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Technological Diversification†

By Miklós Koren and Silvana Tenreyro*

Economies at early stages of development are frequently shaken by large changes in growth rates, whereas advanced economies tend to experience relatively stable growth rates. To explain this pattern, we propose a model of technological diversification. Production makes use of input-varieties that are subject to imperfectly correlated shocks. Endogenous variety adoption by firms raises average productivity and provides diversification benefits against variety-specific shocks. Firm-level and aggregate volatility thus decline as a by-product of the development process. We quantitatively assess the model's predictions and find that it can generate patterns of volatility and development consistent with the data. (*JEL* D21, D24, E23, O33, O47)

Economies at early stages of the development process are often shaken by abrupt changes in growth rates. In his influential paper, Lucas (1988, p. 4) notes that "within the advanced countries, growth rates tend to be very stable over long periods of time," whereas within poor countries "there are many examples of sudden, large changes in growth rates, both up and down."

Motivated by this empirical observation, this paper proposes an endogenous growth model of technological diversification. The model's key idea is that firms using a large variety of inputs can mitigate the impact of shocks affecting the productivity of individual varieties. This process takes place through two channels. First, with a larger number of varieties, each individual variety matters less in production, and productivity thus becomes less volatile by a version of the law of large numbers. Second, whenever a shock hits a particular variety, firms can adjust the use of the other varieties to partially offset the shock. Both channels make the productivity of firms using more sophisticated technologies less volatile. Since firms in richer economies tend to rely on technologies involving a richer menu of inputs, richer countries will also tend to be less volatile.

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Building on the seminal contributions by Romer (1990) and Grossman and Helpman (1991), our model characterizes technological progress as an expansion in the number of input varieties. The number of varieties evolves endogenously in response to producers' incentives to add to the range of inputs they use, and increases in the number of varieties raise the average level of productivity. Our contribution is to make the model stochastic, so that it can be used to study its implications for output volatility. In particular, we assume that each variety can be affected by a productivity shock; thus, the expansion in the number of varieties can provide diversification benefits, and hence reduce the level of volatility of the economy.[1] In other words, the reduction in volatility arises as a likely by-product of firms' incentives to increase productivity. As such, our model highlights a hitherto overlooked implication of expanding-variety growth models, which makes them suitable to explain the decline in volatility that accompanies the development process.

We say "suitable to explain" because, interestingly, once technological diversification is embedded in an endogenous growth model with multiple firms, it is possible to generate examples where volatility and development do not necessarily move in opposite directions. This happens, for example, if a significant number of firms adopt an input that is already widely used by other firms; the economy as a whole may then become highly technologically concentrated and hence exposed to shocks to that particular input, leading to episodic surges in volatility—higher productivity in this case can come at the cost of higher volatility. In practice, however, development and volatility move in opposite directions most of the time, and this is indeed the case in virtually all our numerical experiments. This occurs because the introduction of a *new* variety in the economy always increases the level of development, and raises the degree of technological diversification by reducing the contribution to output of previously existing varieties (thus lowering volatility). A calibrated version of the model can yield a decline in volatility with development quantitatively comparable to that in the data.

A simple example of the mechanism of technological diversification is offered by a comparison of an economy using only labor and an economy using labor and capital. Under standard assumptions on technology, the latter will tend to be more productive on a per capita basis. Our point is that it will also be less volatile. In particular, any shock that reduces the supply of labor (such as an epidemic, a general strike, etc.) will have a bigger negative impact on the economy that does not have scope to substitute labor with capital. Or, to think of a currently more realistic example, consider leading-edge steel producers that have the capacity to process iron ore of a range of qualities as compared to more basic producers who can only accept high-quality ores as input. Clearly the former are more productive, and, in addition, they should be less susceptible to shocks to the (global or local) supply of high-quality iron ore.[2]

[1] Input varieties are broadly construed to encompass both tangible and intangible inputs or technologies. Shocks are variety-specific and to the extent that the varieties are used by a positive measure of firms, they lead to aggregate volatility.

[2] Throughout the paper, we focus the analysis on the case in which different varieties are gross substitutes. In the online Appendix, we show that technological diversification can also lead to lower volatility when varieties are gross complements in production, provided that shocks are not too large. Intuitively, even when goods are complements there can be scope for substitutability in the budget; this is similar to the result that every good (or input) has at least one substitute, even when there is complementarity in utility (or production).

A more drastic example of the lack of technological diversification in less developed economies is offered by the 2011 drought in East Africa and the Horn, where a large fraction of the livestock (one of the main assets of these economies) died, causing large drops in production and threatening the livelihoods of millions of people. (In sharp contrast, more developed and technologically diversified economies count on irrigation systems to cope with droughts.) The stabilizing virtues of technological diversification are also much in evidence in the debate over energy policy in developed economies. The increase in oil prices in the 2000s has led to overwhelming bipartisan support in the United States for the H-prize Act of June 2007, which seeks to incentivize "achievements in overcoming scientific and technical barriers associated with hydrogen energy" in order "to free [the country] from its dependence on foreign oil."[3, 4]

Previous theoretical studies on the relation between volatility and development, including Greenwood and Jovanovic (1990), Saint-Paul (1992), Obstfeld (1994), and Acemoglu and Zilibotti (1997), have focused on *financial*—as opposed to *technological*—diversification. These models feature an inherent trade-off between productivity and risk at the microeconomic level: firms (or decision units) must choose between low-productivity but safe activities and high-productivity but risky ones. Firms in financially underdeveloped countries do not have the facility to pool risks, so in the presence of risk aversion, they minimize risk by choosing low-productivity projects. In financially developed countries, risks can be pooled and hence high-return and high-risk projects are undertaken. Aggregate volatility may still be lower in developed countries if financial development facilitates the creation of new financial diversification opportunities across firms. Thus, as Acemoglu (2005, p. 213) summarizes it, the model of financial diversification implies "a negative relationship between aggregate and firm-level volatility, a positive relationship [between development and] firm-level volatility, a steady increase in firm-level volatility, and a steady decline in aggregate volatility."

Unlike existing models, the expanding-variety model we propose posits no trade-off between productivity and risk at the firm level. Indeed, our point is that there are technological reasons to expect the adoption of a new variety to lead concurrently to an increase in productivity and a decline in volatility. Hence, preferences toward risk, which are crucial in models of "financial diversification," play no role in our story, where firms are uniquely concerned with profit maximization.[5] Furthermore, in our model the process of technological diversification takes place *within the firm*, not *across firms*. Finally, our results do not hinge on financial development.

[3] The first quotation comes from the Act text itself (Senate Committee on Energy and Natural Resources, *H-Prize Act of 2007*, 110th Cong., 1st sess., 2007, H.R. 632, 1, http://www.gpo.gov/fdsys/pkg/BILLS-110hr632rfs/pdf/BILLS-110hr632rfs.pdf). The second comes from its sponsor's statement at the House of Representatives (Committee on Science and Technology, Subcommittee on Energy and Environment, *Prepared Statement of Representative Daniel Lipinski*, Appendix, 110th Cong., 1st. sess., May 10, 2007. http://thomas.loc.gov/cgi-bin/cpquery/?&dbname=cp110&sid=cp1104ZOJZ&refer=&r_n=hr171.110&item=&&&sel=TOC_116398&); the Act was passed with 408 ayes and 8 nays.

[4] Blanchard and Galí (2010) find that in the United States, the share of oil used in production and consumption in the late 1990s was smaller than in the 1970s; that is, the US economy seemed to already be (though slowly) diversifying away from oil.

[5] In particular, if firms were risk-neutral, financial diversification models would predict complete specialization in the most productive and risky sector or activity (and hence extreme volatility), while in our setting risk-neutral firms still want to "diversify" (i.e., expand the number of inputs in production).

These theoretical differences lead to important differences in empirical implications. First, financial diversification models predict an increase in firm-level volatility with the level of development, and a negative comovement between aggregate and firm-level volatility. Instead, our model predicts a decline in firm-level volatility with development and a positive comovement between aggregate and firm-level volatility. On these two predictions, our model finds support in recent work by Davis et al. (2007), who document that in the United States, over time, privately held firms have experienced a substantial decline in volatility; the authors further show that the decline in aggregate volatility in the United States has been overwhelmingly driven by the decline in firm-level volatility (and not by the aggregation of highly volatile firms exposed to increasingly less correlated idiosyncratic shocks). In the next section we discuss new evidence for 17 other countries confirming the tendency for a positive comovement between aggregate and firm-level volatility.

A second testable prediction of models of financial diversification is that the decline in aggregate volatility with development is brought about by financial development. In our model, the decline in volatility takes place independent of the level of financial development. As we argue in the next section, this implication is corroborated by the evidence. The strong negative correlation between volatility and development takes place at all levels of financial development. Put differently, even controlling for the level of financial development, there remains a strong negative correlation between volatility and development that needs explanation. While we view both margins of diversification for the firm, financial and technological, as complementary and empirically plausible, our model will focus exclusively on the second one.[6]

As mentioned, our model posits no trade-off between productivity and volatility at the microeconomic level. The absence of a trade-off is motivated by the finding that countries at early stages of development tend to specialize in low-productivity, high-risk activities, whereas the opposite pattern is observed at later stages of development. Moreover, even within narrowly defined sectors, developing countries tend to feature both lower productivity and higher volatility than developed countries. (See Koren and Tenreyro 2007.)[7, 8]

It is also important to distinguish our mechanism of technological diversification from standard arguments concerning sectoral diversification (or diversification of output), namely that developing countries should reduce their reliance on cash crops or natural resources in order to hedge against fluctuations in these commodities' prices. First, our model concerns the diversification of inputs, not the diversification of outputs or products. Second, and most important, sectoral diversification is usually associated with a move away from comparative advantage, so it tends to

[6] Technological diversification is also complementary to other finance-related mechanisms emphasized in the literature. In particular, shocks can be amplified by introducing financial frictions, a task we do not undertake in the interest of clarity and simplicity. For models with financial frictions, see, among others, Bernanke and Gertler (1990), Kiyotaki and Moore (1997), and Aghion et al. (2010).

[7] The sectoral composition of the economy alone cannot account for the differences in volatility between developed and developing countries; the "within" sector decline in volatility is at least as important in explaining volatility differences between developed and developing economies (Koren and Tenreyro 2007).

[8] In departing from a necessary trade-off between productivity and volatility, our paper is closer to Kraay and Ventura (2007), though the mechanisms are different: in their model, the key idea is that in the event of a shock, terms of trade respond more countercyclically in rich countries than in poor countries.

reduce (average) income. Instead, technological diversification chiefly occurs as a by-product of strategies whose main aim is to increase average income.[9]

Of course, technological diversification is not the only mechanism that can potentially cause a decline in both aggregate and firm-level volatility with the level of development. Indeed, there is a wide literature linking aggregate volatility to different macroeconomic variables, including openness to trade, policy or political volatility, institutions, and the size of governments. We show that even after controlling for all these variables, the correlation between aggregate volatility and the level of development remains strong, suggesting that existing explanations cannot fully account for the large differences in volatility between rich and poor countries. These results hence call for new theories to account for the empirical correlation. This paper offers a new explanation framed within one of the canonical endogenous growth models, and it shows that a calibrated version of the model can quantitatively account for a significant part of the volatility-development relationship.

As in all expanding-variety endogenous growth models, countries at lower levels of development use fewer inputs or technologies; i.e., technology diffusion across countries is not costless or frictionless.[10] Various studies document the slow and delayed diffusion of technology. In a seminal paper, Griliches (1957) documents the slow diffusion of agricultural technology across US regions. Comin and Hobijn (2004) find that most innovations originate in developed countries and spread only gradually to less-developed countries. Caselli and Coleman (2001) find that the adoption of computers depends crucially on the level of development of the economy. Caselli and Wilson (2004) show that this result extends to a broader set of technologies.

