Balanced Growth with Structural Change*

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Abstract

We study a multi-sector model of growth with differences in TFP growth rates across sectors and derive sufficient conditions for the coexistence of a balanced aggregate growth path, with all aggregates growing at the same rate, and structural change, characterized by sectoral labor reallocation. The conditions needed are weak restrictions on the utility and production functions: goods should be poor substitutes and the intertemporal elasticity of substitution should be one. We present evidence from US and UK sectors that is consistent with our conclusions and successfully calibrate the shift from agriculture to manufacturing and services in the United States.

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1 Introduction

This paper analyzes structural change and balanced aggregate growth within a unified model. Structural change is the name normally given to the reallocation of factors

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across different sectors of the economy. The shifts between agriculture, manufacturing and services are the most commonly studied examples of structural change. Structural shifts are usually studied in models that do not satisfy the conditions for balanced aggregate growth. Conversely, balanced aggregate growth is normally studied in models that do not allow structural change. In this paper we extend the one-sector optimizing model of economic growth with exogenous technological progress to many sectors and study structural change in a model that retains all the attractive features of the one-sector model, including, crucially, its steady-state growth equilibrium. The restrictions on functional forms needed to yield balanced growth and structural change in the multi-sector model are weak restrictions on functional forms that are frequently imposed by economists in related contexts.

Structural change in our model is growth-induced. It arises because total factor productivity in different sectors grows at different rates. Our primary objective is to study the implications of this property for the allocation of factors across sectors of the economy in an infinite-horizon optimizing framework and to evaluate it with long time series data. But we have a second objective, which concerns aggregation. The one-sector model with constant rate of productivity growth has proved a useful tool in modern macroeconomics, although it is a fact that modern growth takes place at different rates across different sectors of the economy. We derive a set of sufficient conditions needed for the aggregation of the economy into one sector characterized by steady growth, within the framework of the iso-elastic utility functions and Cobb-Douglas production functions commonly used in the one-sector model.

Pioneering work on the connections between growth and structural change was done by Baumol (1967; Baumol et al., 1985). Baumol divided the economy into two sectors, a "progressive" one that uses capital and new technology and grows at some constant rate and a "stagnant" one that uses labor services as final output (as for example in the arts or the legal profession). He then claimed that because of factor mobility the production costs and prices of the stagnant sector rise indefinitely. So, the stagnant sector attracts more labor to satisfy demand if demand is either income elastic or price inelastic, but vanishes otherwise. Baumol controversially also claimed that if the stagnant sector does not vanish the economy's growth rate is on a declining trend, as more weight is shifted to the stagnant sectors.

We show that Baumol's conclusion, known as "Baumol's cost disease", was overly pessimistic. Although costs rise and resources shift into low-growth sectors, if Baumol's progressive sector that produces the capital goods is used as the numeraire the aggregate economy is on a steady-state growth path with the rate of growth equal to the rate of growth of the progressive sector.[1] Crucially, our economy also satisfies Kaldor's stylized facts of constant rate of return to capital and constant rate of wage growth.

We obtain our results by assuming that capital goods are supplied by only one sector, which we label manufacturing, and which produces also a consumption good. Production functions are identical in all sectors except for their rates of TFP growth and each sector produces a differentiated good that enters a constant elasticity of substitution utility function. We show that a low (below one) elasticity of substitution across goods leads to shifts of employment shares to sectors with low TFP growth. We also show that if in addition the utility function has unit inter-temporal elasticity, the aggregate economy is on a steady-state growth path which is obtained as the solution to two differential equations, one unstable in the control (aggregate consumption) and one stable in the state (the capital stock). Eventually, all sectors vanish in the limit except for manufacturing and the slowest growing sector. But on the adjustment to the limiting state sectors expand or contract according to their relative TFP growth rates, whereas the aggregate economy stays on its balanced growth path.

Our results contrast with the results of Echevarria (1997), who assumed non-homothetic preferences to derive structural change from different rates of sectoral TFP growth. In her economy balanced growth exists only in the limit, when preferences reduce to homotheticity with unit elasticity of substitution, and structural change ceases. In the transition the aggregate growth rate first rises and then falls, in contrast to ours, which is constant. Our results also contrast with the results of Kongsamut et al. (2001), who derive simultaneously constant aggregate growth and structural change. But they obtain their results by imposing a restriction that maps some of the parameters of their Stone-Geary utility function on to the parameters of the production functions, violating one of the most useful conventions of modern macroeconomics, the complete independence of preferences from technologies. Our restrictions are quantitative restrictions that maintain the independence of preferences and technologies.

In the empirical literature two competing explanations (which can coexist) have been put forward for structural change. Our explanation, which is sometimes termed

[1] Ironically, we get our result because of the inclusion of capital, a factor left out of the analysis by Baumol "for ease of exposition ... that is [in]essential to the argument". We show that the inclusion of capital is essential for the growth results, though not for structural change.

"technological" because it attributes structural change to different rates of sectoral TFP growth, and a utility-based explanation, which requires different income elasticities for different goods and can yield structural change even with equal TFP growth in all sectors. Kravis et al. (1983) present evidence that favours the technological explanation, at least when the comparison is between manufacturing and services. Two features of their data that are satisfied by the technological explanation are (a) relative prices have reflected differences in TFP growth rates and (b) real consumption shares have been fairly constant. Our model also has these implications when there is low substitutability across goods. We use multi-sector data for the United States and United Kingdom to show that changes in employment shares, prices and real consumption shares are consistent with our model's predictions. We also evaluate the model's performance in its explanation for the long-run shifts between agriculture, manufacturing and services. We show that although the model tracks the changes well, it predicts a slower decline of agriculture than is observed in the data. This leads us to conclude that although for manufacturing and services the technological explanation may be sufficient to explain structural change, the explanation for the fast decline of agriculture may require something additional, such as a below-unity income elasticity.

Section 2 describes our model of growth with many sectors and derives first the aggregate growth equilibrium and then the conditions for structural change. In section 3 we show some supporting evidence for our results by making use of US and UK sectoral data for 1970-1993. In section 4 we focus on the long-run structural change between manufacturing, agriculture and services and show both analytically and with computations the balanced growth path and the shift from agriculture to manufacturing and services and then from manufacturing to services, with shares matching reasonably well the shares observed in the United States.

2 A growth model with many sectors

We assume the existence of an arbitrary number $n+1$ of sectors. Sectors $i = 1, ..., n$ produce only consumption goods. The last sector, which is denoted by m and labeled manufacturing, produces both a final consumption good and the economy's capital stock. Manufacturing is the numeraire.

The labor force is growing at a constant rate ν and there is full employment. Intersectoral capital and labor mobility are free, so rates of return are equalized across sectors. We denote by k the aggregate capital-labor ratio, by k_i $(i = 1, ..., n, m)$ the ratio of capital to employment in each sector and by n_i the fractions of the labor force employed in each sector. An equilibrium is an optimal allocation of labor and capital across sectors and over time and is derived from the following social planning problem:

$$\max_{\left\{\{c_{it},k_{it},n_{it}\}_{i=1,..,n,m}\right\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} v\left(c_1, ...c_n, c_m\right) dt \tag{1}$$

subject to:

$$\dot{k} = F^m(n_m k_m, n_m) - c_m - (\delta + \nu) k \tag{2}$$

$$c_i = F^i(n_i k_i, n_i) \qquad i = 1, ..., n \tag{3}$$

$$\sum_{i=1,..,n,m} n_i = 1 \tag{4}$$

$$\sum_{i=1,...,n,m} n_i k_i = k. \tag{5}$$

The utility function is defined over the consumption of all goods and denoted by $v(.)$. The state is given by the aggregate capital stock, which is produced entirely in the manufacturing sector and depreciates at rate $\delta > 0$. Equations (4) and (5) are resource constraints.

The planner's controls are the consumption levels and the sectoral allocation of factors. Optimality implies the static efficiency conditions:

$$\frac{v_i}{v_m} = \frac{F_K^m}{F_K^i} = \frac{F_N^m}{F_N^i} \qquad i = 1, ..n. \tag{6}$$

and the dynamic efficiency condition:

$$-\frac{\dot{v}_m}{v_m} = F_K^m - (\delta + \rho + \nu). \tag{7}$$

where F_N^i and F_K^i are the marginal products of labor and capital in sector i.

In order to focus on the implications of different rates of TFP growth across sectors we assume that there are no other differences in sector production functions. We assume:

$$F^i = A_i (n_i k_i)^{\alpha} n_i^{1-\alpha}; \quad \frac{\dot{A}_i}{A_i} = \gamma_i; \quad \alpha \in (0, 1), \quad i = 1, .., n, m. \tag{8}$$

With these production functions, static efficiency (6) implies

$$k_i = k_m \qquad i = 1, ..., n \tag{9}$$

and

$$\frac{v_i}{v_m} = \frac{A_m}{A_i} = p_i \qquad i = 1, ..., n. \tag{10}$$

where p_i is the price of good i in the decentralized economy (in terms of the price of the manufacturing good).

Utility functions are assumed to have constant elasticities both across goods and over time:

$$v\left(c_1, ...c_n, c_m\right) = \frac{\phi\left(.\right)^{1-\theta} - 1}{1-\theta}; \quad \phi\left(.\right) = \left(\sum_{i=1,...,n,m} \omega_i c_i^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \tag{11}$$

where $\theta, \varepsilon > 0$, and $\omega_i > 0$ and $\sum_{i=1,..,n,m} \omega_i = 1$. Of course, if $\theta = 1$, $v(.) = \ln \phi(.)$ and if $\varepsilon = 1$, $\ln \phi(.) = \sum \omega_i \ln c_i$.

We define here an expression that will be useful in our analysis:

$$x_i \equiv \left(\frac{\omega_i}{\omega_m}\right)^{\varepsilon} \left(\frac{A_m}{A_i}\right)^{1-\varepsilon} \qquad i = 1, ...n, m \tag{12}$$

where by definition $x_m = 1$, and so x_i is expressed in terms of the numeraire. Let $X \equiv \sum\limits_{i=1,..n,m} x_i$. We also define aggregate consumption and output in terms of the numeraire:

$$c \equiv c_m + \sum_{i=1}^{n} p_i c_i; \qquad y \equiv y_m + \sum_{i=1}^{n} p_i y_i \tag{13}$$

where y_i is the ratio of output in sector i to the total labor force.

