



# Comment on Braham and Steffen

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# Response to Braham and Steffen

by Moshé Machover

The paper ‘Voting Power in Games with Abstentions’ by Matthew Braham and Frank Steffen is a contribution to a very young and as yet under-developed part of the theory of voting power. I will first make some general remarks in order to put this paper in context. Then I will go on to comment on the substance of the paper.

## 1 Context of the paper

In the literature on social choice, indifference between two or more proposed options is generally regarded as perfectly rational and normal. So, for example, in discussions of the theory of voting procedures, while it is often argued that the ordering of a rational voter’s preferences must be transitive, indifference is not excluded as irrational: the ordering need not be sharp (strict).

It is therefore rather surprising that until quite recently the literature on voting power, almost without exception, assumed that a voter who does not vote for a proposed act automatically votes against it, or at least must count as doing so. Technically speaking, the model exclusively used in this theory was that of a *simple voting game* (SVG), which is *binary* in the sense that it assumes that decisions are made by division of the assembly of all voters into two ‘coalitions’: the ‘yes’ voters — and all the rest, who are presumed to vote ‘no’. Abstention was dismissed implicitly (and on some occasions even explicitly) as ‘irrational’ and therefore undeserving of serious theoretical consideration.

Even more surprising: real-life decision rules (such as those of the US legislature or the UN Security Council) where abstention is in fact a *tertium quid*, were often — in fact, almost always — misreported in the voting-power literature as though they counted abstention as a ‘no’ vote. Apparently, scholars who assumed (quite wrongly, in my opinion!) that abstention is irrational and undeserving of theoretical consideration fell into the trap of assuming that it therefore does not exist. In our papers (1997) and (2001) Felsenthal and I cite many examples of such misreporting (as well as some exceptional cases of correct reporting) from the voting-power literature. In our (2001) and (2001a) we discuss the hypothesis that this widespread misreporting is due to a phenomenon — akin to optical illusion — that philosophers of science have called *theory-laden* (or *theory-biased*) *observation*.

In (1997) and in our book (1998) we propose a model, *ternary voting game* (TVG), which admits abstention as a distinct *tertium quid*, in addition to ‘yes’ and ‘no’ votes, and we outline the rudiments of a theory of voting power based on this model.

I hope I am right in thinking that our little campaign has encouraged others to develop this kind of theory; and that the present paper by Braham and Steffen is to be viewed in this context. This is very welcome.

## 2 Substantive comments

When I just spoke of ‘this kind of theory’ I meant to imply that it does not necessarily have to be exactly the one we have proposed. In fact, the theory proposed by Braham and Steffen is a variant different from ours.

The basic difference is that while we treat abstention on a par with ‘yes’ and ‘no’, they do not. This difference has two aspects: one concerning the structures used to model a decision rule that admits abstention; and the other concerning the assumption made about voters’ behaviour or propensities. I shall deal with these two aspects in turn.

2.1 *Structural aspect.* Here it may be worth noting that in reality abstention can take rather different forms. A voter may fail to turn up for the meeting at which the division takes place; or stalk out of the meeting, with a disgusted or bored expression, before the division. This is abstention *by default*. On the other hand, a voter can actually participate in the division by declaring ‘I abstain’. This is *active* abstention. (An intermediate form of abstention occurs when the voter is present but fails to take part in the division.)

Real-life decision rules that treat abstention as a *tertium quid* do not normally distinguish between the two kinds of abstention. But for some purposes the distinction may be of importance. For example, in legislatures an absent member does not count as part of the quorum, whereas an actively abstaining one normally does.

I am grateful to Detlef Pauly (2001) for bringing to my attention Jeremy Bentham’s discussion in Ch. XIV, §3, of his *Essay on Political Tactics*, where he draws a sharp distinction between the two sorts of abstention. Bentham thoroughly condemns abstention by default:

In regular voting, every member ought to be required to give his vote. This obligation is founded upon the nature of his office... He cannot, as appears to me, neglect this duty, except from indifference, pusillanimity, or corruption.

However, he argues that active abstention is perfectly rational, legitimate and sometimes necessary:

‘No,’ says a wise man, ‘I shall not vote because I am not sufficiently enlightened upon the question: I am equally afraid of error in declaring myself for or against.’

Indecision is a possible state. The mind is as susceptible of this modification as of the two others. To require an affirmative or negative answer from a man who is in doubt, is to substitute constraint for liberty — is to oblige him to tell a lie. The ancient Romans, in penal matters, had seized the distinction of these three states of the mind, and had found formulas for their expression: *absolvo* — *condemno* — *non liquet*. The jurisconsults and legislators, who have drawn so many absurd and atrocious laws from Roman jurisprudence, have never thought of adopting this simple arrangement — this religious homage to truth. ...

