and *it is not an exaggeration to state that a considerable portion of financial innovation over the last 30 years has come from the emergence of derivative markets* (see [1]).

This owes directly to the fact that, for at least a couple of decades, the benefits of using derivatives, as well as of the associated market-based risk sharing model, appeared to be many-sorted and unquestionable. First of all, derivatives have been widely believed to cater for efficient risk management. Secondly, financial derivatives have been acknowledged an important informational role as they could be used as price discovery tools (see, e.g. [21]).

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<sup>1</sup>Representative examples can be found in [5], which also discusses a basic theory of risk sharing in an economy with incomplete markets, and in [23].

Finally, derivatives appeared to provide more liquidity to the markets and also to the general economy.

For all those reasons, the trend in risk allocation has been marked by the securitization and distribution of risk. As this process started to unfold, various types of risks were originated, securitised, rated and allocated by different actors in the financial market. As a fundamental consequence of all this, risks were transferred from the regulated sector to unregulated one. Institutional supervisors welcomed this market-based risk sharing model which, among other things, appeared to distribute risks away from a small number of large and systematically important actors, to a large number of smaller size investors.

Things changed dramatically with the outbreak of the 2007-2008 global financial crisis when a number of serious issues started to be raised with respect to the use of derivative products. Paradigmatic cases have certainly been the instability of markets in the aftermath Lehman Brothers's failure and, to an even larger extent, the need of a bailout of AIG – the largest insurance company in the United States at the time [33]. This much certainly provided enough evidence to the effect that markets can, and sometimes do, fail to deliver the expected risk management performances and the associated desirable social outcomes which largely motivated the enthusiastic endorsement of financial derivatives<sup>2</sup>. Building on this line of thought, a number of authoritative commentators [18, 32, 35] claimed that the global economic and financial crisis was a direct consequence of specific failures of the market-based risk sharing process.<sup>3</sup>

The main contribution of this paper is to provide a simple, yet in our opinion highly flexible, framework for the characterization of what we refer to as *socially efficient* or *coop-*

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<sup>2</sup>For instance, given their capital base and risk management policies, individuals choose their desired level of market risk. However, in setting their level of market risk it is doubtful that individuals internalize the systemic implications of the failure of their firms or the public cost associated with any implicit government safety net.

<sup>3</sup>To worsen the outlook, recent empirical studies established either negative or no relation between derivative usage and firm value. For instance, [24] finds that in firms with greater agency and monitoring problems, derivative usage has a significantly negative impact on firm value measured by their Tobin's Q.

erative derivatives.<sup>4</sup> The main idea articulated across this paper is that a proper evaluation of financial derivatives must take into account their impact on systemic risk.<sup>5</sup> Hence, the key desideratum for a socially efficient derivative consists in providing financial instruments to the market to fund productive but risky projects with the minimum increase in the systemic risk of the whole financial system. This is justified by the obvious observation that capital is a limited resource and so is the capital available to cover for risky investments. Capital must therefore be allocated efficiently toward the highest risk-adjusted productive investments. While doing this, it should be borne in mind that not all risks are created equal, a fact that should be taken into account when risks are diversified and spread across the economy. In particular it is quite natural that those risks which relate to highly productive investments should to be preferred to those bringing lower or zero contribution to the gross national product.

Therefore, the leading intuition consists in defining *efficient* an allocation which allows to finance the maximum amount of productive investments while minimizing systemic risk. This simple approach gives us an opportunity to take into account a number of important factors in the evaluation of the efficiency of a risk allocation, namely (i) the quality of the diversification achieved by the individuals, (ii) the amount of funding of productive investments and (iii) the degree of leverage. Notice that within this framework we can explicitly model the systemic effects induced by the imprecise estimation of risk made by the individuals. Indeed, financial innovations can significantly alter the precision of the estimation of risk, a fact which has clearly important implications for risk allocation. This adjustment of the subjective evaluation of risk is crucial if one aims, as we do, at modelling individual decisions under uncertainty as opposed to risk – the situation in which true and

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<sup>4</sup>Our proposed framework differs substantially from the optimal risk sharing one to be found in the actuarial literature [11, 12], for our main concern is the characterization of a socially efficient derivatives.

<sup>5</sup>The importance of systemic risk is gaining increasingly recognition in the financial community. Several definitions and several measures of systemic risk in the finance and insurance sectors have been proposed (see, e.g., [4, 9, 8, 27]).

maximally precise evaluations of risks are given<sup>6</sup>.

