Pauline Barrieu and Henri Louberge

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# Reinsurance and securitisation of life insurance risk: the impact of regulatory constraints.

Pauline Barrieu[a], Henri Loubergé[b]

[a] *(Corresponding author) LSE, Statistics department, Houghton Street, London WC2A 2AE, United Kingdom. email: p.m.barrieu@lse.ac.uk. Ph: +44 207 955 6016. Fax: +44 207 955 7416.*

[b] *Département des Sciences Economiques, Uni Mail, Bd du Pont d'Arve 40 1211 Geneva 4, Switzerland*

**Abstract**

Large systematic risks, such as those arising from natural catastrophes, climatic changes and uncertain trends in longevity increases, have risen in prominence at a societal level and, more particularly, have become a highly relevant issue for the insurance industry. Against this background, the combination of reinsurance and capital market solutions (insurance-linked securities) has received an increasing interest. In this paper, we develop a general model of optimal risk-sharing among three representative agents – an insurer, a reinsurer and a financial investor, making a distinction between systematic and idiosyncratic risks. We focus on the impact of regulation on risk transfer, by differentiating reinsurance and securitisation in terms of their impact on reserve requirements. Our results show that different regulatory prescriptions will lead to quite different results in terms of global risk-sharing.

*Keywords:* Reinsurance, Risk sharing, Risk measures, Longevity risk, Insurance-Linked securities
JEL and IME codes: G22 ; IM51 ; IM53

The convergence of the insurance industry with capital markets has become ever more important over recent years (see, for instance, the papers by Cowley and Cummins (2005), Cummins (2004) and (2008) or Cummins and Weiss (2009) or the recent handbook by Barrieu and Albertini (2009)). Such convergence has taken many forms. And of the many convergence attempts, some have been more successful than others. The first academic reference to the use of capital markets in order to transfer insurance risk was in a paper by

Goshay and Sandor (1973). The authors considered the feasibility of an organised market, and how this could complement the reinsurance industry in catastrophic risk management. In practice, whilst some attempts have been made to develop an insurance future and option market, the results have, so far, been rather disappointing. In parallel to these attempts, however, the Insurance-Linked Securities (ILS) market has been growing rapidly over the last 15 years. There are many different motivations for ILS, including risk transfer, capital strain relief, boosting of profits, speed of settlement, and duration. Different motives mean different solutions and structures, as the variety of instruments on the ILS market illustrates.

Among the key challenges faced by the insurance industry, the management of longevity risk, i.e. the risk that the trend of longevity improvements significantly changes in the future, is certainly one of the most important. Ever more capital has to be accumulated to face this long-term risk, and new regulations in Europe, together with the recent financial crisis, only amplify this phenomenon. Under the Solvency II rules, put forward by the European Commission, the more stringent capital requirements that have been introduced for banks should also be applied to insurance company operations (see Eling et al. (2007); Harrington (2009); and Geneva Association (2010)). Moreover, in addition to this risk of observing a significant change in the longevity trend, the insurance sector is facing some basis risk, as the evolution of the policyholders' mortality is usually different from that of the national population, due to selection effects. These selection effects have different impacts on different insurance companies' portfolios, as mortality levels and speeds of decrease and increase are very heterogeneous in the insurance industry. This makes it hard for insurance companies to rely on national, or even industry, indices, in order to manage their own longevity risk. Hence, it has become more and more important for insurance companies and pension funds to find a suitable and efficient way to deal with this risk. Recently, various risk mitigation techniques have been attempted. Reinsurance and capital market solutions, in particular, have received an increasing interest (see for instance Blake and Burrows (2001) and Blake et al. (2006)). Even if no Insurance-Linked Securitisation (ILS) related to longevity risk has yet been completed, the development of this market for other insurance risks has been experiencing a continuous growth for several years, mainly encouraged by changes in the regulatory environment and the need for additional capital from the insurance industry. Today, longevity risk securitisation lies at the heart of many discussions, and is widely seen as a potentiality for the future.

The classical and standard framework of risk sharing in the insurance industry, as studied, for instance, by Borch (1960) and (1962), involves two types of agents: primary insurers and a pool of reinsurers. The risk is shared among different agents of the same type, but with both differing sizes and utility functions. The possible financial consequences of some risks, such as large-scale catastrophes or dramatic changes in longevity trends, however, make this sharing process difficult to conduct within a reinsurance pool. In this case, capital markets may improve the risk-sharing process. Indeed, non-diversifiable risks for the insurance industry may be seen as a source of diversification for financial investors, such as a new asset class, enhancing the overall diversification of traditional investment portfolios, particularly in case of low correlations with overall market risk. Even if the correlation is not necessarily low, which may be the case for changes in longevity, the non-diversifiable insurance risks may be shared by a larger population of financial investors, instead of being assumed by reinsurers only. In the first section of the paper, we focus on some insurance risks (for instance, longevity and mortality risks), and, from a general point of view, study the optimal strategy of risk-sharing and risk-transfer between three representative agents (an insurer, a reinsurer and an investor), taking into account pricing principles in insurance and finance within a unified framework. Comments on an optimal securitisation process and, in particular, on the design of an appropriate alternative risk transfer are made. In the second section, we focus on the impact of regulation upon risk transfer, by differentiating reinsurance and securitisation in terms of their impact upon reserves. More precisely, we will study the bias introduced by the regulatory framework, and the subsequent impact upon the aforementioned risk transfer techniques.

## 1. Complementarity of reinsurance and securitisation in insurance risk management

### *1.1. Framework*

In the following section, we consider a simplified economy composed of three different types of agent, namely an insurer, a reinsurer and a representative investor. The problem we would like to study is the risk-transfer of some insurance risk between the three types of agents, which is initially supported by the insurer. The various decisions will take place over a normalised time horizon $[0, 1]$, and, for the sake of simplicity, we neglect interest rates because they are unessential to the analysis. We also introduce a probability space $(\Omega, F, \mathbb{P})$ where $\mathbb{P}$ represents a prior probability measure, typically the historical or statistical probability measure. The expected value under the probability measure $\mathbb{P}$ is simply denoted as $\mathbb{E}(.)$. We first need to introduce the various agents and their respective exposure to this insurance risk.

*1.1.1. The exposure of the insurer*

We consider the following simplified framework: we assume that a representative insurance company is offering some insurance contracts, and makes some payments when a given event occurs (for instance when the insured dies in the case of a life insurance policy). It is, therefore, exposed to some insurance risk. Let us be more specific regarding the overall exposure of the insurer to this insurance risk, introducing a distinction, in the nature of the risk itself, between $\Theta$ representing the systematic component of the risk, and $\Theta^{\perp}$ the specific or idiosyncratic component of the risk in the insurer's portfolio:

$$\begin{aligned} \widehat{X} &= \mathbb{E}\left(\widehat{X}/\Theta\right) + \left(\widehat{X} - \mathbb{E}\left(\widehat{X}/\Theta\right)\right) \\ &\doteq X(\Theta) + \overline{X}\left(\Theta^{\perp}\right) \end{aligned}$$

where $\overline{X}\left(\Theta^{\perp}\right)$ and $X(\Theta)$ are independent. $X(\Theta)$ represents the part of the insurer's exposure that is related to the global insurance risk, while $\overline{X}\left(\Theta^{\perp}\right)$ represents the part of the exposure that is related to the specific nature of the insurer's portfolio, hence, the part that can be diversified within a larger portfolio.

The insurer will transfer part of its exposure to the reinsurance company. The characteristics of the reinsurance risk transfer will be determined later.

