Search and endogenous concentration of liquidity in asset markets

Dimitri Vayanos and Tan Wang

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Abstract

We develop a search-based model of asset trading, in which investors of different horizons can invest in two identical assets. The asset markets are partially segmented: buyers can search for only one asset, but can decide which one. We show that there exists a "clientele" equilibrium where one market has more buyers and sellers, lower search times, higher trading volume, higher prices, and short-horizon investors. This equilibrium dominates the ones where the two markets are identical, implying that the concentration of liquidity in one asset is socially desirable. At the same time, too many buyers decide to search for the liquid asset.

Keywords: Liquidity, Search, Asset pricing

JEL Classification: G1, D8

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1 Introduction

Financial assets differ in their liquidity, defined as the ease of trading them. For example, government bonds are more liquid than stocks or corporate bonds. A large body of research has attempted to measure liquidity and relate it to asset-price differentials. An important and complementary question is why liquidity differs across assets.

A leading theory of liquidity is based on asymmetric information. For example, Glosten and Milgrom (1985) and Kyle (1985) show that market makers can widen their bid-ask spread to compensate for the risk of trading against informed agents. This increases trading costs for all agents, including the uninformed. In many cases, however, asymmetric information cannot be the explanation for liquidity differences. For example, AAA-rated bonds of US corporations are essentially default-free, but are significantly less liquid than Treasury bonds. Since both sets of bonds have essentially riskless cash flows, their value should depend only on interest rates. But information about the latter is generally symmetric, and in any event, possible asymmetries should be common across bonds. An even starker example comes from within the Treasury market: just-issued ("on-the-run") bonds are significantly more liquid than previously issued ("off-the-run") bonds maturing on nearby dates.[1]

In this paper we explore an alternative theory of liquidity based on the notion that asset trading can involve search, i.e., locating counterparties takes time. Search is a fundamental feature of over-the-counter markets, where trade is conducted through bilateral negotiations rather than a Walrasian auction.[2] We show that liquidity, measured by search costs, can differ across otherwise identical assets, and this translates into equilibrium price differentials. We also perform a welfare analysis of the allocation of liquidity, showing that while traders can excessively concentrate in liquid assets, this dominates an equal split across all assets.

We assume that a constant flow of investors enter into a market, seeking to buy one of two identical, infinitely-lived assets. After buying an asset, investors become "inactive" owners, until the time they seek to sell. That event occurs when the investors' valuation of asset

[1]Evidence on the default risk of corporate bonds is in Moody's (2000), on the trading costs of corporate bonds is in Chen, Lesmond, and Wei (2005), on the trading costs of government bonds is in Dupont and Sack (1999), and on the on-the-run phenomenon is in Warga (1992) and Fleming (2002).

[2]Examples of over-the-counter markets are for government, corporate, and municipal bonds, and for many derivatives. We elaborate on the role of search in those markets in Section 2. See also the discussion in Duffie, Garleanu, and Pedersen (2004ab).

payoffs switches to a lower level. The switching rate is inversely related to investors' horizons, and we assume that horizons are heterogeneous across investors. To model search, we adopt the standard framework (e.g., Diamond (1982)) where investors are matched randomly over time in pairs. We also assume that markets are partially segmented in that buyers must decide which of the two assets to search for, and then search only in that asset's market.[3]

We show that there exists an asymmetric equilibrium, where assets differ in their liquidity despite having identical payoffs. The market of the more liquid asset has more buyers and sellers. This results in short search times, i.e., high liquidity, and high trading volume. Moreover, prices are higher in that market, reflecting the premium that buyers are willing to pay for the short search times. The tradeoff between prices and search times gives rise to a clientele effect: buyers with high switching rates, who have a stronger preference for short search times, prefer the liquid asset, while the opposite holds for the more patient, low-switching-rate buyers. The clientele effect is, in turn, what generates the higher trading volume in the liquid asset: high-switching-rate buyers turn faster into sellers, thus generating more turnover.

In addition to the asymmetric ("clientele") equilibrium, there exist symmetric ones, where the two markets are identical in terms of prices and buyers' search times. Comparing the two types of equilibria reveals, in the context of our model, whether the concentration of liquidity in one asset is socially desirable. As a benchmark for this comparison, we determine the socially optimal allocation of entering buyers across the two markets. Under this allocation, the measure of sellers differs across markets, and so do the buyers' search times (which are decreasing in the measure of sellers). Such a dispersion is optimal so that markets can cater to different clienteles: buyers with high switching rates go to the market with the short search times, while the opposite holds for low-switching-rate buyers.

In the symmetric equilibria the buyers' search times are identical across markets, while in the clientele equilibrium some dispersion exists. A sufficient condition for the clientele equilibrium to dominate the symmetric ones is that this dispersion does not exceed the socially optimal level. To examine whether this is the case, we consider the social optimality

[3]The search framework is a stylized representation of over-the-counter markets because it abstracts away from institutional features such as the role of dealers and brokers. We return to dealers/brokers, and to the market-segmentation assumption in Section 2.

of buyers' entry decisions in the clientele equilibrium. We show that despite the higher prices, buyers do not fully internalize the relatively short supply of sellers in the liquid market, and enter excessively in that market. This pushes the measure of sellers in the liquid market below the socially optimal level, and has the same effect on the dispersion in buyers' search times. Thus, the clientele equilibrium dominates the symmetric ones.

This paper is related to Pagano (1989), who studies the concentration of liquidity across two markets. He shows that the markets can coexist, but the equilibrium is generally dominated by shutting one market and concentrating all trade in the other.[4] The main difference with Pagano is that we consider the concentration of liquidity across assets, rather than market venues. In the case of assets, there is no analogue to shutting one market because in equilibrium some investors must hold each asset. Thus, the concentration of liquidity in one asset can hurt the investors in the other, with a possibly ambiguous effect on total welfare.

Admati and Pfleiderer (1988) study the concentration of liquidity under asymmetric information. They show that if uninformed traders have discretion over the timing of their trades, they will all trade when the market is the most liquid. This reduces the informational content of order flow, feeding back into market liquidity. Chowdhry and Nanda (1991) show that uninformed traders can all choose to trade in one of multiple locations for similar reasons. As Pagano (1989), these papers concern the concentration of liquidity across market venues (defined by time or location) rather than assets.[5]

Search-theoretic approaches to liquidity have been explored in the monetary literature following Kiyotaki and Wright (1989) and Trejos and Wright (1995).[6] Aiyagari, Wallace, and Wright (1996) show the coexistence of currencies that differ in liquidity and price, and Wallace (2000) analyzes the relative liquidity of currency and dividend-paying assets. In our model there is no room for currency, and the focus is on the relative liquidity of dividend-paying assets.

Duffie, Garleanu, and Pedersen (2002) and (2004ab) integrate search in models of asset

[4]See also Ellison and Fudenberg (2003) for a general analysis of the coexistence of two markets, and Economides and Siow (1988) for a spatial model of market formation.

[5]See, however, Admati and Pfleiderer (1989) for an asymmetric-information model where two identical assets can differ in liquidity.

[6]See also Lippman and McCall (1986) who link liquidity to search in a partial equilibrium setting.

market equilibrium. This paper builds on their framework, extending it to multiple assets and heterogeneous investors. Independent work by Weill (2005) also considers multiple assets. Investors are homogeneous, however, and differences in liquidity arise because of exogenous differences in assets' issue sizes. Work subsequent to this paper by Vayanos and Weill (2005) shows that differences in liquidity can arise even with identical horizons and issue sizes, provided that there are short-sellers.

Finally, our welfare analysis is related to Diamond (1982). Diamond shows that search can drive a wedge between workers' wages and marginal products, and this can distort the choice between different labor markets. In our model a similar distortion applies to the choice between the markets of different assets.[7]

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 determines investor populations, expected utilities, and prices, taking the allocation of investors across markets as given. Section 4 endogenizes this allocation and determines the set of market equilibria. The welfare analysis is in Section 5. Section 6 considers the case where investment horizons are private information, and Section 7 concludes. All proofs are in the Appendix.

2 Model

Time is continuous and goes from 0 to ∞. There are two assets, 1 and 2, traded in markets 1 and 2, respectively. Both assets pay a constant flow δ of dividends and are in supply S.

Investors are risk-neutral and have a discount rate equal to r. Upon entering the economy, they seek to buy one unit of either asset 1 or 2. After buying the asset, they become "inactive" owners, until the time when they seek to sell. Thus, there are three groups of investors: buyers, inactive owners, and sellers. To model trading motives, we assume that upon entering the economy investors enjoy the full value δ of the dividend flow, but their valuation can switch to a lower level $\delta - x$ with Poisson rate κ. The parameter $x > 0$ can capture, in reduced form, the effect of a liquidity shock or a hedging need arising from a

[7]For search models where agents choose between sub-markets, see also Moen (1997), Mortensen and Pissarides (1998), Inderst and Mueller (2002), and Mortensen and Wright (2002).

position in another market. Buyers and inactive owners enjoy the full value δ of the dividend flow. Buyers experiencing a switch to low valuation simply exit the economy. Inactive owners experiencing the switch become sellers, and upon selling the asset, they also exit the economy.

There is a flow f of investors entering the economy. We assume that investors are heterogeneous in their horizons, i.e., some have a long horizon and some a shorter one. In our model, horizons are inversely related to the switching rates κ to low valuation. Thus, we can describe the investor heterogeneity by a function $\hat{f}(\kappa)$ such that the flow of investors with switching rates in $[\kappa, \kappa + d\kappa]$ is $\hat{f}(\kappa)d\kappa$. Denoting the support of $\hat{f}(\kappa)$ by $[\underline{\kappa}, \overline{\kappa}]$, we have $\int_{\underline{\kappa}}^{\overline{\kappa}} \hat{f}(\kappa)d\kappa = f$. To avoid technicalities, we assume that the function $\hat{f}(\kappa)$ is continuous and strictly positive.

The main feature of our model is that the market operates through search. Search is a fundamental feature of over-the-counter markets, such as those for government, corporate, and municipal bonds, and for many derivatives. Indeed, trades in these markets are negotiated bilaterally between dealers and their customers. And while a customer can easily contact a dealer, dealers often need to engage in search to rebalance their inventories. For example, after acquiring a large inventory from a customer, a dealer needs to unload the inventory to a new customer. This can involve search, and the dealers' ability to search efficiently, by knowing which customers are likely to be interested in a specific transaction, affects the prices they quote in the market.[8]

To model search, we adopt the standard framework (e.g., Diamond (1982)) where buyers and sellers are matched randomly over time in pairs. This framework is, of course, a stylized representation of over-the-counter markets because it abstracts away from the role of dealers. In some fundamental sense, however, dealers come into existence precisely because customers need to search for counterparties. The existence of dealers cannot eliminate the search cost, but only reduce it and express it in a different form, e.g. bid-ask spread. Thus, modelling over-the-counter markets in a "pure" search framework allows us to study the effects of the search friction in a more fundamental manner. Of course, incorporating dealers could be an

[8] According to Garbade (1982, pp.436-437): "Liquidity in the corporate bond market is not derived by knowing what is available and what is being sought in the form of active bids and offerings... Instead, it is derived by knowing what may be available from, or what may be sold to, public investors.... A corporate bond dealer will quote some bid price if a customer wants to sell an issue, but he is likely to quote a better price if he thinks he knows of the existence of another buyer to whom he can quickly resell the same issue."

interesting extension of our research.[9]

We assume that markets are partially segmented in that buyers must decide which of the two assets to search for, and then search only in that asset's market. This assumption is critical: if investors could execute a simultaneous search in both markets, the two assets would have the same set of prospective buyers, and the same liquidity and price. One interpretation of this assumption is that investors are mutual-fund managers who are constrained to hold specific types of assets. (For example, many government-bond funds are restricted from investing in corporate bonds.) Managers can, however, decide between asset types when the fund is incorporated. An alternative interpretation is that dealers/brokers specialize in different asset types. Market segmentation would then follow from the costs of employing multiple dealers. One such cost is complexity: an investor who wants to buy one unit of an asset through multiple dealers would have to give each dealer an order contingent on the other dealers' search outcomes.[10]

Summarizing, we can describe the two markets by the flow diagram in Figure 1. To each market are associated three groups of investors: buyers, inactive owners, and sellers. Investors entering the economy come from the pool of outside investors, and investors exiting the economy return to that pool.

To describe the search process, we need to specify the rate at which buyers meet with sellers. We assume that an investor seeking to trade meets investors from the overall population according to a Poisson process with a fixed arrival rate. Consequently, meetings with investors seeking the opposite side of the trade occur at a rate proportional to the measure of that investor group. Denoting the coefficient of proportionality by λ, and the measures of buyers and sellers in market i by μ_b^i and μ_s^i, respectively, a buyer in market i meets with sellers at the rate $\lambda\mu_s^i$, and a seller meets with buyers at the rate $\lambda\mu_b^i$. Moreover, the overall flow of meetings in market i is $\lambda\mu_b^i\mu_s^i$.

[9]It could also relate our approach to the inventory literature in market microstructure (e.g., Amihud and Mendelson (1980) and Ho and Stoll (1983)). That literature assumes that buyers and sellers arrive randomly in the market and can trade with dealers who face costs to holding inventory.

[10]The two interpretations are somewhat related: dealers could specialize to better serve the investors who are constrained to hold specific asset types.

We should add that our assumption does not preclude investors from searching in one market, and then switching and searching in the other. It rather restricts investors from searching simultaneously in both markets at a given point in time.

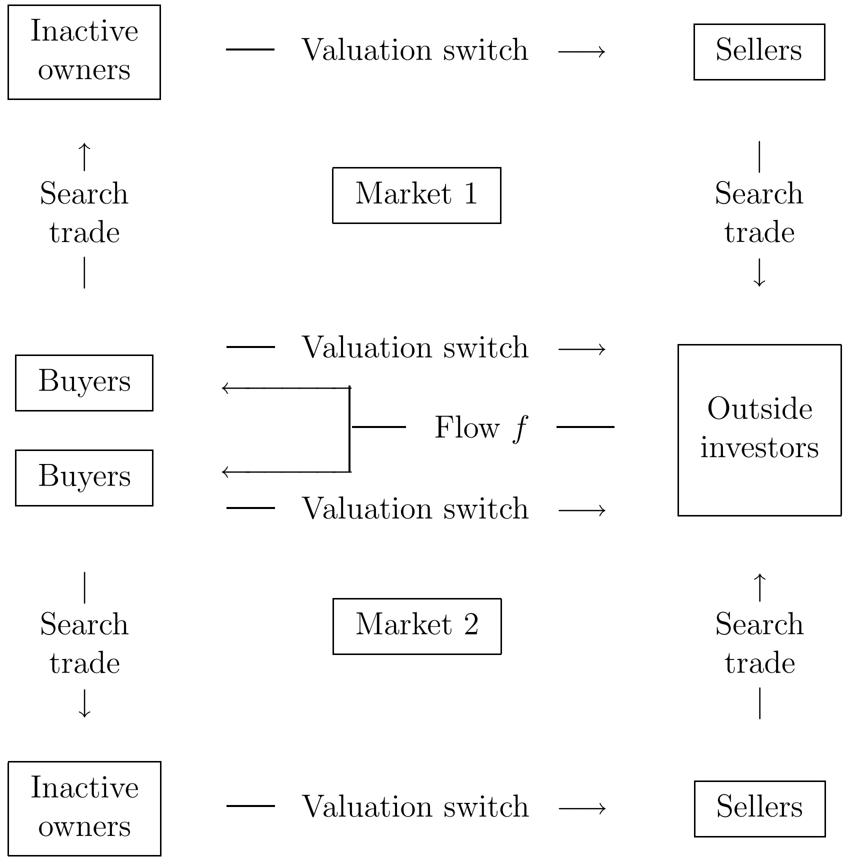

Figure 1: Flow Diagram for the Two Markets

The function $M(\mu_b^i, \mu_s^i) \equiv \lambda \mu_b^i \mu_s^i$ describes the search technology in our model. While the assumed form of M is partly motivated from tractability, it also embodies a notion of increasing returns to scale: doubling the measures of buyers and sellers more than doubles the flow of meetings. Increasing returns to scale seem realistic for financial market search because they imply that an increase in market size reduces search times of both buyers and sellers. This fits with the well-documented notion that trading costs are decreasing with trading volume.