The final steps in the construction of our model consist in defining an *Aggregate Sharpe ratio* and identifying those channels through which a derivative may enhance or reduce the efficiency of the risk allocation in the economy. Having accomplished this we are in a position to say that *a derivative is socially enhancing or cooperative if it leads to a more efficient allocation of risk characterized by a higher Aggregate Sharpe ratio.*

The remainder of the paper is organized as follows. Our model is presented in Section 2. In Section 3 we introduce our definition of a socially efficient derivative and describe the properties of a socially efficient allocation. We illustrate in Section 4 its applicability by means of a qualitative analysis of three kinds of derivatives, namely Plain Vanilla, Asset Backed Securities and Credit Default Swaps. Section 5 concludes.

## 2 The model setup

### 2.1 Portfolio decisions

We begin by considering an economy characterized by a collection of risky investment projects and a group of individuals  $i = 1, \dots, N$  endowed with a given amount of initial (*equity*) capital  $C_i$ . Let  $r_i$  and  $\sigma_i$  be, respectively, the gross return per unit of capital and volatility of the portfolio chosen by agent  $i$  (ROI) and let  $I_i$  be the amount agent  $i$  chooses to invest in the portfolio of risky assets.

As in standard mean-variance theory, each individual, after having selected the desired portfolio, decides how much to invest on it or, equivalently, how much leverage his positions on the basis of his own risk preferences. Hence, each individual chooses the optimal leverage  $\lambda^* = I_i^*/C_i$  ( $C_i$  is considered to be given) which maximizes her utility function. We assume that each agent has a Constant Absolute Risk Aversion (CARA) utility function<sup>7</sup> with different absolute risk aversion parameters  $\alpha_i$ . Assuming for simplicity a zero risk free rate,

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<sup>6</sup>The reader who is unfamiliar with the topic may wish to consult [38] and the excellent selection of references therein contained.

<sup>7</sup>Hence a negative exponential utility function  $U(X) = a - be^{-\alpha X}$ .

each individual  $i$  then maximizes,

$$\max_{\lambda_i} \lambda_i \mu_i - \frac{\alpha_i}{2} \lambda_i^2 \sigma_i^2. \quad (1)$$

The first-order-condition for this utility maximization is

$$\mu_i - \alpha_i \lambda_i \sigma_i^2 = 0 \quad (2)$$

so that the optimal leverage for individual  $i$ ,  $\lambda_i^*$  becomes

$$\lambda_i^* = \frac{\mu_i}{\alpha_i \sigma_i^2} \quad (3)$$

In practice, however, the true values of the portfolio expected return  $\mu_i$  and risk  $\sigma_i$  are not known. Let's call  $\hat{\mu}_i$  the estimate made by agent  $i$  of the unknown expected return  $\mu_i$  and  $\hat{\sigma}_i$  the estimates of the unknown risk  $\sigma_i$ . Hence, the desired leverage can be written as

$$\hat{\lambda}_i^* = \frac{\hat{\mu}_i}{\alpha_i \hat{\sigma}_i^2} = \lambda_i^* \frac{\sigma_i^2}{\hat{\sigma}_i^2} \frac{\hat{\mu}_i}{\mu_i} = \lambda_i^* e_i \quad (4)$$

where  $e_i - 1 = \frac{\sigma_i^2}{\hat{\sigma}_i^2} \frac{\hat{\mu}_i}{\mu_i} - 1$  is the estimation error of the agent  $i$  in evaluating the expected mean and risk of his position.

## 2.2 Individual default probability and distance to default

We assume that an individual  $i$  defaults when the loss on her risky asset position exceeds her initial capital  $C_i$ , which can then be interpreted as the *risk capacity* of agent  $i$ . Thus, the probability of default of agent  $i$  is given by

$$p_i = P(I_i r_i < -C_i).$$

Notice that in a Gaussian world with  $r_i \sim N(\mu_i, \sigma_i^2)$  the return obtained by investing  $I_i$  in such a portfolio is distributed as  $I_i r_i \sim N(I_i \mu_i, I_i^2 \sigma_i^2)$ . Therefore, denoting with  $\Phi$  the distribution function of a standard normal, the true probability of default is  $P(I_i r_i < -C_i) = 1 - \Phi\left(\frac{C_i + I_i \mu_i}{I_i \sigma_i}\right) \simeq 1 - \Phi\left(\frac{C_i}{I_i \sigma_i}\right)$ , i.e. a decreasing function of the so called Distance to Default

( $DD$ ):<sup>8</sup>

$$DD_i^* = \frac{C_i}{I_i \sigma_i} = (\lambda_i \cdot \sigma_i)^{-1}. \quad (5)$$

which can be interpreted as the “number of standard deviations a company is away from its default threshold”.

