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Vulnerability of currency pegs: evidence from Brazil

Working paper

Vulnerability of currency pegs: evidence from Brazil*

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March 2008

Abstract

This paper analyses predictions of a simple model of currency crises in which the peg will be abandoned when the currency overvaluation hits a certain threshold, unknown to the agents. Due to learning about the threshold, some features usually observed in the data and identified with models with multiple equilibria arise in the model. But the model yields distinctive predictions about the behaviour of the probability and the expected magnitude of a currency devaluation. The paper identifies the probability and expected magnitude of a devaluation of Brazilian *Real* in the period leading up to the end of the Brazilian pegged exchange rate regime, using data on exchange rate options. The empirical results are consistent with model predictions.

Keywords: Currency crises, exchange rate, options, probability of devaluation, devaluation size

Jel Classification: F3

1 Introduction

Some puzzling features about currency crises are the inconsinstent and volatile reactions of markets to changes in economic fundamentals, and the alternation between times when currency pegs appear vulnerable to attacks and times when they don't. These features have frequently been taken as signs of multiple equilibria,[1] and some theoretical models in the literature show that this may indeed be so. Those models propose an alternation between periods with a unique equilibrium (when the peg is "safe") and periods with multiple equilibria (when the peg is "vulnerable") as an explanation for the puzzle.[2]

*This paper is partly based on Chapter 2 of my PhD dissertation at Yale University. I am very grateful to Stephen Morris, my advisor, for his guidance. I also thank Fernando Broner, Francesco Caselli, Jaume Ventura and several seminar participants for their comments, BM&F for the data and Nathan Foley-Fisher for superb research assistance financed by STICERD.

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[1] In the words of Angeletos and Werning (2006), "crises can be described as times of high non-fundamental volatility: they involve large and abrupt changes in outcomes, but often lack obvious comparable changes in fundamentals. Many attribute an important role to more or less arbitrary shifts in 'market sentiments' or 'animal spirits', and models with multiple equilibria formalize these ideas".

[2] See Obstfeld (1986, 1996), Jeanne (1997), Chamley (2003), Chari and Kehoe (2003) and Angeletos, Hellwig and Pavan (2007).

Moreover, regressions of the estimates of expectations about currency crises on macroeconomic fundamentals typically fail to find a clear link between those macroeconomic variables and crises.[3] Such failure is often taken as empirical support for multiple-equilibrium models.[4] However, this empirical support is subject to a critique: there may be a fundamental latent variable that determines the outcomes and is missing from the study.[5] The problem is especially complicated by the poor explanatory power of macroeconomic variables for the exchange rate: the weak connection between macroeconomic fundamentals and crises may only be reflecting the exchange rate disconnect puzzle.[6]

In this paper, I examine data on expectations about the Brazilian peg in the period leading up to the currency devaluation of January 1999. Large swings in the probability of a devaluation are present, and periods of higher risk coincide with crises in Asia and Russia. As in the previous studies discussed above, a regression of the risk of a devaluation on Brazilian macro variables would yield nothing whatsoever (Campa et al, 2002) and might be taken as support for multiple equilibria.[7]

But I offer an alternative explanation, a simple model with a unique equilibrium in which the peg is abandoned when the currency overvaluation crosses a threshold. Two latent variables are essential to the model: currency overvaluation and the threshold that leads the government to abandon the peg. By making reasonable and parsimonious assumptions on how the market learns about the threshold, I obtain implications about the probability and expected magnitude of a currency devaluation — which can be empirically tested and are not obtained in multiple-equilibrium models. The implications are consistent with empirical estimates prior to the 1999 Brazilian currency crisis.

I begin with the empirical assessment of expectations. I identify the probability and expected magnitude of a devaluation of the Brazilian *Real* from January 1997 to January

[3] Rose and Svensson (1994) state that "it is difficult to find economically meaningful relationships between realignment expectations and macroeconomic variables". Campa and Chang (1998) and Campa, Chang and Refalo (2002) report that "macroeconomic variables are largely unable to explain intertemporal movements in realignment risk".

[4] For example, the herding model of Chari and Kehoe (2003) aims to explain the "random component of international capital flows", stating that "even after the data on historical crises are conditioned on an extensive list of macroeconomic fundamentals, a sizable nonfundamental, or random, component to the crises remains".

[5] See discussion in Jeanne (1999).

[6] Obstfeld and Rogoff (2000) define the exchange rate disconnect puzzle as "the exceedingly weak relationship (except perhaps in the long run) between the exchange rate and virtually any macroeconomic aggregates". From there, it is fair to conjecture that macroeconomic variables are also loosely connected to the exchange rate which would prevail if the government were to abandon the exchange rate peg at a given moment. But that is the key latent variable in currency crises, because it is (i) the main determinant of the profits/losses agents will make if they decide to attack the currency and (ii) a key determinant of the cost to the government of maintaining the peg.

[7] Paul Krugman summarised this view: "I cannot see any way to make sense of the contagion of 1997-8 without supposing the existence of multiple equilibria, with countries vulnerable to self-validating collapses in confidence, collapses that could be set off by events in faraway economies that somehow served as a trigger for self-fulfilling pessimism." Krugman (1999), page 35. See also the critique in Garber (1999).

1999, when the peg was abandoned, using data on exchange rate options to estimate an asset pricing model. Among emerging economies that suffered currency crises, Brazil has especially good data on options.[8]

Options provide information about the probability density of the exchange rate at different points, so it is possible to disentangle the "thickness of the tail of the distribution" (probability of a devaluation) from the "distance from the tail to the centre" (the expected magnitude of a devaluation). The following example illustrates the identification: suppose the price of an asset tomorrow will be 1 with probability $1 - p$ and 3 with probability p. In a risk-neutral world, a call option with strike price 1 costs $2p$, a call option with strike price 2 costs p. If the probability of a devaluation (p) increases, both options get more expensive but the ratio of their prices remains equal to 2. If the magnitude of the devaluation increases from 3 to 4, the option with strike price 1 will cost $3p$, the option with strike price 2 will cost $2p$ — the ratio changes.

Completely different behaviour for the probability and expected magnitude of a devaluation emerge from the empirical work. The probability of a devaluation was volatile and mostly driven by contagion from external crises, as the Asian and Russian crises triggered by far the greatest increases in the probability of a devaluation. The expected magnitude of a devaluation (conditional on its occurrence) was very stable and entirely unaffected by the Russian episode. Interestingly, the moments in which the probability of a crisis increased sharply coincide with sizable depreciations of flexible exchange rates of countries in similar situations (like Mexico), suggesting a link between the probability of a crisis in Brazil and the Brazilian exchange rate that would prevail in a floating regime.

Following the empirical results, I present the model. The peg is abandoned when the currency overvaluation hits a certain threshold, unknown to private agents. I assume that agents learn about the threshold only when it is "tested". The expected magnitude of a devaluation is the expected value of that threshold. The probability of a devaluation is the likelihood that threshold will be reached.

While the currency overvaluation is low, in a "safe zone", shocks to it bring no information about the threshold. Therefore, shocks to the currency overvaluation normally affect only the probability of a devaluation. The expected magnitude of a devaluation may only be affected when the probability is very high — consistently with the data. That is because when a shock to the currency overvaluation puts it at a point where the peg might be abandoned (leading to a very high probability of devaluation) but the peg is

[8] European exchange rate options between the Brazilian Real and the US Dollar were regularly traded at São Paulo Futures Exchange (BM&F).

maintained, agents learn that the threshold for abandoning the peg is higher than the level just reached. That increases the expected magnitude of a devaluation. Then any decrease in the currency overvaluation leads to a relatively lower probability of devaluation.