When a buyer meets a seller, the price is determined through bilateral bargaining. We assume that the bargaining game takes a simple form, where one party is randomly selected to make a take-it-or-leave-it offer. The probability of the buyer being selected is $z/(1+z)$, where the parameter $z > 0$ measures the buyer's bargaining power.

Because buyers differ in their switching rates κ, they have different reservation values in the bargaining game, and this can introduce asymmetric information. For simplicity we mainly focus on the symmetric information case, where buyers' switching rates are observable to the sellers. For example, switching rates can correspond to buyers' observable institutional characteristics (e.g., insurance companies have a long horizon, while hedge funds a shorter one). We consider the asymmetric information in Section 6, and show that under plausible conditions our results carry through.

3 Analysis

In this section we take as given the investors' decisions as to which asset to search for, i.e., which market to enter. We then determine the measures of buyers, inactive owners, and sellers in each market, the expected utilities of the investors in each group, and the market prices. Throughout, we focus on steady states, where all of the above are constant over time.

3.1 Demographics

We denote by $\nu^i(\kappa)$ the fraction of investors with switching rate κ who decide to enter into market i. We also denote by μ_o^i the measure of inactive owners in market i, and recall that the measures of buyers and sellers are denoted by μ_b^i and μ_s^i, respectively.

Because buyers and inactive owners are heterogeneous in their switching rates κ, we need to consider the distribution of switching rates within each population. (This distribution is not the same as for the investors entering the market, because investors with different switching rates exit the market at different speeds.) To describe the distribution of switching rates within the population of buyers in market i, we introduce the function $\hat{\mu}_b^i(\kappa)$ such that the measure of buyers with switching rates in $[\kappa, \kappa + d\kappa]$ is $\hat{\mu}_b^i(\kappa)d\kappa$. We similarly describe the distribution of switching rates within the population of inactive owners in market i by the function $\hat{\mu}_o^i(\kappa)$. These functions satisfy the accounting identities

$$\int_{\underline{\kappa}}^{\overline{\kappa}} \hat{\mu}_b^i(\kappa)d\kappa = \mu_b^i \tag{1}$$

and

$$\int_{\underline{\kappa}}^{\overline{\kappa}} \hat{\mu}_o^i(\kappa) d\kappa = \mu_o^i. \tag{2}$$

To determine $\hat{\mu}_b^i(\kappa)$, we consider the flows in and out of the population of buyers with switching rates in $[\kappa, \kappa + d\kappa]$. The inflow is $\hat{f}(\kappa)\nu^i(\kappa)d\kappa$, coming from the outside investors. The outflow consists of those buyers whose valuation switches to low and who exit the economy $(\kappa\hat{\mu}_b^i(\kappa)d\kappa)$, and of those who meet with sellers and trade $(\lambda\hat{\mu}_b^i(\kappa)\mu_s^i d\kappa)$. (We are implicitly assuming that all buyer-seller matches result in a trade, a result we show in Proposition 1.) Since in steady state inflow equals outflow, it follows that

$$\hat{\mu}_b^i(\kappa) = \frac{\hat{f}(\kappa)\nu^i(\kappa)}{\kappa + \lambda\mu_s^i}. \tag{3}$$

To determine $\hat{\mu}_o^i(\kappa)$, we similarly consider the flows in and out of the population of inactive owners with switching rates in $[\kappa, \kappa + d\kappa]$. The inflow is $\lambda\hat{\mu}_b^i(\kappa)\mu_s^i d\kappa$, coming from the buyers who meet with sellers, and the outflow is $\kappa\hat{\mu}_o^i(\kappa)d\kappa$, coming from the inactive owners whose valuation switches to low and who become sellers. Writing that inflow equals outflow, and using equation (3), we find

$$\hat{\mu}_o^i(\kappa) = \frac{\lambda\mu_s^i\hat{f}(\kappa)\nu^i(\kappa)}{\kappa\left(\kappa + \lambda\mu_s^i\right)}. \tag{4}$$

Market equilibrium requires that the measure of asset owners in each market is equal to the asset supply. Since asset owners are either inactive owners or sellers, we have

$$\mu_o^i + \mu_s^i = S. \tag{5}$$

Combining equations (2), (4), and (5), we find

$$\int_{\underline{\kappa}}^{\overline{\kappa}} \frac{\lambda\mu_s^i\hat{f}(\kappa)\nu^i(\kappa)}{\kappa\left(\kappa + \lambda\mu_s^i\right)} d\kappa + \mu_s^i = S. \tag{6}$$

Equation (6) determines μ_s^i. Equations (1) and (3) then determine μ_b^i, and equations (2) and (4) determine μ_o^i.

3.2 Expected Utilities and Prices

We denote by $v_b^i(\kappa)$ and $v_o^i(\kappa)$, respectively, the expected utilities of a buyer and an inactive owner with switching rate κ in market i. We also denote by v_s^i the expected utility of a seller, and by $p^i(\kappa)$ the expected price when a buyer with switching rate κ meets a seller. (The actual price is stochastic, depending on which party makes the take-it-or-leave-it offer.)

To determine $v_b^i(\kappa)$, we note that in a small time interval $[t, t+dt]$, a buyer can either switch to low valuation and exit the economy (probability κdt, utility 0), or meet a seller and trade (probability $\lambda\mu_s^i dt$, utility $v_o^i(\kappa) - p^i(\kappa)$), or remain a buyer (utility $v_b^i(\kappa)$). The buyer's expected utility at time t is the expectation of the above utilities, discounted at the rate r:

$$v_b^i(\kappa) = (1 - rdt)\left[\kappa dt 0 + \lambda\mu_s^i dt(v_o^i(\kappa) - p^i(\kappa)) + (1 - \lambda\mu_s^i dt - \kappa dt)v_b^i(\kappa)\right]. \tag{7}$$

Rearranging, we find that $v_b^i(\kappa)$ is given by

$$rv_b^i(\kappa) = -\kappa v_b^i(\kappa) + \lambda\mu_s^i(v_o^i(\kappa) - p^i(\kappa) - v_b^i(\kappa)). \tag{8}$$

The term $rv_b^i(\kappa)$ can be interpreted as the flow utility of being a buyer. According to equation (8), this flow utility is equal to the expected flow cost of switching to low valuation and exiting the economy, plus the expected flow benefit of meeting a seller and trading.

Proceeding similarly, we find that $v_o^i(\kappa)$ and v_s^i are given by

$$rv_o^i(\kappa) = \delta + \kappa(v_s^i - v_o^i(\kappa)), \tag{9}$$

and

$$rv_s^i = \delta - x + \lambda\mu_b^i(E_b^i(p^i(\kappa)) - v_s^i), \tag{10}$$

respectively, where E_b^i denotes expectation under the probability distribution of κ in the population of buyers in market i. According to equation (9), the flow utility of being an inactive owner is equal to the dividend flow from owning the asset, plus the expected flow cost of switching to a low valuation and becoming a seller. Likewise, the flow utility of being a seller is equal to the seller's valuation of the dividend flow, plus the expected flow benefit of meeing a buyer and trading.

The price $p^i(\kappa)$ is the expectation of the buyer's and the seller's take-it-or-leave-it offers. The buyer is selected to make the offer with probability $z/(1+z)$, and offers the seller's revervation value, v_s^i. The seller is selected with probability $1/(1+z)$, and offers the buyer's reservation value, $v_o(\kappa) - v_b(\kappa)$. Therefore,

$$p^i(\kappa) = \frac{z}{1+z} v_s^i + \frac{1}{1+z}(v_o^i(\kappa) - v_b^i(\kappa)). \tag{11}$$

Proposition 1 *Equations (8)-(11) have a unique solution for $(v_b^i(\kappa), v_o^i(\kappa), v_s^i, p^i(\kappa))$. This solution satisfies, in particular, $v_o^i(\kappa) - v_b^i(\kappa) - v_s^i > 0$ for all κ.*

Since $v_o^i(\kappa) - v_b^i(\kappa) - v_s^i > 0$ for all κ, any buyer's reservation value exceeds a seller's. Thus, all buyer-seller matches result in a trade, a result that we have implicitly assumed so far. The intuition is simply that any buyer is a more efficient asset holder than a seller: the buyer values the dividend flow more highly than the seller, and upon switching to low valuation, faces the same rate of meeting new buyers as the seller.

4 Equilibrium

In this section, we endogenize investors' entry decisions, and determine the set of market equilibria. An investor will enter into the market where the expected utility of being a buyer is highest. Thus, the fraction $\nu^1(\kappa)$ of investors with switching rate κ who enter into market 1 is given by

$$\nu^1(\kappa) = 1 \quad \text{if} \quad v_b^1(\kappa) > v_b^2(\kappa) \tag{12}$$

$$0 \le \nu^1(\kappa) \le 1 \quad \text{if} \quad v_b^1(\kappa) = v_b^2(\kappa) \tag{13}$$

$$\nu^1(\kappa) = 0 \quad \text{if} \quad v_b^1(\kappa) < v_b^2(\kappa). \tag{14}$$

Definition 1 *A market equilibrium consists of fractions $\{\nu^i(\kappa)\}_{i=1,2}$ of investors entering in each market, measures $\{(\mu_b^i, \mu_o^i, \mu_s^i)\}_{i=1,2}$ of each group of investors, and expected utilities and prices $\{(v_b^i(\kappa), v_o^i(\kappa), v_s^i, p^i(\kappa))\}_{i=1,2}$, such that* *(a) μ_b^i, μ_o^i, and μ_s^i, are given by equations (1)-(4) and (6).*

(b) $v_b^i(\kappa)$, $v_o^i(\kappa)$, v_s^i, and $p^i(\kappa)$, are given by equations (8)-(11).

(c) $\nu^1(\kappa)$ is given by equations (12)-(14), and $\nu^2(\kappa) = 1 - \nu^1(\kappa)$.

To determine the set of market equilibria, we establish a sorting condition. We consider an investor who is indifferent between the two markets, i.e., a κ^* such that $v_b^1(\kappa^*) = v_b^2(\kappa^*)$, and examine which market the investors with different switching rates will prefer.

Lemma 1 *Suppose that $v_b^1(\kappa^*) = v_b^2(\kappa^*)$. Then, $v_b^1(\kappa) - v_b^2(\kappa)$ has the same sign as $(\mu_s^1 - \mu_s^2)(\kappa - \kappa^*)$.*

According to Lemma 1, the measure of sellers serves as a sorting device across the two markets. If, for example, market 1 has the most sellers, then investors with high switching rates will have a stronger preference for that market than investors with low switching rates. The intuition is that high-switching-rate investors have a stronger preference for short search times, and in a market with more sellers, buyers' search times are short.

Lemma 1 sharply restricts the set of possible equilibria. Equilibria can take one of two forms. First, one market can have more sellers than the other, in which case it attracts the investors with high switching rates. We refer to such equilibria as *clientele* equilibria, to emphasize that each market attracts a different clientele of investors. Second, both markets can have the same measure of sellers, in which case all investors are indifferent between the two markets. We refer to such equilibria as *symmetric* equilibria, to emphasize that markets are symmetric from the viewpoint of all investors.

4.1 Clientele Equilibria

We focus on the case where market 1 is the one with the most sellers. This is without loss of generality as any equilibrium obtained in this case has a symmetric counterpart obtained by switching the indices of the two markets.

Proposition 2 *There exists a unique clientele equilibrium in which market 1 is the one with the most sellers.*

A clientele equilibrium is characterized by the switching rate κ^* of the investor who is indifferent between the two markets. Investors with $\kappa > \kappa^*$ enter into market 1, and investors with $\kappa < \kappa^*$ enter into market 2. According to Proposition 2, such a cutoff κ^* exists and is unique.

Proposition 3 *The clientele equilibrium where market 1 is the one with the most sellers, has the following properties:*

(a) More buyers and sellers in market 1: $\mu_b^1 > \mu_b^2$ *and* $\mu_s^1 > \mu_s^2$.

(b) Higher buyer-seller ratio in market 1: $\mu_b^1/\mu_s^1 > \mu_b^2/\mu_s^2$.

(c) Higher prices in market 1: $p^1(\kappa) > p^2(\kappa)$ *for all* κ.

According to Proposition 3, market 1 has not only more sellers than market 2, but also more buyers, and a higher buyer-seller ratio. Moreover, the price that any given buyer expects to pay is higher in market 1.[11] The intuition is as follows. Since there are more sellers in market 1, buyers' search times are shorter. Therefore, holding all else constant, buyers prefer entering into market 1. To restore equilibrium, prices in market 1 must be higher than in market 2. This is accomplished by a higher buying pressure in market 1, i.e., a higher buyer-seller ratio.

In the resulting equilibrium, there is a clientele effect. Investors with high switching rates, who have a stronger preference for short search times, prefer market 1 despite the higher prices. On the other hand, low-switching-rate investors, who are more patient, value more the lower prices in market 2. The clientele effect is, in turn, what accounts for the

[11] The result that a given buyer expects to pay a higher price in market 1 ($p^1(\kappa) > p^2(\kappa)$ for all κ) does not imply that the average buyer pays a higher price ($E_b^1(p^1(\kappa)) > E_b^2(p^2(\kappa))$) because the buyer populations in the two markets are different. A sufficient condition for the latter is that the discount rate r is small relative to the switching rates and the rates of meeting counterparties. This condition is satisfied for plausible parameter values: for example, switching times in the order of months or years, and meeting times in the order of days or weeks, are shorter than $1/r$ when $r = 5\%$.

larger measure of sellers in market 1, since the high-switching-rate buyers turn faster into sellers.

Our model of search provides a natural measure of liquidity. Since investors cannot trade immediately, they incur a cost of delay. A measure of this cost is the expected time it takes to find a counterparty, and conversely, a measure of liquidity is the inverse of this expected time. Since a buyer in market i meets sellers at the rate $\lambda\mu_s^i$, the expected time it takes to meet a seller is $\tau_b^i \equiv 1/(\lambda\mu_s^i)$. Likewise, the expected time it takes for a seller to meet a buyer is $\tau_s^i \equiv 1/(\lambda\mu_b^i)$. Since the measures of buyers and sellers are higher in market 1, the expected times τ_b^i and τ_s^i are lower in that market, and thus market 1 is more liquid. Note that because there are more buyers and sellers in market 1, the trading volume, defined as the flow $\lambda\mu_b^i\mu_s^i$ at which matches occur, is higher in that market.