*1.1.2. The problem for the reinsurer*

In the economy we consider, there is a representative reinsurance company. It has an initial portfolio with a random value at time 1 equal to $W_R$. The reinsurer is providing reinsurance cover to the insurer for an amount $\widehat{J}$ against the payment of a reinsurance premium $\kappa$. The reinsurer can also transfer part of its risk to the capital markets by sponsoring a insurance-related bond.

This bond is written on a contingent payoff of $M(\Theta)$. Note that the risk covered by the bond is the systematic part only. The idiosyncratic part of the risk is not transferred to the capital markets. Therefore, the type of structure we consider in this paper is index-based. The product is based upon the global insurance risk that can be measured, for instance, using a global index on the population in case of life insurance risk, or a parametric index in case of non-life insurance risks. This avoids issues of asymmetric information between the reinsurer and the investor[1]. The financial investors will be willing to add these products to their existing financial portfolio to enhance diversification. The reinsurer will be interested in issuing such products if they can help her with transferring the non-diversifiable part of the insurance risk. Considering a bond based upon an index enables the issuer to filter out the non-diversifiable risk from the rest, and to end up with a portfolio to which there is a solely asymptotic relationship to the diversifiable part of the risk. At this stage, there is a trade-off between the mitigation of the non-diversifiable risk and the introduction of some basis risk for the issuer. By $\pi$ we denote the initial price of such a (zero-coupon) bond paid at time 0 by the investor.

The exposure for the reinsurer is, therefore

$$W_R - \widehat{J} + \kappa - M(\Theta) + \pi$$

The problem is now a question of how to determine the optimal characteristics for both risk transfers $((\widehat{J}, \kappa)$ and $(M(\Theta), \pi))$ under the participation constraints imposed by the insurer and by the investor.

*1.1.3. Decision criterion*

To make their decision, we assume that each agent will use a conservative choice criterion, which is based upon regulatory constraints, i.e. a risk

[1] Note that this type of approach allows the reinsurer to transfer some of his risk to some agents in the capital markets, likely to be outside of the insurance industry.
The considered structure is that of a bond with contingent payoff $M$. It will typically involve the constitution of a collateral account at the beginning of the transaction using the initial payment by the investor, bringing an additional protection against any default in the transaction. The structure could also be that of a swap, where $M$ represents the contingent cash flow in this case.
Note also, that the risk transfer using the capital markets imposes some constraint upon the type of risk covered by the transaction (systemic part of the risk, and not the idiosyncratic risk). Indeed, this makes the bond a financial contract, and not an insurance contract based upon the actual losses of the protection seeker.

measure. More precisely, the different agents considered assess their risk using a convex risk measure, denoted by $\rho_a$ (where $a = I,\ R$ or $B$). For the sake of simplicity, after the presentation of the general problem, we will consider *entropic risk measures* in order to derive explicit formulae for the different quantities involved. Denoting by $\gamma_a$ ($> 0$) the risk tolerance coefficient of agent $a$ ($a = I, B, R$), her entropic risk measure associated with any terminal investment payoff $\Psi$ is expressed as:

$$\rho_a(\Psi) = \gamma_a \ln \mathbb{E}\left[\exp\left(-\frac{1}{\gamma_a}\Psi\right)\right] \tag{1}$$

We also consider a financial market, typically represented by a set $\mathcal{V}$ of bounded terminal gains[2] at time 1, which is our time horizon, resulting from self-financing investment strategies with a null initial value. The key point is that all agents in the market agree on the initial value of every strategy. In other words, the market value at time 0 of any strategy is null. In particular, an admissible strategy is associated with a derivative contract with a given bounded terminal payoff $\Phi$ only if its forward market price at time 1, $q^m(\Phi)$, is a transaction price for all agents in the market. Then, $\Phi - q^m(\Phi)$ is the bounded terminal gain at time 1 and is an element of $\mathcal{V}$. A typical example of admissible terminal gains is the terminal wealth associated with transactions based on options. Moreover, in order to have coherent transaction prices, we assume, in the following, that the market is arbitrage-free. We do not necessarily require the financial market to be complete.

The set $\mathcal{V}$, previously defined, has also to satisfy some properties to be coherent with some investment principles. The first principle, being the "minimal assumption", is the consistency with the diversification principle. In other words, any convex combination of admissible gains should also be an admissible gain, and, therefore, the set $\mathcal{V}$ is always taken as a *convex set*.

Assuming that the various agents have access to the financial market, each of them will have to determine his or her optimal financial investment by solving the following hedging/investment problem:

$$\min_{\xi \in \mathcal{V}} \rho_a(\Psi - \xi)$$

[2]i.e. the net potential gain corresponds to the spread between the terminal wealth resulting from the adopted strategy and the capitalised initial wealth.

where $\Psi$ is agent $a$'s initial exposure (where $a = I,\ R$ or $B$) and $\xi$ denotes the (bounded) terminal gain of a given financial contract. The functional value of this optimisation problem characterises a new convex risk measure, which corresponds to the risk measure agent $a$ will have after having optimally chosen her financial investment, or hedge, on the market. It is called the *market modified risk measure* of agent $a$ and is denoted by $\rho_a^m$. In the entropic framework, we get:

$$\rho_a^m(\Psi) = \gamma_a \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_a}\Psi\right)\right]$$

where $\widehat{\mathbb{Q}}$ is the minimal entropy probability measure, defined as the probability measure minimising the relative entropy with respect to the prior probability measure $\mathbb{P}$, i.e.

$$\widehat{\mathbb{Q}} = \arg\min \mathbb{E}_{\mathbb{P}}\left(\frac{d\mathbb{Q}}{d\mathbb{P}} \ln \frac{d\mathbb{Q}}{d\mathbb{P}}\right)$$

or equivalently it is the probability measure that is the closest to the original measure in the entropic sense[3].

Therefore, in the following, the risk measure to consider when dealing with any agent is his market modified risk measure, instead of the original one. This allows the agents to take into account simultaneous optimal investment decisions in the financial market, when the agent also trades in the reinsurance and the ILS market. In terms of probability measures, this means that we will not work directly with the historical probability measure $\mathbb{P}$, but instead with the minimal entropy probability measure $\widehat{\mathbb{Q}}$, which is common to all agents, as they have the same access to the financial market.

### *1.2. Optimal risk transfers*

Our risk-sharing model assumes that the reinsurer is the main actor in the risk-sharing transactions. She takes initiatives and minimises her modified risk measure $\rho_R^m$, under the participation constraints of the insurer and of the investor. She also takes optimal decisions concerning the additional financial risk that she faces (as implied by the modified risk measure). However, as shown in the appendix, the same risk-sharing arrangements are observed

[3]For more details, please refer to Barrieu and El Karoui (2003).

if the insurer leads the transactions, instead of the reinsurer. The results are independent of who leads the transactions, the insurer or the reinsurer. A natural question for the reinsurer is then how to optimally mix the risk assumption with the issue of a certain amount of zero-coupon bonds, of the type previously described. Let us assume that, at this stage, the different agents assess their risk using entropic risk measures, and that all of them have access to the financial market so as to invest and hedge optimally. Denoting, as previously, $\rho_a^m$ for $a = I, B, R$ to be the market modified risk measure of the insurer, the buyer and the reinsurer respectively, by $W_a$ we denote the initial position of agent $a$ ($a = I, B, R$), by $\widehat{J}$ the amount covered by the reinsurance contract, by $\kappa$ the premium of the reinsurance contract, by $M$ the contingent payoff of the insurance bond, and by $\pi$ the price of the bond; we have the following problem:

$$
\begin{aligned}
&\min_{\widehat{J},M,\pi,\kappa} \rho_R^m \left( W_R - \widehat{J} - M(\Theta) + \pi + \kappa \right) \\
&\text{s.t.} \quad \rho_I^m \left( W_I - \widehat{X} + \widehat{J} - \kappa \right) \leq \rho_I^m \left( W_I - \widehat{X} \right) \\
&\qquad\;\; \rho_B^m \left( W_B + M(\Theta) - \pi \right) \leq \rho_B^m \left( W_B \right)
\end{aligned}
$$