The perceived risk of a devaluation depends not on the currency overvaluation but on the distance from the expected threshold. So, for the same currency overvaluation, the peg may look safe at times and vulnerable at other times, not because of multiple equilibria but because learning about the threshold can dramatically change the market assessment of risk. Naturally, the effect of the currency overvaluation on the probability of a devaluation is highly non-linear, so shocks to the currency overvaluation generate large probability swings in some times, and small changes in others.

The environment of the model is sufficiently simple that an option can be priced and used to estimate the path of the currency overvaluation. The increases in the Brazilian currency overvaluation, following the crises in Asia and Russia, needed to quantitatively replicate the large swings in the probability of a devaluation present in the data are of a magnitude similar to those experienced by the floating Mexican exchange rate at the same time.

One key distinction between models in this literature is the role of expectations in producing the set of equilibria. In the first generation models (Krugman, 1979, Flood and Garber, 1984), a currency attack is due to bad fundamentals. In contrast, the second generation models (Obstfeld, 1986, 1996) showed how sudden changes in expectations could move an economy to a crisis equilibrium. In the more recent global game models, pioneered by Morris and Shin (1998), expectations are important to agents' decisions, and subtleties in the information structure (whether it is public or private) are important in determining the number and characteristics of equilibria. Therefore, identifying patterns in the behaviour of expectations and connecting them to models is important as a selection criterion, yet relatively little has been done.[9]

Further comments on related empirical and theoretical literature are deferred to the relevant Sections. Section 2 contains the empirical identification of the probability and expected magnitude of a devaluation. The theoretical model is presented in Section 3 and used to obtain the path of Brazilian currency overvaluation in Section 4. Section 5 concludes.

[9] A few contributions to the study of currency crises have attempted to connect theoretical models and empirical data on expectations. Blanco and Garber (1986) generate predictions on expectations about the recurrent devaluation of the Mexican Peso using a variation on the monetary model of Flood and Garber (1984). In one of the few explicit tests for sunspots, Jeanne (1997) performs a likelihood ratio test for the existence of multiple equilibria in the French Franc crisis. He finds some inexplicable shifts between different equilibria.

2 The probability and magnitude of a devaluation

The Brazilian crawling peg was instituted in March 1995 as part of a plan intended to counter the persistent inflation experienced by the economy. Under the peg, the exchange rate could float inside a mini-band that was less than 1% wide. The mini-band was readjusted by about 0.6% each month, distributed over 5 to 7 smaller changes. The objective was to stabilise sustainably the exchange rate, but the sustainability was questionable given the large current account deficits that suggested the Brazilian *Real* was overvalued.

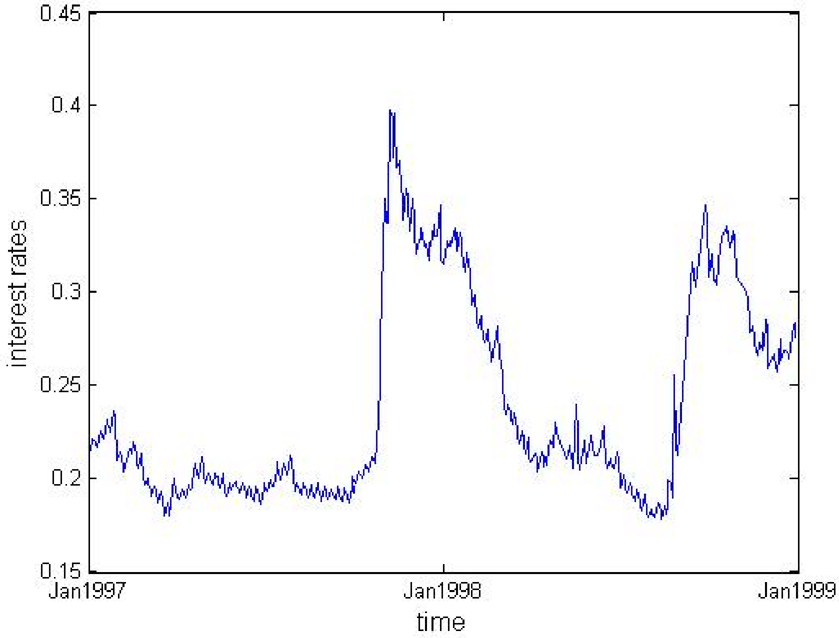

Figure 1: Interest Rates

As Figure 1 shows, interest rates increased by 20 and 15 percentage points due to the Asian (end of 1997) and Russian crises (from August 1998) respectively. This is a testament to the impact these events had on the credibility of the peg — which was finally abandoned in January 1999. The remainder of this Section identifies the probability and expected magnitude of a devaluation of the *Real* from January 1997 to January 1999.

2.1 Empirical identification

The exchange rate risk in a pegged regime depends on the probability that the peg will be abandoned and on the expected size of a consequent currency devaluation. The forward premium is roughly the product of these two variables and may be estimated through some relatively simple calculations. However, observing the forward premium alone does not permit individual identification of the probability of a devaluation and its expected magnitude: a forward premium of 3% a year may refer to an expected devaluation of 30% with probability 10% a year, or an expected devaluation of 5% with probability 60% a year, and so on.

Options are a richer source of data because they provide information about the probability density of the exchange rate at different points, which allows identification of the probability and expected magnitude of a devaluation. Extracting information on the risks of discrete price jumps from data on options is not a novelty. It was the approach taken by Bates (1991) to estimate Merton's (1976) model when testing whether the stock market crash of 1987 had been expected.[10]

However, it is unusual for the empirical literature to identify accurately the probabilities and expected magnitudes of jumps. That is because, in general, the risk of discrete jumps co-exist with the regular disturbances. Subtle changes in options prices correspond to significant changes in the probability distribution of the future value of the asset, so it is difficult to obtain accurate estimates for both the probability and magnitude of a jump.[11] Identification in the case of the Brazilian pegged regime is relatively easier because the volatility of the exchange rate before the crisis was very small, so most of the information contained in option prices relates to the risk of a change in regime. Indeed, the vast majority of options in the sample was worth zero at maturity, which means those options were concerned only with the risk of a large devaluation.

Campa *et al* (2002) estimated the credibility of the Brazilian exchange regime using a non-parametric method. Provided the data were very accurate, their method would obtain the risk neutral densities without any further assumptions. However, in Brazil the lack of liquidity in the market for options leads to substantial noise in the option prices and a purely non-parametric approach cannot be applied. If their methodology were used to construct daily risk-neutral densities, 58% of the days in the sample of this paper would generate probability density functions with some negative values. A better alternative when faced with such noisy data is to use an adequate model to help identify the probability and expected magnitude of a devaluation.

[10] See also Campa and Chang (1996) and Malz (1996) for studies of the credibility of the ERM using data on options.

[11] For example, Bates (1991) estimates the probability and expected size of a jump in the US stock market before the crash of 1987 and finds significant risk of a negative adjustment in parts of his sample. The estimators of the probability and expected magnitude of the jump are very inaccurate but negatively correlated, so estimators of the *product* of them (roughly the price of the risk of a jump) are more precise.

2.2 The asset pricing model

In this paper, a simple asset pricing model is used to estimate the probability and expected magnitude of a devaluation. The model replicates the main features of the Brazilian crawling peg in a simple way.

Denote by S the exchange rate and s its logarithm. Initially, the exchange rate follows a standard Brownian motion with low volatility:[12]

$$ds = \mu_1 dt + \sigma_1 dX$$

In January 1999, the Brazilian exchange rate was allowed to float, and a discrete devaluation took place. Accordingly, in this simple model, the pegged regime may be abandoned at any time. It is convenient to translate the probability of a regime switch into a hazard rate, and the simplest way to do it is to assume that the above process can be interrupted by a Poisson event with hazard rate λ that leads to a discrete jump in the exchange rate and to a new diffusion process that is assumed to last forever.

