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International financial crises

Teaching Resource

Lecture Notes
International Financial Crises

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Abstract

This is a compilation of my lecture notes for different courses. The choice of topics and the way I present them is influenced by my own personal opinions. It probably contains a few mistakes. It is not sufficient.to understand the papers it covers.

Nevertheless, it is useful for my teaching. If you have any comments, suggestions or if you spot any mistakes (or typos), please let me know. If you find it useful for teaching or studying, I will be very glad if you use it and send me an email to let me know.

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1 Currency crises: models

1.1 The first generation models of currency crises

Salant and Henderson (1978) showed that if the government uses a stockpile of an exhaustible resource (e.g., gold) to stabilize its price, eventually a speculative attack will occur: the private investors will suddenly acquire the entire government's stock.

Paul Krugman (1979) showed that if a pegged exchange rate cohexists with budget deficits that need to be financed by money creation, the argument in Salant and Henderson (1978) also applies: a speculative attack will force the government to abandon the pegged regime.

Flood and Garber (1984) develops the concept of "shadow exchange rate" and provide two linear examples of the logic presented by Krugman (1979). This notes covers the deterministic model of that paper. The stochastic model of Flood and Garber (1984) is also interesting.

1.1.1 Flood and Garber (1984)

In the non-stochastic version of Flood and Garber (1984), the exchange rate is initially pegged at $\bar{S}$. Money demand depends negatively on interest rates:

$$\frac{M_t}{P_t} = a_0 - a_1 i_t \tag{1}$$

Money supply equals foreign currency reserves (R_t) plus domestic credit (D_t).

$$M_t = R_t + D_t \tag{2}$$

Domestic credit is expanding:

$$\dot{D}_t = \mu \ , \ \mu > 0 \tag{3}$$

Interest rate parity (IRP) and purchasing power parity (PPP) are also assumed (i_t^* and P_t^* are constants). In the paper, the exchange rate is denoted by S, not by E.

$$P_t = S_t . P_t^* \tag{4}$$

$$i_t = i_t^* + \frac{\dot{S}}{S} \tag{5}$$

Initially, the government has a positive stock of reserves and will keep the peg until reserves reach a given minimum level (say, until $R_t = 0$). Before the peg is abandoned, $\dot{S} = 0$. By PPP (equation 4), $\dot{P} = 0$, and by IRP (equation 5), i_t is constant, equal i_t^*.

Therefore M_t is also constant (equation 1). Define M^H as the demand for money while the peg is kept:

$$\frac{M^H}{\bar{S}.P_t^*} = (a_0 - a_1 i_t^*) \tag{6}$$

In the model, the expansion of domestic credit generates loss of reserves until the moment in which the peg is abandoned. Then, it leads to an increasing trend in the money supply and, consequently, inflation. Therefore, after the peg is abandoned, the demand for real balances is smaller because the nominal interest rate is higher, due to inflation (equations 5 and 1). An arbitrage condition implies that P_t and S_t cannot jump up, and so the discrete reduction in money demand translates in a discrete fall of M_t. Initially, reserves are falling steadily, at a rate μ. When R_t is exactly equal to the difference in money demand in both regimes, all agents exchange part of their domestic currency for foreign currency and the government is forced to abandon the peg. Define M^L as the demand for money right after the peg is attacked:

$$m^L = \frac{M^L}{\bar{S}.P_t^*} = \left(a_0 - a_1\left(i_t^* + \frac{\dot{S}}{\bar{S}}\right)\right) \tag{7}$$

Now, define the shadow exchange rate ($\tilde{S}_t$) as the exchange rate that would prevail if the currency was allowed to float (demand for real balances would be m^L) and foreign reserves vanished (so that $M_t = D_t$). PPP implies that $P_t = P_t^*.\tilde{S}_t$ and we have:

$$m^L = \frac{M_t}{P_t} = \frac{D_t}{\tilde{S}_t P_t^*} \tag{8}$$

Equations 7 and 8 imply:

$$\tilde{S}_t = \frac{\bar{S}}{M^L} D_t \tag{9}$$

As Flood and Garber (1984) show, a speculative attack forces the abandonment of the peg exactly when $\tilde{S}_t = \bar{S}$.

The size of the attack In Flood and Garber (1984), a speculative attack is an instantaneous event: agents exchange some of their local currency for foreign currency (M falls) and deplete the Central Bank stock of reserves (R falls to 0). What is the lost in reserves?

Right before the attack, we have:

$$M^H = \bar{S}.P_t^*.(a_0 - a_1 i_t^*) \tag{10}$$

Right after the attack, we have:

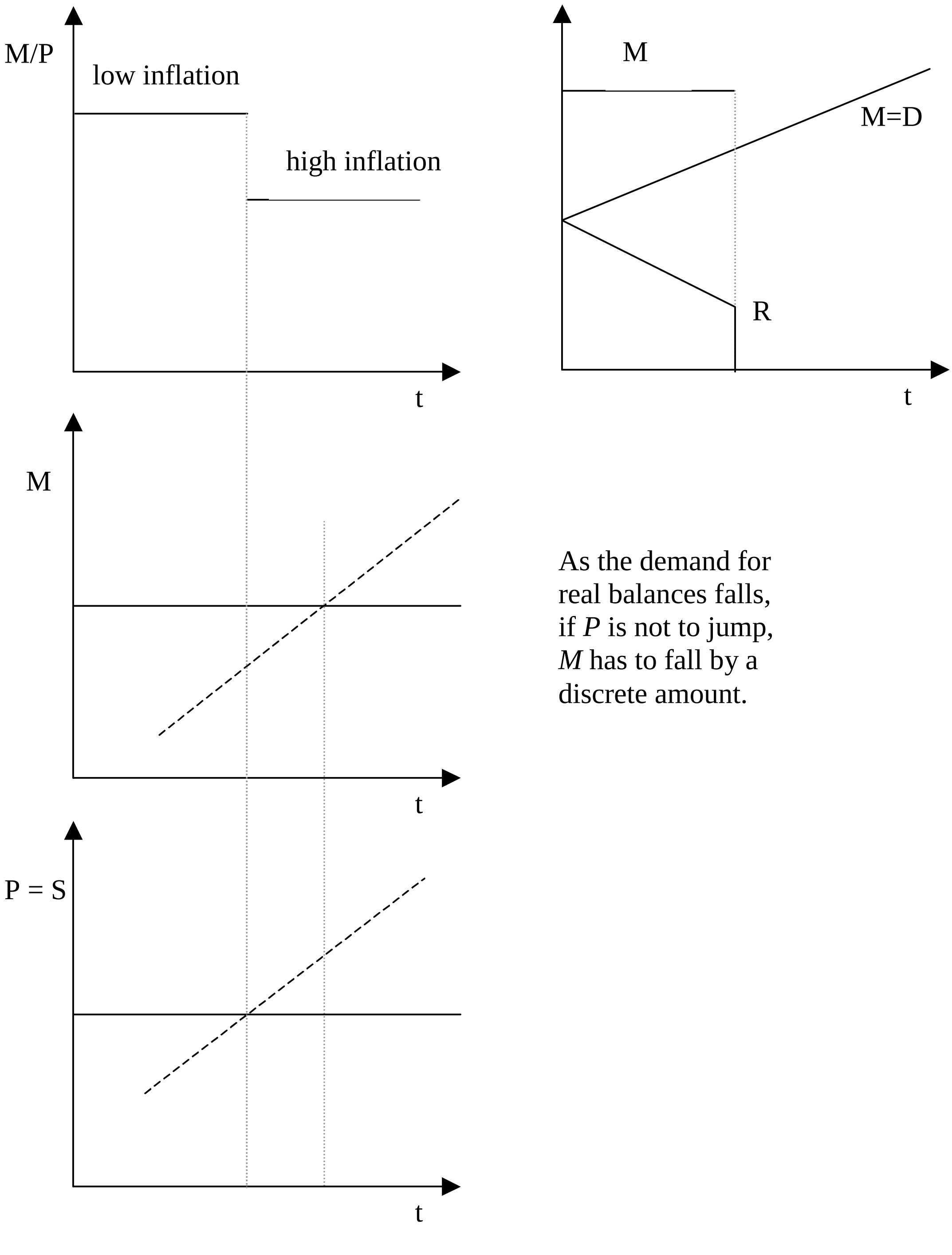

M/P
low inflation
high inflation
t
M
M=D
R
t
M
t
P = S
t
As the demand for
real balances falls,
if *P* is not to jump,
M has to fall by a
discrete amount.

Figure 1:

$$M^L = \bar{S}.P_t^*.\left(a_0 - a_1\left(i_t^* + \frac{\dot{S}}{\bar{S}}\right)\right) \tag{11}$$

Subtracting (11) from (10), we get:

$$\Delta M = M^H - M^L = \bar{S}.P_t^*.a_1.\frac{\dot{S}}{\bar{S}}$$

As $M_t = R_t + D_t$, $\Delta M = \Delta R + \Delta D$. Domestic credit is growing continuously. Therefore, D_t is the same right before and right after the devaluation ($\Delta D = 0$). So, $\Delta M = \Delta R$:

$$\Delta R = \bar{S}.P_t^*.a_1.\frac{\dot{S}}{\bar{S}} = \bar{S}.P_t^*.a_1.\frac{\dot{P}}{P}$$

Interpreting the above equations: the fall in reserves corresponds to the fall in the demand for money. The fall in the demand for money is due to inflation post-devaluation. Inflation occurs because the Central Bank has run out of reserves (so cannot finance the fiscal authority by selling reserves anymore) and thus starts to finance the fiscal authority via inflation.

Some take home points

- Inconsistency between domestic policy and exchange rate policy leads to speculative attacks. Increases in D lead either to decreases in R (reserves dwindle) or to increases in M (monetary expansion, that leads to inflation). Loss of reserves can't go forever (stock of reserves available to Central Banks is finite). At some point, increases in D lead to increases in M.
- A simple demand for money relation, arbitrage in all markets (PPP, IRP) and the increase in domestic credit lead to agents massively sell domestic currency and force the abandonment of the peg.
- A massive speculative attack is not incompatible with rational agents.
- What to do about speculative attacks? The model seems to say: *"don't shoot the messenger!"*
- The model predicts that crises are predictable, antecipated.
- Inflation follows the currency crises.

However...

European Exchange Rate Mechanism, 1993: the bark in the dark that was not heard (Obstfeld, 1996).

1.2 The second generation models of currency crises

The weak links between changes in economic variables and speculative attacks in some recent episodes (e.g., the ERM crises in 1992-3 and the contagion of 1997-8) have stimulated the idea that bad fundamentals may be a pre-condition for a crisis, but its occurrence and timing are somewhat random events. The so called second generation models of currency crisis formalize this view. This literature points out that if fundamentals are not good enough, the optimal strategy for an agent in a currency crisis game depends on expectations: if everybody is expected to attack the currency, it is optimal to attack it, but if everybody is expected to refrain from doing so, then not attacking is the optimal choice. Those models present multiple equilibria. Sudden and exogenous shifts on expectations may trigger a crisis.