From here, we can obtain some bounds on the prices, using the cash invariance property of the various risk measures:

$$
\begin{aligned}
\pi^* \left( M(\Theta) \right) &= \rho_B^m \left( W_B \right) - \rho_B^m \left( W_B + M(\Theta) \right) \\
\kappa^* \left( \widehat{J} \right) &= \rho_I^m \left( W_I - \widehat{X} \right) - \rho_I^m \left( W_I - \widehat{X} + \widehat{J} \right)
\end{aligned}
$$

Using these constraints, and the cash translation invariance property of the various risk measures, the optimisation problem can be rewritten as:

$$
\min_{\widehat{J},M} \left\{ \rho_R^m \left( W_R - \widehat{J} - M(\Theta) \right) + \rho_I^m \left( W_I - \widehat{X} + \widehat{J} \right) + \rho_B^m \left( W_B + M(\Theta) \right) \right\}
$$

*Remark* 1. *As a first step, for the sake of simplicity, we assume that the initial positions are non-random. Otherwise, we can, instead, consider the following normalised risk measure:*

$$
\overline{\rho}^m (\Psi) \triangleq \rho^m (W - \Psi) - \rho^m (W)
$$

*Note that in the entropic framework, $\overline{\rho}^m$ remains an entropic risk measure.*

Considering the initial wealths as a constant, we have:

$$\min_{\widehat{J},M} \left\{ \rho_R^m \left( -\widehat{J} - M(\Theta) \right) + \rho_I^m \left( -\widehat{X} + \widehat{J} \right) + \rho_B^m \left( M(\Theta) \right) \right\} \tag{2}$$

Note that we can also decompose the compensation $\widehat{J}$ into two parts:

$$\begin{aligned} \widehat{J} &= \mathbb{E}\left(\widehat{J}/\Theta\right) + \left(\widehat{J} - \mathbb{E}\left(\widehat{J}/\Theta\right)\right) \\ &\doteq J(\Theta) + \overline{J}\left(\Theta^{\perp}\right) \end{aligned}$$

where $\overline{J}\left(\Theta^{\perp}\right)$ and $J(\Theta)$ are independent. $J(\Theta)$ represents the part of the compensation related to the systematic component of the risk, whilst $\overline{J}\left(\Theta^{\perp}\right)$ represents the part of the compensation related to the specific nature of the reinsured portfolio.

*1.2.1. Entropic framework*

Let us now try to solve this optimisation problem in the particular situation where the three agents have entropic risk measures and constant initial wealth.

$$\rho_a^m(\Psi) = \gamma_a \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_a} \Psi \right) \right]$$

**Proposition 1.1.** *In the entropic framework, the optimal characteristics are (up to a constant):*
*For the reinsurance contract:*

$$\overline{J}^*\left(\Theta^{\perp}\right) = \frac{\gamma_R}{\gamma_R + \gamma_I} \overline{X}\left(\Theta^{\perp}\right) \quad \textit{and} \quad J^*(\Theta) = \frac{\gamma_R + \gamma_B}{\gamma_I + \gamma_R + \gamma_B} X(\Theta)$$

*and for the bond:*

$$M^*(\Theta) = -\frac{\gamma_B}{\gamma_I + \gamma_R + \gamma_B} X(\Theta)$$

**Proof:** To solve the optimisation Programme (2), we proceed in two steps:

- First, we fix the compensation $\widehat{J}$, and solve the optimisation for $M$:

$$\begin{aligned} & \min_{M} \left\{ \rho_R^m \left( -\widehat{J} - M(\Theta) \right) + \rho_I^m \left( -\widehat{X} + \widehat{J} \right) + \rho_B^m \left( M(\Theta) \right) \right\} \\ = & \min_{M} \left\{ \rho_R^m \left( -\widehat{J} - M(\Theta) \right) + \rho_B^m \left( M(\Theta) \right) \right\} \end{aligned}$$

In the entropic framework, the optimisation Programme (2) becomes:

$$\min_M \left\{ \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_R} \left( -\widehat{J} - M(\Theta) \right) \right) \right] + \gamma_B \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_B} M(\Theta) \right) \right] \right\}$$

Taking conditional expectations with respect to $\Theta$, we get:

$$\min_M \left\{ \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_R} \left( -\widehat{J} - M(\Theta) \right) \right) / \Theta \right] \right] + \gamma_B \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_B} M(\Theta) \right) \right] \right\}$$

Using the decomposition of $\widehat{J}$ and the independence property between $J(\Theta)$ and $\overline{J}$, we have:

$$\begin{aligned} & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_R} \left( -\widehat{J} - M(\Theta) \right) \right) / \Theta \right] \right] \\ = & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_R} \left( -J(\Theta) - M(\Theta) \right) \right) \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( \frac{1}{\gamma_R} \overline{J} \left( \Theta^{\perp} \right) \right) \right] \right] \\ = & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_R} \left( -J(\Theta) - M(\Theta) \right) \right) \right] + c \end{aligned}$$

where $c = \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( \frac{1}{\gamma_R} \overline{J} \left( \Theta^{\perp} \right) \right) \right]$ is independent of the optimisation problem. Hence, the programme becomes:

$$\min_M \left\{ \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_R} \left( -J(\Theta) - M(\Theta) \right) \right) \right] + \gamma_B \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_B} M(\Theta) \right) \right] \right\}$$

Using Proposition 2.1 in Barrieu-El Karoui (2005), we obtain:

$$M^*(\Theta) = \frac{\gamma_B}{\gamma_R + \gamma_B} \left( -J(\Theta) \right) + \text{constant}$$

- Then, we fix the contingent payoff $M$ and work on the compensation $\widehat{J}$:

$$\begin{aligned} & \min_{\widehat{J}} \left\{ \rho_R^m \left( -\widehat{J} - M(\Theta) \right) + \rho_I^m \left( -\widehat{X} + \widehat{J} \right) + \rho_B^m \left( M(\Theta) \right) \right\} \\ = & \min_{\widehat{J}} \left\{ \rho_R^m \left( -\widehat{J} - M(\Theta) \right) + \rho_I^m \left( -\widehat{X} + \widehat{J} \right) \right\} \end{aligned}$$

In the entropic framework, the optimisation Programme (2) becomes:

$$\min_{\widehat{J}} \left\{ \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_R} \left( -\widehat{J} - M(\Theta) \right) \right) \right] + \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp \left( -\frac{1}{\gamma_I} \left( -\widehat{X} + \widehat{J} \right) \right) \right] \right\}$$

We can then treat the compensation $\overline{J}\left(\Theta^{\perp}\right)$ and $J\left(\Theta\right)$ separately:

$$\min_{J(\Theta)}\min_{\overline{J}\left(\Theta^{\perp}\right)}\left\{\begin{array}{c}\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\left(J\left(\Theta\right)+\overline{J}\left(\Theta^{\perp}\right)\right)-M\left(\Theta\right)\right)\right)\right] \\ +\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\widehat{X}+\left(J\left(\Theta\right)+\overline{J}\left(\Theta^{\perp}\right)\right)\right)\right)\right]\end{array}\right\}$$