2.1 Aggregate growth

The following results characterize the equilibrium of the aggregate economy:

Proposition 1 *Given any initial k_0, the equilibrium of the aggregate economy is defined as a sequence $\{c_t, k_t\}_{t=0,1,...}$ that satisfies the following two dynamic equations:*

$$\frac{\dot{k}}{k} = A_m k^{\alpha-1} - \frac{c}{k} - (\delta + \nu), \tag{14}$$

$$\theta\frac{\dot{c}}{c} = (\theta - 1)(\gamma_m - \bar{\gamma}) + \alpha A_m k^{\alpha-1} - (\delta + \rho + \nu). \tag{15}$$

where $\bar{\gamma} \equiv \sum_{i=1,...,n,m} \frac{x_i}{X}\gamma_i$ is the weighted average of TFP growth rates.

Proof. All proofs are collected in the Appendix. ■

The key property of our equilibrium is that the contribution of each consumption sector i to aggregate equilibrium is through its weight x_i in the definition of the average TFP growth rate $\bar{\gamma}$. Note that because each x_i depends on the sector's relative TFP level, the weights here are functions of time.

We look for an equilibrium path that satisfies Kaldor's fact of constant rate of return to capital. By the condition of free capital mobility, (9), this requires that $A_m k^{\alpha-1}$ be constant, i.e., k should grow at rate $\gamma_m/(1-\alpha)$. The definition of aggregate output is given in (13) and is in the same units as the aggregate capital stock. Given (9) and (10), we make use of (4) to write aggregate output per worker in the form:

$$y = A_m k^{\alpha}. \tag{16}$$

The technology parameter in (16) is TFP in manufacturing and not an aggregate of all sectors' TFP, so if there is a steady state with k growing at rate $\gamma_m/(1-\alpha)$, y also has to grow at the same rate.[2] But then the state equation (14) implies that c/k must also be constant, so in this steady state aggregate consumption grows at the same rate as well. We define this steady state as the balanced growth path.

By (15), a balanced growth path requires that the expression $(\theta-1)(\gamma_m-\bar{\gamma})$ be a constant, which obtains under some restrictions. We discuss these restrictions in the next section. In the remaining of this section we derive the aggregate balanced growth path by imposing the condition:

$$(\theta-1)(\gamma_m-\bar{\gamma}) \equiv \kappa \qquad \text{constant.} \tag{17}$$

Define aggregate consumption and the aggregate capital-labor ratio in terms of efficiency units

$$c_e \equiv c A_m^{-1/(1-\alpha)}; \qquad k_e \equiv k A_m^{-1/(1-\alpha)}.$$

The dynamic efficiency condition (15) and state equation (14) become

$$\frac{\dot{c}_e}{c_e} = \alpha k_e^{\alpha-1} - \left(\frac{\gamma_m}{1-\alpha} + \delta + \nu + \rho - \kappa\right) \tag{18}$$

$$\frac{\dot{k}_e}{k_e} = k_e^{\alpha-1} - \frac{c_e}{k_e} - \left(\frac{\gamma_m}{1-\alpha} + \delta + \nu\right). \tag{19}$$

[2] Empirically, our aggregate y corresponds to the value of aggregate output at constant manufacturing prices, which is equivalent to using each sector's current relative price as weight. See the Appendix, "matching the model with the data".

Equations (18) and (19) parallel the two differential equations in the control and state of the one-sector model, making the aggregate equilibrium of our many-sector economy identical to the equilibrium of the one-sector Ramsey economy (when $\kappa = 0$) and trivially different from it otherwise. Both models have a saddlepath equilibrium and stationary solutions $\left(\hat{c}_e, \hat{k}_e\right)$ that imply balanced growth in the three aggregates. As anticipated in the aggregate production function (16), a key result is that in our economy the rate of growth in the steady state is equal to the rate of growth of labor-augmenting technological progress in the sector that produces capital goods: the ratio of capital to employment in each sector, aggregate capital per worker, aggregate output per worker, and aggregate consumption per worker grow at rate $\gamma_m/(1-\alpha)$.

2.2 Structural change

Despite the balanced growth properties of our economy, our model can generate sectoral reallocation of labor. We derive in the Appendix (Lemma 5) the behavior of employment shares along the balanced growth path. For the consumption goods sectors, the employment shares satisfy:

$$n_i = \frac{x_i}{X}\hat{c}_e\hat{k}_e^{-\alpha} \qquad i = 1, ..n, \tag{20}$$

where $\hat{c}_e\hat{k}_e^{-\alpha}$ is the ratio of consumption to output in the economy as a whole. For the capital-producing sector:

$$n_m = \left(\frac{1}{X}\hat{c}_e\hat{k}_e^{-\alpha}\right) + \left(1 - \hat{c}_e\hat{k}_e^{-\alpha}\right). \tag{21}$$

The term in the first bracket parallels the term in (20) - noting that $x_m = 1$ - and so represents the employment needed to satisfy the consumption demand for manufacturing goods. The second bracketed term is equal to the savings rate and represents the manufacturing employment needed to satisfy investment demand.

The dynamics of the employment shares along the balanced growth path satisfy:

$$\frac{\dot{n}_i}{n_i} = (1-\varepsilon)\left(\bar{\gamma} - \gamma_i\right); \qquad i = 1, ...n, \tag{22}$$

$$\frac{\dot{n}_m}{n_m} = (1-\varepsilon)\left(\bar{\gamma} - \gamma_m\right)\left(\frac{\hat{c}_e\hat{k}_e^{-\alpha}/X}{\hat{c}_e\hat{k}_e^{-\alpha}/X + 1 - \hat{c}_e\hat{k}_e^{-\alpha}}\right). \tag{23}$$

The growth rate of employment in consumption sector i is proportional to the shortfall of the sector's TFP growth rate from the economy-wide average TFP growth rate.

But in manufacturing the factor of proportionality is not constant as it is multiplied by the fraction of manufacturing employment that is used for consumption purposes, which varies over time.

We define structural change as the state in which at least some of the labor shares change, i.e., $\dot{n}_i \neq 0$ for at least some i. It follows now immediately from (17) and (22)-(23) that:

Proposition 2 *Necessary and sufficient conditions for the existence of an aggregate balanced growth path with structural change are:*

$$\begin{aligned} \theta &= 1, \\ \varepsilon &\neq 1; \ \text{and } \exists i \in \{1,..,n\} \ \text{s.t. } \gamma_i \neq \gamma_m. \end{aligned} \tag{24}$$

Under the conditions of Proposition 2 $\kappa = 0$, and our aggregate economy becomes formally identical to the one-sector Ramsey economy. κ is constant under two other conditions, which give balanced aggregate growth but no structural change, $\gamma_i = \gamma_m$ $\forall i$ and $\varepsilon = 1$. It follows from (22) and (10) that if $\gamma_i = \gamma_m$ we have no quantity or price changes across sectors, whereas if $\varepsilon = 1$ differences in TFP growth rates are absorbed by changes in relative prices and consumption levels and there is no labor reallocation.

Proposition 2 requires the utility function to be logarithmic in the consumption composite ϕ, which implies an intertemporal elasticity of substitution equal to one, but be non-logarithmic across goods, which implies non-unit price elasticities. A noteworthy implication of Proposition 2 is that balanced aggregate growth does not require constant rates of growth of TFP in any sector other than manufacturing. Because both capital and labor are perfectly mobile across sectors, changes in the TFP growth rates of consumption-producing sectors are reflected in immediate reallocations of capital and labor across the sectors (and in price changes), without effect on the aggregate growth path, which grows irrespective of the γ_i at rate γ_m.

Proposition 2 confirms Baumol's (1967) claims about structural change. When demand is price inelastic, the sectors with the low productivity growth rate attract a bigger share of labor, despite the rise in their price. The lower the elasticity of demand, the less the fall in demand that accompanies the price rise, and so the bigger the shift in employment needed to maintain high relative consumption. But in contrast to Baumol's claims, the economy's growth rate is not on an indefinitely declining trend because of the existence of capital goods.

Some further results on the properties of the structural change along the balanced growth path can be obtained as follows. Condition (20) implies that employment in sector i relative to sector j depends only on the ratio x_i/x_j (for $i, j \neq m$). The Appendix shows that this property also holds in the transition to balanced growth. It implies that the rate of change of relative employment depends only on the sectors' TFP growth rates:

$$\frac{\dot{\overline{n_i/n_j}}}{n_i/n_j} = (1-\varepsilon)\left(\gamma_j - \gamma_i\right). \tag{25}$$

But (10) implies that the rate of change of the relative price of good i is (again both along the transition and on the balanced growth path):

$$\frac{\dot{p}_i}{p_i} = \gamma_m - \gamma_i \qquad\qquad i = 1, ..., n, \tag{26}$$

and so,

$$\frac{\dot{\overline{n_i/n_j}}}{n_i/n_j} = (1-\varepsilon)\frac{\dot{\overline{p_i/p_j}}}{p_i/p_j} \tag{27}$$

Proposition 3 *In both the transition to a balanced growth path and on the balanced growth path itself, relative price changes depend only on differences in TFP growth rates; in sectors producing only consumption goods, changes in relative employment shares are proportional to changes in their relative prices, with the factor of proportionality monotonically falling in the price elasticity of demand*

Next, we characterize the set of expanding sectors $(\dot{n}_i \geq 0)$, denoted E_t, and the set of contracting sectors $(\dot{n}_i \leq 0)$, denoted D_t, at any time t, under the restrictions of Proposition 2 and for constant γ_i $\forall i$. Condition (22) implies that the set of expanding and contracting sectors satisfy:

$$\begin{aligned} E_t &= \{i \in \{1, ..., n, m\} : (1-\varepsilon)(\bar{\gamma}_t - \gamma_i) \geq 0\}; \\ D_t &= \{i \in \{1, ..., n, m\} : (1-\varepsilon)(\bar{\gamma}_t - \gamma_i) \leq 0\}. \end{aligned} \tag{28}$$

If goods are poor substitutes ($\varepsilon < 1$), sector i expands if and only if its TFP growth rate is smaller than the weighted average of all sectors' TFP growth rates, and contracts if and only if its growth rate exceeds the weighted average. But if $\varepsilon < 1$, the weighted average $\bar{\gamma}$ is decreasing over time (see Lemma 6 in the Appendix). Therefore, the set of expanding sectors is shrinking over time, as more sectors' TFP growth rates exceed $\bar{\gamma}$.