Since market 1 is more liquid than market 2, the price difference between the two markets can be interpreted as a liquidity premium: buyers are willing to pay a higher price for asset 1 because of its greater liquidity. In generating a liquidity premium, our model is analogous to the literature on asset pricing with transaction costs (e.g., Amihud and Mendelson (1986), Constantinides (1986), Aiyagari and Gertler (1991), Heaton and Lucas (1996), Vayanos (1998, 2005), Vayanos and Vila (1999), Huang (2002), Lo, Mamaysky, and Wang (2004)). The main difference with that literature is that we endogenize transaction costs. In particular, we do not assume that these differ exogenously across assets, but show that differences can arise endogenously in equilibrium, even when the assets are otherwise identical.

Modelling transaction costs through search generates empirical predictions that are unique to our model. For example, the higher prices in the more liquid market are sustained because the buyer-seller ratio is higher in that market. Thus, a unique prediction of our model is that the ratio of sellers' search times to those of buyers (which is equal to the buyer-seller ratio) should be higher in more liquid markets.[12]

[12]While data on search times might not be easily available, one could use data on dealers' inventories. If dealers rebalance their inventories partly through search, the rate at which inventories revert to their long-run mean from below (above) is a measure of the time it takes to find a seller (buyer).

The liquidity premium is determined by the condition that the buyer κ^* is indifferent between the two markets, trading off the higher prices in market 1 with the lower search times. Buyers' search times in a given market are, in turn, determined by the measure of sellers in that market. This measure can be interpreted as an asset's free float since it is equal to the quantity of the asset which is available for sale. In a model with homogeneous investors, Weill (2005) derives an increasing relationship between an asset's liquidity premium and the inverse of its free float. Such a relationship can also be derived in our model. Indeed, using equations (8), (9), (11), and $v_b^1(\kappa^*) = v_b^2(\kappa^*)$, we can show that $p^i(\kappa^*) = A - B/\mu_s^i$, for two constants A and $B > 0$.

4.2 Symmetric Equilibria

In a symmetric equilibrium the measure of sellers is the same across the two markets. For investors to be indifferent between markets, the prices must also be the same. These requirements, however, do not determine a unique symmetric equilibrium.

Proposition 4 *There exist a continuum of symmetric equilibria. In any such equilibrium,* $p^1(\kappa) = p^2(\kappa)$ *for all* κ.

The intuition for the indeterminacy is that there are infinitely many ways to allocate investors in the two markets so that the measure of sellers, and an index of buying pressure (which determines prices), are the same across markets. One trivial example is that for any switching rate, half of the investors go to each market, i.e., $\nu^i(\kappa) = 1/2$ for all κ.

5 Welfare Analysis

In this section we perform a welfare analysis of the allocation of liquidity across assets. We examine, in particular, whether it is socially desirable that liquidity is concentrated in one asset, possibly at the expense of others. In the context of our model, this amounts to comparing the clientele equilibrium, where such concentration of liquidity occurs, to the symmetric equilibria. Prior to performing this comparison, we examine the social optimality of investors' entry decisions in the clientele equilibrium. This gives some insights on the welfare properties of the clientele equilibrium, and is a useful first step for comparing this equilibrium to the symmetric ones.

We use a simple welfare criterion which gives the utilities of all investors present in the market equal weight, and discounts those of the future entrants at the common discount rate r. This discounting is consistent with equal weighting, since future entrants can be viewed as outside investors, whose utility is the discounted value of entering the market. Our welfare criterion thus is

$$\mathcal{W} \equiv \sum_{i=1,2} \left[\int_{\underline{\kappa}}^{\overline{\kappa}} [v_b^i(\kappa)\hat{\mu}_b^i(\kappa) + v_o^i(\kappa)\hat{\mu}_o^i(\kappa)] d\kappa + v_s^i \mu_s^i + \frac{1}{r} \int_{\underline{\kappa}}^{\overline{\kappa}} v_b^i(\kappa) \hat{f}(\kappa) \nu^i(\kappa) d\kappa \right],$$

where the last term reflects the welfare of the stream of future entrants. In Lemma 2, we show that welfare takes a simple and intuitive form.

Lemma 2

$$\mathcal{W} = \frac{2\delta}{r} S - \frac{x}{r}(\mu_s^1 + \mu_s^2). \tag{15}$$

The first term in equation (15) is the present value of the dividends paid by the two assets. Welfare would coincide with this present value if all asset owners enjoyed the full value δ of the dividends. Some owners, however, enjoy only the value $\delta - x$. These are the sellers in the two markets, and welfare needs to be adjusted downwards by their total measure.

5.1 Entry in the Clientele Equilibrium

We begin our welfare analysis with the entry decisions of investors in the clientele equilibrium. These decisions are characterized by a cutoff κ^* such that investors above κ^* enter into market 1 and investors below κ^* into market 2. To examine whether investors' private decisions are socially optimal, we consider the change in welfare if some investors close to κ^* enter into a different market than the one prescribed in equilibrium. More specifically, we assume that at time 0, some buyers with switching rates in $[\kappa^*, \kappa^* + d\kappa]$ are reallocated from market 1 to market 2 (but from then on entry decisions are according to the cutoff κ^*). This reallocation causes the markets to be temporarily out of steady state, and to converge over time to the original steady state.

To compute the change in welfare, we need to evaluate welfare out of steady state. We first consider the non-steady state that results when the measure of buyers with switching rates in $[\kappa, \kappa + d\kappa]$ in market i, is increased by ϵ, relative to the steady state. Denoting welfare in this non-steady state by $\mathcal{W}(\epsilon)$, we set

$$V_b^i(\kappa) \equiv \left. \frac{d\mathcal{W}(\epsilon)}{d\epsilon} \right|_{\epsilon=0}.$$

The variable $V_b^i(\kappa)$ measures the increase in social welfare by adding buyers with switching rate κ in market i. It thus represents the social value of these buyers. Proceeding similarly, we can define the social value $V_o^i(\kappa)$ of owners with switching rate κ, and the social value V_s^i of sellers.

Proposition 5 *The social values $V_b^i(\kappa)$, $V_o^i(\kappa)$, and V_s^i, are given by*

$$rV_b^i(\kappa) = -\kappa V_b^i(\kappa) + \lambda\mu_s^i(V_o^i(\kappa) - V_b^i(\kappa) - V_s^i), \tag{16}$$

$$rV_o^i(\kappa) = \delta + \kappa(V_s^i - V_o^i(\kappa)), \tag{17}$$

$$rV_s^i = \delta - x + \lambda\mu_b^i(E_b^i(V_o^i(\kappa) - V_b^i(\kappa)) - V_s^i). \tag{18}$$

Equations (16)-(18) are analogous to equations (8)-(10) that determine investors' expected utilities. To compare the two sets of equations, we reproduce equations (8)-(10) below, using equation (11) to eliminate the price:

$$rv_b^i(\kappa) = -\kappa v_b^i(\kappa) + \lambda\mu_s^i\frac{z}{1+z}(v_o^i(\kappa) - v_b^i(\kappa) - v_s^i), \tag{19}$$

$$rv_o^i(\kappa) = \delta + \kappa(v_s^i - v_o^i(\kappa)), \tag{20}$$

$$rv_s^i = \delta - x + \lambda\mu_b^i\frac{1}{1+z}(E_b^i(v_o^i(\kappa) - v_b^i(\kappa)) - v_s^i). \tag{21}$$

The key difference between expected utilities and social values concerns the flow benefit of meeting a counterparty. Consider, for example, the flow benefit associated to a buyer. In computing the buyer's expected utility, we multiply the buyer's rate of meeting a seller, $\lambda\mu_s^i$, times the surplus realized by the buyer-seller pair, $v_o^i(\kappa) - v_b^i(\kappa) - v_s^i$, times the fraction of that surplus that the buyer appropriates, $z/(1+z)$. In computing the buyer's social value, however, we need to attribute the full surplus to the buyer. This is because the social value measures an investor's marginal contribution to social welfare. Since a trade involving a specific buyer is realized only because that buyer is added into the market, the buyer's marginal contribution is the full surplus associated to the trade. (The same is obviously true for the seller involved in the trade.)[13] [14]

Proposition 6 *In the clientele equilibrium where market 1 is the one with the most sellers, the social value of the buyer κ^* is higher in market 2, i.e., $V_b^1(\kappa^*) < V_b^2(\kappa^*)$.*

Since the social value of the buyer κ^* is higher in market 2, welfare can be improved by reallocating some buyers close to κ^* from market 1 to market 2. Thus, in the clientele equilibrium, there is excessive entry into market 1, i.e., the more liquid market. The intuition is as follows. Since the buyer κ^* is indifferent between the two markets, the buyer's flow benefit of meeting a seller is the same across markets. A seller's flow benefit of meeting a buyer, however, is higher in market 1. This is because the seller's rate of meeting a buyer involves the measure of buyers rather than that of sellers, and the buyer-seller ratio is higher in market 1. Since a seller's flow benefit is higher in market 1, the discrepancy between the seller's social value and expected utility is larger in that market. (Recall that social value attributes the full benefit of a meeting to each party, while expected utility attributes only a fraction.) Conversely, since buyers bargain on the basis of a seller's expected utility rather than social value, the discrepancy between their own social value and expected utility is

[13] Additionally, in computing the buyer's social value, we need to consider not the buyer's rate of meeting a seller, but the marginal increase in the rate of buyer-seller meetings achieved by adding the buyer in the market. The two coincide, however, because the search technology is linear in the measures of buyers and sellers.

[14] It is worth explaining why our search model generates discrepancies between expected utilities and social values, while the standard Walrasian model does not. In the Walrasian model, the surplus that a buyer-seller pair bargain over is zero, since either party can leave the pair and obtain immediately the market price from another counterparty. In the search model, by contrast, the surplus is non-zero, since finding another counterparty is costly. It is because each party gets only a fraction of this non-zero surplus that discrepancies between expected utilities and social values arise.

smaller in market 1. Given that for the indifferent buyer, expected utility is the same across the two markets, social value is greater in market 2. Intuitively, sellers are more socially valuable in market 1 because they are in relatively short supply in that market. Buyers internalize this through the higher prices, but only partially, and thus they enter excessively into market 1.

While Proposition 6 implies that there is excessive entry into the more liquid market, it does not imply that the concentration of liquidity in that market is excessive. Excessive entry into market 1 implies that the ratio of buyers μ_b^1/μ_b^2 is too high. Thus, when liquidity is measured from a seller's viewpoint, the concentration of liquidity in market 1 is indeed excessive.[15] At the same time, having too many buyers in one market, reduces the number of sellers. Thus, the ratio of sellers μ_s^1/μ_s^2 is too low, which implies that when liquidity is measured from a buyer's viewpoint, the concentration of liquidity in market 1 is not high enough. We next examine whether the concentration of liquidity is desirable in the first place, by comparing the clientele equilibrium to the symmetric ones.

5.2 Clientele vs. Symmetric Equilibria

We start with a methodological observation. Both the clientele and the symmetric equilibria are dynamic steady states, and comparing these can be misleading. Indeed, an action aiming to take the market from an inferior steady state to a superior one, must involve non-steady-state dynamics. For such an action to be evaluated based only on a comparison between steady states, these dynamics must be unimportant relative to the long-run limit. This is the case when the discount rate r is small, which we assume in some of our results below.

Both the clientele and the symmetric equilibria are fully characterized by the decisions of investors as to which market to enter. We next determine, and use as a benchmark, the socially optimal entry decisions in steady state. These are the solution to the problem

$$\max_{\nu^1(\kappa)} \mathcal{W},$$

[15] Recall that liquidity is measured by the inverse of the expected time it takes to find a counterparty. When the expected time is from a seller's viewpoint, the relative liquidity of the two markets is $(1/\tau_s^1)/(1/\tau_s^2) = \mu_b^1/\mu_b^2$, i.e., equal to the ratio of buyers in the two markets.

where $\mathcal{W}$ is given by Lemma 2, μ_s^i by equation (6), and $\nu^2(\kappa) = 1 - \nu^1(\kappa)$. We solve this problem, $(\mathcal{P})$, in Proposition 7.

Proposition 7 *The problem $(\mathcal{P})$ has two symmetric solutions. The first solution satisfies $\mu_s^1 > \mu_s^2$, $\nu^1(\kappa) = 1$ for $\kappa > \kappa_w^*$, and $\nu^1(\kappa) = 0$ for $\kappa < \kappa_w^*$, for a cutoff κ_w^*. The second solution is obtained from the first by switching the indices of the two markets.*

Proposition 7 implies that it is socially optimal to create two markets with different measures of sellers. This is because the two markets can cater to different clienteles of investors: buyers with switching rates above a cutoff κ_w^*, who have a greater preference for lower search times, are allocated to the market with the most sellers, while the opposite holds for buyers below κ_w^*.

The cutoff κ_w^* determines the heterogeneity of the two markets. Increasing κ_w^*, reduces the entry into the more liquid market, say market 1. This increases the ratio of sellers μ_s^1/μ_s^2, and makes the markets more heterogeneous from a buyer's viewpoint.

We next treat the cutoff above which buyers enter into market 1 as a free variable, and denote it by $\hat{\kappa}$. Social welfare is maximized for $\hat{\kappa} = \kappa_w^*$. As $\hat{\kappa}$ decreases below κ_w^*, the two markets become more homogenous from a buyer's viewpoint, and welfare decreases. Consider now two values of $\hat{\kappa}$: the cutoff κ^* corresponding to the clientele equilibrium, and the cutoff κ' for which the measure of sellers is the same across the two markets. Since in the clientele equilibrium there is excessive entry into market 1, markets are not heterogeneous enough from a buyer's viewpoint, and thus $\kappa^* < \kappa_w^*$. At the same time, since there is some heterogeneity, $\kappa^* > \kappa'$. Therefore, welfare under the clientele equilibrium exceeds that under the allocation corresponding to κ'.

Interestingly, welfare under the latter allocation is the same as under any of the symmetric equilibria. To see why, note that both types of allocations have the property that the measure of sellers is the same across the two markets. Consider now an arbitrary allocation with this property, and denote by $\mu_s \equiv \mu_s^1 = \mu_s^2$ the common measure of sellers. The aggregate measure of inactive owners (i.e., the sum across both markets) depends on this allocation only through μ_s, since μ_s is the only determinant of the buyers' matching rate.

Since the aggregate measure of inactive owners, plus that of sellers, must equal the aggregate asset supply, μ_s is uniquely determined regardless of the specific allocation.[16] Since, in addition, welfare depends only on μ_s, it is also independent of the specific allocation. Summarizing, we can show the following proposition:

Proposition 8 *All symmetric equilibria achieve the same welfare. Moreover, for small r, they are dominated by the clientele equilibrium.*

6 Asymmetric Information

In this section, we extend our analysis to the case where buyers' switching rates are not observable to the sellers. Since switching rates affect buyers' reservation values (high-switching-rate buyers have to re-enter the search market as sellers sooner, and thus have lower reservation values), the bargaining game between a buyer and a seller involves asymmetric information.

