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Analysis and control design of sustainable policies for greenhouse gas emissions

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ABSTRACT

Reducing greenhouse gas emissions is now an urgent priority. Systems control theory, and in particular feedback control, can be helpful in designing policies that achieve sustainable levels of emissions of CO_2 (and other greenhouse gases) while minimizing the impact on the economy, and at the same time explicitly addressing the high levels of uncertainty associated with predictions of future emissions. In this paper, we describe preliminary results for an approach where model predictive control (MPC) is applied to a model of the UK economy (UK 4see model) as a test bed to design sustainable policies for greenhouse gas emissions. Using feedback control, the policies are updated on the basis of the actual emissions, rather than on the predicted level of emissions. The basic structure and principle of the UK 4see model is described and its implementation in Simulink is presented. A linearized state space model is obtained and model predictive control is applied to design policies for CO_2 emissions. Simulation results are presented to demonstrate the effectiveness of the proposed method. The preliminary results obtained in this paper illustrate the strength of the proposed design approach and form the basis for future research on using systems control theory to design optimal sustainable policies.

1. Introduction

Global warming is an urgent issue for our planet and finding ways to reduce greenhouse gas (GHG) emissions is now an important research topic. Globally, annual emission now rise to 30.6 gigatonnes of CO_2 per annum, and figures published by the UK Department for Environment, Food and Rural Affairs (DEFRA) show that in 2008, the UK emitted around 533 million tonnes of CO_2 per annum. The draft climate change bill, presented to parliament in 2007, aimed to achieve a 60% cut in emissions by 2050 (compared to 1990 level) as demanded by the Kyoto protocol. This target has been increased to an 80% reduction in emissions by 2050 as set in Climate Change Act 2008 and more recently, an ‘interim target’ of a 34% cut by 2020 was imposed and made legally binding in the April 2009 Budget. Achieving these GHG emission targets without significantly affecting the UK economy is a major challenge.

A lot of research has been done on the evaluation and design of sustainable policies on climate change and economic growth. For example, in Ref. [1] a roadmap for UK carbon capture and storage (CCS) was developed through a combination of a two-phase process of stakeholder engagement and review of the CCS landscape, Refs. [2,3]. introduced different models to evaluate the economic aspect of the CSS technologies for Europe and China, respectively, Refs. [4,5]. explored the trade-offs between alternative energy system pathways, and the cost, energy supply and emissions implications of these alternative pathways under different scenarios based on the UK MARKAL (MARKet ALocation) dynamic optimisation model and incorporated uncertainties through repeated ‘what-if’ sensitivity analysis, Refs. [6,7]. introduced the regional integrated model of climate and the economy (RICE model) and used this model to evaluate the effect of carbon price on the climate change and economic growth across different regions. More references can be found in Refs. [8–16].

It is clear that no single factor will generate the required reduction in emissions. Instead, success will require a combination of policies (by policies we mean the detailed strategies and tactics) for deciding the mix of energy generation and reduction in energy usage through the use of incentives, taxes and quotas, and maximising CO_2 absorption, both naturally by preserving forests and via technical developments, such as carbon capture and storage. It is also clear that implementing these policies will have an impact on economic growth and, as the Stern report emphasises [17], acceptance of these policies will only be achieved if the impact on economic growth is minimized. In addition, it is noted that there are uncertainties/disturbances associated with the effect of the policies on the CO_2 emission rate and economic growth. Therefore, any policy design must take modelling

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uncertainties into account and the policies should be updated based on the current situation (i.e. using a feedback mechanism to update the policies according to actual emissions rather than predicted values) to compensate for the effect of unexpected disturbances.

The design objective can be stated as designing a sustainable policy that achieves sustainable levels of emissions of CO_2 (and other greenhouse gases) while minimizing the impact on the economy. The aim of the designed policy is to adjust factors such as the mix of energy generation methods and policies for reducing emissions from housing, industry and transport based on current situation, in order to achieve a rate of emissions that will allow UK to achieve its emissions targets while maximising economic growth. This can be considered as an optimal control problem and therefore concepts from modern systems control theory (and in particular, feedback) can be used to develop design algorithms. The feedback mechanism updates policies on the basis of the actual rather than predicted emissions, and hence provides a degree of robustness against noise/disturbances. The high levels of uncertainty associated with predictions of future emissions can also be addressed systematically using systems control tools.

The above approach is currently being implemented on the UK 4see model, which describes the dynamic evolution of the UK economy and CO_2 emissions based on the ECCO (Evolution of Capital Creation Options) modelling methodology [18–22]. Compared to other energy economic models (e.g. computable general equilibrium model GEM-E3 [23], or the optimization based MARKAL model [4,5]), the 4see model is a system dynamics model based on general systems theory and thus, is more suitable as an initial test bed for verifying our proposed systems control theory-based design methods. In the future, the tools for control design will be applied to other existing models.

As a preliminary result of this approach, in this paper, Model Predictive Control (MPC) is applied to the UK 4see model to design sustainable policies to demonstrate the potential power of using systems control theory to control greenhouse gas emissions. The paper is organised as follows. In Section 2, the basic structure and principle of the UK 4see model is explained. An initial version of the model in VenSim (a system dynamics modelling simulation software) was converted into Simulink to allow more flexibility in control system analysis and design. The resulting UK 4see model is highly nonlinear and to facilitate the design process, a linearized model is obtained and analysed in Section 3. In Section 4, model predictive control is applied to the linearized model to design sustainable policies and simulation results are presented. Finally, conclusions and future research directions are given in Section 5.

2. UK 4see model and Simulink implementation

2.1. ECCO modelling methodology

The UK 4see model is a dynamic model of the UK national economy based on the ECCO (Evolution of Capital Creation Options) modelling methodology [18–22,24]. It was firstly developed in the Dynamo simulation language by a research group from University of Edinburgh in 1992 and recently, an improved version, on which this paper is based, was developed in VenSim environment by the Innovation & Foresight unit at Ove Arup (where the model was renamed as 4see instead of ECCO [25,26]). ECCO uses a macroeconomic holistic modelling approach for determining the system-wide, long term effect of implementing policy options at the national/regional level. It does this by determining the growth potential of the economy in the context of the existing economic structure and user-defined policies, technology options and environmental objectives. Changes in growth potential in turn alter a wide range of demand and supply terms, and so reflect many other aspects of the evolving economy [24]. The ECCO model emphasises the feedbacks between sectors and the impacts of the policy upon the endogenously determined rate of physical growth. It was originally intended to be used in assessing the compatibility of multiple goals prior to their adoption by policy-makers and mainly addresses the question [18,27], 'What would happen if a set of policies are to be set?' However, in this research the model is used as the basis of an optimal control design.

To develop an ECCO model, the main economic sectors are identified, which include both human-made capital sectors (e.g. industry and agriculture) and natural capital stocks (i.e. energy and material resources). As the states within the system evolve with time, the size of each sector will change and the changes in one sector will affect the growth of other sectors through cross sector interactions. The internal dynamics and interactions between different sectors are characterised through the physical principles of mass and energy balances as measured by embodied energy [24]. The exact parameters of these dynamics are determined either empirically or by validating against historical statistical data for a specific period as obtained from government sources of statistics (e.g. DUKES DUK [28], Blue Book BLU [29] and Pink Book PIN [30]).

2.2. UK 4see model

The UK 4see model consists of thirteen sectors of the UK economy, namely: industry and growth, balance of payments, services, dwellings, standard of living, employment, resource and mining, electricity generation, transport, agriculture, water, global and sectoral coefficients, and carbon dioxide. The model was implemented in VenSim environment into 13 views (subsystems) and each view corresponds to one sector. More details about the model can be found in the Appendix and [24–26] and here, we take the carbon dioxide sector shown in Fig. 1 as an example to illustrate the basic idea and structure of the model.