The simplest way to model the magnitude of the jump is to assume it is a constant (k):

$$\frac{S^{after}}{S^{before}} = (1 + k)$$

The floating regime is described by a Brownian motion with drift and much higher volatility:

$$ds = \mu_2 dt + \sigma_2 dX$$

It is easy to extend the model to incorporate a log-normal jump, and the formula for the price of an option is similar. However, as the standard deviation of the jump plays a role similar to σ_2, it is not possible to get significant estimates for both with the available data for options on Brazilian Real.

The formulae and estimations in this paper consider risk-neutral agents.[13] A call option gives its owner the right to purchase one unit of foreign currency at strike price X. As explained in Appendix A.1, the price of a European call with maturity at time T is:

[12] Such a formulation does not correspond to a mini-band regime, but it serves as a good approximation for the short run path of the Brazilian exchange rate under a maintained peg.

[13] The formula would also be valid for risk averse agents if the risk of a jump were diversifiable and uncorrelated with the market as in Merton (1976). In this case, it would be possible to get an instantaneous zero-beta portfolio and the price of an option would not depend on any individual preferences. In particular, options would have the same value as in a risk-neutral world.

If the risk of a change in the exchange regime is systematic and cannot be diversified, there is no way to get a riskless portfolio and a price independent of agents' risk aversion. Then, using additional assumptions about individuals' preferences and the correlation between their wealth and the underlying assets, it is possible to get richer theoretical models as in Bates (1991, 1996). However, the empirical results would depend on those assumptions. If agents are risk-neutral, observable financial prices are sufficient for the estimations.

$$C^{mod} = e^{-\lambda T} BS\left(Se^{(-q-\lambda k)T}, T; X, r, \sigma_1^2\right) + \int_0^T \lambda e^{-\lambda t} BS\left(Se^{-qT-\lambda kt}(1+k), T; X, r, \frac{(\sigma_1^2 t + \sigma_2^2(T-t))}{T}\right) dt \tag{1}$$

where r is the domestic interest rate, q is the interest rate denominated in foreign currency, X is the strike price, S is the spot exchange rate and $BS(S, T; X, r, \sigma^2)$ denotes the Black and Scholes price of a call option. The first term of Equation 1 represents the value of the option if the peg is not abandoned at time T. The integrand of the second term is the option price given a devaluation at time t multiplied by its probability density function.

The parameter λ reflects the "thickness of the tail of the distribution" and k corresponds to the "distance between the tail and the centre of the distribution". Intuitively, the estimated changes in the expected magnitude of a devaluation are due to changes in the ratio between the jump component of the prices of options with different strike prices. The estimated changes in probability reflect changes in the jump component of the prices of options without changes in their ratios.

To help illustrate identification, consider the following example. For some standard parameter values,[14] options with different λ's and k's, and strike prices equal to 1020 and 1100 are priced as shown in Table 1.

Table 1: Example for identification

λ	k	C(X=1020)	C(X=1100)	ratio
0.10	0.10	0.90	0.28	3.2
0.20	0.10	1.79	0.56	3.2
0.10	0.20	1.86	1.10	1.7

If $\lambda = 0.1$ and $k = 0.1$, an option with strike 1020 is worth 3.2 options with strike 1100. If $k = 0.1$ and $\lambda = 0.2$, both options get roughly twice as expensive because the probability of the option having a positive value at maturity doubles while the probability density of the asset conditional on the occurrence of a jump has not changed. Therefore, the ratio between the option prices remains almost unchanged. On the other hand, if $\lambda = 0.1$ and $k = 0.2$, the higher magnitude of a devaluation translates into a higher expected value of $S - X$ conditional on $S > X$. Crucially, this increase is more pronounced in the case of

[14] $S = 1000$, $T = 0.1$ year, $r = 0.2$/year, $q = 0.1$/year, $\sigma_1 = 0.01$/year and $\sigma_2 = 0.25$/year.

the option with higher strike price and the option-price ratio falls to 1.7. That is the key to identify the probability and expected size of a currency devaluation.

2.3 Data and estimation

The observed price of a call option (C^{obs}) is assumed to be equal to the model price (C^{mod}) plus an error term:

$$C^{obs} = C^{mod}(S, r, q, T; X, k, \lambda, \sigma_1, \sigma_2) + \epsilon \quad (2)$$

where ϵ is a mean-zero error term, independent of the observable variables. The parameters of Equation 2 were estimated by non-linear least squares.

To estimate the parameters of Equation 2, the following data are required: domestic interest rates denominated in domestic and foreign currency; spot exchange rate; and option prices. Interest rate and exchange rate data are available from very liquid markets.[15] Unfortunately, the option market is much less liquid and, since there is no reliable record of the time each option was traded, the price of the last trade for every option must be used.[16] The available data refer to trades realised at potentially different times. Especially in periods when the markets were nervous, this may introduce large measurement error in the dependent variable, as discussed in Appendix B.[17]

The options are European calls, the underlying asset is the US Dollar and the contracts are to be paid in Brazilian *Real*. 75% of the options in the sample were traded less than 45 days before maturity, so the obtained estimates are measures of expectations about the peg in the short run. Options traded too close to maturity (less than 10 days) were discarded, as they contain little information about implicit distributions and their prices are not much greater than the bid-ask spread. In addition, transactions in at least four strike classes with the same maturity were required for each day. Finally, some questionable observations of a few far out-of-the-money option classes were excluded. In the end, there were 3,587 observations in the sample corresponding to 474 days and 25 months. Appendix B provides more details on the data.

λ and k are constants in the model but in the estimations they are allowed to vary over time. This is a potential source of inconsistency, however some Monte Carlo experiments presented in Appendix A.2 show that, for at least some diffusion processes of λ, such a procedure yields reasonable estimates. This is hardly surprising, as prices of European

[15] All data are from contracts traded at São Paulo Futures Exchange (BM&F).

[16] In theory, options were traded at the exchange. In practice, options were traded over the counter and then registered at BM&F.

[17] It is possible to interpret the error term in Equation 2 as measurement error in the dependent variable.

calls do not depend on the particular paths of the hazard rate and magnitude of jump but on the probability distribution of the exchange rate at the maturity date. Indeed, the estimation of different λ's and k's is the standard procedure in the literature (see, for example, Bates (1991, 1996) and Jondeau and Rockinger (2000)). In the empirical work, λ and k are either estimated for each of the 695 sets of options of a certain maturity traded in a given day or constrained to be constant during each of the 25 months.

