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Risk and wealth in a model of self-fulfilling currency attacks

Working paper

Risk and Wealth in a Model of Self-Fulfilling Currency Attacks*

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Abstract

Market participants' risk attitudes, wealth and portfolio composition influence their positions in a pegged foreign currency and, therefore, may have important effects on the sustainability of currency pegs. We analyze such effects in a global game model of currency crises with continuous action choices, generating a rich set of theoretical comparative static predictions related to often discussed but rarely modelled accounts of the onset and timing of currency crises. The model can be solved in closed form and the methods could be used to study other economic issues in which coordination and risk aversion play important roles.

KEYWORDS: Currency crises, global games, risk aversion, wealth, portfolio.

JEL CLASSIFICATION: F3, D8

1 Introduction

Market participants choose their positions in a pegged foreign currency in the light of their beliefs about the sustainability of the peg, their overall portfolios of assets and their risk aversion. So the risk attitudes and portfolio composition of speculators in foreign exchange markets might be expected to have a significant impact on the sustainability of currency pegs. Many popular and academic accounts of the onset and timing of currency crises build on this view. Did hedge funds' ability and willingness to take large short positions play a role in the currency crises of the 1990s? Was the apparent contagion from the Russian crisis of 1998 to Brazil caused by emerging market investors who lost wealth in Russia withdrawing capital from Brazil? Was the apparent contagion from unstable Asian markets like Indonesia to more stable markets like Australia caused by cross-hedging, where investors with illiquid exposure to Indonesia hedged with short positions in the Australian dollar? Did increased (illiquid) foreign direct investment in emerging markets

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create hedging demand for short positions in the currency? Were contingent repo facilities and standstills negotiated by emerging market central banks with New York banks to defend against liquidity crises self-defeating because of the hedging demand for short positions that they created for counterparties?

Standard models of currency crises do not address these issues. In "first generation" models of currency crises (e.g., Krugman (1979)), the timing of the collapse of an unsustainable peg is determined by forward looking arbitrage conditions. In "second generation" models (e.g., Obstfeld (1996)), government actions make currency attacks self-fulfilling giving rise to multiple equilibria. In both cases, each market participant is certain (in equilibrium) if and when an attack will occur. Someone who is certain that an attack will occur will short the currency independent of his attitudes to risk and his underlying portfolio position.

Thus a necessary ingredient of a model of the role of risk and wealth considerations in a currency attack is *strategic uncertainty*: market participants must face uncertainty about whether an attack will be successful or not at the time they take positions in the market.[1] This is surely a realistic feature of these markets. It is also the driving force behind so-called "global games" models of currency crises (Morris and Shin (1998)). In these models, a small amount of uncertainty about fundamentals leads to a large amount of uncertainty about others' actions for the participant on the margin of attacking the currency. Unfortunately, existing global games models have focussed exclusively on the binary actions case, i.e., each participant faces a binary choice between attacking or not attacking, which makes the models inappropriate to understand risk and wealth considerations.[2]

This paper introduces a benchmark static model for analyzing risk, wealth and portfolio composition effects on currency crises. In the model, agents are characterized by the degree of relative risk aversion, the composition of their portfolio of dollar and peso-denominated assets and their propensity to consume in dollar and peso denominated goods. They earn an interest rate premium from holding pesos, but there is a possibility that the exchange rate will be devalued by a known amount. Each investor will choose an optimal portfolio given his beliefs about the likelihood of devaluation. Devaluation occurs if the aggregate net sales of pesos exceeds some stochastic threshold (the "fundamentals"). In section 2, we solve the model assuming that the value of fundamentals is common knowledge. There are multiple equilibria for a range of the fundamentals and we perform comparative statics on the set of equilibria.

[1] One could possibly derive related results in the presence of aggregate uncertainty without idiosyncratic noise, but large amounts of uncertainty would be needed to generate any sizable effect. In this paper, we abstract from aggregate uncertainty.

[2] An important paper by Goldstein and Pauzner (2004) does study the role of risk aversion and wealth in currency crises using a binary action global game model. We will discuss this paper and the limitations of binary action models for this purpose below.

In section 3, we add incomplete information to the model by assuming that each agent has a different noisy signal of fundamentals. That brings strategic uncertainty to the model, which is the key for the results of this paper. We report sufficient conditions for the existence of *threshold equilibria*, where there is a critical value of fundamentals below which the peg always collapses and above which it always survives. The critical threshold will always be in the range where, if there was complete information, there would be both an equilibrium with devaluation and an equilibrium without devaluation. The threshold implicitly determines what we call the "liquidity crisis index": the probability of devaluation conditional on fundamentals being in the multiple equilibrium range of the complete information model. Comparative static effects can then be decomposed into different channels. For any change, we can distinguish between the impact on the complete information equilibria and the impact on the liquidity crisis index. The complete-information equilibria depend on wealth and portfolio composition but not on risk aversion, whereas the liquidity crisis index — intuitively, a measure of which complete-information equilibrium gets selected — depends on risk aversion but not wealth and portfolio composition. The liquidity crisis index in turn depends on the agents' portfolio problem for any given belief about devaluation and the equilibrium distribution of beliefs in the population. The latter depends on the information structure, which does not enter the former.

Our benchmark model, presented in section 4, focuses on the case of a uniform prior — i.e., no public information. This is an example of a global game with continuous actions. Existing theoretical arguments (Frankel, Morris and Pauzner (2003)) establish that there is a unique threshold equilibrium, but no global game with continuous actions has been solved in closed form. We are able to solve the model of this paper in closed form for the case of a uniform prior and — despite the many variables introduced into the model — the solution has a simple economic logic and generates a rich set of theoretical predictions. In particular, when fundamentals are just enough to sustain the peg, the beliefs about the peg are uniformly distributed in the population (regardless of the probability distribution of the noise in private signals). The liquidity crisis index is a function of risk aversion and the relative cost of attacking the currency.

The comparative statics in our benchmark model are very different from comparative statics of the set of equilibria in the corresponding complete information model. We focus on the case where the interest rate differential received from holding pesos is much smaller than the capital loss from holding pesos in the event of a devaluation, and find:

1. Risk Aversion has no impact on the set of complete information equilibria but plays an important role in our benchmark model. If agents are risk neutral, the liquidity crisis probability is close to 1: since the potential payoff to a successful attack is high, agents will attack even when they assign a low probability to success, and, since they are risk neutral, they will take large short positions. But if agents' constant relative

risk aversion is greater than 1, the liquidity crisis probability is less than $\frac{1}{2}$ and, in the limiting case of infinite risk aversion, is close to 0. Under risk neutrality, our model behaves just like the binary action model of Morris and Shin (1998). Chamley (2003) and others have observed that existing global game models are thus models where attacks occur in the unique equilibrium most of the time when they are possible in the complete information multiple equilibrium model. Our results show that this conclusion is dramatically reversed under reasonable levels of risk aversion.

2. Wealth has ambiguous effects under complete information. But in our benchmark incomplete information model, the probability of a crisis is *increasing* in wealth (the opposite of the conventional wisdom underlying some contagion stories). Here it is key that agents can both go long and short pesos. Increased wealth allows agents to take larger short positions, increasing the likelihood of a successful attack.

3. Portfolio effects are essentially the same with complete and incomplete information. An exogenous shift of an agent's wealth out of dollar assets into peso assets is exactly offset by his re-optimized portfolio choice in the currency market.

4. Unlike in the complete information case, increasing the size of the possible devaluation may increase or decrease the likelihood of crisis.

We extend our analysis to the case of exogenous normally distributed public and private signals in section 5. We report a necessary and sufficient condition — depending on the variances of public and private signals — for a unique equilibrium and show, numerically, that (1) with risk aversion, the multiple-equilibrium region shrinks, and (2) the effect of public information is decreasing in risk aversion. Our comparative statics results continue to hold with small amounts of public information. With very accurate public signals, the multiple equilibria converge to the complete information equilibria which have very different comparative statics.

For the sake of expositional clarity, most of the paper focusses on the case of a homogeneous population (representative agent economy), but our results and insights extend to a more general framework with many different heterogeneous agents. The devaluation threshold is *linear* in the distribution of characteristics in the population. Moreover, our benchmark model imposes no restrictions on the positions — long and short — that agents may take in currency markets. We also note how short-sales constraints can be incorporated into our analysis. This reverses some comparative statics predictions: for example, if foreign investors cannot short pesos, then increased wealth will *reduce* the probability of crises. Those important extensions appear in section 6.

Our analysis makes a rich set of empirical predictions from a global games model of currency crises that would probably be hard to replicate with other models that do not build on agents' strategic uncertainty in equilibrium. The task of confronting our model to data is challenging, since the predictions concern micro information about market participants' portfolios that are hard to observe. In section 7, we review some of the empirical and policy debates that we believe our methodology could be employed to help understand, and discuss how they relate to the comparative statics of our model. In order to model the particular debates and develop empirical applications, it would be necessary to model in more detail the richer behavior of hedge funds, banks, governments and other participants in these markets, but we believe that this richer behavior could be incorporated in the analysis because of the nice aggregation properties of this model.

Our analysis follows the approach to modelling currency crises in Morris and Shin (1998), building on the global games analysis of Carlsson and van Damme (1993). Morris and Shin (1998) and other applied papers using these techniques (surveyed in Morris and Shin (2003)) make heavy use of the assumption that each player faces a binary choice (to attack the currency or not). While Frankel, Morris and Pauzner (2003) established existence and uniqueness results in a class of global games with many actions, we have identified a new important class of global games where there is "noise independent selection"; that is, the unique equilibrium is independent of the shape of the noise. In doing so, we demonstrate the robustness of the global game approach to the binary action assumption. Because this selection has an easy and intuitive characterization, it could be used to develop economic insights in other settings. We identify in the body of the paper the features of our model that deliver the noise independence and clean characterization of the equilibrium.