In Fig. 1, each name in the diagram is a variable, e.g. CO_2 *released by oil* (unit: tonnes/year), *oil demand* (unit: VPJ/year, i.e. virtual petajoules calculated using embodied energy [24]) and CO_2 *generated index* (unit: tonnes/year). The value of a variable is either determined by external input data (e.g. energy policies) or by the variables connected to it. The connections between variables represent mathematical operations, which are governed by mass or energy balance. The pipeline connections represent integral operations; for example in this figure according to mass balance the value of CO_2 *generated index* is determined by

$$\frac{\mathrm{d}(CO_2\ \textit{generated index})}{\mathrm{d}t} = RF\,CO_2 - old\,CO_2 \tag{1}$$

where $RF\,CO_2$ is the rate of CO_2 formulation (unit: tonnes/year) and *old* CO_2 is the previous CO_2 production rate. In the *electricity generation* sector the electrical energy generated by thermal power plant measured by gigawatt (GW) is determined by

$$\frac{\mathrm{d}(\textit{Thermal generating capacity})}{\mathrm{d}t} = \textit{rate of building Thermal} - \textit{rate of depreciation Thermal} - \textit{rate of decommissioning of Thermal}. \tag{2}$$

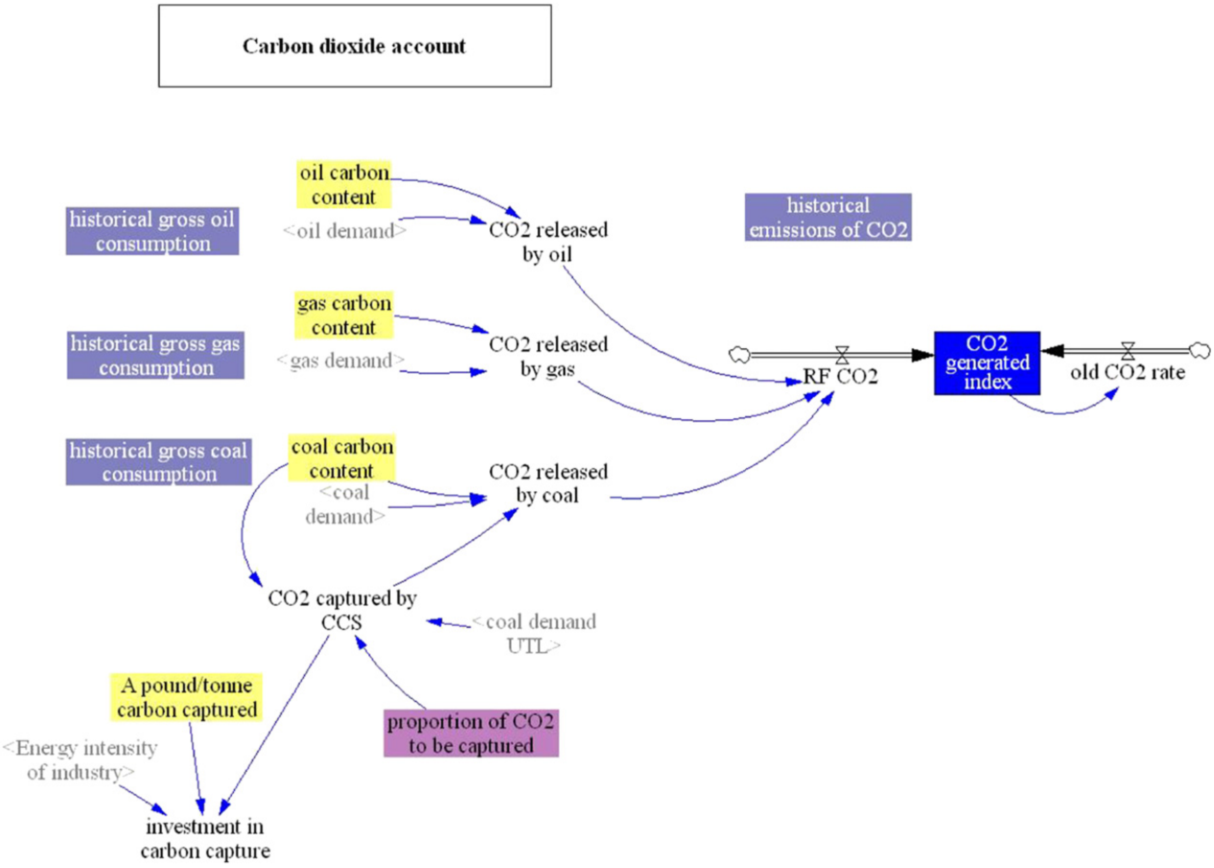

Fig. 1. UK 4see model in VenSim: carbon dioxide sector.

Arrow connections represent all other operations, e.g. basic algebraic operations, lookup tables and ‘if else’ operations. For example, in Fig. 1

$$\begin{aligned} RF\ \ CO_2 \ &= CO_2 \ \textit{released by oil} + CO_2 \ \textit{released by gas} \\ &\quad + CO_2 \ \textit{released by coal} \end{aligned} \tag{3}$$

$$CO_2 \ \textit{released by coal} = \textit{oil demand} \times \textit{oil carbon content}. \tag{4}$$

and in the *industry* sector

$$\begin{aligned} \textit{RCF energy conservation IND} &= \textit{RCF IND } 1{*}\textit{investment in energy} \\ &\quad \textit{efficiency ratio IND}/100, \end{aligned} \tag{5}$$

where *RCF energy conservation IND* is the rate of capital formulation in industry energy conversion (unit: VPJ/year) and the meaning of other two variables can be found in Appendix B.

It can be seen from Fig. 1 that CO_2 emissions come from three sources: CO_2 *released by oil*, CO_2 *released by gas* and CO_2 *released by coal*, which in turn are determined by the fuel demand (oil, gas and coal) and the carbon content, respectively. CO_2 *released by coal* is also affected by the CO_2 *amount captured by CCS in power plant*. It is important to notice that CO_2 emissions heavily depend on fuel demand while fuel demand is determined by all other sectors, e.g. *industry*, *dwelling* and *transport*, which illustrates the interconnections between different sectors.

From a systems point of view, differential equations such as (1) and (2) define the states of the system, while the auxiliary variables defined by algebraic equations (e.g. (3)–(5)) give the interactions of the system states, or define variables of particular interest (e.g. an economic index). As a result, the system can be rewritten in the form

$$\begin{aligned} \dot{x} &= q(x,u,v) \\ 0 &= h(x,u,v) \\ y &= g(x,u,v) \end{aligned} \tag{6}$$

where x represents the system states (e.g. *capital stock of industry*), u is the system input (e.g. energy policies), v stands for the auxiliary (algebraic) variables, y is the system output (e.g. CO_2 emissions and the economic index we are interested in), and $q(\bullet), h(\bullet), g(\bullet)$ are nonlinear functions where the nonlinearity comes from the nonlinear relationships such as (4) and (5). It is noted that by eliminating the auxiliary (algebraic) variable v, system (6) can be further written equivalently into a standard state space form and therefore, can be included into a systems control framework for optimal policy design.

2.3. Simulink implementation

The UK 4see model was developed in the VenSim environment and as a dynamic simulation software, VenSim is well capable of simulating the evolution of the model. However, its functionality is limited for the purpose of control systems design. Indeed, MATLAB Simulink is a widely used dynamic analysis and control design tool. To facilitate the system design process, the UK 4see model is implemented in Simulink following a specific style and procedure. More information on the implementation procedure can be found in Ref. [31]. The final model in Simulink is shown in Fig. 2.