If goods are good substitutes ($\varepsilon > 1$), sector i expands if and only if its TFP growth rate is greater than $\bar{\gamma}$, and contracts otherwise. But $\varepsilon > 1$ implies that $\bar{\gamma}$ is also increasing over time, so, as before, the set of expanding sectors is shrinking over time. This establishes

Proposition 4 *The set of expanding sectors is contracting over time and the set of contracting sectors is expanding over time:*

$$E_{t'} \subseteq E_t \text{ and } D_t \subseteq D_{t'} \qquad \forall t' > t$$

Asymptotically, the economy converges to a two-sector economy consisting of sector m and the sector that has the smallest (largest) TFP growth rate if and only if goods are poor (good) substitutes.

The asymptotic distribution of employment shares in the economy is

$$\begin{aligned} n_l^* &= \hat{c}_e \hat{k}_e^{-\alpha} = 1 - \alpha + \frac{\alpha\rho}{\delta + \nu + \rho + \gamma_m / (1-\alpha)} < 1 \\ n_m^* &= 1 - n_l^* \end{aligned} \tag{29}$$

where sector l denotes the sector with the smallest (largest) TFP growth rate if and only if goods are poor (good) substitutes. We note that n_l^* is equal to the ratio of aggregate consumption to output and so n_m^* is equal to the savings rate (equivalently, to the ratio of investment to output) along the balanced growth path. Denoting the savings rate by $\hat{\sigma}$ we obtain,

$$\hat{\sigma} = n_m^* = \alpha \left(\frac{\delta + \nu + \gamma_m / (1-\alpha)}{\delta + \nu + \rho + \gamma_m / (1-\alpha)} \right).$$

If there is no discounting $\rho = 0$, and the employment share in the capital-producing sector is equal to α, the capital share in the economy as a whole. We can also see from (20), which implies $n_{mt} - n_m^* = n_l^*/X > 0$, that the asymptotic employment share in manufacturing is smaller than its employment share along the balanced growth path at any point in time.

To obtain now the behavior of output and consumption shares we use the results in (9) and (10), to find that output shares at current value are equal to employment shares, so the results obtained for employment shares also hold for them:

$$\frac{p_i Y_i}{Y_m + \sum_{i=1}^{n} p_i Y_i} = n_i \quad \forall i = 1, .., n. \tag{30}$$

The expenditure shares of consumption satisfy the same dynamics as the employment shares,

$$\frac{p_i c_i}{c} = \frac{n_i}{n_l^*}, \tag{31}$$

but the relative real consumption shares satisfy:

$$\frac{\dot{c}_i}{c_i} - \frac{\dot{c}_j}{c_j} = \varepsilon \left(\gamma_i - \gamma_j\right). \tag{32}$$

A comparison of (25) with (32) reveals that a small ε can reconcile the small changes in the relative real consumption shares with the large change in both relative expenditure shares and relative employment shares, a finding that let Kravis et al. (1983) conclude that the technological explanation for structural change was consistent with the data. In the next section, we bring multi-sector data that confirms the small ε.

We conclude this section with an example of a simple economy, characterized by $\varepsilon < 1$, $\omega_i = \omega_m$, and $A_{i0} = A_{m0}$ $\forall i$, i.e., one in which sectors differ only in their rates of TFP growth. Given these assumptions, the weights x_i equal 1 in all sectors at time 0. We rank the consumption sectors according to their TFP growth rate, letting sector n be the slowest growing sector. The weighted average of TFP growth rates at time 0 is the same as the mean TFP growth rate, $\bar{\gamma}_0 = \left(\frac{1}{1+n}\right) \sum \gamma_i$. Thus, initially sectors with a TFP growth rate below the mean are expanding, while sectors with a TFP growth rate above the mean are shrinking. The weight x_i is increasing if and only if $\gamma_i < \gamma_m$. Therefore, over time the weighted average $\bar{\gamma}$ is decreasing and, as claimed in proposition 4, the set of expanding sectors is shrinking and the set of contracting sectors is growing. Asymptotically, all sectors disappear sequentially according to their index until only sectors n and m remain. As x_i equals 1 in all sectors at time 0, the initial employment shares are equal across consumption sectors, i.e. $n_{i0} = (1 - \hat{\sigma}) / (1 + \hat{n})$ $\forall i$. Over time, the consumption sectors that begin by losing employment contract until they disappear and those that begin by expanding eventually contract and disappear except for the nth sector, whose employment share converges to $\hat{\sigma}$ as $t \to \infty$. The only other remaining sector, manufacturing, converges to $1 - \hat{\sigma}$.

3 Supporting evidence

We use sectoral data from the United States and the United Kingdom to provide supporting evidence for our propositions. The three key implications of the model that we examine are summarized in equations (25), (26) and (27), which we re-write in the more convenient form:

$$\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j} = -(1-\varepsilon)\left(\gamma_i - \gamma_j\right) \tag{33}$$

$$\frac{\dot{p}_i}{p_i} - \frac{\dot{p}_j}{p_j} = -(\gamma_i - \gamma_j) \tag{34}$$

$$\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j} = (1-\varepsilon)\left(\frac{\dot{p}_i}{p_i} - \frac{\dot{p}_j}{p_j}\right) \tag{35}$$

Of course, these hold only for the n consumption sectors and the third relation follows from the first two. However, because employment and prices at the sectoral level may be measured with less error than TFP, we report sectoral data for all three relations. Our model is too stylized to justify a full empirical test of its propositions, but if it has predictive value the relations in (33)-(35) should hold, at least on average. The object of the exercise in this section is to show whether these relations hold on average or not.

We use data from the OECD *International Sectoral Database* (ISDB), which covers the whole economy and is annual for the period 1970-1993. The ISDB has been merged with the *STAN Database for Industrial Analysis* and is no longer updated. However, ISDB contain data for sectoral TFP constructed by the OECD whereas STAN does not, so we chose to use the ISDB for the years that it is available. We extracted data for total employment, prices (obtained as the ratio of the sector's value added at current prices to value added at constant prices) and a TFP index for all SIC sectors. We selected the United States and United Kingdom as the two countries least likely to suffer from barriers to inter-sectoral allocations and so be closer to our model specification. For each country we selected from the available sectors those that we considered to be predominantly consumption-goods sectors, by which we mean sectors whose final output is bought primarily by consumers, rather than businesses. These sectors are shown along with the other sectors in the database in Table 1. We excluded altogether from our analysis government services.

The relations in (33)-(35) hold both in and out of steady state but only when our assumptions of full information and full inter-sectoral labor and capital mobility are satisfied. For this reason they are more appropriate descriptions of long-run trends than year-to-year changes. We accounted for this by averaging the annual rates of growth of employment, prices and TFP for each sector over the entire sample and

Table 1: Sectors and Employment Shares

consumption goods			consumption + capital goods		
	empl. share			empl. share	
sector	US	UK	sector	US	UK
agriculture	3.9	3.3	mining	1.0	1.6
food	2.1	3.6	wood	1.5	-
textiles	2.7	3.9	paper	2.4	2.7
trade*	24.7	24.0	chemical	2.5	3.5
transport	5.3	7.7	non metallic	0.8	1.3
finance	5.4	10.4	metal	1.9	-
real est.$^+$	8.9		machinery	10.1	15.1
services	17.4	9.4	other manuf.	0.5	0.5
			utilities	1.0	1.7
			construction	6.7	8.1

Notes. The time period for the US is 1970-93 and for the UK, 1970-90. * In the US, trade includes retail and wholesale trade; in the UK it includes in addition restaurants and hotels. + In the UK finance and real estate are grouped into one sector. There are no TFP data for the wood and metal sectors in the UK

report results obtained with these averages. With 8 consumption sectors there are 28 sector pairs (in the UK there are 7 sectors and 21 pairs). Figure 1, panel (a), plots the differentials in the growth rates of the three variables against each other for the United States and figure 2, panel (a), repeats the same for the United Kingdom.

The three slopes in each diagram are as predicted by the model. In Table 2 we report estimates for the slopes in (33)-(34). The slope of the line in the price-TFP space is not significantly different from 1, in either the US or the UK. The employment-TFP regression line gives $\varepsilon = 0.28$ for the United States and $\varepsilon = -0.34$ for the United Kingdom, but one that is not significantly different from zero. Re-estimating the two-equation system (33)-(34) by imposing a unit coefficient on (34) we obtain $\varepsilon = 0.29$ (s.e.=0.19) for the United States and $\varepsilon = -0.01$ (s.e.=0.35) for the United Kingdom. So the values of ε obtained from these regressions are very small.[3] We argued that a small ε is crucial if our model is to explain the coexistence of large changes in employment shares with small changes in consumption shares.

As a further test of the model, we repeat the same exercise for the non-consumption

[3] We also estimated the slope of the line in employment-price space for comparison with the other slopes. We obtained consistent estimates. In the US, the slope gives $\varepsilon = 0.36$ (s.e.=0.20) and in the UK it gives $\varepsilon = 0.27$ (s.e.=0.19).

