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Portfolio credit risk of default and spread widening

Research paper

Portfolio Credit Risk of Default and Spread Widening

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Abstract

This paper introduces a new model for portfolio credit risk incorporating default and spread widening in a simple and consistent framework. Credit spreads are modelled by geometric Brownian motions with a dependence structure powered by a $t-$copula. Their joint evolution drives the spreads widening and triggers defaults, and then the loss can be calculated accordingly. It is a heterogeneous model that takes account of different credit ratings and term structures for each underlying spread. This model is applicable to portfolio credit risk management, stress test, or to fit into regulatory capital requirements. The procedures of parameter calibration and scenario simulation are provided. A detailed example is also given to see how this proposed model can be implemented in practice.

Keywords: Portfolio credit risk, Stress test, Economic capital, Default risk, Spread widening risk, Copula, Basel III
JEL: Primary: G32; Secondary: G18; C30; C51
2010 MSC: 91G40

1. Introduction

Credit risk and market are intrinsically related to each other and, more importantly, they are not separable (Jarrow and Turnbull 2000). During the recent financial crisis, especially after the collapse of Lehman Brothers in September 2008, the interaction of credit risk and market becomes more evident. It has exposed some severe problems in the original systems of risk assessment, management and supervision, due to inadequate recognition of this interaction. This is also one of the main issues that have pushed G20 to propose a major agreement of the *Basel III* for a global banking reform. It is a new global regulatory standard on bank capital adequacy and liquidity, and will impose new capital requirements on the world's banking system, in an effort to strengthen the ability of absorbing shocks from economic stress and to avoid future financial meltdowns.

Modelling the credit risk itself has already been a challenge, as more credit products are more frequently bought and sold, instead of being held to maturity. Nowadays, researchers, practitioners and regulators start to explore the complexity of the interactive behavior of credit risk and market, and attempt to develop more sophisticated models to capture the credit risk and

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other forms of risk in a consistent methodological framework. Simply calculating the market risk and credit risk separately and then summing them up could either underestimate or overestimate the overall risk involved (BCBS, 2009). The evidences were provided by Breuer, Jandačka, Rheinberger and Summer (2010), and Alessandri and Drehmann (2010). Gupton, Finger and Bhatia (1997) developed a mark-to-market credit migration model, *CreditMetrics*, widely used in practice for portfolio credit risk management and economic capital calculation. Kupiec (2007) extended the *CreditMetrics* model to additionally incorporate the valuation effects of market risk on non-default credits. Altman, Brady, Resti and Sironi (2005) and Bruche and González-Aguado (2010) investigated the negatively correlated interaction between default and recovery rate. Alessandri and Drehmann (2010) implemented the stress test by integrating the default and interest rate risk. Tang and Yan (2010) analysed the impact of the interaction between market and default on corporate credit spreads based on macroeconomic factors, such as the GDP growth rate and its volatility, and consumer confidence. In particular, inspiring discussion and comprehensive summary on this issue can also be found from the conference on "Interaction of Market and Credit Risk" (IMCR), held by *Deutsche Bundesbank* and the working group of Research Task Force established by *Basel Committee on Banking Supervision* (BCBS) in Berlin in 2007 (Hartmann, 2010; BCBS, 2008).

In this paper, we attempt to model the interaction of default risk and spread widening risk, which has been often neglected before the recent financial crisis. Credit spread risk as defined by BCBS (2008) is the risk of potential loss due to a change in an instrument's credit spread (defined as the instrument's yield relative to that of a comparable-duration default-free instrument) that is not attributable to defaults or credit migrations (e.g. a change in liquidity premia). During the recent credit market turmoil, banks were hit hard by these structured credit instruments they held in their trading books, with significant losses coming from spread widening (Madigan 2010). Calculating this risk has been required by new financial capital regulation, in particular, the incremental risk charge (IRC). The IRC expands the scope of the capital charge to capture not only price changes due to defaults but also other sources of price risk, such as significant moves of credit spreads (BCBS, 2008). Banks must conduct stress tests that include widening credit spreads in recessionary scenarios, as also proposed by Basel III.

This paper introduces a new portfolio credit risk model incorporating default and spread widening in one consistent framework over a single horizon. It is applicable to credit risk management, stress test, or to fit into the new regulatory requirements being raised from the recent financial crisis, in particular, the IRC. This model is similar to the seminal Merton (1974)'s model, a structural model based on an unobservable underlying firm's value. The main difference is that we assume the underlying processes are the firms' credit spreads rather than the firms' values, and they have a dependence structure powered by a copula. Their joint evolution drives the spreads widening and triggers defaults simultaneously, and then the loss can be calculated accordingly. The credit spreads are observable from credit market and can be calibrated from historical data. They are also closely linked to the underlying default risk by implying the default probability under risk-neutral measure (Hull, Predescu and White, 2005). This is a simple heterogeneous model that can take account of different credit ratings and term structures for each underlying spread, and also can integrate different marginal distributions for credit spreads with a flexible dependence structure via a copula function. The setting of heterogeneity is similar to the *CreditMetrics* model by Gupton, Finger and Bhatia (1997). The parameters can be calibrated from the historical daily time series of corporate bond spreads, and our model then can be implemented by Monte Carlo simulation for a bond portfolio in practice.

The paper is organised as follows. Section 2 gives a mathematical description of our framework, including the marginal distribution for each credit spread in Section 2.1, and a dependence structure for all spreads in a bond portfolio in Section 2.2. Based on this common framework, the credit risk of spread widening and default can be modelled respectively in Section 3.1 and Section 3.2, and an integrated model of overall credit risk is given in Section 3.3. We use the database of historical corporate bond spreads as given in Section 4 and calibrate parameters in Section 5. In Section 6 we provide a detailed example to demonstrate how this proposed model can be implemented in industrial practice. Section 7 concludes this paper and gives some suggestions for future research.

2. Model Framework

The whole framework for modelling the joint evolution of credit spreads consists of two parts:

1. a marginal distribution for the spread evolution of each bond in the portfolio (in Section 2.1);
2. a copula function for the dependence structure of these spreads (in Section 2.2).

We have investigated this framework by using different types of marginal distributions and copulas. To illustrate the modelling idea, in this paper, we only use log-normal marginal distributions with a $t-$copula dependency as an example.

2.1. Marginal Distribution for the Individual Spread

There are N bonds in the portfolio, and we assume $\left\{S_t^{<j>}\right\}_{t\geq 0}$, the spread of bond $j \in \{1, 2, ..., N\}$, follows a geometric Brownian motion,

$$\frac{\mathrm{d}S_t^{<j>}}{S_t^{<j>}} = \mu_j \mathrm{d}t + \sigma_j \mathrm{d}W_t^{<j>}$$

where the drift μ_j, volatility σ_j are positive constants, and $W_t^{<j>}$ is a Brownian motion. It is well known that the analytic solution is given by

$$S_t^{<j>} = S_0^{<j>} \exp\left[\left(\mu_j - \frac{1}{2}\sigma_j^2\right) t + \sigma_j \mathrm{d}W_t^{<j>}\right]$$

and $S_t^{<j>}$ follows the log-normal distribution,

$$\ln S_t^{<j>} \sim \mathcal{N}\left(\ln S_0^{<j>} + \left(\mu_j - \frac{1}{2}\sigma_j^2\right) t, \sigma_j^2 t\right).$$

Since $\mu_j - \frac{1}{2}\sigma_j^2$ is very small[1], we simply assume $\mu_j - \frac{1}{2}\sigma_j^2 \approx 0$, then,

$$\ln S_t^{<j>} \sim \mathcal{N}\Big(\ln S_0^{<j>}, \sigma_j^2 t\Big),$$

[1] This can be verified by the observation from the spread database in Section 4.

namely,

$$S_t^{<j>} = S_0^{<j>} e^{\sigma_j \sqrt{t} Z_j} \tag{1}$$

where $Z_j \sim \mathcal{N}(0,1)$.