2.4 Results

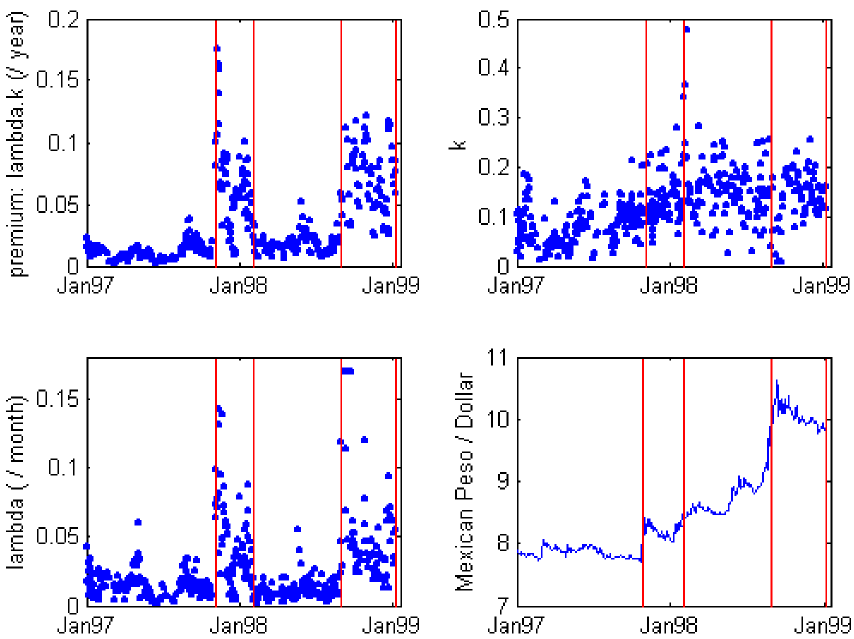

Figure 2: Daily Estimates

Figure 2 shows the results when λ and k are allowed to vary across dates and maturity dates, assuming $\sigma_1 = 0.75\%$ per year and $\sigma_2 = 25.5\%$ per year.[18] Among the 695 $\{\lambda, k\}$ estimated, 442 pairs have a t-statistic higher than 2 for both estimates. Figure 2 shows just the results for those 442 'significant' days. The estimates of λ higher than 0.17 are plotted as if they were equal to 0.17 (five cases yield significant estimates of λ between 0.25 and 0.40).[19] The vertical lines mark the periods in which the 'devaluation premium' is high.

[18] 25.5% is the standard deviation of the observed daily changes in the exchange rate from 1/19/99 to 12/31/99

[19] Nothing substantial changes in the Figures if the lower bound for t-statistics and the censorship limit is altered.

The top left graph shows the devaluation premium, λk. Unsurprisingly, it resembles Figure 1: the two major shocks in the series follow the Asian and the Russian crises.[20] The options allow us to disentangle and determine the relative importance of the two key components of the forward premium. A dramatic increase in the probability of a devaluation follows both crises, and is the predominant cause of the rise in the devaluation premium. The expected magnitude appears lower in 1997 than in 1998, but shows no sign of being affected by the foreign crises.

Figure 3 presents the estimates when λ and k are constrained to be constant within each month, whilst maintaining the assumption that $\sigma_1 = 0.75\%$ per year and $\sigma_2 = 25.5\%$ per year. If there are substantial variations in the probability and expected magnitude during a month, it is not clear how mixing different option dates will impact the estimates. Nonetheless, it is a useful exercise to help understand the daily estimates.

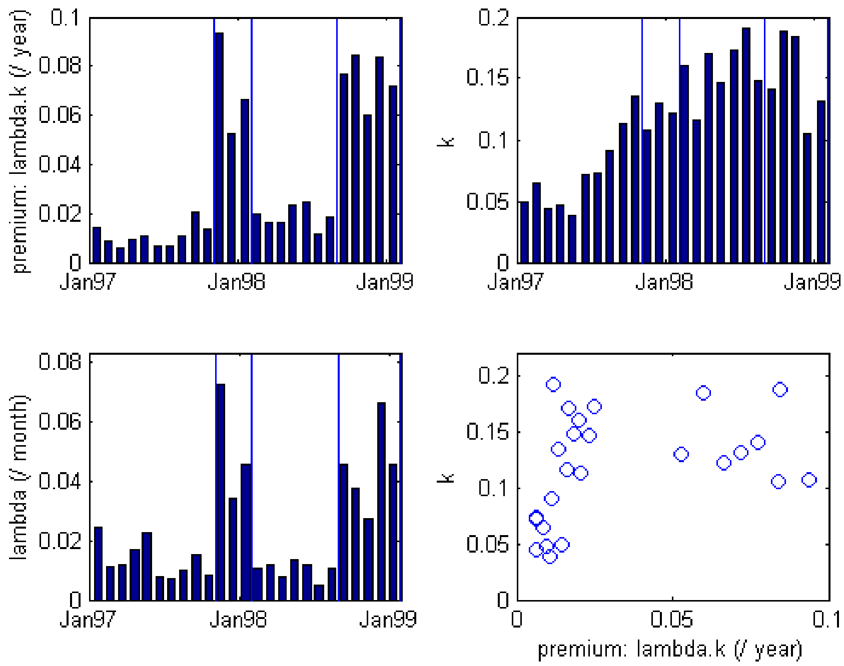

Figure 3: Monthly estimates

Figures 2 and 3 show expectations about the Brazilian pegged regime from January 1997 to January 1999. At the end of October 1997, the Asian crisis strongly affected the credibility of the *Real*. The probability of a devaluation reached its peak in November 1997 but had returned to previous levels by February 1998. It remained low until August,

[20] Actually, the interest rate rise is greater than the increase in the devaluation premium due to the additional increase in the risk of default.

when Russia defaulted on its debt, upon which it sharply rose and remained around 5% per month until January 1999, when the peg was removed. The parameter k increased at some point in 1997 and remained roughly stable around 15% after the Asian crisis until the end of the pegged regime. In 1998, virtually all changes in the devaluation premium were due to movements in the probability of a devaluation; the Russian crisis appears to have had no effect on the expected magnitude.

Both the Asian and Russian crises strongly affected the probability of a devaluation but had little or no effect on its expected magnitude. More generally, fluctuations in the devaluation premium are largely explained by movements in λ alone. The correlation between λ and λk is 92%, while the correlation between k and λk is only 37%.

Table 2 shows the value of estimates and standard errors in the monthly exercise. The lowest pseudo-t-statistic is 2.96 and the average pseudo-t-statistic is 7.6.

Interestingly, as shown in Figure 2, the greatest jumps in the Mexican exchange rate coincided with the largest movements in the probability of a devaluation in Brazil. Like Brazil, Mexico had large current account deficits by that time and few direct links with Russia, Korea or Hong Kong, but its currency was floating. It is reasonable to expect that the Brazilian "shadow exchange rate" and the Mexican floating rate would respond to the Asian and Russian crises in similar ways: had the Brazilian currency been floating, it would have depreciated.[21]

The monthly probability of a devaluation was almost always below 10% and remained around 5% from September 1998 until January 1999. Even as the regime break approached, the estimates of λ did not increase sharply.[22] Indeed, Brazilian interest rates were decreasing (from 2.93% per month in October 1998 to 2.38% per month in December 1998), the government entered into an arrangement with the IMF towards the end of 1998, and some macroeconomic reports were pointing to an increase in the credibility of the currency by December 1998.[23]

The results also show that agents underestimated the size of the jump: while the expected depreciation is never greater than 20%, the observed devaluation was as high as 60%. Some comments on the discrepancy between the expected and the observed devaluation are in Appendix C.

[21] Actually, the crises of 1997-8 negatively affected all the main Latin American synchronically floating currencies. The Chilean *Peso*, despite the good economic performance of Chile, was adversely hit by the Asian crisis. The Colombian *Peso* lost 10% of its value in the month following the Russian default (its average monthly devaluation over the period was 2%).

[22] There are estimates for λ until 01/08/99, 3 business days before the jump. Options get slightly more expensive 1 or 2 days before the devaluation.

[23] For example, the December 1998 economic analysis bulletin of IPEA (the Brazilian institute for research in applied economics) states that '(...) the pressure on the exchange rate got milder, and now a speculative attack is less likely to occur', IPEA (1998, in Portuguese), page 6.