Our model makes progress relative to existing models of aggregate fluctuations in that it endogenizes the link between the level of development and the susceptibility of the economy to shocks.[11] To focus on this link, which is the novel contribution of the paper, the model is more stylized in other dimensions that have been emphasized in the real business cycle (RBC) or the New Keynesian literature. The paper does, on the other hand, speak to other regularities that are not addressed in the RBC or New Keynesian literature, which we discuss in the next section.

The paper is organized as follows. Section I documents a set of empirical observations that motivate our model and differentiate it from alternative explanations. Section II presents the model of technological diversification and derives its implications for aggregate dynamics. Section III presents a quantitative analysis of the model. Section IV offers concluding remarks. The online Appendix provides additional evidence supporting the regularities in Section I. It then presents the proofs, generalizes the model, and discusses its robustness under different assumptions. In particular, it studies the conditions under which technological diversification takes

[9] In fact, sectoral diversification as a hedging strategy is dominated by financial hedging on commodity-futures markets. As discussed, no such (better) substitute exists for technological diversification.

[10] Moreover, trade liberalization per se does not ensure that a country would adopt the technologies or inputs of other (more developed) countries. Similarly, the fact that a firm uses a more sophisticated technology in a given country does not assure that all other producers will be able to use that technology, unless they invest in adoption (e.g., know-how).

[11] Implicitly, in real business cycle models, the source of aggregate fluctuations and the *level* of development are considered unrelated phenomena or, put differently, the level of development plays no role in determining fluctuations.

place when varieties are gross complements, as is the case in the O-ring theory formulated by Kremer (1993), and it works out the implications of the model under different assumptions regarding technology and risk preferences.

I. Empirical Motivation

This section presents the main empirical observations that motivate the theoretical model, and along which we shall later evaluate it. It also discusses a set of auxiliary empirical results that justify the search for new models. In the interest of space, most of the supporting tables and figures are reported in the online Appendix.

A. *Empirical Observations*

Empirical Observation 1: GDP volatility declines with development, both in the cross section and for a given country over time.

The negative association between aggregate volatility and the level of development, noted in Lucas's (1988) seminal paper, is one of the stylized facts in the macro-development literature and the starting motivation of this paper. The relation is summarized in the first column of Table 1, which reports the results from a regression of the (log) level of volatility, measured as the standard deviation of the annual growth rate of real GDP per capita over nonoverlapping decades from 1960 through 2007, on the average (log) level of real GDP per capita of the corresponding decade. The data come from the Penn World Tables (PWT, version 6.3) and are adjusted for purchasing-power parity (PPP). The second column displays the regression results after controlling for country-specific fixed effects; it indicates that for a given country over time, growth and changes in volatility are also negatively correlated.[12] The third and fourth columns show the corresponding results when the data are not adjusted for PPP.[13] In all cases, the slope coefficients are statistically significant at the 1 percent level, and become larger when fixed effects are included.

The model of technological diversification we present generates a negative correlation between volatility and development as countries using a larger number of input varieties are both more productive and typically better-diversified across varieties. The high volatility that characterizes early stages of development results from the relatively low number of varieties used in the production process.

Empirical Observation 2: Firm-level volatility declines with firm size.

The volatility of an individual firm's sales growth and the size of the firm, whether gauged by the average volume of sales or the number of employees, appear to be negatively correlated. This finding was first documented by Hymer and Pashigian (1962) for the US economy, and later corroborated by a number of empirical studies (see, for example, Hall 1987 and Sutton 2002).

[12] In related work, Ramey and Ramey (1995) study the link between volatility and growth. We focus instead on the links between volatility and *development* or between *changes* in volatility and growth, to be consistent with the predictions of the model we later develop.

[13] The non-PPP-adjusted data correspond to the series of GDP per capita in constant US dollars from the World Bank's *World Development Indicators* (WDI).

Table 1—Volatility and Development

Dependent variable: SD of GDP per capita growth rates	PPP-adjusted data		Non-PPP-adjusted data	
Real GDP per capita (PPP-adjusted, PWT)	−0.206***	−0.467***		
	[0.032]	[0.072]		
Real GDP per capita (WDI)			−0.118***	−0.456***
			[0.023]	[0.088]
Constant	−1.482***	0.746	−2.542***	−0.005
	[0.274]	[0.612]	[0.174]	[0.659]
Country fixed effects				
Observations	714	714	706	706
R^2	0.108	0.564	0.069	0.467

Notes: All variables are in logs. The dependent variable is measured as the standard deviation of annual real GDP per capita growth rates over nonoverlapping decades from 1960 to 2007. The regressor is computed as the average over the decade. The data in the first two columns come from PWT and are PPP-adjusted. The data in the last two columns come from WDI and are not adjusted for PPP. Clustered (by country) standard errors in brackets.
*** Significant at the 1 percent level.
** Significant at the 5 percent level.
* Significant at the 10 percent level.

Table 2—Firm-Level Volatility and Size

	Size measure			
Dependent variable: SD of sales growth rates	Number of employees		Volume of sales	
Size	−0.226***	−0.134***	−0.192***	−0.157***
	[0.003]	[0.013]	[0.002]	[0.009]
Constant	−1.078***	−1.427***	−1.779***	−1.791***
	[0.029]	[0.082]	[0.019]	[0.037]
Firm fixed effects	No	Yes	No	Yes
Observations	38,168	38,168	50,308	50,308
R^2	0.246	0.713	0.244	0.675
Number of clusters		16,961		19,529

Notes: All variables are in logs. The equations use the five-year standard deviation of annual (real) sales growth rates from 1975 to 2007. The two size measures (number of employees and volume of sales) are computed at their mean values over the lustrum. Year fixed effects included in all regressions. Clustered (by firm) standard errors in brackets.
*** Significant at the 1 percent level.
** Significant at the 5 percent level.
* Significant at the 10 percent level.

The relation is illustrated in Table 2 for US firms included in Standard and Poor's *Compustat* 2010 database. The table shows the coefficients from a regression of (log) volatility of sales growth on average size, measured as either the (log) number of employees or the (log) volume of sales. Volatility is calculated for nonoverlapping five-year periods from 1975 through 2005. The negative correlation remains strong even if we include firm-fixed effects to consider within-firm variation only. In Table A1 of the online Appendix we report new evidence on the cross-sectional relation between firm-level volatility and size for a broad group of countries at different stages of development. The size-volatility relationship is consistently negative in all countries.

There is also evidence that the share of small firms in the economy (measured in terms of output or employment) correlates negatively with income per capita both across countries (Leidholm and Mead 1987, and Banerji 1978) and within countries over time (Little, Mazumdar, and Page 1987, and Steel 1993). This will be the case in our model: economies with lower income per capita have a higher share of small and highly volatile firms (i.e., firms using a relatively small number of varieties).

As previously stated, in our model technological diversification stems from the diversification of (broadly construed) inputs, not outputs. It is thus pertinent to note that the decline in firm volatility with size is not driven simply by large firms operating in a bigger number of business segments; in other words, diversification in output alone does not account for the negative correlation. We investigate this issue in the online Appendix, where we confirm that the results in Table 2 are robust to controlling for the number of business segments in which firms operate and that these results also hold for a sample of firms operating in a single business segment—that is, firms with no diversification along the output dimension.

Empirical Observation 3: Firm-level and aggregate volatility tend to display a positive comovement.

Aggregate volatility and the volatility of privately owned firms tend to comove positively. As shown by Davis et al. (2007), the decline in aggregate volatility in the US economy that took place from the mid-1980s until the mid-2000s has been overwhelmingly driven by the decline in volatility of nonlisted firms and not by the aggregation of increasingly more volatile firms displaying progressively lower correlation in their performance.[14,15] A similarly positive comovement between firm-level and aggregate volatility is documented for France by Thesmar and Thoenig (2011) and for Germany by Buch, Döpke, and Strotmann (2006). In the online Appendix we study a relatively long time series of firm-level data for Hungary, confirming a positive comovement between firm and aggregate volatility. The results are reported in Figure A1. In addition, the online Appendix reports further evidence for 14 other countries, for which we have shorter time series of firm-level data. The results, while only suggestive given the data limitations, indicate that firm-level and aggregate volatility tend to move in the same direction.

B. *Alternative Explanations and Additional Evidence*

Financial Development.—The positive comovement between firm-level and aggregate volatility is one distinguishing feature of our mechanism vis-à-vis models of financial diversification. In addition, in financial diversification models, the decline in aggregate volatility with development is brought about by financial development. In the data, however, the volatility-development relationship holds at different levels of financial development, measured as private credit over GDP.[16] This relationship is illustrated in Figure A2 of the online Appendix, where we split

[14] Comin and Philippon (2006) had previously documented that publicly traded US firms experienced an increase in volatility during the same period. Publicly traded firms are only a small fraction of all firms, however. Since a majority of firms in most countries are privately held, the evidence from Davis et al. (2007) is more informative for our purposes.

[15] Models of financial diversification require a decrease in cross-firm correlation over time in order to generate a decline in aggregate volatility. In our model, this correlation will be constant.

[16] The data come from the World Bank's Financial Structure Database (v.4) and correspond to the series private credit by deposit money banks and other financial institutions over GDP.

the level of financial development into different quartiles. The plots show that the decline in volatility with development is not sensitive to the country's level of financial development, the key mediating mechanism in financial diversification models. Put differently, even controlling for the level of financial development, there remains a strong negative correlation between volatility and development that needs explanation. Equally important, while the univariate regressions between volatility and financial development yield a negative coefficient, the correlation vanishes once other controls are added to the specification. Instead, the relation between volatility and development appears robust to the same controls and is not altered by the inclusion of a proxy for financial development. This is illustrated in Tables A4 and A5 of the online Appendix, which report the results from regressions of volatility on a number of covariates.[17]

Other Covariates.—The main goal of this paper is to account for the negative association between volatility in production and economic development. It is important to stress that this negative correlation is not explained away by other covariates that have been suggested in the literature. Previous studies have stressed policy variability, openness to trade, and political instability as potential sources of volatility.[18] In the online Appendix we study the robustness of the volatility-development relation to the inclusion of alternative determinants of volatility. The results are reported in Tables A4 and A5. The main message from our analysis is that there is a strong relationship between volatility and development that remains statistically unexplained, even after controlling for a wide range of covariates. We hence need new channels to explain the data and in this paper we theoretically explore the extent to which technological differences, or, more concretely, differences in the degree of technological sophistication across countries, can quantitatively account for the observed correlation, and at the same time match the firm-level evidence discussed in this section.

Skewness.—An additional observation consistent with our model is that the time series of growth rates exhibit negative skewness.[19] While in our model there are both positive and negative "fundamental" shocks at the firm level, positive shocks add up to a smooth aggregate process. This captures the empirical observation that the growth process seems to be more gradual, with positive growth rates clustered around the median growth rate. In contrast, falls (or negative deviations from trend) do not wash out and generate sharp aggregate fluctuations.

[17] An additional difference with financial models concerns the relation between productivity and volatility at the microeconomic level. Koren and Tenreyro (2007) find a negative correlation between productivity and volatility across and within sectors for a broad sample of countries, which is at odds with standard assumptions in financial diversification models. Our model imposes no microeconomic trade-off.

[18] See Becker and Mauro (2006) for an analysis of the sources of crises. Note that institutional instability can itself be the result of economic shocks. We take a broad interpretation of the sources of shocks: as in the RBC literature's tradition, changes in policies (through taxes, or regulations) can be the source of changes in inputs' or technologies' productivity.

[19] McQueen and Thorley (1993), Sichel (1993), and Jovanovic (2006) highlighted this asymmetry using US data on industrial production and GDP growth rates. We found this asymmetry to be present in the majority of countries in the sample (see online Appendix, Section A4).