Within the 13 sectors of the model there are 946 variables, of which 61 are national policies (e.g. *fraction of renewables* and *investment in energy efficiency ratio in the dwelling sector*) that could be adjusted to affect CO_2 emissions and economic growth. These 61 policies serve as manipulated variables or inputs to the system and

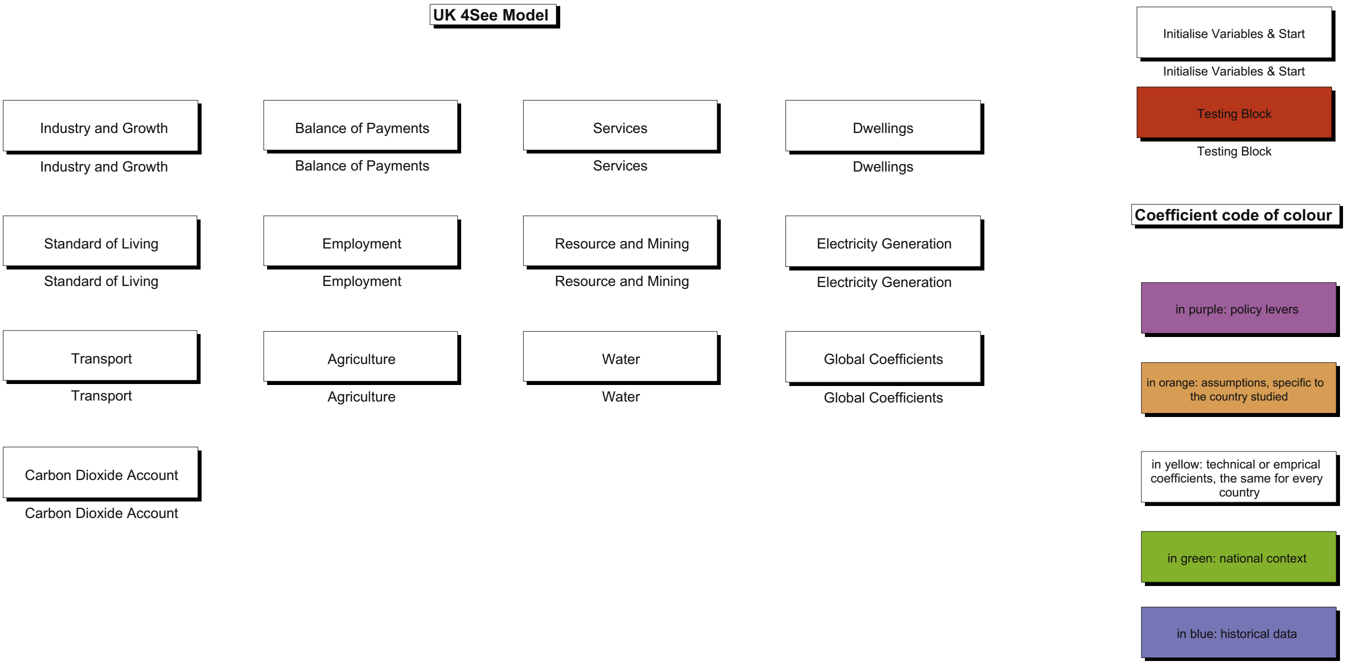

Fig. 2. UK 4see model in MATLAB Simulink. This figure shows the top level view of the UK 4see model, which consists of 13 subsystems and illustration blocks. It is important to point out that there are strong interconnections between different subsystems, which are implemented using From/Goto blocks (equivalent to line connections) within subsystems to avoid messy line links between different sectors.

will be designed at a later stage. The outputs we are interested in are CO_2 emissions and economic growth. While CO_2 emissions are predicted in the model by CO_2 *generated index*, the characterisation of economic growth is not straight forward. The economic growth can be observed from a combination of factors, for example, employment situation and standard of living. As a starting point, standard of living, which is characterised by *index material standard of living per capita* in the model, is used as a measure of economic growth. Therefore, our objective can then be stated as to design policy that optimally keeps CO_2 emissions (CO_2 *generated index*) at a sustainable level, while maximising the economic growth as measured by *index material standard of living per capita*.

From a systems point of view, we now have a model with 61 inputs and 2 outputs. We can simulate the model with an empirically predesigned set of policies for the 'business as usual' scenario. These two outputs are shown in Figs. 3 and 4 for the period until 2025. It can be seen from these figures that under the designed policies CO_2 emissions clearly will not achieve the objective of 34% cut in emissions by 2020 (i.e. 593 × 66% = 391.4 million tonnes/year), which demonstrates the need for an optimal sustainable policy design.

3. UK 4see: preliminary analysis results

3.1. A linearized model

As mentioned in Section 2, the UK 4see model is a nonlinear model and to simplify the analysis and design, a linearized model is firstly obtained. The system can be linearized either at a single point (usually an equilibrium point or a steady-state operating point) or along a specified trajectory; linearizing at a point will

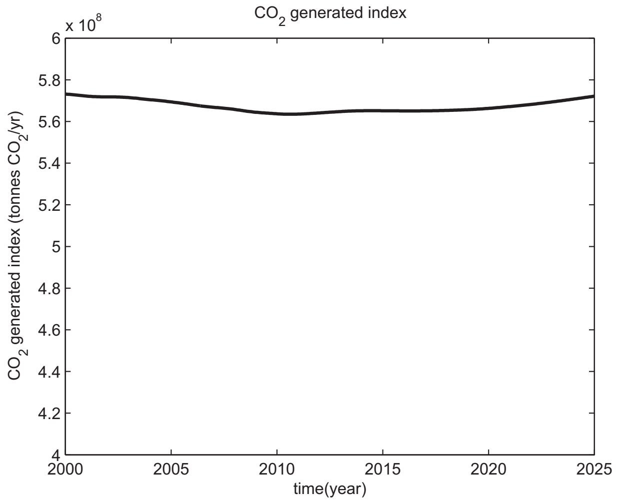

Fig. 3. CO_2 emissions until 2025 under empirically predesigned 'business as usual' policies.

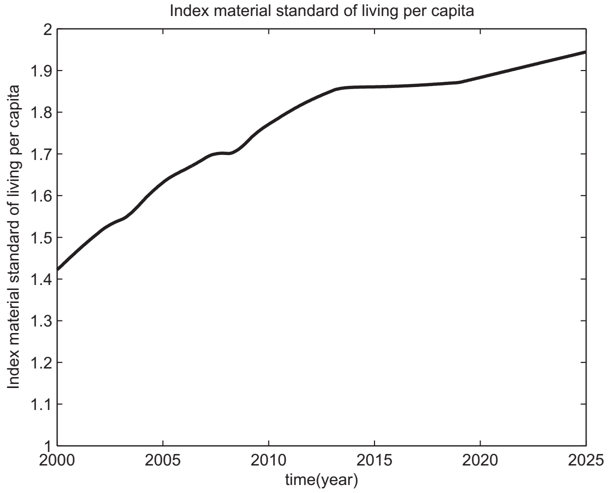

Fig. 4. Economic growth as measured by *index material standard of living per capita* until 2025 under empirically predesigned 'business as usual' policies.

result in a linear time invariant (LTI) system, while linearizing along a trajectory will lead to a linear time varying (LTV) system. LTV systems are more complex than LTI systems in systems analysis and design, so for simplicity, the system is linearized at the current year ($t = 2010$), even though this is not an equilibrium point under the empirically predesigned policies, to obtain an LTI state space model of the following form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{7}$$

where $x \in \mathbb{R}^{41}$ is the state vector, $u \in \mathbb{R}^{61}, y \in \mathbb{R}^2$ are input and output vectors, A, B, C, D are matrices of appropriate dimensions. Detailed data are omitted here for brevity.

To validate the linearized model (7), a step input (of amplitude 0.05) is added onto one particular input (*fraction of nuclear power*) at $t = 2010$ and the output of the original nonlinear model and the linearized model are shown in Figs. 5 and 6. From Figs. 5 and 6, it can be seen that the response of the linearized model (solid line) is very close to the original nonlinear model (dashed line), which demonstrates the linearized model obtained provides a good approximation of the original nonlinear system. The response of the linearized model was validated against the response of the full nonlinear model for a range of inputs. The linearized model will be used in future systems analysis and design.