However, taking into account estimation errors on the determination of the optimal level of investment and, thus, on the optimal leverage, will affect the volatility of the portfolio chosen by the individuals. In fact, the variance of the leveraged portfolio per unit of capital invested, that was  $\lambda_i^2 \sigma_i^2$  in absence of estimation errors, now is (assuming independence between estimation errors and returns)

$$V[\hat{\lambda}_i^* r_i] = V[\lambda_i^* e_i r_i] = \lambda_i^{*2} (E[e_i]^2 \sigma_i^2 + \mu_i^2 V[e_i] + V[e_i] \sigma_i^2). \quad (6)$$

Clearly for  $E[e_i] = 1$  and  $V[e_i] = 0$  we recover the variance of the portfolio with no estimation errors, while the higher are the mean and variance of the estimation errors the higher will be the variance of the leveraged portfolio.

Therefore, the individual  $DD$  in presence of estimation error becomes:

$$DD_i = \left( \lambda_i \sqrt{E[e_i]^2 \sigma_i^2 + \mu_i^2 V[e_i] + V[e_i] \sigma_i^2} \right)^{-1}. \quad (7)$$

Hence, the  $DD_i$ 's, and thus the individual probabilities of default, are made of three components: the degree of leverage  $\lambda_i$ , the true riskiness of the portfolio  $\sigma_i$ , and the statistical properties of the estimation error  $e_i$ .

## 2.3 Systemic Risk

Following the popular measure proposed by [14] we define systemic risk as the sum of the expected capital shortage over the whole population of financial institutions. Thus, according

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<sup>8</sup> $DD$  is a convenient measure that emerged as the industry standard for estimating and evaluating the default risk in credit risk modelling (Moody's KMV model). Obviously, the larger is  $DD$  the smaller is the probability of default. The  $DD$  measure does not include the contribution of the mean of  $r_i$  being typically negligible on the short-medium horizon and very difficult to estimate. Moreover, it does not consider the effects of the higher order moments of the distribution of  $r_i$ .



to this prominent literature, systemic risk is measured by the total amount of capital that the government would have to provide in order to bailout the financial system in case of distress.

Specifically, in our framework, Systemic Risk ( $SR$ ) can be defined as

$$SR \equiv \sum_{i=1}^N ES_i$$

where  $ES_i$  is the expected (capital) shortfall defined as

$$ES_i = -E [I_i r_i | I_i r_i < -C_i]$$

i.e. the expected loss given the default of the financial institution  $i$  (expressed as a positive value for convenience).

### 3 Evaluation of the social efficiency of derivatives

#### 3.1 Definition of socially efficient derivatives

Let us denote

$$Y = \sum_{i=1}^N I_i r_i$$

to be the total gross return of the whole economy. We are now in the position to present our definition of a socially efficient allocation. We say that an allocation is *socially efficient* if, for any given level of  $E[Y]$ , it minimizes  $SR$ ,

$$\min SR \quad \text{subject to} \quad E[Y] = E[Y_0].$$

Note that this is equivalent to requiring that for any given level of  $SR$ , it maximizes the expected value of  $E[Y]$ , i.e.

$$\max E[Y] \quad \text{subject to} \quad SR = SR_0.$$

Hence, the socially efficient frontier will be a curve in the plane  $SR-E[Y]$  which maximizes

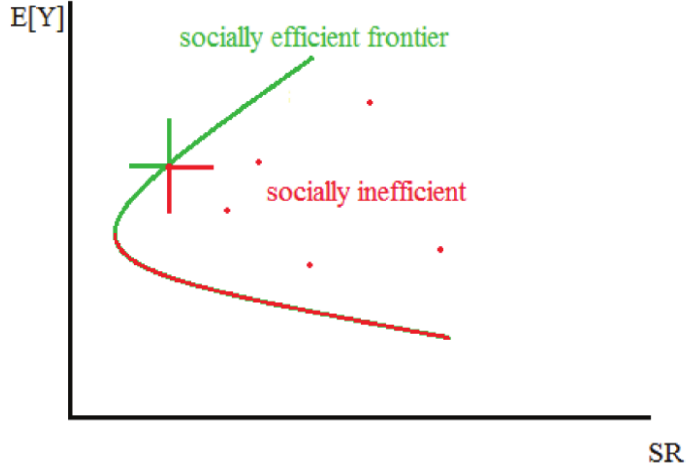


Figure 1: A stylized representation of the efficient frontier in the aggregate  $SR-E[Y]$  plane.

$E[Y]$  for any level of  $SR$  or, alternatively, which minimizes  $SR$  for any level of  $E[Y]$ .