Table 2: Monthly estimates of k and λ

	k		λ (year^{-1})	
Jan-1997	0.0491	(0.0042)	0.2929	(0.0166)
Feb-1997	0.0646	(0.0136)	0.1307	(0.0196)
Mar-1997	0.0442	(0.0060)	0.1378	(0.0099)
Apr-1997	0.0471	(0.0092)	0.2060	(0.0245)
May-1997	0.0389	(0.0055)	0.2702	(0.0229)
Jun-1997	0.0715	(0.0112)	0.0914	(0.0119)
Jul-1997	0.0735	(0.0132)	0.0876	(0.0135)
Ago-1997	0.0904	(0.0239)	0.1212	(0.0324)
Sep-1997	0.1135	(0.0122)	0.1812	(0.0233)
Oct-1997	0.1350	(0.0401)	0.0987	(0.0334)
Nov-1997	0.1076	(0.0131)	0.8690	(0.1252)
Dec-1997	0.1299	(0.0154)	0.4054	(0.0586)
Jan-1998	0.1216	(0.0121)	0.5472	(0.0667)
Feb-1998	0.1606	(0.0205)	0.1247	(0.0193)
Mar-1998	0.1154	(0.0112)	0.1411	(0.0148)
Apr-1998	0.1707	(0.0138)	0.0961	(0.0090)
May-1998	0.1464	(0.0254)	0.1589	(0.0316)
Jun-1998	0.1723	(0.0164)	0.1436	(0.0153)
Jul-1998	0.1913	(0.0280)	0.0612	(0.0097)
Ago-1998	0.1481	(0.0330)	0.1252	(0.0303)
Sep-1998	0.1411	(0.0217)	0.5461	(0.0935)
Oct-1998	0.1877	(0.0216)	0.4509	(0.0600)
Nov-1998	0.1845	(0.0274)	0.3251	(0.0554)
Dec-1998	0.1051	(0.0157)	0.7966	(0.1362)
Jan-1999	0.1319	(0.0149)	0.5431	(0.0733)

Standard deviations in parentheses.

3 A simple model

This section presents a stylised model of currency crises that is consistent with the evidence above. The key ingredients are: (i) the devaluation is triggered when the currency overvaluation crosses a threshold; and (ii) the threshold is unknown, agents are learning about it, and they have the opportunity to learn when the currency overvaluation goes above its previous maximum, as it indicates to the agents that the threshold is higher than they previously thought.

Many models imply that a peg is abandoned when a threshold is crossed. The first generation models of currency crises (Krugman, (1979), Flood and Garber (1984)) predicted that a currency attack would occur when fundamentals crossed a fundamental threshold. Some recent dynamic models of currency crises based on that framework also yield fundamental thresholds for a currency devaluation (Guimaraes (2006), Broner (2007)), although others lead to different implications (e.g., Pastine (2002)).

Obstfeld and Rogoff (1995) defend the idea that the costs and benefits for the government to keep the peg are the key determinants for the fate of the exchange rate regime, as countries usually have enough reserves to sustain a currency peg. If those costs and benefits are not affected by agents' expectations, then the logic of Obstfeld and Rogoff (1995) implies that a peg is abandoned when some economic fundamentals are "bad enough" — e.g., when the currency overvaluation crosses a threshold.

Assuming that the costs and benefits for the government to maintain the peg depend on what agents do, Obstfeld (1986, 1996), among others, developed a multiple-equilibria explanation for the puzzling behaviour of markets with respect to currency crises. Obstfeld (1986, 1996) get a coordination game between the agents and, with complete information, multiple equilibria. But Morris and Shin (1998, 1999) added incomplete information to the coordination game and obtained a unique equilibrium in which a currency crisis occurs if economic fundamentals go beyond a threshold. Adding uncertainty about the government costs and benefits to Morris and Shin (1999) would lead to a model with an uncertain threshold, with implications similar to the model in this paper.

3.1 The model

In Section 2, the probability and expected magnitude of a devaluation are exogenous variables to be estimated. Here, they will be obtained endogenously. The primitives of the model are the paths of the exchange rate and the currency overvaluation, and the distribution of the threshold that triggers a currency devaluation. All of these are observed by the agents. The model impose restrictions on the behaviour of the probability and expected magnitude of a devaluation that may be inconsistent with the analysis presented in Section 2.

The exchange rate process before the peg is abandoned is identical to that of the previous section:

$$ds = \mu_1 dt + \sigma_1 dX_1$$

Currency overvaluation in logs is denoted by θ and follows a similar stochastic process:[24]

$$d\theta = \mu_\theta dt + \sigma_\theta dX_\theta \tag{3}$$

Denoting by ϕ the shadow exchange rate, i.e. the exchange rate if the government decided to abandon the peg:

$$\phi = s + \theta$$

Thus, when the pegged regime is abandoned, the de facto magnitude of the devaluation will equal to θ — the exchange rate jump from s to ϕ.

The peg will be abandoned whenever the currency overvaluation hits a threshold, θ^*, unknown to the agents. An important implication of this assumption is that the expected magnitude of a devaluation conditional on its occurrence is substantially different from the unconditional expectation of θ. The devaluation occurs when θ crosses θ^* and conditioning the expected value of θ on that information makes a big difference.

Denote the maximum value that θ has achieved up to time t by $\theta^{\min}$. We know that $\theta^* > \theta^{\min}$, otherwise the peg would have been abandoned before. Agents have common uncertainty about θ^*, $g(\theta^*|\theta^{\min})$.

The probability of a devaluation before time τ is then:

$$prob = \int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^*).d\theta^*$$

where $preach(\theta^*)$ is the probability that θ^* will be reached before time τ. As long as $\theta < \theta^{\min}$, it can also be written as:

$$\begin{aligned} prob &= \int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^* \\ &= preach(\theta^{\min}).\int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^* \end{aligned}$$

where $preach(\theta^{\min})$ is the probability that $\theta^{\min}$ will be reached before time τ and $preach(\theta^*|\theta^{\min})$ is the probability that θ^* will be reached before τ conditional on $\theta^{\min}$

[24] Assuming some slow mean reversion would not qualitatively change the results.

being reached before τ. The second equality arises because $preach(\theta^{\min})$ is independent of θ^*.

Provided θ remains below $\theta^{\min}$:

$$\left.\frac{\partial prob}{\partial \theta}\right|_{\theta<\theta^{\min}} = \frac{\partial preach(\theta^{\min})}{\partial \theta} \int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^* > 0$$

As long as $\theta < \theta^{\min}$, increases in θ drive the economy closer to the region where the devaluation is possible, increasing the probability of a devaluation.

The expected magnitude of a devaluation conditional on its occurrence up to time τ is:

$$\begin{aligned} E(magn_t) &= \frac{\int_{\theta^{\min}}^{\infty} \theta^*.g(\theta^*|\theta^{\min}).preach(\theta^*).d\theta^*}{\int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^*).d\theta^*} \\ &= \frac{\int_{\theta^{\min}}^{\infty} \theta^*.g(\theta^*|\theta^{\min}).preach(\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^*}{\int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^*} \\ &= \frac{\int_{\theta^{\min}}^{\infty} \theta^*.g(\theta^*|\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^*}{\int_{\theta^{\min}}^{\infty} g(\theta^*|\theta^{\min}).preach(\theta^*|\theta^{\min}).d\theta^*} \end{aligned}$$

which is independent of θ.

Provided θ remains below $\theta^{\min}$, movements in θ do not affect the expected magnitude of a devaluation:

$$\left.\frac{\partial Emagn}{\partial \theta}\right|_{\theta<\theta^{\min}} = 0$$

While $\theta < \theta^{\min}$, increases in θ provide no extra information about θ^*, causing therefore no impact on the expected magnitude of a devaluation. The unconditional expected currency overvaluation is different from the expected magnitude of a devaluation conditional on its occurrence.