He therefore proposes

... a new form of voting. There have hitherto been only two lists, or two ballots — the one for the *ayes*, the other for the *noes*; I would establish a third, for the *neuters*.

But it may be asked, why require a man to vote, whilst he is permitted to give a vote which will have effect neither on the one side nor the other?

It is replied, that a *neuter* vote subjects the individual who gives it to the judgment of public opinion. By abstaining from voting, he may escape observation, or he may excuse himself upon divers grounds. But admit a *neuter* vote in a case in which the public interest is manifest, the voter cannot withdraw himself from censure — it will exhibit either his crime or his incapacity in as clear a manner as if he had decidedly taken the wrong side.

In cases which admit of honest doubts, the number of neuter votes would serve to enlighten the assembly, by showing that its deliberations had not yet reached maturity.

The TVG model proposed by us can perhaps be regarded as assimilating all abstentions to those of the active kind: it treats abstention on a par with ‘yes’ and ‘no’, like Bentham’s neuter vote or the Roman ‘*non liquet*’. Technically speaking, a TVG is a direct

ternary analogue of an SVG. In an SVG, the outcome is determined by any given *bi-partition* of the set of all voters into two camps: ‘yes’ voters and ‘no’ voters. Analogously, in a TVG the outcome is determined by any given *tripartition* of the set of voters into three camps: ‘yes’ voters, abstainers and ‘no’ voter.

By the way, this set-up lends itself naturally to generalization. Instead of allowing voters two or three options (as in an SVG or a TVG, respectively) we could allow them  $k$  options (with  $k > 3$ ), representing  $k$  degrees of support for a proposed act, from total support to total opposition, with  $k - 2$  intermediate degrees. (I believe that J. Freixas and W. Zwicker have been working in this direction.)

The model proposed by Braham and Steffen, on the contrary, can be regarded as assimilating all abstentions to those that occur by default. In this model, abstention does not really figure as expressing an intermediate or even indeterminate degree of support between ‘yes’ and ‘no’, but as opting not to participate in a division. Voting is thus conceptualized as a two-stage affair: first, a voter chooses *whether* to vote at all, and if s/he opts to vote s/he must then choose between casting a ‘yes’ or a ‘no’ vote. Technically speaking, such a model is in effect a whole bundle of SVGs: if the set of all (potential) voters is  $N$ , then for every subset  $S \subseteq N$  we must have an SVG whose set of actual voters is  $S$ , while the remaining members of  $N$  are regarded as absent.

For this to work, one must extend the usual notion of SVG to the case where the set  $S$  of actual voters is empty, because one has to allow for the eventuality that all members of  $N$  opt to abstain; but this is a minor technical matter. With this proviso, such a bundle of SVGs can provide a reasonable way to model a decision rule that admits abstentions as a *tertium quid*, although one might perhaps object that it is unnecessarily complicated.

*2.2 Voters’ propensities.* In the theory of a priori voting power for the binary case — in which abstention is not admitted — it is normally assumed that each voter votes ‘yes’ and ‘no’ with equal probability of  $1/2$ . This is mathematically the simplest and most natural assumption. But it also has a clear conceptual justification: there is an inherent symmetry of duality between ‘yes’ and ‘no’ votes, corresponding to the symmetry of duality in classical two-valued logic between propositions and their negations.

In the rudimentary theory of a priori voting power proposed for the ternary case by Felsenthal and me in (1997) and (1998), we assign — in strict analogy with the binary case — equal probability of  $1/3$  to each of the three options, ‘yes’, abstain and ‘no’. Braham and Steffen are critical of this — with good reason. As we ourselves have admitted, while this assumption is again the simplest and most natural in a purely mathematical sense, it lacks solid conceptual justification. Conceptually, it is still correct to assign equal a priori probability to ‘yes’ and ‘no’; but why should abstention be assigned the same a priori probability as these two options? Of course, as we have pointed out, our theory can be modified by assigning probability  $p$  to abstention and  $1/2(1 - p)$  to each of the other two options. But what should  $p$  be? The only non-arbitrary choice of a value for  $p$  is  $1/3$ , which has at least a formal mathematical justification, although this value seems unrealistically high.

Unfortunately, the situation is no better for the theory proposed by Braham and Steffen: here too there is no conceptual reason for choosing any particular value for  $p$ . Moreover, since in their model the choice whether to participate or abstain is made by the voter at an independent preliminary stage, the only non-arbitrary value for  $p$  in the formal sense is  $1/2$  — which is perhaps even less realistic than  $1/3$ .

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In conclusion, I would like to say that both variants of the theory of voting power for decision rules that allow abstention are legitimate; they seem to embody alternative,

perhaps equally valid, concepts of abstention. And both variants are in need of much further development.

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