We begin our analysis by examining whether a clientele equilibrium exists. Such an equilibrium is characterized by the cutoff κ^* of the indifferent investor. Without loss of generality, we assume that investors above the cutoff enter into market 1. We denote by κ^i the maximum switching rate of an investor in market i, i.e., $\kappa^1 = \overline{\kappa}$ and $\kappa^2 = \kappa^*$. Since reservation values decrease in switching rates, the buyer with switching rate κ^i has the lowest reservation value in market i.

For simplicity, we restrict the clientele equilibrium to be in pure strategies, i.e., all sellers in a given market make the same offer. In a pure-strategy equilibrium, the sellers' offer must be accepted by all buyers entering a market. Indeed, suppose that the buyers with switching rates above a cutoff $\hat{\kappa}^i < \kappa^i$ reject the sellers' offer in market i. Then, the density function $\mu_b^i(\kappa)$ of buyers in market i would increase discontinuously at $\hat{\kappa}^i$, as the buyers above $\hat{\kappa}^i$

[16] To show this formally, we add equation (6) for market 1 to the same equation for market 2, and find

$$\int_{\underline{\kappa}}^{\overline{\kappa}} \frac{\lambda \mu_s \hat{f}(\kappa)}{\kappa\left(\lambda \mu_s + \kappa\right)} d\kappa + 2\mu_s = 2S.$$

This equation determines μ_s uniquely, regardless of the specific allocation.

would exit the buyer pool at lower rates.[17] This discontinuity would induce the sellers to slightly lower their offer, as to trade with buyers above $\hat{\kappa}^i$.

Since all buyer-seller matches result in a trade, the equations for the measures of buyers, inactive owners, and sellers in each market are as in the symmetric information case. The equations for the expected utilities and prices are, however, somewhat different, because the price is now the same for all buyers entering a market. More specifically, the sellers' offer in market i is $v_o^i(\kappa^i) - v_b^i(\kappa^i)$, i.e., the reservation value of the highest-switching-rate buyer, and the buyers' offer is v_s^i, i.e., the reservation value of a seller. Since buyers are selected to make the offer with probability $z/(1+z)$, and sellers with probability $1/(1+z)$, the expected price in market i is

$$p^i = \frac{z}{1+z} v_s^i + \frac{1}{1+z}(v_o^i(\kappa^i) - v_b^i(\kappa^i)). \tag{22}$$

The expected utility of a buyer in market i is given by

$$r v_b^i(\kappa) = -\kappa v_b^i(\kappa) + \lambda \mu_s^i (v_o^i(\kappa) - p^i - v_b^i(\kappa)), \tag{23}$$

the expected utility of a seller is given by

$$r v_s^i = \delta - x + \lambda \mu_b^i (p^i - v_s^i), \tag{24}$$

and the expected utility of an inactive owner is given by equation (9) as in the symmetric information case.

For a clientele equilibrium to exist, each seller must find it optimal to make an offer which is accepted by all buyers. Suppose that upon meeting a buyer, a seller decides to make an offer which is accepted only when the buyer's switching rate is up to κ. Then, the offer is $v_o^i(\kappa) - v_b^i(\kappa)$, and if it is rejected the seller re-enters the search process with expected utility v_s^i. Thus, the seller finds it optimal to trade with all buyers if

$$\kappa^i \in \operatorname{argmax}_\kappa \left[P_b^i(\kappa)(v_o^i(\kappa) - v_b^i(\kappa)) + (1 - P_b^i(\kappa)) v_s^i \right], \tag{25}$$

where $P_b^i(\kappa)$ denotes the probability that a buyer in market i has switching rate up to κ.

[17] More specifically, $\mu_b^i(\kappa)$ would be given by equation (3) for $\kappa < \hat{\kappa}^i$, and $\mu_b^i(\kappa) = \hat{f}(\kappa)\nu^i(\kappa)/\kappa$ for $\kappa > \hat{\kappa}^i$, as the buyers above $\hat{\kappa}^i$ would exit the buyer pool only because of a switch to low valuation.

Additionally, in a clientele equilibrium, the buyer κ^* must be indifferent between the two markets. In the asymmetric information case, an indifferent buyer might not exist. Indeed, suppose that the seller has all the bargaining power ($z = 0$). Then, the buyer κ^* receives zero surplus in market 2 (because the price is equal to his reservation value), but a positive surplus in market 1. To formulate a sufficient condition for the existence of an indifferent buyer, we treat the cutoff above which investors enter into market 1 as a free variable, and consider population measures and expected utilities as functions of that variable. We also consider the value κ' of the cutoff for which the measures of sellers are equal in the two markets. Then, the sufficient condition is that when the cutoff takes the value κ', the buyer κ' prefers entering into market 2. We refer to this condition as Condition (C). Proposition 9 confirms that a clientele equilibrium exists under Conditions (25) and (C). Moreover, these conditions are satisfied for plausible values of the exogenous parameters, as shown in Proposition 10.

Proposition 9 *If Conditions (25) and (C) hold, a clientele equilibrium exists and has the following properties:*

(a) More buyers and sellers in market 1: $\mu_b^1 > \mu_b^2$ *and* $\mu_s^1 > \mu_s^2$.

(b) Higher buyer-seller ratio in market 1: $\mu_b^1/\mu_s^1 > \mu_b^2/\mu_s^2$.

(c) Higher prices in market 1: $p^1 > p^2$.

According to Proposition 9, the conditions which ensure the existence of a clientele equilibrium also ensure that this equilibrium has the same properties as in the symmetric information case. In particular, the market with the high-switching-rate investors has more sellers, more buyers, a higher buyer-seller ratio, and higher prices.[18]

Conditions (25) and (C) hold in the natural case where the rates of meeting counterparties are high relative to the switching rates and the discount rate. One way to derive this case is to assume that asset supply and demand are large, so that the measures of buyers

[18] In fact, some properties of a clientele equilibrium can be proven more generally, without using the sufficient conditions for existence. These are that the market with the high-switching-rate investors has more buyers and higher trading volume, and has a higher buyer-seller ratio and higher prices if it has more sellers.

and sellers are large. In particular, we assume that the supply S of each asset grows large, and the flow of investors into the economy is $\hat{f}(\kappa) = S\hat{F}(\kappa)$, where $\hat{F}(\kappa)$ is held fixed. We further assume that

$$\int_{\underline{\kappa}}^{\overline{\kappa}} \frac{\hat{F}(\kappa)}{\kappa} d\kappa = 2, \tag{26}$$

a condition ensuring that asset supply and demand grow at the same rate.[19]

Proposition 10 *Suppose that $z \in (0, \infty)$, and $\hat{f}(\kappa) = S\hat{F}(\kappa)$ for $\hat{F}(\kappa)$ satisfying Condition (26). Then, for S large enough, Conditions (25) and (C) hold.*

Having established the existence of a clientele equilibrium, we next examine its welfare properties. As shown in Section 5, a sufficient condition for the clientele equilibrium to dominate the symmetric ones is that entry into market 1 is at or above the socially optimal level. To examine whether this condition holds in the asymmetric information case, we compare entry decisions with the symmetric information case. Under asymmetric information, the buyer κ^* receives a positive surplus from the seller's offer when entering into market 1, because the same offer must also be accepted by the buyer $\overline{\kappa}$. This induces more entry into market 1. At the same time, a seller's outside option is reduced by his inability to price-discriminate, and this lowers the offer a buyer can make, thus raising the buyer's utility. Whether this induces more or less entry into market 1 depends on the relative heterogeneity of investors in the two markets. When, for example, κ^* is close to $\overline{\kappa}$, market 1 is more homogeneous. Thus, the inability to price-discriminate hurts more the sellers in market 2, inducing more entry into that market. The overall effect is ambiguous. Suppose, for example, that $\hat{F}(\kappa) = c\kappa^{\alpha}$, where $\alpha \in \mathbb{R}$ measures the tilt of the distribution towards high switching rates, c is a normalizing constant (so that equation (26) holds), and $\overline{\kappa}/\underline{\kappa} = 2$. Then, entry into market 1 is greater in the asymmetric information case, as long as α is smaller than 0.51.

Even when entry into market 1 is lower in the asymmetric information case, it can still be socially excessive, because it is so under symmetric information. For example, when $\hat{F}(\kappa) = c\kappa^{\alpha}$, entry into market 1 is socially excessive for all values of α and $\overline{\kappa}/\underline{\kappa}$.[20]

[19] The two assets are in total supply $2S$. The total demand is the measure of high-valuation investors, i.e., buyers and inactive owners, across the two markets. From equations (1)-(4), this is $\int_{\underline{\kappa}}^{\overline{\kappa}} \frac{\hat{f}(\kappa)}{\kappa} d\kappa$.

[20] There might, perhaps, be counterexamples for more complicated distributions.

7 Conclusion

In this paper we explore a theory of asset liquidity based on the notion that trading involves search. We assume that investors of different horizons can invest in two identical assets. The asset markets are partially segmented, in that buyers must decide which asset to search for, and then search only in that asset's market. We show that there exists a "clientele" equilibrium where one market has more buyers and sellers, lower search times, higher trading volume, higher prices, and short-horizon investors. Thus, our theory provides one explanation for why assets with very similar cash flows can differ substantially in their liquidity (e.g., AAA-rated corporate bonds vs. Treasury bonds, and on- vs. off-the-run Treasury bonds). This phenomenon is inconsistent with theories based on asymmetric information.

Our model produces a non-trivial welfare analysis of the allocation of liquidity across assets. We show that the clientele equilibrium dominates the ones where the two markets are identical, implying that the concentration of liquidity in one asset is socially desirable. At the same time, too many buyers decide to search for the liquid asset, implying that the two markets are not heterogeneous enough from the buyers' perspective.

Appendix

Proof of Proposition 1: Plugging equation (11) into (8) and (10), we find

$$rv_b^i(\kappa) = -\kappa v_b^i(\kappa) + \lambda\mu_s^i \frac{z}{1+z}(v_o^i(\kappa) - v_b^i(\kappa) - v_s^i), \tag{27}$$

and

$$rv_s^i = \delta - x + \lambda\mu_b^i \frac{1}{1+z}(E_b^i(v_o^i(\kappa) - v_b^i(\kappa)) - v_s^i). \tag{28}$$

Subtracting equation (27) from (9), we find

$$r(v_o^i(\kappa) - v_b^i(\kappa)) = \delta + \kappa(v_s^i - v_o^i(\kappa) + v_b^i(\kappa)) - \lambda\mu_s^i \frac{z}{1+z}(v_o^i(\kappa) - v_b^i(\kappa) - v_s^i)$$

$$\Rightarrow \quad v_o^i(\kappa) - v_b^i(\kappa) = \frac{\delta + \left(\kappa + \lambda\mu_s^i \frac{z}{1+z}\right) v_s^i}{r + \kappa + \lambda\mu_s^i \frac{z}{1+z}}. \tag{29}$$

Plugging equation (29) into (28), we find

$$rv_s^i = \delta - x + \lambda\mu_b^i \frac{1}{1+z}(\delta - rv_s^i) E_b^i \left[\frac{1}{r + \kappa + \lambda\mu_s^i \frac{z}{1+z}}\right]$$

$$\Rightarrow \quad v_s^i = \frac{\delta}{r} - \frac{x}{r} \frac{1}{1 + \lambda\mu_b^i \frac{1}{1+z} E_b^i \left[\frac{1}{r+\kappa+\lambda\mu_s^i \frac{z}{1+z}}\right]}. \tag{30}$$

Given v_s^i, the variables $v_o^i(\kappa)$, $v_b^i(\kappa)$, and $p^i(\kappa)$, are uniquely determined from equations (9), (27), and (11), respectively. In the rest of the proof, we compute $v_b^i(\kappa)$ and $p^i(\kappa)$, because we use them in our subsequent analysis. Plugging equation (30) into (29), we find

$$v_o^i(\kappa) - v_b^i(\kappa) = \frac{\delta}{r} - \frac{x}{r} \frac{\kappa + \lambda\mu_s^i \frac{z}{1+z}}{\left(r + \kappa + \lambda\mu_s^i \frac{z}{1+z}\right)\left[1 + \lambda\mu_b^i \frac{1}{1+z} E_b^i \left[\frac{1}{r+\kappa+\lambda\mu_s^i \frac{z}{1+z}}\right]\right]}. \tag{31}$$

Subtracting equation (30) from (31), we find

$$v_o^i(\kappa) - v_b^i(\kappa) - v_s^i = \frac{x}{\left(r + \kappa + \lambda\mu_s^i \frac{z}{1+z}\right)\left[1 + \lambda\mu_b^i \frac{1}{1+z} E_b^i \left[\frac{1}{r+\kappa+\lambda\mu_s^i \frac{z}{1+z}}\right]\right]} > 0. \tag{32}$$

Plugging equation (32) into (27), we can compute $v_b^i(\kappa)$:

$$rv_b^i(\kappa) = -\kappa v_b^i(\kappa) + \lambda\mu_s^i \frac{z}{1+z} \frac{x}{\left(r+\kappa+\lambda\mu_s^i\frac{z}{1+z}\right)\left[1+\lambda\mu_b^i\frac{1}{1+z}E_b^i\left[\frac{1}{r+\kappa+\lambda\mu_s^i\frac{z}{1+z}}\right]\right]}$$

$$\Rightarrow \quad v_b^i(\kappa) = \frac{\lambda\mu_s^i\frac{z}{1+z}x}{(r+\kappa)\left(r+\kappa+\lambda\mu_s^i\frac{z}{1+z}\right)\left[1+\lambda\mu_b^i\frac{1}{1+z}E_b^i\left[\frac{1}{r+\kappa+\lambda\mu_s^i\frac{z}{1+z}}\right]\right]}. \tag{33}$$

Plugging equations (30) and (31) into (11), we can compute $p^i(\kappa)$:

$$p^i(\kappa) = \frac{\delta}{r} - \frac{x}{r}\frac{1-\frac{r}{1+z}\frac{1}{r+\kappa+\lambda\mu_s^i\frac{z}{1+z}}}{1+\lambda\mu_b^i\frac{1}{1+z}E_b^i\left[\frac{1}{r+\kappa+\lambda\mu_s^i\frac{z}{1+z}}\right]}. \tag{34}$$

∎

Proof of Lemma 1: Since $v_b^1(\kappa^*) = v_b^2(\kappa^*) > 0$, the difference $v_b^1(\kappa) - v_b^2(\kappa)$ has the same sign as

$$\frac{v_b^1(\kappa)}{v_b^1(\kappa^*)} - \frac{v_b^2(\kappa)}{v_b^2(\kappa^*)}.$$