Adoption of Varieties.—In our model firms grow by expanding the set of technologies or inputs they use. In the online Appendix we discuss extensive evidence supporting this assumption. For an overview, see Granstrand (1998), who summarizes the results from several studies using data from Japanese, European, and American companies, and argues that technology diversification (defined as the firm's expansion of its technology base into a wider range of technologies) was a fundamental causal variable behind corporate growth; this was also the case when controlling for product diversification and acquisitions. Granstrand, Pavitt, and Patel (1997) provide additional case-study analysis of the phenomenon of technological diversification in the growth of a firm and point out that technological diversification took place even in firms whose "product base" shrank, following an emphasis on "focus" and "back to basics" during the 1980s. Oskarsson (1993) documents an increase over time in technological diversification in Organisation for Econonic Co-operation and Development (OECD) countries at various levels of aggregation (industry, firm, product). He finds a strong positive correlation between sales growth and growth in technology diversification at all levels of aggregation. Gambardella and Torrisi (1998) measure technological diversification of the largest US and European electronics firms by calculating the Herfindahl index of each firm's number of patents in 1984–1991. Their main findings are that better performance (in terms of sales and profitability) is associated with increased technological diversification and *lower* product diversification. They conclude that technological diversification is the key covariate positively related with various measures of performance.

Feenstra, Markusen, and Zeile (1992) provide evidence that input diversification leads to growth and productivity gains. Using data on South Korean conglomerates, they find that the entry of new input-producing firms into a conglomerate increases the productivity of that conglomerate. In farming, there are multiple examples of inputs leading to productivity gains and faster growth. The World Bank (2011) reports that in larger-scale crop production, the two short-term interventions with the greatest impact in productivity are the use of high-quality seed and chemical fertilizers. The same study lists a number of inputs that both increase agricultural productivity and lower volatility, including fertilizers, modern seeds, agronomic skills, irrigation systems, and cell phones (useful to transmit information on weather news).

In the online Appendix we discuss additional studies and present evidence from input-output tables in different countries showing that purchases (direct or indirect) by a given sector from itself relative to total purchases by that sector have fallen significantly over time in OECD countries from 1970 to 2007. The trend towards higher usage of inputs from other sectors is another manifestation of the technological diversification mechanism.

II. A Model of Technological Diversification

Before specifying the model in detail, we offer a brief informal preview of the main features. Monopolistically competitive firms produce goods using a variety of inputs (or, more broadly, technologies). There is free entry by firms and new firms start up with no varieties. Firms can add new varieties to the range of inputs they use by engaging in some adoption effort (e.g., to learn how to use it). In particular, they can invest resources in an adoption process, which succeeds sooner the more resources the firm invests. In deciding how much to invest in adoption, each firm seeks to maximize the present discounted value of profits. (Since firms are risk-neutral, profit maximization is the only goal of this process.) Hence the adoption part of the model is very similar to standard expanding-variety models, except that the adoption goes on simultaneously in multiple firms and, due to the random elements of the model, it implies that different firms will have different numbers of varieties at a given point in time.

The model's innovative feature is that varieties are subject to productivity shocks. In particular, once a new variety has been added to a firm's range of inputs or technologies, it becomes a permanent part of its productive process until a random shock causes a drop in its productivity. (This assumption is motivated below.) This shock is variety-specific, so it affects all firms that happen to be using that particular variety. The aggregate effects of such shocks depend, therefore, on the distribution of varieties across different firms. Thus, to study the evolution of volatility over time it is necessary to keep track of this distribution.

To sustain long-term balanced growth, the model features an entry-exit margin of firms and allows for external effects in production. Balanced growth is not needed for the technological-diversification channel to operate, but it facilitates tractability.

On the household side, identical agents supply labor effort inelastically in competitive labor markets and seek to maximize the present discounted stream of consumption of the final good, which is a composite of the individual goods produced by all firms in the economy. Households own the firms in the economy.

A. *The Economy*

There is a continuum of monopolistically competitive firms, indexed by j, each producing a differentiated product. The output of the final good is a constant-elasticity-of-substitution (CES) aggregate of firm-level outputs,

$$Y(t) = \left[\int_0^{M(t)} y(j,t)^{(\varepsilon-1)/\varepsilon} \, dj \right]^{\varepsilon/(\varepsilon-1)}, \tag{1}$$

where $y(j,t)$ is the output produced by firm j at time t, $M(t)$ is the mass of firms at time t, and $\varepsilon \in (1,\infty)$ is the elasticity of substitution across firms. Each individual firm produces output by combining a variety of inputs through the CES production function,

$$y(j,t) = A(t) \left[\sum_{i \in \mathcal{I}(j,t)} [\chi_i(t) \, l_i(j,t)]^{1-1/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)}, \tag{2}$$

where $\chi_i(t)$ is the productivity of variety i at time t; $l_i(j,t)$ is the number of workers allocated to the operation of input-variety i by firm j at time t; $\mathcal{I}(j,t)$ is the set of varieties used by firm j at t; and $A(t)$ is an aggregate productivity factor, which will vary due to external effects, introduced below.

For analytical convenience, we assume the elasticity of substitution between varieties ε in equation (2) to be the same as the elasticity of demand in equation (1). This assumption will ensure that profits are linear in the number of varieties, simplifying the algebra of aggregation. It can, however, be dispensed with at the cost of additional algebra. In the online Appendix we relax this assumption and characterize the equilibrium conditions when the elasticities are different. As usual in most endogenous growth models (Romer 1990, Grossman and Helpman 1991), we assume that $\varepsilon > 1$; that is, technologies are gross substitutes. The online Appendix derives the conditions under which technological diversification can lead to lower volatility when $\varepsilon \leq 1$; that is, when inputs are complements.

One could reinterpret the varieties in our model not as inputs but as disembodied technologies to turn labor into output. Expanding the number of varieties of such technologies is also likely to both increase productivity and provide technological diversification. Hence, in the rest of the paper we refer to the varieties interchangeably as inputs and as technologies.[20]

Notice that we are implicitly assuming that the firm uses each variety in constant quantities, here normalized to one. What varies is the number of varieties, the quantity of labor assigned to each of them—capacity utilization—(both of which depend on the firm's decisions) and the productivity of each variety (which will be random). In reality, the quantity of each input variety will also vary, but abstracting from this decision allows us to focus on technological diversification, which comes from an expansion in the number of varieties, without overly complicating the analysis. Under the technology interpretation, the assumption would be fine as is.

We assume that varieties have a constant productivity during their random lifetime; when a shock hits the variety, it ceases to contribute to production. (A variety can potentially be readopted if firms incur new adoption costs—see below.) The arrival of shocks for a given variety i is common to all firms using this variety, and it follows a Poisson process with arrival rate γ. Shocks are independent across varieties.

Because a shock arrives with a Poisson process, the input's productive lifetime follows an exponential distribution with parameter γ. Hence, conditional on variety i working at time 0, the distribution of $\chi_i(t)$ is given by

$$\chi_i(t) = \begin{cases} 1 & \text{with prob. } e^{-\gamma t}, \\ 0 & \text{with prob. } 1 - e^{-\gamma t}. \end{cases}$$

Let $aJ(bt)$ be a Poisson process with arrival rate b and jumps of size a.[21] With this notation, the dynamics of a variety's productivity can be written as

$$d\chi_i(t) = -\chi_i(t)\, dJ_i(\gamma t), \tag{3}$$

where the subindex i in $J_i(\gamma t)$ highlights that the Poisson processes are variety-specific and independent across varieties. Productivity is constant (at 1) before a

[20] Another interpretation is that the production function takes the form $y = A\left[\sum_i x_i^{1-1/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)}$ where x_i is the intermediate good produced by the firm by transforming labor through $x_i = \chi_i l_i$. Nothing substantial changes either if the inputs are produced by specialized producers and sold to the firm at arms' length; in this case, shocks to χ_i map into input price shocks.

[21] See, for example, Cox and Isham (1980).

jump occurs, and it jumps down to zero with the first arrival of $dJ_i > 0$. Substituting this productivity into the production function of the firm, we obtain

$$y(j,t) = A(t)\Big[\sum_{i:\chi_i(t)=1} l_i(j,t)^{1-1/\varepsilon}\Big]^{\varepsilon/(\varepsilon-1)}.$$

We denote the number of productive varieties used by firm j at time t by $n(j,t)$. Given that all productive varieties enter symmetrically in the production function, firms will allocate the same number of workers to each; hence, $l_i(j,t) = l(j,t)/n(j,t)$, where $l(j,t)$ is the total number of workers employed by firm j at time t.[22] We can then write the production function as

$$y(j,t) = A(t)\, l(j,t)\, n(j,t)^{1/(\varepsilon-1)}. \tag{4}$$

Our main motivation for the choice of the stochastic process in equation (3) is analytical tractability. It dramatically simplifies the firm's decision problem, because there is only one firm-level state variable to keep track of: since the productivity of each variety can only take the values zero and one, firms only care about the *set* of varieties that are still productive. Moreover, the stochastic process, together with the symmetry of the varieties, ensures that it is only the *number* of productive technologies that matters. Importantly, although the Poisson process may suggest irreversibility, in practice the failure of a given variety in the model does not need to be completely irreversible, since a variety can in principle be put back into place, provided that firms pay the adaptation (or adoption) costs, which we shall describe later.[23]

While analytical tractability is a main consideration, equation (3) does describe a class of relevant input-specific shocks, namely shocks that make an input completely unavailable (at least for a discrete period of time). This can occur and has occurred in the case of some natural resources that exist in finite quantities. The canonical example familiar from history textbooks is the nineteenth century "guano crisis." Guano was widely used as a fertilizer to increase crop yields during the early nineteenth century all over the world. In the second half of the century the reserves ran out (largely due to the Peruvian government's mismanagement) and the fertilizer became unavailable, causing a major disruption in agriculture—particularly in countries that did not use a more diversified set of fertilizers such as nitrates and mined rock phosphate. Oil is another example; while oil reserves have not (yet) been depleted, oil disruptions have been a recurrent and important source of output fluctuations in the last half century.

An input does not need to be an exhaustible natural resource to become (temporarily or permanently) unavailable. In the 2011 drought in East Africa a large fraction of the livestock died, causing drastic drops in output. In 1993 an explosion

[22] This formula implicitly assumes that labor can be reallocated at no cost after a shock is realized. This is exclusively done for simplicity; introducing reallocation costs will magnify the loss from technology shocks and mitigate the immediate gain from successful adoption, but will not alter the main results.

[23] For completeness, the online Appendix analyzes the case in which productivity drops, but not to zero. It shows that the key intuition and implications of the technological diversification mechanism go through as in the baseline model. This alternative setting adds a new state variable (because we have to keep track of unproductive varieties), but does not add any new insight. To focus on the key mechanism of the model we hence relegate this variation to the online Appendix.

in a Sumitomo plant in Japan led to the destruction of two-thirds of the world supply of the high-grade epoxy resin used to seal most computer chips, causing shortages and price hikes in the semiconductor industry for several months. More generally, various disasters can destroy the output of intermediate goods producers. Similarly, government policies can hinder the production or use of certain intermediate products. Human capital is not immune from such shocks either: Pol Pot and Mao Zedong wiped out the human capital of an entire generation in their respective countries.

Even if not taken literally, the process described in equation (3) can also be seen as a shortcut to model less radical disruptions; in that spirit, shocks to χ_i can result, for example, from changes in taxes or regulatory policies, increases in the cost of production or the import price of a variety (or from the price of an input needed to use that variety, such as the price of fuels), trade disruptions, weather-related shocks that render a variety useless or severely hinder its transportation to its destination, and so on.[24]

B. *A Firm's Static Decisions*

Since firms engage in monopolistic competition, each firm faces an isoelastic demand with elasticity ε:

$$y(j,t) = Y(t)p(j,t)^{-\varepsilon}, \tag{5}$$

where aggregate output $Y(t)$ is taken as the numeraire, and $p(j,t)$ is the price charged by firm j at time t.

The production function pins down the number of workers necessary to satisfy this demand,

$$l(j,t) = y(j,t)n(j,t)^{1/(1-\varepsilon)}/A(t) = Y(t)p(j,t)^{-\varepsilon}n(j,t)^{1/(1-\varepsilon)}/A(t).$$

Firms with more varieties of inputs are more productive (a standard love-of-variety effect) and hence can produce a given level of output with fewer workers.