We can now state our definition of a socially efficient derivative: *a derivative is said to be socially efficient or cooperative if it permits to move from an interior and inefficient point to a superior aggregate risk allocation having a higher “Aggregate Sharp ratio”  $E[Y]/SR$ .*

### 3.2 Properties of a socially efficient allocation

Under normality of the return distribution the expected shortfall, being simply the first moment of a truncated Normal distribution, can be explicitly written as:<sup>9</sup>

$$ES_i = -E[I_i r_i | I_i r_i < -C_i] = -C_i E[-\hat{\lambda}_i^* r_i | \hat{\lambda}_i^* r_i < -1] = C_i \cdot DD_i^{-1} \cdot M(DD_i) \quad (8)$$

where  $M(z)$  is the inverse Mills ratio defined as

$$M(z) = \frac{\phi(z)}{1 - \Phi(z)}. \quad (9)$$

with  $\phi(z)$  and  $\Phi(z)$  being, respectively, the density and distribution function of the standard normals.

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<sup>9</sup>Again, we are neglecting the contribution of the mean of returns, as it is customary for computations at short-medium horizons.

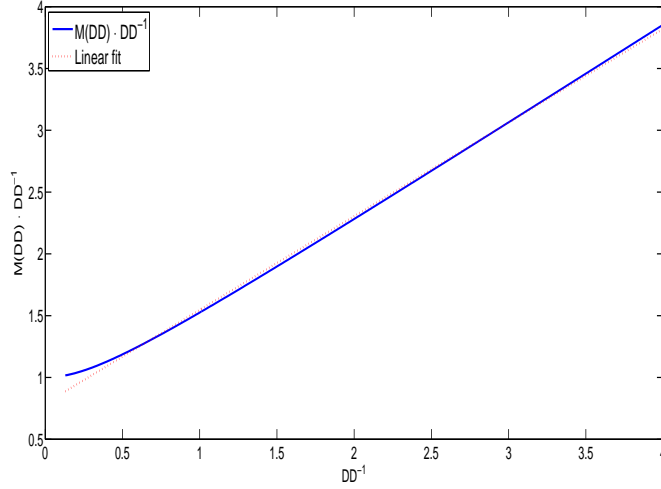


Figure 2: Plot of  $DD_i^{-1} \cdot M(DD_i)$  as a function of  $DD_i^{-1}$  and its linear fit obtained by standard OLS (giving intercept 0.7892 and slope 0.7579).

The product  $DD_i^{-1} \cdot M(DD_i)$  is an increasing function of  $DD_i^{-1}$  and it is virtually linear in  $DD_i^{-1}$ , as illustrated in Figure 2. As pointed out in section 2.2, the individual  $DD_i$ 's, are made of three components: the degree of leverage  $\lambda_i$ , the true riskiness of the portfolio  $\sigma_i$ , and the statistical properties of the estimation error  $e_i$ . Therefore, each individual expected shortfall can be very well described as:

$$ES_i \propto C_i \cdot DD_i^{-1} = C_i \lambda_i \sqrt{E[e_i]^2 \sigma_i^2 + \mu_i^2 V[e_i] + V[e_i] \sigma_i^2}. \quad (10)$$

Thus, the expected shortfall of individual  $i$  increases with the degree of leverage  $\lambda_i$ , with the true riskiness of the portfolio  $\sigma_i$ , with the positive bias of the estimation error, i.e. the underestimation of risk ( $\hat{\sigma}_i < \sigma_i \Rightarrow E[e_i] > 1$ ) and the overestimation of the expected return ( $\mu_i < \hat{\mu}_i \Rightarrow E[e_i] > 1$ ), and, finally, with the variance of the estimation error  $V[e_i]$ .

Hence, ceteris paribus  $SR$ , being the sum of individual expected shortfall, is an increasing function of the individual values  $\sigma_i$  and  $\lambda_i$ . Concerning the estimation error components, at the aggregate level it seems, in general, reasonable to assume that individual errors are uncorrelated and hence their estimation errors tend to average out over distinct individuals. On the other hand, the variance of the estimation error  $V[e]$  continues to play an important

role on systemic risk. In fact, the larger the variance of the estimation error  $V[e]$ , the larger the individual expected shortfall, and hence the higher will be the  $SR$  of the economy.

Summarizing, an efficient allocation is reached in the economy when: (i) the idiosyncratic risks of the single projects are optimally diversified (reducing  $\sigma_i$ 's); (ii) the remaining non-diversifiable risk is optimally estimated (minimum  $V[e]$  and  $E[e] = 1$ ), (iii) and optimally shared among the investors according to their risk capacities (minimization of the aggregate leverage of the economy). This definition of an efficient risk allocation makes also clear that only those risks which directly or indirectly contribute to elevating  $E[Y]$  should be born by the society. As they only increase  $SR$ , but not  $E[Y]$ , the other risks lead to a less efficient allocation point in the  $SR-E[Y]$  plane.