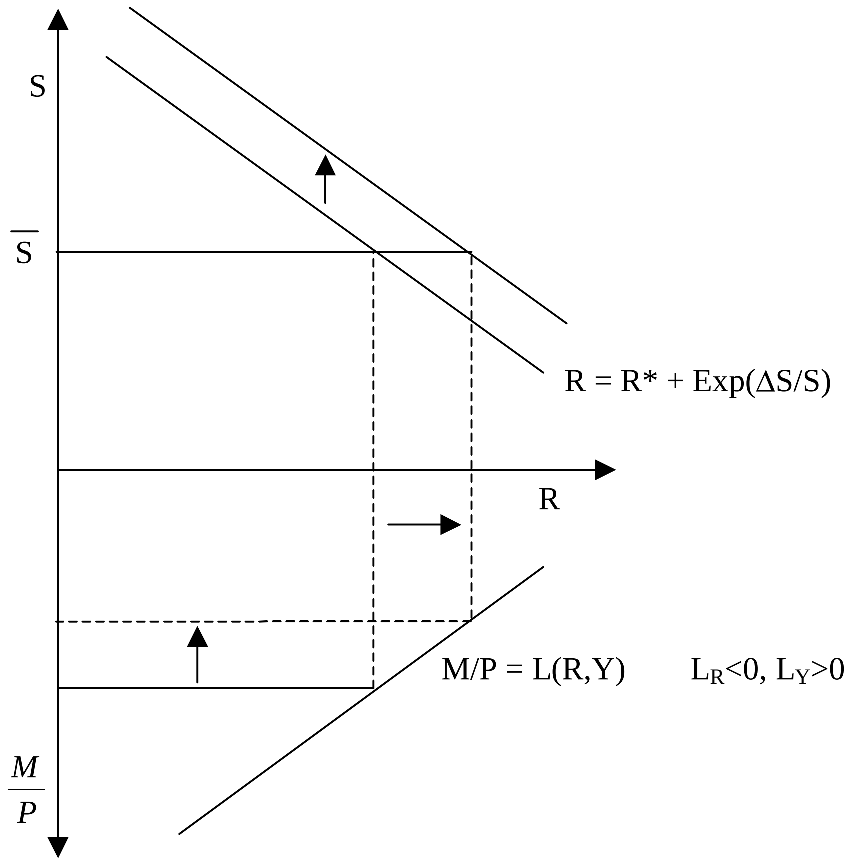


Figure 2:

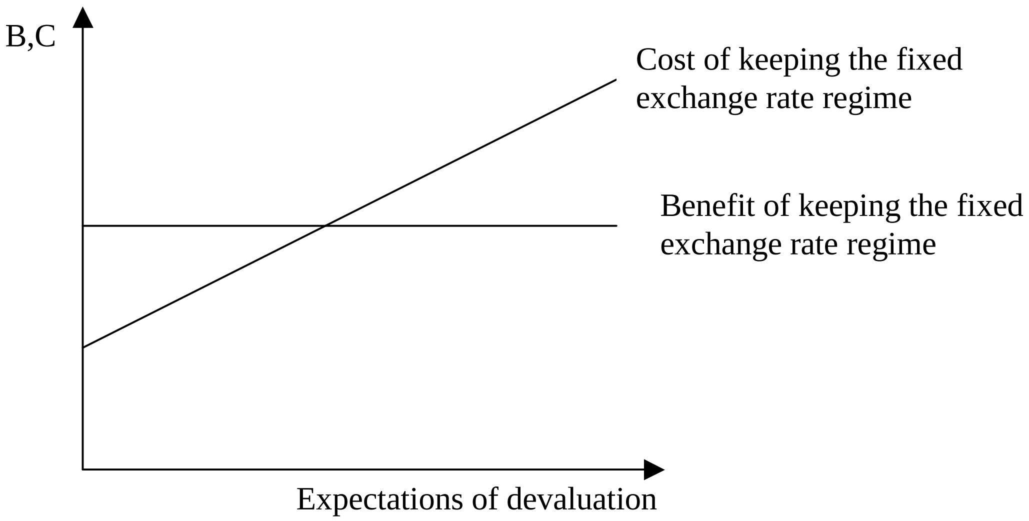
A second generation model of currency needs:

1. A reason why the government want to abandon its fixed exchange rate regime,

2. A reason why the government want to keep its fixed exchange rate regime,

3. Cost of defending the fixed exchange rate regime must be increasing in expectations of devaluation.

Figure 3: Government's decision on the exchange rate regime



What are the benefits of keeping the fixed exchange rate regime?

- Removing volatility of the exchange rate regime is good for trade, investment.
- Nominal anchor - inflation.
- Reputation.

What are the costs of keeping the fixed exchange rate regime? (See Obstfeld, 1996)

- Increases in the interest rate may slow down economy, increase unemployment.
- Distribution effects: hikes in the interest rate make mortgages more expensive, bond holders wealthier, indebted companies poorer.
- Banks may suffer when interest rates increases (we will discuss this issue further in a couple of weeks).
- If a country is highly indebted, its fiscal burden increases if expectations of a devaluation push up interest rates.
- Government may want to inflate away its debt.

Why would the costs of keeping the fixed exchange rate regime be increasing:

- The higher is the expected devaluation, the higher is the hike in the interest rates.
- Seeing from a different perspective, if one expects a currency to depreciate, he/she will sell it (or short it). The pressure for devaluation is proportional to this amount sold (or short) as the government will have to buy it or to increase incentives (interest rates) for others to hold it.

1.2.1 Obstfeld (1996)

As government's decision depends on how many agents attack the currency, self-fulfilling crises may occur. Everybody expects that the peg will be abandoned, so everybody attacks the currency. And the peg is abandoned because everybody attacked the currency. This kind of circular logic is characteristic of the second generation models of currency crises.

A simple example from Obstfeld (EER 1996) helps to clarify this point: a government that wants to fix its currency and two private holders of domestic currency who can sell it (attack the currency) or hold it (not attack). The government has R reserves to defend the peg. Each trader has domestic money resources of 6 which can be sold for reserves. To sell and take a position against the government, there is a cost of 1 (assumed to be irrespective of the amout sold, but that is not important for the results). In the event of giving up its peg, the government devalues by 50 percent (so, the traders get 1/2 unit of money for each unit they bought in the event of a successful attack).

The "high-reserve" game: $R = 20$

Trader 1 \ Trader 2	A	N
A	$-1, -1$	$-1, 0$
N	$0, -1$	$0, 0$

The "low-reserve" game: $R = 6$

Trader 1 \ Trader 2	A	N
A	$1/2, 1/2$	$2, 0$
N	$0, 2$	$0, 0$

The "intermediate-reserve" game: $R = 10$

		Trader 2	
		A	N
Trader 1	A	3/2, 3/2	$-1, 0$
	N	0, -1	$0, 0$

Take home points

- There are self-fulfilling crises in a model with very rational investors.
- *Sunspots*, events completely disconnected from the economy, may change expectations and trigger a currency crisis.
- Fixed exchange rate regimes that would work well in the absence of the speculative attack may fall without major fundamental imbalances.
- There are *strategic complementarities* between agents' actions: incentives for an agent to attack the currency are increasing in the share of agents that choose to attack the currency.
- Crises are not fully predictable.

However...

- Assumption that agents know what others are doing in equilibrium is very strong (is it?)
- Expectations are given exogenously in the model. How would you form your own expectations about what other players would do? Compare it with your own decision in the game played in class.

1.2.2 A side point: effects of increases in R^*

The effect of an increase in foreign interest rates (R^*) can also be seen at figure **??** If R^* increases...

In 1994, the US Federal Reserve Bank sharply increased interest rates. Interest rates in Mexico had to follow. Clearly, that increases the cost of keeping the fixed exchange rate regime for the Mexican government. The currency crises occurred in December 1994.

1.3 Expectations and higher order beliefs

1.3.1 The common knowledge assumption

If an event (say $\theta > 0$) is common knowledge, then everybody knows it, everybody knows that everybody knows it, everybody knows that everybody knows that everybody knows it, everybody knows that everybody knows that everybody knows that everybody knows it, and so on.

It is not easy to see why that would be different from the simple "everybody knows it". The example in Geanakoplos (1992, page 54) illustrates the strenght of the common knowledge hypothesis.

1.3.2 Beauty contests

Consider the following game:

		Player 2 A	Player 2 N
Player 1	A	θ, θ	$\theta - 1, 0$
	N	$0, \theta - 1$	$0, 0$

If $\theta \in (0, 1)$, the game has 2 Nash Equilibria: (A, A) and (N, N). What would you choose?

The optimal choice depends on the probability attached to the other agent choosing A. Let's denote this probability by p, that is, $p = \Pr(s_2 = A)$. Then, my payoff from choosing A is:

$$\begin{aligned} \pi(s_1 &= A) = p.\theta + (1 - p).(\theta - 1) \\ &= p + \theta - 1 \end{aligned}$$

So, A is the optimal choice if:

$$p > 1 - \theta$$

A natural question is: what does p depend on? What is the probability that the other player will choose A? Naturally, that depends on the probability player 2 assigns for player 1 choosing A...

Keynes, in the "General Theory of Employment, Interest and Money" (1936) wrote that: "...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgement, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees."

In a Nash equilibrium, an agent *knows* which action the other players will be choosing. So, the above mentioned reasoning is not incorporated in the second generation models of currency crises.

1.3.3 The model with incomplete information

Carlsson and Van Damme (Econometrica, 1993) show that multiplicity of equilibrium comes from two modelling assumptions:

- In equilibrium, agents know what others will do;
- All information is common knowledge.

What happens when we remove those assumptions and agents are uncertain about what others will do? An agent has to estimate the likelihood of other players attacking the currency. So, they try to assess the others' information — what do they know? what are they expecting? Like in the beauty contest example, an agent is guessing what the others guess that she knows.

Consider that agents are playing the same game they played before:

		Player 2: A	Player 2: N
Player 1	A	θ, θ	$\theta - 1, 0$
	N	$0, \theta - 1$	$0, 0$

However, θ is not observed. The *only* information agents have about θ is a noisy signal x. Formally, the prior on θ is uniformly distributed in the real line (or think of a normal distribution with $\sigma \to \infty$) For each agent i:

$$x_i = \theta + \epsilon_i$$

The error term, ϵ_i, is independent accross agents. The striking result first proved by Carlsson and Van Damme is that the above game has a unique equilibrium *even if the* *variance of the error term is very small* — for example, even if ϵ_i is distributed uniformly between -0.01 and 0.01, that is: $\epsilon_i \sim U(-0.01, 0.01)$.