3.2. Preliminary analysis

The UK 4see model is a large scale model with strong interactions between the 13 consisting sectors. Having obtained a linearized model, by exploring the structure of the system matrix A, a graph illustrating the interactions between the states of different sectors can be obtained, as shown in Fig. 7(a), which illustrates the high degree of interactions in the UK economic system.

From Fig. 7(a), it can be seen there are two different types of interactions. For example, the industry and service sectors have strong interactions and each of them affects the growth of the other one; some other sectors, for example, the carbon dioxide sector, is affected by, but does not significantly affect other sectors. It is important to point out that, the figure illustrates the interactions between the states of different sectors, but the states of a sector can not represent all the information of the sector and therefore, the exact relationship of the 13 sectors should be analysed using a different method. An example of this is that the sector *balance of payments* has no state in the UK 4see model and therefore, is not shown in Fig. 7(a). However, it does relate to industry, service and other sectors. The *transport* sector, is another example. In the 4see model, the information about conventional transport methods (e.g. cars) which affect the use of fuel (e.g. oil) and in turn affect the carbon dioxide emissions, are not included in the states. Therefore, there is no direct link in Fig. 7(a) from sector transport to carbon dioxide emissions.

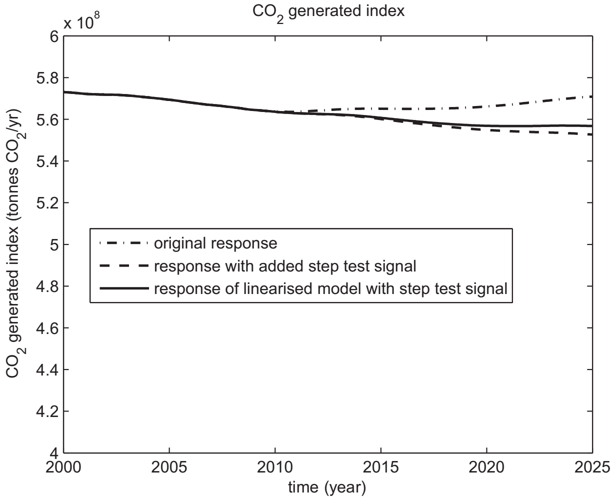

Fig. 5. CO$_2$ emissions: comparison of responses for the nonlinear and linearized models.

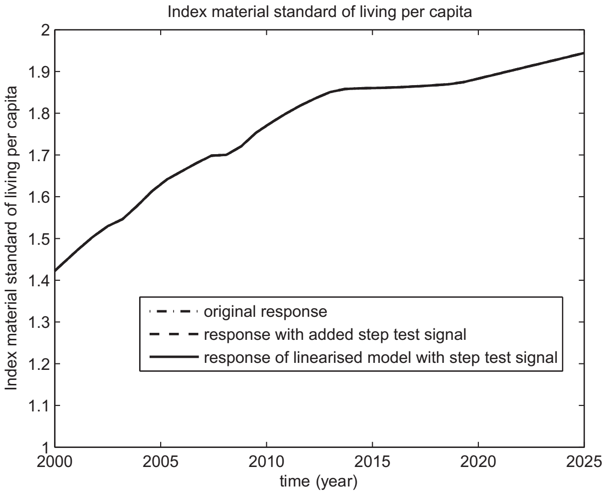

Fig. 6. Economic growth as measured by *index material standard of living per capita*: Comparison of responses for the nonlinear and linearized models. Note that the three responses are very close and almost indistinguishable.

The linearized model (7), however, is unstable and has an unstable pole at $s = 0.014$ Further analysis shows that all the subsectors are stable, therefore, the instability is caused by the strong interactions between different sectors. To identify these 'strong' interactions, an energy based method similar to [32,33] utilizing the results from systems theory is used to characterise the strength of the connections.

Suppose system (7) has been re-ordered and partitioned according to the sector size. Now denote $x_i,\ i = 1, \ldots, 12$ the state vector of sector i (e.g. states of *industry* sector), then e_{ij} the effect of jth sector on ith sector (e.g. the effect of *industry* sector on *service* sector), is $A_{ij}x_j$ as can be seen from the system equation. We now try to find the worst-case energy flow on the graph in Fig. 7(a), (i.e. $\max\sum_{i,j} e_{i,j}^2$). To do this, an output map corresponding to all the connections e_{ij} is appended to the autonomous system as

$$\begin{aligned} \dot{x} &= Ax \\ z &= Fx \end{aligned} \tag{8}$$

where F is a diagonal matrix with diagonal elements $A_{ij} \neq 0,\ i,j = 1, \ldots, 12$. In (8), the first equation describes the evolution of the system and the second equation gives the interactions between different sectors. The energy flow on the graph (which characterises the amount of interactions between different sectors) from a given initial condition x_0 (i.e. an initial state of the economy) can now be computed using results from systems theory as

$$\|e\|_2^2 = x_0^T Q x_0$$

where Q is the observability grammian defined by [34]

$$A^T Q + QA + F^T F = 0$$

which characterises the observability of the system and in our case, the amount of interactions between the sectors that can be

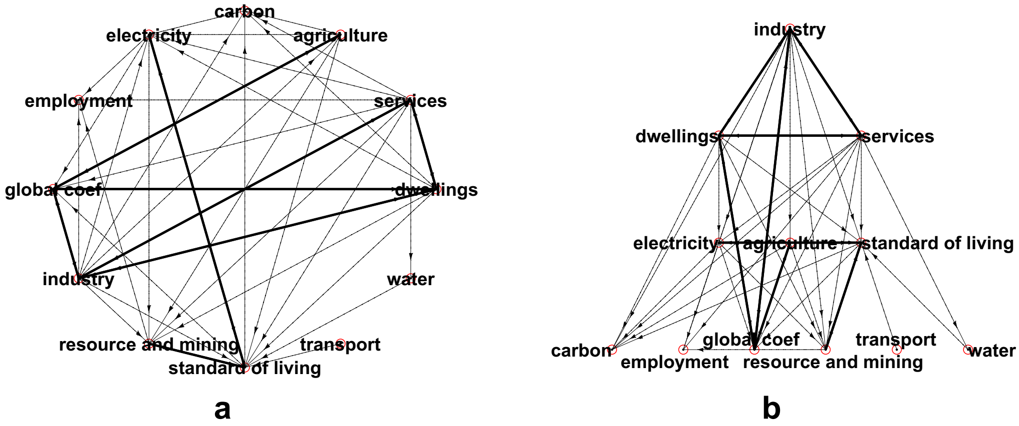

Fig. 7. UK 4see model. (a) This figure shows the interconnections between different sectors of the UK 4see model. Thin line arrows represent single direction effects and bold lines represent bi-directional effects. It can be seen that there are strong interconnections between different subsystems in the UK economic system. (b) A hierarchical structure. This figure shows the interconnections between different sectors of the UK 4see model in a hierarchical structure. Thin line arrows represent single direction effects and bold lines represent bi-directional effects.

observed. The unit-norm initial condition $\bar{x}_0$ that produces the most output energy is then the eigenvector of Q corresponding to the largest eigenvalue. Starting from this initial condition, the energy flow e_{ij} from sector j to sector i (e.g. the effect of *industry* sector on *service* sector) can be computed as

$$\|e_{ij}\|_2^2 = x_0^T Q^{(ij)} x_0$$

where $Q^{(ij)}$ is the observability grammian defined by

$$A^T Q^{(ij)} + Q^{(ij)} A + A_{ij}^T A_{ij} = 0.$$

Note that when the system is unstable (as in our case for the UK 4see model), a finite horizon method should be used.