Equation (33) implies that

$$\begin{aligned}
\frac{v_b^1(\kappa)}{v_b^1(\kappa^*)} - \frac{v_b^2(\kappa)}{v_b^2(\kappa^*)} &= \frac{r+\kappa^*}{r+\kappa}\left[\frac{r+\kappa^*+\lambda\mu_s^1\frac{z}{1+z}}{r+\kappa+\lambda\mu_s^1\frac{z}{1+z}} - \frac{r+\kappa^*+\lambda\mu_s^2\frac{z}{1+z}}{r+\kappa+\lambda\mu_s^2\frac{z}{1+z}}\right] \\
&= \frac{r+\kappa^*}{r+\kappa}\frac{\lambda(\mu_s^1-\mu_s^2)\frac{z}{1+z}(\kappa-\kappa^*)}{\left(r+\kappa+\lambda\mu_s^1\frac{z}{1+z}\right)\left(r+\kappa+\lambda\mu_s^2\frac{z}{1+z}\right)},
\end{aligned}$$

which proves the lemma. ∎

To prove Proposition 2, we first prove the following lemma:

Lemma 3 *Suppose that investors' entry decisions are given by $\nu^1(\kappa) = 1$ for $\kappa > \kappa^*$, and $\nu^1(\kappa) = 0$ for $\kappa < \kappa^*$, for some cutoff κ^*. Then, μ_s^1 and μ_s^2 are uniquely determined, μ_s^1 is increasing in κ^*, and μ_s^2 is decreasing in κ^*.*

Proof: Using equation (6), and setting $i = 1$, $\nu^1(\kappa) = 1$ for $\kappa > \kappa^*$, and $\nu^1(\kappa) = 0$ for $\kappa < \kappa^*$, we find

$$\int_{\kappa^*}^{\overline{\kappa}} \frac{\lambda \mu_s^1 \hat{f}(\kappa)}{\kappa\,(\kappa + \lambda \mu_s^1)} d\kappa + \mu_s^1 = S. \tag{35}$$

The LHS of this equation is strictly increasing in μ_s^1, is zero for $\mu_s^1 = 0$, and is infinite for $\mu_s^1 = \infty$. Therefore, equation (35) has a unique solution μ_s^1. Moreover, differentiating implicitly w.r.t. κ^*, we find

$$\frac{d\mu_s^1}{d\kappa^*} = \frac{\frac{\lambda \mu_s^1 \hat{f}(\kappa^*)}{\kappa^*(\kappa^* + \lambda \mu_s^1)}}{1 + \int_{\kappa^*}^{\overline{\kappa}} \frac{\lambda \hat{f}(\kappa)}{(\kappa + \lambda \mu_s^1)^2} d\kappa} > 0.$$

Proceeding similarly, we find that μ_s^2 is uniquely determined by

$$\int_{\underline{\kappa}}^{\kappa^*} \frac{\lambda \mu_s^2 \hat{f}(\kappa)}{\kappa\,(\kappa + \lambda \mu_s^2)} d\kappa + \mu_s^2 = S. \tag{36}$$

Differentiating implicitly w.r.t. κ^*, we find

$$\frac{d\mu_s^2}{d\kappa^*} = -\frac{\frac{\lambda \mu_s^2 \hat{f}(\kappa^*)}{\kappa^*(\kappa^* + \lambda \mu_s^2)}}{1 + \int_{\underline{\kappa}}^{\kappa^*} \frac{\lambda \hat{f}(\kappa)}{(\kappa + \lambda \mu_s^2)^2} d\kappa} < 0.$$

$\blacksquare$

Proof of Proposition 2: The cutoff κ^* is determined by the indifference condition $v_b^1(\kappa^*) = v_b^2(\kappa^*)$. Using equation (33), we can write this condition as

$$\frac{\mu_s^1}{r + \kappa^* + \lambda \mu_s^1 \frac{z}{1+z} + \lambda \mu_b^1 \frac{1}{1+z} E^1} = \frac{\mu_s^2}{r + \kappa^* + \lambda \mu_s^2 \frac{z}{1+z} + \lambda \mu_b^2 \frac{1}{1+z} E^2}, \tag{37}$$

where

$$E^i \equiv E_b^i \left[\frac{r + \kappa^* + \lambda \mu_s^i \frac{z}{1+z}}{r + \kappa + \lambda \mu_s^i \frac{z}{1+z}} \right].$$

Multiplying by the denominators, we find

$$(r + \kappa^*)(\mu_s^1 - \mu_s^2) + \lambda \frac{1}{1+z} (\mu_s^1 \mu_b^2 E^2 - \mu_s^2 \mu_b^1 E^1) = 0. \tag{38}$$

Since

$$E^1 = \frac{1}{\mu_b^1}\int_{\underline{\kappa}}^{\overline{\kappa}} \frac{r+\kappa^*+\lambda\mu_s^1\frac{z}{1+z}}{r+\kappa+\lambda\mu_s^1\frac{z}{1+z}}\hat{\mu}_b^1(\kappa)d\kappa = \frac{1}{\mu_b^1}\int_{\kappa^*}^{\overline{\kappa}} \frac{\hat{f}(\kappa)\left(r+\kappa^*+\lambda\mu_s^1\frac{z}{1+z}\right)}{(\kappa+\lambda\mu_s^1)\left(r+\kappa+\lambda\mu_s^1\frac{z}{1+z}\right)}d\kappa,$$

and

$$E^2 = \frac{1}{\mu_b^2}\int_{\underline{\kappa}}^{\kappa^*} \frac{\hat{f}(\kappa)\left(r+\kappa^*+\lambda\mu_s^2\frac{z}{1+z}\right)}{(\kappa+\lambda\mu_s^2)\left(r+\kappa+\lambda\mu_s^2\frac{z}{1+z}\right)}d\kappa,$$

equation (38) can be written as

$$\begin{aligned}\mu_s^1-\mu_s^2 \quad &+ \quad \mu_s^1\frac{1}{(r+\kappa^*)(1+z)}\int_{\underline{\kappa}}^{\kappa^*} \frac{\lambda\hat{f}(\kappa)\left(r+\kappa^*+\lambda\mu_s^2\frac{z}{1+z}\right)}{(\kappa+\lambda\mu_s^2)\left(r+\kappa+\lambda\mu_s^2\frac{z}{1+z}\right)}d\kappa \\ &- \quad \mu_s^2\frac{1}{(r+\kappa^*)(1+z)}\int_{\kappa^*}^{\overline{\kappa}} \frac{\lambda\hat{f}(\kappa)\left(r+\kappa^*+\lambda\mu_s^1\frac{z}{1+z}\right)}{(\kappa+\lambda\mu_s^1)\left(r+\kappa+\lambda\mu_s^1\frac{z}{1+z}\right)}d\kappa = 0. \end{aligned} \tag{39}$$

To prove the proposition, we consider equation (39) as a function of the single unknown κ^*, i.e., we assume that μ_s^1 and μ_s^2 are implicitly defined, given κ^*, from Lemma 3. To show that an equilibrium exists, it suffices to show that equation (39) has a solution κ^*, for which $\mu_s^1 > \mu_s^2$. For $\kappa^* = \underline{\kappa}$, the LHS is negative, since equation (36) implies that $\mu_s^2 = S > \mu_s^1$. Conversely, for $\kappa^* = \overline{\kappa}$, the LHS is positive. Therefore, equation (39) has a solution $\kappa^* \in (\underline{\kappa}, \overline{\kappa})$. To show that $\mu_s^1 > \mu_s^2$, we first note that

$$\begin{aligned}&\int_{\kappa^*}^{\overline{\kappa}} \frac{\lambda\hat{f}(\kappa)\left(r+\kappa^*+\lambda\mu_s^1\frac{z}{1+z}\right)}{(\kappa+\lambda\mu_s^1)\left(r+\kappa+\lambda\mu_s^1\frac{z}{1+z}\right)}d\kappa - \int_{\kappa^*}^{\overline{\kappa}} \frac{\lambda\hat{f}(\kappa)\kappa^*}{(\kappa+\lambda\mu_s^1)\kappa}d\kappa \\ = \quad &\int_{\kappa^*}^{\overline{\kappa}} \frac{\lambda\hat{f}(\kappa)\left(r+\lambda\mu_s^1\frac{z}{1+z}\right)(\kappa-\kappa^*)}{(\kappa+\lambda\mu_s^1)\kappa\left(r+\kappa+\lambda\mu_s^1\frac{z}{1+z}\right)}d\kappa > 0. \end{aligned} \tag{40}$$

Similarly,

$$\int_{\underline{\kappa}}^{\kappa^*} \frac{\lambda\hat{f}(\kappa)\left(r+\kappa^*+\lambda\mu_s^2\frac{z}{1+z}\right)}{(\kappa+\lambda\mu_s^2)\left(r+\kappa+\lambda\mu_s^2\frac{z}{1+z}\right)}d\kappa - \int_{\underline{\kappa}}^{\kappa^*} \frac{\lambda\hat{f}(\kappa)\kappa^*}{(\kappa+\lambda\mu_s^2)\kappa}d\kappa < 0. \tag{41}$$

Combining equations (40) and (41) with (39), we find

$$\mu_s^1-\mu_s^2+\frac{\kappa^*}{(r+\kappa^*)(1+z)}\left[\mu_s^1\int_{\underline{\kappa}}^{\kappa^*}\frac{\lambda\hat{f}(\kappa)}{(\kappa+\lambda\mu_s^2)\kappa}d\kappa - \mu_s^2\int_{\kappa^*}^{\overline{\kappa}}\frac{\lambda\hat{f}(\kappa)}{(\kappa+\lambda\mu_s^1)\kappa}d\kappa\right] > 0.$$

Combining this equation with (35) and (36), we find

$$\mu_s^1 - \mu_s^2 + \frac{\kappa^*}{(r+\kappa^*)(1+z)} \left[\mu_s^1 \left(\frac{S}{\mu_s^2} - 1\right) - \mu_s^2 \left(\frac{S}{\mu_s^1} - 1\right)\right] > 0$$

$$\Rightarrow \quad (\mu_s^1 - \mu_s^2) \left[1 + \frac{\kappa^*}{(r+\kappa^*)(1+z)} \left[\frac{S(\mu_s^1 + \mu_s^2)}{\mu_s^1 \mu_s^2} - 1\right]\right] > 0.$$

Since the term in brackets is positive, we have $\mu_s^1 > \mu_s^2$.

To show that the equilibrium is unique, it suffices to show that for any κ^* that solves equation (39), the derivative of the LHS w.r.t. κ^* is strictly positive. Denoting the LHS by $F(\kappa^*, \mu_s^1, \mu_s^2)$, we have

$$\frac{dF(\kappa^*, \mu_s^1, \mu_s^2)}{d\kappa^*} = \frac{\partial F(\kappa^*, \mu_s^1, \mu_s^2)}{\partial \kappa^*} + \frac{\partial F(\kappa^*, \mu_s^1, \mu_s^2)}{\partial \mu_s^1} \frac{d\mu_s^1}{d\kappa^*} + \frac{\partial F(\kappa^*, \mu_s^1, \mu_s^2)}{\partial \mu_s^2} \frac{d\mu_s^2}{d\kappa^*}.$$

We will show that the partial derivatives w.r.t. κ^* and μ_s^1 are strictly positive, while that w.r.t. μ_s^2 is strictly negative. Since $d\mu_s^1/d\kappa^* > 0$ and $d\mu_s^2/d\kappa^* < 0$, this will imply that $dF(\kappa^*, \mu_s^1, \mu_s^2)/d\kappa^* > 0$. Setting

$$g^i(\kappa) \equiv \frac{\lambda \hat{f}(\kappa)}{(\kappa + \lambda\mu_s^i)\left(r + \kappa + \lambda\mu_s^i \frac{z}{1+z}\right)},$$

we have

$$\begin{aligned}
\frac{\partial F(\kappa^*, \mu_s^1, \mu_s^2)}{\partial \kappa^*} \quad &= \quad \frac{\lambda\mu_s^1 \hat{f}(\kappa^*)}{(r+\kappa^*)(1+z)(\kappa^* + \lambda\mu_s^2)} + \frac{\lambda\mu_s^2 \hat{f}(\kappa^*)}{(r+\kappa^*)(1+z)(\kappa^* + \lambda\mu_s^1)} \\
&\quad + \frac{\lambda\mu_s^1\mu_s^2 \frac{z}{1+z}}{(r+\kappa^*)^2(1+z)} \left[\int_{\kappa^*}^{\overline{\kappa}} g^1(\kappa)d\kappa - \int_{\underline{\kappa}}^{\kappa^*} g^2(\kappa)d\kappa\right],
\end{aligned}$$

$$\begin{aligned}
\frac{\partial F(\kappa^*, \mu_s^1, \mu_s^2)}{\partial \mu_s^1} \quad &= \quad 1 + \frac{r + \kappa^* + \lambda\mu_s^2 \frac{z}{1+z}}{(r+\kappa^*)(1+z)} \int_{\underline{\kappa}}^{\kappa^*} g^2(\kappa)d\kappa - \mu_s^2 \frac{\lambda \frac{z}{1+z}}{(r+\kappa^*)(1+z)} \int_{\kappa^*}^{\overline{\kappa}} g^1(\kappa)d\kappa \\
&\quad + \mu_s^2 \frac{r + \kappa^* + \lambda\mu_s^1 \frac{z}{1+z}}{(r+\kappa^*)(1+z)} \int_{\kappa^*}^{\overline{\kappa}} g^1(\kappa) \left[\frac{\lambda}{\kappa + \lambda\mu_s^1} + \frac{\lambda \frac{z}{1+z}}{r + \kappa + \lambda\mu_s^1 \frac{z}{1+z}}\right] d\kappa,
\end{aligned}$$

and

$$\frac{\partial F(\kappa^*,\mu_s^1,\mu_s^2)}{\partial \mu_s^2} = -1-\frac{r+\kappa^*+\lambda\mu_s^1\frac{z}{1+z}}{(r+\kappa^*)(1+z)}\int_{\kappa^*}^{\overline{\kappa}} g^1(\kappa)d\kappa+\mu_s^1\frac{\lambda\frac{z}{1+z}}{(r+\kappa^*)(1+z)}\int_{\underline{\kappa}}^{\kappa^*} g^2(\kappa)d\kappa$$

$$-\mu_s^1\frac{r+\kappa^*+\lambda\mu_s^2\frac{z}{1+z}}{(r+\kappa^*)(1+z)}\int_{\underline{\kappa}}^{\kappa^*} g^2(\kappa)\left[\frac{\lambda}{\kappa+\lambda\mu_s^2}+\frac{\lambda\frac{z}{1+z}}{r+\kappa+\lambda\mu_s^2\frac{z}{1+z}}\right]d\kappa.$$

Consider first the equation for $\partial F(\kappa^*,\mu_s^1,\mu_s^2)/\partial\kappa^*$. To show that the RHS is positive, it suffices to show that the term in brackets is positive. The latter follows by writing equation (39) as

$$\mu_s^1-\mu_s^2+\frac{\mu_s^1-\mu_s^2}{1+z}\int_{\underline{\kappa}}^{\kappa^*} g^2(\kappa)d\kappa-\mu_s^2\frac{r+\kappa^*+\lambda\mu_s^1\frac{z}{1+z}}{(r+\kappa^*)(1+z)}\left[\int_{\kappa^*}^{\overline{\kappa}} g^1(\kappa)d\kappa-\int_{\underline{\kappa}}^{\kappa^*} g^2(\kappa)d\kappa\right]=0,$$

and recalling that $\mu_s^1>\mu_s^2$. Consider next the equation for $\partial F(\kappa^*,\mu_s^1,\mu_s^2)/\partial\mu_s^1$. To show that the RHS is positive, it suffices to show that the sum of the first three terms is positive. The latter follows by writing equation (39) as

$$\mu_s^1\left[1+\frac{r+\kappa^*+\lambda\mu_s^2\frac{z}{1+z}}{(r+\kappa^*)(1+z)}\int_{\underline{\kappa}}^{\kappa^*} g^2(\kappa)d\kappa-\mu_s^2\frac{\lambda\frac{z}{1+z}}{(r+\kappa^*)(1+z)}\int_{\kappa^*}^{\overline{\kappa}} g^1(\kappa)d\kappa\right]$$

$$-\quad \mu_s^2\left[1+\frac{1}{1+z}\int_{\kappa^*}^{\overline{\kappa}} g^1(\kappa)d\kappa\right]=0.$$

Consider finally the equation for $\partial F(\kappa^*,\mu_s^1,\mu_s^2)/\partial\mu_s^2$. To show that the RHS is negative, it suffices to show that the sum of the first three terms is negative. The latter follows in the same way as for $\partial F(\kappa^*,\mu_s^1,\mu_s^2)/\partial\mu_s^1$. ∎

Proof of Proposition 3: Property (a) follows from $\mu_s^1>\mu_s^2$ and property (b). To prove property (b), we note that since $\kappa>\kappa^*$ in market 1 and $\kappa<\kappa^*$ in market 2, $E^1<1$ and $E^2>1$. Equation (38) then implies that

$$(r+\kappa^*)(\mu_s^1-\mu_s^2)+\lambda\frac{1}{1+z}(\mu_s^1\mu_b^2-\mu_s^2\mu_b^1)<0$$

$$\Rightarrow\ \lambda\frac{1}{1+z}(\mu_s^2\mu_b^1-\mu_s^1\mu_b^2)>(r+\kappa^*)(\mu_s^1-\mu_s^2)>0,$$

which, in turn, implies property (b).