## 4 Discussion of specific cases

The theoretical framework developed in the previous sections allows us to identify the various channels through which a given type of derivative enhances or reduces the efficiency of the risk allocation according to the Aggregate Sharpe ratio of the economy. In this Section we present a qualitative analysis of some types of derivatives and show how our framework could be applied to evaluate different types of derivative contracts. A more detailed quantitative analysis of the various types of derivatives will be the topic for future, empirically oriented, work.

### 4.1 Case 1: Plain vanilla derivatives

The main characteristic of the most standard and simple derivative contracts (such as futures, plain vanilla options, or swaps) is the possibility to transfer specific types of risks. This possibility of relocating risks from one subject to another, who is more willing to bear it, is often considered sufficient to enhance risk allocation and, hence, to be socially beneficial. However, this depends on whether the final allocation entails a lower systemic risk compared to the initial one. The reduction of systemic risk, in turn, depends on whether risk is

transferred to individuals with larger  $DD$ , i.e. with either higher risk capacity (lower leverage  $\lambda$ ), better portfolio diversification (lower  $\sigma_i$ ), or better information (smaller estimation error  $e$ ). This is particularly unlikely to be the case when risk transfer is motivated by regulatory arbitrages or moral hazard behaviors.

Moreover, even in many plain vanilla derivatives, it is well possible that none of the counterparties is transferring a preexisting risk. In this case, since no risk is reallocated, there cannot be any social efficiency gain coming from a better risk allocation. However, new risks are created. In fact, although such contracts are zero-sum transactions, they are certainly not “zero-sum risk”, in the sense that the risk generated by the two-side bets does not compensate. Hence, even though the risks produced by these speculative bets do not contribute to  $E[Y]$ , they have to either absorb capital diverted from other investments projects (which instead contribute to  $Y$ ) or increase the position leverage of the agents and, therefore, the systemic risk of the economy. So, even simple and usually beneficial derivatives products, if misused, can considerably increase systemic risk and thus reduce the Aggregate Sharpe ratio of the economy.

## 4.2 Case 2: Asset backed securities

An Asset Backed Security (ABS) is a security whose cash flows are derived from, and collateralized by, a specified pool of receivables or other financial assets. Those underlying assets are typically represented by illiquid and risky assets which individually possess a very low rating score. The securitization process precisely consists in pooling various types of contractual debt (mortgages, loans or credit card debt) and selling those debts to various investors. Pooling those assets together and slicing such a pool into different risk classes, or tranches, categorized into varying degrees of subordination, reduces the risk of the so called “senior” tranches, since these rely on the protection of the junior tranches which would be the first to suffer the losses in the pool. Through this complex mechanism of pooling and splitting into different risk classes, the senior tranches were able to achieve AAA-ratings. Prior to the 2007-2008 financial crisis, this extremely high rating induced many banks and

other investors to hold large amount of AAA-tranches of a vast variety of ABS products. As it was apparent after the burst of the crisis, those risk evaluations of ABS were severely biased, since the products turned out to be much riskier than implied by their credit scores.

Hence, the complex and opaque structure of the ABS and the related Collateralize Debt Obligations (CDOs) [13] not only increased uncertainty on the level of risk of the underlying assets, thus increasing the variance of the estimation error  $e$ , but also caused a systematical underestimation of such a risk giving rise to a strong bias in  $e$  arose; this in turn led to an unwarranted rise in leverages which further increased individual risk. The separation between the subject originating the risk and those bearing it, also gave rise to large scale moral hazard problems, which greatly increased the size and misspricing of the generated risks. Moreover, being present in the portfolio of many institutions, ABSs and CDOs highly increased the correlation among the individual probability of default.<sup>10</sup>

Therefore, our model suggests that this type of products exposed financial institutions to more systemic risk through:

- (i) an increase in the variance of the estimation error  $V[e]$  (lack of transparency);
- (ii) a positive bias in  $e$ , i.e.  $E[e] > 1$  (misspricing);
- (iii) an increase in individual leverages  $\lambda_i$  (moral hazard) and
- (iv) an increase in the correlation of the probability of default which magnify the probability of contagion.

As a consequence, according to our definition, this type of derivatives were not (in that form) cooperative or socially efficient: even if  $E[Y]$  might increase,  $SR$  is also increased, and the net change in the position of the economy in the  $E[Y] - SR$  space is not superior to (i.e. does not dominate) the initial one.

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<sup>10</sup>See [7] accessible yet very informative discussion of the increasing asset correlations and of the interconnectedness of markets (and the world economy).