Table 2: Changes in Employment Shares and Prices, United States

	United States				United Kingdom			
	indep. vars.				indep. vars.			
dep. var.	cons.	$\gamma_i - \gamma_j$	R^2	obs.	cons.	$\gamma_i - \gamma_j$	R^2	obs.
consumption sectors								
$\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j}$	-1.41	-0.72	0.33	28	-0.74	-1.34	0.40	21
	(0.41)	(0.19)			(0.30)	(0.36)		
$\frac{\dot{p}_i}{p_i} - \frac{\dot{p}_j}{p_j}$	-0.56	-0.94	0.80	28	-0.22	-1.21	0.40	21
	(0.19)	(0.09)			(0.27)	(0.32)		
manufacturing sectors								
$\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j}$	0.91	0.11	0.01	45	0.59	0.66	0.05	28
	(0.30)	(0.23)			(0.50)	(0.59)		
$\frac{\dot{p}_i}{p_i} - \frac{\dot{p}_j}{p_j}$	-0.15	-0.30	0.05	45	0.07	-1.98	0.84	28
	(0.24)	(0.19)			(0.45)	(0.17)		

Notes. Estimation of the coefficients is by SUR. Results are very similar to OLS. Numbers in brackets are standard errors.

sectors. The model assumes the existence of only one capital-producing sector, so it is silent about the relations that should hold between capital-producing sub-sectors. But in general when a sector produces capital goods the relations in (33)-(35) should not hold, as (22) makes clear. Panels (b) in figures 1 and 2 give results comparable to those of panels (a) but for capital-producing sectors. The results contrast sharply with those found for the consumption sectors. For the United States none of the three diagrams shows a significant relation between the variables. For the United Kingdom, the points in the two diagrams with employment are again not showing significant relations and the only significant relation, between the growth differential of prices and TFP, gives a slope very close to 2, instead of the 1 obtained for the consumption sectors.

4 A Three-sector economy

We now focus on the nature of long-run structural change predicted by the model, by computing the balanced growth path for an economy with three sectors, agriculture (sector a), services (sector s) and manufacturing (sector m). If the ranking of the TFP growth rates is such that $\gamma_a > \gamma_m > \gamma_s$, then the TFP growth rate in agriculture is always above the weighted average of TFP growth rates while the TFP growth rate in services is always below it, i.e. $\gamma_a > \bar{\gamma}_t > \gamma_s$ for all t. Therefore, the model predicts that if the three goods are poor substitutes, the agricultural employment share should decline indefinitely and the service sector employment share should rise. The manufacturing employment share may rise before it starts to decline if its TFP growth rate is lower than the initial economy-wide weighted average of TFP growth rates. But even if the share of manufacturing increases at first, eventually it should decline, as the weighted average of the TFP growth rates falls over time. Asymptotically, the three-sector economy converges to a two-sector economy with manufacturing and services only, with the employment share of manufacturing equal to the investment to output ratio along the balanced growth path.

From (20), the employment shares at any time t obey

$$\begin{aligned} n_{it} &= (1-\hat{\sigma})\frac{x_{it}}{X_t} \qquad i = a, s \\ n_{mt} &= 1 - n_{at} - n_{st}. \end{aligned} \tag{36}$$

Therefore, the initial weight $(x_{a0,}x_{s0})$, for given initial distribution of employment shares (n_{a0}, n_{s0}, n_{m0}) is,[4]

$$x_{a0} = \frac{n_{a0}}{n_{m0} - \hat{\sigma}}; \qquad x_{s0} = \frac{n_{s0}}{n_{m0} - \hat{\sigma}}. \tag{37}$$

With information on the parameter ε and the TFP growth rates, the model generates the distribution of employment shares over time: Given x_{i0}, ε and the $\gamma_i's$, we use (12) to derive x_{it}, then use (36) to derive n_{it}.

Consider now a plausible scenario for industrialized countries, an investment rate of 20 percent and an aggregate growth rate of 2 percent. Also, let initially half the labor force be in agriculture and the other half divided equally between manufacturing and services.[5] As demonstrated in figure 3, the employment share of agriculture in this scenario falls while the employment share of services rises, both monotonically. The employment share in manufacturing first rises slightly, then it flattens and finally it declines. The decline is more noticeable when the agricultural employment share becomes small.

[4]Given (37), n_m is first increasing if $\gamma_m < \frac{n_{a0}\gamma_a + n_{s0}\gamma_s}{n_{a0}+n_{s0}}$. Therefore, initial employment shares and TFP growth rates are necessary and sufficient to determine whether n_m increases before it starts to decline.

[5]In other words, $\hat{\sigma} = 0.2$ and the implied x_{a0} and x_{s0} are derived from (37). The 2 percent aggregate growth rate implies $\gamma_m = 0.012$ for $\alpha = 0.4$. Note that this implies that the labor productivity in the manufacturing sector is also growing at 2 percent. The rest of the parameters are $\gamma_a = 0.025$, $\gamma_s = 0.005$, $\varepsilon = 0.2$.

The pattern implied by this scenario is a typical pattern of structural change observed in industrialized countries.[6]. The "shallow bell shape" for manufacturing that was found by Maddison (1980, p. 48) for each of the 16 OECD countries in his sample is a prediction that we believe is unique to our model. Figure 4 shows that the same patterns also hold when the employment shares are plotted against GDP for the 16 OECD countries in cross sections, using data from 1870 to 2001.[7]

To evaluate the quantitative implications of our model, we calibrate our balanced growth path to the US economy from 1869 to 1998. We describe how we conducted the calibration in the Appendix. Our model makes predictions about the aggregate economy, relative prices and employment shares. The strategy is to choose parameters to match the first two and let the model determine the dynamics of employment shares. In brief, we set $\hat{\sigma}$ to match the aggregate investment rate and γ_m to match the manufacturing growth rate, and (γ_s, γ_a) to match the average growth rate for the relative prices of agriculture and services in terms of manufacturing. We use values of ε that are consistent with our estimates in Table 2. Given these parameters, we use (37) to match the employment shares in 1869, and examine how the predictions of the model compare with the employment shares in the data. We exclude the government sector in the calculation of employment shares, as the services provided by government are not priced optimally.[8] Thus, the baseline parameters are

$\hat{\sigma}$	γ_m	γ_a	γ_s	ε
0.2	0.013	0.023	0.003	0.3

Figure 5, panel (a), reports the results for our baseline parameters. Although the

[6] For example, Kuznets (1966) documented this pattern for 13 OECD countries and the USSR between 1800 and 1960. The 13 OECD countries are Australia, Belgium, Canada, Denmark, France, Great Britain, Italy, Japan, Netherlands, Norway, Sweden, Switzerland and US. Maddison (1980) and (1991) documented this pattern for 16 OECD countries from 1870 to 1987. The 16 countries include the 13 countries in Kuznets (1966), Austria, Finland and Germany.

[7] GDP per capita in 1990 international dollar are from Maddison (2001). Agriculture includes agriculture, forestry, and fishing; industry includes mining, manufacturing, electricity, gas and water supply, and construction. Services is a residual which includes government. The 16 OECD countries are the same as in Maddison (1980). The figure includes data for all countries in 1870, 1913, 1950, 1960, 1973, 1987 and 2001 with two exceptions: (1) only agriculture shares in Denmark, Japan and Switzerland for 1870, and (2)1913 only has France, Germany, Netherlands, Germany, UK and US.

[8] The employment shares are caculated using data from Historical Statistics for 1869-1959 and ISDB for 1960-1996.

model captures the general features of the data, it fails to capture the full extent of the decline of agriculture. Figure 5, panel (b) allows for a lower elasticity of substitution, $\varepsilon = 0.1$, which improves the prediction for agriculture.[9] However, the model still predicts too high an employment share for agriculture, above 10 percent for 1990 to 1996, while it was smaller than 5 percent in the data. This suggests productivity growth alone is not sufficient to account for the decline in agriculture, but the model predicts well the allocations of non-agricultural employment between manufacturing and services. In the case of $\varepsilon = 0.3$ we overpredict agricultural employment by 16 percentage points and underpredict services and manufacturing employment by 14 and 2 points respectively. If we were to redistribute the 16 point surplus share from agriculture to manufacturing and services according to their existing share proportions, we obtain a share of manufacturing of 29 percent and a share of services of 68 percent, which compare favorably with their actual shares of 27 and 70 percent respectively.

A reason for the failure to match the decline of agriculture may be the unit income elasticity that we assumed. There seems to be a consensus in the literature that the income elasticity of demand for agricultural products is below unity, so a more appropriate utility function for agricultural goods may be one that includes a subsistence level, e.g., one that takes the form $v(c_a - \bar{c}_a, c_m, c_s)$, with $\bar{c}_a > 0$. A constant $\bar{c}_a$ would contribute to the fast decline of agricultural employment in the first stages of development, when most of consumption is accounted for by subsistence.[10]

5 Conclusion

Economic growth takes place at uneven rates across different sectors of the economy. This paper had two objectives related to this basic fact, (a) to show that even taking into account the different sectoral rates of productivity growth there can still be balanced growth in the aggregate economy, and (b) to derive the implications of uneven

[9] Note that this value is within one standard error of our estimates in Table 2.

[10] However, a subsistence level alone is still not capable of explaining the fast decline in agricultural employment. if the elasticity of substitution between the three goods is unity. It appears that less than unit elasticity of substitution is also needed, as in the models of Laitner (2000) and Gollin et al. (2002) where the elasticity of substitution is effectively 0 after a subsistence level of agricultural consumption has been satisfied. In contrast Caselli and Coleman (2002), assume a unit elasticity of substitution but match the fast decline of agricultural employment by assuming that the cost of moving out of agriculture fell because of the increase of education in rural areas.

sectoral growth for structural change, the shifts in sectoral employment shares that take place over long periods of time. We have shown that balanced growth requires some quantitative restrictions on parameters, the most important being a logarithmic intertemporal utility function. Predicted sectoral change that is consistent with the facts requires in addition low substitutability between the final goods produced by each sector. We have shown that underlying the balanced aggregate growth there is a shift of employment away from sectors with high rate of technological progress towards sectors with low growth, and eventually, in the limit, only two sectors survive, the sector producing capital goods and the sector with the lowest rate of productivity growth.

An examination of the facts for the United States and the United Kingdom has shown that our predictions are consistent with the facts, and that focusing on uneven sectoral growth and abstracting from all other causes of structural change (such as different capital intensities and non-unit income elasticities) can explain a large fraction of the observed employment shifts. More specifically, it can explain large parts of the shift of employment from agriculture to manufacturing and services and subsequently from manufacturing to services, albeit at a lower rate than is observed in the data.