Taking into account conditional expectations with respect to $\Theta$, we get:

$$\min_{J(\Theta)}\min_{\overline{J}\left(\Theta^{\perp}\right)}\left\{\begin{array}{c}\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\widehat{J}-M\left(\Theta\right)\right)\right)/\Theta\right]\right] \\ +\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\widehat{X}+\widehat{J}\right)\right)/\Theta\right]\right]\end{array}\right\}$$

Using the decomposition of $\widehat{J}$ and $\widehat{X}$ and the independence property between $J\left(\Theta\right)$ and $\overline{J}$ on the one hand, and between $X\left(\Theta\right)$ and $\overline{X}$ on the other hand, we have:

$$\begin{aligned}
& \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\left(J\left(\Theta\right)+\overline{J}\left(\Theta^{\perp}\right)\right)-M\left(\Theta\right)\right)\right)/\Theta\right]\right] \\
= \; & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-J\left(\Theta\right)-M\left(\Theta\right)\right)\right)\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\overline{J}\left(\Theta^{\perp}\right)\right)\right]\right] \\
= \; & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-J\left(\Theta\right)-M\left(\Theta\right)\right)\right)\right]+c
\end{aligned}$$

and

$$\begin{aligned}
& \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\widehat{X}+\widehat{J}\right)\right)/\Theta\right]\right] \\
= \; & \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-X\left(\Theta\right)+J\left(\Theta\right)\right)\right)\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\overline{X}\left(\Theta^{\perp}\right)+\overline{J}\left(\Theta^{\perp}\right)\right)\right)\right]\right] \\
= \; & \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-X\left(\Theta\right)+J\left(\Theta\right)\right)\right)\right]+c'.
\end{aligned}$$

Both $c=\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\overline{J}\left(\Theta^{\perp}\right)\right)\right]$ and $c'=\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\overline{X}\left(\Theta^{\perp}\right)+\overline{J}\left(\Theta^{\perp}\right)\right)\right)\right]$ are independent of the optimisation problem in $J$. Hence the programme becomes:

$$\min_{J}\left\{\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-J\left(\Theta\right)-M\left(\Theta\right)\right)\right)\right]+\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-X\left(\Theta\right)+J\left(\Theta\right)\right)\right)\right]\right\}$$

Using a simple change in variables $J' \doteq J - X$, we obtain from Proposition 2.1 in Barrieu-El Karoui (2005):

$$J'^{*}(\Theta) = \frac{\gamma_I}{\gamma_R + \gamma_I}(-X(\Theta) - M(\Theta)) + \text{ constant}$$

Hence

$$J^{*}(\Theta) = \frac{\gamma_R}{\gamma_R + \gamma_I}X(\Theta) - \frac{\gamma_I}{\gamma_R + \gamma_I}M(\Theta) + \text{ constant}$$

Conditioning now, with respect to $\Theta^{\perp}$, we have to minimise with respect to $\overline{J}$ and $J$:

$$\min_{\overline{J},J}\left\{\begin{array}{c} \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\widehat{J} - M(\Theta)\right)\right)/\Theta^{\perp}\right]\right] \\ +\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\widehat{X} + \widehat{J}\right)\right)/\Theta^{\perp}\right]\right] \end{array}\right\}$$

Using the decomposition of $\widehat{J}$ and $\widehat{X}$ and the independence property, between $J(\Theta)$ and $\overline{J}$ on the one hand, and between $X(\Theta)$ and $\overline{X}$ on the other hand, we have:

$$\begin{aligned} & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\widehat{J} - M(\Theta)\right)\right)/\Theta^{\perp}\right]\right] \\ = \; & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\overline{J}(v)\right)\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}(-J(\Theta) - M(\Theta))\right)\right]\right] \\ = \; & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\overline{J}\left(\Theta^{\perp}\right)\right)\right] + \overline{c} \end{aligned}$$

and

$$\begin{aligned} & \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\widehat{X} + \widehat{J}\right)\right)/\Theta^{\perp}\right]\right] \\ = \; & \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\overline{X}\left(\Theta^{\perp}\right) + \overline{J}\left(\Theta^{\perp}\right)\right)\right)\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}(-X(\Theta) + J(\Theta))\right)\right]\right] \\ = \; & \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\overline{X}\left(\Theta^{\perp}\right) + \overline{J}\left(\Theta^{\perp}\right)\right)\right)\right] + \overline{c}' \end{aligned}$$

where the two constants $\overline{c} = \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}(-J(\Theta) - M(\Theta))\right)\right]$ and $\overline{c}' = \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}(-X(\Theta) + J(\Theta))\right)\right]$ are independent of the optimisation problem in $\overline{J}$. Hence the programme becomes:

$$\min_{\overline{J}} \left\{ \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( \frac{1}{\gamma_R} \overline{J}\left(\Theta^{\perp}\right) \right) \right] + \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_I} \left( -\overline{X}\left(\Theta^{\perp}\right) + \overline{J}\left(\Theta^{\perp}\right) \right) \right) \right] \right\}$$

We then get:

$$\overline{J}^{*}\left(\Theta^{\perp}\right) = \frac{\gamma_R}{\gamma_R + \gamma_I} \overline{X}\left(\Theta^{\perp}\right)$$

- Finally, using the various results, we have (up to a constant):

$$J^{*}(\Theta) = \frac{\gamma_R + \gamma_B}{\gamma_I + \gamma_R + \gamma_B} X(\Theta)$$

and

$$M^{*}(\Theta) = -\frac{\gamma_B}{\gamma_I + \gamma_R + \gamma_B} X(\Theta)$$

□

*Remark* 2. ($i$) *It is interesting to note that $\widehat{J}$ is a proper reinsurance compensation, provided that $\widehat{X}$ is positive, i.e. the insurer experiments with some losses with a positive amount, then $0 < \widehat{J} < \widehat{X}$, and the compensation is positive but smaller than the losses of the insurer.*
($ii$) *Note also that the amount of risk taken by the reinsurer, $\widehat{J} + M(\Theta)$ is also reasonable from an insurance point of view as $0 < \widehat{J} + M(\Theta) < \widehat{X}$. Moreover, since $M(\Theta)$ is negative, this contingent payoff is smaller than the risk directly transferred by the insurer. The issue of the bond is, therefore, helping the reinsurer by reducing its exposure.*
($iii$) *The results in Proposition 1.1 are obtained up to constant terms that do not play a role in the rest of the analysis, since they do not alter the risk-sharing process. These constant terms represent "side payments" that the reinsurer - who organises the risk-transfer process - must accept in order to induce participation by the insurer and the investor. They reflect the impact of the two participation constraints faced by the optimising reinsurer.*

*1.2.2. Comments on the pricing and feasibility of the securitisation*

The pricing of such an insurance bond is not trivial. We cannot use a standard risk-neutral pricing, since the market containing this new security is clearly not complete; and, even if no arbitrage is assumed, this particular security is not attainable by a replicating portfolio. A direct consequence is that the pricing rule one may obtain is not unique, but, in fact, depends on the decision criterion of the different agents. Typically, the buyer will have a maximum price, $\pi_B$, which he does not want to exceed for a given product, whilst the seller will have a minimum price $\pi_R$ below which he does not want to sell the product. A transaction will take place if the minimum seller's price is less than the maximum buyer's price. Moreover, any price within this range can be an acceptable transaction price. One may then discuss the interest of various decision criteria. Assuming that both agents (the investor and the reinsurer) have access to the financial market, each of them will base his decision on his market modified risk measure, which corresponds to the risk measure agent $a$ will have after having optimally chosen her financial investment, or hedge, on the market.