To build a heterogeneous model that can take account of different credit ratings and term structures for each bond in the portfolio, we assume the parameter volatility is a function of credit rating $\mathcal{R}$ and maturity bucket $\mathcal{T}$, i.e.

$$\sigma_j = \sigma(\mathcal{R}, \mathcal{T})$$

where $\mathcal{R} \in \{AAA, AA, A, BBB\}$[2], and $\mathcal{T} \in \{00Y-05Y, 05Y-10Y, 10Y-15Y, 15Y-20Y, 20Y^{+}\}$ grouped according to the bond's time to maturity.

2.2. *Dependence Structure for All Spreads*

We adopt the copula approach for modelling the dependence structure because of its flexibility of incorporating different marginal distributions and the ease of simulation (Li, 2000; Nelsen, 2006; Kole, Koedijk and Verbeek, 2007). There are a variety of other techniques for dependency modelling in the literature, such as the methodology of credit contagion introduced by Jarrow and Yu (2001), Errais, Giesecke and Goldberg (2009), and more recently Dassios and Zhao (2011).

The dependence structure of all spreads $\vec{S}_t =: \left(S_t^{<1>}, ..., S_t^{<N>}\right)'$ given by (1) is built via a vector of dependent standard normal distributed random variables $\vec{Z} =: (Z_1, ..., Z_N)'$, with a dependence structure following a $t-$copula parameterised by a $N \times N$ correlation matrix

$$\Sigma =: \begin{bmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,N} \\ \rho_{2,1} & 1 & \cdots & \rho_{2,N} \\ \vdots & & \ddots & \vdots \\ \rho_{N-1,1} & \rho_{N-1,2} & \cdots & \rho_{N-1,N} \\ \rho_{N,1} & \rho_{N,2} & \cdots & 1 \end{bmatrix} \tag{2}$$

and degree of freedom ν. $\rho_{i,j}$ is the correlation coefficient for bond i and j ($i \neq j$) in the portfolio.

We adopt the procedure by Romano (2002) to construct the vector $\vec{Z}$ as follows:

1. generate a vector of independent random variables $\vec{X} =: (X_1, ..., X_N)'$ where $X_j \sim \mathcal{N}(0,1)$;
2. obtain a vector of dependent random variables $\vec{Y} =: (Y_1, ..., Y_N)'$ with a joint distribution $\Phi_\Sigma(\vec{Y})$ where

$$\vec{Y} = \mathbb{A}\vec{X},$$

   and matrix $\mathbb{A}$ is the Cholesky decomposition of Σ, i.e.

$$\Sigma = \mathbb{A}\mathbb{A}^{\mathrm{T}};$$

[2] We assume each bond is in the investment grade.

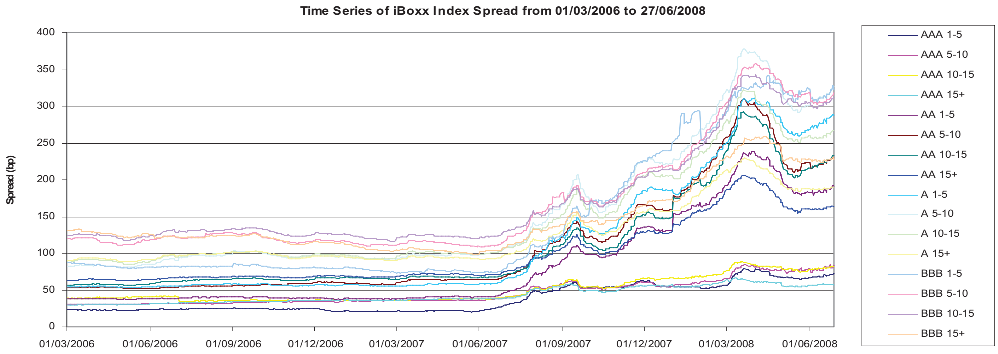


Figure 1: The Historical Time Series of Benchmark Spreads of iBoxx Index *01/03/2006-27/06/2008*

3. obtain a vector of dependent random variables $\vec{t} =: (t_1, ..., t_N)'$ by

$$\vec{t} = \frac{\vec{Y}}{\sqrt{\frac{c}{\nu}}}$$

where c is a random variable following a chi-squared distribution χ^2_ν, and independent of $\vec{X}$;

4. transform vector $\vec{t}$ to a unit space $[0, 1]^N$ by

$$(U_1, ..., U_N) = \Big(t_\nu(t_1), ..., t_\nu(t_N)\Big), \tag{3}$$

and then vector $\vec{U} =: (U_1, ..., U_N)'$ has a $t-$copula dependence structure;

5. $\vec{Z}$ is constructed via

$$(Z_1, ..., Z_N) = \Big(\Phi^{-1}(U_1), ..., \Phi^{-1}(U_N)\Big) \tag{4}$$

where $\Phi^{-1}(\cdot)$ is the inverse function of the accumulative standard normal distribution.

We assume the correlation coefficient for bond i and j $(i \neq j)$ is a function of their credit ratings, i.e.

$$\rho_{i,j} = \rho(\mathcal{R}_1, \mathcal{R}_2) \quad (i \neq j)$$

where $\mathcal{R}_1, \mathcal{R}_2 \in \{AAA, AA, A, BBB\}$, and $\rho_{i,j} = 100\%$ for $i = j$. This assumption is based on the observation of the joint evolution of spreads from, for instance, the historical time series of benchmark spreads of *iBoxx Index 01/03/2006–27/06/2008* for rating-maturity buckets as given by *Figure 1*. Their dependence pattern of joint movement is particularly evident during the period of credit crunch.

Remark 2.1. We use a $t-$copula rather than Gaussian copula, since the dependence structure in a $t-$copula is controlled by not only a $N \times N$ correlation matrix Σ given by (2) (which is the same as Gaussian copula) but also an extra parameter – degree of freedom ν. It provides higher flexibility to capture joint extreme events, which are the center concern of the risk management (Embrechts, McNeil and Straumann, 1999). Also, the Gaussian copula can also be recovered by setting ν to infinity.

In this paper, we initially assume degree of freedom $\nu = 3$ for a $t-$copula for instance, then compare the results by choosing different ν. Based on the assumptions given above, the parameters in our model needed to be calibrated are volatility $\sigma_j = \sigma(\mathcal{R}, \mathcal{T})$ for each bond and correlation $\rho_{i,j} = \rho(\mathcal{R}_1, \mathcal{R}_2)$ $(i \neq j)$ for each pair of bonds. The detail for this calibration is provided in Section 5.

3. Portfolio Credit Risk of Spread Widening and Default

Based on the framework for the joint evolution of credit spreads given by Section 2, now we can model the portfolio credit risk of spread widening and default consistently. In particular, we provide two types of models:

1. stand-alone models for the spread widening risk (in Section 3.1) and the default risk (in Section 3.2);
2. an integrated model for overall risk of spread widening and default (in Section 3.3).

We assume there is no default (loss) in the stand-alone model for spread widening risk, and there is no spread widening loss for a bond given default in the integrated model.

3.1. Credit Risk of Spread Widening

In the short term, the major contribution of credit risk for a portfolio is from the spread widening, rather than default. The credit spread widens, the value of the bond decreases, by mark-to-market valuation. This risk is usually measured daily, monthly, or yearly for risk management.