The firm hires workers in competitive labor markets. At time t it faces a wage rate $w(t)$, which depends on the aggregate state of the economy, and is taken as given by individual firms. Flow profits are revenue minus labor cost, so the operating profit of the firm is

$$\pi(j,t) = Y(t)p(j,t)^{1-\varepsilon} - n(j,t)^{1/(1-\varepsilon)}w(t)\,Y(t)p(j,t)^{-\varepsilon}/A(t). \tag{6}$$

Since at a given point in time the number of varieties $n(j,t)$ is predetermined, the only choice variable of the firm that can affect current profits is the price $p(j,t)$. The optimal price will in general be a function of $n(j,t)$, aggregate demand $Y(t)$, the wage rate $w(t)$, and aggregate productivity $A(t)$. The aggregate variables $Y(t)$, $w(t)$, and $A(t)$ all depend on the state of the economy in ways that will be specified below.

[24] A transportation or trade disruption might make a technology or variety temporarily unavailable, but the variety can potentially come back into use after reinvestment.

C. *Technology Adoption and Risk Preferences*

As in Romer (1990) and Grossman and Helpman (1991), adopting new varieties is a costly activity. Adoption costs can also be thought of as the cost of research and development of new varieties; however, for most producers in most countries, "adoption or adaptation" is probably a more realistic description of the investment effort fuelling growth. Often, technologies are available in principle, but firms do not invest in adoption.[25]

For analytical convenience, we assume that the investment in adoption pays off after a random time period. Higher investment in adoption results in a shorter expected waiting time for the next variety. Specifically, following Klette and Kortum (2004), we assume that the adoption of a new variety requires both a stock of knowledge (embedded in current technologies, n) and a flow of investment. If firm j spends $I(j)$ units of the final good to adopt a new variety, the adoption will be successful with a Poisson arrival rate $f[I(j)/L, n(j)]$, where $f(.,.)$ is a standard neoclassical production function subject to constant returns to scale and satisfying the Inada conditions; and L is the size of the labor force (assumed to be constant throughout the paper).[26]

To draw an example from agriculture, a firm that seeks to adopt, say, a new variety of fertilizer, will need to engage in costly activity, which might include the effort to find the appropriate type and dose for its crop and soil conditions, the buildup of infrastructure to spread it, and so on. The more the firm invests and the more productive or bigger (and hence more knowledgeable) the firm is, the sooner the new variety will be put into place.

Prospective entrants (whom we model as firms with no varieties) have to spend κL units of the final good per unit of time in order to adopt their first technological variety. The adoption of the first variety will then be successful with a Poisson arrival rate η; that is, the expected waiting time of a new entrant to become a productive firm is $1/\eta$. The entrant may also exit at any point in time.[27]

Risk-neutral firms are indifferent as to which variety to choose, since all varieties enter symmetrically in their profit function and their sole goal is profit maximization. Since the choice is indeterminate, as a tiebreaker, we assume that firms try to adopt technologies with lower indexes first. A firm of size n thus has access to technologies $1, 2, \dots, n$ and would, upon success, adopt technology $n + 1$ next. This

[25] Griliches (1957), Caselli and Coleman (2001), Caselli and Wilson (2004), and Comin and Hobijn (2004), among others, present examples of technologies or inputs that existed but were only slowly adopted in both developed and developing countries. We do not draw a distinction between innovation, adoption, adaptation, or imitation. In practice, all four processes have two features in common: (i) they need an investment for the variety to be operational (e.g., in the case of adoption or adaptation, the know-how or training to operate a technology or input variety; in the case of innovation, the effort to develop the technology and design its implemention; in the case of imitation, the effort to redevelop or reverse-engineer the technology); and (ii) there is some uncertainty with regards to the timing in which these technologies will be fully operational.

[26] The random, "memoryless" adoption process ensures that we do not have to track past adoption investment flows of the firm. This is a standard simplifying assumption in endogenous growth models.

The scaling by labor force L is made to rule out weak scale effects at the country level. Without this rescaling, the model would counterfactually predict that countries with larger population are proportionally richer (see discussion in Jones 1995). Note also that because of the model will allow for a firm entry margin, the model will not feature *strong* scale effects; i.e., long-run growth will be independent of country size. This is true whether or not adoption costs are scaled by L.

[27] In equilibrium, free entry pins down the value of a prospective entrant at zero. Hence the marginal entrant will be indifferent between continuing to spend on adoption costs or exiting.

tiebreaking condition captures the notion that some technologies or inputs are easier to adopt and hence tend to be adopted first by most firms.[28]

Let $\lambda(n) = f[I(n)/L, n]/n$, for $n > 0$, denote the adoption *intensity* of a size-*n* firm. Because *f* is homogeneous of degree one, the flow cost of this adoption intensity is

$$I(n) = g[\lambda(n)]Ln, \quad n > 0, \tag{7}$$

where $g(\cdot)$ is the inverse of $f(.,1)$, an increasing, convex function. For prospective entrants with $n = 0$, the flow cost of adopting the first variety is simply $I(0) = \kappa L$.

State Variables.—Because *n* is the only firm-level state variable, we introduce a change of variables and index firms by *n*. A type-*n* firm has exactly *n* working varieties at its disposal. Because we only need to keep track of the working varieties, whenever a variety is hit by a shock, the index of all varieties with a higher index is readjusted so as to leave no holes in the ordering.[29]

Define as $m_k(t)$ the measure of firms having exactly $k = 0, 1, 2, \ldots$ working varieties at time *t*. Let $\mathcal{M}(t) = \{m_0(t), m_1(t), m_2(t), \ldots\}$ denote the firm-size mass distribution at time *t*. Hence, the total mass of firms at time *t* is given by $M(t) = \sum_{k=0}^{\infty} m_k(t)$.[30] Even though entrants have zero productivity and hence do not contribute to output or employment, they may become successful in adopting their first variety, so it is important to track them.

The mass distribution $\mathcal{M}(t)$ sufficiently characterizes the state of the economy, both in terms of aggregate allocations and prices, and in terms of dynamics. Note that $\mathcal{M}(t)$ is *random*: the firm-size mass distribution will depend on the realization of adverse technology shocks. Let $\mathcal{S}$ denote the set of all possible firm-size mass distributions. We assume that $\mathcal{M}(t)$ follows a Markov process with deterministic trends and jumps (we later verify this to be true in equilibrium):

$$dm_k = F_k(\mathcal{M})\,dt + \sum_{i=1}^{\infty} G_{ki}(\mathcal{M})\,dJ_i(\gamma t), \tag{8}$$

where $F_k : \mathcal{S} \to \mathbb{R}$ is a function capturing the deterministic change in $m_k(t)$ for all $k \geq 1$; $G_{ki} : \mathcal{S} \to \mathbb{R}$ is a function capturing the jump in m_k due to a shock to variety *i*; and the $J_i(\gamma t)$s are independent Poisson processes, each with arrival rate γ. As we shall show, the mass of new entrants at time *t*, $m_0(t)$, will be pinned down by the free-entry condition at time *t*. The process starts from an initial firm-size mass distribution $\mathcal{M}(0) = \mathcal{M}_0$.

[28] This could alternatively be modeled by assuming a functional form for fixed costs of investment, whereby different varieties have different costs of investments, and hence lower-cost varieties are adopted first. The core results will be similar to the ordering assumption in the text.

[29] That is, if an economy has varieties $k = 1, 2, 3, 4$ and variety 3 fails, then, variety 4 is reindexed 3 and the new set of varieties has indexes $k' = 1, 2, 3$.

[30] Note that $\mathcal{M}(t)$ is not a probability distribution as the total mass $M(t)$ is in general different from 1; the probability (share) of firms with *k* varieties, is given by $\frac{m_k(t)}{M(t)}$.

Denoting the vector of elements F_k by $\mathbf{F} = \{F_1, F_2, \ldots\}$, and the vector of elements G_{ki} by $\mathbf{G}_i = \{G_{1i}, G_{2i}, \ldots\}$, equation (8) can be written as

$$d\mathcal{M} = \mathbf{F}(\mathcal{M})\,dt + \sum_{i=1}^{\infty} \mathbf{G}_i(\mathcal{M})\,dJ_i(\gamma t). \tag{9}$$

Firm-Level Stochastic Dynamics.—The stochastic dynamics of n can be summarized as follows. Any one of the varieties fails with arrival rate γ, decreasing n by 1. A firm may become successful in adopting a new input with arrival rate $\mu(n)$ (where $\mu(n) = \lambda n$ for $n > 0$ and $\mu(0) = \eta$), increasing n by 1. Hence,

$$dn = dJ_+[\mu(n)t] - \sum_{i=1}^{n} dJ_i(\gamma t), \tag{10}$$

where $J_+(\mu t)$ is a Poisson process with arrival rate μ, governing the success of adoption. It is independent across firms and from the J_is. Because there is a continuum of firms, a nonstochastic fraction of firms are going to be successful in adoption at any point in time. This means that, in this setup, adoption does not contribute to aggregate uncertainty.[31] At the same time, negative technology shocks affect all firms using the affected varieties and hence the Poisson process J_i in equation (10) is the same for all firms, and the same as in equation (9). This ensures that negative technology shocks have an aggregate impact. The asymmetry in the aggregate impact of positive and negative microeconomic shocks generates negative skewness in the distribution of growth rates, a feature consistent with the data, as discussed earlier.

As mentioned, technological diversification in this model is not driven by risk aversion. To stress this point, we next characterize the optimal rate of technology adoption in the case of risk-neutral agents.[32] Identical risk-neutral households maximize the present value of consumption, discounted at the rate ρ:

$$\mathcal{U} \equiv \int_{t=0}^{\infty} e^{-\rho t} C(t)\,dt.$$

The Euler equation pins down the riskless rate of return in the economy at $r(t) = \rho$. Investors maximize the expected present value of profits, discounted at the rate ρ. To ensure nonnegative growth and a finite value for the firm, we impose the following parameter restrictions on γ, and the cost of adoption:

$$g'(\gamma) \leq \frac{\kappa}{\eta} \quad \text{and} \quad \lim_{x \to \gamma + \rho} g(x) = \infty. \tag{11}$$

[31] This also implies that each new technology is adopted gradually: frontier technologies will be used by a bigger and bigger fraction of firms over time. This is consistent with the evidence discussed earlier.

[32] In the online Appendix we characterize adoption under complete financial autarky and risk-averse investors. We do this to highlight that there is technological diversification in both cases and that the incentive to diversify does not hinge on the financial structure of the economy or the degree of risk aversion (though quantitatively they may affect these incentives).

The first condition ensures that a variety is profitable enough that it is worth investing in adoption costs when a variety suffers a shock. The second condition ensures that adoption is costly enough that the growth rate of the economy will never exceed ρ, the subjective discount rate.

Bellman Equation.—Let $V(n, \mathcal{M})$ denote the value of a size-n firm when the state of the economy is $\mathcal{M}$. It is the expected present value of the stream of future profits, coming from net operating revenues minus the costs of adoption,

$$V(n, \mathcal{M}) = \max_{\{p,I\}} E \int_{t=0}^{\infty} e^{-\rho t} \{\pi [n(t), \mathcal{M}(t)] - I[n(t)]\} dt,$$

where $\mathcal{M}(t)$ and $n(t)$ evolve subject to the laws of motion described in equations (9) and (10), respectively.

From the perspective of a firm, there is a firm-level state variable, n, and an aggregate state variable, $\mathcal{M}$, the two of which contain all the information relevant in its decision. The firm chooses the price of its product (taking aggregate demand, the production function and wages as given), and the intensity with which it invests in adopting new varieties. The policy variables are thus p and λ.