In this framework, the buyer will impose an upper bound to the transaction price by looking at the following relationship, since he wants to reduce his risk by performing the transaction:

$$\rho_B^m\left(M\left(\Theta\right)-\pi\right) \quad \leq \quad \rho_B^m\left(0\right)$$

Hence, by binding this constraint at the optimum, we obtain the maximum buyer's price, as:

$$\pi_B\left(M\right)=\rho_B^m\left(0\right)-\rho_B^m\left(M\left(\Theta\right)\right)$$

The issuer imposes a lower bound to the transaction price, since he wants to reduce his risk by performing the transaction:

$$\rho_R^m\left(-\widehat{J}-M\left(\Theta\right)+\pi+\kappa\right)\leq\rho_R^m\left(-\widehat{J}+\kappa\right)$$

Hence, by binding this constraint at the optimum, we obtain the minimum seller's price as:

$$\pi_R\left(M\right)=\rho_R^m\left(-\widehat{J}-M\left(\Theta\right)\right)-\rho_R^m\left(-\widehat{J}\right)$$

For a transaction to take place, one should have:

$$\pi_R\left(M\right)\leq\pi_B\left(M\right)$$

and any price $\pi$ in the range $[\pi_R,\pi_B]$ would be acceptable as a transaction price. The following result ensures the feasibility of the securitisation:

**Proposition 1.2.** *The indifferent buyer's price is always higher than the indifferent seller's price, ensuring the feasibility of the insurance bond transaction.*

**Proof:** Let us consider $\pi_B - \pi_R \doteq \pi_B(M) - \pi_R(M)$. We have:

$$\begin{aligned}\pi_B - \pi_R &= -\gamma_B \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_B}M^*\right)\right] + \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\widehat{J}^*\right)\right] \\ &- \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\left(\widehat{J}^* + M^*\right)\right)\right]\end{aligned}$$

Using the optimal values for $M^*$ and $\widehat{J}^*$, we have:

$$\begin{aligned}\pi_B - \pi_R &= -\gamma_B \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_I + \gamma_R + \gamma_B}X\right)\right] \\ &\quad +\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\left(\frac{\gamma_R + \gamma_B}{\gamma_I + \gamma_R + \gamma_B}X + \frac{\gamma_R}{\gamma_I + \gamma_R}\overline{X}\right)\right)\right] \\ &\quad -\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_I + \gamma_R + \gamma_B}X + \frac{1}{\gamma_I + \gamma_R}\overline{X}\right)\right]\end{aligned}$$

Using the independence between $X$ and $\overline{X}$, the formula can be simplified as:

$$\begin{aligned}\pi_B - \pi_R &= -\gamma_B \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_I + \gamma_R + \gamma_B}X\right)\right] \\ &\quad +\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\left(\frac{\gamma_R + \gamma_B}{\gamma_I + \gamma_R + \gamma_B}X\right)\right)\right] \\ &\quad -\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_I + \gamma_R + \gamma_B}X\right)\right]\end{aligned}$$

which can be rewritten as:

$$\begin{aligned}\pi_B - \pi_R &= -\left(\gamma_B + \gamma_R\right) \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_I + \gamma_R + \gamma_B}X\right)\right] \\ &\quad +\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\left(\frac{\gamma_R + \gamma_B}{\gamma_I + \gamma_R + \gamma_B}X\right)\right)\right]\end{aligned}$$

or equivalently:

$$\begin{aligned}\pi_B - \pi_R &= -\left(\gamma_B + \gamma_R\right) \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_B + \gamma_R}\left(\frac{\gamma_B + \gamma_R}{\gamma_I + \gamma_R + \gamma_B}X\right)\right)\right] \\ &\quad +\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\left(\frac{\gamma_R + \gamma_B}{\gamma_I + \gamma_R + \gamma_B}X\right)\right)\right]\end{aligned}$$

Denoting by $\chi \doteq \frac{\gamma_B+\gamma_R}{\gamma_I+\gamma_R+\gamma_B} X$, it becomes:

$$\pi_B - \pi_R = -\left(\gamma_B + \gamma_R\right) \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_B+\gamma_R}\chi\right)\right] + \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_R}\chi\right)\right]$$

We can prove that the function $f\left(\gamma\right) \doteq \gamma \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma}\chi\right)\right]$ is strictly decreasing. Since $\gamma_B + \gamma_R > \gamma_R$, we immediately obtain the desired result:

$$\pi_B - \pi_R > 0$$

□

## 2. Impact of regulatory constraints

In this section, we are interested in studying the impact of having different regulatory constraints for risk transfer, when using reinsurance and alternative risk transfer solutions. We consider differing regulatory frameworks, introducing, in particular, some asymmetry regarding the treatment of the reinsurance and the mortality bond for the reinsurer. The risk measure of the reinsurer is no longer

$$\rho_R^m\left(-\widehat{J} - M\left(\Theta\right) + \pi + \kappa\right)$$

but becomes either asymmetric or weighted in the following way:

$$\rho_R^m\left(-\widehat{J} + \kappa\right) + \alpha\rho_R^m\left(-M\left(\Theta\right) + \pi\right) \qquad \text{or} \qquad \rho_R^m\left(-\widehat{J} + \kappa + \alpha\left(-M\left(\Theta\right) + \pi\right)\right)$$

where $\alpha$ is a specific weight applied for alternative risk transfer solutions. $\alpha$ may be greater than 1 but also smaller, depending on the regulatory constraint and on the level of retention imposed upon the reinsurance company.

As proved in the next two subsections, the following result holds true, emphasising the impact of the regulatory framework upon the reinsurance market and the risk transfer capacity. Therefore, a poor regulatory constraint may lead to a negative impact on reinsurance capacity.

**Proposition 2.1.** *Depending on the type of regulatory constraint, the reinsurance capacity can be increased or decreased.*

*2.1. Distinct regulatory treatments*

Let us first consider the situation where the risk measure of the reinsurer is expressed as:

$$\rho_R^m\left(-\widehat{J}+\kappa\right)+\alpha\rho_R^m\left(-M\left(\Theta\right)+\pi\right)$$

In this case, there is a distinct regulatory treatment for both risk transfer solutions and the optimisation programme becomes:

$$\begin{aligned}
&\min_{\widehat{J},M,\pi,\kappa}\rho_R^m\left(-\widehat{J}+\kappa\right)+\alpha\rho_R^m\left(-M\left(\Theta\right)+\pi\right)\\
&\text{s.t.}\quad \rho_I^m\left(-\widehat{X}+\widehat{J}-\kappa\right)\ \leq\ \rho_I^m\left(-\widehat{X}\right)\\
&\qquad\ \ \rho_B^m\left(M\left(\Theta\right)-\pi\right)\ \leq\ \rho_B^m\left(0\right)
\end{aligned}$$

We bind both constraints at the optimum, and use the cash translation invariance property to get:

$$\begin{aligned}
\pi^*\left(M\left(\Theta\right)\right) &= \rho_B^m\left(0\right)-\rho_B^m\left(M\left(\Theta\right)\right)\\
\kappa^*\left(\widehat{J}\right) &= \rho_I^m\left(-\widehat{X}\right)-\rho_I^m\left(-\widehat{X}+\widehat{J}\right)
\end{aligned}$$

Therefore, the optimisation problem becomes:

$$\min_{\widehat{J},M}\left\{\rho_R^m\left(-\widehat{J}\right)+\rho_I^m\left(-\widehat{X}+\widehat{J}\right)+\alpha\rho_R^m\left(-M\left(\Theta\right)\right)+\alpha\rho_B^m\left(M\left(\Theta\right)\right)\right\}$$

or equivalently:

$$\min_{\widehat{J}}\left\{\rho_R^m\left(-\widehat{J}\right)+\rho_I^m\left(-\widehat{X}+\widehat{J}\right)\right\}+\alpha\min_{M}\left\{\rho_R^m\left(-M\left(\Theta\right)\right)+\rho_B^m\left(M\left(\Theta\right)\right)\right\}$$