Our results were obtained under some restrictions, which future work could relax. Extensions worth pursuing are the marrying of our model with models that emphasize different rates of factor intensity and different income elasticities of demand as well as models with more than one capital-producing sector, for example, models that draw a distinction between capital equipment and information technology.

Finally, our model has implications for studies that take structural change as a fact and calculate its contribution to overall growth (Broadberry, 1998, Temple, 2001). For example, Broadberry and others calculate an economy's growth rate under the counterfactual of no structural change. They then attribute the difference between the actual growth rate and their hypothetical rate to structural change. Their approach has parallels with Baumol's approach (see also Triplett and Bosworth, 2003) who claim that the shift of weight in the aggregate economy to low growth service sectors should reduce the overall growth rate of the economy. Our model shows that structural change is a necessary part of aggregate growth and may shed new light on how to design accounting exercises of this kind. Temple uses growth accounting to calculate the contribution of structural change to overall growth, on the premise that labor moves from sectors which have low marginal product of labor to sectors that have high marginal product. He focuses on the shift out of agriculture and into manufacturing and services. But our analysis shows that labor moves because technological progress raises the marginal product of labor in the origin sectors and the prices of sectors in the receiving sectors. This reallocation mechanism, which is quite distinct from the one that he assumed, may shed new light into the kind of decomposition that he does. Ultimately, the objective of these exercises is to understand the causes of growth and our approach suggests that structural change is an outcome for growth, not a cause.

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6 Appendix

Proof of Proposition 1. The iso-elastic utility function implies a simple expression for marginal utility:

$$v_i = \phi^{-\theta} \omega_i \left(\frac{\phi}{c_i} \right)^{1/\varepsilon} \qquad i = 1, .., n, m \tag{38}$$

which simplifies (6) to

$$\frac{c_i}{c_m} = \frac{x_i A_i}{A_m} = \frac{x_i}{p_i} \qquad i = 1, ...n, \tag{39}$$

and $c = Xc_m$.Using (4) and (9), rewrite (2) as:

$$\frac{\dot{k}}{k} = A_m k^{\alpha-1}(1 - \sum_{i=1}^{n} n_i) - \frac{c_m}{k} - (\delta + \nu). \tag{40}$$

But (39) and (8) imply

$$c_m x_i = A_m k^{\alpha} n_i, \qquad i = 1, ...n.$$

Thus, (40) becomes:

$$\frac{\dot{k}}{k} = A_m k^{\alpha-1} - \frac{c}{k} - (\delta + \nu),$$

Finally, (38) implies $v_m = \frac{\omega_m \phi^{1-\theta}}{c}$, and from (39), $\phi = \omega_m^{\varepsilon/(\varepsilon-1)} X^{1/(\varepsilon-1)} c$. Therefore, (7) becomes

$$\theta \frac{\dot{c}}{c} = (\theta - 1)(\gamma_m - \bar{\gamma}_t) + \alpha A_m k^{\alpha-1} - (\delta + \rho + \nu)$$

Matching the model with the data The real output per capita (using t_0 as the fixed base year) is

$$y_t^R = P_{mt_0} \left(\sum_{i=1,..,n,m} p_{it_0} y_{it} \right)$$

where the upper-case P_i is the actual price level of sector i, and the lower-case p_i is the relative price of sector i in terms of manufacturing goods, i.e. $p_{mt_0} = 1$. We can rewrite it as

$$y_t^R = P_{mt_0} y_t Q_t$$

where $y_t \equiv y_{mt} + \sum_{i=1}^{n} p_{it} y_{it}$ is the measure of aggregate output per capita that we used in the model and $Q_t \equiv \sum_{i=1,..n,m} \frac{p_{it_0} y_{it}}{y_t}$. The model implies y_t is growing at the rate $\frac{\gamma_m}{1-\alpha}$, thus

$$\frac{\dot{y}_t^R}{y_t^R} = \frac{\gamma_m}{1-\alpha} + \frac{\dot{Q}_t}{Q_t}.$$

Note that $c_{it} = y_{it}$, $\forall i \neq m$, and $c_t / y_t = 1 - \hat{\sigma}$, we can rewrite Q_t as

$$Q_t = \hat{\sigma} + (1 - \hat{\sigma}) q_t$$

where

$$q_t \equiv \sum_{i=1,..,n,m} \left(\frac{p_{it_0}}{p_{it}} \right) \frac{p_{it} c_{it}}{c_t}$$

is the weighted average of the ratio between relative price (price in terms of manufacturing goods) in date 0 and in date t, where the corresponding expenditure share is used as the weight. Our model implies expenditure goes to one for the sector with the fastest growth in relative price, therefore, q_t will eventually fall and converge to zero.

To see this formally, use the results that relative prices $p_i = A_m/A_i$, and expenditure share i equal to x_i/X, which grows at rate $(1-\varepsilon)(\bar{\gamma}-\gamma_i)$,

$$q_t = \sum_{i=1,,,n,m} \left(\frac{x_{it_0}}{X_{t_0}}\right) \exp\left\{\left(\varepsilon\gamma_i + (1-\varepsilon)\bar{\gamma}_t - \gamma_m\right)(t-t_0)\right\}$$

Given $\bar{\gamma}_t$ is decreasing over time, then $q_t \to 0$ if $\min\{\gamma_i\}_{i=1,..n} < \gamma_m < \max\{\gamma_i\}_{i=1,..n}$. Therefore, Q_t converges to constant $\hat{\sigma}$, and the growth rate of y_t^R converges to $\gamma_m/(1-\alpha)$.

For any finite t, to compute the growth rate y_t^R, we have to compute $\dot{Q}$, which is:

$$\dot{Q}_t = n_l^* \sum_{i=1,2,..,n,m} \left(\varepsilon\gamma_i + (1-\varepsilon)\bar{\gamma}_t - \gamma_m\right)\left(\frac{x_{it_0}}{X_{t_0}}\right) e^{(\varepsilon\gamma_i+(1-\varepsilon)\bar{\gamma}_t-\gamma_m)(t-t_0)}.$$

In general, Q_t is not a constant. However, for reasonable (γ_i, γ_m), the growth rate of Q_t is very small.

Lemma 5 *The employment shares along the balanced growth path are*

$$\begin{aligned} n_i &= \left(\frac{x_i}{X}\right)\hat{c}_e\hat{k}_e^{-\alpha} \qquad i = 1,..n, \\ n_m &= \left(\frac{1}{X}\right)\hat{c}_e\hat{k}_e^{-\alpha} + \left(1-\hat{c}_e\hat{k}_e^{-\alpha}\right); \end{aligned}$$

and the dynamics of the employment share are:

$$\dot{n}_i = (1-\varepsilon)(\bar{\gamma}-\gamma_i)\,\hat{c}_e\hat{k}_e^{-\alpha}\frac{x_i}{X}; \qquad i = 1,...n,m$$

Proof. The employment share in sector $i = 1,...,n$, follows from the substitution of the production function for c_i into (39), and the employment share in sector m is derived from the labor resource constraint (4), where the steady state $\left(\hat{c}_e, \hat{k}_e\right)$ solves $\frac{\dot{c}_e}{c_e} = 0$ and $\frac{\dot{k}_e}{k_e} = 0$ and $c_m = c/X$.

Given $x_i \equiv \left(\frac{\omega_i}{\omega_m}\right)^{\varepsilon}\left(\frac{A_m}{A_i}\right)^{1-\varepsilon}$ and $X \equiv \sum_{i=1,..,n,m} x_i$, we have:

$$\frac{\dot{x}_i}{x_i} = (1-\varepsilon)(\gamma_m - \gamma_i);$$

$$\dot{X} = \sum_{i=1,\dots,n,m} x_i \frac{\dot{x}_i}{x_i} = (1-\varepsilon)(\gamma_m - \bar{\gamma}) X.$$

The growth rate of the employment share in consumption sector i is:

$$\frac{\dot{n}_i}{n_i} = \frac{\dot{x}_i}{x_i} - \frac{\dot{X}}{X} = (1-\varepsilon)(\bar{\gamma} - \gamma_i) \qquad i = 1,..n$$

which implies

$$\dot{n}_i = (1-\varepsilon)(\bar{\gamma} - \gamma_i) \frac{x_i}{X} \hat{c}_e \hat{k}_e^{-\alpha} \qquad i = 1,...n$$

By the labor resource constraint, $\sum_{i=1,..n,m} n_i = 1$,

$$\begin{aligned} \dot{n}_m &= -\sum_{i=1}^{n} \dot{n}_i \\ &= -(1-\varepsilon) \frac{\hat{c}_e \hat{k}_e^{-\alpha}}{X} \sum_{i=1}^{n} x_i (\bar{\gamma} - \gamma_i) \\ &= (1-\varepsilon)(\bar{\gamma} - \gamma_m) \frac{x_m}{X} \hat{c}_e \hat{k}_e^{-\alpha} \end{aligned}$$

The result follows. ■

Transitional dynamics of relative employment shares Differentiating the consumption employment shares obtained in Lemma 5 we obtain

$$\frac{\dot{n}_i}{n_i} = \frac{\dot{c}_e}{c_e} - \alpha \frac{\dot{k}_e}{k_e} + \frac{\dot{\overline{x_i/X}}}{x_i/X}; \qquad i = 1,..,n, \forall t,$$

where $\frac{\dot{\overline{x_i/X}}}{x_i/X}$ is the growth rate of the employment shares along the balanced growth path. The first two terms are independent of i, so the growth rate of n_i relative to n_j along the transition path is the same as the relative growth rate along the balanced growth path.