After computing the energy flow e_{ij} of the interactions between sectors, we now define the strength of the interaction $\tilde{e}_{ij}$ as

$$\tilde{e}_{ij} = \frac{\|e_{ij}\|_2^2}{\sum_{j=1}^{12} \|e_{ij}\|_2^2}$$

which characterises the relative importance of a particular link e_{ij} out of the many interactions affecting sector i It is important to notice that this relative effect is more important that the absolute energy flow value e_{ij} We have now characterized the strength of the interactions between sectors. Inspection of the data shows that some of the connection strengths are extremely small (i.e. $\tilde{e}_{ij} \leq 1e-4$), e.g. the effect of *water* on *standard of living*, while others are significant, e.g. the effect of *industry* on *standard of living*. Rearranging the graph according to the interaction strengths (details are omitted here for brevity), we obtain a hierarchical structure of the system, as shown in Fig. 7(b).

In Fig. 7(b), the upper sectors have significantly more effect on the lower sectors, while the effect of the lower sectors on the upper sectors is very small and can be neglected. Three sectors, industry, dwellings and services, are at the top of the graph, which implies that they are the dominant sectors of the UK economy. This is consistent with our recent results based on a model reduction approach, details of which can be found in Ref. [35]. Further analysis on the data reveals that there are strong positive feedback interactions between the dwelling and service sectors, representing the requirement that the two sectors should develop in a compatible manner. This is the cause of the instability and needs careful consideration in the design process.

4. UK 4see: model predictive control design

4.1. Problem formulation

As stated earlier, the objective is to design optimal policies that achieve sustainable levels of emissions of CO_2 (and other greenhouse gases) and minimizes the impact on the economy. This can be formulated into an optimal control framework as follows

$$u^* = \arg\min_{u \in U} J(y,u) \tag{9}$$

where $J(y,u)$ is a performance index defined as

$$J(y,u) := \int_{t=2011}^{\infty} f(y,u)\mathrm{d}t \tag{10}$$

that measures the long term effect of the output, i.e. y_{CO_2} and y_{SOL} and input u, while U is the set of admissible inputs that defines the constraints the input policies should satisfy, e.g. the investment values should be positive and the fraction of electricity provided by the renewable energy should be within the range $[0,1]$. The above optimization problem is subject to the system dynamics described by the 4see model and the optimal feedback control law is then defined as

$$u^* = k(x,t) \tag{11}$$

where x is the state vector of the 4see model.

The solution of the above optimization problem gives the optimal policies. However, directly solving the optimization problem is difficult due to: (1) the performance index has an infinite horizon, (2) the optimization problem is subject to the constraint $u \in U$, and (3) the system dynamics defined by the 4see model are nonlinear. Model Predictive Control (MPC) provides as a suboptimal solution of the above problem by sequentially solving the following finite horizon optimization subproblem

$$u^* = \arg\min_{u \in U} \int_{t=t_0}^{t_0+N_p} f(y,u)\mathrm{d}t \tag{12}$$

which in discrete time becomes

$$u^* = \arg\min_{u \in U} \sum_{t=t_0}^{t_0+N_p} f(y,u), \tag{13}$$

where N_p is the predictive horizon. This problem is of small size and can be solved effectively using standard optimization techniques (e.g. the MATLAB optimization toolbox).

The implementation of the MPC approach consists of the following steps:

1. At current time t_0 use the system model to predict the system output $y(t),\ t_0 < t \le t_0 + N_p$ based on the current state x_{t_0}
2. Solve the constrained optimization problem (13) to obtain $u^*(t),\ t_0 \le t < t_0 + N_p$
3. Implement the current input $u^*(t_0)$ to the system
4. Repeat the above steps.

It is important to note that this is a feedback control mechanism (11) since in Step 1, the current system state is fed into the optimization problem. Therefore, the MPC algorithm has some degree of robustness against model uncertainties and disturbances. As a result, instead of using complex nonlinear accurate models, simplified models can be used in predicting system future output, which can reduce the computational load in the optimization process. These important features make MPC a very successful and widely used advanced control technique in many practical applications. More information on MPC can be found in Refs. [36–39].

Remark 1. *There are some results in the literature on the design of sustainable policies [4,5]. However, most of these methods do not (or at least not explicitly) design feedback mechanisms that update the policies on the basis of the actual emissions rather than the predicted emissions. Designing policies solely on the basis of predicted emissions is sensitive to disturbances (e.g. unexpected disruptive events). By contrast, our proposed feedback-based approach is robust against disturbances and the model uncertainty can also be systematically incorporated into the design process, which is more likely to achieve better performance and is one of the most appealing features of our proposed method.*

Remark 2. *It is possible to choose different predictive horizons N_u and N_y for input and output in the performance index. By choosing different input and output predictive horizons, different system performance can be achieved. In this paper, for presentation simplicity, the input and output predictive horizons are chosen to be the same. More details on this can be found in Ref. [36].*

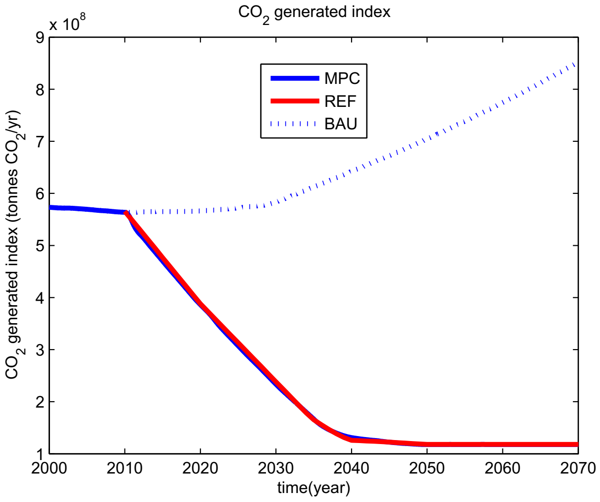

Fig. 8. CO_2 emissions until 2070 using MPC design.

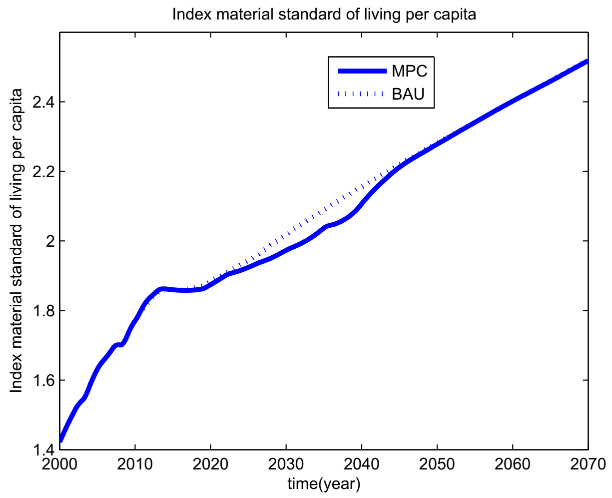

Fig. 9. Economic growth as measured by *index material standard of living per capita* until 2070 using MPC design.

4.2. Simulation results

The design of function $f(y,u)$ in (10) is important since it directly defines the system performance. In this paper, as an initial design, $f(y,u)$ is chosen to be

$$f(y,u) = \lambda_{CO_2} \times \left|y_{CO_2} - r_{CO_2}\right|^2 + \lambda_{SOL} \times \left|y_{SOL} - r_{SOL}^{BAU}\right|^2 + \lambda_u \times \|u - u_{base}\|^2, \quad (14)$$

where r_{CO_2} is a reference trajectory for the CO_2 emissions representing the sustainable levels/government legal requirements, r_{SOL}^{BAU} is the ‘business as usual’ case standard of living and $\lambda_{CO_2}, \lambda_{SOL}, \lambda_u$ are weighting parameters. By minimizing (13), the first term in (14) requires the CO_2 emissions to follow the given reference trajectory as close as possible, the second term aims to minimize the negative effect on economic growth and the last term aims to ensure that the input is not be too different from the baseline level u_{base} and is included for caution and robustness.