1.3.4 Unique equilibrium: an intuition

Suppose that $\epsilon_i \sim N(0, \sigma)$.

From the point of view of agent 1, that got signal x_1

- $\theta \sim N(x_1, \sigma)$.
- $x_2 \sim N(x_1, \sqrt{2}\sigma)$. (Why? From the point of view of agent 1, $\theta \sim N(x_1, \sigma)$. and $x_2 \sim N(\theta, \sigma)$, so...)

The expected payoff from choosing A for an agent that got signal x_1 is:

$$E(\pi(s_1 = A)) = p + x_1 - 1$$

Dominant regions

- If $x < 0$, $E(\pi(s_1 = A)) < p - 1 < 0$. Regardless of what player 2 is choosing, player 1 is better off by choosing N.
- If $x > 1$, $E(\pi(s_1 = A)) > p > 0$. Regardless of what player 2 is choosing, player 1 is better off by choosing A.

Now, what if $0 < x < 1$? Suppose for example that $\sigma = 0.1$.

If $x_1 = 0.1$, what is the probability that agent 2 will play A?

$$\begin{aligned} \Pr(x_2 &< 0) = \Pr\left(\frac{x_2 - 0.1}{0.1\sqrt{2}} < \frac{-0.1}{0.1\sqrt{2}}\right) \\ &= \Phi\left(\frac{1}{\sqrt{2}}\right) = 0.24 \end{aligned}$$

where Φ is the standard normal cumulative distribution function. So, with probability 0.24, agent 2 got a signal smaller than 0 and will play N. If agent 2's signal is positive, we don't know what he will do, but we can say that:

$$p < 0.76$$

So, agent 1's expected payoff of attacking is:

$$E(\pi(s_1 = A)) < 0.76 + 0.1 - 1 = -0.14$$

so, agent 1 will *not attack*.

Using a similar argument, if $x_1 = 0.9$, then:

$$\begin{aligned}\Pr(x_2 &> 1) = \Pr\left(\frac{x_2 - 0.9}{0.1\sqrt{2}} > \frac{1 - 0.9}{0.1\sqrt{2}}\right) \\ &= 1 - \Phi\left(\frac{1}{\sqrt{2}}\right) = 0.24\end{aligned}$$

So, with probability 0.24, agent 2 got a signal bigger than 1 and will play A. If agent 2's signal is smaller than 1, we don't know what he will do, but we can say that:

$$p > 0.24$$

So, agent 1's expected payoff of attacking is:

$$E(\pi(s_1 = A)) > 0.26 + 0.9 - 1 = 0.14$$

so, agent 1 will *attack*.

Now, suppose that $x_1 = 0.2$. Then the probability that agent 1 gets a signal smaller than 0 is just 0.08. If all we can say is that $p < 92$, then we cannot say anything about agent 1's optimal decision in this case because

$$\begin{aligned}E(\pi(s_1 &= A)) = p + x_1 - 1 \Rightarrow \\ E(\pi(s_1 &= A)) < 0.92 + 0.2 - 1 = 0.12\end{aligned}$$

that is, from this calculation, we can't tell whether $E(\pi(s_1 = A))$ is positive or negative. How can agent 1 decide in this case?

Agent 2 is also rational and is doing the same calculations agent 1 is doing. Therefore, agent 1 knows that if $x_2 = 0.10$, agent 2 will not attack. Why is that? If agent 2 got signal $x_2 = 0.10$, he will consider that the probability that agent 1 got a negative signal is 0.24 and will not attack. Agent 1 knows that. And given that his signal $x_1 = 0.20$, he knows that agent 2 will have got a signal $x_2 \leq 0.10$ with probability 0.24 because:

$$\Pr(x_2 < 0.1) = \Pr\left(\frac{x_2 - 0.2}{0.1\sqrt{2}} < \frac{0.1 - 0.2}{0.1\sqrt{2}}\right)$$

So, he knows that $p < 0.76$ and therefore, agent 1's expected payoff of attacking is:

$$E(\pi(s_1 = A)) < 0.76 + 0.2 - 1 = -0.04$$

$E(\pi(s_1 = A))$ is negative! If $x_1 = 0.2$, agent 1 will choose N.

Now, suppose that we know that players 1 and 2 choose N if $x \leq x_L$, for some x_L.

Suppose $x_1 = x_L + \eta$, where η is a positive and very small constant, $\eta << \sigma$.

Then:

$$\begin{aligned} \Pr(x_2 &\leq x_L) = \Pr\left(\frac{x_2 - x_1}{\sqrt{2}\sigma} \leq \frac{x_L - x_1}{\sqrt{2}\sigma}\right) \\ &= \Pr\left(z \leq \frac{\eta}{\sqrt{2}\sigma}\right) \approx 0.5 \end{aligned}$$

So,

$$\begin{aligned} p &= \Pr(s_2 = A) \leq 1 - \Pr(x_2 \geq x_L) \\ &\leq 0.5 \end{aligned}$$

Then,

$$\begin{aligned} E(\pi(s_1 &= A)) = p + x_1 - 1 \\ &\leq x_1 - 0.5 \end{aligned}$$

Therefore, we can iteratively delete $s = A$ whenever $x_1 < 0.5$.

At the other side, suppose that we know that players 1 and 2 choose A if $x \geq x_H$, for some x_H.

Suppose $x_1 = x_H - \eta$, where η is a positive and very small constant, $\eta << \sigma$.

Then:

$$\begin{aligned} \Pr(x_2 &\geq x_H) = \Pr\left(\frac{x_2 - x_1}{\sqrt{2}\sigma} \geq \frac{x_H - x_1}{\sqrt{2}\sigma}\right) \\ &= \Pr\left(z \geq \frac{\eta}{\sqrt{2}\sigma}\right) \approx 0.5 \end{aligned}$$

So,

$$\begin{aligned} p &= \Pr(s_2 = A) \geq \Pr(x_2 \geq x_H) \\ &\geq 0.5 \end{aligned}$$

Then,

$$\begin{aligned} E(\pi(s_1 &= A)) = p + x_1 - 1 \\ &\geq x_1 - 0.5 \end{aligned}$$

Therefore, we can iteratively delete $s = N$ whenever $x_1 > 0.5$.

So, the unique equilibrium that survives strategically elimination of strictly dominated strategies is:

- $s_i = A$ if $x_i > 0.5$
- $s_i = N$ if $x_i < 0.5$

The argument works in the same way even if the support of ε is bounded, for example, even if $\epsilon_i \sim U(-0.01, 0.01)$. Note the higher order beliefs in action: if you get a low signal (say $x = 0.05$) you will end up playing N even though you know that θ is positive and you know that the other player knows that θ is positive. That is because he may think that you may think that he may think that .... that θ is negative. Although agents in this case know that $\theta > 0$, so the (A, A) equilibrium would yield positive payoffs for them, that is not *common knowledge*.

1.3.5 Morris and Shin (1998)

Morris and Shin (1998), building on Carlsson and Van Damme (Econometrica, 1993), endogenize expectations in a model of currency attacks.

Consider an economy with a continuum of players and denote by l the proportion of players that choose to attack. The government has a constant benefit for holding the peg and a cost that depends negatively on fundamentals (θ) and positively on the proportion of agents that choose to attack (l). We will consider that the cost will exceed the benefit if $l - \theta > 0$. So, the government abandons the peg if $l > \theta$.

The information structure is as before ($x_i = \theta + \epsilon_i$). An agent chooses between 'attack' (A) and 'not attack' (N). If the agent chooses N, she does not win or lose anything, her payoff is 0 regardless of what others do. The cost of attacking is t. If she attacks and there is a devaluation, she gets 1. So, if she chooses A, her payoff if $1 - t$ is there is a devaluation and $-t$ otherwise. Suppose that $\epsilon_i \sim N(0, \sigma)$.

Here, if θ is negative, the government abandons the peg regarless of what agents do and if $\theta > 1$, the peg if kept even if all agents decide to attack. The agent will attack the currency if she perceives that fundamentals are weak, that is, only if she thinks that θ is small.

The model with common knowledge of fundamentals Suppose for a while that all agents observe θ. Then, the model is a second generation model.

Suppose everyone attacks the currency ($l = 1$). Then the government abandons the peg if $\theta < 1$. In this case, it is optimal for an agent to attack the currency if $\theta < 1$.

Suppose that noone attacks the currency ($l = 0$). The the government abandons the peg if $\theta < 0$. In this case, it is optimal for an agent not to attack the currency if $\theta > 0$.

Thus, if $\theta < 0$, fundamentals are too weak, everybody attacks the currency and the government leave the peg. If $\theta > 1$, nobody attacks and the peg survives. But if $0 < \theta < 1$, both equilibria exist.

The model with imperfect information about θ Now, let's return to the case when θ is not observed and agents have just imperfect information about it.

Morris and Shin (1998) show that there is a unique equilibrium, characterized by two thresholds:

1. an agent attacks only if her signal x_i is smaller than x^*.
2. the government abandons the exchange rate peg only if $\theta < \theta^*$.

2 conditions pin down the equilibrium:

1. An agent who gets signal x^* is indifferent between attacking or not attacking.
2. When $\theta = \theta^*$, the fraction of agents that attack the currency is just enough to make the government abandon the peg.

An agent that gets signal x^* asks himself: what is the probability that the attack will succeed?

- The question is: what is the probability that $\theta < \theta^*$?
- From that agent's point of view, $\theta \sim N(x^*, \sigma)$. Saying differently, $\theta = x^* - \epsilon_i$.
- So, $\Pr(\theta < \theta^*) = \Pr(x^* - \epsilon_i < \theta^*) = \Pr(-\epsilon_i < \theta^* - x^*) = \Pr(\epsilon_i < \theta^* - x^*)$. Thus:
$$\Pr(\theta < \theta^*) = \Phi(\frac{\theta^* - x^*}{\sigma})$$
where Φ is the cumulative standard normal distribution.

So, The expected payoff of attaking is:

$$\begin{aligned} E(payA) &= (1-t).\Phi(\frac{\theta^* - x^*}{\sigma}) - t(1 - \Phi(\frac{\theta^* - x^*}{\sigma})) \\ &= \Phi(\frac{\theta^* - x^*}{\sigma}) - t \end{aligned}$$

The agent is indifferent when:

$$\Phi(\frac{\theta^* - x^*}{\sigma}) - t = 0 \tag{12}$$

The second equilibrium condition: when $\theta = \theta^*$, the fraction of agents that attack the currency is just enough to make the government abandon the peg. But when $\theta = \theta^*$, what is the proportion of agents that choose to attack?