Lemma 6 *The weighted average of TFP growth rates $\bar{\gamma}$ is monotonic, and*

$$\frac{d\bar{\gamma}}{dt} \leq 0 \Leftrightarrow \varepsilon \leq 1$$

Proof. Totally differentiating $\bar{\gamma}$ as defined in Proposition 1 we obtain

$$\frac{d\bar{\gamma}}{dt} = \sum_{i=1,\dots,n,m} \frac{x_i \gamma_i}{X} \left(\frac{\dot{x}_i}{x_i} - \frac{\sum \dot{x}_j}{X} \right)$$

and from (12)

$$\begin{aligned}\frac{d\bar{\gamma}}{dt} &= (1-\varepsilon)\sum_{i=1,\ldots,n,m}\frac{x_i\gamma_i}{X}\left(\gamma_m-\gamma_i-\sum\frac{x_j}{X}(\gamma_m-\gamma_j)\right)\\ &= (1-\varepsilon)\left(\bar{\gamma}^2-\sum_{i=1,\ldots,n,m}\frac{x_i}{X}\gamma_i^2\right)\\ &= -(1-\varepsilon)\sum_{i=1,\ldots,n,m}\frac{x_i}{X}(\gamma_i-\bar{\gamma})^2\end{aligned}$$

Since the summation term is always positive the result follows. ■

Proof of Proposition 4

Lemma 7 *If $\varepsilon \lesseqgtr 1$, the employment share n_i is non-monotonic if and only if $\bar{\gamma}_0 \gtreqless \gamma_i$. The non-monotonic employment share first increases at a decreasing rate for $t < t_i$, reaches a maximum at t_i, then decreases and converge to constant n_i^* asymptotically, where t_i is such that $\bar{\gamma}_{t_i} = \gamma_i$, $i = 1,..n,m$. The monotonic employment share is decreasing and converges to zero asymptotically.*

Proof. Using Lemma 5,

$$\dot{n}_i = (1-\varepsilon)(\bar{\gamma}-\gamma_i)\,\hat{c}_e\hat{k}_e^{-\alpha}\frac{x_i}{X} \qquad i = 1,\ldots n,m$$

thus, n_i is increasing if and only if $\bar{\gamma}_t > \gamma_i$. Using Lemma 6, $\bar{\gamma}_t$ is decreasing over time, so n_i eventually decreases. Therefore, n_i is non-monotonic if and only if $\bar{\gamma}_0 > \gamma_i$. ■

Corollary 8 *If $\varepsilon < 1$, $t_s \to \infty$ where s is such that $\gamma_s = \min\{\gamma_i\}_{i=1,..n,m}$. If $\varepsilon > 1$, $t_f \to \infty$ where f is such that $\gamma_f = \max\{\gamma_i\}_{i=1,..,n,m.}$*

To establish now the claims in Proposition 4, assume, without loss of generality, $\varepsilon < 1$, $\gamma_1 > .. > \gamma_n$and $\gamma_m < \gamma_h = \bar{\gamma}_0$ where $1 < h < n$. Then, Lemma 7 implies $t_i = 0$, $\forall i \leq h$, and E_0 contains all the sectors $i \geq h$, moreover,

$$E_{t_{h+1}} \cup \{h+1\} = E_0;\ \text{and}\ D_{t_h+1} = D_0 \cup \{h+1\}$$

Thus,

$$E_{t_{h+1}} \subseteq E_0 \text{ and } D_0 \subseteq D_{t_{h+1}}$$

The result follows for any arbitrary $t > 0$.

Next we prove that the economy converges to a two-sector economy. Without loss of generality, consider the case of $\varepsilon < 1$. Given

$$\frac{X}{x_i} = \sum_{j=1,..n,m} \left(\frac{\omega_j}{\omega_i}\right)^{\varepsilon} \left(\frac{A_i}{A_j}\right)^{1-\varepsilon}$$

and

$$\frac{A_i}{A_j} \to 0 \Leftrightarrow \gamma_i < \gamma_j,$$

we obtain,

$$\frac{X}{x_i} \to 1 \Leftrightarrow \gamma_i = \min \left\{\gamma_j\right\}_{j=1,..n,m}$$

Therefore, the asymptotic distribution of employment shares in the economy is

$$n_l^* = \hat{c}_e \hat{k}_e^{-\alpha}; \qquad n_m^* = 1 - n_l^*.$$

where sector l is the sector with the smallest TFP growth rate.

Next, we demonstrate that these results hold also in the transition to the balanced growth path. Along the transition, (18) and (19) imply that aggregate consumption and capital are increasing at a decreasing rate. The dynamics of consumption to capital can also be derived. Let $z \equiv \frac{c_e}{k_e}$, (18) and (19) imply

$$\frac{\dot{z}}{z} = (\alpha - 1)\, k_e^{\alpha-1} + z - \rho$$

together with (19),

$$\frac{\dot{k}_e}{k_e} = k_e^{\alpha-1} - z - \left(\frac{\gamma_m}{1-\alpha} + \delta + \nu\right),$$

A phase diagram can be drawn in (k_e, z) space, with z decreasing along the transition.

Given z is decreasing, the growth rate of employment shares is decreasing along the transition. This implies that if sector $i \in E_0$, then i must be increasing at a decreasing rate along the transition. Therefore, Lemma 7 also holds in the transition. In other words, given any arbitrary initial condition, if sector i is a declining sector to begin with, then it is always a declining sector. If sector i is an expanding sector (but it is not sector l) to begin with, then it must be first increasing up to t_i, then it asymptotically converges to zero. Finally, sector l is always increasing at a decreasing rate.

Calibration The parameters are the preference parameters $(\omega_a, \omega_m, \omega_s, \rho)$, the technology parameters $(\gamma_m, \gamma_s, \gamma_a, A_{a0}, A_{m0}, A_{s0,}\alpha, \delta)$ and the labor force growth rate ν. Given $\gamma_m/(1-\alpha)$, the roles of the parameters (δ, ν, ρ) are summarized through $\hat{\sigma}$ while the roles of $(\omega_a, \omega_m, \omega_s)$ and (A_{a0}, A_{m0}, A_{s0}) are summarized through the initial weights (x_{a0}, x_{s0}) which are set to match the employment shares in 1870 using (37). Therefore, there are only 5 parameters, $(\hat{\sigma}, \gamma_m, \gamma_s, \gamma_a, \alpha)$ to calibrate to the US economy.

$(\hat{\sigma})$: We set $\hat{\sigma}$ to 0.2 which is the average investment rate during 1950-2000 from the Penn World Tables. The investment rate fluctuated within a band of 16.5 to 24.3% during this period.

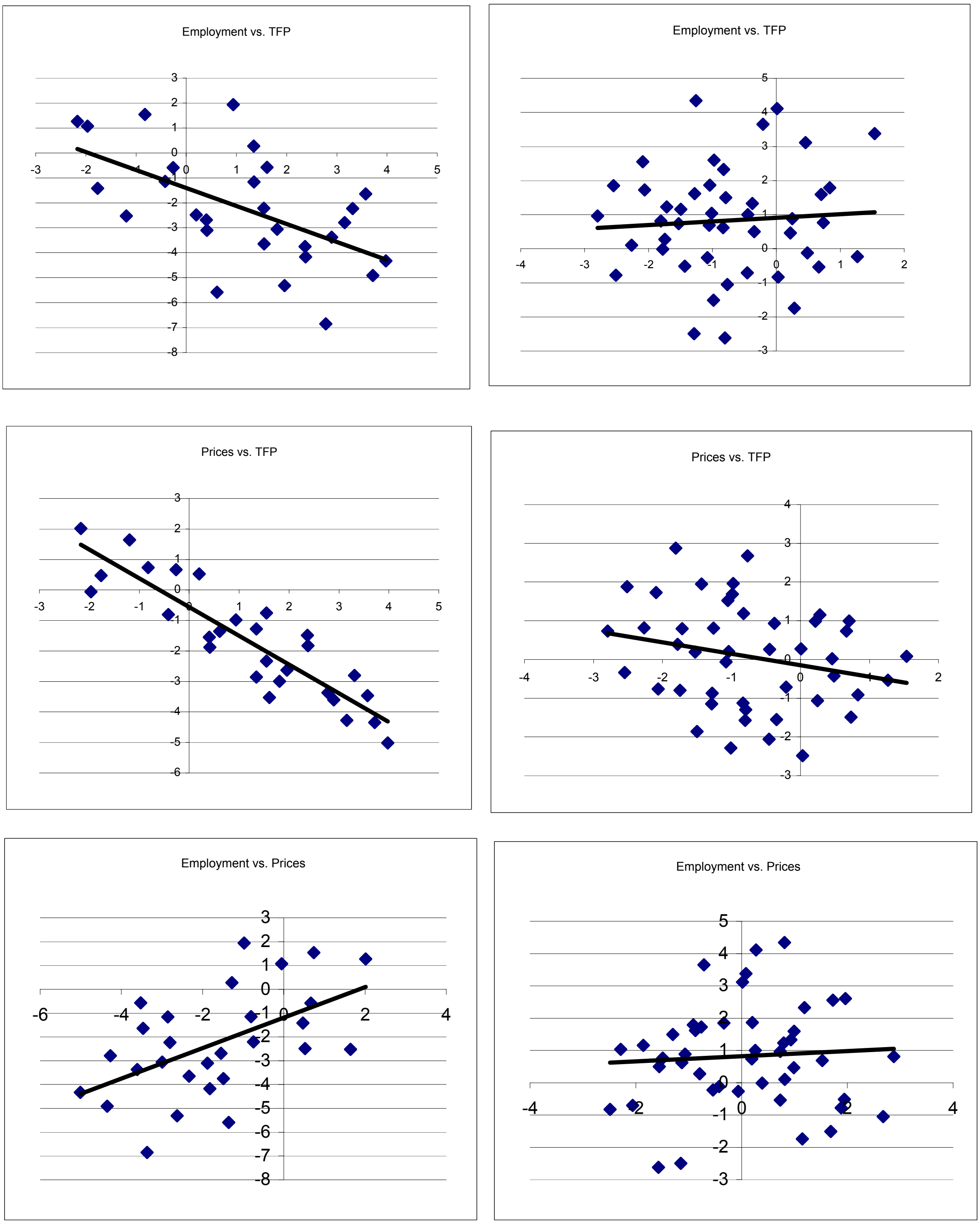
To determine $(\gamma_a, \gamma_m, \gamma_s)$, we use data from two main sources:

- *Historical Statistic of the United States: Colonial Times to 1970*, Part 1 and 2: for the sectoral employment (series F250-258), relative prices (series E17, E23-25, E42, E52-E53) and index of manufacturing production (series P13-17).
- *Economic Report of the President*: for the relative prices and index of manufacturing production.