### 4.3 Case 3: Credit Default Swaps

A Credit Default Swap (CDS) is a contract in which one party, the protection seller, sells protection to a second party, the protection buyer, against a credit event of a third party issuing a debt. In case of a default the protection buyer is compensated for the loss generated by the failure of the third party. The protection buyer pays, on a regular basis, a premium to the protection seller usually expressed in percentage points of the notional, the so called CDS spread. CDS are then simply a form of insurance against default. However, unlike insurance contracts, CDS do not require an exposure to the underlying credit risk. If the protection buyer does not hold the underlying security, CDS are said to be *naked*. Naked CDS can then be used to build speculative bets on the default of the third party: the naked protection buyer is betting on default while the other is betting against. The large amount of volume of the CDS market (in principle even larger than the total debt of the underlying entity) indicates that a substantial portion of contracts are naked [29, 39].

CDS have come to play an important role in conveying information about the market consensus on the creditworthiness of the underlying<sup>11</sup>, especially in cases when the underlying debt market is not particularly liquid. These price discovery properties of the CDS, if viewed in the light of our model, translate in a reduction of the variance (and possibly bias) of the estimation errors  $e$ .

Non-naked CDS simply imply transferring credit risk from the protection buyer to the protection seller, leaving the total amount of credit risk in the economy unaltered. So the systemic risk implication of this relocation of risk depends on the relative risk capacity of the protection seller compared to the one of the protection buyer. If the protection seller has a greater risk capacity (in terms of larger capital, smaller leverage or better diversified portfolio which results in greater  $DD$ ) than the protection buyer, systemic risk is reduced, otherwise it will be increased.

However, in the presence of naked CDS where no risk shifting is involved, the total

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<sup>11</sup> Empirical studies on how new information is incorporated in bond and CDS prices shows that information mainly flows from CDS to bond prices (see [10]).

amount of counterparty risk in the economy is increased by the two-sided bets on the default event. These newly created risk are unproductive and need to either absorb risk capacity or increases the systemic risk. Naked CDS can also encourage investors to divert their capital away from financing real investment leading in some cases to the selection of riskier ventures with lower expected returns [15].

Moreover, being the market of CDS highly concentrated in few large protection sellers, the counterparty risk generated by the default of one of these dominant actors can generate default contagion and domino effects. In the presence of a large market of naked CDS, the risk is not only given by the loss generated by the default of the issuer of debt, but also by the counterparty risk of all the protection sellers who wrote CDS on that entity. In other words, if CDS protection sellers have insufficient capital to cover CDS losses, the default of the issuer of the debt also causes the default of protection sellers, hence widening the scope for contagion and systemic risk [16].

This risk was clearly illustrated by the AIG case during the 2008 crisis which exerted domino effects through enormous CDS contractual links. In fact, AIG's collapse was caused largely by its \$526 billion portfolio of CDSs. Federal Reserve Chairman Ben Bernanke has characterized AIG operations in derivative markets as the behaviour of a "quasi-hedge fund" that "made irresponsible bets and took huge losses"<sup>12</sup>.

Therefore, the introduction of CDS have both positive and negative effects on systemic risk and  $E[Y]$ :

- (i) they allow for a better price discovery on the creditworthiness of the issuer so that, in the notation of our model,  $V[e]$  and hence  $SR$  is reduced;
- (ii) if credit risk is allocated to more capitalized and diversified subjects, it could permit a better allocation of risk, i.e. either decreasing  $SR$  or increasing  $Y$ ;
- (iii) if, however, CDS transfer credit risk to more leveraged and systemically important institutions  $SR$  will increase;

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<sup>12</sup>David Stout and Brian Knowlton, Fed Chief Says Insurance Giant Acted Irresponsibly, N.Y. TIMES, Mar. 3, 2009, available at <http://www.nytimes.com/2009/03/04/business/economy/04webecon.html>



(iv) in addition, CDS can magnify the underlying credit risk by compounding it with the counterparty risk of the protection sellers, thus increasing the total amount of risk in the system (without financing more productive projects) and the possibility of contagion ( $\bar{\rho}$  which elevates  $P(D_N/N > \vartheta)$ ).

Therefore, the social efficiency of CDS remains unclear, depending on which of the above effects eventually dominates. However, some of the negative effects which tend to increase  $SR$  could be mitigated by moving the trading of CDS from the OTC market to a centralized clearinghouse which would greatly reduce the counterparty risk of CDS.

## 5 Conclusions

Today, the first and most fundamental problem in risk allocation is to understand the complex effects induced by financial derivatives and how they could be so badly misused to ignite a global crisis. We argued that part of the answer lies in the relation among individual risk, systemic risk and the related notions of correctly assessing risk capacity and leverage.