Given the flow profit function in equation (6), the cost function for adoption in equation (7), and the law of motion for $\mathcal{M}$ in equation (9), the Bellman equation for the firm's profit maximization problem can be written as

$$\text{(12)} \quad \rho V(n, \mathcal{M}) = \max_{p,I} \Bigg\{ \pi(p, n, \mathcal{M}) - I + \mu(n)[V(n+1, \mathcal{M}) - V(n, \mathcal{M})] + \gamma \sum_{i=1}^{n} [V(n-1, \mathcal{M} + \mathbf{G}_i(\mathcal{M})) - V(n, \mathcal{M})] + \gamma \sum_{i=n+1}^{\infty} [V(n, \mathcal{M} + \mathbf{G}_i(\mathcal{M})) - V(n, \mathcal{M})] + V_{\mathcal{M}} \mathbf{F}(\mathcal{M}) \Bigg\}.$$

The opportunity cost of time is compensated by flow profits, $\pi - I$, where π is given by equation (6) for $n > 0$ and is zero for $n = 0$, and by expected capital gains. With arrival rate $\mu(n)$, a new variety is developed, and firm value changes by $V(n+1, \mathcal{M}) - V(n, \mathcal{M})$. Because adoption success is idiosyncratic, the aggregate state of the economy does not change when a new variety is adopted. With arrival rate γ, a variety i is lost. If this is among the varieties used by the firm ($i = 1, 2, \ldots, n$), firm value changes by $V(n-1, \mathcal{M} + \mathbf{G}_i(\mathcal{M})) - V(n, \mathcal{M})$. The firm will have one variety less, but also the aggregate state of the economy will jump. If variety i is not used by the firm ($i = n+1, n+2, \ldots$), then only the aggregate state is affected. The last term, $V_{\mathcal{M}}\mathbf{F}(\mathcal{M})$, captures the expected changes in value due to smooth changes in $\mathcal{M}$ alone, holding n fixed.

The first-order conditions for optimal pricing and optimal adoption for $n = 1, 2, \ldots$ are

$$\text{(13)} \qquad p = \frac{\varepsilon}{\varepsilon - 1} w(\mathcal{M}) n^{1/(1-\varepsilon)} / A(\mathcal{M}),$$

$$g'(\lambda)L = V(n+1,\mathcal{M}) - V(n,\mathcal{M}). \tag{14}$$

The firm's optimal price is a constant markup over unit cost. The unit cost decreases in the number of varieties, and increases with the prevailing wage rate. The marginal cost of adoption of new varieties has to equal the marginal benefit: the potential jump in value when adoption is successful. Firms with $n = 0$ do not produce, so they do not have a pricing decision to make, and their adoption intensity is given by η. Their key decision is on the entry-exit margin, which is explained next.

Free Entry.—There is an unbounded mass of potential entrants who can start a new firm at no cost. This pins down the value of new entrants at zero for all possible states of the economy,

$$V(0,\mathcal{M}) = 0. \tag{15}$$

We next describe the aggregate variables in the economy.

External Effects.—Building on the insights of Arrow (1962) and Romer (1986), we allow for external effects stemming from the stock of knowledge embedded in the aggregate number of varieties, conditional on the mass of firms in the economy. External effects, in combination with the exit-entry margin will allow the economy to achieve a balanced (expected) growth path (BEGP), to be defined below.[33] We stress that while a BEGP is convenient for the analysis, the technological diversification mechanism also operates outside the BEGP and hence it is not needed for the mechanism emphasized in this paper.

Denote by $N(\mathcal{M})$ the aggregate number of varieties defined as

$$N(\mathcal{M}) \equiv \int_{\mathcal{M}} n d\mathcal{M} = \sum_{i=1}^{\infty} i m_i, \tag{16}$$

where $\int_{\mathcal{M}} d\mathcal{M}$ stands for the Lebesgue integral over firms with different sizes, with respect to the firm-size measure $\mathcal{M}$.[34]

Aggregate productivity $A(\mathcal{M})$ is given by a positive function $A(N, m_0) > 0$ that depends on the total number of varieties N used by productive firms, and on the mass

[33] Dixit-Stiglitz formulations with external economies go back to Ethier (1982), and have been used to match other features of the data in more recent contributions; see, for example, Grossman and Rossi-Hansberg (2010), and the references therein. As shall become clear, an entry-exit adjustment alone as in Rossi-Hansberg and Wright (2007) is not sufficient to ensure a BEGP in our model and so we allow the entry margin to have an external effect on the profitability of other firms.

[34] The sum $\sum_{i=1}^{\infty} i m_i$ will be finite with probability one at any point in time, as long as we start from an initial firm-size mass distribution $\mathcal{M}_0$ with finite N. This is because at any point in time t, $N(t)$ has a finite upper bound. From condition (11), adoption intensity by incumbents cannot be greater than $\gamma + \rho$ for any firm. Hence varieties used by incumbent firms can at most grow at the rate $\gamma + \rho$. As will become clear, the growth stemming from the creation of new firms is $\eta \frac{m_0}{N(\mathcal{M})}$, which is bounded from above by $\eta \frac{\varepsilon \overline{\pi}}{\kappa}$, where $\overline{\pi}$ is finite. Hence, for any t, $N(t) \leq N(0) e^{(\gamma + \rho + \frac{\varepsilon \overline{\pi}}{\kappa})t}$. Note that after a positive amount of time has passed, n will have full support with probability one. This is because successful adoption follows a Poisson process, which makes the number of new varieties a Poisson-distributed random variable.

of zero-size firms m_0, which do not contribute to production. We assume that $A(N, m_0)$ satisfies the following properties: $\theta_N(\mathcal{M}) \geq 0$, $\theta_{m_0}(\mathcal{M}) < 0$, and

$$\frac{1}{\varepsilon - 1} + \theta_N(\mathcal{M}) < 1 - \theta_{m_0}(\mathcal{M}), \tag{17}$$

where $\theta_N(\mathcal{M}) \equiv \frac{\partial \ln A(N, m_0)}{\partial \ln N}$ is the elasticity of $A(N, m_0)$ with respect to N (holding m_0 fixed), and $\theta_{m_0}(\mathcal{M})$ is the corresponding elasticity with respect to m_0 (holding N fixed). These conditions are jointly *sufficient* for the existence and uniqueness of a BEGP. The assumption that A is nondecreasing in N ($\theta_N(\mathcal{M}) \geq 0$) embeds the idea that there can be knowledge spillovers across productive firms. Note that the inequality is weak, so $\theta_N(\mathcal{M}) = 0$ is a possibility. The assumption that A is decreasing in m_0 ($\theta_{m_0}(\mathcal{M}) < 0$) implies that for a given number of varieties N, whenever there are too many new entrants relative to equilibrium, profits per firm fall, reducing the incentives to enter. Intuitively, unproductive firms with no varieties contribute negatively to the average stock of knowledge of the economy. The final inequality condition ensures that the contribution of new firms to GDP growth vanishes as the economy grows and guarantees a positive measure of new firms in equilibrium.

Note that these conditions are sufficient, but not necessary for a BEGP. In particular, if $\theta_N(\mathcal{M}) = \theta_{m_0}(\mathcal{M}) = 0$, the economy features a BEGP when $\varepsilon = 2$.[35] Similarly, if $\theta_{m_0}(\mathcal{M}) = 0$, the economy features a BEGP when $\theta_N(\mathcal{M}) = 1 - \frac{1}{\varepsilon - 1}$. In the baseline quantitative exercise, we allow for very small external effects, consistent with the empirical literature. The entry margin m_0 adjusts so as to prevent explosive growth in the case of low substitutability ($\varepsilon < 2$), or to prevent stagnation, in the case of high substitutability ($\varepsilon > 2$). Finally, we reiterate that a BEGP is not needed for the technological diversification channel to operate and therefore neither external effects nor parametric restrictions on ε are necessary to yield a decline in volatility with development over a finite time period.

D. *Equilibrium*

In what follows, we first define the equilibrium in the economy and then establish the conditions for existence.

DEFINITION 1: *A recursive equilibrium in this economy is* (*i*) *a price policy function* $p(n, \mathcal{M})$; (*ii*) *an innovation policy function* $\lambda(n, \mathcal{M})$; (*iii*) *a value function* $V(n, \mathcal{M})$; (*iv*) *a wage function* $w(\mathcal{M})$; (*v*) *a final output function* $Y(\mathcal{M})$; (*vi*) *a mass of entrants* $m_0(\mathcal{M})$; *and* (*vii*) *a law of motion* $\mathbf{F}(\mathcal{M})$ *and* $\mathbf{G}(\mathcal{M})$ *for the variety distribution such that* (*i*) *given the law of motion*, $V(n, \mathcal{M})$ *satisfies the firm's Bellman equation*, (12); (*ii*) *the policy functions* $p(n, \mathcal{M})$ *and* $\lambda(n, \mathcal{M})$ *maximize firm value*, (13) *and* (14); (*iii*) *entrants make zero value* (15); (*iv*) *labor and final good markets clear*; (*v*) *the law of motion coincides with the Markov process characterized by the adoption function* $\lambda(n, \mathcal{M})$ *and the technology shock* γ.

[35] If $\theta_N = \theta_{m_0} = 0$ and $\varepsilon = 2$, the aggregate demand externality and the competition effect cancel out and profits per variety are constant, which is sufficient for a BEGP.

This equilibrium definition already makes use of the production function, and the demand curve for individual products, that are both embodied in the profit formula in the Bellman equation.

Income Accounting.—In the model's economy, GDP is equal to consumption plus investment (in adopting new varieties), which equals the output of the final good,

$$Y(\mathcal{M}) = C(\mathcal{M}) + I(\mathcal{M}), \tag{18}$$

where

$$I(\mathcal{M}) = \int_{\mathcal{M}} I(n) d\mathcal{M} = \kappa L m_0 + \int_{\mathcal{M}} g[\lambda(n,\mathcal{M})] L n d\mathcal{M},$$

and $\int_{\mathcal{M}} d\mathcal{M}$, as before, stands for a Lebesgue integral defined over $\mathcal{M}$. By Walras's law, this equation will hold whenever labor markets clear.

The income side of GDP is made of wage income and profits, which accrue to households owning the monopolistically competitive firms,

$$Y(\mathcal{M}) = w(\mathcal{M}) L + \int \pi(n,\mathcal{M}) d\mathcal{M}. \tag{19}$$

As the next proposition makes clear, all static allocations and prices are a function of N only.

PROPOSITION 1: *A recursive equilibrium exists. In equilibrium, firm value is*

$$V(n,\mathcal{M}) = vn, \tag{20}$$

where $v \equiv \frac{\kappa}{\eta} L$ *is the firm value per variety, and innovation policy is a constant* λ *implicitly defined by*

$$g'(\lambda) = \frac{\kappa}{\eta}. \tag{21}$$

Wages and final output are linear in N. Wages are

$$w(\mathcal{M}) = (\varepsilon - 1)\overline{\pi} N(\mathcal{M}); \tag{22}$$

final output is

$$Y(\mathcal{M}) = \varepsilon \overline{\pi} N(\mathcal{M}) L, \tag{23}$$

where $\overline{\pi} = (\rho + \gamma - \lambda)\frac{\kappa}{\eta} + g(\lambda)$ *is per capita profits per variety, which is constant. Firm prices are*

$$p(n,\mathcal{M}) = \left[\frac{N(\mathcal{M})}{n}\right]^{1/(\varepsilon-1)}. \tag{24}$$

The mass of new entrants m_0 satisfies

$$\overline{\pi} = \frac{1}{\varepsilon} N(\mathcal{M})^{\frac{\varepsilon-2}{\varepsilon-1}} A(N, m_0). \tag{25}$$

The law of motion for $\mathcal{M}$ is Markov with F_i and G_{ik} defined as

$$F_i(\mathcal{M}) = \begin{cases} \lambda(i-1)m_{i-1} - \lambda i m_i & \text{if } i > 1, \\ \eta m_0 - \lambda m_1 & \text{if } i = 1. \end{cases} \tag{26}$$

$$G_{ik}(\mathcal{M}) = \begin{cases} m_{i+1} - m_i & \text{if } k \leq i, \\ m_{i+1} & \text{if } k = i+1, \\ 0 & \text{if } k > i+1. \end{cases} \tag{27}$$

The proof of this and all other propositions are in the online Appendix.