An approximate formula for calculating the value loss for any bond j due to spread widening is given by

$$L_j^W = PV_j(1 + \Delta_j)^{-D_j} - PV_j \tag{5}$$

where PV_j is the present value, Δ_j is the change of spread, $S_t^{<j>} - S_0^{<j>}$, and D_j is the duration. By (1), the changes of spreads can be expressed as $\vec{\Delta} =: (\Delta_1, ..., \Delta_N)'$ where

$$\Delta_j = S_0^{<j>} \left(e^{\sigma_j \sqrt{t} Z_j} - 1 \right). \tag{6}$$

Therefore, based on the assumption of no default (loss) in this stand-alone model, the portfolio loss L^W due to spread widening is given by

$$L^W = \sum_{j=1}^{N} L_j^W, \tag{7}$$

or, the portfolio loss in percentage

$$L^W\% = \frac{\sum_{j=1}^{N} L_j^W}{\sum_{j=1}^{N} PV_j}. \tag{8}$$

3.2. Credit Risk of Default

In the long term, the main portfolio credit risk is from default. Default model can be consistently incorporated into the spread widening model above. The approach used here is similar to the credit barrier models, developed by Iscoe, Kreinin and Rosen (1999), Hull and White (2001), and later extended by Albanese and Chen (2006).

We assume the default probabilities $\vec{p} =: (p_1, ..., p_N)'$ are given, for instance, by Moodys (2009), and depend on the credit rating and time to maturity, i.e.

$$p_j = p(\mathcal{R}, T)$$

where $\mathcal{R} \in \{AAA, AA, A, BBB\}$ and T is time to maturity.

Based on the *inverse transform sampling method* from probability theory, there are three types of equivalent boundaries for triggering the default of bond j in the portfolio:

1. if the uniform distributed random variable U_j overshoots the boundary $1 - p_j$ (i.e. the survival probability) ;
2. if the standard normal distributed random variable Z_j overshoots the boundary b_j where

$$b_j =: \Phi^{-1}(1 - p_j);$$

3. if the change of spread Δ_j excesses the boundary c_j where

$$c_j =: S_0^{<j>} \left(e^{\sigma_j \sqrt{t} b_j} - 1 \right).$$

U_j, Z_j and Δ_j are all latent random variables with a common dependence structure powered the t−copula constructed in Section 2.2. $1 - p_j$, b_j and c_j are called the corresponding *implied default boundaries*.

Then, the default probability for bond j is equivalently given by

$$p_j = \mathbb{P}\{U_j > 1 - p_j\} = \mathbb{P}\{Z_j > b_j\} = \mathbb{P}\{\Delta_j > c_j\} \tag{9}$$

and the default indicator for bond j is given by

$$I_j = \mathbb{I}\{U_j > 1 - p_j\} = \mathbb{I}\{Z_j > b_j\} = \mathbb{I}\{\Delta_j > c_j\}$$

where $\mathbb{I}$ is an indicator function. Hence, we have the portfolio default scenario $\vec{I} =: (I_1, ..., I_N)'$, and the loss due to default for bond j can be calculated by

$$L_j^D = -I_j PV_j (1 - \delta_j)$$

where δ_j is the recovery rate from the present value of bond j. The whole portfolio loss due to default is given by

$$L^D = \sum_{j=1}^{N} L_j^D,$$

or, the portfolio loss in percentage

$$L^D\% = \frac{\sum_{j=1}^{N} L_j^D}{\sum_{j=1}^{N} PV_j}. \tag{10}$$

3.3. Overall Credit Risk

The stand-alone credit risk assessment for spread widening and default risk separately are given above in Section 3.1 and Section 3.2. Now we consider the integrated model for overall credit risk involved.

To avoid double-counting two types of risk, in the integrated model, we assume there will be no any further loss of spread widening for a bond given default. This is the major difference between the stand-alone models and the integrated risk model. Hence, the aggregate portfolio loss of default and spread widening is given by

$$L = \sum_{j=1}^{N} L_j$$

where

$$L_j = (1 - I_j)L_j^W + I_j L_j^D, \tag{11}$$

or, the portfolio loss in percentage

$$L\% = \frac{\sum_{j=1}^{N} L_j}{\sum_{j=1}^{N} PV_j}. \tag{12}$$

Remark 3.1. The loss of spread widening in this integrated model is capped by the loss due to default. The worst scenario, or the maximum total loss in percentage, is given by

$$\lim_{\vec{S}_t \to \infty} L\% = 1 - \bar{\delta} \tag{13}$$

where $\bar{\delta} = \frac{\sum_{j=1}^{N} \delta_j}{N}$ is the average recovery rate and $\vec{S}_t \to \infty$ means all spreads in the portfolio simultaneously widen to infinity, i.e. $S_t^{<j>} \to \infty$ for all j, and trigger joint default of all bonds.

The stand-alone default loss $L^D\%$ in Section 3.2 is actually the loss of default without spread interaction, and the overall loss $L\%$ can be decomposed by

$$L\% = L^D\% + L^I\%,$$

where $L^I\%$ is the extra loss due to spread interaction, upon the stand-alone default loss $L^D\%$.

4. Data Description

We use two types of data:

- **Spread Data**: the historical daily time series of bond spreads over Gilts in the *iBoxx Sterling Universe 01/01/1999–13/05/2008* (including a period of credit crunch), provided by *Deutsche Bank*.
- **Default Data** (*Table 1*): the *2008*'s annual issuer-weighted corporate default rates $p(\mathcal{R}, T = 1\text{year})$ from Moodys (2009).

Table 1: The *2008*'s Annual Issuer-weighted Corporate Default Rates $p(\mathcal{R}, T = 1\text{year})$

Rating	AAA	AA	A	BBB	BB	B	CCC
Year 2008	0.000%	0.515%	0.333%	0.454%	1.058%	1.985%	14.532%

Table 2: A Sample from `Description.csv`

`ISIN`	`Maturity Date`	`Maturity Bucket`	`Type`	`Sector`	`Rating`	`Issuer`
BE0118988667/IDX	24/04/2018	05Y-10Y	Corp	Industrial	AA	SNCVP

In particular, the spread data includes two separate `csv`-files:

- `SpreadData.csv`: a (2443 dates × 1159 issuers) matrix of the daily spreads (in basis point) over Gilts for all *GBP iBoxx* bonds from *01/01/1999 to 13/05/2008* including all business days.
- `Description.csv`: a (1159 issuers × 7 descriptions) matrix provides information of `ISIN`, `Maturity Date`, `Maturity Bucket`, `Type`, `Sector`, `Rating` and `Issuer`.

A specified sample of time series of spreads in `SpreadData.csv` and the associated information in `Description.csv` is given by *Figure 2* and *Table 2*, respectively. Note that, there is no data at the beginning of the plot in *Figure 2*, as this bond had not yet been issued.

Remark 4.1. We assume the two rating systems by *Moody's* and *Standard & Poor's* have the equivalence: $Aaa \equiv AAA$, $Aa \equiv AA$, $A \equiv A$, $Baa \equiv BBB$.

Remark 4.2. There are two types of bonds in the database: `Corp` and `Govt`, here we only select `Corp`-type bonds for calibration, as we assume there is no credit risk for the bonds issued by governments.

5. Parameter Calibration

We calibrate the parameters (by programming in `R`) from the spread database by using a procedure similar to Blamont, Hauviller and Prieul (2007). The calibration is based on the daily

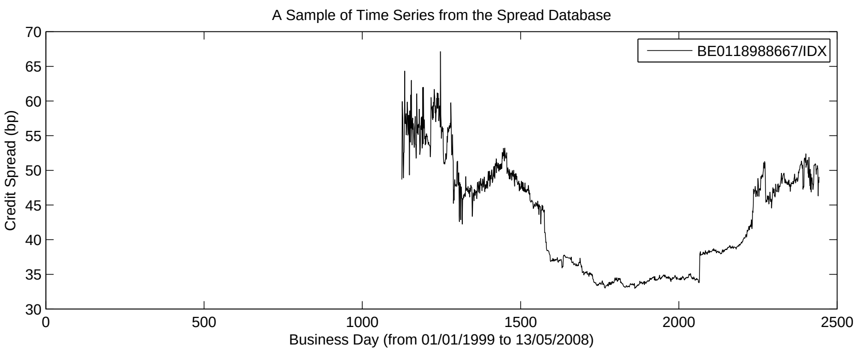


Figure 2: A Sample of Time Series from `SpreadData.csv`

time series of credit spreads excluding the dates before the bond offering.