- As we have many agents, the question is: what is the proportion of agents that get a signal x such that $x < x^*$?
- If $\theta = \theta^*$, $x = \theta^* + \epsilon_i$.
- So, $\Pr(x < x^*) = \Pr(\theta^* + \epsilon_i < x^*) = \Pr(\epsilon_i < x^* - \theta^*)$. Thus:

$$\Pr(x < x^*) = \Phi(\frac{x^* - \theta^*}{\sigma})$$

When $\theta = \theta^*$, the cost and the benefit of government keeping the peg are the same. So:

$$\theta = l \Rightarrow \theta^* = \Phi(\frac{x^* - \theta^*}{\sigma}) \qquad (13)$$

Remember that:

$$\Phi(\frac{\theta^* - x^*}{\sigma}) = 1 - \Phi(\frac{x^* - \theta^*}{\sigma})$$

So, the two equilibrium conditions (equations 12 and 13) yield:

$$1 - \theta^* - t = 0$$

So:

$$\theta^* = 1 - t$$

1.3.6 A few take home points

- If agents are trying to guess what others are trying to do in a currency crises situation, expectations are crucial for the final outcome but are not disconnected from economic fundamentals.
- In this simple model, expectations depend only on prices (t) and fundamentals (θ). However, more elaborated models using this technique are able to show other interesting features of expectations (for example, the effect of public information in crises).

1.4 Rational herd behavior

1.4.1 Bikhchandani, Hirshleifer and Welch (1992)

Here I go through the model at section 2 of BHW with a slight modification: BHW assume that if the agent is indifferent, she will adopt either action with probability 50%. I will assume that if the agent is indifferent, he will follow its own signal. It makes no difference in the general message but simplifies the algebra a bit.

Suppose agent 1 receives signal H (without loss of generality). Then, from his point of view:

$$\begin{aligned}\Pr(V &= 1|H) = p \\ \Pr(V &= 0|H) = 1 - p\end{aligned}$$

Since $p > 0.5$, he chooses to *adopt*. Thus, the following players know he has got signal H.

Suppose agent 2 receives signal H. From her point of view:

$$\Pr((V = 1)|(H, H)) = \frac{\Pr((V = 1) \cap (H, H))}{\Pr((H, H) \cap (V = 0)) + \Pr((H, H) \cap (V = 1))}$$

So:

$$\Pr((V = 1)|(H, H)) = \frac{\frac{1}{2}p^2}{\frac{1}{2}p^2 + \frac{1}{2}(1 - p)^2}$$

Similarly,

$$\Pr((V = 0)|(H, H)) = \frac{\frac{1}{2}(1 - p)^2}{\frac{1}{2}p^2 + \frac{1}{2}(1 - p)^2}$$

And, as $p > \frac{1}{2}$, she chooses to *adopt*. If she receives signal L, then, from her point of view:

$$\begin{aligned}\Pr((V &= 1)|(H, L)) = \frac{\frac{1}{2}p(1 - p)}{\frac{1}{2}p(1 - p) + \frac{1}{2}p(1 - p)} = \frac{1}{2} \\ \Pr((V &= 0)|(H, L)) = \frac{\frac{1}{2}p(1 - p)}{\frac{1}{2}p(1 - p) + \frac{1}{2}p(1 - p)} = \frac{1}{2}\end{aligned}$$

She is indifferent. So, according to my tie-breaking convention, she follows her own signal and decides to *reject* — in the paper, due to their tie-breaking rules, she takes either action with 50% probability.

Now, consider agent 3. If agents 1 and 2 have taken different actions. Agent 3 knows that they have got different signals. Thus, before agent 3 sees her own signal, she thinks that: $\Pr(V = 0) = \Pr(V = 1) = \frac{1}{2}$. So, the game is as in the beginning.

What if agent 1 and 2 have both decided to *adopt*? Agent 3 knows that both have received signal H. If she also receives signal H, she will also decide to adopt, as from her point of view, $\Pr(V = 1) > \Pr(V = 0)$ — is it clear for you without doing the calculations? But what if she receives signal L? Then, she will think: "3 signals, 2 H and 1 L. Which is more likely: $(V = 0)$ or $(V = 1)$?"

$$\begin{aligned}\Pr((V &= 1)|(H, H, L)) = \frac{\frac{1}{2}p^2(1 - p)}{\frac{1}{2}p^2(1 - p) + \frac{1}{2}p(1 - p)^2} \\ \Pr((V &= 0)|(H, H, L)) = \frac{\frac{1}{2}p(1 - p)^2}{\frac{1}{2}p^2(1 - p) + \frac{1}{2}p(1 - p)^2}\end{aligned}$$

Again, as $p > \frac{1}{2}$, she chooses to adopt, although she got signal L. The intuition is: more signals pointing to $(V = 1)$ makes $(V = 1)$ more likely.

What will agent 4 do? She knows that agent 3 would *adopt* anyway, so agent 3's action is uninformative. She knows that agents 1 and 2 have received signal H. Thus, she is in the same situation of agent 3 and will *adopt* regardless of her signal. Do I need to repeat this spiel for agent 5?

That characterizes an information cascade. Agents act regardless of their own information, not because they are crazy but precisely because they are rational — they extract information from others' actions instead of ignoring it.

A key (and interesting) feature of the equilibrium is that agents may observe many others moving before them, but only the actions of the first ones are informative. The others are just following the herd. So, the information they had is not passed ahead.

Discussion of the key assumptions:

- Action: 0 or 1. If there was a continuous set of actions and agents' optimal action depended on the probability they assigned to true value being 1, everybody would infer the signal received by an agent from her action. Thus, there will be no information cascade: individuals would always consider their own information and eventually the true state would be known.
- Exogenous order of movements (which can be relaxed, see Caplin and Leahy, 1994)
- My signal and your signal have (almost) the same weight in my decisions. What do you think of this assumption? Is it rational?

But let's be careful: conformity of behavior does not need to imply an informational cascade.

Examples:

- If my payoff depends on your actions, we may choose the same action due to the strategic complementarities discussed above, not because of herding.
- Conformity of behavior may be related to preferences: my willingness to go to a party may be an increasing function of the others' decisions.
- Following that idea, suppose that you work for an asset management company. If you lost money in Asia and all your peers did the same, or if you all profit from investing there, this is business as usual for you. If you are the only one that lost

money there, your job may be in danger. On the other hand, if you are the only one that got it right, you may get an extra bonus. However, you may prefer the former safe choice to the latter risky lottery, so it may be rational for you to mimic market behavior.

1.5 Banking crises and moral hazard

1.5.1 The moral hazard effect

The game: 'heads I win, tails the taxpayeres lose':

Suppose that there is a potential investment that will cost \$70 million up front. If all goes well, the project will yield \$90 million. That will occur with probability 1/2. But with probabilty 1/2, the return will only be \$30 million. The expected payoff then is $(1/2 \times \$90m) + (1/2 \times \$30m) = \$60$ million. Ordinarily, this investment would never be made.

However, bailout garantees change the result. Suppose that an investor is able to borrow the entire \$60 million because everyone (including him and the lenders) knows that the government will protect them if his project fails and he cannot repay. Then, from his point of view, he will make \$20 million $(= \$90m - \$70m)$ with probability half and walk away with nothing with probability half.

The solution seems to be simple: the government cannot bailout such projects. But that is not simple. Diaz-Alejandro (1985), studies the example of Chile in the 80's. "Good-bye financial repression, hello financial crash", that is what happened. Lots of bank regulations were removed with the aim of ending financial repression and the government of Chile had pledged not to bail-out banks if they crashed. However...

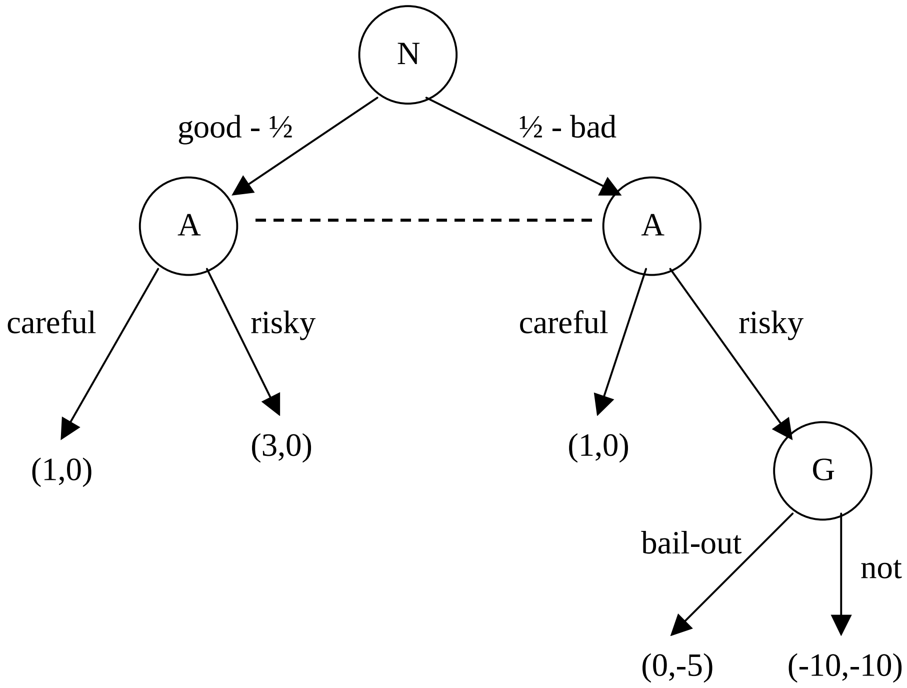
The externality A bank is a borrower and a lender. Its assets are bonds, loans, other financial assets. Its liabilities are loans, bonds and other financial instruments. If a bank crashes and it cannot pay its debts, its bankrupcy can lead to the failure of other banks and companies — it may be the first domino to fall — because its liabilities are other banks and companies assets.

So, bailing-out a failing bank is costly. But not bailing-out may be worse, due to the negative externalities spread to the rest of the economy.

The credibility issue Consider the game shown at figure 4.

Players:

Figure 4: The game



- N: nature,
- A: agent (banks, big companies, lenders...),
- G: government.

When the government says: "I will not bail out insolvent banks/companies", the government is saying: "I will play *not*". If the government is indeed playing not, the agent gets payoff of 1 if he plays *careful* and expected payoff of -3.5 if he plays *risky.* Thus, the agent playing *careful* and the government playing *not* is a Nash equilibrium: nobody has any incentive to deviate.

However, this Nash equilibrium is not credible (it is not a Subgame Perfect Nash Equilibrium). If we get into a situation in which the government is called to action, choosing *not* yields -10, while choosing *bail-out* yields -5. None is great, but *bail-out* causes less damage. So, that is what the government chooses. Now, the agent knows this, the threat of not bailing-out is not *credible.* Thus, the agent gets payoff of 1 if he plays *careful* and expected payoff of 1.5 if he plays *risky.* Thus, the agent plays *risky* and, in the event of a bad state of nature, hello financial crash.

