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**IS DUVERGER'S 'LAW' BASED ON A MISTAKE?
WHY THE 'LAW' MISATTRIBUTES THE IMPACT
OF CHANGES IN EFFECTIVE COMPETITION SPACE
TO ELECTORAL SYSTEMS**

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Abstract:

In the Duverger's Law (DL) literature, any effects detected in holding down the number of parties in plurality rule or majoritarian systems are conventionally ascribed *tout court* to the electoral system, and vice versa for proportional systems allegedly encouraging more parties. By contrast, we argue that a DL effect can only be identified when tested against a much more sophisticated null hypothesis that starts by recognizing fundamental variations in the *effective competition space*, driven by the *number of observable parties* (or candidates, or coalitions) that enter competition at some low but significant level of support, say, receiving 1 per cent of the vote each. The appropriate null hypothesis has three parts, each of which must be refuted in order for a DL effect to be established:

1. *The patterns of election district outcomes observed do not differ from those that would be expected under equiprobability, given the number of observable parties (N_{op}) competing and the effective competition space (ECS) that this creates.*
2. *The deviations from equiprobability found do not show two-party drift as DL predicts (but either no pattern, or unipolar drift, or multi-party drift).*
3. *Measured as deviations from equiprobability, the extent of two-party drift is no greater in plurality/majority systems than in proportional systems, (for example, because two-party drift occurs quite evenly in all systems, or because there are individual electoral system variations in two party drift).*

We operationalize the first two parts of the test for some recent plurality rule elections in India, Great Britain and the USA, by mapping empirical district outcomes onto the logically feasible competition space for districts with different numbers of observable parties. We develop new criteria for assessing Duvergerian versus equi-probability patterning of district outcomes including: the proportions of all districts in an election spread across different N_{op} levels; the minimum level of combined support for V_3 to V_N parties; the degree of patterning of the outcome distribution by a two-party relationship, versus the degree of random scatter of results; the divergence of outcomes in two-party contests from a bi-nomial distribution; the divergence of outcomes in a three-party contest from a multi-nomial distribution; and finally in four or more party contests the clustering of outcomes in relation to the peak densities of 'non-equivalent distributions' across the V_1V_2 'floorplate' plot. Our analysis is necessarily preliminary, because the third part of the null hypothesis above inherently requires a cumulative research effort beyond the bounds of any single paper. But the tools and measures we set out here make this next stage of research one that is eminently feasible to achieve.

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Few propositions in political science are as well known as Duverger's association of plurality rule systems with two (or few) party competition and an accompanying 'hypothesis' (more tentatively) linking proportional representation systems to multi-party systems.¹ Both claims have generated a large mass of valuable research in electoral analysis. Yet we none the less want to argue that the 'Law' has lead the field up a methodological blind alley, by failing to discriminate between a number of separate influences. The 'Law' should be concerned with the *distinctive* impacts of electoral systems. But in the formulations of Duverger and his followers this focus becomes muddled up with some analytically separate 'competition space' effects. Duverger's 'Law' only makes sense as a scientific proposition if it is tested against a null hypothesis – one where electoral system effects on voting patterns are only measured as deviations from the outcomes that one would expect to occur anyway, given the number of parties competing.

We begin by briefly reviewing the Duverger's Law (hereafter DL) literature, from the original proposal through to some influential modern reformulations. We suggest that there has been only a shifting and inadequate specification of what properly testing DL would look like. In its place we set out a three-stage null hypothesis whose refutation would adequately found DL claims. Section 2 begins to specify how this concept can be operationalized, introducing the concept of the 'number of observable parties', which in turn specifies different 'effective competition spaces'. Using district-level datasets from recent elections in three plurality rule systems - India, the UK and the United States - we show how equi-probability outcomes can be specified when the 'number of observable parties' is either 2 or 3, comparing these outcomes with DL predictions. Section 3 looks at multi-party situations, where predicting equi-probability outcomes becomes a more complex task. The massing of 'non-equivalent distributions' away from the two-party zones automatically creates an equi-probability effect yielding a greater scatter of district outcomes. The conclusions show how these predictions and related tests make possible a thorough-going change in how DL effects are searched for and established.