This paper is related to work on contagion by Goldstein and Pauzner (2001). In their model, catastrophic losses in Russia, say, reduce the wealth of investors. If those same investors are also investing in Brazil, and those investors have decreasing absolute risk aversion, then they will reduce their risky exposure to Brazil, thus generating a wealth contagion mechanism. Goldstein and Pauzner emphasize that risk aversion has a large impact on the unique equilibrium even though there may be an arbitrarily small amount of uncertainty about fundamentals, and the same mechanism underlies our results. But by working with a binary action model, they are forced as a modelling assumption to decide if attacking or defending the currency is the riskier action. By allowing for a continuum of actions, we are able to endogenize the amount of "hot money" available in currency attacks and endogenize whether attacking or defending the currency is the riskier action.[3] Our results show how the Goldstein-Pauzner model — and the underlying intuition about contagion — rely on a (perhaps empirically plausible) incomplete markets

[3]Calvo and Mendoza (2000) modelled this type of contagion using an informational story. Kyle and Xiong (2001), like Goldstein-Pauzner, modelled a wealth effect version of the contagion story, but the mechanism is different, relying on a significant amount of uncertainty in equilibrium. These papers also rely on explicit or implicit assumptions that "attacking" (selling pesos) rather than "defending" (buying pesos) is the safe action.

assumption that people who lost money in Russia were unable to short the Brazilian real. In a complete markets model, their loss of wealth in Russia should reduce their ability to short the real, and under the one way bet assumption and plausible risk aversion, this would actually decrease the likelihood of a Brazilian crisis.

A number of recent papers have examined if, when and how the global game approach is robust to a wide variety of features, including public information (exogenous and endogenous), asymmetric players and dynamic effects (with or without signalling),[4] with both positive and negative results. We do not incorporate these robustness concerns (other than exogenous public information) into our analysis. However, we believe that the key features of the model are the strategic complementarities in actions and the friction (incomplete information) that prevents agents from perfectly coordinating their actions. We expect other frictions ensuring strategic uncertainty — such as the timing frictions from Calvo (1983) — to deliver similar economic insights.[5]

2 The Complete Information Model

2.1 The Agent's Problem

A continuum of agents (measure 1) will realize wealth w_D denominated in dollars and wealth w_P denominated in pesos next period. Each agent must decide his net demand for dollars today, y, with $-y$ being the dollar value of the agent's net demand for pesos. Dollar investments earn an interest rate normalized to zero. Peso investments earn an interest rate of r. The initial peso/dollar exchange rate is fixed at e_0, but there is a possibility that the exchange rate will be devalued next period. Thus the exchange rate next period (e_1) will be either e_0 or $E > e_0$.[6]

The agent may consume both foreign goods (x_D, denominated in dollars) and domestic goods (x_P, denominated in pesos), and we assume their prices are constant. The agent's von-Neumann Morgenstern utility function over foreign and domestic goods is Cobb-Douglas,

$$u\left(x_D, x_P\right) = x_D^{\alpha} x_P^{1-\alpha},$$

with $\alpha \in [0, 1]$. A higher α implies a larger home consumption bias. The agent has

[4]A few examples are Atkeson (2000), Dasgupta (2006), Corsetti, Dasgupta, Morris and Shin (2004), Tarashev (2006), Angeletos, Hellwig and Pavan (2006), Angeletos and Werning (2006) and Hellwig, Mukherji and Tsyvinski (2005).

[5]See Frankel and Pauzner (2000) for an elegant model of timing frictions generating strategic uncertainty in a game theoretic setting, and Guimaraes (2006) for a model of currency crises using such timing frictions.

[6]If the exchange rate at period 1, conditional on the occurrence of a devaluation, was assumed to be a decreasing function of θ ($e_1(\theta)$), we would not be able to get an analytical solution for the problem unless we restricted the noise to have finite support and let the support shrink to zero. Even in this case, for $\rho > 1$, there could be multiple multiple equilibria in the game and additional restrictions — e.g., upper bounds on the slope of $e_1(.)$ — would be needed to restore uniqueness (see the working paper version, Guimaraes and Morris (2003), for details).

constant relative risk aversion ρ over his vNM index.[7] He chooses y to maximize the expected value of:

$$\frac{1}{1-\rho}\left(\left(\frac{e_1}{e_0}\right)^{1-\alpha}\widetilde{w}\left(y,e_1\right)\right)^{1-\rho}$$

where $\widetilde{w}$ is his final period wealth (denominated in dollars):

$$\widetilde{w}\left(y,e_1\right)=w_D+\frac{w_P}{e_1}+y\left(1-\frac{e_0}{e_1}(1+r)\right).$$

His indirect utility is thus given by:[8]

$$v\left(y,e_1\right)=\left(\frac{e_1}{e_0}\right)^{1-\alpha}\widetilde{w}\left(y,e_1\right).$$

We assume that the agent will always hold non-negative wealth next period — he cannot promise to deliver more than he will have available. Thus $\widetilde{w}\left(y,E\right)\geq 0$ implies that

$$y>\underline{y}=-\frac{w_D+\frac{w_P}{E}}{1-(1+r)\frac{e_0}{E}}; \tag{1}$$

and $\widetilde{w}\left(y,e_0\right)\geq 0$ implies

$$y<\overline{y}=\frac{w_D+\frac{w_P}{e_0}}{r}. \tag{2}$$

The above "resource constraints" follow from optimizing behavior when there is uncertainty about whether a devaluation will occur or not. As we approach the complete information limit via more accurate public signals (as described in section 5), the resource constraints arise endogenously through the Inada conditions. We assume there are no other constraints on the agents' positions. The case with short-selling constraints is analyzed in section 6.2.

We will assume that the net return to attacking the currency by buying a dollar (and going short in pesos to do so) if there is a devaluation is positive, so

$$v_A=\frac{dv\left(y,E\right)}{dy}=\left(1-(1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{1-\alpha}>0;$$

and the net return to defending the currency by selling a dollar (and purchasing pesos) if there is no devaluation is

$$v_D=-\frac{dv\left(y,e_0\right)}{dy}=r>0.$$

[7]The constant relative risk aversion assumption plays an important part in our analysis. However, many qualitative conclusions go through with more general function forms (see the working paper version, Guimaraes and Morris (2003), for details).

[8]See appendix A.

Much of our analysis depends on the relative returns to attacking and defending, and we find it convenient to parameterize this as

$$t = \frac{v_D}{v_A + v_D} = \frac{r}{\left(1 - (1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)} + r};$$

t can be interpreted as the "transaction cost" of attacking the current (in terms of foregone interest in domestic currency) for each unit of profit from a successful attack. We will often want to make the "one way bet" assumption that the returns to a successful attack are bigger than the costs of an unsuccessful one, i.e.,[9]

$$v_A > v_D$$

and thus

$$t < \frac{1}{2}.$$

2.2 Exchange Rate Determination

We assume that a devaluation occurs if the aggregate net demand for dollars exceeds a threshold θ. This assumption can be understood as a reduced form description of an optimizing decision by the government whether to abandon the peg. Morris and Shin (1998) has a slightly more detailed modelling of government behavior — the government pays an exogenous reputational cost of abandoning the peg — that would give the same results in this setting. A natural interpretation is that θ is the amount of dollars available to the country and the peg can be sustained only if this is enough to cover the demand for dollars.

2.3 The Complete Information Benchmark

We first analyze the equilibria and comparative statics of this model under the assumption that θ is common knowledge. This provides a useful benchmark for our equilibrium and comparative static analysis in the incomplete information case.

We can first ask when there is an equilibrium where every agent attacks the currency to the maximum extent possible, i.e., sets $y_i = \overline{y}$. This will lead to devaluation only if $\overline{y} > \theta$. Thus there is an equilibrium with everyone attacking and devaluation if and only if $\theta < \overline{y}$. On the other hand, if every agent defends the currency to the maximum extent possible, i.e., sets $y_i = \underline{y}$, then there will be devaluation only if $\underline{y} > \theta$. Thus there is an equilibrium with everyone defending and no devaluation if and only if $\theta \geq \underline{y}$. Thus there are multiple equilibria if $\underline{y} \leq \theta < \overline{y}$. This is the standard type of multiple equilibria arising from self-fulfilling beliefs described, for example, in Obstfeld (1996).

[9] Betting in favor of a devaluation is often seen as a one-way bet because the opportunity cost of taking a temporary short position in the currency is small relative to the potential gains from devaluation. See, e.g., Krugman (1979).

It is a little hard to interpret comparative statics in multiple equilibrium models, without a theory of which equilibrium gets chosen, but we can analyze what happens to the whole set of equilibria by seeing how $\underline{y}$ and $\overline{y}$ vary as the parameters change. We focus on four comparative statics in this paper. In each case, we simply report the effect of changing parameters on the resource constraints reported in equations (1) and (2), omitting the straightforward algebra.

1. **Risk Aversion** (increases in ρ): The resource constraints do not depend on risk aversion, so the set of equilibria do not vary with risk aversion. Under complete information pure strategy equilibria, selling pesos is either a profitable strategy or it is isn't, and risk aversion does not effect the calculation. In the complete information model with no strategic uncertainty, risk aversion is irrelevant.

2. **Wealth** (increases in w_D and w_P): Increases in wealth (either domestic or foreign) expand the range of multiplicity in both directions ($\underline{y}$ decreases and $\overline{y}$ increases). Increases in wealth allow agents to take larger positions *both* in attacking the currency or defending the currency. Thus there are more resources committed to attacking in the attack equilibria, and more resources committed to defense in the defense equilibria. More hot money implies more multiplicity of equilibria.

3. **Portfolio Shifts (**decrease in w_D and increase in w_P holding $w_D e_0 (1+r) + w_P$ constant): Suppose that the agent had made an investment that reduced his income tomorrow by one dollar and increases his peso income tomorrow by $e_0 (1+r)$. This is what he could earn if he committed dollars today to an investment earning pesos at the domestic interest rate. Then the range of multiplicity moves up (both $\overline{y}$ and $\underline{y}$ increase by one).[10] This is simply an accounting observation: the agent can exactly reverse his increased exposure to pesos by increasing the quantity of dollars he sells. Thus an increase in illiquid exposure of the agent translates into more resources available to attack the currency.