Equation(20)shows the firm value as a function of the adoption cost of new entrants. Each new entrant spends κL for an expected $1/\eta$ units of time before becoming a productive firm and achieving a value of $V(1, \mathcal{M}) = v$. The rest of the firm value function is linear in n. Equation (21) is the first-order condition for optimal adoption. This condition pins down a unique, constant λ that is independent of n. Equation (22) shows how wages depend on the aggregate number of varieties. When the economy uses more input varieties, aggregate labor productivity is higher, and wages are higher in terms of the final good (the numeraire). We have already substituted out the equilibrium value of the external productivity A (equation (25)). Equation (23) expresses final output as a function of N. An increase in the number of varieties leads to higher GDP. Both wages and GDP are linear in N, because the per variety profit per capita $\overline{\pi} = \frac{\pi(n,\mathcal{M})}{nL}$ is constant. Equation (24) describes optimal pricing of a size-n firm: because firm-level productivity increases in n and firms charge a constant markup, firm prices will decrease in n. On the other hand, because wages increase in the aggregate number of varieties, N, prices increase in N. Equation (25) is the zero-value condition that pins down the equilibrium mass of entrants, so that prospective entrants are indifferent between entering and not entering the market. Equation (26) captures the deterministic component of the law of motion for $\mathcal{M}$. A measure $\lambda(i-1)m_{i-1}$ of firms will be successful in adopting variety i, and become size i. A measure $\lambda i m_i$ of size-i firms will be successful in adopting variety $i+1$, and will no longer be size i. A measure ηm_0 of new entrants will be successful in adopting their first variety and will become size one. Equation (27) captures the jump component of the law of motion for $\mathcal{M}$. If any of the first i varieties fail ($dJ_k > 0$), then size-i firms become size $i-1$, and, at the same time, firms of size $i+1$ see their size reduced to i.[36] Hence the change in the mass of firms. If variety $i+1$ fails, all those firms become size i, adding to m_i.

[36]Note that with the relabelling rule, i indexes firm sizes. (Recall that, if variety i fails, we relabel varieties with higher indexes so that $i+1$ becomes i, etc.)

E. *Firm-Level and Aggregate Dynamics*

PROPOSITION 2: *In a recursive equilibrium, the expected growth of sales for a firm of size $n > 0$ is constant,*[37]

$$\frac{E(dn/n)}{dt} = \lambda - \gamma, \tag{28}$$

and the variance of sales growth is decreasing in n,

$$\frac{\mathrm{Var}(dn/n)}{dt} = \frac{\lambda + \gamma}{n}. \tag{29}$$

It follows from equation (24) that sales are a linear function of n, hence their growth rate equals the growth rate of n. The expected growth in the number of varieties equals the rate of technology adoption minus the rate of technology failure, $\lambda - \gamma$. The variance of sales growth is driven by the two shocks the firm faces: the randomness of the adoption process and variety failures. Hence the variance of an individual variety is $\lambda + \gamma$. Total sales volatility then declines with n by the law of large numbers. The formal proof follows directly from equation (10) by Lemma 1 (in the online Appendix).

Aggregate Dynamics.—To understand the dynamics of aggregate GDP, we need to characterize the dynamics of N (see equation (23)). There are two types of shocks affecting N. First, successful adoption by some firms will move them from n varieties to $n + 1$ varieties. Recall that a size-n firm adopts new varieties with arrival rate $\mu(n) = \lambda n$ for $n \geq 1$ and $\mu(0) = \eta$. At every point in time, a measure

$$\int_{\mathcal{M}} \mu(n)\, d\mathcal{M}\, dt = \lambda N(\mathcal{M})\, dt + \eta m_0(\mathcal{M})\, dt$$

of firms becomes successful in adopting the next variety.

The second type of shock is the failure of a particular technology k. This shock decreases the number of varieties by 1 for all firms that use variety k. Because there is a positive mass of these firms, this shock induces an instantaneous *jump* in N. The aggregate impact of the shock (and, ultimately, aggregate volatility) will depend on the measure of firms using technology k. Note that because technology shocks are common across firms, they will also induce correlations across firms. This is why there is aggregate uncertainty even with a continuum of firms.[38]

Let M_i denote the mass of firms using variety i. Because firms adopt lower-indexed varieties first, this is the same as the mass of firms with i or more varieties,

[37] We focus on the behavior of sales growth, for which data are available at the firm level.

[38] Because positive shocks (technology adoptions) are independent across firms, while negative shocks (technology failures) are common, aggregate shocks will have a negatively skewed distribution. This is consistent with evidence presented by for the US economy. See the online Appendix for futher references as well as new evidence on skewness in other countries.

$M_i = \sum_{k=i}^{\infty} m_k$. Then, using the aggregation above and the Markov dynamics of $\mathcal{M}$ as given by equations (26)–(27), we can write the dynamics of N as follows:

$$dN = [\lambda N + \eta m_0]\, dt - \sum_{i=1}^{\infty} M_i\, dJ_i(\gamma t). \tag{30}$$

The first two terms are the effects of innovation. Each firm's adoption is subject to an independent Poisson process, the sum of which is a deterministic process by the law of large numbers. The second term captures adverse productivity shocks, which are common across firms and are not washed out by aggregation. Because variety i is used by a measure M_i of firms, a shock dJ_i reduces the total number of varieties by M_i.

PROPOSITION 3: *In a recursive equilibrium, the expected growth rate of the number of varieties N (and hence of output Y) is*

$$\frac{E(dN/N)}{dt} = \lambda + \eta \frac{m_0}{N} - \gamma, \tag{31}$$

and its instantaneous variance is

$$\frac{\mathrm{Var}(dN/N)}{dt} = \gamma \sum_{k=1}^{\infty} s_k^2, \tag{32}$$

where $s_k = M_k/N$ *measures the contribution of variety k to GDP.*

Intuitively, the average firm innovates with intensity $\lambda + \eta \frac{m_0}{N}$, which then gives the growth rate of N until a shock occurs. Shocks occur with arrival rate γ, which brings about an expected decline in the total number of varieties at the same rate. In this sense, γ is akin to a (stochastic) depreciation rate.

To understand the intuition for the variance, consider a shock hitting variety k. This reduces N by a fraction s_k. Given that this has probability γdt, the aggregate variance contributed by this shock is $\gamma s_k^2 dt$. Because variety-specific shocks are independent, we can simply add up the individual variances.[39] To gain more intuition for formula (32), consider some simple examples. If all firms use just one variety, the sum on the right-hand side is one. This leads to the highest possible level of aggregate volatility, γdt. If all firms use N different varieties, the contribution of each variety to GDP is $s_k = 1/N$ and the sum equals $1/N$. In this case, the sum decreases inversely with the number of varieties and volatility (the standard deviation of growth rates) hence declines at the rate $1/\sqrt{N}$.[40] In general, not all firms will use all varieties and the distribution of varieties will be uneven. This, combined with the smaller number

[39] Note that m_0 does not contribute to instantaneous volatility (i.e., over an infinitesimal period of time); however, by altering the (expected) time derivative of N, it affects volatility over a discrete period. This will be reflected in the discrete-time simulations performed in the next section. In our numerical exercises, the quantitative contribution of new entrants to aggregate volatility turns out to be negligible even at annual frequency.

[40] Note that since firms are not symmetric ex post (only a fraction of firms is successful in adoption), this result cannot hold at every point in time.

of varieties in less developed economies, slows down the effect of the law of large numbers (LLN) and leads to higher volatility in poor countries.[41]

The term $\sum_{k=1}^{\infty} s_k^2$ can be construed as an index of the economy's technological concentration or the inverse of an index of technological diversification; this is the key determinant of volatility. In a multiple-firm economy, volatility depends not only on the overall number of varieties $N(t)$, but also on the degree of diversification in the usage of different varieties. As mentioned, both $N(t)$ and the shares s_k are history-dependent.

F. *Balanced-Expected-Growth Path*

The BEGP of the economy is defined as follows.

DEFINITION 2: *A BEGP is a recursive equilibrium in which the expected growth rates of output, consumption, investment, real wages, and the number of varieties converge to a positive constant.*

As said, a BEGP is not needed for the technological diversification channel to operate. It is, however, standard practice to focus the analysis of growth models on the BEGP, and in the rest of this paper, we adhere to this practice.[42]

PROPOSITION 4: *For all initial variety distributions, $\mathcal{M}(0) = \mathcal{M}_0$, the recursive equilibrium converges to a balanced-expected-growth path. The expected long-run growth rate x is implicitly defined by*

$$g'(\gamma + x) = \frac{\kappa}{\eta}, \tag{33}$$

with $x \in [0, \rho)$ and $\frac{\kappa}{\eta}$ gives the per capita firm value per variety. In the long run, as $N \to \infty$, the contribution of entrants m_0 to growth vanishes as $\frac{m_0}{N} \to 0$.

The proof makes use of the sufficient conditions $\theta_N(\mathcal{M}) \geq 0$, $\theta_{m_0}(\mathcal{M}) < 0$ and equation (17). As stressed earlier, these conditions are jointly sufficient but not necessary for a BEGP. Note, in particular, that there are two alternative ways to achieve a BEGP in our setup. The first, which does not rely on external effects, is to impose a parametric restriction on the elasticity of substitution. Specifically, when $\theta_N(\mathcal{M}) = \theta_{m_0}(\mathcal{M}) = 0$, a BEGP exists if $\varepsilon = 2$.[43] The second alternative that does not rely on external effects from entry (i.e., $\theta_{m_0}(\mathcal{M}) = 0$) is to impose the restriction that $\theta_N(\mathcal{M}) = 1 - \frac{1}{\varepsilon - 1}$.[44] These two options cause output to be linear in N, the key condition for a BEGP. In the baseline quantitative exercise we allow for small external

[41] In related work, Gabaix (2011), Carvalho (2010), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2010) offer examples in which the LLN is attenuated; however, none of these studies is concerned with the relation between economic development and volatility.

[42] Growiec (2010) has argued against the BGP focus in the literature; he points out that the long run with $t \to \infty$ is irrelevant, and proposes that only finite time spans be analyzed instead.

[43] In this case, profits per variety are constant and there is no entry of firms: $m_0 = 0$.

[44] As before, in this case, there is no entry and $m_0 = 0$.

effects, consistent with the literature, and in the online Appendix we present the results without external effects.

G. *GDP Dynamics along the Balanced-Expected-Growth Path*

Since at any time t (instantaneous) GDP growth $dY(t)/Y(t)$ is a random variable, it not only has an expected value but also a variance. This variance is not constant, even on the BEGP. (Notice that if it were, the model would have no hope of explaining the cross-sectional patterns of volatility and development that motivate the paper.) Instead, it depends on the set of technologies in use, as well as their distribution among firms. In general, these depend on the particular history of shocks that have hit the economy, so the variance must be computed by numerical simulation. Before we turn to this task, we offer some theoretical results that both help to understand the simulations and provide some intuition on the main mechanism at play.

The volatility of N (and hence of Y) depends on the whole distribution of varieties used by firms. If some varieties are used by more firms than others, then shocks affecting these varieties are going to have a larger impact on GDP. Through the introduction of *new* varieties, technological progress increases the degree of technological diversification (and hence lowers volatility) while increasing the level of development. This imparts a natural tendency for a negative correlation between volatility and development that will be prevalent in our numerical analysis. Note, however, that in principle the relationship between volatility and development does not always need to be strictly negative. To understand this point, it is convenient to distinguish between the two forces that shape the behavior of aggregate volatility and development in the model. The first is the increase in usage of a variety in the economy, which results from firms' adoption processes; the second is a shock that destroys a variety. We discuss these forces in Propositions 5 and 6, respectively.