The reference trajectory r_{CO_2} is chosen to be the signal shown in Fig. 8 to reflect the UK CO_2 emissions reduction goal of 34% by 2020 and 80% by 2050. There are a range of different views on the correct trajectory – a representative trajectory is used here. Other trajectories will be considered in future research (e.g. the EU roadmap

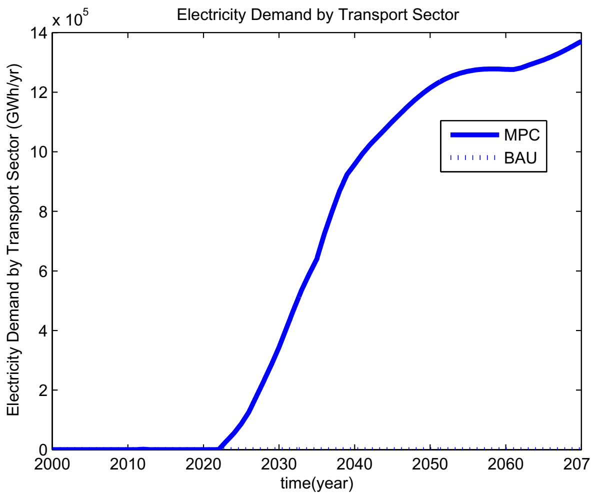

Fig. 10. Electricity demand by transport sector: MPC design.

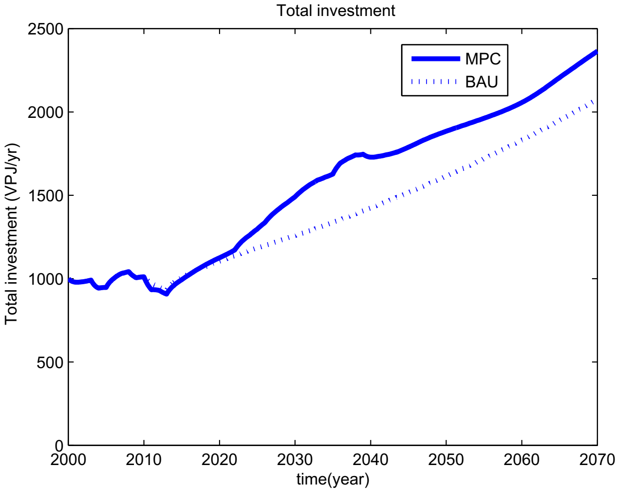

Fig. 11. Total investment into the economy: MPC design.

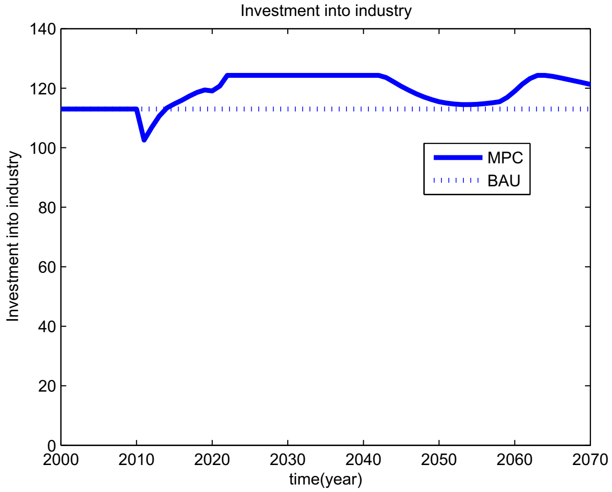

Fig. 13. Investment into industry: MPC design.

targets for moving to a low-carbon economy in 2050). Of the 61 input policies, 13 policies are chosen to be controlled in our simulation, among which the investment into industry is allowed for fluctuations of 10 per cent. Note that the nuclear power as an alternative electricity generation method is not included into the 13 controlled policies in this initial design due to considerations regarding operation safety, nuclear waste disposal, etc. The weighting parameters are chosen to be $\lambda_{CO_2} = 0.2, \lambda_{SOL} = 1, \lambda_u = 1$ and the predictive horizon $N_p = 5$. The system model used in the prediction is the linearized model (7) discretised with a sampling time $T_s = 1$ year and real-time state feedback is used. The simulation results using MPC design and the 'business as usual' (BAU) case are shown in Figs. 8–12 and several typical policies are shown in Figs. 13–15. In fact, much more information can be inspected from the MPC design. Due to space considerations, only some key observations are shown here.

From Figs. 8 and 9, it can be seen that under the MPC design, the output of CO_2 emissions follows the reference and achieves the design objective, while at the same time, the standard of living maintains its growth and remains close to the business as usual scenario. It is noted that the standard of living exhibits a decrease during years 2025–2045. This is because during this period, the economy is trying to reduce the CO_2 emissions of the transport sector by increasing the use of electric vehicles (as can been seen from Fig. 15), which will create significant increase in the electricity demand (Fig. 10) and in turn, require big investment into the economy to build renewable power station and related facilities (as seen in Fig. 11) and thus limit the amount of materials that people can consume. Therefore, the standard of living is decreased. It is also noticed that the capital stock of industry grows faster than the business as usual case (Fig. 12), in order to produce more goods to compensate for the increasing demand in investment and to maintain economy growth.

Further exploring the designed optimal input policies indicates that, in order to reduce CO_2 emissions and minimize the effect on economic growth, the main policies we will need to apply are: to (1) put more investments in industry (Fig. 13) to maintain economic growth (as seen in Fig. 12); (2) increase the fraction of electricity produced by renewable energy (Fig. 14); and (3) shift from fossil fuel-based cars to electric vehicles in transportation (Fig. 15). These observations are consistent with our intuition and could provide useful suggestions for policymakers.

Because this is an initial design, it must be pointed out that the designed policies may not be realizable and certain limitations should be noted. For example, in Fig. 13, the designed policy of investment in industry may not be acceptable in practice; in Figs. 14 and 15, it might not be possible to increase the fraction of electricity produced by renewable energy to 99.8% by the year 2022 and to

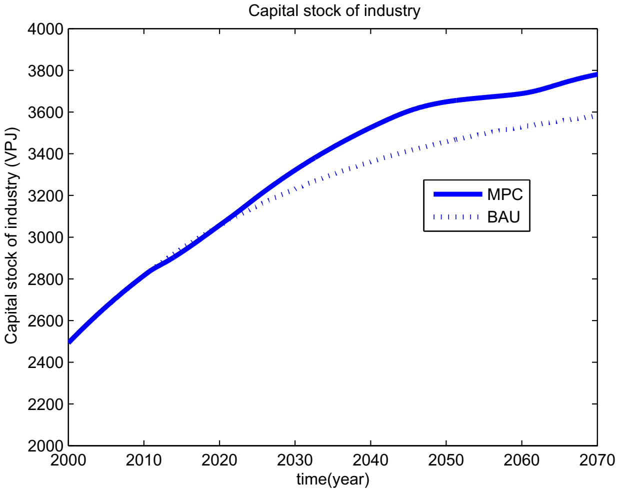

Fig. 12. Capital stock of industry: MPC design.

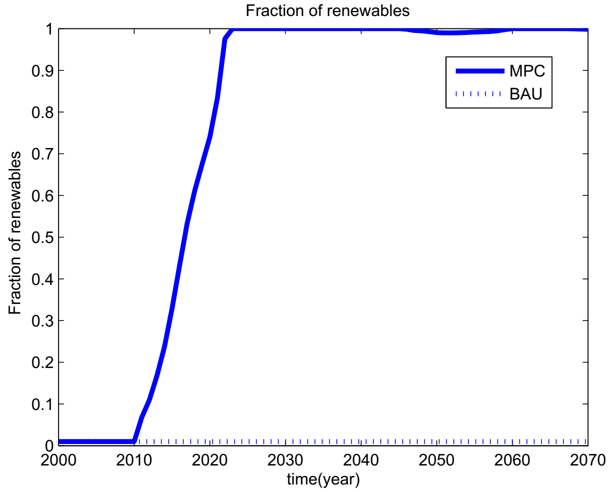

Fig. 14. Fraction of electricity produced by renewable energy: MPC design.