The key assumption in this game is that, once a banking crisis takes place, the payoff of not bailing out the banks is smaller than the payoff of helping them. Diaz Alejandro (1985) argues that is true in reality: promises to play *not* at the last node are not credible.

Policy implication: regulation, ex-ante actions are needed.

1.5.2 The Asian crisis of 1997

Many analysts have considered that moral hazard — the game 'heads I win, tails the taxpayers lose' — played an important role in the Asian crisis of 1997. The paragraphs below are taken from Corsetti, Pesenti and Roubini (1999), with minor changes.

"In interpreting the Asian meltdown, one should consider three different, yet strictly interrelated dimensions of the moral hazard problem at the corporate, financial, and international level. At the corporate level, political pressures to maintain high rates of economic growth had led to a long tradition of public guarantees to private projects, some of which were effectively undertaken under government control, directly subsidized, or supported by policies of directed credit to favored firms and/or industries. In light of the record of past government intervention, the production plans and strategies of the corporate sector largely overlooked costs and riskiness of the underlying investment projects. With financial and industrial policy enmeshed within a widespread business sector network of personal and political favoritism, and with governments that appeared willing to intervene in favor of troubled firms, markets operated under the impression that the return on investment was somewhat 'insured' against adverse shocks.

Such pressures and beliefs accompanied a sustained process of capital accumulation, resulting into persistent and sizable current account deficits. While common wisdom holds that borrowing from abroad to finance domestic investment should not raise concerns about external solvency – it could actually be the optimal course of action for undercapitalized economies with good investment opportunities – the evidence for the Asian countries in the mid-1990s highlights that the profitability of new investment projects was low.

Investment rates and capital inflows in Asia remained high even after the negative signals sent by the indicators of profitability. Consistent with the financial side of the moral hazard problem in Asia, the crucial factor underlying the sustained investment rates was excessive borrowing by national banks abroad, corresponding to high and excessive investment at home. Financial intermediation played a key role in channeling funds toward projects that were marginal if not outright unprofitable.

The adverse consequences of these distortions were crucially magnified by the rapid process of capital account liberalization and financial market deregulation in the region during the 1990s, which increased the supply-elasticity of funds from abroad. The extensive liberalization of capital markets was consistent with the policy goal of providing a large supply of low-cost funds to national financial institutions and the domestic corporate sector. The same goal motivated exchange rate policies aimed at reducing the volatility of the domestic currency in terms of the US dollar, thus lowering the risk premium on dollar-denominated debt.

The international dimension of the moral hazard problem hinged upon the behavior of international banks, which over the period leading to the crisis had lent large amounts of funds to the region's domestic intermediaries, with apparent neglect of the standards for sound risk assessment. Underlying such overlending syndrome may have been the presumption that short-term interbank cross-border liabilities would be effectively guaranteed by either a direct government intervention in favor of the financial debtors, or by an indirect bailout through IMF support programs. A very large fraction of foreign debt accumulation was in the form of bank-related short-term, unhedged, foreign-currency denominated liabilities: by the end of 1996, a share of short-term liabilities in total liabilities above 50% was the norm in the region. Moreover, the ratio of short-term external liabilities to foreign reserves – a widely used indicator of financial fragility – was above 100% in Korea, Indonesia and Thailand."

The banking crisis and Flood and Garber (1984) What is the link between the banking crisis and the currency crisis? Corsetti, Pesenti and Roubini (1999) say:

"To satisfy solvency, the government must then undertake appropriate domestic fiscal reforms, possibly involving recourse to seigniorage revenues through money creation (that means, inflation, increase in P, that leads to the exchange rate devaluation). Speculation in the foreign exchange market, driven by expectations of inflationary financing, causes a collapse of the currency and anticipates the event of a financial crisis. Financial and currency crises thus become indissolubly interwoven in an emerging economy characterized by weak cyclical performances, low foreign exchange reserves, and financial deficiencies eventually resulting into high shares of non-performing loans."

In Flood and Garber (1984), the speculative attack results from expectations of inflationary financing. This pushes up the "shadow exchange rate" (in Flood and Garber, as PPP is assumed, that occurs instantaneously) and that leads to a speculative attack. Here, expectations of money creation come not through the expansion of domestic credit but from the fragility of the economy and the banking system (they point the expectations of bail-outs as key factor for that).

Other analysts will argue that those Asian countries had not bad economic fundamentals and that a self-fulfilling crisis in the form described by the second generation models happened. Radelet and Sachs (1998) is a well known example of a paper defending such view.

1.6 Contagion

There is large evidence that a crisis in a country makes other countries more subject to crises. The Mexican crisis in the end of 1994 had large impacts in other Latin American countries, especially in Argentina. The Asian crisis started in Thailand in July/1997 and was spread to many other countries in Southern Asia. Latin American countries were also affected (the Asian Flu). The Russian crisis in 1998 also had strong impacts in other developing countries like Brazil and Mexico (the Russian virus). The Brazilian devaluation in 1999 let Argentina in a more fragile position.

What are the possible linkages between countries?

- Trade links.
- Information.
- The magnifying effect of public information.
- Changes in "first world" prices: major economic shifts in developed countries may have strong effects on developing countries. Examples are the impacts in Latin America of changes in US interest rates and the effects of the devaluation of the Yen with respect to the dollar in 1995-1996.
- Other financial links: if countries are financially integrated, it is natural to expect that a crisis in a country will have effects on the other for the same reason that the performance of different sectors in an economy affect each other.
- Liquidity: I lost money in a country, need to withdraw from some other...

1.6.1 The effect of public information

Let's look at the magnifying effect of public information.

Consider again the following game:

		Player 2	
		A	N
Player 1	A	θ, θ	$\theta - 1, 0$
	N	$0, \theta - 1$	$0, 0$

Suppose that agents do not observe θ. The information agents have about θ is a noisy private signal x_i and a public signal y. For each agent i:

$$x_i = \theta + \varepsilon_i \ , \ \varepsilon_i \sim N(0, \sigma^2)$$

and y is the same for all agents:

$$y = \theta + \eta \ , \ \eta \sim N(0, \sigma^2)$$

The private signal is the result of your own analysis of the economy. The public signal is in the first page of the Financial Times and you are sure that everybody will read it.

For concreteness, let's suppose an example in which $\theta = 0.5$, $x_1 = 0.4$, $x_2 = 0.4$ and $y = 0.6$. Remember that in the case with only private information, agents would be indifferent when $x_i = 0.5$. The question is: what happens in this case?

From the point of view of agent 1:

- $E(\theta) = 0.5$.
- $E(x_2) = E(\theta + \varepsilon_2) = E(\theta) + E(\varepsilon_2) = 0.5 + 0 = 0.5$

Now, what does agent 1 expect about agent 2's expected x_1?

- Agent 1 considers that agent 2 has 2 pieces of information: x_2 and y.
- From the point of view of agent 1, $E(x_2) = 0.5$ and $E(y) = 0.6$.
- So, $E_1(E_2(x_1)) > 0.5$ (it is a (weighted) average between 0.5 and 0.6).

Notice that both agents think that θ is 0.5 plus an error term. But both agents think that the other agent's estimate of θ is bigger than 0.5.

In this case, agents would be indifferent if all their information were private. But both expect that the other will attack because the information in the front page of the Financial Times matters more for what I think that you think.

Notice that here there is no herding: agents are not learning from what others are doing but trying to infer what others are expecting. Public information matters more than private information because it has a higher impact on expectations and expectations matter in a coordination game (as well as in a currency crisis situation).

2 Currency crises: empirics

Currency crises are rare events, so data explicitly relating to them are relatively scarce. But that problem may be overcome using financial price data, which are abundant and reflect expectations about currency devaluations.

2.1 Expectations implicit in financial prices

2.1.1 Rose and Svensson (1994)

Rose and Svensson (EER 1994) estimate expectations of changes in the exchange rate before the ERM crisis in 1992. Key questions are: (i) was the crisis expected? (ii) how much of the change on expectations are explained by macroeconomic variables? Their answers are: (i) basically no and (ii) basically nothing.

Measuring credibility Notation: δ_t is the interest rate differential between a given country and Germany and s_t is the exchange rate (price of a Deutschemark in domestic currency). Uncovered interest parity implies:

$$\delta_t = \frac{E\left[\Delta s_t\right]}{\Delta t}$$

Now, the exchange rate was allowed to fluctuate inside a band. Thus, s may be written as $s = x + c$, where c is the (log of) the central parity and x denotes deviations of the spot rate from c. Expected depreciation can be separated in 2 parts:

$$\frac{E\left[\Delta s_t\right]}{\Delta t} \equiv \frac{E\left[\Delta x_t\right]}{\Delta t} + \frac{E\left[\Delta c_t\right]}{\Delta t}$$

They use 2 measures of realignment expectations:

1. measure is δ_t. It assumes $E\left[\Delta x_t\right]/\Delta t = 0$.
2. Estimates $E\left[\Delta x_t\right]/\Delta t$ with a linear regression:

$$\frac{\Delta x_t}{\Delta t} = \sum \alpha_i + \beta . x_t + \gamma . \delta_t + u_{t'}$$

How to interpret it? $g = 10\%$ may mean an expected realignment of 10% with hazard rate 1/year or an expected realignment of 1% with hazard rate 10/year...

Fundamentals? Rose and Svensson try to assess the impacts of macroeconomic fundamentals on some macroeconomic variables with:

- a regression of g in a set of macroeconomic variables (with country and time dummies).
- the estimation of a VAR.

They find that macroeconomic variables have very little impact on credibility.

What to take out of this? Positive results would be hard to be explained given the lack of a structural model and, especially, the endogeneity of many of the regressors. The authors claim that it is more difficult to dismiss negative results — if there was some important relation between those macroeconomic variables and the credibility of the exchange rate parity, their analysis should have detected it.

The analysis is very simple. Both the estimation of credibility and the assessment of the importance of fundamentals are quite atheoretical. How can it be improved?

2.2 Expectations and option prices

The exchange rate risk in a pegged regime depends on the probability that the peg will be abandoned and on the expected size of a consequent currency devaluation. To a first order approximation, the forward premium (or the interest rate differential) is roughly the product of these two variables. However, observing the forward premium alone does not permit individual identification of the probability of a devaluation and its expected magnitude: a forward premium of 3% a year may refer to an expected devaluation of 30% with probability 10% a year, or an expected devaluation of 5% with probability 60% a year, and so on.

Options are a richer source of data because they provide information about the probability density of the exchange rate at different points. So it is possible to disentangle the "thickness of the tail of the distribution" (probability of a devaluation) and the "distance from the tail to the center" (the expected magnitude of a devaluation).