4. **Size of Devaluation** (an increase in E). An increase in the size of devaluation will not affect the total resources that can be used to attack the currency ($\overline{y}$). Thus the set of parameters where a successful attack is possible will remain the same. However, the fact that a possible devaluation would be large will reduce the amount of resources that agents who want to defend the currency can devote to selling dollars short (so $\underline{y}$ will increase). Thus the range of multiplicity will be reduced and there will be an increased range of parameters where an attack will be inevitable. Overall, an increase in E makes devaluation more likely. But it is important to note that

[10] Formally,

$$-\frac{d\overline{y}}{dw_D} + e_0 (1+r) \frac{d\overline{y}}{dw_P} = -\frac{d\underline{y}}{dw_D} + e_0 (1+r) \frac{d\underline{y}}{dw_P} = 1$$

this complete information has nothing to do with increasing the returns to a possible attack (i.e., a price effect). It operates by reducing the resources that could be devoted to defending the currency.

3 The Incomplete Information Model

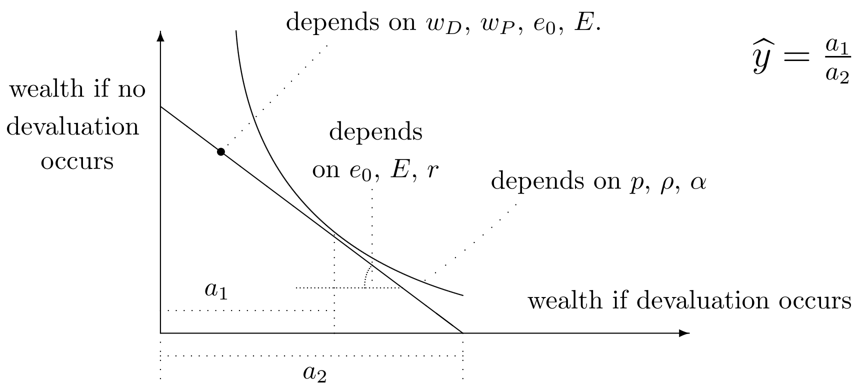
Now we add incomplete information to the model, in the style of the global games literature (Carlsson and van Damme (1993) and Morris and Shin (1998, 2003)). Let the prior distribution of θ be described by a probability density function g and a cumulative distribution function G. Each agent i observes a signal x_i, $x_i = \theta + \varepsilon_i$, where the ε_i are distributed in the population according to probability density function f (cumulative distribution F).

Combined with the environment described in the previous section, we have a game of incomplete information. A pure strategy for each agent is a function mapping signals to dollar demands. A profile of strategies is going to give rise to a set of fundamentals where devaluation will occur. Each agent's private signal and the equilibrium set of fundamentals where devaluation occurs will imply a probability that the agent assigns to the peg being maintained in equilibrium. Thus the first ingredient of our incomplete information analysis is the partial equilibrium analysis of agents' net dollar demands for a given probability that the peg is maintained.

3.1 The Single Agent's Portfolio Problem

Consider an agent who believes that the peg will be maintained with probability p. His problem can be represented by figure 1.

Figure 1: Agent's maximization problem, given p



Prices (E, e_0 and r) determine the slope of the budget constraint. The indifference curve depends on p, ρ and α. For close-to-risk-neutral agents, the indifference curve is close to linear and its slope is determined by p. At the limit, the agent will be basically comparing prices and probability and choosing (a point close to) one of the corners. As ρ increases, p gets less important in shaping the indifference curve, which gets more curvature. So, the optimal choice is closer to the 'middle of the graph'. For low risk aversion, comparative statics are dominated by substitution effects. For high risk aversion, comparative statics will be dominated by income effects. Changes in w_D and w_P change the endowment point, but not the slope. Formally, let $y^*(p)$ be the dollar position of an agent who believes that the peg will be maintained with probability p,

$$y^*(p) = \underset{y \in \left[\underline{\theta}, \overline{\theta}\right]}{\arg\max} \left[p\left(\widetilde{w}(y, e_0)\right)^{1-\rho} + (1-p)\left(\left(\frac{E}{e_0}\right)^{1-\alpha} \widetilde{w}(y, E)\right)^{1-\rho} \right]. \tag{3}$$

An insightful change of variables allows us to report a more explicit expression for y^* in terms of the underlying parameters. Recall that $\underline{y}$ and $\overline{y}$ are the smallest and largest position an agent will take given the resource constraints and let

$$\widehat{y}(p) = \frac{y^*(p) - \underline{y}}{\overline{y} - \underline{y}};$$

thus $\widehat{y}(p) \in [0,1]$ can be interpreted as the proportion of potentially usable resources (given resource constraints) that the agent puts into dollars. We will refer to $\widehat{y}$ as the agent's *market stance*. If $\widehat{y} = 1$, the agent is attacking the currency to the maximum extent possible; if $\widehat{y} = 0$, the agent is defending it.

Thus

$$y^*(p) = \widehat{y}(p)\,\overline{y} + (1 - \widehat{y}(p))\,\overline{y}; \tag{4}$$

In the graphical representation of the portfolio choice problem, $\widehat{y}$ is measured by the proportion of the distance down the budget constraint to the optimal choice, as shown at figure 1. Simple optimization and algebraic manipulation (see appendix B) give the following very simple closed form characterization of $\widehat{y}(p)$, which will be extensively used in our analysis:

$$\widehat{y}(p) = \frac{1}{1 + \left(\frac{p}{1-p}\right)^{\frac{1}{\rho}} \left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}} \tag{5}$$

A very convenient feature of this expression is that it depends only on the determinants of t (r, $\frac{E}{e_0}$ and α) and risk aversion ρ, not on portfolio variables (w_D and w_P). Changes in w_D and w_P alter the agent's budget constraint but not its slope, so they do not influence $\widehat{y}$, because preferences are homothetic.

3.2 Threshold Equilibria

The information structure is said to satisfy first order stochastic dominance (FOSD) if a higher signal lead an agent to shift up his beliefs about fundamentals (in FOSD sense). We denote by $H(\cdot|x)$ the cumulative distribution over θ of an agent who observes signal x,

$$H(\theta|x) = \frac{\int\limits_{\widetilde{\theta}=-\infty}^{\theta} g\left(\widetilde{\theta}\right) f\left(x-\widetilde{\theta}\right) d\widetilde{\theta}}{\int\limits_{\widetilde{\theta}=-\infty}^{\infty} g\left(\widetilde{\theta}\right) f\left(x-\widetilde{\theta}\right) d\widetilde{\theta}},$$

and write $h(\cdot|x)$ for the corresponding density.

Definition 1 *The information structure satisfies first order stochastic dominance (FOSD) if $H(\theta|x)$ is decreasing in x for each θ.*

This condition is sufficient for the existence of particularly simple "threshold equilibria", characterized by a critical θ^* such that above that θ^*, the peg will survive and below that θ^*, there will be a devaluation.[11] To see this, suppose that each agent believes that devaluation will occur only if θ is less than θ^*. Then an agent observing x will attach probability $1 - H(\theta^*|x)$ to the peg surviving and will thus demand $y^*(1 - H(\theta^*|x))$ dollars. Now FOSD and $\frac{dy^*}{dp} < 0$ imply that $y^*(1 - H(\theta^*|x))$ is decreasing in x. Now the total demand for dollars in equilibrium, if the true state is θ and everyone expects the θ^* threshold, will be

$$\int\limits_{x=-\infty}^{\infty} y^*(1 - H(\theta^*|x)) f(x-\theta)\, dx. \tag{6}$$

A necessary condition for a θ^* threshold equilibrium is that if the true state is in fact θ^*, total dollar demand is just enough to cause a devaluation, i.e.,

$$\theta^* = \int\limits_{x=-\infty}^{\infty} y^*(1 - H(\theta^*|x)) f(x-\theta^*)\, dx. \tag{7}$$

But this condition is also sufficient for the existence of a threshold equilibrium, since expression (6) is decreasing in θ for fixed θ^*, so if (7) holds, we will have

$$\int\limits_{x=-\infty}^{\infty} y^*(1 - H(\theta^*|x)) f(x-\theta)\, dx > \theta$$

[11] This condition is not typically explicitly assumed in the global games literature as it holds for free in the limit as noise becomes small (as in Frankel, Morris and Pauzner (2003) and Section 4) and under the normality assumptions typically assumed in applications away from the limit. But it is required to prove existence away from the limit for general distributions. Two recent papers on global games with many actions, Mathevet (2005) and Oury (2005), use such a property away from the limit.

only if $\theta < \theta^*$. Thus we have

Proposition 1 *Suppose that the information structure satisfies FOSD. If θ^* satisfies (7), then there is a threshold equilibrium where an agent observing signal x demands $y^*(1 - H(\theta^*|x))$ dollars and there is devaluation only if $\theta < \theta^*$. There always exists at least one threshold equilibrium.*

Existence is guaranteed by the fact that the right hand side of (7) is continuous and bounded in the interval $\left[\underline{y}, \overline{y}\right]$.