(γ_m, α) : The model implies that the aggregate growth rate is the same as the growth rate of labor productivity in manufacturing, and both are equal to $\gamma_m/(1-\alpha)$. The average annual growth rate of labor productivity in manufacturing is 2.2 percent between 1869 and 1998, which is consistent with the finding for the aggregate growth rate. The role of α in the quantitative analysis is through its influence on the implied γ_m, which is between 0.013 ($\alpha = 0.4$) and 0.014 ($\alpha = 1/3$). The results are robust to this range. We only report results with $\gamma_m = 0.013$.

(γ_a, γ_s) : The model implies the growth rate of relative price p_i is equal to $\gamma_m - \gamma_i$. The price data for agriculture and manufacturing start from 1869. However, the price data for services start in 1929. The average annual growth rate for the relative price of services in terms of manufacturing is -0.01 for the period 1929-1998, which implies $\gamma_s = 0.003$. The average annual growth rate of agriculture relative to manufacturing price for 1869-1998 is -0.003, which implies $\gamma_a = 0.016$. However, if we use the same period as for the service sector, i.e. for the period 1929-1998, the annual growth rate becomes -0.01, which implies $\gamma_a = 0.023$. What is important for the shift between the agriculture and service sector is the difference $\gamma_a - \gamma_s$, so we use the same period for both prices series, $\gamma_a = 0.023$ and $\gamma_s = 0.003$.

(ε) : Ideally, we would like to have an estimate for the elasticity of substitution for the period 1869-1998. Without this measure, we use $\varepsilon = 0.3$ as baseline. This is the value implied by our results in Table 2 for the period 1970-1993. But we also report results for a lower value of $\varepsilon = 0.1$.

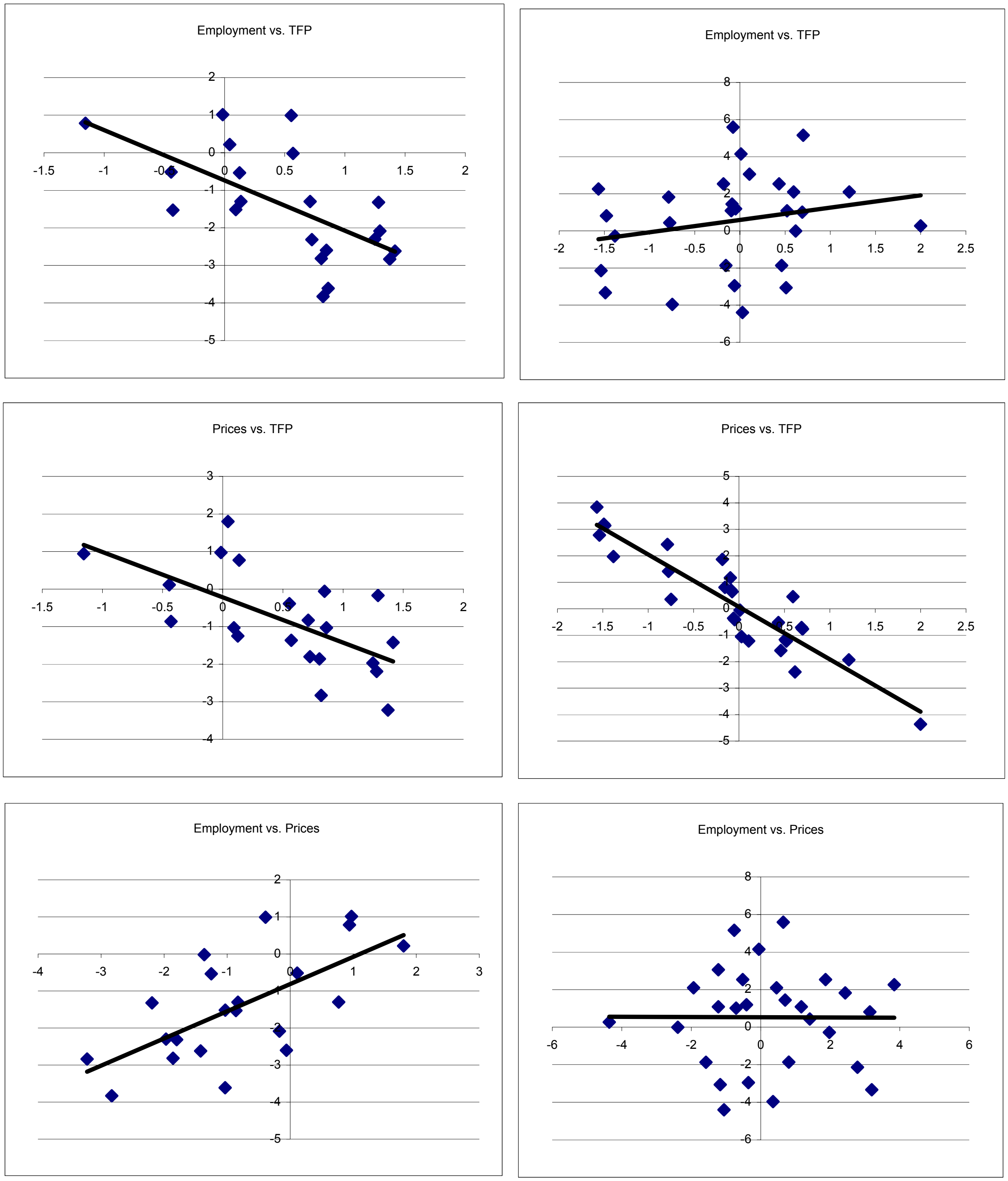


(a) "Consumption" sectors

(b) "Manufacturing" sectors

Figure 1

Changes in relative employment shares, relative TFP, and relative prices.
(percent, United States, averages for 1970-1993)



(a) "Consumption" sectors

(b) "Manufacturing sectors

Figure 2

Changes in relative employment shares, relative TFP, and relative prices
(percent, United Kingdom, averages for 1970-1990)

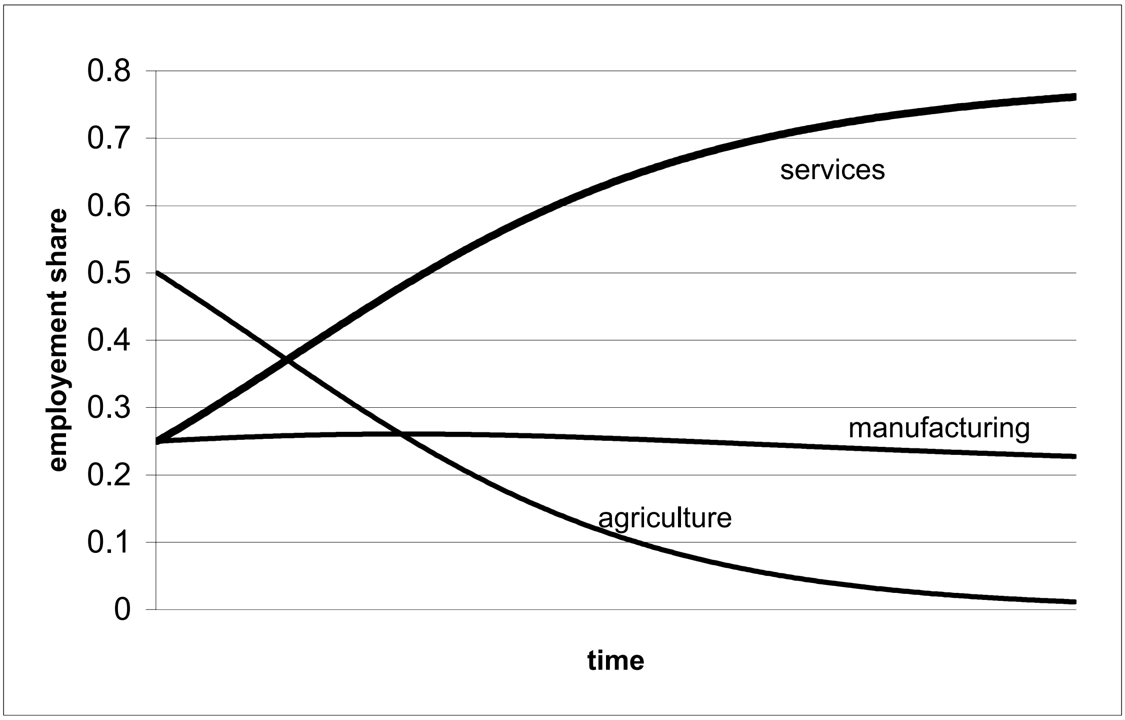
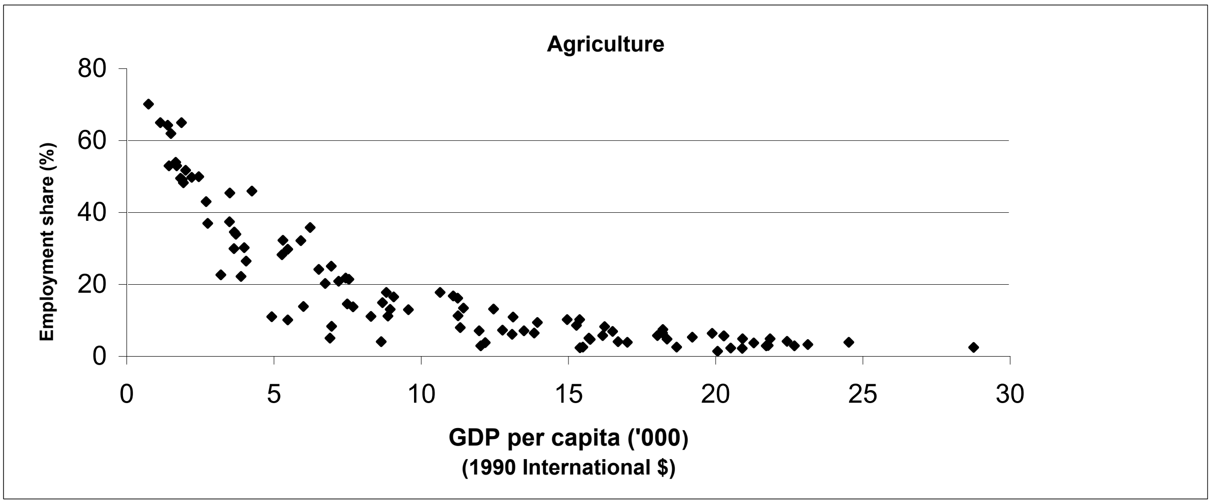
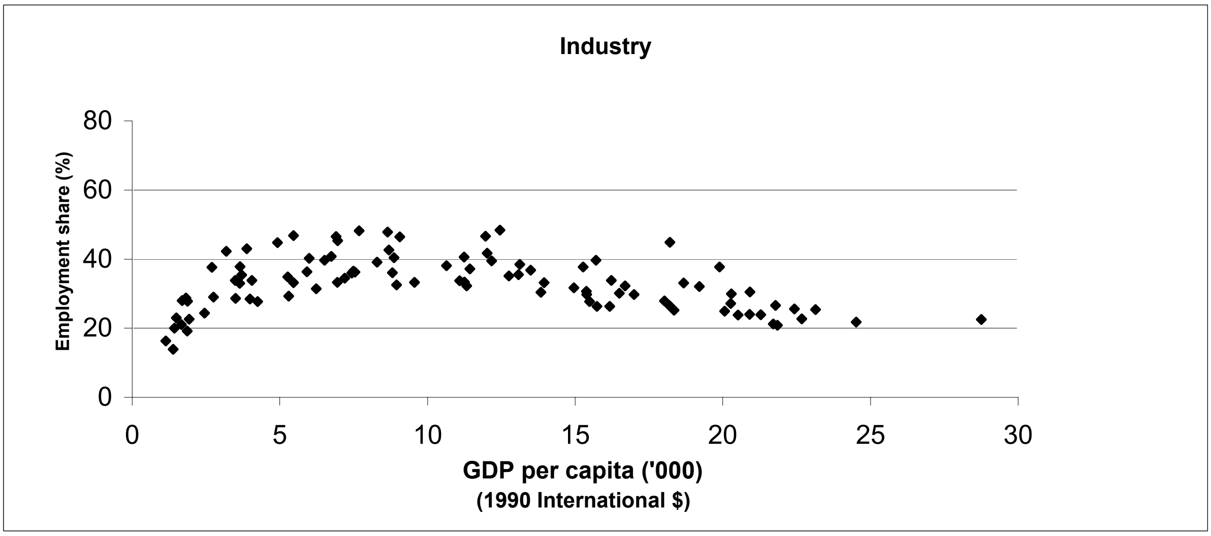