1. THE EVOLUTION OF METHODS FOR IDENTIFYING DUVERGER'S LAW EFFECTS

The development of research into Duverger's Law (DL) falls into five main phases. The classical formulation of the 'Law' is famously rather vaguely expressed:

The simple-majority single-ballot system favours the two-party system. Of all the hypotheses that have been defined in this book, this approaches the most nearly perhaps to a true sociological law (Duverger, 1959, 217).

This proposition resonated widely because Duverger offered a two-part explanation of how plurality rule shaped people's voting behaviour and induced two-party local competition by:

- discouraging smaller parties from forming or standing, since their leaders and supporters know that plurality rule comprehensively discriminates against parties that cannot win more votes than any rival in at least some constituencies (the 'mechanical effect'); and
- discouraging voters (and potential leaders) from supporting small parties for fear of 'wasting' their vote (the 'psychological effect').

The key insight here is that the electoral system can affect which parties stand and how voters distribute their support, thereby exerting an ongoing (permanently in-place) influence on the patterns of how voters vote - especially in plurality rule systems building up support for the two leading contenders at the expense of the rest.

Yet from the start these propositions were controversial:

'The reason that Duverger's Law has stuck in the craw of so many political scientists of a sociological bent is that it seems to set up some sort of "institutional determinism," wherein markedly different social cleavage structures are hypothetically all mashed into one final outcome (a "two-party system") merely upon [the] application of a particular set of electoral laws' (Cox, 1997, pp. 15-16).

A succession of 'sociological' critics pointed to plurality rule systems exhibiting more-than-two ('serious') parties, the 'deviant' cases appearing to increase over time, especially in Canada, India, New Zealand and the UK, all of which transitioned decisively in the post-war period towards multi-party competition. Other critics focused on the Law's vagueness and poor empirical operationalization. According to Sartori: '[B]oth the methodological and substantive febleness of the laws of Duverger are patent and easily demonstrated' (Sartori, 1986, p. 45). Duverger was also accused of developing an

argument about effects that should be measured at the electoral district level, but then justifying them empirically only with national level data (Wildavsky, 1959).

An early revisionist phase in the fortunes of DL propositions was inaugurated by attempts to integrate it into rational choice approaches and to protect the ‘Law’ against sociological critics. Rae (1971, p. 95) suggested a modified DL as follows: ‘plurality formulae are always associated with two-party competition, except where strong local minority parties exist’. Partly incorporating this and partly responding to the early post-war Indian experience, Riker (1982, p. 761) replaced Rae’s caveat above with two caveats:

‘except in countries where (1) third parties nationally are continually one of two parties locally, and (2) one party among several is almost always the Condorcet winner in elections’.

Riker also took over Wildavsky’s critique, but now reversed to act as a defence mechanism, insisting that the proof of the DL pudding lay only in the district-level patterning of votes. At a stroke national multi-partism was rendered irrelevant to the Law’s standing. Riker also suggested that DL was a ‘probabilistic’ rather than a ‘determinant’ law. The ‘association’ of plurality rule and two-party systems is a tendency that operates universally in plurality rule systems, but without any requirement that its effects must necessarily offset other (sociological, diversifying) tendencies towards multi-partism. None of the revisionists’ changes mollified the ‘Law’s’ strongest critics. Sartori (1986, p. 48) complained:

‘[W]e are simply left with saying that two-party-ism does not materialize when something goes wrong (eventually in Condorcet’s name). If so, Riker’s law turns out to be more objectionable and even feebler than the one of Duverger’.

From an alternative perspective, Gunnell (1986, pp. 57, 58) commented on Riker’s free-and-easy way of claiming a ‘scientific’ basis for the re-formulated DL in verificationist mode, while yet citing Popper and Kuhn in support of his interpretation:

‘Popper... has consistently rejected the notion of verification, which is basically a vestigial positivist concept. What Riker describes as proposition replacement and theory adjustment is essentially what Popper has criticized as the use of ad hoc hypotheses to sustain theories whose elements have been falsified. Riker presents his history of Duverger's law as an example of what Kuhn has called "normal science"—a notion that Popper has condemned as detrimental to the growth of scientific knowledge and that Kuhn advanced in the context of an argument about the logic of science that is antithetical to the basic assumptions informing Riker's

claims about the "accumulation" of scientific knowledge demonstrated in his account of Duverger's hypothesis...