We will exploit a very useful and insightful re-writing of the threshold equilibrium condition (7). Given a threshold θ^* and probability p of the peg being maintained, the monotonicity of H ensures that there is a unique signal x such that

$$1 - H(\theta^*|x) = p,$$

i.e., such that the agent observing x assigns probability p to the peg being maintained. Write $\xi(\theta^*, p)$ for that critical x. Now write $\Gamma(p|\theta^*)$ for the proportion of the population who assign probability p or less to the peg being maintained when the true state is θ^* and the threshold is θ^*. Thus

$$\Gamma(p|\theta^*) = \int\limits_{x=-\infty}^{\xi(\theta^*,p)} f(x - \theta^*)\, dx = F(\xi(\theta^*, p) - \theta^*).$$

We write $\gamma(\cdot|\theta^*)$ for the corresponding density. Now the threshold equilibrium condition (7) can be re-written as

$$\theta^* = \int\limits_{p=0}^{1} y^*(p)\, \gamma(p|\theta^*)\, dp. \tag{8}$$

Let us interpret this condition. For any given true θ^*, we can ask what is the population density over the probability that agents assign to the true state being above θ^* when the true value of fundamentals happens to be exactly θ^*. This is the density $\gamma(\cdot|\theta^*)$. Thus the expression on the right hand side of (8) is total dollar demand when the true state is θ^* and agents expect the peg to be maintained only if θ exceeds θ^*. Thus we see directly that (8) is the threshold equilibrium condition. Note also that (8) can also be derived from (7) by changing the variable of integration from x to $p = 1 - H(\theta^*|x)$.

3.3 A Decomposition of Comparative Statics

In a complete information model, there are multiple equilibria in the range $\theta \in \left[\underline{y}, \overline{y}\right]$. The threshold $\theta^* \in \left[\underline{y}, \overline{y}\right]$ of a threshold equilibrium divides this range in two, with devaluation occurring only if fundamentals are less than θ^*. Now

$$\widehat{\theta} = \frac{\theta^* - \underline{y}}{\overline{y} - \underline{y}}$$

measures the proportion of fundamentals, in the multiple equilibrium region, where devaluation will occur in the θ^* threshold equilibrium, and

$$\theta^* = \widehat{\theta}\overline{y} + \left(1 - \widehat{\theta}\right)\underline{y}. \tag{9}$$

We will label $\widehat{\theta}$ the *liquidity crisis index*, since it measures how often the currency crisis collapses because of self-fulfilling beliefs, when the peg might have survived if there was complete information (and we were in a good equilibrium). The liquidity crisis index varies from 0 and 1, with higher values indicating a high proportion of unnecessary self-fulfilling currency crises.

Now substituting (4), (5) and (9) into (8), we can derive an equilibrium condition for the liquidity crisis index $\widehat{\theta}$ that is independent of the bounds $\underline{y}$ and $\overline{y}$:

$$\widehat{\theta} = \int_{p=0}^{1} \widehat{y}\,(p)\,\gamma\,(p|\theta^*)\,dp \tag{10}$$

This equation is crucial in carrying out and understanding comparative statics. If we fix an equilibrium and vary parameters, there are three distinct channels through which the equilibrium threshold θ^* will vary:

1. The Complete Information channel: The bounds $\underline{y}$ and $\overline{y}$ — and thus complete information comparative statics — depend on wealth (w_D and w_P), interest rate (r) and shadow exchange rate (E) as described in equations (1) and (2).

2. The Portfolio Choice channel: the stance of an individual agent who assigns probability p to the peg being maintained ($\widehat{y}\,(p)$, given by equation 5) depends only on risk aversion (ρ) and the cost of attacking (t).

3. The Equilibrium Beliefs channel: Associated with a given threshold equilibrium θ^*, there is a distribution of beliefs in the population about the probability of the peg being maintained when the true value of fundamentals equals θ^* ($\gamma\,(p|\theta^*)$).

Whether or not there are multiple equilibria, it is possible to carry out comparative statics of any equilibrium using this decomposition. In the next section, we will examine a uniform prior benchmark model where there is a unique equilibrium and comparative statics are driven by the interaction between the complete information channel and the portfolio choice channel.

4 The Uniform Prior Benchmark

In this section, we assume that the prior distribution of θ is uniform. This allows us to establish uniqueness of equilibrium, obtain a closed form solution and determinate comparative statics. As is well known from the global games literature (Carlsson and van Damme (1993), Morris and Shin (2003)), results that are true for a uniform prior are also true for an arbitrary smooth prior as long as private signals are sufficiently accurate. Thus the key substantive assumption in this section is that private signals are accurate relative to any information in the prior. In the next section, we will return to the question of how robust our conclusions are to this assumption.

4.1 Equilibrium Uniqueness and Noise Independence

Suppose that the peg will be maintained only if $\theta \geq \theta^*$. An agent observing signal x knows that $\theta \leq \theta^*$ only if

$$\varepsilon \geq x - \theta^*$$

where ε is the noise in his private signal. Under the uniform prior assumption, he thus assigns probability

$$H(\theta^*|x) = 1 - F(x - \theta^*).$$

to $\theta \leq \theta^*$. So he assigns probability

$$F(x - \theta^*)$$

to the peg being maintained. Thus any agent observing a signal

$$\xi(\theta^*, p) = \theta^* + F^{-1}(p)$$

attaches probability p to the peg being maintained; and any agent observing a signal less than $\xi(\theta^*, p)$ attaches a probability less than p to the peg being maintained. Thus if the true state is in fact θ^*, the proportion of agents assigning probability p or less to the peg being maintained will be

$$\begin{aligned}
\Gamma(p|\theta^*) &= \int_{x=-\infty}^{\xi(\theta^*,p)} f(x - \theta^*)\,dx \\
&= F(\xi(\theta^*, p) - \theta^*) \\
&= F\left(F^{-1}(p)\right) \\
&= p.
\end{aligned}$$

The corresponding density is $\gamma(p|\theta^*) = 1$ and thus we immediately have, from (10), that

$$\widehat{\theta} = \int_{p=0}^{1} \widehat{y}(p)\, dp. \tag{11}$$

Two important results emerge. First, clearly there is a unique $\widehat{\theta}$ that satisfies equation (11), so the model has exactly one threshold equilibrium. There are no other equilibria. A general result in Frankel, Morris and Pauzner (2003) can be used to show this.[12] Because the game is one of strategic complementarities, there will be a largest and smallest strategy surviving iterated deletion of strictly dominated strategies. The strategic complementarities also ensure that the largest and smallest strategies are both monotonic — i.e., an agent's dollar position is decreasing in his signal. Thus both are threshold equilibria characterized by a critical value θ^*. But the argument above established that there is at most one such equilibrium, so the largest and smallest strategies surviving iterated deletion must be the same. The same argument ensures that the unique equilibrium is also the unique strategy surviving iterated deletion of strictly dominated strategies.

Second, the equilibrium beliefs $\gamma(\cdot|\theta^*)$ do not depend on the distribution of noise F. The arguments in Frankel, Morris and Pauzner (2003) establish that there is a unique equilibrium, but they show that, in general, the form of the unique equilibrium will depend on the noise distribution F. The continuum action game studied in this paper turns out to have the attractive feature that there is "noise independent selection".

These observations are summarized in the following proposition.

Proposition 2 *If the prior is uniform, then the unique strategy profile surviving iterated deletion of strictly dominated strategies is a threshold equilibrium with threshold given by (11).*

4.2 Comparative statics

4.2.1 Risk Aversion

We start by analyzing the impact of risk aversion. Recall that risk aversion (ρ) does not influence the resource constraints and thus there was no comparative static effect of changing risk aversion on the set of equilibria under complete information.

Under incomplete information, risk aversion matters because it impacts the position of agents who are uncertain, in equilibrium, whether the peg will be maintained or not. We briefly summarize three leading cases to give some intuition about the role of risk aversion.

[12] Frankel, Morris and Pauzner (2003) main result is a limit uniqueness result for global games with strategic complementarities and general priors as noise becomes small. However, a step in the argument establishes uniqueness of equilibrium in a model with uniform priors and applies to the model in this paper.

First, consider the case with no risk aversion; formally, we let $\rho \to 0$. In this case, an agent will go long in dollars up to the resource constraints as long as his expected return is positive and short dollars up to the opposite bound as long as the expected return is negative. Thus

$$\widehat{y}(p) \to \begin{cases} 0 & \text{if} \quad p > 1 - t \\ 1 & \text{if} \quad p < 1 - t \end{cases}$$

and

$$\hat{\theta} = \int_{p=0}^{1} \widehat{y}(p)\, dp \to 1 - t$$

This corresponds to the equilibrium in models with risk neutral agents and binary actions following Morris and Shin (1998). In applications, t is often considered to be close to 0. That implies $\hat{\theta}$ close to 1 in the risk neutral case — conditional on θ being in the multiple equilibrium region, the probability of a liquidity crisis is very high.

The importance of risk aversion can be easily seen by considering the case of log utility (the limit as $\rho \to 1$). In this case, $\widehat{y}(p) = 1 - p$: with logarithmic utility, the proportion invested in dollars is equal to the probability of a devaluation and does not depend on anything else — prices are irrelevant. Then, $\hat{\theta} = \frac{1}{2}$. Note the dramatic impact of risk aversion on $\hat{\theta}$. For example: if $t = 0.05$, the liquidity crisis index is 0.95 in the risk neutral case but equals only 0.5 if agents have logarithmic utility.

Finally, as the agents become infinitely risk averse ($\rho \to \infty$), $\hat{y}(p) \to t$ and $\hat{\theta} \to t$. When the agent has no exposure to peso risk ($w_P = 0$ and $\alpha = 0$), $t = \underline{y} / \overline{y}$ and so $\widehat{\theta} = t$ implies that $\theta^* = 0$: very risk averse agents will take zero positions.