On the other hand, when θ is at $\theta^{\min}$, upward movements in θ increase $\theta^{\min}$ and the expected magnitude of a devaluation conditional on its occurrence is thus affected. It can be shown, and is intuitive, that the expected magnitude of a devaluation is increasing in $\theta^{\min}$. When $\theta = \theta^{\min}$, the probability of a devaluation is at its highest since the last time θ reached $\theta^{\min}$ and the expected magnitude increases with any shock to θ.[25]

Thus the model predicts that the increases in Brazilian currency overvaluation should normally affect only the probability of a devaluation, but will affect its expected magnitude when the probability of a devaluation reaches new heights. These predictions are borne

[25] The possibility of discrete jumps in the currency overvaluation would weaken this result: the expected magnitude of a devaluation would then be somewhat affected by movements in θ. But if the frequency of the jumps were small, the effect would not be large. The empirical results suggest that the possibility of jumps in the currency overvaluation might be important, but for simplicity I abstract from them in the analysis.

out by the movements of the probability and expected magnitude in 1998 and are close to the pattern observed in 1997: virtually all movement occurs in the probability, except for the increase in the magnitude of a devaluation when the Asian crisis erupted.

Both the empirical results and the model predictions contrast with the predictions of an extreme sunspot model, where maintaining the peg is entirely independent of currency overvaluation. In Appendix D I show, in that case, there is a disconnection between θ and the probability of a devaluation that implies a connection between θ and the expected magnitude of a devaluation. Those predictions are at odds with the empirical results.

In the model, agents do not have any information about θ^* besides $\theta^{\min}$ and its distribution, so they just update their beliefs about θ^* when θ reaches $\theta^{\min}$. One could think that, in reality, agents could access other sources of information about θ^*. However, it is actually difficult for the government to communicate its commitment to the peg because the incentive to assert the peg will only be abandoned in dramatic circunstances (abnormally high values of θ) is always present. That is because the devaluation premium (the product of probability and expected magnitude) depends negatively on the expected value of θ^*, which can be interpreted as a higher perceived commitment to the peg. A higher $\theta^{\min}$ (or a higher expected θ^*) leads to a lower devaluation premium because it lowers the probability of a devaluation and increases its expected magnitude, but the effect on the probability dominates, due to the strong non-linear dependance of the probability of a devaluation on the distance between θ_t and $\theta^{\min}$.[26]

4 Empirical estimation of the model

The environment of the model is simple enough to price an option. Using the data on option prices, we can infer the path of θ. This exercise serves two purposes: (i) to check whether the simple model can generate values for the probability and expected magnitude of the devaluation consistent with the data, under reasonable assumptions on parameters; and (ii) to examine the path of the shadow exchange rate implied by the model and the option data.

Some simplifying assumptions are necessary to facilitate the estimation process. I

[26] Going a bit outside the model, the negative impact of $\theta^{\min}$ on the devaluation premium explains why governments are keen to declare they will never leave the peg unless the world ends – in other words, θ^* is close to infinity.

assume that dX_1 and dX_θ are independent, so:[27]

$$d\phi = (\mu_\theta + \mu_1)dt + \sqrt{\sigma_\theta^2 + \sigma_1^2} dX_\phi$$

and the distribution of θ^*, $g(\theta^*|\theta^{\min})$, is exponential

$$g(\theta^*|\theta^{\min}) = \delta e^{-\delta(\theta^* - \theta^{\min})}$$

Given the exponential distribution, the probability of a devaluation depends only on $\theta - \theta^{\min}$. When $\theta = \theta^{\min}$, the probability is at its maximum, independent of $\theta^{\min}$.

Denote by θ_t and $\theta_t^{\min}$ the values of θ and $\theta^{\min}$ at date t.

In a risk-neutral world, the price of the option with strike price X and maturity at date τ is equal to:

$$c = \int_{\theta^{\min}}^{\infty} \delta e^{-\delta\left(\theta^* - \theta^{\min}\right)} c_2(\theta^*) d\theta^*$$

where $c_2(\theta^*)$ is the price of a call option conditional on a given value of θ^*.

$c_2(\theta^*)$ is given by:

$$c_2(\theta^*) = \int_t^\tau c_1(\theta^*, T)h(\theta^*, T)dT + \left(1 - \int_t^\tau h(\theta^*, T)dT\right) BS\left(S^{-q\tau - prob.E(magn)}, \tau; X, r, \sigma_1^2\right)$$

where $c_1(\theta^*, T)$ is the price of a call option conditional on θ^* being reached at time T, $h(\theta^*, T)$ is the probability density that θ^* will be reached at time T, BS is the Black-Scholes price of an option, *prob* and $E(magn)$ are the probability and expected magnitude of a devaluation, given by the formulae presented at the last section.

Lastly, $c_1(\theta^*, T)$ is worth:

$$c_1(\theta^*, T) = e^{-r(\tau - t)} \int_X^\infty \left(e^{\theta_\tau} - X\right) f(\theta_\tau | \theta_T = \theta^*) d\theta_\tau$$

where $f(\theta_\tau|\theta_T = \theta^*)$ is the probability density of θ_τ conditional on $\theta_T = \theta^*$.

The densities f and h depend on the diffusion process of θ. Thus, the option price can be calculated as a function of θ_t, $\theta_t^{\min}$, the other parameters $(\mu_1, \sigma_1^2, \mu_\theta, \sigma_\theta^2, \delta)$ and observables (S, X, r, q, τ).

The parameters $(\mu_1, \sigma_1^2, \mu_\theta, \sigma_\theta^2, \delta)$ are calibrated. The values of μ_1 and σ_1 are taken from the crawling mini-band regime, in the period of January-1997 to January-1999. I set $\mu_\theta = 0$ for simplicity. I choose σ_θ and δ so that the currency overvaluation in the first half of 1997 is small but usually positive. As long as that restriction is respected, different choices of parameters yield very similar results. I present results using $\sigma_\theta =$

[27] This assumption is made for simplicity, the process of s is only important to price some few options with lower strike price, so that should not make a significant difference in the estimation of θ. But it is worth mentioning that in the end of 1997, while the risk of a devaluation was sky rocketing, s stayed at the bottom of the "mini-band" (perhaps surprisingly).

10%/year and $\delta = 6$ (which implies that the $Pr\left(\theta^* \in [\theta^{\min}, 1.1 \times \theta^{\min}]\right) = 45\%$ and $Pr\left(\theta^* \in [\theta^{\min}, 1.2 \times \theta^{\min}]\right) = 70\%$).

I estimate a value of θ_t for every day and $\theta_0^{\min}$. Then, $\theta_t^{\min} = \max\left\{\theta_{t-1}^{\min}, \theta_t\right\}$. The values of θ_t are estimated sequentially, for the sake of simplicity. The results are shown in Figure 4.