It would be overkill to pursue the fact that this notion of a "probabilistic" law does not, on its face, conform to the formulations of Popper and Hempel or to any other standard treatment of the subject and that Riker does not even attempt to demonstrate such congruence... [Riker's] claims [do not] bear any significant relationship to explanation in natural science and what might be construed as theories and laws in these fields....It may very well be that social science should emulate natural science, but it is a mistake to believe that what Riker, for example, presents as a law is in fact an analogue of a claim in natural science'.

The third tack in the development of the DL literature was an empiricist one. In an effort to unify Duverger's separate Law (about plurality rule systems) and Hypothesis (about PR systems) into a single empirical proposition, Taagepera and Shugart (1989) deployed the 'effective number of parties' (ENP) concept – arguing that the ENP scores across countries increase in line with the national average district magnitude. Here the alleged association of plurality rule and two-party systems appears as just a 'special case', one end of the line for a wider empirical generalization. Yet this seeming advance towards empirical operationalization was one step forward, three steps back. In line with the whole analysis of *Seats and Votes*, the unified proposition was again framed only in terms of national election data, with no district level evidence deployed. The causal relationship between district magnitude and ENP was left unspecified: the strength of the relationship is set by whatever the data shows, rather than independently. Finally, the authors' enthusiasm for ENP acknowledged no problems in using it as a measure of party fragmentation – even though a given ENP score can be produced in many different ways.

The fourth stage of the literature swung back towards the revisionists, with Andrew Cox's (1997) reformulation of DL predicting that the number of 'serious' parties *at district-level* in all electoral systems should be no more than $M + 1$, where M is the district magnitude. Cox provided a (soft) public choice justification for the $M + 1$ rule, arguing that under condition of perfect 'strategic' co-ordination by both elites (candidates or parties) and voters then the (purely local) equilibrium number of parties in a single-ballot plurality rule systems is two. For him 'strategic refers to actions that are primarily instrumental as opposed to consummatory, that is, actions taken because of their perceived impact on the final outcome of the election, rather than because of any intrinsic value they may have' (Cox, 1997, p. 149). A 'non-Duvergerian equilibrium' could also come into existence (either through ill fortune or for sociological reasons) and be

sustained, where several closely-matched parties approximated each other's support and hence voters, potential leaders and party financial backers might not be able to converge on clear front-runners or might retain confidence that their choices were still viable.

Cox cited two main kinds of supporting empirical evidence. The first were ENP scores in district-level data, backing up with data Riker's insistence on judging DL effects only at local level. By also separating off entirely issues of party 'nationalization' from issues about district-level competition, Cox decisively insulated his DL reformulation from most of the sociologists' apparently disconfirming cases. Second, Cox argued that the confusing labelled 'SF ratio' of the second losing party's vote divided by the first losing party's vote (that is, V_3/V_2) should tend to polarize between results close to 0 (a Duvergerian equilibrium where V_2 is way ahead of V_3) and results close to 1 (a non-DL equilibrium where V_2 and V_3 are almost equal). These moves also seemed to provide operational tests of countries' conformity to one or another situation in an empirical way that stood up Riker's earlier position that DL is 'probabilistic'. Following Cox, Benoit (2006, p. 76) claimed: 'It is now standard in the study of electoral institutions to treat institutional characteristics as producing tendencies in party systems that are probabilistic, not deterministic, in nature'.

Despite these advances, Cox's approach had three limits. The $M + 1$ prediction directed attention only to vaguely defined 'serious' parties, a defensive move apparently designed to protect the proposition against the fact that the only plurality rule countries to retain pure 2-party systems into the twenty-first century were the United States plus a few small Caribbean polities. Cox also argued that in 'very large' electoral districts, defined as those with six or more seats, the $M + 1$ limit stopped operating. This caveat was apparently inserted to protect the $M + 1$ rule from counter-examples of systems with large electoral districts but without multitudes of parties. It is difficult to see both moves as anything other than ad hoc qualifications. In addition, analysts quickly pointed out that Cox's SF ratio (V_3/V_2) is well nigh useless as an indicator of non-Duvergerian situations, generating results close to 1 when *either* relatively even multi-party competition prevails, *or* when the winning party V_1 is so far ahead as to push the next two competitors into parity at low levels of support (which might be seen as DL consistent) (Gaines, 1999; Diwakar, 2007, p.541).