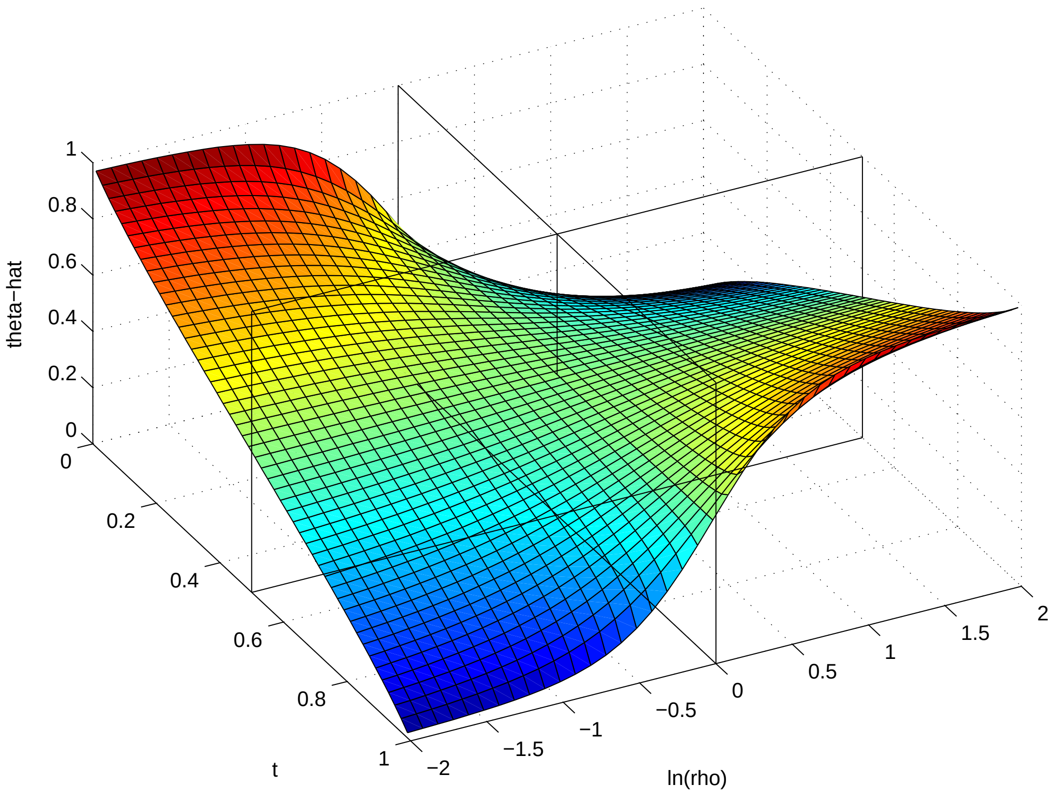
As suggested by the above closed form solutions, under the one way bet assumption ($t < \frac{1}{2}$), risk aversion reduces $\hat{\theta}$ and makes investors less willing to attack the currency. While risk aversion leads optimistic agents (those who expect the peg to be maintained) to take smaller positions selling dollars and pessimistic agents (those who expect to the peg to collapse) to take smaller positions buying dollars, the one way bet assumption and the uniform distribution of beliefs at the equilibrium threshold imply that the reduction in the positions of pessimistic agents predominates and a devaluation is less likely. Figure 2 plots $\hat{\theta}$ as a function of $\log(\rho)$ and t.[13] Note that if the one way bet assumption failed ($t > \frac{1}{2}$), risk aversion would *increase* the likelihood of devaluation.

4.2.2 Wealth

The liquidity crisis index $\widehat{\theta}$ is independent of wealth, which only affects the bounds $\underline{y}$ and $\overline{y}$. As we noted in our analysis of complete information comparative statics, increasing wealth

[13] Analytically, we are able to show that for $t < \frac{1}{2}$, $\hat{\theta}$ is decreasing in ρ for $\rho \geq 1$ — we couldn't prove it for $\rho < 1$ although we believe it also holds in this case. Analogously, for $t > \frac{1}{2}$, we can show that $\hat{\theta}$ is increasing in ρ only for $\rho \leq 1$.

Figure 2: $\hat{\theta}$ as a function of t and $\log(\rho)$



decreases $\underline{y}$ and increases $\overline{y}$: more wealth increases the absolute size of the positions agents could take, either attacking or not attacking the currency. In our incomplete information analysis, the size of the liquidity crisis index — which is independent of wealth — will determine which effect dominates. For simplicity, we focus on the case where $w_P = 0$, $w_D > 0$ and $\alpha = 1$.[14] We have:

$$
\begin{aligned}
\theta^* &= \hat{\theta}\overline{y} + \left(1 - \hat{\theta}\right)\underline{y} \\
&= w_D \left[\frac{\hat{\theta}}{r} - \frac{1 - \hat{\theta}}{1 - \frac{e_0}{E}(1+r)}\right] \\
\frac{d\theta^*}{dw_D} &= \frac{\hat{\theta}}{r} - \frac{1 - \hat{\theta}}{1 - \frac{e_0}{E}(1+r)} \\
&= \left(\frac{1 - \hat{\theta}}{r}\right)\left(\frac{\hat{\theta}}{1 - \hat{\theta}} - \frac{t}{1 - t}\right) \qquad (12)
\end{aligned}
$$

[14] The result holds for increases in w_D or w_P more generally.

Under the one way bet assumption ($t < \frac{1}{2}$), $\hat{\theta} > t$ for any ρ and so

$$\frac{d\theta^*}{dw_D} > 0.$$

Thus increased wealth *increases* the probability of a successful currency attack. This result is different from the usual intuition and from the complete information comparative static. As we discuss in section 6.2, the more conventional intuition that increases in the amount of "hot money" (wealth in our model) will decrease the likelikhood of a crisis relies on the assumption of short-selling constraints.

4.2.3 Portfolio

As we did in the case of complete information, consider a simultaneous increase in w_D and decrease in w_P holding $w_D e_0 (1 + r) + w_P$ constant. For any probability p the agent attaches to the peg being maintained, this shift does not change the agent's budget constraint. Thus, for *any* p, the agent's optimal net demand for dollars $y^*(p)$ would increase by exactly the reduction in his dollar wealth w_D. Thus the agent would perfectly hedge his overall exposure. Since this is true for each p, the overall effect of the portfolio shift would be increase θ^* by the amount of the increase in w_D.

4.2.4 The Size of Devaluation

An increase in E represents an increase in the size of the devaluation, if it occurs. This has two effects. First, as noted at section 2.3, it changes the resource constraints: while $\overline{y}$ stays constant, $\underline{y}$ will be reduced, as an increase in E will reduce the amount of resources that can be devoted to a successful defense of the currency and so will increase the probability of devaluation via the resource constraints. But an increase in E will also change t by reducing the price of consumption in the state of the world where devaluation occurs. This price effect does not appear in the complete information analysis.

What is the direction of the effect of the price change? The substitution effect of the price change will make devaluation more likely. But there will be a countervailing income effect: an increase in E means that less resources must be put into attacking the currency in order to secure a given level of consumption in the event of a devaluation. Risk neutrality implies that indifference curves are linear and the substitution effect predominates. The income effect becomes more important as agents become more risk averse. This can be seen in figure 2. An increase in E decreases t, which leads to a increase in the liquidity crisis index ($\widehat{\theta}$) if $\rho < 1$ but to a decrease in $\widehat{\theta}$ if $\rho > 1$. Thus for empirically plausible levels of risk aversion (greater than 1), a higher value of E leads to a *lower* liquidity crisis probability.

Combining the two effects, we know that for $\rho \leq 1$, we must have θ^* increasing in E. But for $\rho > 1$, the combined effect may go either way for reasonable values of the parameters.

These comparative statics illustrate a wide range of possible interactions between complete information comparative statics and global game comparative statics. We can have determinate comparative statics under incomplete information from variables that do not even enter the complete information analysis (e.g., risk aversion). We can have ambiguous comparative statics with respect to the set of complete information equilibria but determinate comparative statics under incomplete information (e.g., wealth). We can have comparative statics that are essentially the same for multiple complete information equilibria and the unique global game equilibrium (e.g., portfolio effects). And incomplete information comparative statics may incorporate both a determinate complete information comparative static and a distinctive but indeterminate comparative static arising from incomplete information (the size of devaluation). Apart from the substantive interest in these comparative statics, we hope they illustrate the richness of the incomplete information analysis.[15]

5 The Public Information Case

As we move away from the benchmark uniform prior case, it is convenient to focus on the case with normally distributed priors and signals.[16] Thus we let the prior G be given by a normal distribution with mean μ and variance σ_η^2. There is common knowledge of μ, the mean of θ, which can thus be interpreted as a public signal. In addition, each agent i observes a signal x_i, where the signals x_i are normally distributed in the population with mean θ and variance σ_ε^2. We will analyze the resulting incomplete information game.[17]

[15] In the working paper version of this paper, Guimaraes and Morris (2003), we report other comparative statics. An increase in the home consumption bias (α) does not effect the resource constraints but does decrease the cost of attacking t; for $\rho > 1$ and $t < \frac{1}{2}$, this implies an increase in the probability of devaluation. Changes in the interest rate (r), like changes in the shadow exchange rate (E), effect both the resource constraints and the cost of attacking and have ambiguous comparative statics.

[16] Little is known about properties of global game equilibria away from the no noise limit without the normality assumption: see Morris and Shin (2005b) for one preliminary analysis.

[17] Here, we study the case of exogenous public information. A series of recent papers — Tarashev (2006), Angeletos and Werning (2006), Hellwig et al (2005) and Morris and Shin (2005a) — have examined what happens when public information is endogenously generated in global game models.