To give a simple intuition for identification, suppose the price of an asset tomorrow will be 1 with probability $1-p$ and 3 with probability p. In a risk-neutral world, a call option with strike price 1 costs $2p$, a call option with strike price 2 costs p. If the probability of a devaluation (p) increases, both options get more expensive but the ratio of their prices remains equal to 2. If the magnitude of the devaluation increases from 3 to 4, the option with strike price 1 will cost $3p$, a call option with strike price 2 will cost $2p$ — the ratio changes.

2.2.1 The Black and Scholes model

Before we start studying the options, let's take a look at the basic asset pricing model.

The Black and Scholes model assumes that the asset follows:

$$\frac{dS}{S} = \mu.dt + \sigma.dZ$$

where:

- μ is the instantaneous expected return on the asset (ex-dividend);
- σ is the instantaneous variance conditional on no jumps and
- Z is a standard Wiener process;

The B&S model is the benchmark model in financial applications. According to the model, the distribution of returns on an asset is log-normal. The data, however, does not fully comply with the B&S formula. In particular, the tails of the distribution are too thick.

The B&S price of an (European) option depends on observed variables (interest rate, spot value of the asset, strike price, time to maturity) and the volatility. With the price of an option, we can calculate its implicit volatility. Usually, we observe that the implicit volatility increases with $|S - X|$. This generates the so called *volatility smiles*.

2.2.2 Campa, Chang and Refalo (2002)

Campa, Chang and Refalo (JDE 2002) use options to measure the credibility of Brazilian exchange rate regime. Among financial prices, options are better sources of information on the expectations about a peg than future prices (or the interest rate differential) because their value at maturity is nonzero only if the exchange rate goes beyond a certain level (the strike price). So, if there is data on options of different strike classes, there is information about the probability density of the exchange rate at different points, and it is possible to uncover more information about the expectations on the path of the exchange rate. For example, it is possible to identify the probability that the currency peg will be abandoned and the expected magnitude of a devaluation (conditional on its occurrence). Campa *et al* employ a very interesting non-parametric approach (see also Campa and Chang, 1996).

Market expectations and option prices Under risk neutrality, the price of a call with strike price K and expiration date T is:

$$C_{K,T} = \frac{1}{1+i_t} \int_K^{\infty} (S_T - K).f(S_T).dS_T$$

Differentiating if we respect to K, we get:

$$\frac{\partial C_{K,T}}{\partial K} = -\frac{1}{1+i_t}.\left[\int_K^{\infty} f(S_T).dS_T\right] = -\frac{1}{1+i_t}.\left[1 - F(K)\right]$$

Differentiating again, we get:

$$\frac{\partial^2 C_{K,T}}{\partial K^2} = \frac{1}{1+i_t}.f(K)$$

Intuition:

A one-dollar increase in the strike price decreases the value of the call by (the present value of) an amount equal to the probability that the option will finish "in-the-money". The higher the probability the option finishes in the money, the more likely a one-dollar increase in the strike price will matter to the option holder and the greater the decrease in the option price.

Thus, the second partial derivative of the option price with respect to K yields the probability density function of the exchange rate at date T.

The probability functions derived from option prices are the so called "risk-neutral" probabilities. They can differ from the real pdf's due to risk considerations, but nevertheless they reveal important information about expectations about an asset.

Estimation If we had a continuous of options (or, if we had a lot), we could just evaluate numerically the derivatives. The available data is definitely not enough for that.

We could do some interpolation (e.g., spline) and calculate the pdf. Problem: the call prices are not always a convex function (even without interpolation). We do not want negative pdf's.

Methodology Campa et al employ: they obtain the implied volatility of each option as a function of the strike price. That yields a "volatility smile". Then, they transform it into a continuous call price function that is twice-differential in strike — which can be done either by fitting the implied volatility as a quadratic function or by some cubic spline interpolation. Having the price of a call option as a continuum function of the strike prices, we apply the formula and get the densities.

Again, fundamentals? They ask the question: can realignment "intensity" be explained by the usual macro variables? They regress their measure of intensity of devaluation in a bunch of macro variables. The intensity measure is the following:

$$G(T) = \int_{\bar{S}}^{\infty} (S_T - \bar{S}).f(S_T).dS_T$$

which implies:

$$G(T) = C_{\bar{S},T}.(1 + i_T)$$

Results: the level of international reserves is the only significant variable in the regression (and it is endogenous). They conclude that results are consistent with past evidence: "macroeconomic variables are largely unable to explain intertemporal movements in realignment risk".

There is no theory behind their regression. Should the macro variables impact probability, expected magnitude, or both?

2.2.3 Bates (1991)

The question David Bates is asking is: was the crash of '87 expected? That is: were put options too expensive prior to the crash?

In the first section of the paper, Bates examines the skewness of the implicit distribution. He finds that in the year leading up to the crash the probability of a fall was higher than the probability of a large increase. I will jump to the second section in which he presents a model (actually very similar to Merton, 1976) and estimates its parameters implicit in the prices of options.

The model The asset (in this specific case, the S&P 500 index) is assumed to follow:

$$\frac{dS}{S} = (\mu - \lambda.k).dt + \sigma.dZ + k.dq$$

where:

- μ is the instantaneous expected return on the asset (ex-dividend);
- σ is the instantaneous variance conditional on no jumps;
- Z is a standard Wiener process;
- k is the random percentage jump conditional on its occurrence. It is lognormally distributed:
  - $\ln(1+k) \sim N(\gamma - \delta^2/2, \delta^2) \equiv N(\gamma', \delta^2)$.
  - $E(k) \equiv \bar{k} = e^{\gamma} - 1$
- λ is the hazard rate of the Poisson event and
- dq is the Poisson counter: $\Pr(dq = 1) = \lambda.dt$

His proposition 2 shows that contingent claims are priced as if investors were risk-neutral and the asset price followed a similar jump diffusion (page 1025) with different parameters. Saying differently, by estimating the above model, we are obtaining the risk-adjusted parameters.

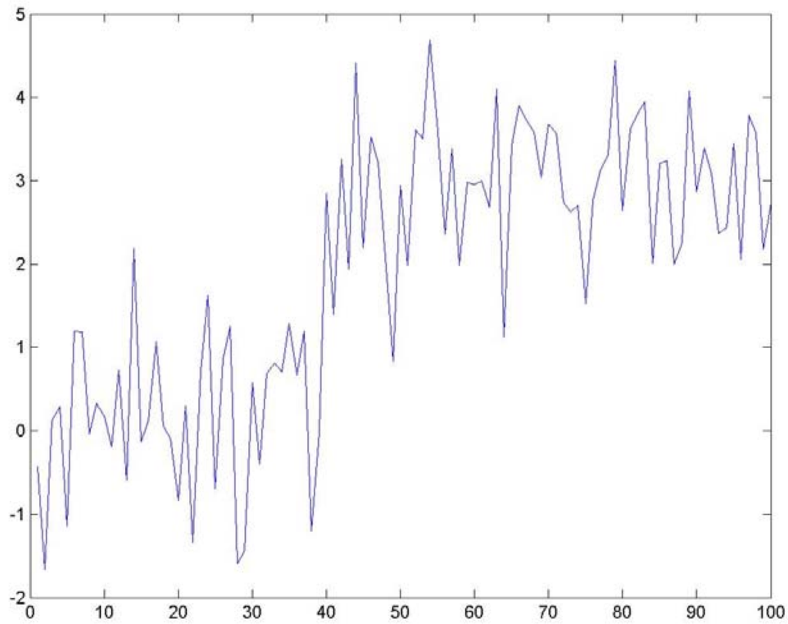


Figure 5:

The asset follows a Black and Scholes diffusion path almost all the time. Sometimes, a Poisson event happens (hazard rate is λ) and there is a discrete jump.

The parameters of this model are:

- the volatility σ;
- the hazard rate λ;
- the mean of the jump γ; and
- the standard deviation of the jump δ.

How changes in each parameter changes the distribution of probability of an European option:

- σ: increases standard deviation of the no-jump scenario;
- λ: increases the weight of the jump scenario;
- γ: pushes the part of the pdf that refers to the sump scenario further away from the no-jump scenario;
- δ: increases standard deviation of the jump scenario.

How each pararameter influences the prices of call options?

Estimation The model yields a closed form solution for the price of a call option (equation 11 in the paper). Then, it is assumed that the observed price of an option equals the corresponding model price plus an additive distrubance term (it could be a multiplicative error term also, both have some inconveniences).

Then, a cross-sectional data sample with identical maturities was used and implicit parameters were estimated via non-linear least squares for all days in the sample. The parameters are not constrained to be constant over time.

Results It is hard to argue that the estimation succeeded in making a clear distinction between the probability and the expected magnitude of the devaluation (figure 6). It seems that the estimation is picking up something happening at the tail, but it really can't tell what is probability and what is magnitude.

Figure 7 shows the "price of risk" ($\lambda.k$). We can see that $\lambda.k < 0$ especially from June-1987 to August-1987 (dates are not shown in the horizontal axis, June to August is the period with higher risk of a jump down.

Interestingly, in the 2 months right before the crash, the risk of a jump implicit in option prices is much smaller. Even in the Friday right before the crash, there is no sign of risks of an immediate collapse of stock prices.

2.2.4 Guimaraes (2007)

Time for me to talk about my own work!

Guimaraes (2007) presents procedure for testing whether currency crises depend on sunspots or are triggered when the overvaluation hits a threshold.

The idea is the following: if crises are triggered by currency overvaluation crossing a threshold, the expected magnitude of a devaluation, conditional on its occurrence, is equal to the threshold value, which may differ substantially from the unconditional expected currency overvaluation. On the other hand, if crises are triggered by sunspots, uncorrelated with the economic variables that determine the exchange rate in a floating regime, then the expected magnitude of a devaluation conditional on its occurrence is similar to the unconditional expected currency overvaluation.

The probability and the expected magnitude of a devaluation are not observable but can be estimated using data on exchange rate options. Guimaraes(2007) identifies the probability and expected magnitude of a devaluation of Brazilian *Real* in the period leading up to the end of the Brazilian pegged exchange rate regime and contrasts the estimates to the predictions from a simple model of currency crises under different assumptions about the trigger.

The model As in Bates (1991), a parametric approach is employed. The asset pricing model is the following:

Denote by S the exchange rate and s its logarithm. Initially, the exchange rate follows a standard Brownian motion with low volatility:

$$ds = \mu_1 dt + \sigma_1 dX$$

The pegged regime may be abandoned at any time. The interruption is a Poisson event with hazard rate λ. It leads to a discrete jump in the exchange rate and to a new diffusion process, assumed to last forever.