Both optimisation problems are now disjointed, and can, thus, be solved separately. In the entropic framework, the following result prevails:

**Proposition 2.2.** *With differential regulatory constraints acting upon the risk measure for different activities, in the entropic framework, the optimal reinsurance cover is given by:*
*For the reinsurance contract:*

$$\widehat{J}^*=\frac{\gamma_R}{\gamma_R+\gamma_I}\widehat{X}$$

*and the optimal contingent payoff for the bond is:*

$$M^*\left(\Theta\right)=0$$

**Proof:** This is an immediate consequence of Proposition 2.1 in Barrieu-El Karoui (2005). $\square$

It is, therefore, optimal not to issue any bond. Comparing this result with the previous one where no distinction in the regulatory treatment is introduced (Proposition 1.1), we can see a reduction in the reinsurance capacity**.** This result is highly meaningful. Separating both regulatory treatments for traditional reinsurance and risk transfer through capital market solutions would have an overall negative impact on reinsurance capacity. A regulation trying to address the securitisation activity independently of the insurance-reinsurance activity would introduce severe distortions in the insurance market. Indeed, whatever the value of $\alpha$ in our model, securitisation is brought to zero. Even with $\alpha < 1$, meaning a more favourable treatment of insurance securitisation, when compared to reinsurance transfer, securitisation disappears. The reason is that the link between securitisation and insurance risk is broken. The diversification effect of the entropic risk measure is lost. This underlines the non-speculative motivation behind the sponsorship of an ILS instrument as well as the complementary role of both solutions in risk transfer.

*2.2. Similar regulatory treatment with different weights*

Let us now consider the situation where the risk measure of the reinsurer is expressed as:

$$\rho_R^m \left( -\widehat{J} + \kappa + \alpha \left( -M\left(\Theta\right) + \pi \right) \right)$$

In this case, there is a similar regulatory treatment, but different weights are applied depending upon the risk transfer methods, and the optimisation programme becomes:

$$\begin{array}{lll} \min\limits_{\widehat{J},M,\pi,\kappa} \rho_R^m \left( -\widehat{J} + \kappa + \alpha \left( -M\left(\Theta\right) + \pi \right) \right) & & \\ \text{s.t.} \quad \rho_I^m \left( -\widehat{X} + \widehat{J} - \kappa \right) & \leq & \rho_I^m \left( -\widehat{X} \right) \\ \qquad \rho_B^m \left( M\left(\Theta\right) - \pi \right) & \leq & \rho_B^m \left( 0 \right) \end{array}$$

We bind both constraints at the optimum, and use the cash translation invariance property to get:

$$\begin{aligned} \pi^* \left( M\left(\Theta\right) \right) &= \rho_B^m \left( 0 \right) - \rho_B^m \left( M\left(\Theta\right) \right) \\ \kappa^* \left( \widehat{J} \right) &= \rho_I^m \left( -\widehat{X} \right) - \rho_I^m \left( -\widehat{X} + \widehat{J} \right) \end{aligned}$$

Therefore, the optimisation problem becomes:

$$\min_{\widehat{J},M}\left\{\rho_R^m\left(-\widehat{J}-\alpha M\left(\Theta\right)\right)+\rho_I^m\left(-\widehat{X}+\widehat{J}\right)+\alpha\rho_B^m\left(M\left(\Theta\right)\right)\right\}$$

In the entropic framework, the following result prevails:

**Proposition 2.3.** *With a regulatory constraint acting upon the securitisation activity, in the entropic framework, the optimal reinsurance cover is given by: For the reinsurance contract:*

$$\overline{J}^*\left(\Theta^{\perp}\right)=\frac{\gamma_R}{\gamma_R+\gamma_I}\overline{X}\left(\Theta^{\perp}\right)\quad \text{and}\quad J^*\left(\Theta\right)=\frac{\gamma_R\left(\gamma_R+\alpha\gamma_B\right)}{\gamma_R\left(\gamma_R+\alpha\gamma_B\right)+\gamma_I\left(\gamma_R+\left(\alpha-1\right)\gamma_B\right)}X\left(\Theta\right)$$

*and the optimal contingent payoff for the bond is:*

$$M^*\left(\Theta\right)=-\frac{\gamma_B\gamma_R}{\gamma_R\left(\gamma_R+\alpha\gamma_B\right)+\gamma_I\left(\gamma_R+\left(\alpha-1\right)\gamma_B\right)}X\left(\Theta\right)$$

**Proof:** In the entropic framework, this optimisation problem is equivalent to that of Proposition 1.1, i.e.

$$\min_{\widehat{J},M}\left\{\rho_R^m\left(-\widehat{J}-M\left(\Theta\right)\right)+\rho_I^m\left(-\widehat{X}+\widehat{J}\right)+\rho_B^m\left(M\left(\Theta\right)\right)\right\}$$

with a modified risk tolerance coefficient for the investor: $\widehat{\gamma}_B\equiv\alpha\gamma_B$ instead of $\gamma_B$ and a modified nominal amount $\widehat{M}\left(\Theta\right)\equiv\alpha M\left(\Theta\right)$ instead of $M\left(\Theta\right)$, as

$$\alpha\rho_B^m\left(M\left(\Theta\right)\right)=\alpha\gamma_B\ln\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_B}M\left(\Theta\right)\right)\right]=\alpha\gamma_B\ln\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\alpha\gamma_B}\alpha M\left(\Theta\right)\right)\right]$$

Therefore, all the previous results of Proposition 1.1 still hold true and in particular the risk-sharing of the idiosyncratic risk is not affected, but that of the systematic risk is given now as:

$$M^*\left(\Theta\right)=-\frac{1}{\alpha}\frac{\alpha\gamma_B}{\gamma_R+\alpha\gamma_B}J^*\left(\Theta\right)$$

Hence the result. □
The impact of the level of regulation $\alpha$ is not simple as shown in the proposition below:

**Proposition 2.4.** *A stricter regulation, limiting the beneficial effect of securitisation ($\alpha < 1$) will decrease the amount of bond to be transferred as well as the reinsurance supply. Setting $\alpha > 1$ will, in contrast, increase the regulatory efficiency of transfers to the capital market and increase the reinsurance capacity.*

**Proof:** *As far as the reinsurance cover is concerned*, let us consider the following function of $\alpha$:

$$f(\alpha) = \frac{1}{\frac{\gamma_R(\gamma_R+\alpha\gamma_B)}{\gamma_R(\gamma_R+\alpha\gamma_B)+\gamma_I(\gamma_R+(\alpha-1)\gamma_B)}} = \frac{\gamma_R\left(\gamma_R+\alpha\gamma_B\right)+\gamma_I\left(\gamma_R+(\alpha-1)\gamma_B\right)}{\gamma_R\left(\gamma_R+\alpha\gamma_B\right)} = 1+\frac{\gamma_I\left(\gamma_R+(\alpha-1)\gamma_B\right)}{\gamma_R\left(\gamma_R+\alpha\gamma_B\right)}$$

The derivative of $f$ with respect to $\alpha$ is given by:

$$\begin{aligned} f'(\alpha) &= \frac{\gamma_B\gamma_I\gamma_R\left(\gamma_R+\alpha\gamma_B\right)-\gamma_I\left(\gamma_R+(\alpha-1)\gamma_B\right)\gamma_R\gamma_B}{\left(\gamma_R\left(\gamma_R+\alpha\gamma_B\right)\right)^2} \\ &= \frac{\alpha\left(\gamma_B^2\gamma_I\gamma_R-\gamma_R\gamma_B^2\gamma_I\right)+\gamma_B\gamma_I\gamma_R^2-\gamma_I\gamma_R^2\gamma_B-\gamma_I\gamma_B^2\gamma_R}{\left(\gamma_R\left(\gamma_R+\alpha\gamma_B\right)\right)^2} \\ &= \frac{-\gamma_I\gamma_B^2\gamma_R}{\left(\gamma_R\left(\gamma_R+\alpha\gamma_B\right)\right)^2} < 0 \end{aligned}$$