5.1 Equilibrium

By standard properties of the normal distribution, an individual observing signal x has posterior beliefs that θ is normally distributed with mean

$$\frac{\sigma_{\varepsilon}^{2}\mu+\sigma_{\eta}^{2}x}{\sigma_{\varepsilon}^{2}+\sigma_{\eta}^{2}}$$

and variance

$$\frac{\sigma_{\varepsilon}^{2}\sigma_{\eta}^{2}}{\sigma_{\varepsilon}^{2}+\sigma_{\eta}^{2}}.$$

Thus the probability he assigns to $\theta\leq\theta^{*}$ is

$$H\left(\theta^{*}|x\right)=\Phi\left(\sqrt{\frac{\sigma_{\varepsilon}^{2}+\sigma_{\eta}^{2}}{\sigma_{\varepsilon}^{2}\sigma_{\eta}^{2}}}\left[\theta^{*}-\frac{\sigma_{\varepsilon}^{2}\mu+\sigma_{\eta}^{2}x}{\sigma_{\varepsilon}^{2}+\sigma_{\eta}^{2}}\right]\right)$$

Now observe that $\xi\left(\theta^{*},p\right)$, the critical signal at which an agent assigns probability p to $\theta\geq\theta^{*}$, is thus the unique value of x solving

$$p=1-\Phi\left(\sqrt{\frac{\sigma_{\varepsilon}^{2}+\sigma_{\eta}^{2}}{\sigma_{\varepsilon}^{2}\sigma_{\eta}^{2}}}\left[\theta^{*}-\frac{\sigma_{\varepsilon}^{2}\mu+\sigma_{\eta}^{2}x}{\sigma_{\varepsilon}^{2}+\sigma_{\eta}^{2}}\right]\right). \tag{13}$$

If the true state is θ^{*}, the proportion of the population believing that the probability that $\theta\geq\theta^{*}$ is less than p is

$$\Gamma\left(p|\theta^{*}\right)=\Phi\left(\frac{\xi\left(p,\theta^{*}\right)-\theta^{*}}{\sigma_{\varepsilon}}\right) \tag{14}$$

and there is a unique threshold equilibrium only if there is only one value of θ^{*} that satisfies equation (8). We can establish that if private signals are sufficient accurate relative to public signals, there is a unique equilibrium.

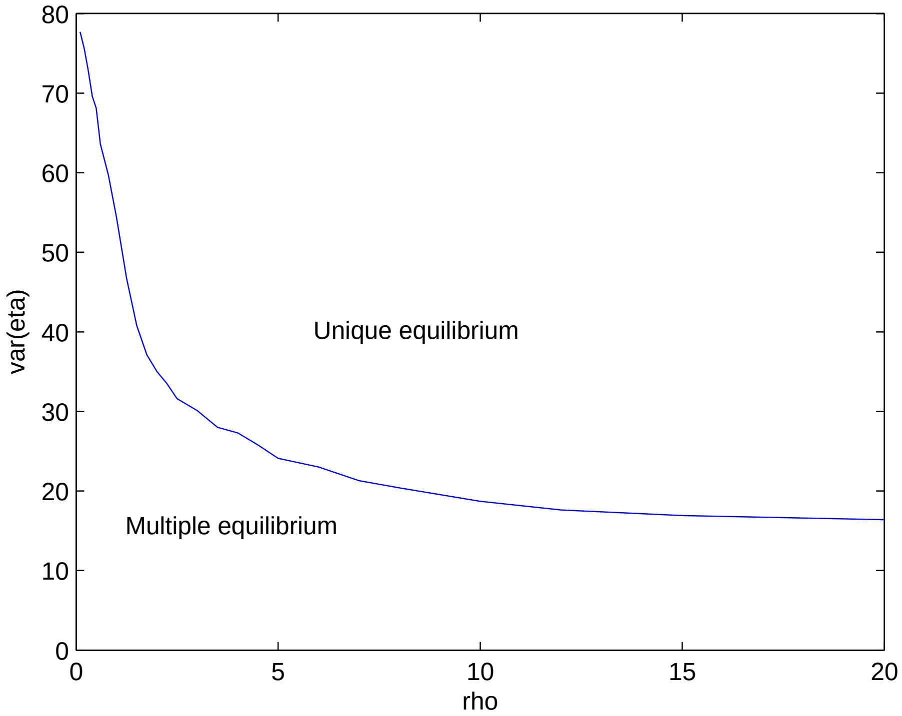
Proposition 3 *There is a unique rationalizable equilibrium in the model (for all values of μ and ρ) if*

$$\frac{\sigma_{\eta}^{2}}{\sigma_{\varepsilon}}>\left(\overline{y}-\underline{y}\right)\sqrt{2\pi}. \tag{15}$$

If the condition fails, then for some μ and ρ sufficiently close to 0, there are multiple equilibria.

This is the uniqueness condition identified in Morris and Shin (2004), which essentially corresponds to the model in this paper in the case of risk neutral agents. The idea of the proof, presented in the appendix, is to establish that risk neutrality is the hardest case in which to prove uniqueness. When condition (15) fails, it is possible to show the existence of multiple equilibria for some public signal μ as long as ρ is sufficiently close to 0. By adding in risk aversion to the model, agents' portfolio choices depend more smoothly on signals and thus uniqueness is easier to obtain. Figure 3 confirms numerically that the threshold for multiple equilibria decreases as ρ increases.[18]

Figure 3: Multiplicity and uniqueness regions



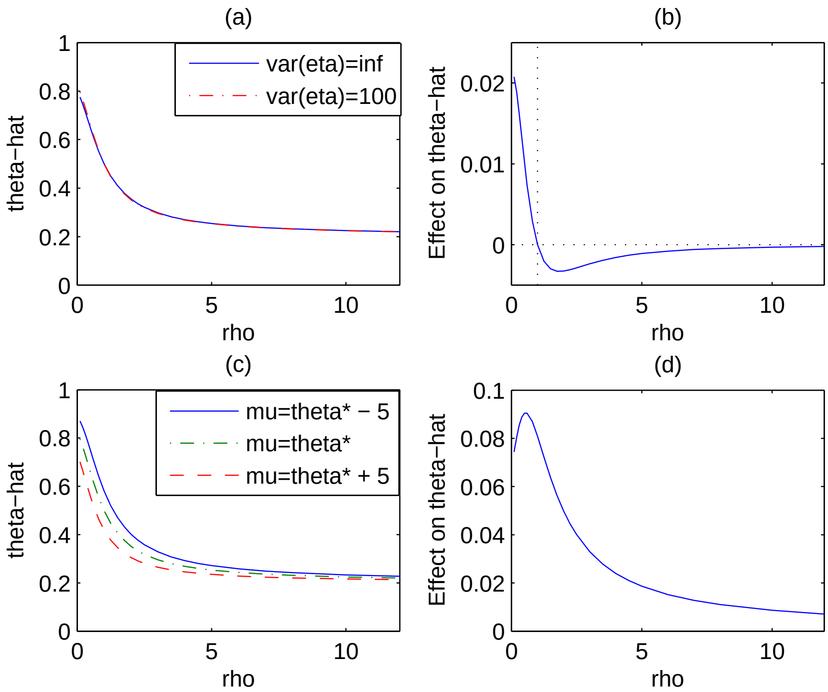
This result says that there is a unique equilibrium if private information is sufficiently accurate relative to public information. But how should we interpret this condition? Our view is the private signal / public signal modelling should not be taken too literally — after all, the prior and signal distributions are not common knowledge in practise, as the model assumes. Rather, condition (15) is sufficient to ensure that agents' beliefs about how others' beliefs differ from theirs do not change much as their beliefs vary. It is this property that ensures uniqueness and also this property that will make the comparative statics derived under the uniform prior assumption hold more generally.

5.2 Comparative Statics

With public information, the third channel for comparative statics identified in section 3.3 comes into play. A change in the public signal μ has no effect on the resource con-

[18]Parameters in this example: $\sigma_\varepsilon^2 = 10$, $w_D = 1$, $w_P = 1$, $\alpha = 0.5$, $r = 0.04$ and $E = 1.2$. A grid for p was constructed and the line separating the 2 regions shows the maximum σ_ε^2 for which multiple equilibria were found for, at least, one value of p.

Figure 4: Public information and risk aversion



straints and no effect on the choice of an agent that assigns probability p to the peg being maintained, $\widehat{y}(p)$. The only impact in the equilibrium threshold is via the equilibrium distribution over the probability attached to the peg being maintained, $\gamma\left(\cdot|\theta^*\right)$.

The comparative statics reported in section 4.2 will continue to hold as long as the equilibrium distribution over beliefs about devaluation remain close to uniform. This will be true in the region where there is a unique equilibrium. Our simulations also confirm that the effect of public information is decreasing in risk aversion. Figure 4-(a) shows the liquidity crisis index $\hat{\theta}$ for parameters $\sigma_{\varepsilon}^2 = 10$, $t = 0.2$ in two situations: $\sigma_{\eta}^2 = 100$ and $\sigma_{\eta}^2 \to \infty$ (that is, no public information), imposing the condition $\mu = \theta^*$ (when $\theta = \theta^*$, public information is not misleading the agents). The two lines are not distinguishable at figure 4-(a) but figure 4-(b) plots the difference between them. For $\rho < 1$, $\hat{\theta}$ would be bigger than 0.5 and, with public information, it is even bigger. For $\rho > 1$, $\hat{\theta}$ would be smaller than 0.5 and, with public information, it is even smaller. So, public information brings the equilibrium closer to the extremes. However, the effect is quite small for large values of ρ, because when agents are more risk averse, changes in p have smaller effects on their decisions.

Figure 4-(c) compares the equilibrium threshold when $\mu = \theta^*$, $\mu = \theta^* - 5$ (worse public

information) and $\mu = \theta^* + 5$ (better public information).[19] When public information is bad, the equilibrium threshold is higher, we need better fundamentals to compensate the bad public information. Figure 4-(d) shows the effect of a bad public signal ($\hat{\theta}(\mu = \theta^* - 5) - \hat{\theta}(\mu = \theta^*)$). It is again true that for high levels of risk aversion ρ, the effect of public signals is reduced.

The expression for wealth effects shown at section 4.2 still holds with public information, but now the value of $\widehat{\theta}$ may not belong in the interval $[t, 1-t]$, so we cannot sign the comparative statics effect.

5.3 Limiting Results

If σ_η^2 is fixed and $\sigma_\varepsilon^2 \to 0$, then public information becomes less important and it is as if agents have a uniform prior. Thus for small σ_ε^2, this model converges to the uniform prior model of section 4.

If σ_ε^2 is fixed and $\sigma_\eta^2 \to 0$, then the incomplete information case converges to the complete information case. Consider the case where $\underline{y} < \mu < \overline{y}$. For sufficiently small σ_η^2, there are three threshold equilibria with the three thresholds converging to $\underline{y}$, μ and $\overline{y}$ as $\sigma_\eta^2 \to 0$.