To be consistent with our bucketing assumption given in Section 2, we group the bonds in the database by (4 ratings × 5 maturity buckets) 20 rating-maturity buckets. The mean of the spreads is calculated based on the actual spreads[3]. The volatility and correlation are calibrated on the daily changes of log-spreads (or continuously compounded rate).

The calibration procedure and the corresponding results are given as below:

- **Mean Calibration** (*Table 3*):
  1. calculate the average of daily spreads for each bond;
  2. average by each rating-maturity bucket.

- **Volatility Calibration** (*Table 4*):
  1. calculate the volatility of daily log-spread changes for each bond;
  2. average by each rating-maturity bucket, i.e. $\sigma(\mathcal{R}, \mathcal{T})$ where $\mathcal{R} \in \{AAA, AA, A, BBB\}$ and $\mathcal{T} \in \{00Y - 05Y, 05Y - 10Y, 10Y - 15Y, 15Y - 20Y, 20Y^+\}$;
  3. convert the daily volatility $\sigma(\mathcal{R}, \mathcal{T})$ to the yearly volatility $\sigma(\mathcal{R}, \mathcal{T}) \times \sqrt{t}$ where time $t = 260$ days[4].

- **Correlation Calibration** (*Table 5*):
  1. calculate the correlation for each pair of daily log-spread changes, and obtain a full (1159 issuers × 1159 issuers) correlation matrix;
  2. average this full matrix by each rating-rating bucket, and obtain a concentrated (4 ratings × 4 ratings) correlation matrix $\rho(\mathcal{R}_1, \mathcal{R}_2)$ where $\mathcal{R}_1, \mathcal{R}_2 \in \{AAA, AA, A, BBB\}$.

Table 3: The Mean of Spreads (bp) by Rating-maturity Bucket

	00Y-05Y	05Y-10Y	10Y-15Y	15Y-20Y	$20Y^+$
AAA	49.36	51.76	67.18	70.46	54.76
AA	92.65	114.84	118.49	109.30	91.67
A	118.13	150.81	139.28	126.76	126.3
BBB	148.32	151.78	170.30	186.83	138.15

6. Model Implementation

In this section, we use a specified example portfolio to demonstrate how this heterogeneous model can be implemented to capture the portfolio credit risk of default and spread widening in a single consistent framework, by using the mathematical framework in Section 2, 3 and the calibrated parameters in Section 5.

[3]The mean calibration is only used to investigate the average level of spreads for each bucket and provide a board picture of the database used for calibration, but it will not be implemented in our model.

[4]We assume there are 260 business days in one year.

Table 4: The Yearly Volatility $\sqrt{260}\sigma(\mathcal{R},\mathcal{T})$ of Log-spread Changes by Rating-maturity Bucket

	00Y-05Y	05Y-10Y	10Y-15Y	15Y-20Y	$20Y^+$
AAA	85.81%	59.69%	71.86%	50.50%	68.42%
AA	42.98%	34.94%	31.48%	39.01%	56.30%
A	45.35%	34.33%	27.99%	33.13%	32.80%
BBB	42.48%	30.11%	38.57%	33.24%	50.40%

Table 5: The Correlation Matrix $\rho(\mathcal{R}_1,\mathcal{R}_2)$ of Log-spread Changes by Rating Bucket

	AAA	AA	A	BBB
AAA	18.26%	15.84%	8.50%	13.47%
AA	15.84%	14.92%	9.10%	11.48%
A	8.50%	9.10%	10.43%	7.15%
BBB	13.47%	11.48%	7.15%	10.93%

One year's period of time well combines the short and long term perspectives, and it is also a convention for risk management and regulatory requirements. Hence, we assess the credit risk over one-year capital horizon for an example portfolio composed of 20 corporate bonds with the detail specified by *Figure 3*.

We model the volatility and correlation for each bond in the example portfolio by matching its rating and maturity to the corresponding bucket of calibrated volatility and correlation provided by *Table 3* and *Table 4*, respectively. Hence, the results of yearly volatilities $\left\{\sqrt{260}\sigma_j\right\}_{j=1,2,\ldots,20}$ and the 20×20 correlation matrix Σ for this example portfolio are specified by *Figure 4*.

6.1. Credit Risk of Spread Widening

According to Section 3.1, now we have all information needed for calculating the portfolio percentage loss $L^W\%$ due to one year's spread widening:

- the calibrated parameters for spreads: the yearly volatility $\sqrt{260}\sigma_j$ as given by the first column of the table in *Figure 4*;
- the calibrated parameters for $t_{\nu=3}-$copula: the 20×20 full correlation matrix Σ for the whole portfolio as given by *Figure 4*;
- the information for each bond in the portfolio: present value PV_j, duration D_j, and current spread $S_0^{<j>}$ as given by *Figure 3*.

Due to the heterogeneity of this model, there is no analytic solution for the whole loss of the portfolio, hence we implement Monte Carlo simulation for $\vec{Z}$ with $500,000$ sample paths. The percentiles of loss percentage $L^W\%$ are given by *Table 6*.

To investigate the underlying scenarios, for each percentile of $L^W\%$ in *Table 6*, we present a sample of realised underlying spread changes in *Figure 5* from the simulation. Take the percentile $100^{th}\%$ for instance, we can observe from the last row of *Figure 5* that, it is mainly the significant spread increase of these specific bonds in the portfolio that contributes to a total loss of 39.15%.

Table of Sample Portfolio Detail on 13/05/2008

Index	Rating	PV	Duration	Maturity Bucket	Current Spread	Nominal	Term
1	AA	953,557	5.96	05Y-10Y	258.39	1,069,375	7.13
2	A	279,189	6.49	05Y-10Y	427.07	331,875	8.34
3	BBB	606,531	3.81	00Y-05Y	84.34	590,000	4.25
4	AA	310,798	5.67	05Y-10Y	9.67	295,000	6.58
5	A	1,954,039	7.20	05Y-10Y	209.93	1,750,000	9.81
6	A	1,619,433	7.80	10Y-15Y	113.46	1,585,625	10.25
7	A	2,149,645	8.66	10Y-15Y	210.25	2,000,000	12.71
8	AAA	3,449,815	12.35	15Y-20Y	54.93	3,262,000	18.92
9	A	83,994	6.62	05Y-10Y	351.17	100,000	7.99
10	A	2,070,471	2.78	00Y-05Y	285.22	2,101,875	3.04
11	AA	1,987,433	2.88	00Y-05Y	152.42	2,000,000	3.12
12	AA	872,450	6.45	05Y-10Y	354.08	950,000	8.38
13	BBB	1,676,105	4.97	05Y-10Y	133.28	1,733,125	5.60
14	AA	1,920,598	0.80	00Y-05Y	113.68	1,900,000	1.01
15	BBB	668,915	9.79	10Y-15Y	138.46	663,750	14.35
16	AA	9,321,789	6.90	05Y-10Y	446.81	10,000,000	9.65
17	A	8,086,788	13.93	20Y+	190.26	8,100,000	29.68
18	A	4,889,691	8.15	10Y-15Y	346.26	5,000,000	11.93
19	AA	349,339	10.94	20Y+	146.81	290,000	29.77
20	BBB	7,357,536	10.79	10Y-15Y	154.80	3,000,000	13.42