In this paper we put forward a new theoretical framework within which the social efficiency of the risk allocation determined by a given derivative product can be assessed. To this purpose, we investigate the determinants of the default risk of a single agent identifying three components: (i) the degree of leverage, (ii) the riskiness of the individual portfolio and (iii) the estimation error in evaluating the expected return and risk of the portfolio. Then, after defining systemic risk as the sum of individual expected loss given default, we introduced an aggregate risk-return plane for the risk allocations and define an Aggregate Sharpe ratio for the whole economy. Finally, we proposed our criteria to evaluate the social efficiency of a derivative by looking at whether it increases or reduces the Aggregate Sharpe ratio of the system.

This approach, albeit simple, permits to take into account many important factors in the evaluation of the efficiency of a risk allocation: the quality of the diversification achieved by the individuals, the amount of funding of productive investments, the degree of leverage

and, importantly, it brings to the foreground the systemic effects induced by the imprecise estimation of risk made by the individuals.

Applying this theoretical framework to some of the most controversial derivatives we can conclude that a generic derivative contract is socially cooperative if : (i) informational gains are provided by improving transparency and price discovery, (ii) preexisting risks are reallocated only to more capitalized, diversified or informed subjects, (iii) new risks are not created.

We believe that our framework may be employed by regulators and supervising authorities for policy evaluations as well as in the design of future financial reform. In order to fully explore such a possibility, however, we must bring to completion the empirically oriented work which is currently under way on this topic.

## **Acknowledgments**

A preliminary version of this work was presented at the *Enterprise Risk Management and Corporate Governance for Insurance Firms* conference, held on 17 May 2011 at EDHEC Business School in Lille. We are very grateful to the organisers, discussant and the conference audience for stimulating discussions and encouraging feedback. We would also like to thank two anonymous referees whose unrestrained comments gave us an opportunity to produce a much better paper.

## References

- [1] Viral Acharya, Menachem Brenner, Robert Engle, Anthony Lynch and Matthew Richardson, *Derivatives - The Ultimate Financial Innovation* chapter 10 in *Restoring Financial Stability: How to Repair a Failed System*, edited by Viral Acharia and Matt Richardson , Wiley , 2009.
- [2] Acharya, V.V. and Pedersen, L.H. and Philippon, T. and Richardson, M., *Measuring systemic risk*, chapter 4 in *Regulating Wall Street: The Dodd-Frank Act and the New Architecture of Global Finance*, edited by Viral Acharia, Wiley, 2010.
- [3] Adrian, T. and Brunnermeier, M., *CoVaR*, Federal Reserve Bank of New York Staff Reports, volume 348, pages 1–8, 2009.
- [4] Hamed Amini, Rama Cont and Andreea Minca, *Stress testing the resilience of financial networks* International Journal of Theoretical and Applied Finance, to appear, 2011
- [5] Franklin Allen and Douglas Gale, *Financial Innovation and Risk Sharing*, MIT Press, 1994
- [6] Franklin Allen, Douglas Gale, *Systemic Risk and Regulation*, chapter 7 in *The Risks of Financial Institutions* Mark Carey and Ren M. Stulz editors, University of Chicago Press, 2007
- [7] John Authers, *The Fearful Rise of Markets: Global Bubbles, Synchronized Meltdowns, and How To Prevent Them in the Future* FT Press, 2010
- [8] Battiston, Stefano, Delli Gatti, Domenico, Gallegati, Mauro, Greenwald, Bruce C. and Joseph E. Stiglitz, *Liasons Dangereuses: Increasing Connectivity, Risk Sharing, and Systemic Risk* NBER Working Paper 15611, 2009
- [9] Billio, Monica, Getmansky, Mila, Lo, Andrew W. and Pelizzon, Lorian, *Econometric Measures of Systemic Risk in the Finance and Insurance Sectors*. Journal of Financial Economics, volume 104, Issue 3, pages 535559, 2012

- [10] Blanco, R. and Brennan, S. and Marsh, I.W., *An Empirical Analysis of the Dynamic Relation between Investment-Grade Bonds and Credit Default Swaps*, Journal of Finance, volume 60, pages 2255–2281, 2005.
- [11] K. Borch, *Reciprocal Reinsurance Treaties*. Astin Bulletin, vol. 1, 170-191, 1960
- [12] K. Borch, *Equilibrium in a Reinsurance Market*, Econometrica, 30, 424-444, 1962
- [13] Brigo, Damiano, Pallavicini, Andrea and Torresetti, Roberto, *Credit Models and the Crisis, or: How I Learned to Stop Worrying and Love the CDOs*. Available at SSRN: <http://ssrn.com/abstract=1529498> (December 29, 2009)
- [14] Brownlees, Christian and Engle, Robert *Volatility, Correlation and Tails for Systemic Risk Measurement*. Available at SSRN 1611229, 2012
- [15] Yeon-Koo Che and Rajiv Sethi, *Economic Consequences of Speculative Side Bets: The Case of Naked Credit Default Swaps*. Available at SSRN: <http://ssrn.com/abstract=1654222> 2010
- [16] Rama Cont, *Credit default swaps and financial stability*, Financial Stability Review (Banque de France), 2010.
- [17] *Hanbook of Mathematical Economics, Vol 4*, vol. 56, Elsevier, 1991.
- [18] John Cassidy, *How Markets Fail*, Farrar, Straus and Giroux, 2009.
- [19] Cummins, John David and Weiss, Mary A., *Systemic Risk and the U.S. Insurance Sector*. Available at SSRN: <http://ssrn.com/abstract=1725512>, 2010
- [20] Bruno De Finetti, *Impostazione individuale e impostazione collettiva del problema della riassicurazione*, Giornale dell'Istituto Italiano degli Attuari, Roma 1942.
- [21] Diamond, D.W. and Verrecchia, R.E., *Constraints on short-selling and asset price adjustment to private information*. Journal of Financial Economics, 1987, vol. 18, pages=277–311.