The fifth and most recent phase in the development of the DL literature was a new wave of empiricist studies, fuelled by the relatively new availability of large amounts of

district-level data. Only a few studies picked up on Cox's flawed SF ratio and none tried to operationalize his 'serious' parties. Instead, from the 1990s attention focused on how to operationalize DL criteria purely in ENP terms. It is worth quoting at length the somewhat involved and partly self-reproachful justification offered by Chhibber and Kollman (2004, p. 48) for deciding to count any ENP (votes) score of 2.5 or less as a 'Duvergerian' outcome:

'Any criteria we use to determine that a particular district does not confirm to the predictions of Duverger's Law, other than a standard that says only two parties or candidates will get votes and all others (if they exist) will get 0 per cent, will be arbitrary. For our purpose, we settle on an effective number of parties [in votes] of 2.5. If a district has more than 2.5 effective number of parties, we will say for present purpose that it violates Duverger's Law. There are good reasons for using such a cut-off. Cox (1997) argues persuasively that Duvergerian logic reduces the number of serious competitors in plurality elections but that the logic does not lead to any conclusions about whether plurality elections increase the number of competitors above one. For example, having fewer than 2 parties in a single member, simple plurality district does not necessarily contradict the predictions from modern versions of Duverger's Law, whereas having 4 parties does. Thus, it makes sense to divide districts into categories, not by whether they confirm to Duverger's Law within a range around 2 (say, between 1.75 and 2.25)..., but whether they fit below a particular value... A relatively liberal criteria such as a 2.5 cut-off enables us to present a strong case that deviations from Duverger's Law are not random. Using strict criteria - for example, any deviation from 2 - would lead us to reject the value of the law too hastily (a type II error, loosely speaking). Further, .. only at an N of 2.5 could the vote share of the third party in the average district have influenced the outcome of the typical election, as it is only then that the vote share of the third party [empirically] becomes larger than the difference in the vote shares of the first- and the second-placed parties'.²

This approach has now been picked up by a substantial literature, which also looks at measures of spread across districts within countries. Early work concluded that:

"focusing on the means and modes in the data leads one to conclude that Duverger's law works well in our countries, while focusing on the deviations around those means and modes leads to the conclusion that there are important and systematic exceptions to Duverger's Law" (Chhibber and Kollman 1998, 53).

Later work has generally been more sceptical of the continued existence of 'Duvergerian' district-level effects, as in Diwakar's (2007, 2006) work on post-war Indian elections where competition by multiple parties is shown to have greatly increased.

Cumulatively the DL literature has stimulated a lot of useful work, especially with the fairly recent arrival of large N district-level studies. Nonetheless we would argue that it is in Lakatos's sense a degenerating research programme, for several reasons:

- (i) Both the original Law and most reformulations are vaguely phrased in terms of plurality rule ‘always’ ‘favouring’ or ‘being associated with’ a two-party system (originally nationally and now at district level), and recently modified to mean two ‘serious’ parties or even further stretched to include any ENP score of 2.5 or less (whatever pattern of party fragmentation underlies it).
- (ii) All the studies (and their sociological critics) assess the existence of DL effects or tendencies in terms of the presence or not of particular substantive patterns (ENP numbers or substitute indicators) – but these are gross outcome indicators influenced by a very wide range of variables and not just by electoral systems. This approach commits a serious fallacy in compounding all unexplained residual causal influences into an aggregate ‘Duvergerian’ effect, which is then credited to the electoral system.
- (iii) Operationalizing DL effects in terms of absolute ENP numbers makes matters worse, because any given ENP score can be produced in many different ways (Dunleavy and Boucek, 2003). Noisy data can then fog up the empirical assessment in ways favourable to DL. For instance, ENP numbers can be low because of the two-party drift that DL predicts, or because of a quite separate pattern of ‘unipolar’ drift where votes concentrate in a single party but there are a large number of smaller competitors.
- (iv) The existing literature has no clear criteria for the Law to be judged as falsified, chiefly because of the repeated efforts by DL defenders to insulate it against disconfirmation, whether by restricting its range of application or by loose talk of large-scale disconfirming results (like Indian patterns) as ‘Duvergerian exceptionalism’. No clearly specified null hypothesis has been defined, against which a serious effort is made to falsify the ‘Law’. In this sense it is hard to see the existing body of research as scientifically established.