Figure 3: The Detail of the Example Portfolio

Full Correlation Matrix

Volatility	Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
34.94%	1	100%	15.84%	11.48%	14.92%	15.84%	15.84%	15.84%	9.10%	15.84%	15.84%	14.92%	14.92%	11.48%	14.92%	11.48%	14.92%	15.84%	15.84%	14.92%	11.48%
34.33%	2	15.84%	100%	13.47%	15.84%	18.26%	18.26%	18.26%	8.50%	18.26%	18.26%	15.84%	15.84%	13.47%	15.84%	13.47%	15.84%	18.26%	18.26%	15.84%	13.47%
42.48%	3	11.48%	13.47%	100%	11.48%	13.47%	13.47%	13.47%	7.15%	13.47%	13.47%	11.48%	11.48%	10.93%	11.48%	10.93%	11.48%	13.47%	13.47%	11.48%	10.93%
34.94%	4	14.92%	15.84%	11.48%	100%	15.84%	15.84%	15.84%	9.10%	15.84%	15.84%	14.92%	14.92%	11.48%	14.92%	11.48%	14.92%	15.84%	15.84%	14.92%	11.48%
34.33%	5	15.84%	18.26%	13.47%	15.84%	100%	18.26%	18.26%	8.50%	18.26%	18.26%	15.84%	15.84%	13.47%	15.84%	13.47%	15.84%	18.26%	18.26%	15.84%	13.47%
27.99%	6	15.84%	18.26%	13.47%	15.84%	18.26%	100%	18.26%	8.50%	18.26%	18.26%	15.84%	15.84%	13.47%	15.84%	13.47%	15.84%	18.26%	18.26%	15.84%	13.47%
27.99%	7	15.84%	18.26%	13.47%	15.84%	18.26%	18.26%	100%	8.50%	18.26%	18.26%	15.84%	15.84%	13.47%	15.84%	13.47%	15.84%	18.26%	18.26%	15.84%	13.47%
50.50%	8	9.10%	8.50%	7.15%	9.10%	8.50%	8.50%	8.50%	100%	8.50%	8.50%	9.10%	9.10%	7.15%	9.10%	7.15%	9.10%	8.50%	8.50%	9.10%	7.15%
34.33%	9	15.84%	18.26%	13.47%	15.84%	18.26%	18.26%	18.26%	8.50%	100%	18.26%	15.84%	15.84%	13.47%	15.84%	13.47%	15.84%	18.26%	18.26%	15.84%	13.47%
45.35%	10	15.84%	18.26%	13.47%	15.84%	18.26%	18.26%	18.26%	8.50%	18.26%	100%	15.84%	15.84%	13.47%	15.84%	13.47%	15.84%	18.26%	18.26%	15.84%	13.47%
42.98%	11	14.92%	15.84%	11.48%	14.92%	15.84%	15.84%	15.84%	9.10%	15.84%	15.84%	100%	14.92%	11.48%	14.92%	11.48%	14.92%	15.84%	15.84%	14.92%	11.48%
34.94%	12	14.92%	15.84%	11.48%	14.92%	15.84%	15.84%	15.84%	9.10%	15.84%	15.84%	14.92%	100%	11.48%	14.92%	11.48%	14.92%	15.84%	15.84%	14.92%	11.48%
30.11%	13	11.48%	13.47%	10.93%	11.48%	13.47%	13.47%	13.47%	7.15%	13.47%	13.47%	11.48%	11.48%	100%	11.48%	10.93%	11.48%	13.47%	13.47%	11.48%	10.93%
42.98%	14	14.92%	15.84%	11.48%	14.92%	15.84%	15.84%	15.84%	9.10%	15.84%	15.84%	14.92%	14.92%	11.48%	100%	11.48%	14.92%	15.84%	15.84%	14.92%	11.48%
38.57%	15	11.48%	13.47%	10.93%	11.48%	13.47%	13.47%	13.47%	7.15%	13.47%	13.47%	11.48%	11.48%	10.93%	11.48%	100%	11.48%	13.47%	13.47%	11.48%	10.93%
34.94%	16	14.92%	15.84%	11.48%	14.92%	15.84%	15.84%	15.84%	9.10%	15.84%	15.84%	14.92%	14.92%	11.48%	14.92%	11.48%	100%	15.84%	15.84%	14.92%	11.48%
32.80%	17	15.84%	18.26%	13.47%	15.84%	18.26%	18.26%	18.26%	8.50%	18.26%	18.26%	15.84%	15.84%	13.47%	15.84%	13.47%	15.84%	100%	18.26%	15.84%	13.47%
27.99%	18	15.84%	18.26%	13.47%	15.84%	18.26%	18.26%	18.26%	8.50%	18.26%	18.26%	15.84%	15.84%	13.47%	15.84%	13.47%	15.84%	18.26%	100%	15.84%	13.47%
56.30%	19	14.92%	15.84%	11.48%	14.92%	15.84%	15.84%	15.84%	9.10%	15.84%	15.84%	14.92%	14.92%	11.48%	14.92%	11.48%	14.92%	15.84%	15.84%	100%	11.48%
38.57%	20	11.48%	13.47%	10.93%	11.48%	13.47%	13.47%	13.47%	7.15%	13.47%	13.47%	11.48%	11.48%	10.93%	11.48%	10.93%	11.48%	13.47%	13.47%	11.48%	100%

Figure 4: The Yearly Volatilities $\{\sqrt{260}\sigma_j\}$ and The Full Correlation Matrix Σ for the Portfolio

Table 6: The Percentiles of Loss Percentage $L^W\%$ Due to One Year's Spread Widening

Percentile	$50^{th}\%$	$90^{th}\%$	$95^{th}\%$	$97.5^{th}\%$	$99^{th}\%$	$99.5^{th}\%$	$100^{th}\%$
Loss in Percentage ($L\%$)	0.52%	5.39%	7.27%	9.17%	11.75%	13.77%	39.15%

Sample of Simulated Spread Change Δ

Loss% \ Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
50th%	-4	253	27	-1	6	15	-21	-7	-1	151	89	29	31	10	70	96	-32	31	-36	-37
90th%	101	82	-28	6	67	51	58	-15	201	216	38	147	43	103	33	120	77	-11	5	85
95th%	-116	67	15	6	179	-38	152	-29	34	82	40	38	30	43	47	509	51	-114	-42	64
97.5th%	-24	-217	66	-1	148	48	108	-20	-65	563	-13	38	42	41	77	208	6	282	-77	242
99th%	301	147	-48	3	142	43	81	142	255	-3	-44	208	-55	164	125	333	176	303	-97	5
99.5th%	-147	-292	-50	-6	154	18	102	203	830	227	426	520	122	-76	313	-322	396	283	744	469
100th%	-181	1069	-34	35	574	179	418	482	1188	-240	604	1198	-96	463	-111	1349	606	928	1498	606

Figure 5: Sample: The Simulated One-Year Spread Changes Δ (bp) for the Corresponding Percentile $L^W\%$ in *Table 6*