We finally prove property (c). Substituting the price from equation (34), we have to prove that

$$\frac{1-\frac{r}{1+z}\frac{1}{r+\kappa+\lambda\mu_s^1\frac{z}{1+z}}}{1+\lambda\mu_b^1\frac{1}{1+z}E_b^1\left[\frac{1}{r+\kappa+\lambda\mu_s^1\frac{z}{1+z}}\right]}<\frac{1-\frac{r}{1+z}\frac{1}{r+\kappa+\lambda\mu_s^2\frac{z}{1+z}}}{1+\lambda\mu_b^2\frac{1}{1+z}E_b^2\left[\frac{1}{r+\kappa+\lambda\mu_s^2\frac{z}{1+z}}\right]}.$$

Dividing both sides by equation (37), we can write this inequality as $G(\mu_s^1)<G(\mu_s^2)$, where the function $G(\mu)$ is defined by

$$G(\mu)\equiv\frac{\left[1-\frac{r}{1+z}\frac{1}{r+\kappa+\lambda\mu\frac{z}{1+z}}\right]\left[r+\kappa^*+\lambda\mu\frac{z}{1+z}\right]}{\mu}.$$

Given that $\mu_s^1>\mu_s^2$, the inequality $G(\mu_s^1)<G(\mu_s^2)$ will follow if we show that $G(\mu)$ is decreasing. Simple calculations show that

$$G'(\mu)=-\frac{r+\kappa^*}{\mu^2}\left[1-\frac{r}{(1+z)\left(r+\kappa+\lambda\mu\frac{z}{1+z}\right)}-\frac{r\lambda\mu\frac{z}{1+z}\left(r+\kappa^*+\lambda\mu\frac{z}{1+z}\right)}{(1+z)\left(r+\kappa+\lambda\mu\frac{z}{1+z}\right)^2(r+\kappa^*)}\right].$$

The term in brackets in increasing in both κ and κ^*, and is equal to $z/(1+z)>0$ for $\kappa=\kappa^*=0$. Therefore, it is positive, and thus $G(\mu)$ is decreasing. ∎

Proof of Proposition 4: In a symmetric equilibrium, equation (37) must hold for all κ^*. This is equivalent to $\mu_s^1=\mu_s^2=\mu_s$ (from Lemma 1), and

$$\mu_b^1E_b^1\left[\frac{1}{r+\kappa+\lambda\mu_s\frac{z}{1+z}}\right]=\mu_b^2E_b^2\left[\frac{1}{r+\kappa+\lambda\mu_s\frac{z}{1+z}}\right]. \tag{42}$$

It is easy to check that there is a continuum of functions $\nu^1(\kappa)$ such that the two scalar equations $\mu_s^1=\mu_s^2=\mu_s$ and (42) hold. Additionally, plugging these equations in equation (34), we find $p^1(\kappa)=p^2(\kappa)$ for all κ. ∎

Instead of proving Lemma 2, we prove a more general lemma that (i) covers non-steady states (where population measures, expected utilities, and prices, vary on time), and (ii) does not require that the measures of inactive owners and sellers add up to the asset supply, as must be the case in equilibrium. We extend our welfare criterion to non-steady states as

$$\begin{aligned}\mathcal{W}_t \;\equiv\; & \sum_{i=1,2}\left[\int_{\underline{\kappa}}^{\overline{\kappa}}[v^i_{b,t}(\kappa)\hat{\mu}^i_{b,t}(\kappa)+v^i_{o,t}(\kappa)\hat{\mu}^i_{o,t}(\kappa)]d\kappa+v^i_{s,t}\mu^i_{s,t}\right.\\ & \left.+\int_t^{\infty}\left[\int_{\underline{\kappa}}^{\overline{\kappa}}v^i_{b,t'}(\kappa)\hat{f}(\kappa)\nu^i(\kappa)d\kappa\right]e^{-r(t'-t)}dt'\right],\end{aligned}$$

using the second subscript to denote time, and considering, for generality, welfare at any time t.

Lemma 4

$$\mathcal{W}_t=\sum_{i=1}^{2}\int_t^{\infty}\left[\delta(\mu^i_{o,t'}+\mu^i_{s,t'})-x\mu^i_{s,t'}\right]e^{-r(t'-t)}dt'. \tag{43}$$

Proof: To prove the lemma, it suffices to show that

$$\frac{d(\mathcal{W}_te^{-rt})}{dt}=-\sum_{i=1}^{2}\left[\delta(\mu^i_{o,t}+\mu^i_{s,t})-x\mu^i_{s,t}\right]e^{-rt}, \tag{44}$$

since we can integrate this equation to equation (43). Using the definition of $\mathcal{W}_t$, we find

$$\frac{d(\mathcal{W}_te^{-rt})}{dt}=\sum_{i=1,2}A^ie^{-rt},$$

where

$$\begin{aligned}A^i \;=\; & \int_{\underline{\kappa}}^{\overline{\kappa}}\left[\frac{dv^i_{b,t}(\kappa)}{dt}\hat{\mu}^i_{b,t}(\kappa)+v^i_{b,t}(\kappa)\frac{d\hat{\mu}^i_{b,t}(\kappa)}{dt}+\frac{dv^i_{o,t}(\kappa)}{dt}\hat{\mu}^i_{o,t}(\kappa)+v^i_{o,t}(\kappa)\frac{d\hat{\mu}^i_{o,t}(\kappa)}{dt}\right]d\kappa\\ & +\frac{dv^i_{s,t}}{dt}\mu^i_{s,t}+v^i_{s,t}\frac{d\mu^i_{s,t}}{dt}\\ & -r\left[\int_{\underline{\kappa}}^{\overline{\kappa}}[v^i_{b,t}(\kappa)\hat{\mu}^i_{b,t}(\kappa)+v^i_{o,t}(\kappa)\hat{\mu}^i_{o,t}(\kappa)]d\kappa+v^i_{s,t}\mu^i_{s,t}\right]\\ & -\int_{\underline{\kappa}}^{\overline{\kappa}}v^i_{b,t}(\kappa)\hat{f}(\kappa)\nu^i(\kappa)d\kappa.\end{aligned} \tag{45}$$

To simplify equation (45), we compute the derivatives of the population measures and of the expected utilities. The derivative of a population measure is equal to the difference between the inflow and the outflow associated to that population. Proceeding as in section 3.1, we find

$$\frac{d\hat{\mu}^i_{b,t}(\kappa)}{dt} = \hat{f}(\kappa)\nu^i(\kappa) - \kappa\hat{\mu}^i_{b,t}(\kappa) - \lambda\hat{\mu}^i_{b,t}(\kappa)\mu^i_{s,t}, \tag{46}$$

$$\frac{d\hat{\mu}^i_{o,t}(\kappa)}{dt} = \lambda\hat{\mu}^i_{b,t}(\kappa)\mu^i_{s,t} - \kappa\hat{\mu}^i_{o,t}(\kappa), \tag{47}$$

and

$$\frac{d\mu^i_{s,t}}{dt} = \int_{\underline{\kappa}}^{\overline{\kappa}} \left[\kappa\hat{\mu}^i_{o,t}(\kappa) - \lambda\hat{\mu}^i_{b,t}(\kappa)\mu^i_{s,t}\right] d\kappa. \tag{48}$$

To compute the derivatives of the expected utilities, consider, for example, $v_{b,t}(\kappa)$. For non-steady states, equation (7) generalizes to

$$v^i_{b,t}(\kappa) = (1 - rdt)\left[\kappa dt0 + \lambda\mu^i_s dt(v^i_{o,t}(\kappa) - p^i_t(\kappa)) + (1 - \lambda\mu^i_s dt - \kappa dt)v^i_{b,t+dt}(\kappa)\right]. \tag{49}$$

Rearranging, we find

$$rv^i_{b,t}(\kappa) - \frac{dv^i_{b,t}(\kappa)}{dt} = -\kappa v^i_{b,t}(\kappa) + \lambda\mu^i_{s,t}(v^i_{o,t}(\kappa) - p^i_t(\kappa) - v^i_{b,t}(\kappa)). \tag{50}$$

We similarly find

$$rv^i_{o,t}(\kappa) - \frac{dv^i_{o,t}(\kappa)}{dt} = \delta + \kappa(v^i_{s,t} - v^i_{o,t}(\kappa)), \tag{51}$$

and

$$rv^i_{s,t} - \frac{dv^i_{s,t}}{dt} = \delta - x + \int_{\underline{\kappa}}^{\overline{\kappa}} \lambda\hat{\mu}^i_{b,t}(\kappa)(p^i_t(\kappa) - v^i_{s,t})d\kappa. \tag{52}$$

Plugging equations (46)-(48) and (50)-(52) into (45), and canceling terms, we find

$$A^i = -\delta\mu^i_{o,t} - (\delta - x)\mu^i_{s,t},$$

which proves equation (44). ∎

Proof of Proposition 5: We only derive equation (16), as equations (17) and (18) can be derived using the same procedure. Suppose that at time t, the measure of buyers with switching rates in $[\kappa, \kappa + d\kappa]$ in market i, is increased by ϵ, while all other measures remain as in the steady state. That is,

$$\hat{\mu}^i_{b,t}(\kappa) = \hat{\mu}^i_b(\kappa) + \frac{\epsilon}{d\kappa}, \tag{53}$$

$\hat{\mu}^i_{b,t}(\kappa') = \hat{\mu}^i_b(\kappa')$ for $\kappa' \notin [\kappa, \kappa + d\kappa]$, $\hat{\mu}^i_{o,t}(\kappa') = \hat{\mu}^i_o(\kappa')$ for all κ', and $\mu^i_{s,t} = \mu^i_s$, where measures without the time subscript refer to the steady state.

We will determine the change in population measures at time $t + dt$. Consider first the buyers with switching rates in $[\kappa, \kappa + d\kappa]$. Equation (46) implies that

$$\hat{\mu}^i_{b,t+dt}(\kappa) = \hat{\mu}^i_{b,t}(\kappa) + \left[\hat{f}(\kappa)\nu^i(\kappa) - \kappa\hat{\mu}^i_{b,t}(\kappa) - \lambda\hat{\mu}^i_{b,t}(\kappa)\mu^i_{s,t}\right] dt. \tag{54}$$

Plugging equations (53) and $\mu^i_{s,t} = \mu^i_s$ into (54), and using the steady-state version of (54), i.e.,

$$\hat{f}(\kappa)\nu^i(\kappa) - \kappa\hat{\mu}^i_b(\kappa) - \lambda\hat{\mu}^i_b(\kappa)\mu^i_s = 0,$$

we find

$$\hat{\mu}^i_{b,t+dt}(\kappa) = \hat{\mu}^i_b(\kappa) + \frac{\epsilon}{d\kappa}(1 - \kappa dt - \lambda\mu^i_s dt).$$

Thus, the measure of buyers with switching rates in $[\kappa, \kappa + d\kappa]$, increases by $\epsilon(1 - \kappa dt - \lambda\mu^i_s dt) \equiv \epsilon\Delta^i_b(\kappa)$. In a similar manner, equation (47) implies that the measure of inactive owners with switching rates in $[\kappa, \kappa + d\kappa]$ increases by $\epsilon\lambda\mu^i_s dt \equiv \epsilon\Delta^i_o(\kappa)$, and equation (48) implies that the measure of sellers decreases by $\epsilon\lambda\mu^i_s dt \equiv \epsilon\Delta^i_s$. Finally, the measures of buyers and inactive owners with $\kappa' \notin [\kappa, \kappa + d\kappa]$ do not change in order dt.