Hence $J^*(Q) = \frac{1}{f(\alpha)}X(Q)$ is an increasing function of $\alpha$. The optimal reinsurance coverage will, therefore, decrease with a stricter regulation, i.e. when $\alpha$ becomes smaller, and, in particular, smaller than 1.
*As far as the nominal of the bond is concerned*, let us consider the following function of $\alpha$:

$$g(\alpha) = \gamma_R\left(\gamma_R+\alpha\gamma_B\right)+\gamma_I\left(\gamma_R+(\alpha-1)\gamma_B\right)$$

This function is increasing in $\alpha$. Therefore, $M^*(Q) = -\frac{1}{g(\alpha)}X(Q)$ is an increasing function of $\alpha$. $\square$

*Remark* 3. *Note that we cannot solve in $M$ the optimisation problem for the limit case $\alpha = 0$, as any value for $M$ could be optimal since the programme no longer depends upon $M$.*

## Appendix: what happens if the insurer initiates the transactions?[4]

Let us consider the case where the insurer directly transfers some share of the systematic risk to the investor. In this case, the insurer initiates the transaction under the participation constraints for the reinsurer and the investor. He shares the systematic risk with the reinsurer, on one hand, and with the investor, on the other. Simultaneously, the specific risk is shared between the insurer and the reinsurer.
In this case we have the following problem:

$$\begin{array}{lll} \min_{\widehat{J},M,\pi,\kappa} \rho_I^m \left( W_I - \widehat{X} + \widehat{J} - M(\Theta) + \pi - \kappa \right) & & \\ \text{s.t.} \quad \rho_R^m \left( W_R - \widehat{J} + \kappa \right) & \leq & \rho_R^m (W_R) \\ \qquad \rho_B^m (W_B + M(\Theta) - \pi) & \leq & \rho_B^m (W_B) \end{array}$$

From here, we can obtain some bounds on the prices, using the cash invariance property of the various risk measures:

$$\begin{array}{rcl} \pi^* (M(\Theta)) & = & \rho_B^m (W_B) - \rho_B^m (W_B + M(\Theta)) \\ \kappa^* \left( \widehat{J} \right) & = & \rho_R^m \left( W_R - \widehat{J} \right) - \rho_R^m (W_R) \end{array}$$

Using these constraints and the cash translation invariance property of the various risk measures, the optimisation problem can be rewritten as:

$$\min_{\widehat{J},M} \left\{ \rho_I^m \left( W_I - \widehat{X} + \widehat{J} - M(\Theta) \right) + \rho_R^m \left( W_R - \widehat{J} \right) + \rho_B^m (W_B + M(\Theta)) \right\}$$

Considering the initial wealths as constant, we have:

$$\min_{\widehat{J},M} \left\{ \rho_I^m \left( -\widehat{X} + \widehat{J} - M(\Theta) \right) + \rho_R^m \left( -\widehat{J} \right) + \rho_B^m (M(\Theta)) \right\}$$

As before, we can also decompose the compensation $\widehat{J}$ into two parts:

$$\begin{array}{rcl} \widehat{J} & = & \mathbb{E} \left( \widehat{J}/\Theta \right) + \left( \widehat{J} - \mathbb{E} \left( \widehat{J}/\Theta \right) \right) \\ & \doteq & J(\Theta) + \overline{J} \left( \Theta^{\perp} \right) \end{array}$$

[4]We thank Martin Boyer for suggesting this alternative approach.

where $\overline{J}\left(\Theta^{\perp}\right)$ and $J\left(\Theta\right)$ are independent. $J\left(\Theta\right)$ represents the part of the compensation related to the systematic component of the risk, while $\overline{J}\left(\Theta^{\perp}\right)$ represents the part of the compensation related to the specific nature of the insured portfolio.

Let us now try to solve this optimisation problem in the particular situation where the three agents have entropic risk measures.

**Proposition A.1.** *In the entropic framework, the optimal characteristics are (up to a constant):*
*For the reinsurance contract:*

$$\overline{J}^{*}\left(\Theta^{\perp}\right)=\frac{\gamma_I}{\gamma_R+\gamma_I}\overline{X}\left(\Theta^{\perp}\right)\quad \textit{and}\quad J^{*}\left(\Theta\right)=\frac{\gamma_R}{\gamma_I+\gamma_R+\gamma_B}X\left(\Theta\right)$$

*and for the bond:*

$$M^{*}\left(\Theta\right)=-\frac{\gamma_B}{\gamma_I+\gamma_R+\gamma_B}X\left(\Theta\right)$$

**Proof:** To solve the optimisation Programme, we proceed in two steps:
• First, we fix the compensation $\widehat{J}$, and solve the optimisation for $M$:

$$\begin{aligned}&\min_{M}\left\{\rho_I^m\left(-\widehat{X}+\widehat{J}-M\left(\Theta\right)\right)+\rho_R^m\left(-\widehat{J}\right)+\rho_B^m\left(M\left(\Theta\right)\right)\right\}\\ =\;&\min_{M}\left\{\rho_I^m\left(-\widehat{X}+\widehat{J}-M\left(\Theta\right)\right)+\rho_B^m\left(M\left(\Theta\right)\right)\right\}\end{aligned}$$

In the entropic framework, the optimisation Programme (2) becomes:

$$\min_{M}\left\{\gamma_I\ln\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\widehat{X}+\widehat{J}-M\left(\Theta\right)\right)\right)\right]+\gamma_B\ln\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_B}M\left(\Theta\right)\right)\right]\right\}$$

Taking conditional expectations with respect to $\Theta$, we get:

$$\min_{M}\left\{\gamma_I\ln\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\widehat{X}+\widehat{J}-M\left(\Theta\right)\right)\right)/\Theta\right]\right]+\gamma_B\ln\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_B}M\left(\Theta\right)\right)\right]\right\}$$

Using the decomposition of $\widehat{J}$, and the independence property between $J\left(\Theta\right)$ and $\overline{J}$, we have:

$$\begin{aligned}&\gamma_I\ln\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\widehat{X}+\widehat{J}-M\left(\Theta\right)\right)\right)/\Theta\right]\right]\\ =\;&\gamma_I\ln\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-X\left(\Theta\right)+J\left(\Theta\right)-M\left(\Theta\right)\right)\right)\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_I}(\overline{X}-\overline{J})\left(\Theta^{\perp}\right)\right)\right]\right]\\ =\;&\gamma_I\ln\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-X\left(\Theta\right)+J\left(\Theta\right)-M\left(\Theta\right)\right)\right)\right]+c\end{aligned}$$

where $c$ is a constant independent of the optimisation problem. Hence, the programme becomes:

$$\min_{M}\left\{\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-X\left(\Theta\right)+J\left(\Theta\right)-M\left(\Theta\right)\right)\right)\right]+\gamma_B \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_B}M\left(\Theta\right)\right)\right]\right\}$$

Using Proposition 2.1 in Barrieu-El Karoui (2005), we obtain:

$$M^*\left(\Theta\right)=\frac{\gamma_B}{\gamma_I+\gamma_B}\left(-X\left(\Theta\right)+J\left(\Theta\right)\right)+\text{constant}$$