PROPOSITION 5: *The increase in use of variety k in the economy increases output unambiguously and decreases volatility if and only if* $\sum_{i=1}^{\infty} s_i^2 > s_k$, *where* s_i *is the contribution to output of variety i.*

By an increase in the use of variety k in the economy we refer to a marginal increase in the mass of firms using that variety (all else equal).[45] Intuitively, as long as a variety is not widely used in the economy, increasing its usage provides diversification benefits against other variety-specific shocks and hence lowers aggregate volatility. In contrast, when a variety is already used intensely, increasing its usage makes the economy more vulnerable to shocks affecting that variety. Note that because $\lim_{i\to\infty} s_i = 0$, there is always an index K (a *frontier variety*) above which all varieties are rare enough to satisfy this condition. Adopting frontier varieties therefore always leads to lower volatility.

PROPOSITION 6: *A shock that destroys variety k decreases output unambiguously and increases volatility if and only if* $\sum_{i=1}^{\infty} s_i^2 > \frac{s_k}{2 - s_k}$.

[45] This leads to an increase in s_k and the consequent relative decrease in the contribution of other varieties: recall the shares s_k add to 1 by construction.

In words, as long as s_k is not too big, expected volatility increases with the destruction of variety k. This happens together with the unambiguous decline in output caused by the destruction of that variety. Volatility might decrease only if the production process relies strongly on variety k. In this case, the disappearance of that variety leads to higher diversification for the economy. As before, there exists a frontier variety K such that the destruction of all varieties $k > K$ leads to an increase in volatility and a decline in income.

While one can construct examples where the negative relationship between volatility and development breaks, the model dynamics tend to generate a negative correlation. This is because the growth process, through the steady introduction of *frontier* varieties, leads, on average, to both higher levels of development and higher degrees of technological diversification. In the long run, as per capita GDP grows without bound, volatility approaches zero.

PROPOSITION 7: *As per capita GDP increases without bound*, *volatility tends to zero.*

The intuition is straightforward: long-run growth of per capita GDP is achieved by the addition of frontier varieties, which reduces volatility. As time progresses, volatility vanishes and the economy converges to a stable deterministic growth path with rate $\lambda - \gamma$. The decline in volatility thus results as a by-product of the development process.

Before moving to the quantitative results, a comment on the asymmetry in the sources of aggregate volatility is in order. In the model, positive shocks at the micro level average up to a smooth aggregate process, whereas negative shocks at the micro level generate aggregate volatility. While of course this is a modelling simplification, the asymmetry leads to negative skewness in the distribution of growth rates, a prediction that is consistent with the data (see online Appendix).

III. Volatility and Development: A Quantitative Assessment

Our analysis so far has shown that volatility declines monotonically with the degree of technological diversification and that, ceteris paribus, the introduction of a new variety in the economy increases the level of development and the degree of technological diversification, thus lowering volatility. We have also argued that the growth process, through the expansion in the number of varieties, tends to impart a negative correlation between volatility and development, though this tendency may be overturned under certain histories of shocks; specifically, it is conceivable that countries that use a few varieties very intensely display both a relatively high level of development and high volatility due to their lack of diversification. To establish whether these occurrences are frequent or rare, one has to simulate the model.

Our strategy is to generate artificial data by simulating the model 1,000 times for 64 different economies (countries) from 1870 through 2007. All economies start at the stage of development they were at in 1870, according to Maddison (2010). (There are 64 countries at different stages of development with data on GDP per capita in 1870; see the online Appendix for the list of countries.) An initial (single-parameter) logarithmic firm-size distribution for each economy is calibrated so as to match the level of development of the country in 1870 (we shall elaborate on this later). All parameters characterizing the evolution of the economies are identical. Shocks are country-specific, however, and different realizations of shocks lead to potentially different growth paths. We analyze the relation between volatility and the level of development for the simulated economies, and compare patterns of volatility and development in the last 48 years of our simulations to the corresponding patterns in the cross-sectional data that we already examined in Section I, covering the period 1960–2007. Note that because the volatility of aggregate GDP depends on the distribution of technologies across firms, our simulations need to keep track of the entire distribution of technology usage across firms at all points in time.

We emphasize that in reality there are several additional mechanisms driving a country's economic development and its patterns of volatility. The goal of this numerical exercise is to study how the model behaves with reasonable parameter values, not to run a horse race among potential explanations.

A. *Parametrization and Computation*

Technology and Growth.—We compute a discrete-time approximation of the continuous-time model. In calibrating the discrete-time approximation, a period is interpreted as a year. We need to set values for γ, the arrival rate of negative shocks, and ρ, the rate of time preference. In principle, we also need to specify and parametrize the cost of adoption function $g(\cdot)$, the entry cost κ, and the size of the labor force L. Note, however, that g, κ, and L only serve to pin down the search intensity λ, which is constant in the BEGP. Hence we let g, κ, and L unspecified and calibrate λ directly.

The model's key parameter, γ, is in principle difficult to calibrate without observing technology shocks directly. Our strategy is thus to simulate the model for a reasonably wide grid of values for γ, ranging from 0.05 to 0.20. Values below 0.05 imply very low aggregate volatility; values above 0.20 are unlikely, as they would imply that technologies on average last less than five years.[46] Because γ is also the (stochastic) depreciation rate of the economy, $\gamma = 0.10$ would be a natural choice, but we find it useful to investigate the model's outcomes for a wider parameter range. Recalling that in the long run the expected growth rate converges to $\lambda - \gamma$, we can then set λ so that the long-run annual growth rate is 0.02 (that is, $\lambda = \gamma + 0.02$).

In the baseline model we set the elasticity of substitution ε equal to 3, and in the online Appendix we report results for ε between 1.6 and 5 (Table A11).

Entrants and External Effects.—We also need to calibrate the success rate of entrants η and the external effects they generate, captured by $A(\cdot)$. In the baseline calibration, we set the success rate of new firms η so as to match the median age of nonemployer firms at seven years (Davis et al. 2009).[47] In our model, the median

[46] Technologies with such short duration exist in practice, but are unlikely to be the norm. One way to extend the model is to allow for heterogeneity in the probability of failure across technologies. We concentrate on what we think are the first-order insights of the model by assuming a constant γ.

[47] In our model, nonemployers do not generate revenues. In practice they do, but as Davis et al. (2009) document, nonemployer firms account for a modest four percent of aggregate US business revenue. As in our model, the

age of a nonemployer firm is $\ln 2/\eta$, which implies $\eta = 0.1$. For robustness, we also experimented with a wider grid of values for η, ranging from 0.05 to 0.20. Perhaps not surprisingly, the results were not sensitive to the parametric choice.[48] Indeed, as Proposition 4 indicates, the contribution of new entrants to growth vanishes in the long run. This is because $\frac{m_0}{N}$, which is endogenously determined in the model, declines as the economy grows, and the success rate of entrants matters less and less for growth dynamics.

We assume a simple power function for the external effect $A = N^{\theta_N} m_0^{\theta_{m_0}}$, with $\theta_N \geq 0$ and $\theta_{m_0} < 0$ satisfying the sufficient conditions for long-term growth. In words, firms' productivity increases (or remains invariant) with the aggregate number of varieties in the economy and decreases with the number of entrants. We do not have direct estimates of θ_N and θ_{m_0}; the literature has pointed to small but nonzero external effects. For example, Combes, Duranton, and Gobillon (2011a, 2011b) estimate both positive urban externalities and congestions costs to be of the order of 0.03. Hence we have selected $\theta_N = -\theta_{m_0} = 0.03$ as the baseline. (Note that the magnitudes do not need to coincide to satisfy the sufficient conditions for BEGP.) For robustness, we experiment with different values for θ_N and $(-\theta_{m_0})$ between 0 and 1 satisfying equation (17). We found that the choice of these parameters was not important for the relation between volatility and development; the robustness results are available in the online Appendix. (Note that in the limit case in which $\theta_N = \theta_{m_0} = 0$, a necessary and sufficient condition for balanced growth is that $\varepsilon = 2$. The results for this case are in Table A12 of the online Appendix.)

Initial Conditions.—We initialize the model in 1870 and assume that in each country, the initial firm-size distribution is logarithmic. (This is the distribution of firms' size in Klette and Kortum 2004, discussed thoroughly in the context of firms' sizes by Ijiri and Simon 1977.)[49] We calibrate the parameter of the country-specific distribution so as to match the country's level of development in 1870, according to Maddison (2010). Hence, all countries start at the level of development they had in 1870. More specifically, the logarithmic distribution is given by

$$p_k = \frac{-1}{\ln(1-\nu)} \frac{\nu^k}{k}, \quad k \geq 1, \tag{34}$$

where p_k is the fraction of firms using k varieties. We assume all countries start with a unit mass of productive firms, M_1, in 1870, and let the distribution of firms vary

authors find that a substantial share of employers originate as nonemployer firms.

[48] Given that the parameter choices for entrants do not affect our results in any significant way, we omit the results for alternative parameter choices, but these are available in the online Appendix.

[49] The logarithmic distribution is appealing in this context, both because it can match important features of the firm-size distribution (Klette and Kortum 2004), and because it relies on a single parameter, which we can calibrate using aggregate data.

across countries with a country-specific parameter ν_c.[50] Thus, p_k maps into m_k, the mass of firms with $k \geq 1$ varieties. The mean of the size distribution is

$$\sum_{k=1}^{\infty} k \cdot p_k = \frac{-1}{\ln(1-\nu)} \frac{\nu}{1-\nu}, \tag{35}$$

which is increasing in ν. This mean maps into N, which is linearly related to GDP per capita: $Y(\mathcal{M})/L = \varepsilon \overline{\pi} N(\mathcal{M})$. Hence, from data on real GDP per capita in 1870 ($GDPPC_{c,1870}$), for a given ε, we can obtain $\overline{\pi} N_{c,1870}$, where $N_{c,1870}$ is the average number of varieties in country c in 1870:[51]

$$GDPPC_{c,1870} = \varepsilon \overline{\pi} N_{c,1870}. \tag{36}$$

To pin down $\overline{\pi}$, we match an additional moment in the data as follows. Note first that, given the initial distribution of varieties, the (initial) instantaneous variance of real GDP growth is given by

$$\gamma \sum_{k=1}^{\infty} M_k^2/N^2 = \gamma \frac{(1-\nu)(1+\nu)\ln(1-\nu^2) - 2\nu(1-\nu)\ln(1-\nu)}{\nu^2}, \tag{37}$$

where $M_k = \sum_{i=k}^{\infty} m_k$ is as before, the mass of firms using variety k (those that are at least k large). Hence, for a given γ one could exactly pin down ν using data on the variance of per capita GDP growth. The limitation is that only a small set of countries in have uninterrupted series of real GDP per capita in the early period, necessary to compute variances.[52] We thus use data on the United States, for which the series of Maddison (2010) is uninterrupted, and calculate the variance of per capita GDP growth during 1870–80.

Thus, for a given γ, we can pin down ν_{US} from equation (37) and hence the average number of varieties in the United States $N_{US,1870}$ from equation (35). Using the latter, together with data on real GDP per capita for the United States, we can obtain $\overline{\pi}$ from equation (36). Having $\overline{\pi}$, we can calculate $N_{c,1870}$ for all the remaining countries using equation (36). For each $N_{c,1870}$, there is a parameter ν_c for the logarithmic distribution that fits equation (35). This gives the initial firm-size distribution in equation (34) for each country.

Simulation.—We simulate the model in each economy 1,000 times from 1870 through to 2007. To do so, we resort to discrete-time methods. (Note that the state space is already discrete.) We approximate the continuous-time adoption and failure processes as follows. Over a period Δt, a firm of size n_t adopts $q_{t,\Delta t}$ new varieties, where $q_{t,\Delta t}$ has a Poisson distribution with expected value $\lambda n_t \Delta t$. As to new firms, they become successful with probability $1 - \exp(-\eta \Delta t)$, so this fraction of size-zero firms will become size one. Similarly, we discretize the failure process by

[50] Over time, the size of $M_1(t)$ will adjust through the entry margin.