Equation (43) implies that

$$\mathcal{W}_t = \sum_{i=1}^{2} \left[\delta(\mu^i_{o,t} + \mu^i_{s,t}) - x\mu^i_{s,t}\right] dt + (1 - rdt)\mathcal{W}_{t+dt}.$$

The derivative of $\mathcal{W}_t$ w.r.t. ϵ at $\epsilon = 0$ is $V^i_b(\kappa)$. The derivative of the term in brackets is zero, since $\mu^i_{o,t} = \mu^i_o$ and $\mu^i_{s,t} = \mu^i_s$. Finally, the derivative of $\mathcal{W}_{t+dt}$ is $\Delta^i_b(\kappa)V^i_b(\kappa) + \Delta^i_o(\kappa)V^i_o(\kappa) - \Delta^i_s V^i_s$. Thus,

$$V^i_b(\kappa) = (1 - rdt)\left[\Delta^i_b(\kappa)V^i_b(\kappa) + \Delta^i_o(\kappa)V^i_o(\kappa) - \Delta^i_s V^i_s\right].$$

Rearranging this equation, we find equation (16). ∎

Proof of Proposition 6: Equations (16)-(18) are the same as equations (19)-(21), except that $z/(1+z)$ and $1/(1+z)$ are both replaced by 1. Therefore, to compute $V_b^i(\kappa)$, we can follow the same steps as when computing $v_b^i(\kappa)$ in the proof of Proposition 1. Instead of equation (33), we now find

$$V_b^i(\kappa) = \frac{\lambda \mu_s^i x}{(r+\kappa)\left(r+\kappa+\lambda\mu_s^i\right)\left[1+\lambda\mu_b^i E_b^i\left[\frac{1}{r+\kappa+\lambda\mu_s^i}\right]\right]}.$$

Using this equation, the inequality $V_b^1(\kappa^*) < V_b^2(\kappa^*)$ is equivalent to

$$\frac{\mu_s^1}{r+\kappa^*+\lambda\mu_s^1+\lambda\mu_b^1 E_b^1\left[\frac{r+\kappa^*+\lambda\mu_s^1}{r+\kappa+\lambda\mu_s^1}\right]} < \frac{\mu_s^2}{r+\kappa^*+\lambda\mu_s^2+\lambda\mu_b^2 E_b^2\left[\frac{r+\kappa^*+\lambda\mu_s^2}{r+\kappa+\lambda\mu_s^2}\right]}. \tag{55}$$

Dividing both sides by equation (37), we obtain the equivalent inequality

$$\frac{r+\kappa^*+\lambda\mu_s^1\frac{z}{1+z}+\lambda\mu_b^1\frac{1}{1+z}E^1}{r+\kappa^*+\lambda\mu_s^1+\lambda\mu_b^1 E_b^1\left[\frac{r+\kappa^*+\lambda\mu_s^1}{r+\kappa+\lambda\mu_s^1}\right]} < \frac{r+\kappa^*+\lambda\mu_s^2\frac{z}{1+z}+\lambda\mu_b^2\frac{1}{1+z}E^2}{r+\kappa^*+\lambda\mu_s^2+\lambda\mu_b^2 E_b^2\left[\frac{r+\kappa^*+\lambda\mu_s^2}{r+\kappa+\lambda\mu_s^2}\right]}. \tag{56}$$

Since for $\kappa > \kappa^*$,

$$\frac{r+\kappa^*+\lambda\mu_s^1}{r+\kappa+\lambda\mu_s^1} > \frac{r+\kappa^*+\lambda\mu_s^1\frac{z}{1+z}}{r+\kappa+\lambda\mu_s^1\frac{z}{1+z}},$$

we have

$$E_b^1\left[\frac{r+\kappa^*+\lambda\mu_s^1}{r+\kappa+\lambda\mu_s^1}\right] > E^1.$$

We similarly have

$$E_b^2\left[\frac{r+\kappa^*+\lambda\mu_s^2}{r+\kappa+\lambda\mu_s^2}\right] < E^2.$$

Therefore, to show equation (56), it suffices to show that

$$\frac{r+\kappa^*+\lambda\mu_s^1\frac{z}{1+z}+\lambda\mu_b^1\frac{1}{1+z}E^1}{r+\kappa^*+\lambda\mu_s^1+\lambda\mu_b^1 E^1} < \frac{r+\kappa^*+\lambda\mu_s^2\frac{z}{1+z}+\lambda\mu_b^2\frac{1}{1+z}E^2}{r+\kappa^*+\lambda\mu_s^2+\lambda\mu_b^2 E^2},$$

which is equivalent to

$$(r+\kappa^*)\left[(\mu_s^1-\mu_s^2)\frac{1}{1+z}+(\mu_b^1E^1-\mu_b^2E^2)\frac{z}{1+z}\right]+\lambda(\mu_s^2\mu_b^1E^1-\mu_s^1\mu_b^2E^2)\frac{z-1}{1+z}>0. \quad (57)$$

Equation (38) implies that

$$\lambda\frac{1}{1+z}(\mu_s^2\mu_b^1E^1-\mu_s^1\mu_b^2E^2)=(r+\kappa^*)(\mu_s^1-\mu_s^2). \quad (58)$$

Using this equation to substitute $\mu_s^2\mu_b^1E^1-\mu_s^1\mu_b^2E^2$ in equation (57), we find the equivalent equation

$$(\mu_s^1-\mu_s^2)\frac{z^2}{1+z}+(\mu_b^1E^1-\mu_b^2E^2)\frac{z}{1+z}>0.$$

This equation holds because (i) $\mu_s^1>\mu_s^2$, and (ii) $\mu_b^1E^1>\mu_b^2E^2$ (which follows from equation (58) and $\mu_s^1>\mu_s^2$). ∎

Proof of Proposition 7: From Lemma 2, maximizing $\mathcal{W}$ is equivalent to minimizing $\mu_s^1+\mu_s^2$. We will first minimize $\mu_s^1+\mu_s^2$ through the choice of a "trigger" allocation, i.e., through the choice of a cutoff κ_w^* such that $\nu^1(\kappa)=1$ for $\kappa>\kappa_w^*$, and $\nu^1(\kappa)=0$ for $\kappa<\kappa_w^*$. We will show that this constrained problem, $(\mathcal{P}_c)$, has a unique solution. We will next show that this solution, together with the symmetric solution obtained by switching the indices of the two assets, are the only solutions to the unconstrained problem $(\mathcal{P})$.

Lemma 3 implies that the derivative of $\mu_s^1+\mu_s^2$ w.r.t. κ_w^* is

$$\frac{\lambda\hat{f}(\kappa_w^*)}{\kappa_w^*}\left[\frac{\mu_s^1}{\kappa_w^*+\lambda\mu_s^1+\int_{\kappa_w^*}^{\overline{\kappa}}\frac{\lambda\hat{f}(\kappa)(\kappa_w^*+\lambda\mu_s^1)}{(\kappa+\lambda\mu_s^1)^2}d\kappa}-\frac{\mu_s^2}{\kappa_w^*+\lambda\mu_s^2+\int_{\underline{\kappa}}^{\kappa_w^*}\frac{\lambda\hat{f}(\kappa)(\kappa_w^*+\lambda\mu_s^2)}{(\kappa+\lambda\mu_s^2)^2}d\kappa}\right]. \quad (59)$$

Multiplying by the denominators, we find that the term in brackets has the same sign as

$$F\equiv\mu_s^1-\mu_s^2+\mu_s^1\frac{1}{\kappa_w^*}\int_{\underline{\kappa}}^{\kappa_w^*}\frac{\lambda\hat{f}(\kappa)(\kappa_w^*+\lambda\mu_s^2)}{(\kappa+\lambda\mu_s^2)^2}d\kappa-\mu_s^2\frac{1}{\kappa_w^*}\int_{\kappa_w^*}^{\overline{\kappa}}\frac{\lambda\hat{f}(\kappa)(\kappa_w^*+\lambda\mu_s^1)}{(\kappa+\lambda\mu_s^1)^2}d\kappa.$$

Proceeding as in the existence proof of Proposition 2, we can show that there exists $\kappa_w^*\in(\underline{\kappa},\overline{\kappa})$ such that $F=0$, and moreover, that for any κ_w^* solving $F=0$, we have $\mu_s^1>\mu_s^2$.

Proceeding as in the uniqueness proof of Proposition 2, we can show that for any κ_w^* solving $F=0$, the derivative of F w.r.t. κ_w^* is positive. This implies that κ_w^* is unique. It also implies that F is negative and then positive, and thus κ_w^* corresponds to a minimum of $\mu_s^1+\mu_s^2$.

To show that the solution to $(\mathcal{P}_c)$, together with its symmetric solution, are the only solutions to $(\mathcal{P})$, we proceed by contradiction, and assume that there is a third allocation $\nu^1(\kappa)$ for which $\mu_s^1+\mu_s^2$ is weakly lower. Since assets 1 and 2 are symmetric, we can assume that $\nu^1(\kappa)$ satisfies $\mu_s^1 \geq \mu_s^2$. Given the first part of the proof, we can also assume that $\nu^1(\kappa)$ is not a trigger allocation. Define κ' by

$$\int_{\underline{\kappa}}^{\kappa'} \frac{\hat{f}(\kappa)}{\kappa(\kappa+\lambda\mu_s^2)}d\kappa = \int_{\underline{\kappa}}^{\overline{\kappa}} \frac{\hat{f}(\kappa)\nu^2(\kappa)}{\kappa(\kappa+\lambda\mu_s^2)}d\kappa, \tag{60}$$

and consider the corresponding trigger allocation. From the definition of κ', the measure μ_s^2 of sellers in market 2 under the allocation $\nu^1(\kappa)$, solves also equation (6) under the new allocation. Therefore, this measure is the same under both allocations. We next show that

$$\int_{\kappa'}^{\overline{\kappa}} \frac{\hat{f}(\kappa)}{\kappa(\kappa+\lambda\mu_s^1)}d\kappa \geq \int_{\underline{\kappa}}^{\overline{\kappa}} \frac{\hat{f}(\kappa)\nu^1(\kappa)}{\kappa(\kappa+\lambda\mu_s^1)}d\kappa, \tag{61}$$

and that the inequality is strict if $\mu_s^1 > \mu_s^2$. This will imply that the measure μ_s^1 of sellers in market 1 under $\nu^1(\kappa)$ is greater than the solution to equation (6) under the new allocation, and strictly so if $\mu_s^1 > \mu_s^2$. This will contradict the fact that $\nu^1(\kappa)$ is a solution to $(\mathcal{P})$. (The contradiction for $\mu_s^1 = \mu_s^2$ is because $\nu^1(\kappa)$ will be equivalent to a trigger allocation with $\mu_s^1=\mu_s^2$, but such an allocation is not optimal from the first part of the proof.)

Equation (60) implies that

$$\int_{\kappa'}^{\overline{\kappa}} \frac{\hat{f}(\kappa)}{\kappa(\kappa+\lambda\mu_s^2)}d\kappa = \int_{\underline{\kappa}}^{\overline{\kappa}} \frac{\hat{f}(\kappa)\nu^1(\kappa)}{\kappa(\kappa+\lambda\mu_s^2)}d\kappa. \tag{62}$$

Equation (62), together with the fact that $\nu^1(\kappa)$ gives non-zero weight to values below κ' (since it is not a trigger allocation), and the fact that the function $1/(\kappa+\lambda\mu_s^1)$ is decreasing in κ, imply that

$$\int_{\kappa'}^{\overline{\kappa}} \frac{\hat{f}(\kappa)}{\kappa(\kappa+\lambda\mu_s^2)(\kappa+\lambda\mu_s^1)}d\kappa < \int_{\underline{\kappa}}^{\overline{\kappa}} \frac{\hat{f}(\kappa)\nu^1(\kappa)}{\kappa(\kappa+\lambda\mu_s^2)(\kappa+\lambda\mu_s^1)}d\kappa. \tag{63}$$

Multiplying equation (63) by $\lambda(\mu_s^1 - \mu_s^2)$, and subtracting it from equation (62), we obtain equation (61). ∎

Proof of Proposition 8: Consider the trigger allocation with cutoff $\hat{\kappa}$, and denote by $\mathcal{W}(\hat{\kappa})$ the welfare under this allocation. From Proposition 7, $\mathcal{W}(\hat{\kappa})$ is increasing for $\hat{\kappa} < \kappa_w^*$, and decreasing for $\hat{\kappa} > \kappa_w^*$.

We next show that the cutoff κ^* in the clientele equilibrium satisfies $\kappa^* < \kappa_w^*$. For this, it suffices to show that the function F of Proposition 7 is negative for κ^*. Using equation (59), and noting that

$$E_b^i\left[\frac{1}{\kappa + \lambda\mu_s^i}\right] = \frac{1}{\mu_b^i}\int_{\underline{\kappa}}^{\overline{\kappa}} \frac{\hat{f}(\kappa)}{\kappa + \lambda\mu_s^i}\hat{\mu}_b^i(\kappa)d\kappa,$$

equation $F < 0$ is equivalent to

$$\frac{\mu_s^1}{\kappa^* + \lambda\mu_s^1 + \lambda\mu_b^1 E_b^1\left[\frac{\kappa^*+\lambda\mu_s^1}{\kappa+\lambda\mu_s^1}\right]} < \frac{\mu_s^2}{\kappa^* + \lambda\mu_s^2 + \lambda\mu_b^2 E_b^2\left[\frac{\kappa^*+\lambda\mu_s^2}{\kappa+\lambda\mu_s^2}\right]}.$$

For small r, this equation is equivalent to (55), which holds from Proposition 6.

The cutoff κ' for which the measure of sellers is the same across the two markets obviously satisfies $\kappa' < \kappa^*$. Since $\kappa' < \kappa^* < \kappa_w^*$, and the function $\mathcal{W}(\hat{\kappa})$ is increasing for $\hat{\kappa} < \kappa_w^*$, we have $\mathcal{W}(\kappa') < \mathcal{W}(\kappa^*)$. Since, in addition, $\mathcal{W}(\kappa')$ is equal to the welfare $\mathcal{W}_s$ in any symmetric equilibrium (by the argument in Footnote 16), we have $\mathcal{W}_s < \mathcal{W}(\kappa^*)$. ∎

Proof of Proposition 9: To show that a clientele equilibrium exists, it suffices to show that there exists κ^* such that when (a) entry decisions are given by $\nu^1(\kappa) = 1$ for $\kappa > \kappa^*$, and $\nu^1(\kappa) = 0$ for $\kappa < \kappa^*$, (b) population measures are given as in Section 3.1, and (c) expected utilities and prices are given by equations (9) and (22)-(24), $v_b^1(\kappa) - v_b^2(\kappa)$ has the same sign as $\kappa - \kappa^*$. Simple algebra shows that the solution to the system of equations (9) and (22)-(24) is

$$v_b^i(\kappa) = \frac{\lambda\mu_s^i\left[\frac{r}{r+\kappa}\left(r + \kappa^i + \lambda\mu_s^i\frac{z}{1+z}\right) - \frac{r}{1+z}\right]x}{r\left(r + \kappa + \lambda\mu_s^i\right)\left(r + \kappa^i + \lambda\mu_b^i\frac{1}{1+z} + \lambda\mu_s^i\frac{z}{1+z}\right)}, \tag{64}$$

$$v_o^i(\kappa) = \frac{\delta}{r} - \frac{\kappa\left(r + \kappa^i + \lambda\mu_s^i \frac{z}{1+z}\right)x}{r(r+\kappa)\left(r + \kappa^i + \lambda\mu_b^i \frac{1}{1+z} + \lambda\mu_s^i \frac{z}{1+z}\right)}, \tag{65}$$

$$v_s^i = \frac{\delta}{r} - \frac{\left(r + \kappa^i + \lambda\mu_s^i \frac{z}{1+z}\right)x}{r\left(r + \kappa^i + \lambda\mu_b^i \frac{1}{1+z} + \lambda\mu_s^i \frac{z}{1+z}\right)}, \tag{66}$$

$$p^i = \frac{\delta}{r} - \frac{\left(r\frac{z}{1+z} + \kappa^i + \lambda\mu_s^i \frac{z}{1+z}\right)x}{r\left(r + \kappa^i + \lambda\mu_b^i \frac{1}{1+z} + \lambda\mu_s^i \frac{z}{1+z}\right)}. \tag{67}$$

We next show that there exists κ^* such that $v_b^1(\kappa^*) = v_b^2(\kappa^*)$. For this, we treat κ^* as a free variable, κ^i, μ_s^i, and μ_b^i, as functions of κ^*, and look for a solution to equation

$$v_b^1(\kappa^*) - v_b^2(\kappa^*) = 0. \tag{68}$$

The LHS of this equation is negative for $\kappa^* = \kappa'$, from condition (C). To show that it is positive for $\kappa^* = \overline{\kappa}$, we write it as

$$H(\kappa^1, \mu_s^1, \mu_b^1) - H(\kappa^2, \mu_s^2, \mu_b^2),$$

where the function H is defined by

$$H(\ell, \mu_s, \mu_b) \equiv \frac{\lambda\mu_s\left[\frac{r}{r+\kappa^*}\left(r + \ell + \lambda\mu_s\frac{z}{1+z}\right) - \frac{r}{1+z}\right]}{(r + \kappa^* + \lambda\mu_s)\left(r + \ell + \lambda\mu_b\frac{1}{1+z} + \lambda\mu_s\frac{z}{1+z}\right)},$$

and is increasing in ℓ and μ_s, and decreasing in μ_b. For $\kappa^* = \overline{\kappa}$, we have $\kappa^1 = \kappa^2 = \overline{\kappa}$, $\mu_s^1 = S$, $\mu_s^2 < S$, $\mu_b^1 = 0$, and $\mu_b^2 > 0$. Therefore, the LHS of equation (68) is positive, and thus this equation has a solution $\kappa^* \in (\kappa', \overline{\kappa})$. Since $\kappa^* > \kappa'$, and since μ_s^1 is increasing in κ^* and μ_s^2 is decreasing in κ^*, we have $\mu_s^1 > \mu_s^2$.