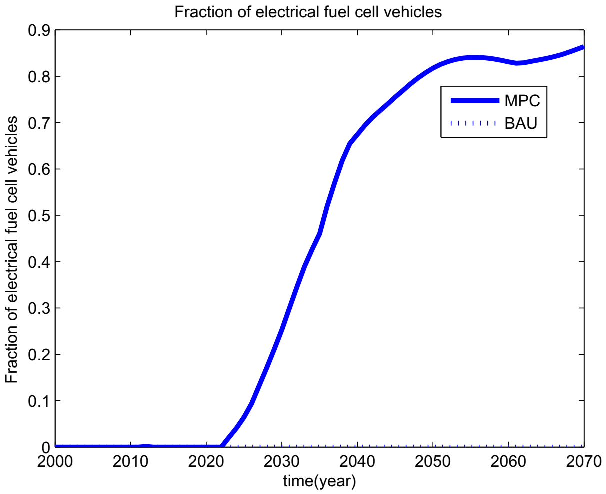

Fig. 15. Fraction of electrical vehicles: MPC design.

increase the proportion of electrical vehicles in transportation from 0 to 80% within twenty years, which will require significant investments into building renewable power stations and related facility construction. These practical considerations can be incorporated into MPC design by imposing an appropriate constraint on the possible input policy change rate, which is currently being investigated and will be reported elsewhere.

5. Conclusions

Reducing greenhouse gas emissions is now important and urgent. In this paper, we used the UK 4see model as a test bed and applied model predictive control to design sustainable policies to demonstrate the benefit of using systems control theory to regulate greenhouse gas emissions. The proposed approach can also be applied to other existing models. The basic structure of the UK 4see model was described and an initial VenSim version of the model was introduced. Since the functionality of VenSim is limited for system design, the model was re-implemented in Simulink. A linearized state space model was then obtained and analysed using energy based methods, and a hierarchical structure of the system was derived, illustrating the dominant structure of the UK economic system.

The objective of designing sustainable policies that achieve sustainable levels of CO_2 emissions (and other greenhouse gases) while minimizing the impact on the economy was then formulated into a constrained infinite horizon optimal feedback control problem. The feedback mechanism increases the robustness of the designed policies and thus is more likely to achieve better performance. Directly solving this problem is difficult and model predictive control was applied to obtain a suboptimal solution. The results showed that the designed policies achieve the desired CO_2 reduction goals and their effect on economic growth is also small.

Although model predictive control was used in this paper to design sustainable policies, other design methods from systems control theory can also be applied. For example, distributed control can be adopted to take advantage of the hierarchical structure of the system to increase robustness and reduce computational load. Also, there are model uncertainties associated with inaccuracies in the model parameters and the linearization process, but these uncertainties have not been included into the control design in this paper. These important questions form the basis of future research and will be reported separately.

Finally, in this paper we uses systems control theory to design a set of sustainable policies that if adopted, will achieve the CO_2 emissions reduction and economic growth goals. However, we have not taken into account the behaviour of the policymakers and thus the question of whether or not these designed policies will be or likely to be adopted by the policymakers, has not been addressed in this paper. Modelling and incorporating the behaviour of the policymakers into the design process is an equally, if not more, important question to the design approach itself and would be considered in our future research.

Acknowledgements

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Appendix A. A brief introduction on sectoral details of the UK 4see model

In Section 2, the basic structure and principle of the UK 4see model is explained. Here, we give a more detailed description on the model for the 13 sectors. For more information, please refer to [18–22,24–26].

Agriculture. This sector describes the supply and demand of agricultural production. The demand is calculated using the population and national nutritional goal level, as well as self sufficiency in food production requirement. The production is a function of the capital stock of the sector. Energy (i.e. thermal energy and electricity energy) is used during the process, which will further determine the fossil fuel demand and electricity demand. The sector also includes a model for biofuels production.

Industry. Industry is the core of the UK 4see model and its output is the basic generator of wealth. The output of the sector is a function of its capital stock, which can be either consumed by the population, or provide the means of investment to grow the economy. The industry sector is energy intensive and demands thermal and electricity energy in producing goods, which determines the fossil fuels and electricity demand for the *resource and mining* and *electricity generation* sectors. Industry mainly grows by investment into the sector, which is one of the main controlled policies in the UK 4see model.

Services. The *service* sector is one of the major sectors in the UK 4see model, in the sense of capital stock, energy use and employee number. Service is demanded by several sectors, e.g. industry, dwelling and itself. Adding these together gives the total service demand, which is to be met by growing the service capital stock. The service sector is also energy demanding. Thermal and electricity demand are calculated according to the capital stock of the sector, which further determines the fossil fuel and electricity requirement.

Dwellings. Similar to the *service* sector, the *dwellings* sector is another important sector in the UK 4see model. The demand of this sector is determined by the population and standard of living (as an index of life quality). The *dwellings* sector requires thermal and electricity energy, e.g. for cooking and heating, which determines the fossil fuel and electricity demand of this sector. It also includes models for other heating methods, e.g. heat pump, biomass and solar water.

Resource and mining. This sector produces the main fossil fuel for the whole economy, which is further used to produce thermal or electrical energy. The sector mainly considers three natural sources: coal, gas and oil. For each resource, there is a natural capital stock (the growth of which depends on the discovery rate and mining rate) and a man-made extraction capital stock (which determines the production). It also models its relationship with the world resource through import/export. In the model, the Uranium demand is purely met by imports. The energy used during the production of the resource is also characterised in the sector.

Electricity generation. This sector is the most important sector for energy generation. It first calculates all the electricity required by the economy by adding up the electricity demand of each sector and subtracts the electricity imported from other countries, which results in the total domestic electricity generation demand after taking into account of electricity transmission efficiency. This demand is met by both traditional power plant (i.e. thermal power plant, combined-cycle plant and nuclear power plant) and renewable power source (i.e. ocean wave, solar electricity, onshore/offshore wind power, hydropower, tidal power), each of which has its own capital stock. It also calculates the amount of fossil fuel (e.g. coal, gas and oil) required during the generation of electricity.

Water. Each sector of the UK 4see model generates a water demand and meeting these demand is very important for economic survival. The *water* sector characterizes the water demand of all other sectors by evaluating their capital stock, water demand intensity and water use efficiency and meets this demand by growing its capital stock. It also includes a model describing the capital required by water distribution and delivery.

Transport. This sector includes both passenger and freight transportation. It includes major transportation options, e.g. car, rail, flight and water. Its demand is determined by other sectors, e.g. *industry* and *service*. Transportation is one of the major contributions to carbon dioxide emissions and thus, its energy use (i.e. thermal and electricity energy demand) is characterised in a detailed manner. It also includes a model for electrical fuel cell vehicles, which provides possible ways of reducing carbon dioxide emissions.

Employment. The sector describes the employment situation in the UK economy. Employment opportunities mainly come from *industry*, *service*, energy (e.g. *electricity generation*, *resource and mining*) and *agriculture* sectors. The contribution of each sector to employment is determined by its capital stock and labour use intensity. It also includes information for potential working population and unemployment rate, etc.

Global and sectoral coefficients. The sector aggregates all the important coefficients together for convenience, which includes global coefficients e.g. system gross energy requirement (SYSGER) and fuel requirement for electricity (FEREL) and sectoral coefficients, e.g. water demand intensity by industry and thermal energy demand intensity by dwellings.

Balance of payment. The sector describes UK's balance of payment situation by including the import/export information for resources (i.e. gas, coal, oil and Uranium), goods from industry, service. It also includes information on income, transfers, liabilities, assets and international investment position.

Standard of living. The sector characterises the standard of living in the UK economy, which represents the quality of life in the UK from a material point of view. It divides the gross material for consumption of the economy by the total population and compares the results with the 1990 level to give an index for standard of living (i.e. index material standard of living *per capita*). This sector is probably the one of the most interest for various purposes.