To see why this is the case, fix any $\theta < \mu$ and ask what happens to $\Gamma(p|\theta)$ as $\sigma_\eta^2 \to 0$. We must have $\Gamma(p|\theta) \to 1$ for all $p > 0$: in the limit, everyone will be convinced that the true state is μ which is greater than θ. Thus the only possible candidate threshold equilibrium below μ is one where everyone is convinced that the peg will survive. In this case, all agents will choose $\underline{y}$. A symmetric argument establishes that the only possible equilibrium above μ is $\overline{y}$ and is associated with beliefs $\Gamma(p|\theta) \to 0$ for all $p < 1$. In the interior equilibrium, beliefs will be such that each agent attaches probability $[y^*]^{-1}(\mu)$ to the peg surviving in the limit.

Thus the largest and smallest threshold equilibria correspond to the largest and smallest complete information equilibrium and will have the same comparative statics in the limit as we discussed at section 2.3.

6 Extensions

6.1 Heterogeneous agents

Our analysis and comparative statics considered ex ante identical agents who differ only in their private signals. However, the model can deal with heterogeneous agents in a very simple way: there is linear aggregation across the population. This feature ensures

[19] Parameters of this example: $\sigma_\varepsilon^2 = 10$, $t = 0.2$ and $\sigma_\eta^2 = 100$.

that it is trivial to extend the comparative statics to heterogeneous population and offers hope for empirical work and policy analyses that carry out a detailed treatment of market participants. For simplicity, we will report this extension for the uniform prior benchmark, but the analysis can be straightforwardly extended to general priors.[20]

Consider an agent of type i, characterized by a coefficient of constant relative risk aversion ρ_i, dollar-denominated wealth w_{Di}, peso-denominated wealth w_{Pi}, and preference parameter α_i. The proportion of type i agents is denoted by λ_i. Let's denote the bounds for agent i as $\overline{y}_i$ and $\underline{y}_i$. Define:

$$\overline{\theta} = \sum_i \lambda_i.\overline{y}_i \quad , \quad \underline{\theta} = \sum_i \lambda_i.\underline{y}_i$$

Then, with complete information, if $\theta < \underline{\theta}$, a devaluation would occur, if $\underline{\theta} \leq \theta < \overline{\theta}$, there would be multiple equilibria and if $\theta \geq \overline{\theta}$, the peg would be maintained.

With incomplete information, if agent i assigned probability p to the peg being maintained, his demand for dollars would be:

$$y_i^*(p) = \underset{y \in [\underline{y}_i, \overline{y}_i]}{\arg\max} \left[\begin{array}{l} p \left(w_{Di} + \frac{w_{Pi}}{e_0} - yr \right)^{1-\rho_i} \\ + (1-p) \left(\left(\frac{E}{e_0} \right)^{1-\alpha_i} \left(w_{Di} + \frac{w_{Pi}}{E} + y \left(1 - \frac{e_0}{E}(1+r) \right) \right) \right)^{1-\rho_i} \end{array} \right]. \tag{16}$$

If there was a homogenous continuum of agents of type i, we know that the critical threshold would be

$$\theta_i^* = \int_{p=0}^{1} y_i^*(p)\, dp.$$

But if there was a heterogeneous population, with proportion λ_i of type i, then there would be a uniform distribution of probabilities of devaluation at the threshold within each population. Applying the reasoning described to obtain equation (8), we get the resulting threshold:

$$\theta^* = \sum_{i=1}^{I} \lambda_i \int_{\pi=0}^{1} y_i^*(p)\, dp = \sum_{i=1}^{I} \lambda_i \theta_i^*.$$

Interestingly, the threshold θ^* is just an average of θ_i^* weighted by λ_i. In some models in which risk aversion plays an important role, the existence of a few close-to-risk-neutral agents is enough to make the system behaves as if there were no risk averse agents because the "close-to-risk-neutral" agents provide the hedge to all others. Here, the agents with low ρ will be very aggressive in their bets but they know that the others will not and that is crucial in determining the threshold θ^*.

[20] Guimaraes and Morris (2005) show how to extend this analysis to normal prior and signals and also allow heterogeneous accuracy of private information. The necessary and sufficient condition for equilibrium uniqueness reported in proposition 3 continues to hold in this case.

6.2 The Case with Short Selling Constraints

Our analysis up to now has assumed that agents are free to short any amount of dollars and/or pesos. The analysis continues to apply to the case where there are short sales constraints. Thus we could add the constraint that each agent must choose $y \in [\underline{\theta}, \overline{\theta}]$, where $\underline{y} \leq \underline{\theta} < \overline{\theta} \leq \overline{y}$. In this case, there would be multiple equilibria in the complete information case only if $\theta \in [\underline{\theta}, \overline{\theta}]$. The equilibrium will still be given by equation 8, but $y^*(p)$ would be different because the agent's options are constrained and he knows all other agents are also constrained. Public information will have the same effect of changing the distribution of beliefs in equilibrium (γ) and nothing else. The equilibrium with heterogeneous agents is again an average of θ_i^*'s weighted by λ_i's.

Comparative statics change, however.[21] First, while we can analogously define the liquidity crisis index as

$$\widehat{\theta} = \frac{\theta^* - \underline{\theta}}{\overline{\theta} - \underline{\theta}},$$

now changes in w_D and w_P also affect $\widehat{\theta}$. The simple decomposition of comparative statics into the separate channels no longer operates. Nonetheless there are some important distinctive comparative static effects we can identify.

The analysis of the risk neutral case is essentially unchanged with short sales constraints: as $\rho \to 0$, $\hat{\theta} \to 1 - t$. But now the impact of risk aversion depends on which action is "risky". Without short-sales constraints, it was endogenous whether more agents were long or short in pesos, and the one-way bet assumption ensured that, when $\theta = \theta^*$, most of them would be short. So the overall effect of risk aversion was to reduce the likelihood of devaluation. With short-sales constraints, our modelling choices will determine which action is risky. Suppose, for example, that agents cannot go short in either currency. If all investors consume only foreign goods and have all their wealth in dollars ($\alpha = 1$ and $w_P = 0$), buying pesos is the risky action and thus risk aversion increases the probability of attacks (independent of t). Conversely, if all agents consume only domestic goods and have all their wealth in pesos ($\alpha = 0$ and $w_D = 0$), risk aversion reduces the probability of attacks (independent of t), because defending the peg is the safe action by assumption.

A similar logic applies to the comparative statics of wealth: the key determinant of the direction of the effects is whether agents are in equilibrium mostly short or long in pesos. In the first scenario described above (no shorting either currency, $\alpha = 1$ and $w_P = 0$), an increase in wealth will always increase agents' appetite for risk (given our constant relative risk aversion assumption), thus making them more willing to hold deposits in pesos. This result is similar to one obtained in the contagion model by Goldstein and Pauzner (2004). Short selling constraints are implicitly assumed in their paper and agents decide to invest

[21] See the working paper version of this paper, Guimaraes and Morris (2003), for details.

or not one unit of money in the country. Their conclusion still holds when agents have a continuum of actions but depends on positive investments in the country — agents need to be investing, not attacking.

7 Relation to Empirical Work

Our model delivers some striking comparative static findings. While the comparative statics of risk aversion are interesting, it is not clear that we observe the variation in risk aversion in empirical evidence that would enable us to test our findings. On the other hand, significant empirical and policy debates concern the role of wealth and hedging motives in currency crises, and our comparative static findings for those variables inform those debates. While our theoretical model is too stylized to be related directly to empirical evidence, it is nonetheless useful to sketch how we think various empirical findings relate to the model in this paper.

It is often said that a negative wealth shock may threaten a currency peg because investors are forced to withdraw their money. For example, when Russia defaulted its debt in 1998, Brazil experienced a large capital outflow. The contagion from Russia to Brazil is studied by Baig and Goldfajn (2001).[22] They argue that the "compensatory liquidation of assets story", according to which institutional investors withdraw their money from Brazil to compensate losses in Russia, does not find empirical support from the data because those who had lost more money in Russia did not present higher rates of withdrawals from Brazil. But our analysis in section 6.1 shows that if agents face short-selling constraints, a decrease in some agents' wealth would decrease θ^*, increase the likelihood of a crisis and, therefore, lead *all* agents to decrease their exposure in pesos. Likewise, Broner, Gelos and Reinhart (2006) find that funds that underperform tend to reduce their exposure in the countries they were *overweighted*, and a suggested interpretation is that the negative wealth effect reduces their risk appetite, bringing them closer to the mean portfolio. Interestingly, they also find that the funds that overperform tend to reduce their exposures in countries they were *underweighted*, which could be interpreted as an increase in their risk appetite leading them to move away from the mean portfolio. Consistently with their empirical evidence, our model shows that if some agents who would have some preferences to invest in country A (say, because of their α) lose money, all agents would reduce exposure in country A due to coordination reasons.

Sometimes wealthy investors are said to have triggered a crisis. Large short positions taken by hedge funds might have played an important role in the British Pound devaluation in 1992 and, more generally, in the ERM crisis in 1992-3. In the complete markets case, wealth is indeed positively related to the likelihood of a devaluation (if $t < \frac{1}{2}$).

[22] For a popular account of this contagion event, see Blustein (2001, chapter 12).

Thus, a message from this paper is that wealth effects exist regardless of any short selling constraint, but the direction of such effects on the probability of a crisis depends on institutional constraints on agents' position.