To re-emphasize point (iv) in a more positive way, consider the algorithm in Figure 1 which sets out a sequence of tests that in our judgement would satisfactorily establish the existence of a DL effect. Our start point is that any assessment of electoral system influences needs to be framed against a counter-factual, specifying what patterning of district-level outcomes we would expect to see from the basic structure of a competitive situation. We need to start from a baseline concept of what patterns of district-level

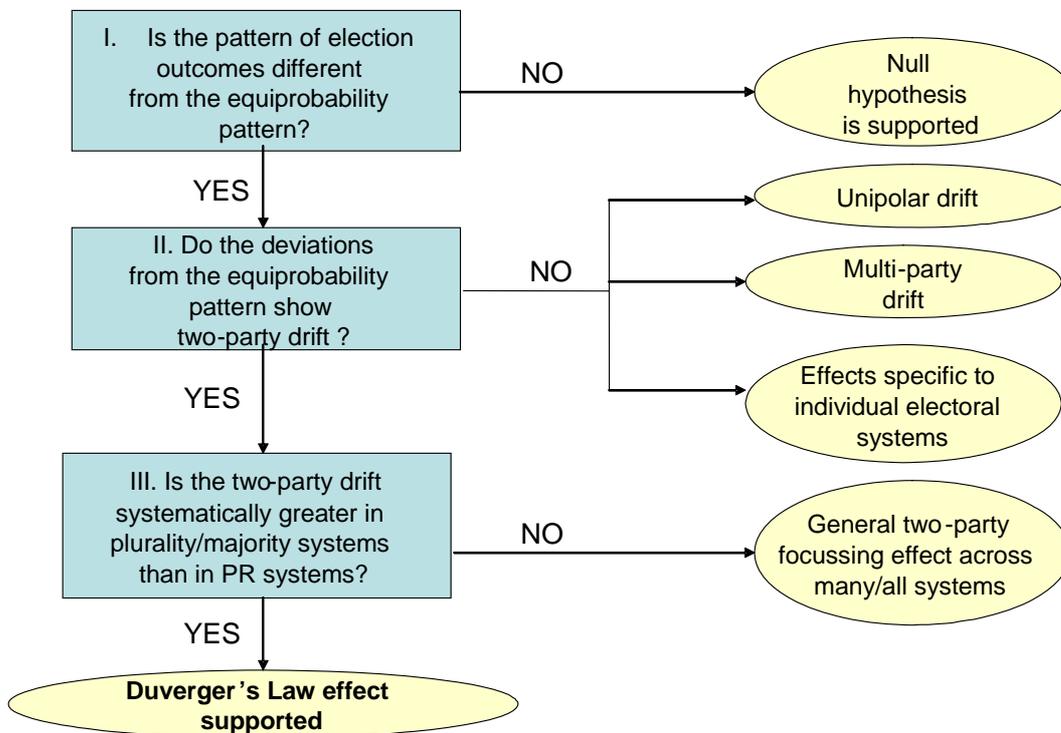
outcomes are logically feasible, given the number of parties (or candidates) that enter competition and secure a minimum level of votes (say 1 per cent). As a start point against which to measure empirical deviations, we also need to assume that all logically feasible outcomes are equally likely to occur – the standard equi-probability assumption. We next need to clearly operationalize a concept of ‘two party drift’ and distinguish it from other deviations from equi-probability. Finally we would need to establish that two-party drift from equi-probability occurs differentially in plurality rule systems compared with PR systems.