Figure 3

Structural transformation in a three-sector economy





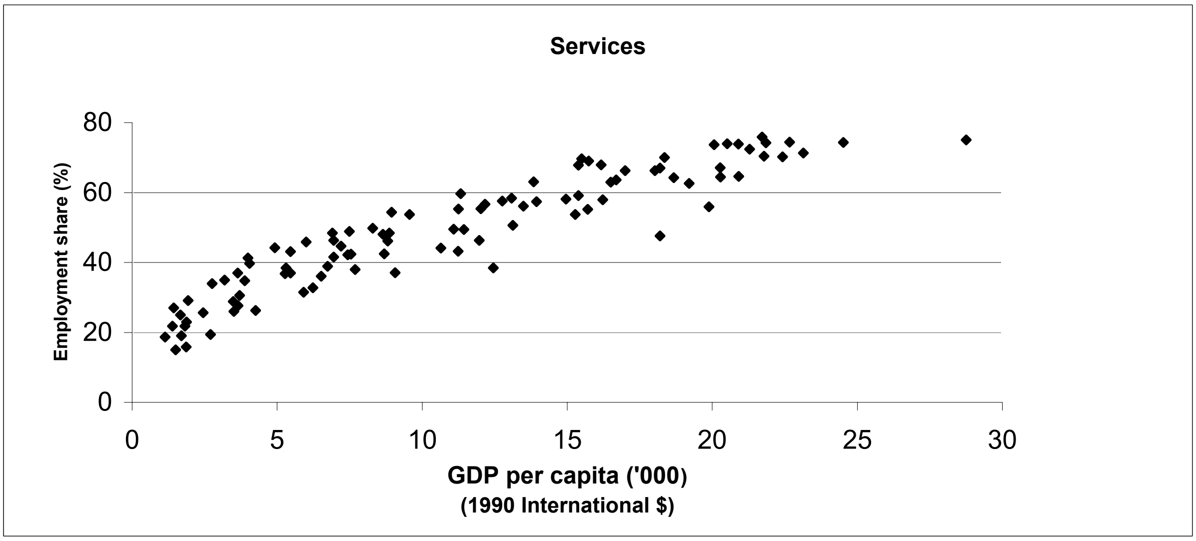
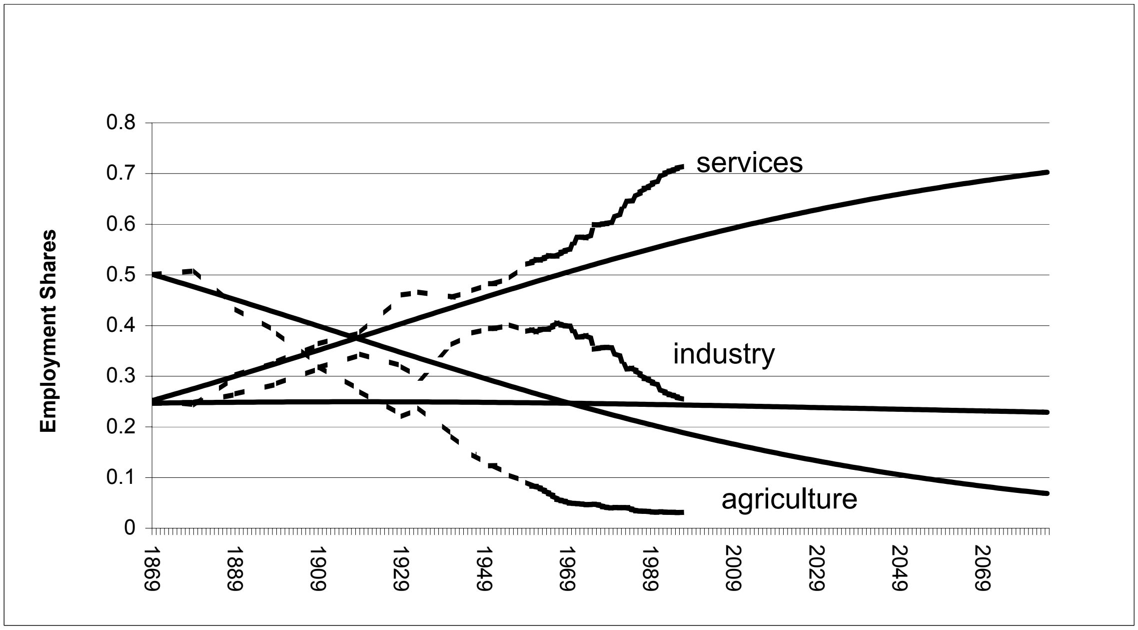
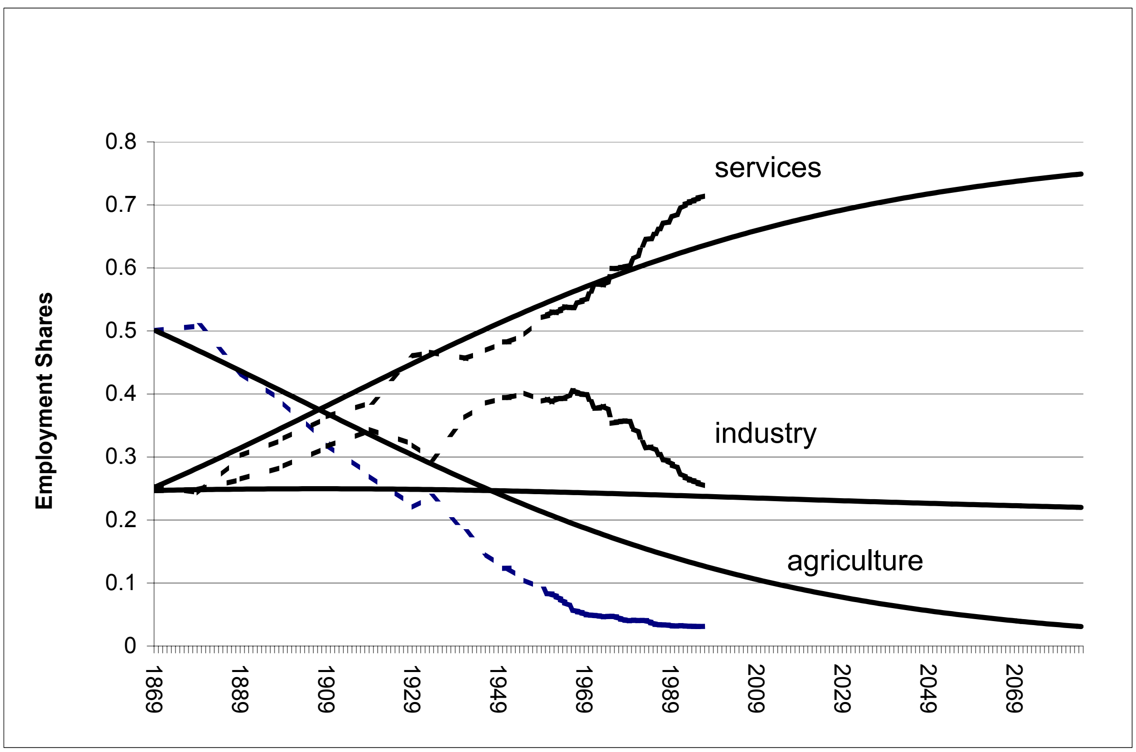


Figure 4

Sectoral Employment Shares 1870-2001
(Sixteen OECD countries and seven years)



(a) epsilon = 0.3



(b) epsilon = 0.1

Figure 5

Structural transformation in the US economy

___________ model calibration …………. data