Popular and academic informal accounts of agents' behavior in currency crises often consider portfolio-hedging issues. For example, foreign direct investment is sometimes said to be more resilient in the face of financial crises.[23] However, agents with illiquid investments in a country will have incentives to hedge. Indeed, IMF (1998) points that "hedging by multinational corporations was ascribed a significant role by market participants in generating the pressures on the Brazilian real in late 1997" (page 16) and mentions similar effects during the Asian crisis. The effects of portfolio composition are surely not related only to FDI. A report from the Financial Stability Forum describes a perhaps curious setting in which portfolio effects led to *cross-hedging*: during the Asian crisis, agents with investment in Indonesia that became illiquid were selling Australian dollars short to (imperfectly) hedge their positions.[24]

Hedging motivations may also have played an important role in some recent experiences with private sector involvement. In March 1999, there was a voluntary agreement by Brazil's creditor banks to maintain interbank and trade lines at existing levels. The banks stuck to their agreement but partially hedged by shorting the liquid Brazilian C-bond (IMF (2000) page 135). Prior to the collapse of the Argentine currency board, the Central Bank of Argentina negotiated contingent repurchase agreements with international banks for government bonds and other Argentine domestic financial assets. The policy was intended to provide insurance against systemic liquidity shocks, and at one point amounted to around \$6 billion in assets. One concern in the design of the policy was the possibility that the international banks would limit their (contingent) country exposure by hedging in other markets. In the end, the facility was triggered in August 2001, with the then \$1.8 billion commitments honored by the international banks and later fully repaid by the central bank.[25]

Our benchmark model shows that atomistic agents fully undo the portfolio shift in the currency market when the amount received in pesos compensates them for the risk of a devaluation (the $(1 + r)$ factor) and there are no restrictions or costs to hedge their positions. In the above examples, hedges are imperfect and/or costly. A model addressing one of those specific questions would need to take this into account.

[23]World Bank (1999) states that "FDI is less subject to capital reversals (...) since the presence of large, fixed, illiquid assets makes rapid disinvestment more difficult than the withdrawal of short-term bank lending or the sale of stock holdings".

[24]See Financial Stability Forum (2000), page 117.

[25]See Powell (2002, page 283) on the Argentinian experience and Tirole (2002, page 21) and Giannini (2002, page 534) on theoretical difficulties with such repo agreements.

8 Conclusion

We built a 'global-games' model of currency crisis in order to analyze the impact on agents behavior of issues related to risk and wealth. While our analysis concerns currency crises, the modelling may be relevant to a wide array of macroeconomic issues. The analysis of risk and wealth is central to macro. Self-fulfilling beliefs and strategic complementarities play an important role in many macroeconomic settings. In the marriage of these two strands in this paper, risk, wealth and portfolio effects play a central role in determining how strategic complementarities translate into economic outcomes.

Under our naive, static, complete markets model of agents' portfolio choices, we were able to derive a number of striking predictions about the likelihood of currency crises. However, our conclusions were sensitive to the market assumptions: plausible sounding incomplete market restrictions can have a dramatic impact on comparative statics. Real currency markets reflect the transaction, hedging and speculative demands of many private traders, the policy interventions of central banks and the strategies of large institutions such as hedge funds that may be hard to explain and model as the aggregation of individual utility maximizing behavior. One message of this paper is that if currency crises are self-fulfilling, the motives and strategies of market participants may be important in a way they are not in models where an arbitrage condition (and not strategic considerations) pins down the equilibrium.

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A Agent's indirect utility

The agent's final period wealth is:

$$\begin{aligned} \widetilde{w}(y, e_1) &= w_D + y + \left(\frac{w_P}{e_1} - y \frac{e_0}{e_1}(1+r) \right) \\ &= w_D + \frac{w_P}{e_1} + y \left(1 - \frac{e_0}{e_1}(1+r) \right). \end{aligned}$$

Letting q_D and q_P be the constant prices of dollar and peso denominated goods, respectively, indirect vNM utility is

$$\begin{aligned} &\left(\frac{\alpha \widetilde{w}(y, e_1)}{q_D} \right)^{\alpha} \left(\frac{(1-\alpha) e_1 \widetilde{w}(y, e_1)}{q_P} \right)^{1-\alpha} \\ = &\left(\frac{\alpha}{q_D} \right)^{\alpha} \left(\frac{1-\alpha}{q_P} \right)^{1-\alpha} e_1^{1-\alpha} \widetilde{w}(y, e_1). \end{aligned}$$

Dividing through by the constant

$$\left(\frac{\alpha}{q_D} \right)^{\alpha} \left(\frac{1-\alpha}{q_P} \right)^{1-\alpha} e_0^{1-\alpha},$$

we obtain his normalized indirect vNM utility.

B Derivation of $\widehat{\theta}$

The first order conditions for (3) imply that

$$p (\widetilde{w}(y, e_0))^{-\rho} v_D = (1-p) \left(\left(\frac{E}{e_0} \right)^{1-\alpha} \widetilde{w}(y, E) \right)^{-\rho} v_A.$$

Thus

$$\left(\frac{\left(\frac{E}{e_0}\right)^{1-\alpha}\widetilde{w}\left(y,E\right)}{\widetilde{w}\left(y,e_0\right)}\right)^{-\rho} = \left(\frac{p}{1-p}\right)\left(\frac{t}{1-t}\right),$$

$$\frac{\widetilde{w}\left(y,E\right)}{\widetilde{w}\left(y,e_0\right)} = \left(\frac{1-p}{p}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha},$$

and

$$y^*\left(p\right) = \frac{\left(\frac{1-p}{p}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}\left(w_D+\frac{w_P}{e_0}\right)-\left(w_D+\frac{w_P}{E}\right)}{\left(\frac{1-p}{p}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}r+1-\frac{e_0}{E}\left(1+r\right)}.$$

But observe that

$$\underline{y} = y^*\left(1\right) \text{ and } \overline{y} = y^*\left(0\right),$$

so

$$\begin{aligned}
\widehat{y}\left(p\right) &= \frac{y^*\left(p\right)-y^*\left(1\right)}{y^*\left(0\right)-y^*\left(1\right)} \\
&= \frac{\frac{\left(\frac{1-p}{p}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}\left(w_D+\frac{w_P}{e_0}\right)-\left(w_D+\frac{w_P}{E}\right)}{\left(\frac{1-p}{p}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}r+1-\frac{e_0}{E}\left(1+r\right)}+\frac{\left(w_D+\frac{w_P}{E}\right)}{1-\frac{e_0}{E}\left(1+r\right)}}{\frac{w_D+\frac{w_P}{e_0}}{r}+\frac{\left(w_D+\frac{w_P}{E}\right)}{1-\frac{e_0}{E}\left(1+r\right)}} \\
&= \frac{1}{1+\frac{1-\frac{e_0}{E}\left(1+r\right)}{r}\left(\frac{p}{1-p}\right)^{\frac{1}{\rho}}\left(\frac{t}{1-t}\right)^{\frac{1}{\rho}}\left(\frac{E}{e_0}\right)^{(1-\alpha)}} \\
&= \frac{1}{1+\left(\frac{p}{1-p}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}}.
\end{aligned}$$

Thus

$$\begin{aligned}
\widehat{\theta} &= \frac{\int\limits_{p=0}^{1} y^*\left(p\right)\gamma\left(p|\theta^*\right)dp-\underline{y}}{\overline{y}-\underline{y}} \\
&= \int\limits_{p=0}^{1}\widehat{y}\left(p\right)\gamma\left(p|\theta^*\right)dp \\
&= \int\limits_{p=0}^{1}\frac{1}{1+\left(\frac{p}{1-p}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}}\gamma\left(p|\theta^*\right)dp
\end{aligned}$$

C Sufficient condition for uniqueness with normal signals

From equation (8), we get that a sufficient condition for uniqueness is that for any μ and θ^*,

$$\frac{d}{d\theta^*}\int\limits_{p=0}^{1} y^*(p)\,\gamma(p|\theta^*,\mu)\,dp < 1$$

But observe that $y^*(p) \in \left[\underline{y}, \overline{y}\right]$, so

$$\int\limits_{p=0}^{1} y^*(p)\,\gamma(p|\theta^*+\varepsilon)\,dp - \int\limits_{p=0}^{1} y^*(p)\,\gamma(p|\theta^*)\,dp \leq \left(\overline{y}-\underline{y}\right)\max_p\left(\Gamma(p|\theta^*+\varepsilon)-\Gamma(p|\theta^*)\right)$$

Thus

$$\frac{d}{d\theta^*}\int\limits_{p=0}^{1} y^*(p)\,\gamma(p|\theta^*)\,dp < \left(\overline{y}-\underline{y}\right)\max_p\left(\frac{d\Gamma(p|\theta^*)}{d\theta^*}\right)$$

Thus a sufficient condition for uniqueness is

$$\left(\overline{y}-\underline{y}\right)\max_p\left(\frac{d\Gamma(p|\theta^*)}{d\theta^*}\right) < 1 \tag{17}$$

From equation (13), we get

$$x_i(p) = \theta^* + \frac{\sigma_\varepsilon^2}{\sigma_\eta^2}(\theta^*-\mu) - \sigma_\varepsilon^2\sqrt{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}}\Phi^{-1}(1-p)$$

And using equation (14)

$$\Gamma(p|\theta^*) = \Phi\left(\frac{1}{\sigma_\varepsilon}\left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}(\theta^*-\mu) - \sigma_\varepsilon^2\sqrt{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}}\Phi^{-1}(1-p)\right)\right)$$

Now

$$\frac{d\Gamma(p|\theta^*)}{d\theta^*} \leq \frac{\sigma_\varepsilon}{\sigma_\eta^2}\phi\left(\frac{1}{\sigma_\varepsilon}\left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}(\theta^*-\mu) - \sigma_\varepsilon^2\sqrt{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}}\Phi^{-1}(1-p)\right)\right) \leq \frac{\sigma_\varepsilon}{\sigma_\eta^2\sqrt{2\pi}}$$

Therefore, using inequality (17), a sufficient condition for uniqueness is that

$$\frac{\sigma_\varepsilon}{\sigma_\eta^2\sqrt{2\pi}}\left(\overline{y}-\underline{y}\right) < 1$$