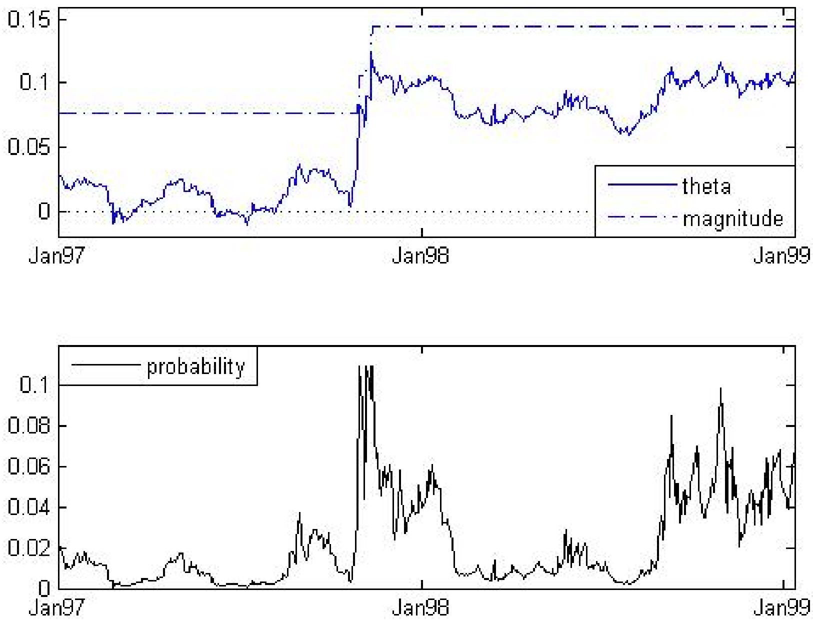

Figure 4: Path of θ, probability and expected magnitude

The top graph shows θ_t and the expected magnitude of a devaluation conditional on its occurrence in a month. The latter is equal to $\theta^{\min}$ plus a constant that depends on δ, the parameters of the stochastic processes, and the time span. The value of θ_t at the end of 1998 is close to the peak reached at the end of 1997. Consistent with the results obtained in Section 2, the expected magnitude of a devaluation conditional on its occurrence increases following the Asian crises and stays constant from then on. The bottom graph shows that the path of the probability of a devaluation is very similar to that obtained in Section 2. The model thus generates sensible values for the probability and expected magnitude of a devaluation.

These results suggest the following story: in 1997, fluctuations in θ had small or moderate impacts on the probability of a devaluation until the end of October 1997, when a large shock to the currency overvaluation occurred: θ increased by around 10% — a similar devaluation was experienced by the Mexican Peso. Then, the current value of θ_t had surpassed $\theta_{t-1}^{\min}$ and reached the region where an immediate devaluation was possible. The probability of a change in regime was very high, agents were uncertain whether the government would let the currency float or not. The government did not, despite the very high interest rates resulting from the high risk of a devaluation. Agents learnt, $\theta^{\min}$ increased, so the expected θ^* and hence the expected magnitude of a devaluation became higher than before the crisis.[28]

By the end of February 1998, the probability of a devaluation was back to low levels and fell even further during the year — to a trough where it was close to the lowest levels observed in 1997. Currency overvaluation, θ, never receded as much, though it decreased somewhat and, although much higher than before the crisis, was far enough from $\theta^{\min}$ to generate a low probability of devaluation since agents knew that it would take a relatively higher θ to make the government abandon the peg. A similar currency overvaluation in 1997 would have corresponded not only to a higher probability of devaluation but also to a substantially higher devaluation premium, because of the strong non-linear effects of $(\theta^{\min} - \theta)$ on the probability of a devaluation, as explained above.

The arrival of the first signs of trouble from Russia led to a 5% increase in the currency overvaluation, driving θ very close to $\theta^{\min}$ and triggering a massive increase in the probability of a devaluation. The expected magnitude of a devaluation did not change — because θ did not go beyond $\theta^{\min}$ — but any sharp movement in θ might have put it beyond $\theta^{\min}$ and, perhaps, triggered a devaluation.

In Angeletos et al (2007), the change in the mood of the market is due to changes from a unique equilibrium to multiple equilibria. Here, instead, the market learns that the government is willing to sustain the peg at a certain level, which decreases the likelihood of a devaluation. It is a model with a unique prediction so changes in expectations require changes in the currency overvaluation, but those can be small (a fall of 3-4% in the currency overvaluation takes the probability from the highest level to a very low level).

5 Concluding Remarks

The Asian crisis in 1997 and the Russian crisis in 1998 have shaken financial markets around the world. This paper shows that they affected the risk of a devaluation in Brazil mostly through the probability of a devaluation, rather than the expected magnitude. It offers the following explanation: by driving the currency overvaluation to a region

[28] Similar learning effects after a "fire test" are present in other papers (e.g., Chari and Kehoe, 2003).

where agents were unsure about whether the peg would be kept, those shocks increased the probability of a devaluation. The defence of the *Real* by the Brazilian government following the Asian crisis convinced the agents that the threshold for abandoning the peg was higher, which increased the expected magnitude of a devaluation, conditional on its occurrence, but allowed for a decrease in the probability when the economic climate got slightly better. The subsequent negative shock in the second half of 1998 drove the exchange rate close to the threshold again, and once more the probability of a devaluation soared. Moderate shifts of Brazilian currency overvaluation due to the Asian and Russian crises can account for these massive increases in the probability of a devaluation.

Some theoretical mechanisms that can lead to currency crises are now well understood, but the empirical literature hasn't caught the pace of sophistication of the recent theoretical literature. Progress in distinguishing between models with multiple equilibria and models driven by latent variables is crucial for our understanding of crises. That requires models, however simple, that can be directly connected to data and yield testable predictions.

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A The asset pricing model

A.1 Formula for the price of a call

This section provides an intuitive explanation of Equation 1:

$$C^{mod} = e^{-\lambda T} BS\left(Se^{(-q-\lambda k)T}, T; X, r, \sigma_1^2\right) + \int_0^T \lambda e^{-\lambda t} BS\left(Se^{-qT-\lambda kt}(1+k), T; X, r, \frac{(\sigma_1^2 t + \sigma_2^2(T-t))}{T}\right) dt$$

where $BS\left(S, T; X, r, \sigma^2\right)$ denotes the Black-Scholes price of a European call option if the underlying asset follows a Brownian motion with drift $\left(\frac{dS}{S} = \mu.dt + \sigma.dX\right)$, r is the interest rate, X is the strike price, S is the spot exchange rate and T is the time to maturity.

The price of an exchange rate option with the above characteristics is:

$$BS\left(S.e^{-qT}, T; X, r, \sigma^2\right) \tag{4}$$

where q is the interest rate denominated in foreign currency.

The first term of Equation 1 is the value of the option if there is no devaluation until time T. With probability $e^{-\lambda T}$, the value of a call option will be given by Equation 4 with the spot exchange rate S multiplied by $e^{-\lambda kT}$. This latter term accounts for the devaluation premium:

$$BS\left(Se^{(-q-\lambda k)T}, T; X, r, \sigma_1^2\right)$$

The second term of Equation 1 integrates the products of the probability of a devaluation at time t $(\lambda e^{-\lambda t})$, and the value of a call option at that time which is given by:

$$BS\left(Se^{-qT-\lambda kt}(1+k), T; X, r, \frac{(\sigma_1^2 t + \sigma_2^2(T-t))}{T}\right)$$

The exchange rate in this case is distributed as if it followed a regular Brownian motion starting from $Se^{-qT-\lambda kt}(1+k)$ and with volatility $\frac{\left(\sigma_1^2 t+\sigma_2^2(T-t)\right)}{T}$. The spot exchange rate needs to be corrected by the jump (multiplied by $(1+k)$) and by the devaluation premium up to time t (multiplied by $e^{-\lambda kt}$). The volatility is just a weighted average of the variances in the 2 regimes.

A.2 Theoretical option price under variable λ

Although the model assumes a fixed hazard rate λ, my estimations do not impose this constraint. This Appendix answers the question of how different theoretical option prices would be if λ were allowed to vary.

Since the answer may depend on the underlying process for λ, I used Monte Carlo simulations to approximate option prices for a particular case, when the hazard rate λ behaves according to the following equation:

$$d\log(\lambda) = \sigma_\lambda . dX$$

The table below shows the prices of options with 0.2 year to maturity for different σ_λ's but same expected λ after 0.1 year [29].