We next consider the sign of $v_b^1(\kappa) - v_b^2(\kappa)$. This sign is the same as that of

$$\frac{v_b^1(\kappa)}{v_b^1(\kappa^*)} - \frac{v_b^2(\kappa)}{v_b^2(\kappa^*)},$$

and from equation (64) it is the same as that of

$$\frac{\left(r + \overline{\kappa} + \lambda\mu_s^1\frac{z}{1+z} - \frac{r+\kappa}{1+z}\right)(r + \kappa^* + \lambda\mu_s^1)}{\left(r + \overline{\kappa} + \lambda\mu_s^1\frac{z}{1+z} - \frac{r+\kappa^*}{1+z}\right)(r + \kappa + \lambda\mu_s^1)} - \frac{\left(r + \kappa^* + \lambda\mu_s^2\frac{z}{1+z} - \frac{r+\kappa}{1+z}\right)(r + \kappa^* + \lambda\mu_s^2)}{\left(r + \kappa^* + \lambda\mu_s^2\frac{z}{1+z} - \frac{r+\kappa^*}{1+z}\right)(r + \kappa + \lambda\mu_s^2)}.$$

Multiplying by the denominators, this has the same sign as

$$\left(r+\kappa^*+\lambda\mu_s^2\right)\left[\left(r+\overline{\kappa}+\lambda\mu_s^1\frac{z}{1+z}-\frac{r+\kappa}{1+z}\right)\left(r+\kappa^*+\lambda\mu_s^1\right)\left(r+\kappa+\lambda\mu_s^2\right)\frac{z}{1+z}\right.$$
$$\left.-\left(r+\overline{\kappa}+\lambda\mu_s^1\frac{z}{1+z}-\frac{r+\kappa^*}{1+z}\right)\left(r+\kappa+\lambda\mu_s^1\right)\left(r+\kappa^*+\lambda\mu_s^2\frac{z}{1+z}-\frac{r+\kappa}{1+z}\right)\right].$$

Simple algebra shows that the term in brackets is equal to

$$(\kappa-\kappa^*)\left[(\overline{\kappa}-\kappa^*)\frac{1}{1+z}(r+\kappa+\lambda\mu_s^1)+\lambda(\mu_s^1-\mu_s^2)\frac{z}{1+z}(r+\overline{\kappa}+\lambda\mu_s^1)\right],$$

and has the same sign as $\kappa-\kappa^*$, since $\mu_s^1>\mu_s^2$. Therefore, a clientele equilibrium exists.

We next prove properties (a)-(c). Property (a) follows from $\mu_s^1>\mu_s^2$ and property (b). To prove property (b), we note that since the function H is increasing in ℓ,

$$H(\kappa^*,\mu_s^1,\mu_b^1)<H(\overline{\kappa},\mu_s^1,\mu_b^1)=v_b^1(\kappa^*)=v_b^2(\kappa^*)=H(\kappa^*,\mu_s^2,\mu_b^2).$$

Since

$$H(\kappa^*,\mu_s,\mu_b)=\frac{\lambda\mu_s\frac{r}{r+\kappa^*}\frac{z}{1+z}}{r+\kappa^*+\lambda\mu_b\frac{1}{1+z}+\lambda\mu_s\frac{z}{1+z}},$$

we can write the above inequality as

$$\frac{\mu_s^1}{r+\kappa^*+\lambda\mu_b^1\frac{1}{1+z}+\lambda\mu_s^1\frac{z}{1+z}}<\frac{\mu_s^2}{r+\kappa^*+\lambda\mu_b^2\frac{1}{1+z}+\lambda\mu_s^2\frac{z}{1+z}}.$$

Multiplying by the denominators, and using the property $\mu_s^1>\mu_s^2$, we find property (b).

We finally prove property (c). Substituting the price from equation (67), we have to prove that

$$\frac{r\frac{z}{1+z}+\kappa^1+\lambda\mu_s^1\frac{z}{1+z}}{r+\kappa^1+\lambda\mu_b^1\frac{1}{1+z}+\lambda\mu_s^1\frac{z}{1+z}}<\frac{r\frac{z}{1+z}+\kappa^2+\lambda\mu_s^2\frac{z}{1+z}}{r+\kappa^2+\lambda\mu_b^2\frac{1}{1+z}+\lambda\mu_s^2\frac{z}{1+z}}.$$

Dividing both sides by equation $v_b^1(\kappa^*)=v_b^2(\kappa^*)$, we can write this inequality as $G(\kappa^1,\mu_s^1)<G(\kappa^2,\mu_s^2)$, where the function $G(\ell,\mu)$ is defined by

$$G(\ell,\mu)\equiv\frac{\left(r\frac{z}{1+z}+\ell+\lambda\mu\frac{z}{1+z}\right)(r+\kappa^*+\lambda\mu)}{\left(r+\ell+\lambda\mu\frac{z}{1+z}-\frac{r+\kappa^*}{1+z}\right)\lambda\mu}.$$

Since $G(\ell, \mu)$ is decreasing in ℓ and μ, and since $\kappa^1 > \kappa^2$ and $\mu_s^1 > \mu_s^2$, inequality $G(\kappa^1, \mu_s^1) < G(\kappa^2, \mu_s^2)$ holds. ∎

Proof of Proposition 10: We first show Condition (25). This condition can be written as

$$\begin{aligned} & v_o^i(\kappa^i) - v_b^i(\kappa^i) \geq \left[P_b^i(\kappa)(v_o^i(\kappa) - v_b^i(\kappa)) + (1 - P_b^i(\kappa))v_s^i\right] \\ \Leftrightarrow \quad & (1 - P_b^i(\kappa))(v_o^i(\kappa) - v_b^i(\kappa) - v_s^i) \geq v_o^i(\kappa) - v_b^i(\kappa) - (v_o^i(\kappa^i) - v_b^i(\kappa^i)). \end{aligned} \tag{69}$$

To show that this equation holds, we use equations (64)-(66). Combining equations (64) and (65), we find

$$v_o^i(\kappa) - v_b^i(\kappa) = \frac{\delta}{r} - \frac{\left[\left(r + \kappa^i + \lambda\mu_s^i\frac{z}{1+z}\right)(\kappa + \lambda\mu_s^i) - \lambda\mu_s^i\frac{r}{1+z}\right]x}{r\left(r + \kappa + \lambda\mu_s^i\right)\left(r + \kappa^i + \lambda\mu_b^i\frac{1}{1+z} + \lambda\mu_s^i\frac{z}{1+z}\right)}, \tag{70}$$

which for $\kappa = \kappa^i$ simplifies into

$$v_o^i(\kappa^i) - v_b^i(\kappa^i) = \frac{\delta}{r} - \frac{\left(\kappa^i + \lambda\mu_s^i\frac{z}{1+z}\right)x}{r\left(r + \kappa^i + \lambda\mu_b^i\frac{1}{1+z} + \lambda\mu_s^i\frac{z}{1+z}\right)}. \tag{71}$$

Equations (66) and (70) imply that

$$v_o^i(\kappa) - v_b^i(\kappa) - v_s^i = \frac{(r + \kappa^i + \lambda\mu_s^i)\,x}{(r + \kappa + \lambda\mu_s^i)\left(r + \kappa^i + \lambda\mu_b^i\frac{1}{1+z} + \lambda\mu_s^i\frac{z}{1+z}\right)},$$

and equations (70) and (71) imply that

$$v_o^i(\kappa) - v_b^i(\kappa) - (v_o^i(\kappa^i) - v_b^i(\kappa^i)) = \frac{r(\kappa^i - \kappa)x}{r\left(r + \kappa + \lambda\mu_s^i\right)\left(r + \kappa^i + \lambda\mu_b^i\frac{1}{1+z} + \lambda\mu_s^i\frac{z}{1+z}\right)}.$$

Equation (69) is thus equivalent to

$$\frac{1 - P_b^i(\kappa)}{\kappa^i - \kappa}\left(r + \kappa^i + \lambda\mu_s^i\right) \geq 1. \tag{72}$$

To show that equation (72) holds for S large enough, we determine the asymptotic behavior of μ_s^i. Equations (1) and (3) imply that

$$\mu_b^i = \int_{\ell^i}^{\kappa^i} \frac{S\hat{F}(\kappa)}{\kappa + \lambda\mu_s^i}d\kappa \tag{73}$$

and equation (6) implies that

$$\int_{\ell^i}^{\kappa^i} \frac{\lambda \mu_s^i S \hat{F}(\kappa)}{\kappa(\kappa + \lambda \mu_s^i)} d\kappa + \mu_s^i = S, \tag{74}$$

where ℓ^i denotes the minimum switching rate of a buyer in market i (i.e., $\ell^1 = \kappa^*$ and $\ell^2 = \underline{\kappa}$). Adding equation (74) for $i = 1$ to the same equation for $i = 2$, and using equation (26), we find

$$-\int_{\kappa^*}^{\overline{\kappa}} \frac{S \hat{F}(\kappa)}{\kappa + \lambda \mu_s^1} d\kappa - \int_{\underline{\kappa}}^{\kappa^*} \frac{S \hat{F}(\kappa)}{\kappa + \lambda \mu_s^2} d\kappa + \mu_s^1 + \mu_s^2 = 0. \tag{75}$$

Suppose now that μ_s^1 goes to a finite limit when S goes to ∞. Equation (75) implies that μ_s^2 is of order S, and equation (73) implies that μ_b^1 is of order S and μ_b^2 is of order 1. Equation (23) then implies that $v_b^1(\kappa^*)$ goes to zero and $v_b^2(\kappa^*)$ goes to $x/(r+\kappa^*) > 0$, which is inconsistent with $v_b^1(\kappa^*) = v_b^2(\kappa^*)$. Therefore, μ_s^i must go to ∞ when S goes to ∞, and

$$P_b^i(\kappa) = \frac{\int_{\ell^i}^{\kappa} \frac{S \hat{F}(y)}{y + \lambda \mu_s^i} dy}{\int_{\ell^i}^{\kappa^i} \frac{S \hat{F}(y)}{y + \lambda \mu_s^i} dy} \to \frac{\int_{\ell^i}^{\kappa} \hat{F}(y) dy}{\int_{\ell^i}^{\kappa^i} \hat{F}(y) dy}.$$

Equation (72) thus holds for S large enough if the function

$$L^i(\kappa) \equiv \frac{1}{\kappa^i - \kappa} \frac{\int_{\kappa}^{\kappa^i} \hat{F}(y) dy}{\int_{\ell^i}^{\kappa^i} \hat{F}(y) dy}$$

is bounded away from zero for $\kappa \in [\ell^i, \kappa^i]$. This follows because L^i is continuous in the compact set $[\ell^i, \kappa^i]$ (it can be extended by continuity for $\kappa = \kappa^i$), and is strictly positive since $\hat{F}(\kappa) > 0$.

We next show Condition (C). From the definition of κ', we have $\mu_s^1 = \mu_s^2 \equiv \mu_s$. Using equation (64), we can then write inequality $v_b^1(\kappa') < v_b^2(\kappa')$ as

$$\frac{r + \overline{\kappa} + \lambda \mu_s \frac{z}{1+z} - \frac{r+\kappa'}{1+z}}{r + \overline{\kappa} + \lambda \mu_b^1 \frac{1}{1+z} + \lambda \mu_s \frac{z}{1+z}} < \frac{r + \kappa' + \lambda \mu_s \frac{z}{1+z} - \frac{r+\kappa'}{1+z}}{r + \kappa' + \lambda \mu_b^2 \frac{1}{1+z} + \lambda \mu_s \frac{z}{1+z}}.$$

Simple algebra shows that this equation is equivalent to

$$\lambda(\mu_b^1 - \mu_b^2)\frac{z}{1+z}(r + \kappa' + \lambda\mu_s) > (\overline{\kappa} - \kappa')(r + \kappa' + \lambda\mu_b^2). \tag{76}$$

To show that equation (76) holds for S large enough, we determine the asymptotic behavior of μ_s and μ_b^i. For $\mu_s^1 = \mu_s^2 \equiv \mu_s$, equation (75) becomes

$$-\int_{\underline{\kappa}}^{\overline{\kappa}} \frac{S\hat{F}(\kappa)}{\kappa + \lambda\mu_s} d\kappa + 2\mu_s = 0,$$

and it implies that

$$\mu_s \sim \sqrt{\frac{S\int_{\underline{\kappa}}^{\overline{\kappa}} \hat{F}(\kappa)d\kappa}{2\lambda}}.$$

Equation (73) then implies that

$$\mu_b^i \sim \sqrt{\frac{2S}{\lambda\int_{\underline{\kappa}}^{\overline{\kappa}} \hat{F}(\kappa)d\kappa}} \int_{\ell^i}^{\kappa^i} \hat{F}(\kappa)d\kappa.$$

Therefore, the LHS of equation (76) is of order

$$S\lambda\frac{z}{1+z}\left(\int_{\kappa'}^{\overline{\kappa}} \hat{F}(\kappa)d\kappa - \int_{\underline{\kappa}}^{\kappa'} \hat{F}(\kappa)d\kappa\right), \tag{77}$$

while the RHS is of order $\sqrt{S}$. Equation (76) thus holds if the term in parenthesis in equation (77) is strictly positive. Since μ_s goes to ∞ when S goes to ∞, equation (74) implies that κ' goes to a limit such that

$$\int_{\kappa'}^{\overline{\kappa}} \frac{\hat{F}(\kappa)}{\kappa} d\kappa = \int_{\underline{\kappa}}^{\kappa'} \frac{\hat{F}(\kappa)}{\kappa} d\kappa = 1.$$

Therefore,

$$\int_{\kappa'}^{\overline{\kappa}} \hat{F}(\kappa)d\kappa > \kappa' \int_{\kappa'}^{\overline{\kappa}} \frac{\hat{F}(\kappa)}{\kappa} d\kappa = \kappa' \int_{\underline{\kappa}}^{\kappa'} \frac{\hat{F}(\kappa)}{\kappa} d\kappa > \int_{\underline{\kappa}}^{\kappa'} \hat{F}(\kappa)d\kappa.$$

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