Default Rates and Implied Boundaries for Triggering Default

Index	Rating	Volatility	Current Spread	One Year Default Rate (*P*)	Implied Boundary for Default Trigger		
					Uniform (*1-P*)	Normal (*B*)	Spread Widening (*C*)
1	AA	34.94%	258.39	0.515%	99.485%	0.8963	374.82
2	A	34.33%	427.07	0.333%	99.667%	0.9316	657.09
3	BBB	42.48%	84.34	0.454%	99.546%	1.1083	171.13
4	AA	34.94%	9.67	0.515%	99.485%	0.8963	14.03
5	A	34.33%	209.93	0.333%	99.667%	0.9316	323.00
6	A	27.99%	113.46	0.333%	99.667%	0.7595	129.04
7	A	27.99%	210.25	0.333%	99.667%	0.7595	239.12
8	AAA	50.50%	54.93	0.000%	100.000%	Inf	Inf
9	A	34.33%	351.17	0.333%	99.667%	0.9316	540.31
10	A	45.35%	285.22	0.333%	99.667%	1.2306	691.15
11	AA	42.98%	152.42	0.515%	99.485%	1.1028	306.74
12	AA	34.94%	354.08	0.515%	99.485%	0.8963	513.64
13	BBB	30.11%	133.28	0.454%	99.546%	0.7857	159.13
14	AA	42.98%	113.68	0.515%	99.485%	1.1028	228.77
15	BBB	38.57%	138.46	0.454%	99.546%	1.0063	240.31
16	AA	34.94%	446.81	0.515%	99.485%	0.8963	648.15
17	A	32.80%	190.26	0.333%	99.667%	0.8901	273.07
18	A	27.99%	346.26	0.333%	99.667%	0.7595	393.81
19	AA	56.30%	146.81	0.515%	99.485%	1.4445	475.63
20	BBB	38.57%	154.80	0.454%	99.546%	1.0063	268.66

Figure 6: The One-Year Default Rates and Equivalent Implied Boundaries for Triggering Defaults for the Portfolio

Remark 6.1. Note that, the underlying scenarios corresponding to each loss percentile are not necessary unique, particularly for these lower percentiles. We only provide one of the samples in *Figure 5* from the simulation.

6.2. Credit Risk of Default

According to Section 3.2, we can model the default rate p_j for each bond, by matching its rating to one year's default rate in *Table 1*, and use latent variables against the implied boundaries for triggering defaults. For any bond j, by (9), we can provide three types of equivalent implied boundaries, $1 - p_j$, b_j and c_j, under distributions of uniform U_j (as given by (3)), normal Z_j (as given by (4)) and change of spread Δ_j (as given by (6)), respectively. The results are given by *Figure 6*.

We can simply compare the change of spread Δ_j already simulated in Section 6.1 with the corresponding implied boundary c_j, and determine whether bond j has defaulted or not. By using the information of the present value for each defaulted bond as given by *Figure 3* and recovery rate $\delta = 40\%$, the percentiles of loss percentage $L^D\%$ due to one year's bond defaults are given

Table 7: The Percentiles of Loss Percentage $L^D\%$ Due to One-Year Defaults

Percentile	$50^{th}\%$	$90^{th}\%$	$95^{th}\%$	$97.5^{th}\%$	$99^{th}\%$	$99.5^{th}\%$	$100^{th}\%$
Loss in Percentage ($L^D\%$)	0.00%	0.00%	0.00%	1.99%	9.59%	13.36%	53.36%

Sample of Simulated Bond Defaults

Loss% \ Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
50th%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
90th%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
95th%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
97.5th%	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
99th%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
99.5th%	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0
100th%	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1

Figure 7: Sample: The Simulated One-Year Defaults (Indicated by ***1***) for The Corresponding Percentile $L^D\%$ in *Table 7*

by *Table7*.

For each percentile in *Table 7*, we provide one sample of the underlying scenarios $\vec{I}$ in *Figure 7*. In each scenario, the default is indicated by ***1*** and the no default by ***0***. Each ***1*** or ***0*** is determined by comparing the underlying realised spread change Δ_j with the corresponding implied boundary c_j given by *Figure 6*. If Δ_j overshoots c_j, then it generates a number of ***1***; otherwise, it gives ***0***. Take the worst scenario $100^{th}\%$ for instance, we can identify that it is the default of these specific bonds indicated by ***1*** in the last row of *Figure 7* that contributes to the total loss as much as 53.36%.

6.3. Overall Credit Risk

According to Section 3.3, now we assess the overall portfolio credit risk of spread widening and default. The distribution and percentiles [5] of integrated portfolio loss *L%* are given by *Figure 8* and *Table 8*, respectively.

For each percentile of *L%* in *Table 8*, we provide one sample of the underlying scenarios of spreads widening and defaults in *Figure 9*, with their corresponding loss contributions given by *Figure 10*[6].

In *Figure 9*, similarly, if the spread increment Δ_j has exceeded the corresponding implied boundary c_j (given by *Figure 6*), then the bond is assumed to have defaulted. Each defaulted

[5] We provide the result of $99.5^{th}\%$ instead of the regulatory $99.9^{th}\%$ standard, because for such a small portfolio of only 20 names, the scenario $99.9^{th}\%$ is almost the same severe as $100^{th}\%$ where most of the bonds have defaulted. Hence, we suggest calculate $99.9^{th}\%$ when implementing this model to a real portfolio of larger scale.

[6] *Figure 10* provides 14 percentile cases, more than the number given in *Table 8* and *Figure 9*.

Table 8: The Percentiles of Integrated Loss Percentage *L%*

Percentile	$50^{th}\%$	$90^{th}\%$	$95^{th}\%$	$97.5^{th}\%$	$99^{th}\%$	$99.5^{th}\%$	$100^{th}\%$
Loss in Percentage ($L\%$)	0.54%	5.51%	7.57%	9.88%	14.03%	18.36%	54.12%

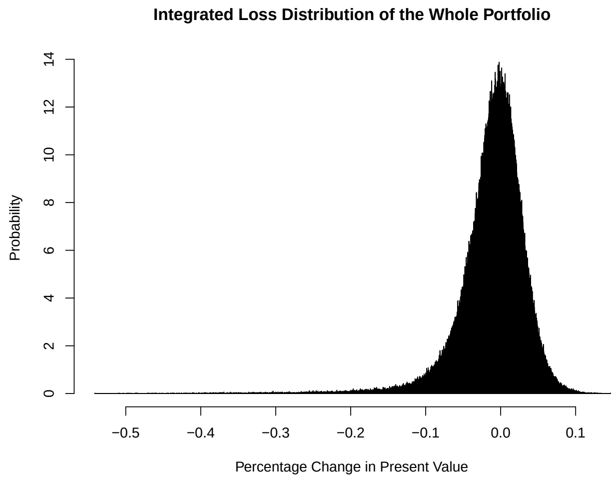


Figure 8: The Distribution of Integrated Loss $L\%$ of the Whole Portfolio

Sample of Simulated Spread Change Δ

Loss% \ Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
50th%	13	162	2	1	136	2	-4	-4	-34	-48	-5	-99	9	22	-9	-89	41	-32	-39	59
90th%	135	13	-28	6	53	-5	-12	-6	130	219	-58	111	0	-31	-19	259	89	-36	-57	37
95th%	30	-132	32	-3	11	32	4	3	70	184	11	97	-23	42	47	262	115	66	109	75
97.5th%	176	361	8	5	-78	8	129	-18	258	325	246	42	4	170	-50	241	148	199	214	100
99th%	502	303	182	-3	48	105	-47	-42	0	-222	-92	232	-53	127	-43	-127	313	347	-113	164
99.5th%	-25	673	66	1	94	-41	114	120	326	526	-18	-195	79	61	206	213	265	465	272	176
100th%	643	1241	313	26	414	179	-85	142	691	1215	606	1084	188	512	403	804	467	674	795	477

Figure 9: Sample: The Simulated One Year's Spread Changes Δ (bp) and Defaults for Corresponding Percentile of $L\%$ in *Table 8*