$E(\lambda \mid t = 0.10)$	$\sigma_\lambda = 0$	$\sigma_\lambda = 0.5$
0.15	3.499 (0.008)	3.487 (0.010)
0.20	4.602 (0.009)	4.600 (0.008)
0.25	5.651 (0.009)	5.664 (0.009)

The lack of sensitivity to σ_λ is not due to little volatility. If $\sigma_\lambda = 0.5$ and $\lambda(t = 0) = 0.1975$, $E(\lambda|t = 0.10) = 0.20$ but the 95% confidence interval for $\lambda(t = 0.20)$ is wide: $[0.127, 0.306]$ — λ varies significantly in the 0.2-year period.

The results show that, at least for this particular case, changes in the standard deviation of the diffusion process for λ have no impact on option prices. This example seems to confirm the intuition that estimates of λ obtained in this work should be close to what agents perceived as an average hazard rate.

B The Data

Table 3 shows the data for the last week in October 1997 and the first week in November 1997. All information refers to contracts with maturity on the last day of November. The data contains 695 rows like the 10 presented in Table 3.

The first column shows the trading day. Columns 2 to 7 show the prices of options with strike price shown in the first line of the table: for example, on 10/27, options that give its holder the right to buy US\$1000 for BR\$1115 were traded at price BR\$2.25. F denotes the future exchange rate: on 10/27, US\$1000 on the last day of November were

[29] Some simplifications were made to reduce computations cost of this exercise, so all prices are probably slightly overestimated. The parameters used were: $\sigma_1 = .01, \sigma_2 = .10; k = .20, S = 1000, X = 1100, \tau = .20, r = .22, q = .11$.

Table 3: A subset of the data

Day	X									
	1113	1115	1120	1150	1170	1200	F	S	τ	DI
10/27		2.25	2.20	1.40	1.50	1.30	1115.8	1102.7	32	97958
10/28		3.50	3.50	1.40	2.00	2.10	1116.9	1106.4	31	97841
10/29		3.00	4.50		2.00	2.20	1118.2	1102.4	30	97746
10/30	12.00	11.00	12.50	5.00	5.00	5.00	1125.8	1106.3	29	97473
10/31	11.00	7.00	11.00	4.00	4.50	3.30	1124.9	1103.1	28	97056
11/03		7.00	8.00	5.51	4.50	3.00	1121.6	1103.0	25	97123
11/04	5.50	5.50	6.40	3.51	3.49	2.00	1116.9	1104.1	24	97338
11/05	4.30	3.50	3.25	2.60	1.75	2.00	1118.1	1104.1	23	97402
11/06	8.00	7.00	6.00	4.30	2.70	3.00	1118.4	1106.9	22	97541
11/07	13.70	10.50	11.00	7.50	8.00	8.00	1123.5	1108.2	21	97392

priced at BR$1115.80. S is the spot exchange rate: on 10/27, US$1000 cost BR$1102.70. τ in this table is the number of days until maturity and DI is an interest rate derivative contract: on 10/27, BR$100,000 on the first day of December were worth BR$97,958. The information on future contracts of interest rate and exchange rate allows us to calculate interest rates denominated in domestic and foreign currency.

The peg was not abandoned in November 1997, so on the maturity date of those options, the exchange rate was BR$1109 for US$1000 and all options shown in Table 3 were worth 0.

Option prices show huge daily variations, which suggests that large intra-day fluctuations may also occur. As the data refer to options traded in potentially different times, this may lead to severe measurement error in the dependent variable of Equation 2. In an extreme example, on 10/31/97, the price of a call with strike 1115 (Reais/US$1000) and maturity 12/01/97 is 7.00 and a call with strike 1120 and same maturity costs 11.00. The sum of the absolute measurement error is therefore greater than 4.00! There are plenty of examples like this, though less dramatic.

The price of a call option must be (weakly) convex as function of the strike price, otherwise there are arbitrage opportunities (see Campa *et al* (2002)). Violation of such properties is evidence of noise in the data on options, probably due to trades being realised at different times, and generate probability density functions with negative values if the methodology of Campa *et al* (2002)) is applied. In our sample, convexity is violated in 58% of the 695 days in our sample, and two thirds of those 695 days consist of only 4 or 5 points.

Table 4: Spot exchange rate in January-99

Day	Exchange Rate	Jump
1/11/99	1.211	
1/12/99	1.211	
1/13/99	1.319	8.9%
1/14/99	1.319	8.9%
1/15/99	1.466	21.0%
1/18/99	1.538	27.0%
1/19/99	1.558	28.6%
1/20/99	1.574	29.9%
1/21/99	1.660	37.0%
1/22/99	1.705	40.7%
1/25/99	1.761	45.3%
1/26/99	1.877	54.9%
1/27/99	1.889	55.9%
1/28/99	1.921	58.5%
1/29/99	1.983	63.7%

C Expected and observed jump size

On 02/01/99, the first maturity day of options after the devaluation, the exchange rate was at 1.983 R$/US$, 63.7% higher than 3 weeks before. According to our estimates, agents were expecting a substantially smaller jump.[30] Actually, this belief is confirmed by the exchange rate path directly after the devaluation. Table 4 shows the spot exchange rates in January 1999 — future rates display the same pattern. On January 13^{th}, the end of the pegged regime was announced and the Central Bank tried to impose a new upper bound of fluctuation, at R$1.32/US$.[31] Two days later, the new-born band was abandoned and the exchange rate started to float. On the 15^{th}, even though Brazilian *Real* had lost this first battle, the US Dollar was still just 21% more expensive than before the jump. The spot rate would go up gradually and increase every single day until the end of the month.

The behaviour of the exchange rate in the very short run after the devaluation is interesting: there seems to be a clear and strong upward trend for the price of the US Dollar, suggesting either that bad news for Brazilian economy was arriving every single

[30] Malz (1996) estimates the expected devaluation of the Sterling Pound in 1992, conditional on its occurrence, and finds that the expected jump was much smaller than the observed depreciation of 12.5%, which suggests that the market was also surprised by the extent of the Sterling devaluation.

[31] That would mean a devaluation of around 9%.

day or that the market took a couple of weeks to update its more optimistic prior. A look at the newspapers of January 1999 favours the latter explanation.[32]

D A simple "sunspot" model

In models with sunspots, switches between equilibria are the main determinants of crises (Obstfeld (1996), Jeanne (1997)). In the context of this paper, it is instructive to look at an extreme case of a sunspot model, where the maintenance of the peg is entirely independent of the currency overvaluation, but depends on some i.i.d. sunspot shocks.

In that case, the probability of a devaluation is unrelated to θ (by assumption): it depends only on the sunspot shocks, unrelated to the currency overvaluation. That is:

$$\frac{\partial prob}{\partial \theta} = 0$$

As the probability of a devaluation is disconnected from θ, the expected magnitude of a devaluation conditional on its occurrence at time t is equal to the unconditional expected value of θ at time t, because conditioning on a currency devaluation yields no extra information about θ:

$$E(magn_t) = E(\theta_t)$$

From Equation 3, it follows that changes in θ do not depend on its level and, therefore:

$$\frac{\partial E(magn_t)}{\partial \theta} = 1 > 0$$

This is an extreme model, but it highlights the fact that a disconnection between θ and the probability of a devaluation implies a connection between θ and the expected magnitude of a devaluation.

[32] For example, the magazine Epoca published on 01/18/99, when the devaluation was already above 20%, brought Finance Minister Pedro Malan arguing that Brazilian currency overvaluation was slightly *lower than 10%* — he cited studies from institutions such as Morgan, Lloyds, IMF and Goldman Sachs that confirmed his opinion. He dismissed the estimations of an overvaluation of "20%, 25%, 30% and *even* 40%" as based on some "simplistic calculations". On that day, 40% sounded like a bad joke. Reality proved to be different.