Carbon dioxide. This sector describes the carbon dioxide emissions of the UK economy during the production of wealth, and is one of our major concerns for the environment. Carbon dioxide emissions mainly come from three sources, i.e. oil, coal and gas. The sector calculates the carbon dioxide emissions by working out the demand for the three sources and multiplying them by their different carbon contents. It also includes a model for carbon capture and storage, which is a technique that could be used to reduce coal carbon content.

Appendix B. A list of states and input policies in the UK 4see model

In the UK 4see model, there are 946 variables. Some of the important states and input policies are listed in Table B.1 and Table B.2, respectively and others are omitted here for space reasons.

Table B.1
A list of important states.

Name	Meaning	Unit
CS agriculture	Capital stock of agriculture	VPJ
CO_2 generated index	CO_2 generated index	tonne carbon/y
CS dwellings	Capital stock of dwellings	VPJ
CS energy conservation DWL	Capital stock of energy conversion in dwellings	VPJ
CCT generating capacity	Generating capacity of combined-cycle plant	MW
CS uranium	Capital stock of uranium	tonne
CS waste nuc	Capital stock of uranium waste	VPJ
Hydro generating capacity	Generating capacity of hydropower	MW
Nuclear generating capacity	Generating capacity of nuclear power station	MW
Offshore wind generating capacity	Generating capacity of offshore wind power	MW
Onshore wind generating capacity	Generating capacity of onshore wind power	MW
PV generating capacity	Generating capacity of solar power PV	MW
Tidal stream generating capacity	Generating capacity of tidal power	MW
Wave generating capacity	Generating capacity of ocean wave	MW
Bio-elec generating capacity	Generating capacity of bio-electricity	MW
Solar desert generating capacity	Generating capacity of solar power in desert	MW
Thermal generating capacity	Generating capacity of thermal power station	MW
CS electricity distribution	Capital stock of electricity distribution	VPJ
JOB industry	Employees in industry	capita
JOB service	Employees in service	capita
JOB energy	Employees in energy	capita
CS energy conservation IND	Capital stock of energy conversion in industry	VPJ
CS industry	Capital stock of industry	VPJ
CS gas extraction	Capital stock of gas extraction	VPJ
CS coal extraction	Capital stock of coal extraction	VPJ
CS oil extraction	Capital stock of oil extraction	VPJ
CS gas extraction	Capital stock of gas extraction	VPJ
Coal stocks	Stock of coal	PJ
Oil stocks	Stock of oil	PJ
Gas stocks	Stock of gas	PJ

(*continued on next page*)

Table B.1 (*continued*)

Name	Meaning	Unit
CS energy conservation SER	Capital stock of energy conversion in service	VPJ
CS services	Capital stock of service	VPJ
Gross material standard of living	Gross material for consumption	VPJ
CS electrical vehicle	Capital stock of electrical vehicles	VPJ
CS water	Capital stock of water	VPJ
CS water distribution	Capital stock of water distribution	VPJ

Table B.2
A list of input policies.

Name	Meaning	Unit
Coal fraction in TED AGR	Fraction of thermal energy demand met by coal in agricultural	N/A
Gas fraction in TED AGR	Fraction of thermal energy demand met by gas in agricultural	N/A
Land dedicated to biofuels per biomass	Land dedicated to biofuels per biomass	1000 ha
Proportion of CO_2 to be captured	Proportion of CO_2 to be captured by carbon capture and storage	N/A
Biomass heating substitution coeff DWL	Fraction of thermal energy demand met by biomass heating in dwellings	N/A
HP substitution coeff DWL	Fraction of thermal energy demand met by heat pump in dwellings	N/A
Services to dwellings ratio	Services to dwellings ratio in rate of capital stock formation	N/A
Solar water substitution coeff DWL	Fraction of thermal energy demand met by solar water in dwellings	N/A
Coal fraction in TED DWL	Fraction of thermal energy demand met by coal in dwellings	N/A
Electricity heating substitution coeff DWL	Fraction of thermal energy demand met by electricity heating in dwellings	N/A
Gas fraction in TED DWL	Fraction of thermal energy demand met by gas in dwellings	N/A
Investment in energy efficiency ratio DWL	Ratio of investment into energy efficiency in dwelling	N/A
km between UK and solar plants	Distance between UK and solar plants	KM
Coal fraction TED TH	Fraction of thermal energy demand met by coal in thermal plant	N/A
Coal fraction TED CCT	Fraction of thermal energy demand met by coal in combined-cycle plant	N/A
Fraction PV	Fraction of electricity produced by solar power PV	N/A
Fraction S nuclear	Fraction of electricity produced by nuclear power	N/A
Fraction S offshore wind	Fraction of electricity produced by offshore wind power	N/A
Fraction S CCT1	Fraction of electricity produced by combined-cycle plant	N/A
Fraction S tidal stream	Fraction of electricity produced by tidal power	N/A
Fraction S hydro	Fraction of electricity produced by hydropower	N/A
Fraction S renewables	Total fraction of electricity produced by renewable source	N/A
Fraction bio-elec	Fraction of electricity produced by bio-electricity	N/A
Fraction solar desert	Fraction of electricity produced by solar power in the desert	N/A
Fraction wave	Fraction of electricity produced by solar ocean wave	N/A
Gas fraction TED CCT	Fraction of thermal energy demand met by gas in combined-cycle plant	N/A
Gas fraction TED TH	Fraction of thermal energy demand met by gas in thermal plant	N/A
Interconnector load factor	Interconnector load factor	N/A
Nuclear policy	Rate of building nuclear power station	MW/year
rd cct	Rate of decommissioning combined-cycle plants	MW/year
rdnuc	Rate of decommissioning nuclear power plants	MW/year
rdth	Rate of decommissioning thermal power plants	MW/year
HP substitution coeff IND	Fraction of thermal energy demand met by heat pump in industry	N/A
Industrial decay	Industrial decay	%
RF IND 1	Rate of input of re-investment in industry	VPJ/y
Coal fraction TED IND	Fraction of thermal energy demand met by coal in industry	N/A
Current account less services and fuel in pound	Current account less services and fuel in pound	Pound
Gas fraction TED IND	Fraction of thermal energy demand met by coal in industry	N/A
Investment in energy efficiency ratio IND	Ratio of investment into energy efficiency in industry	N/A
Coal fraction TED mining	Fraction of thermal energy demand met by coal in mining	N/A
Gas fraction TED mining	Fraction of thermal energy demand met by coal in mining	N/A
Self sufficiency in coal production	Self sufficiency in coal production	N/A
Self sufficiency in gas production	Self sufficiency in gas production	N/A
Self sufficiency in oil production	Self sufficiency in oil production	N/A
Penetration of heat pump in SER heating	Fraction of penetration of heat pump in service heating	N/A
Coal fraction TED SER	Fraction of thermal energy demand met by coal in service	N/A
Gas fraction TED SER	Fraction of thermal energy demand met by gas in service	N/A
Investment in energy efficiency ratio SER	Ratio of investment into energy efficiency in service	N/A
Consumption modulation	Consumption modulation	N/A
Carpool	Average passengers per car	N/A
PASS fraction air	Fraction of passengers that travel by air	N/A
PASS fraction water	Fraction of passengers that travel by water	N/A
PASS fraction rail	Fraction of passengers that travel by rail	N/A
Fraction car to pedestrian	Fraction of passengers by walk	N/A
Fraction freight air	Fraction of freight by air	N/A
Fraction freight rail	Fraction of freight by rail	N/A
Fraction freight water	Fraction of freight by water	N/A
Fraction of electrical vehicles	Fraction of electrical vehicles	N/A
Fuel efficiency coefficient	Fuel use efficiency coefficient	N/A
Oil price modulation	Oil price modulation coefficient	N/A
Water efficiency coeff	Water use efficiency coefficient	N/A

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