The jump is constant (k):

$$\frac{S^{after}}{S^{before}} = (1 + k)$$

The floating regime is described by a Brownian motion with drift and higher volatility:

$$ds = \mu_2 dt + \sigma_2 dX$$

Results The empirical results unveil completely different patterns for the probability and expected magnitude of a devaluation (conditional on its occurrence). The probability was volatile and mostly driven by contagion from external crises, as the Asian and Russian crises triggered by far the greatest increases in the probability that the peg would be abandoned. In contrast, the expected magnitude was stable and entirely unaffected by the Russian episode.

In addition, these data suggest that the Asian and Russian crises negatively impacted the Brazilian shadow exchange rate. They explicitly show that the crises coincided with both the greatest increases in the risk of a devaluation in Brazil and the largest depreciations of other Latin American currencies, like the Mexican Peso. Since the crises were fairly exogenous to the Latin American economies, it is natural to assume that if the Brazilian currency were allowed to float, it would also have depreciated.

Conclusion The empirical findings favour thresholds and learning over sunspots.

2.2.5 Parametric × non-parametric method

Some advantages of the parametric method:

- it imposes a structure that makes sense economically.
- interpretation of the results is immediate.
- in the non-parametric case, you need to impose some structure anyway (quadratic, interpolation).
- it is good if there is not much data (market is not so liquid), and if the model is appropriate.

Some disadvantages of parametric method:

- it imposes a structure that may differ from the true data generating process.
- the parametric model is not exactly what you end up estimating (as you vary λ's and μ's).
- the non-parametric approach may yield accurate results if we have very good data (or if we do not have a good model).

2.2.6 Extensions

Jondeau and Rockinger (2000) describe alternative methods to infer risk-neutral densities. Their paper brings a collection of techniques. It is worth discussing a few examples.

Stochastic volatility The diffusion process is the following:

$$\begin{aligned} \frac{dS_t}{S_t} &= \mu.dt + \sqrt{\nu_t}.\sigma.dW_1 \\ d\nu_t &= \kappa(\theta - \nu_t)dt + \gamma.\sqrt{\nu_t}.dW_2 \end{aligned}$$

Parameters:

- θ: long-run volatility;
- κ: mean-reversion speed;
- γ: volatility of the volatility.

The instantaneous volatility ν_t is not exactly a parameter, is the realization of a random variable, but it is natural to estimate it as well.

Sum of log-normals Another way to imposing some structure in the probability density functions is to consider that the option price is a mixture of M log-normal densities. That is:

$$C_t = e^{-rT} \sum_{i=1}^{M} \alpha_i \int_{S_\tau = K}^{\infty} (S_\tau - K).l(S_\tau; \mu_i, \sigma_i).dS_\tau$$

That gives us a formula not-much more complicated than the Black-Scholes formula and we can estimate the implicit parameters.

Now, given that we want to impose some structure, what is good about this one? Or, which kind of model could yield some distribution similar to this?

Stochastic interest rate Bakshi, Cao and Chen (1997) present a model with stochastic volatitly, jumps (hazard rate is given by Poisson distribution and the size of the jump is logn-normal) and *stochastic interest rate*. They estimate the implicit parameters of the model and check what matters.

2.3 A likelihood test of self-fulfilling crises

2.3.1 Jeanne (1997)

Jeanne (1997) executes a likelihood ratio test for the existence of multiple equilibria in the French Franc crisis. Some unexplainable shifts on expectations seem to be present, allowing him to conclude that jumps between different equilibria were playing a role. The empirical support for multiple equilibria in his paper comes from the existence of mysterious changes that fundamentals do not seem to account for.

Setup of the model

- The currency is pegged at a fixed rate with a foreign currency.
- Policymaker can defend the peg (possibly at some cost) or abandon it. He may be in:
  - "soft" mood, with probability μ: mantains the peg if the net benefit of doing so is positive.
  - "tough" mood, with probability $1 - \mu$: maintains the peg whatever is the circunstance.

  He needs this changes in moods to fit better the data, the model would go through without it.

- Net benefit of mantaining the peg:

$$B_t = b_t - \alpha.\pi_{t-1}$$

where b_t is the gross benefit of the fixed peg and π_{t-1} is the probability evaluated by the private sector that the peg will be abandoned next period. So, b_t depends on economic fundamentals but B_t also depends on expectations. We assume that:

$$b_t = \phi_{t-1} + \epsilon_t$$

where $\phi_{t-1} = E_t b_{t+1}$ and ϵ_t is an error term.

Equilibria

- The devaluation probability must be equal to the probability that the government is soft and the net benefit of mantaining the peg is negative, that is:

$$\begin{aligned}\pi_t &= \mu.\Pr\left(B_{t+1} < 0\right) \\ &= \mu.\Pr\left(\epsilon_{t+1} < \alpha.\pi_t - \phi_t\right) \\ &= \mu.F\left(\alpha.\pi_t - \phi_t\right)\end{aligned}$$

There may be multiple equilibria or not, depending on $f(0)$.

Estimation To check for self-fulfilling speculation, we need to check if:

1. the estimated parameters satisfy $\mu\alpha f(0) > 1$;
2. the estimated fundamental lies in the multiple-equilibrium interval;
3. there is evidence of jumps in the probability of a devaluation accross different states.

It is assumed:

$$\begin{aligned}\pi_t &= \hat{\pi}_t + \eta_t \\ \hat{\pi}_t &= \mu F_\sigma\left(\hat{\pi}_t - \phi_t\right) \\ \phi_t &= \gamma' x_t\end{aligned}$$

There are 3 possible equilibria. The equilibrium that is currently being played is described by a Markov process:

$$\Theta = \begin{pmatrix} \theta(1,1) & 0 & \theta(1,3) \\ 0 & 1 & 0 \\ \theta(3,1) & 0 & \theta(3,3) \end{pmatrix}$$

Now, we can write a likelihood function as a product of (i) the likelihood of the model prediction and (ii) the likelihood of the state:

$$L = L_\eta L_s$$

The case of the French Franc The data: he picks some fundamental macro variables (vector x_t). To get $\hat{\pi}_t$, he uses the drift adjustment method of Rose and Svensson (EER 1994) and assumes that the expected magnitude of a devaluation is 5%.

Jeanne (1997) concludes that self-fulfilling speculation was at work in the case of the French Franc (1992-3).

3 Sovereign debt and default

3.1 Models

3.1.1 Arellano (2006)

Arellano's quantitative analysis builds on Eaton and Gersovitz (RES 1981).

Setup of the model

- Small open economy
- Households are identical and have preferences given by

$$E_o \sum_{t=0}^{\infty} \beta^t u(c_t)$$

- and $u(c_t)$ respects the Inada condition. In the numerical simulation:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

- Stochastic endowment: y
- Goverment is benevolent and maximizes utility of households.
- One international asset: one period discount bonds (B).
  - Price of bond: q
  - Borrowing qB' implies repayment of B' next period
  - notation: $B > 0$ means positive assets, $B < 0$ means the country is indebted.

- Government chooses between defaulting or repaying
  - no commitment to repay
- If government chooses to repay, the resource constraint is:

$$c = y + B - qB'$$

- Default implies:
  - current debts are erased;
  - temporary exclusion from international financial markets (borrowing and lending);
  - output costs: endowment is lower, equal to $y^{def} = h(y) \leq y$, h is an increasing function;
  - so:

$$c = y^{def}$$

- Country re-enters the financial market with an exogenous constant probability θ.
- Probability of default: δ
- Creditors are risk-neutral and behave competitively.
  - International risk-free interest rate: r
  - An arbitrage condition pins down q:

$$\begin{aligned} R^f &= R^d \\ 1 + r &= \frac{1}{q}(1 - \delta) \\ &\Rightarrow \quad q = \frac{1 - \delta}{1 + r} \end{aligned}$$

- Timing:
  - government starts with assets B, observes realization of endowment y and decides about repaying or not;
  - if the govenent decides to repay, borrowing takes place.
  - consumers consume.

The value function depends on whether the country repays or defaults on its debt. Denote by v^c the value function condtional on repayment and by v^d the value function conditional on default.

The government chooses what is best for the agents. Suppose we start the period at state (B, y). Denote the value function by $v^o(B, y)$. Then:

$$v^o(B, y) = \max \left\{ v^c(B, y), v^d(y) \right\}$$

If the country decides to repay, than the value function is given by:

$$v^c(B, y) = \max_{B'} \left\{ u(c) + \beta \int_{y'} v^o(B', y') f(y'|y) dy' \right\}$$

where $c = y + B - qB'$.

If the country chooses to default, the value function is given by:

$$v^d(y) = u(y^{def}) + \beta \int_{y'} \left[\theta v^o(0, y') + (1 - \theta) v^d(y') \right] f(y'|y) dy'$$

Equilibrium In a recursive equilibrium:

1. Risk-neutral creditors are indifferent between lending to the domestic country or not (so q is obtained through the above arbitrage condition).

2. Government maximizes.

3. Households eat.

All the action comes from the government's decision.

Arellano shows some analytical results (that are already in Eaton in Gersovitz, 1981):

1. Take $B^1 < B^2$. If there is default at state (B^2, y), then there is default at state (B^1, y).

2. Suppose that shocks are i.i.d and $y_1 < y_2$. If there is default at state (B, y_2), then there is default at state (B, y_1).

3. For a given y, there is a maximum value of B that is incentive compatible to repay.

Computation of equilibrium The problem here is partial equilibrium in the sense that we do not model the outside world (foreign interest rates are taken as given). However, prices depend on the probabilities of default (arbitrage condition for the risk-neutral creditors) and although the government internalizes the effects of its actions on the price of debt, we can't solve for it directly. We have to find q as a fixed point — q is a function.

Figure 6 shows the basic structure of an algorythm to compute the equilibrium of the model.

3.1.2 Guimaraes (2006)

Another opportunity for me to talk about a paper of mine!

Guimaraes (2006) analyses whether sovereign default episodes can be seen as contingencies of optimal international lending contracts. The model considers a small open economy with capital accumulation and without commitment to repay debt.

The model An open economy can borrow from abroad, but cannot commit to repay its debts. The economy is populated by a continuum of infinitely lived agents whose preferences are aggregated to form the usual representative agent utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

Default implies a permanent output cost, γ. The fraction of output lost due to default is γ, so production is given by:

$$y_t = \begin{cases} A.f(k_t) \text{ , if no default} \\ A(1-\gamma).f(k_t) \text{ , if default} \end{cases}$$

Default also implies permanent exclusion from international capital markets.

Default costs are permanent, which captures the loss that a country suffers by taking an antagonistic position towards the rest of the world and never repaying its debts. In the model this is out-of-equilibrium behaviour, which corresponds to never observing such action in reality.

There is a continuum of risk-neutral lenders that, in equilibrium, lend to the country as long as the expected return on their assets is not lower than the risk-free interest rate in international markets, r^*. The price of a bond that delivers one unit of the good next period with certainty, $(1+r^*)^{-1}$, will be denoted q^*. There is a maximum amount of debt the country can contract that prevents it from running Ponzi schemes but it is never reached in equilibrium.