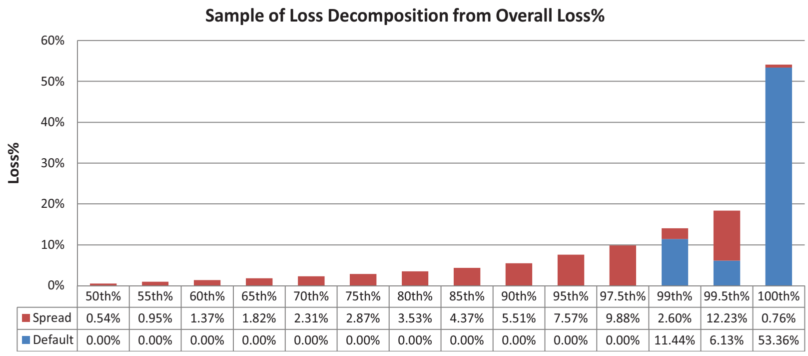
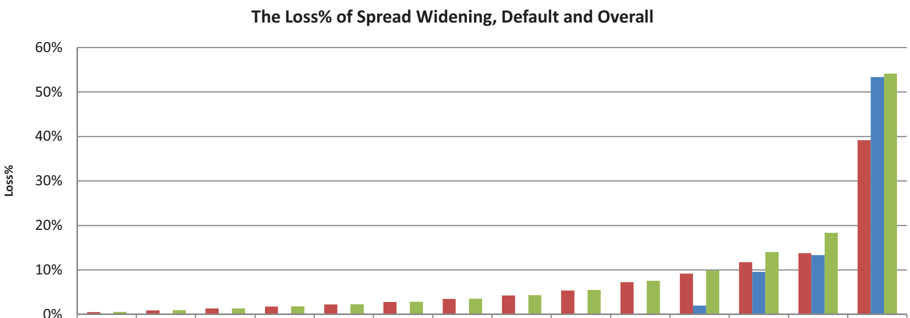


Figure 10: Sample: The Loss Decomposition for the Percentiles of the Integrated Loss $L\%$ (*Table 8*)



	50th%	55th%	60th%	65th%	70th%	75th%	80th%	85th%	90th%	95th%	97.5th%	99th%	99.5th%	100th%
Spread	0.52%	0.93%	1.35%	1.79%	2.28%	2.83%	3.48%	4.29%	5.39%	7.27%	9.17%	11.75%	13.77%	39.15%
Default	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.99%	9.59%	13.36%	53.36%
Overall	0.54%	0.95%	1.37%	1.82%	2.31%	2.87%	3.53%	4.37%	5.51%	7.57%	9.88%	14.03%	18.36%	54.12%

Figure 11: The Percentiles of the Loss of Spread Widening, Default and Overall in the Integrated Model

bond is indicated by a grey grid, and its loss of spread widening should not be accounted.

In *Figure 10*, the decomposition of the underlying spread widening loss and default loss from the integrated loss $L\%$ is not stable and not necessarily unique. This is because one single value of integrated loss $L\%$ could be decomposed into two types of loss with various possible combinations.

The difficulty of risk decomposition raised here in *Figure 10* is in line with the a conclusion from the IMCR conference summarised by Hartmann (2010): Credit risk and other types of risk could be defined distinctly, but their differences should not be overstated. Drawing a clear distinction between them in practical risk measurement and management is very difficult, and we need to account for their joint influence. Hence, for calculating the economic capital, we suggest use the results of integrated loss as given *Table 8*.

Alternatively, we can investigate the underlying risk contributions of spread widening and default by calculating their loss percentiles separately in this integrated model and the results are given by *Figure 11*. We can observe that the relative contribution of default increases more significantly than the one from spread widening when approaching towards the (negative) left-tail of the loss distribution.

In *Figure 11*, the spread widening loss from the integrated model is less than the spread widening loss $L^W\%$ (in *Table 6*) from the stand-alone model (in Section 6.1), because for some scenarios in the integrated model, the spread widening loss could be capped by default, under the additional assumption of no spread widening loss for a bond given default (formally as given by (11)).

Note that, calculating the percentile of the integrated loss by just simply summing up the spread widening loss and default loss for the corresponding percentile in *Figure 11* is not appropriate, as this summation involves double-counting of two types of risk and does not agree with

Table 9: Risk Contribution of Spread Interaction

Percentile	$97.5^{th}\%$	$99^{th}\%$	$99.5^{th}\%$	$100^{th}\%$
Integrated Loss $L\%$	9.88%	14.03%	18.36%	54.12%
Loss of Default without Spread Interaction $L^D\%$	1.99%	9.59%	13.36%	53.36%
Extra Loss due to Spread Interaction$L^I\%$	7.89%	4.45%	5.00%	0.76%
Spread-Interaction Multiplier $L^I\%/L^D\%$	3.97	0.46	0.37	0.01

the corresponding percentile of overall loss for the integrated model. This would overestimate the whole credit risk for higher percentiles, and this is similar to the finding by Alessandri and Drehmann (2010).

To see the risk contribution from spread interaction, we decompose the integrated credit risk by

$$L\% = L^D\% + L^I\%$$

where the percentiles of $L\%$ and $L^D\%$ are already given by *Table 8* and *Table 7* (see also *Figure 11*), respectively, and the higher percentiles of $L^I\%$ is provided in *Table 9*. In this table, the spread-interaction multiplier $L^I\%/L^D\%$, is a simple measure for the relative risk contribution from spread interaction upon the original default risk, which is often neglected in credit risk management, particularly before the 2007-2008's credit crunch. We observe that the value of the multiplier decreases when the percentile increases. This output is consistent with the reality: in a short period of one year, for a relatively normal economic condition, the major risk comes from spread widening whereas for a more severe environment the main risk originates from default.

6.4. Overall Credit Risk by Alternative Dependence Structures

The issue of dependence structure is of central importance in all portfolio credit risk modelling methodologies (Jarrow and Turnbull 2000), especially that concerns the high-percentile loss in the tail. We provide some alternative settings for the dependency in our model for assessing the overall credit risk.

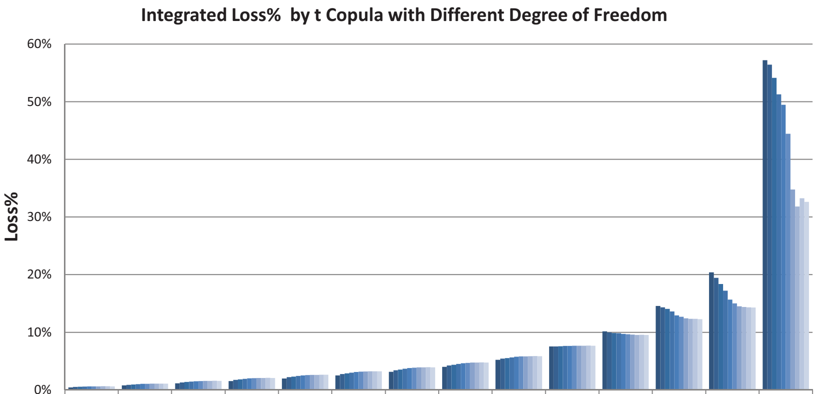
6.4.1. Degree of Freedom ν

The results above are all based on a $t-$copula with degree of freedom $\nu = 3$. In *Figure 12*, we calculate the overall loss $L\%$ by choosing different ν for a comparison. We can observe from this sensitivity study that, when the dependency becomes weaker (ν larger), the loss $L\%$ increases for lower percentiles but decreases for higher percentiles. The critical point for loss $L\%$ as an increasing or decreasing function of ν is approximately $95^{th}\%$. By probability theory, when ν goes to infinity, it will eventually converge to a Gaussian copula model which is widely used in practice.

6.4.2. Correlation Σ

Calibrating the correlation remains a challenge for researchers and practitioners. In our model we calibrate correlation parameters for driving spreads widening and defaults based on the spread database, because credit spread provides us an instantaneous market view of the risk premium for the underlying credit, with the majority [7] implying the level of default risk (Hull, Predescu

[7] There are other types of risk implied from credit spreads, such as liquidity.