- Then, we fix the contingent payoff $M$ and work on the compensation $\widehat{J}$:

$$\begin{aligned}
&\min_{J}\left\{\rho_I^m\left(-\widehat{X}+\widehat{J}-M\left(\Theta\right)\right)+\rho_R^m\left(-\widehat{J}\right)+\rho_B^m\left(M\left(\Theta\right)\right)\right\}\\
=\;&\min_{J}\left\{\rho_I^m\left(-\widehat{X}+\widehat{J}-M\left(\Theta\right)\right)+\rho_R^m\left(-\widehat{J}\right)\right\}
\end{aligned}$$

In the entropic framework, the optimisation Programme becomes:

$$\min_{\widehat{J}}\left\{\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\widehat{X}+\widehat{J}-M\left(\Theta\right)\right)\right)\right]+\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\widehat{J}\right)\right)\right]\right\}$$

We can then treat the compensation $\overline{J}\left(\Theta^{\perp}\right)$ and $J\left(\Theta\right)$ separately:

$$\min_{J(\Theta)}\min_{\overline{J}(\Theta^{\perp})}\left\{\begin{array}{c}\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\left(X\left(\Theta\right)+\overline{X}\left(\Theta^{\perp}\right)\right)+\left(J\left(\Theta\right)+\overline{J}\left(\Theta^{\perp}\right)\right)-M\left(\Theta\right)\right)\right)\right]\\ +\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\left(J\left(\Theta\right)+\overline{J}\left(\Theta^{\perp}\right)\right)\right)\right)\right]\end{array}\right\}$$

Taking conditional expectations with respect to $\Theta$, we get:

$$\min_{J(\Theta)}\min_{\overline{J}(\Theta^{\perp})}\left\{\begin{array}{c}\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\widehat{X}+\widehat{J}-M\left(\Theta\right)\right)\right)/\Theta\right]\right]\\ +\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\widehat{J}\right)\right)/\Theta\right]\right]\end{array}\right\}$$

Using the decomposition of $\widehat{J}$ and $\widehat{X}$ and the independence property between $J\left(\Theta\right)$ and $\overline{J}$, on the one hand, and between $X\left(\Theta\right)$ and $\overline{X}$, on the other hand, we have:

$$\begin{aligned}
&\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-\left(X\left(\Theta\right)-J\left(\Theta\right)+\overline{X}\left(\Theta^{\perp}\right)-\overline{J}\left(\Theta^{\perp}\right)\right)-M\left(\Theta\right)\right)\right)/\Theta\right]\right]\\
=\;&\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-X\left(\Theta\right)+J\left(\Theta\right)-M\left(\Theta\right)\right)\right)\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_I}\left(\overline{X}\left(\Theta^{\perp}\right)-\overline{J}\left(\Theta^{\perp}\right)\right)\right)\right]\right]\\
=\;&\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_I}\left(-X\left(\Theta\right)+J\left(\Theta\right)-M\left(\Theta\right)\right)\right)\right]+c
\end{aligned}$$

and

$$\begin{aligned}
& \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_R} \left( -\widehat{J} \right) \right) / \Theta \right] \right] \\
= \; & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_R} \left( -J\left(\Theta\right) \right) \right) \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_R} \left( -\overline{J}\left(\Theta^{\perp}\right) \right) \right] \right] \right] \\
= \; & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_R} \left( -J\left(\Theta\right) \right) \right) \right] + c'
\end{aligned}$$

where $c$ and $c'$ are two constants independent of the optimisation problem in $J$. Hence, the programme becomes:

$$\min_{J} \left\{ \begin{array}{c} \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_I} \left( -X\left(\Theta\right) + J\left(\Theta\right) - M\left(\Theta\right) \right) \right) \right] \\ + \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_R} \left( -J\left(\Theta\right) \right) \right) \right] \end{array} \right\}$$

We obtain from Proposition 2.1 in Barrieu-El Karoui (2005):

$$J^{*}\left(\Theta\right) = \frac{\gamma_R}{\gamma_I + \gamma_R} \left( -X\left(\Theta\right) - M\left(\Theta\right) \right) + \text{ constant}$$

Conditioning, now, with respect to $\Theta^{\perp}$, we have to minimise with respect to $\overline{J}$ and $J$:

$$\min_{J(\Theta)} \min_{\overline{J}(\Theta^{\perp})} \left\{ \begin{array}{c} \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_I} \left( -\widehat{X} + \widehat{J} - M\left(\Theta\right) \right) \right) / \Theta^{\perp} \right] \right] \\ + \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_R} \left( -\widehat{J} \right) \right) / \Theta^{\perp} \right] \right] \end{array} \right\}$$

Using the decomposition of $\widehat{J}$ and $\widehat{X}$ and the independence property between $J\left(\Theta\right)$ and $\overline{J}$, on the one hand, and between $X\left(\Theta\right)$ and $\overline{X}$, on the other hand, we have:

$$\begin{aligned}
& \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_I} \left( -\widehat{X} + \widehat{J} - M\left(\Theta\right) \right) \right) / \Theta^{\perp} \right] \right] \\
= \; & \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( \frac{1}{\gamma_I} \left( \overline{X}\left(\Theta\right) - \overline{J}\left(\Theta\right) \right) \right) \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( -\frac{1}{\gamma_I} \left( -X\left(\Theta\right) + J\left(\Theta\right) - M\left(\Theta\right) \right) \right) \right] \right] \\
= \; & \gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}} \left[ \exp\left( \frac{1}{\gamma_I} \left( \overline{X}\left(\Theta^{\perp}\right) - \overline{J}\left(\Theta^{\perp}\right) \right) \right) \right] + \overline{c}
\end{aligned}$$

and

$$\begin{aligned}
& \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\widehat{J}\right)\right)/\Theta^{\perp}\right]\right] \\
= \; & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\overline{J}\left(\Theta^{\perp}\right)\right)\right)\mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-J\left(\Theta\right)\right)\right)\right]\right] \\
= \; & \gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\overline{J}\left(\Theta^{\perp}\right)\right)\right)\right] + \overline{c}'
\end{aligned}$$

where $\overline{c}$ and $\overline{c}'$ are two constants independent of the optimisation problem in $\overline{J}$. Hence, the programme becomes:

$$\min_{\overline{J}}\left\{\gamma_I \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(\frac{1}{\gamma_I}\left(\overline{X}\left(\Theta^{\perp}\right)-\overline{J}\left(\Theta^{\perp}\right)\right)\right)\right]+\gamma_R \ln \mathbb{E}_{\widehat{\mathbb{Q}}}\left[\exp\left(-\frac{1}{\gamma_R}\left(-\overline{J}\left(\Theta^{\perp}\right)\right)\right)\right]\right\}$$

We then get:

$$\overline{J}^*\left(\Theta^{\perp}\right)=\frac{\gamma_R}{\gamma_R+\gamma_I}\overline{X}\left(\Theta^{\perp}\right)$$

• Finally, using the various results, we have (up to a constant):

$$J^*\left(\Theta\right)=\frac{\gamma_B+\gamma_R}{\gamma_I+\gamma_R+\gamma_B}X\left(\Theta\right)$$

and

$$M^*\left(\Theta\right)=-\frac{\gamma_B}{\gamma_I+\gamma_R+\gamma_B}X\left(\Theta\right)$$

□

*Remark* 4. *We note that the results are unchanged. The same amount of systematic risk is transferred to the capital market, and the reinsurer retains the same net amount of this risk. The same risk-sharing transaction is also observed between the insurer and the reinsurer concerning the specific risk. Hence, our results also apply to the case where a large insurer is directly in contact with the investors in order to transfer a part of his endowed systematic risk to the capital market.*

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