Stochastic interest rates Let's analyze the case of stochastic q^*. The price of a riskless bond in international markets is q^{*h} in the high state and q^{*l} in the low state, $q^{*h} >$

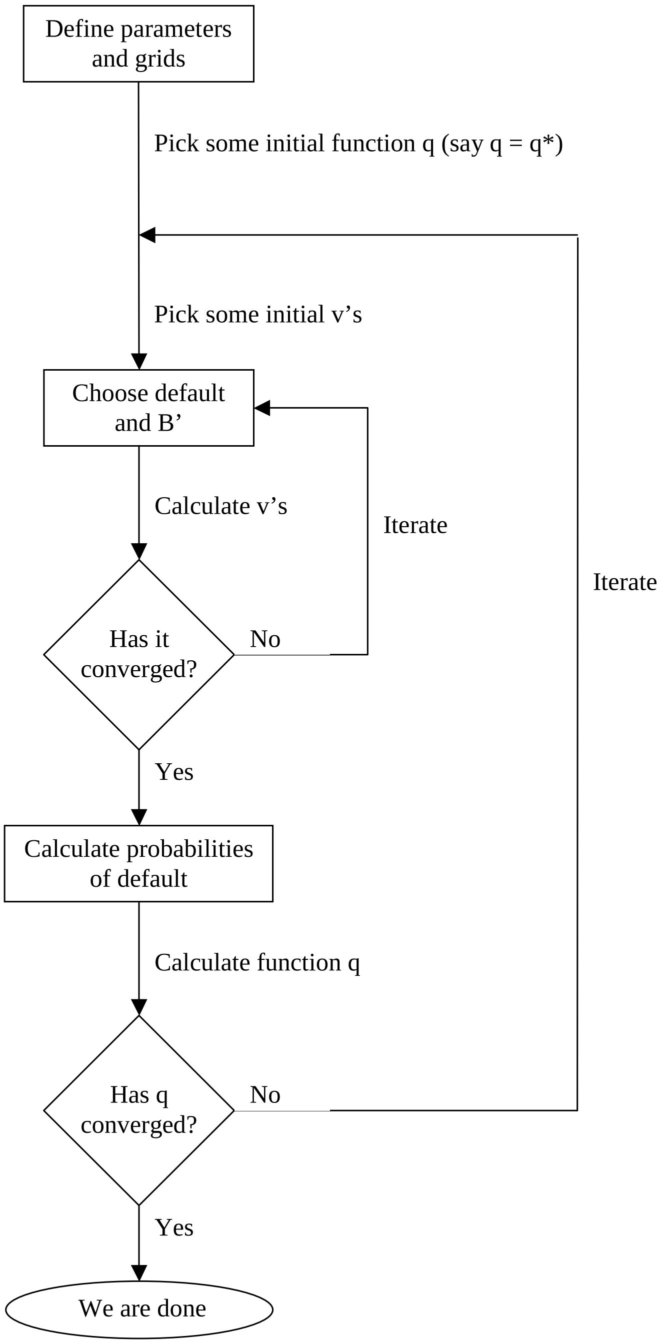


Figure 6: Flow diagram for Arellano (2006)

q^{*l}. A risk-neutral creditor that lends q^*d' must get an expected repayment equal to d'. Denote by d^h and d^l the repayment conditional on high and low state, respectively, and $\Delta d = d^h - d^l$. If $s_{t-1} = h$, a country that borrowed $q^{*h}d'$ has debt d^h if $s_t = h$ and d^l if $s_t = l$ such that $d^h(1-\psi) + d^l\psi = d'$. Hence, $d^h = d' + \psi\Delta d$ and $d^l = d' - (1-\psi)\Delta d$. If $s_{t-1} = l$, a country borrowing $q^{*l}d'$ has debt d^h if $s_t = h$ and d^l if $s_t = l$ such that $d^l(1-\psi) + d^h\psi = d'$. Hence, $d^l = d' - \psi\Delta d$ and $d^h = d' + (1-\psi)\Delta d$.

The economy's flow budget constraint is then given by:

$$c_t + k_{t+1} = \begin{cases} A.f(k_t) + (1-\delta)k_t - d_t + q_t d_{t+1} \text{ , if no default} \\ A(1-\gamma).f(k_t) + (1-\delta)k_t \text{ , if default} \end{cases}$$

Where q_t is the (endogenously determined) price of debt.

In each period, the central planner chooses between repaying or defaulting. Each option yields a different value function and the planner chooses the maximum of the two:

$$V(k,d) = \max\{V_{pay}(k,d), V_{def}(k,\gamma)\}$$

The value functions conditional on repayment are:

$$\begin{aligned} V^h_{pay}(k,d) &= \max_{k',d',\Delta d}\left\{u(c) + \beta\left[(1-\psi)V^h(k', d'+\psi\Delta d) + \psi V^l(k', d' - (1-\psi)\Delta d)\right]\right\} \\ V^l_{pay}(k,d) &= \max_{k',d',\Delta d}\left\{u(c) + \beta\left[(1-\psi)V^l(k', d'-\psi\Delta d) + \psi V^h(k', d' + (1-\psi)\Delta d)\right]\right\} \end{aligned}$$

where $c = Af(k) + (1-\delta)k - k' - d + q^i(k', d')d'$.

In case of default, the value function in both states is:

$$V_{def}(k,\gamma) = \max_{k'}\{u((1-\gamma)Af(k) + (1-\delta)k - k') + \beta V_{def}(k',\gamma)\}$$

Equilibrium Taking first order approximations of Bellman equations, Guimaraes (2006) derives analytical expressions for the equilibrium level of debt and the optimal debt contract.

The following result is proven: as we approach a deterministic steady state around, $\bar{k}$, $\bar{d}$ and $\bar{q}$, such that q^{*h} and q^{*l} are close to $\bar{q}$ and $\bar{q} = \left(q^{*h} + q^{*l}\right)/2$, as long as the borrowing constraint is binding, a linear approximation of the value functions tell us that d^h and d^l are chosen so that:

$$\frac{d^h - d^l}{\bar{d}} = \frac{q^{*h} - q^{*l}}{1 - \beta(1-2\psi)}$$

where $\bar{d} = \left(d^h + d^l\right)/2$.

The paper claims that debt relief prescribed by the model following the interest rate hikes of 1980-81 accounts for more than half of the debt forgiveness obtained by the main Latin American countries through the Brady agreements.

Stochastic technology In a model with stochastic technology (and constant world interest rates), d^h and d^l are chosen so that:

$$\frac{d^h - d^l}{\bar{d}} = \frac{(1-q^*)}{1-\beta(1-2\psi)} \frac{A^h - A^l}{\bar{A}}$$

Compare both expressions: debt relief generated by reasonable fluctuations in productivity is at least an order of magnitude below that generated by shocks to world interest rates.

References

[1] Arellano, Cristina, 2006, "Default risk and income fluctuations in emerging economies", forthcoming, *American Economic Review.*

[2] Bakshi, Gurdip; Cao, Charles and Chen, Zhivu, 1997, "Empirical performance of alternative option pricing models", *Journal of Finance* 52, 2003-2049.

[3] Bates, David S., 1991, "The crash of '87: Was it expected? The Evidence from Options Markets", *Journal of Finance* 46, 1009-1044.

[4] Bikhchandani, Sushil; Hirshleifer, David and Welch, Ivo, 1992, " A theory of fads, fashion, custon, and cultural change as informational cascades", *Journal of Political Economy* 100, 992-1026.

[5] Campa, Jose M., Chang, P. H. Kevin, 1996, "Arbitrage-based tests of target zone credibility: evidence from ERM crossrate options", *American Economic Review* 86, 726–740.

[6] Campa, José M.; Chang, P. H. Kevin and Refalo, James F., 2002, "An options-based analysis of emerging market exchange rate expectations: Brazil's Real plan, 1994 - 1997", *Journal of Development Economics* 69, 227-253.

[7] Caplin, Andrew and Leahy, John, 1994, "Business as usual, market crashes and wisdom after the fact", *American Economic Review* 84, 548-565.

[8] Carlsson, Hans; and van Damme, Eric, 1993, "Global games and equilibrium selection", *Econometrica* 61, 989-1018.

[9] Corsetti, Giancarlo; Pesenti, Paolo and Roubini, Nouriel, 1999, "Paper tigers? A model of the Asian crisis", *European Economic Review* 43, 1211-1236.

[10] Diaz-Alejandro, Carlos, 1985, "Good-bye financial repression, hello financial crash", *Journal of Development Economics* 19, 1-24.

[11] Eaton, Jonathan and Gersovitz, Mark, 1981, "Debt with potential repudiation: theoretical and empirical analysis", *Review of Economic Studies* 48, 289-309.

[12] Flood, Robert and Garber, Peter, 1984, "Collapsing exchange-rate regimes: some linear examples", *Journal of International Economics* 17, 1-13.

[13] Geanakoplos, John, 1992, "Common Knowledge", *Journal of Economic Perspectives* 6, 53-82.

[14] Guimaraes, Bernardo, 2006, "Optimal external debt and default", mimeo.

[15] Guimaraes, Bernardo, 2007, "Currency crisis triggers: sunspots or thresholds?", mimeo.

[16] Jeanne, Olivier, 1997, "Are currency crises self-fulfilling? A test", *Journal of International Economics* 43, 263-286.

[17] Jondeau, Eric and Rockinger, Michael, 2000, "Reading the smile: the message conveyed by methods which infer risk neutral densities", *Journal of International Money and Finance* 19, 885-915.

[18] Krugman, 1979, "A Model of Balance-of-Payments Crises", *Journal of Money, Credit and Banking* 11, 311-325.

[19] Merton, Robert C., 1976, "Option pricing when underlying stock returns are discontinuous", *Journal of Financial Economics* 3, 125-144.

[20] Morris, Stephen and Shin, Hyun, 1998, "Unique equilibrium in a model of self-fulfilling currency attacks", *American Economic Review* 88, 587-597.

[21] Obstfeld, Maurice, 1996, "Models of currency crises with self-fulfilling features", *European Economic Review* 40, 1037-1047.

[22] Radelet, Steven and Sachs, Jeffrey, 1998,."The East Asian Financial Crisis: Diagnosis, Remedies, Prospects", *Brookings Papers on Economic Activity*, 1998 #1, 1-90.

[23] Rose, Andrew and Svensson, Lars, 1994, "European exchange rate credibility before the fall", *European Economic Review* 38, 1185-1216.

[24] Salant, Stephen and Henderson, Dale, 1978, "Market antecipation of government policy and the price of gold", *Journal of Political Economy* 86, 627-648.