	50th%	55th%	60th%	65th%	70th%	75th%	80th%	85th%	90th%	95th%	97.5th%	99th%	99.5th%	100th%
v=1	0.44%	0.79%	1.15%	1.54%	1.99%	2.51%	3.15%	4.00%	5.23%	7.54%	10.18%	14.55%	20.38%	57.21%
v=2	0.51%	0.89%	1.31%	1.74%	2.22%	2.76%	3.42%	4.26%	5.43%	7.55%	9.98%	14.30%	19.43%	56.41%
v=3	0.54%	0.95%	1.37%	1.82%	2.31%	2.87%	3.53%	4.37%	5.51%	7.57%	9.88%	14.03%	18.36%	54.12%
v=5	0.57%	1.00%	1.44%	1.91%	2.43%	3.00%	3.67%	4.50%	5.63%	7.63%	9.83%	13.61%	17.18%	51.27%
v=10	0.60%	1.05%	1.51%	2.00%	2.53%	3.12%	3.80%	4.63%	5.75%	7.64%	9.71%	12.95%	15.64%	49.46%
v=20	0.61%	1.06%	1.53%	2.03%	2.57%	3.17%	3.86%	4.69%	5.81%	7.67%	9.67%	12.70%	14.99%	44.42%
v=50	0.62%	1.08%	1.56%	2.06%	2.61%	3.22%	3.91%	4.74%	5.84%	7.66%	9.60%	12.42%	14.49%	34.75%
v=100	0.63%	1.09%	1.57%	2.07%	2.62%	3.22%	3.92%	4.76%	5.87%	7.67%	9.55%	12.34%	14.38%	31.80%
v=200	0.64%	1.10%	1.58%	2.09%	2.63%	3.24%	3.94%	4.78%	5.89%	7.71%	9.58%	12.33%	14.30%	33.24%
v=500	0.62%	1.08%	1.56%	2.08%	2.63%	3.23%	3.93%	4.76%	5.87%	7.67%	9.53%	12.28%	14.27%	32.63%

Figure 12: The Integrated Loss $L\%$ by $t-$Copula with Different Degree of Freedom ν

and White, 2005; Longstaff, Mithal and Neis, 2005).

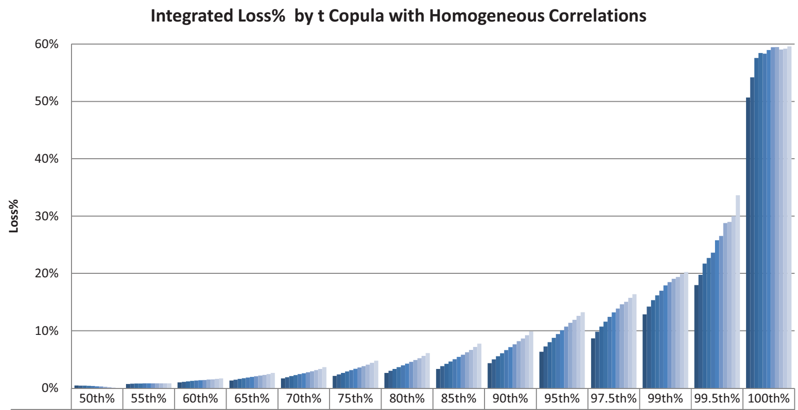
There are other approaches for correlation calibration. Düllmann, Küll and Kunisch (2010) calibrated the correlations from the time series of the underlying equity prices and default rates, and also had a comparison between them. The Internal Ratings-Based (IRB) approach for Basel II by BCBS (2001) assumed a (homogeneous) average around 20% for correlations (comparing with an average of 18.45% in our model given by *Figure 4*).

To be more convenient to compare our model with these models based on different calibration methods for correlation, we provide a reasonable approximation by setting one homogeneous correlation $\bar{\rho}$ to all pairs of bonds in Σ in our model,

$$\{\rho_{ij}\}_{i\neq j} \equiv \bar{\rho}$$

to replace the calibrated heterogeneous correlations in *Table 5*. All other parameters remain the same. We can see how these percentiles of the loss $L\%$ migrate accordingly when increasing $\bar{\rho}$ from 0% to almost 100% as given by *Figure 13*.

Remark 6.2. As we assume the recovery rate $\delta = 40\%$ for all bonds, by (13) it implies that the upper limit of total portfolio loss percentage is 60%. It is approximately the worst scenario (59.58%) simulated by setting the homogeneous correlation $\rho = 99.99\%$ for percentile $100^{th}\%$ as given by *Figure 13*.



	50th%	55th%	60th%	65th%	70th%	75th%	80th%	85th%	90th%	95th%	97.5th%	99th%	99.5th%	100th%
0.00%	0.48%	0.75%	1.04%	1.36%	1.71%	2.14%	2.66%	3.35%	4.38%	6.35%	8.66%	12.85%	17.97%	50.69%
10.00%	0.46%	0.78%	1.12%	1.50%	1.92%	2.42%	3.03%	3.84%	5.02%	7.25%	9.81%	14.20%	19.75%	54.20%
20.00%	0.45%	0.82%	1.20%	1.63%	2.11%	2.67%	3.36%	4.26%	5.58%	8.02%	10.73%	15.35%	21.70%	57.56%
30.00%	0.41%	0.83%	1.27%	1.75%	2.29%	2.92%	3.69%	4.67%	6.11%	8.76%	11.59%	16.18%	22.70%	58.44%
40.00%	0.38%	0.85%	1.34%	1.87%	2.47%	3.16%	4.00%	5.08%	6.66%	9.41%	12.42%	16.98%	23.60%	58.31%
50.00%	0.35%	0.86%	1.39%	1.97%	2.62%	3.38%	4.29%	5.46%	7.14%	10.09%	13.20%	17.91%	25.79%	58.96%
60.00%	0.29%	0.85%	1.44%	2.08%	2.79%	3.61%	4.59%	5.85%	7.65%	10.71%	13.87%	18.47%	26.54%	59.45%
70.00%	0.24%	0.86%	1.50%	2.20%	2.97%	3.85%	4.93%	6.28%	8.17%	11.38%	14.61%	19.06%	28.78%	59.48%
80.00%	0.18%	0.85%	1.55%	2.31%	3.15%	4.11%	5.24%	6.67%	8.65%	11.91%	15.05%	19.37%	28.99%	59.03%
90.00%	0.10%	0.84%	1.62%	2.45%	3.38%	4.42%	5.65%	7.16%	9.21%	12.59%	15.74%	19.88%	30.00%	59.20%
99.99%	0.01%	0.85%	1.73%	2.66%	3.68%	4.82%	6.13%	7.75%	9.88%	13.22%	16.38%	20.25%	33.62%	59.58%

Figure 13: The Integrated Loss $L\%$ by $t-$Copula with Different Homogeneous Correlation $\bar{\rho}$

7. Conclusions

The model developed in this paper provides a simple and fast way to assess the credit risk of a portfolio on spread widening and default under a heterogeneous and consistent framework. It is also able to capture the behavior of each underlying bond by distinguishing its credit rating and maturity. The procedure of calibration and implementation are simple and easy to be extended. Alternative marginal distributions and copula functions can be easily replaced under different assumptions tailored to meet different purposes for portfolio risk management.

For future research, instead of using a simple log-normal process to model the evolution of credit spread, we could use other non-negative processes, such as the mean-reverting process. For the dependency modelling, we could consider other copula functions, such as Gaussian and family of Archimedean copulas. For more general portfolio risk management, this framework powered by a copula could possibly be generalised to consistently integrate other types of risk, such as liquidity risk, or to incorporate the negatively related behavior between default and recovery rate. Moreover, rather than fixing a one-year horizon, we could set different time horizons in this model, and then would be able to see the evolution of the portfolio credit risk of default and spread widening over short, median and long terms. We could also extend our approach for pricing credit derivatives under an appropriate probability measure, or a dynamic model to capture the credit risk by calibrating the parameters from instantaneously daily updating database.

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