- [22] Duffie, Darrell , *Innovations in Credit Risk Transfer: Implications for Financial Stability*. BIS Working Paper No. 255; Rock Center for Corporate Governance Working Paper No. 6. Available at SSRN: <http://ssrn.com/abstract=1165484> (July 1, 2008)
- [23] Duffie Darrell and Rahi Rohit, *Financial Market Innovation and Security Design: An Introduction* Journal of Economic Theory Volume 65, Issue 1, February 1995, Pages 1-42
- [24] Larry Fauver and Andy Naranjo *Derivative usage and firm value: The influence of agency costs and monitoring problems* Journal of Corporate Finance, 2010, Volume 16, Pages 719–735.
- [25] Irving Fisher, *The Nature of Capital and Income*. New York: Macmillan, 1906.
- [26] Giesecke, K. and Kim, B., *Systemic Risk: What Defaults Are Telling Us*, Working Paper, Stanford University, 2010. Available at SSRN: [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1474055](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1474055)
- [27] Khandani, Amir E., Lo, Andrew W. and Merton, Robert C., *Systemic Risk and the Refinancing Ratchet Effect*. MIT Sloan Research Paper No. 4750-09; Harvard Business School Finance Working Paper No. 1472892. Available at SSRN: <http://ssrn.com/abstract=1472892> (September 15, 2009).
- [28] Paul Klumpes, Peter Welch and Andres Reibel, *Bank cash flows a source of new insight?* Journal of Financial Transformation, 2009, vol. 26, pages 69-78
- [29] Lewis, M. *The big short: inside the doomsday machine*, WW Norton & Company, 2010.
- [30] Paolo Medolaghi, *Il rischio nelle assicurazioni*, Atti dell'Istituto Nazionale delle Assicurazioni, Roma 1930.
- [31] Giuseppe Ottaviani, *Sul problema della riassicurazione*, Giornale dell'Istituto Italiano degli Attuari, Roma 1952
- [32] Robert J. Shiller, *The Subprime Solution: How Today's Global Financial Crisis Happened and What to Do about It*, Princeton University Press, 2008.

- [33] Sjostrom, William K., *The AIG Bailout* Washington and Lee Law Review, Vol. 66, p. 943, 2009. Available at SSRN: <http://ssrn.com/abstract=1346552>
- [34] *Systemic Risk in Insurance. An analysis of insurance and financial stability* Special Report of The Geneva Association Systemic Risk Working Group, available at the link [http://www.genevaassociation.org/PDF/BookandMonographs/Geneva\\_Association\\_Systemic\\_Ris](http://www.genevaassociation.org/PDF/BookandMonographs/Geneva_Association_Systemic_Ris) 2010
- [35] Nassim N. Taleb, *Why did the Crisis of 2008 Happen?*. Available at SSRN: <http://ssrn.com/abstract=1666042>, 2010
- [36] Taleb, Nassim Nicholas and Tapiero, Charles S. , *The Risk Externalities of Too Big to Fail* (November 1, 2009). NYU Poly Research Paper. Available at SSRN: <http://ssrn.com/abstract=1497973>
- [37] *The Turner Review: A regulatory response to the global banking crisis*. Available at the link [http://www.fsa.gov.uk/pubs/other/turner\\_review.pdf](http://www.fsa.gov.uk/pubs/other/turner_review.pdf), 2009
- [38] Wakker, Peter *Uncertainty* in Blume, L. & Durlauf, S. (Eds.) *The New Palgrave: A Dictionary of Economics* 8, 428-439, The MacMillan Press, London.
- [39] Zuckerman, G. *The greatest trade ever: The behind-the-scenes story of how John Paulson defied Wall Street and made financial history*, Crown Business, 2010.