[51] We start in 1870 because the data coverage in that year is particularly good: there are many missing observations in the early years of Maddison's data.

[52] 1870 was a benchmark year with data for 64 countries, but this is not the case for subsequent years.

assuming that each variety has the $\gamma \Delta t$ probability of failing during a period of time. As Δt tends to zero, these processes converge to the continuous-time processes described in Section II. We take Δt to be a year. (Since both λ and γ are fairly small, Δt does not need to be too small for the above approximation to be accurate.) In addition to the discretization of time, we also put an upper bound on the support of n_{it}, as the computer program cannot handle unbounded support. We set the upper bound at $n_{\max} = 900$, which implies that even in the richest country, 99.999 percent of firms remain within this bounded support during the 138 years of simulation.

At any point in time, we can take a snapshot of the economy by counting the number of firms in each size bin, $m_{1t}, m_{2t}, \ldots$ Real GDP per capita at time t can then be calculated as $\varepsilon \overline{\pi} \sum_{i=1}^{n_{\max}} i m_{it}$. To construct statistics that have the same interpretation as those in the empirical analysis of Section I, we compute decade averages of (the log of) GDP per capita and (logs of) standard deviations of per capita GDP growth (our measure of volatility). Recall that in Section I we run a regression of country-level volatility on income for the nearly five decades between 1960 and 2007. To run a similar regression on our simulated panel of countries, we used the last 48 years of data generated in our simulations. To reduce simulation error, we report the means from 1,000 simulations.

B. *Results*

This section presents and discusses the results from the baseline calibration, along with the three main regularities motivating the model.

1. *GDP Volatility Declines with Development, Both in the Cross Section and for a Given Country over Time.*—The first set of rows in Table 3 shows for each value of γ, the model-generated slope coefficients and the corresponding standard errors from ordinary least squares regressions of decade (log) volatility on the average (log) GDP per capita of the decade, pooling data from all simulated countries in the last five decades. The last two columns in the row show the corresponding figures using two samples of PPP-adjusted data from the PWT. The first sample uses the set of countries for which Maddison's data are available in 1870—the countries for which we pin down the initial conditions. We refer to this subsample as the Maddison sample and the results are reported in the next-to-last column. The last column uses the whole sample, reproducing the results in the first column of Table 1.

The second set of rows shows, correspondingly, the within-country slopes and the standard deviations resulting from the model-generated data for different γs, as well as the empirical results based on the Maddison sample, and the whole sample (the latter corresponding to the second column of Table 1). As in the data, the time-series slopes generated by the model tend to be larger in magnitude than the corresponding cross-sectional slopes. These results indicate that for the parameter values analyzed, the coefficients in both the pooled and within-country regressions are negative and significant at standard confidence levels and quantitatively comparable to those in the data.

To assess whether and to what extent the model can account for the decline in volatility with development seen in the data, it is important to know not only the slope coefficients but also the degree of dispersion in GDP generated by the model.

TABLE 3—VOLATILITY AND DEVELOPMENT: QUANTITATIVE RESULTS FOR DIFFERENT γ

	Poisson parameter γ				Data	
	0.05	0.10	0.15	0.20	Maddison sample	All countries
Cross-sectional slope of volatility on development	−0.272 (0.101)	−0.262 (0.036)	−0.213 (0.033)	−0.169 (0.032)	−0.270 (0.056)	−0.205 (0.032)
Time-series slope of volatility on development	−0.487 (0.054)	−0.455 (0.056)	−0.402 (0.060)	−0.355 (0.066)	−0.421 (0.105)	−0.496 (0.073)
SD of log-GDP per capita in 1960	0.894 (0.436)	0.729 (0.069)	0.688 (0.058)	0.648 (0.053)	0.977	0.970
Percent variation in volatility due to a 1-SD increase in log GDP per capita	−24.4%	−19.1%	−14.6%	−10.9%	−26.4%	−19.9%

Notes: The table shows, correspondingly, the cross-sectional and within-country slope coefficients and standard deviations (in parentheses) from regressions of (log) volatility of annual growth rates computed over nonoverlapping decades on the average (log) level of development in the decade; a constant (not reported) is included in each regression. The cross-sectional regressions are based on pooled data for five decades. The third set of rows shows the standard deviation of average logged GDP per capita over the whole decade (and the standard deviation over 1,000 simulations). The fourth line shows the percent variation in volatility generated by a one standard deviation increase in the logged GDP per capita. "Maddison sample" is a subset of the whole sample that includes the countries with data on GDP in 1870 (from Maddison 2010). The results correspond to the baseline calibration; the parameter values are: $\epsilon = 3$; $\Theta_N = -\theta_{m_0} = 0.03$; and $\eta = 0.10$. See text for explanations.

In the 1960s, the standard deviation of (log) per capita GDP across countries in the data was 0.970 (the corresponding value was 0.977 in Maddison's sample).[53] This is shown in the third row of Table 3, along with the corresponding statistics based on model-generated data.[54] The dispersion generated by the model is smaller than that in the data. Because the model has no mechanism to generate convergence, over time, cross-country GDP dispersion tends to either increase or remain constant, as appears to be the case in the data.[55]

An appealing way to measure the statistical variation of volatility with the level of development in the data is given by $\hat{\beta} \cdot \sigma_{GDP}$, where $\hat{\beta}$ is the slope regression coefficient and σ_{GDP} is the standard deviation of (log) per capita GDP. It indicates the percent decline in volatility generated by a one-standard deviation increase in (log) per capita GDP. We can construct similar statistics in the model-generated data. The results are reported in the last row of Table 3. As shown, the model is capable of generating a significant variation in volatility with respect to economic development. One could use these figures to assess what fraction of the statistical variation in the data can be generated by the model: $\frac{\hat{\beta}(\gamma) \cdot \sigma_{GDP}(\gamma)}{\hat{\beta} \cdot \sigma_{GDP}}$, where $\hat{\beta}(\gamma)$ and $\sigma_{GDP}(\gamma)$ are the model-generated slope coefficient and the standard deviation of (log) GDP, for different values of γ. For example, the model's explanatory power at $\gamma = 0.10$ is

[53] This is the standard deviation across countries of the decade-average (log) GDP per capita.

[54] Note that GDP dispersion is slightly decreasing in γ. This is because in order to match the US level of volatility in 1870, a lower γ implies a lower value for the initial ν in equation (37). This leads to a lower initial number of varieties in the United States, and, from equations (35) and (36) in all other countries. Hence, while on the one hand, a lower γ reduces country-level volatility, it necessitates a lower initial number of varieties, which tends to increase it. The net effect over time can only be assessed quantitatively.

[55] In the data, the standard deviation of log GDP increased from 0.97 at the beginning of the regression sample (1960s) to 1.2 by the end of the sample (2000s). For the Maddison subsample, the standard deviation was 0.56 in 1870 and reached 0.977 in the 1960s, remaining relatively constant thereafter.

around $\frac{\hat{\beta}(\gamma)\cdot\sigma_{GDP}(\gamma)}{\hat{\beta}\cdot\sigma_{GDP}}100 = 73$ percent. A value of $\gamma = 0.10$ means that technologies have a ten-year average lifetime.

2. *Firm-Level Volatility Declines with the Size of the Firm.*—In the model's BEGP, instantaneous firm-level volatility (the standard deviation of sales growth) is given by $\sqrt{[\lambda + \gamma]/n(j,t)}$, which declines monotonically with the size $n(j,t)$ of the firm. Hence, the model mechanically generates a negative relationship between firm-level volatility and size like the one in the data.

The model-generated slope is 0.5, while in the data the slope coefficient is estimated to be between 0.1 and 0.3, depending on the country (Tables 2 and A1). There are, of course, many possible explanations for the discrepancy between model and data, including measurement error in firm-level data. Within the model, a potential way to generate a smaller slope coefficient is to allow for variation in the intensity of use of different input varieties by firms, in the same way as the overall intensity of use varies for the economy as a whole. (Recall that, in the baseline model, at the firm level, all productive varieties are used in equal quantities, normalized to 1.) Note that it is precisely variations in the intensity of usage of input varieties for the economy as a whole that leads to a slope coefficient smaller than 0.5 in the cross-country regressions using model-generated data. We leave this extension for future work.

As noted in Section II, there is also evidence that the share of small firms in the economy correlates negatively with income per capita. This is also the case in our model. A regression of the share of small firms[56] on log GDP per capita in the model yields negative and significant coefficients, ranging from -0.049 (standard error $= 0.011$) for $\gamma = 0.05$ to -0.025 (s.e. $= 0.009$) for $\gamma = 0.20$.

3. *Firm-Level and Aggregate Volatility Tend to Display Positive Comovement.*—In the model-generated data, firm-level volatility, measured as the standard deviation of sales growth for the median firm, and aggregate volatility are positively correlated. The mean correlations (and the standard deviations over 1,000 simulations—in parentheses) are, correspondingly, 0.489 (0.048) for $\gamma = 0.05$; 0.421 (0.042) for $\gamma = 0.10$; 0.359 (0.055) for $\gamma = 0.15$; and 0.274 (0.059) for $\gamma = 0.20$. Interestingly, the results also suggest that when the volatility of shocks is higher (that is, γ is higher), the correlation between micro and macro volatility becomes weaker. This model prediction can potentially be tested in the future, as longer time series on firm-level data for different countries are gathered. In all, the positive comovement generated by the model is consistent with the available evidence (see Section I and the online Appendix).

Finally, as noted earlier, in a majority of countries, the distribution of growth rates is negatively skewed (skewness is measured as the sample third standardized moment). The model is capable of generating this negative skewness, as only negative shocks at the micro level contribute to aggregate volatility (positive microeconomic shocks add up to a deterministic aggregate process). The average skewness for countries in the model-generated data ranges from -0.285 when $\gamma = 0.05$ to

[56] Small firms are defined as those having five or fewer input varieties.

-0.076 when $\gamma = 0.20$. When $\gamma = 0.10$, skewness is -0.141. The average skewness coefficient in the data is higher: -0.390. The model could yield higher skewness if negative shocks were not independent across varieties.

In all, the quantitative exercise leads us to conclude that the technological-diversification model, though stylized, can potentially account for a substantial part of the decline in volatility with development observed in the data. The model offers an alternative channel to account (at least partially) for the volatility-size relationship observed at the firm level, and generates a positive correlation between firm-level data and aggregate volatility that appears in line with recent empirical findings in this area.

IV. Concluding Remarks

We argue that technological diversification offers a promising (yet so far overlooked) explanation for the negative relation between volatility and development. We do so by proposing a model in which the production process makes use of different varieties subject to imperfectly correlated shocks. As in Romer (1990) and Grossman and Helpman (1991), technological progress takes place as an expansion in the number of input varieties, increasing productivity. The new insight in the model is that the expansion in input varieties can also lead to lower volatility in production. First, as each individual variety matters less and less in production, the contribution of variety-specific fluctuations to overall volatility declines. Second, each additional variety provides a new adjustment margin in response to external shocks, making productivity less volatile. In the model, the number of varieties evolves endogenously in response to profit incentives and the decrease in volatility results as a by-product of firms' incentives to increase profits. We simulate the model for plausible parameter values and find that it can quantitatively account for a substantial fraction of the statistical variation in volatility with respect to development observed in the data.

There are three natural directions for further investigation. First, extending the setup to a multi-sector model that explicitly distinguishes between within- and across-sector diversification. Second, allowing for international trade to analyze the trade-off between higher sectoral specialization (possibly brought about by increased trade openness), and the scope for input or technology diversification facilitated by trade.[57] The third direction entails extending the model to match the regularities emphasized in the RBC literature, with a focus on poor countries. Some of the frictions (and shocks) needed to augment our model will be similar to the extensions made to the RBC (or New Keynesian) model. The key contribution of our model will be on the endogenous link between a country's development and its susceptibility to shocks, a link that is not addressed by the RBC literature.

[57] See Caselli et al. (2010) for an exploration of these mechanisms.

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