The appropriate null hypothesis then has three parts, each of which must be refuted for a DL effect to be established:

- I. The patterns of constituency election outcomes observed do not differ from those that would be expected under equiprobability.
- II. The deviations from equiprobability found do not show two-party drift as DL predicts (but either no pattern, or unipolar drift, or multi-party drift).
- III. Measured as deviations from equiprobability, the extent of two-party drift is no greater in plurality/majority systems than in proportional systems, (for example, because two-party drift occurs quite evenly in all systems, or because there are individual electoral system variations in two party drift).

On test (i) we show below that assessing performance cannot be done intuitively by ‘eyeballing’ charts of district outcomes or checking ENP numbers to see if some blanket numerical criterion is met or not. Rather the counter-factual needs to be carefully established in a context-sensitive way. On test (ii) we need to measure the deviations in observed results carefully from the equiprobability counter-factual and demonstrate that they do indeed show evidence of a two-party drift. In the rest of this paper we concentrate on showing how these two key stages can be implemented, using empirical data from India to illustrate the methods involved. Test (iii) here involves the accumulation of properly measured evidence of two-party drift deviations across many different electoral systems, so as to demonstrate whether they are generally greater in plurality rule systems as a whole (and by how much), or not. Inherently this cumulation stage cannot be undertaken in a single paper, but rather is a research agenda on which the wider profession will need to engage, hopefully using the conceptual framework and methods set out below.

Figure 1: The key steps need to establish a Duverger’s Law effect, and alternative possible outcomes



2. THE NUMBER OF OBSERVABLE PARTIES, EFFECTIVE COMPETITION SPACE AND EQUIPROBABILITY OUTCOMES IN ELECTIONS WITH FEW PARTIES

Every voting contest creates a *competition space*, defined as all the outcomes that are logically feasible between parties (or blocs or candidate or actors) involved in the competition. The shape and patterning of this space varies in important and systematic ways depending on the number of ‘observable’ parties in competition (N_{op}), which we have defined elsewhere (Dunleavy et al, 2007) as those with at least 1 per cent support. (Here, as in our earlier paper, we assume for simplicity’s sake an ‘integer universe’ where party outcomes are denominated solely in integers. We can envisage also an error term that bundles together and expresses to the nearest integer number the vote shares of all parties too small to be ‘observable’). Figure 2 shows how this influence from N_{op} levels

operates in determining the ‘effective competition space’ (ECS). The overall space of all possible elections (the Nagayama triangle) is shown here in blue. But at any given N_{op} level, only a portion of this overall space will be feasible for district outcomes, as the Figure shows for 3, 10 and 50 parties. Variations in the feasible ECS area are critical for all our subsequent proposed tests of the Duverger’s Law hypothesis. By not recognizing and controlling for ECS variations the existing DL literature systematically misinterprets their effects as if they were electoral system effects.

Given the importance of N_{op} levels in driving radical ECS changes, a first very simple but illuminating test of whether DL is applying in a country or not is to look at the distribution of district outcomes across N_{op} levels. Figure 3 shows data for three recent countries’ elections – US Senate elections across 2002, 2004 and 2006 (where we need to take a complete cycle of 3 elections to get enough cases to analyse); the UK general election of 2005; and two rather contrasting Indian general elections in 1996 and 2004. In all cases, we have carefully reconstructed these datasets from original Election Commission district data so as to cover all parties gaining 1 per cent or more in any district. (This is a far higher level of accuracy than is normal in comparative elections research, where the results for smaller parties or those contesting only a few districts are conventionally aggregated into a single ‘Other parties’ category, greatly coarsening data analysis at a very fundamental stage).³

An appropriate test for Duverger’s Law might focus on the proportion of districts with N_{op} scores of either 2 or 3, which if DL is correct should account for the great bulk of the total. Yet Figure 3 shows sharp differences between these three plurality rule countries. While over three quarters of recent US Senate contests still have an N_{op} level of 3 or less, this is true in India in fewer than a quarter of recent election contests. And in the UK it applied in only one in 30 contests in the 2005 general election. In both the India elections and in the UK the median constituency had five observable parties.

Figure 2: How the shape of the effective space for competition between the top two parties varies with changes in the number of observable parties

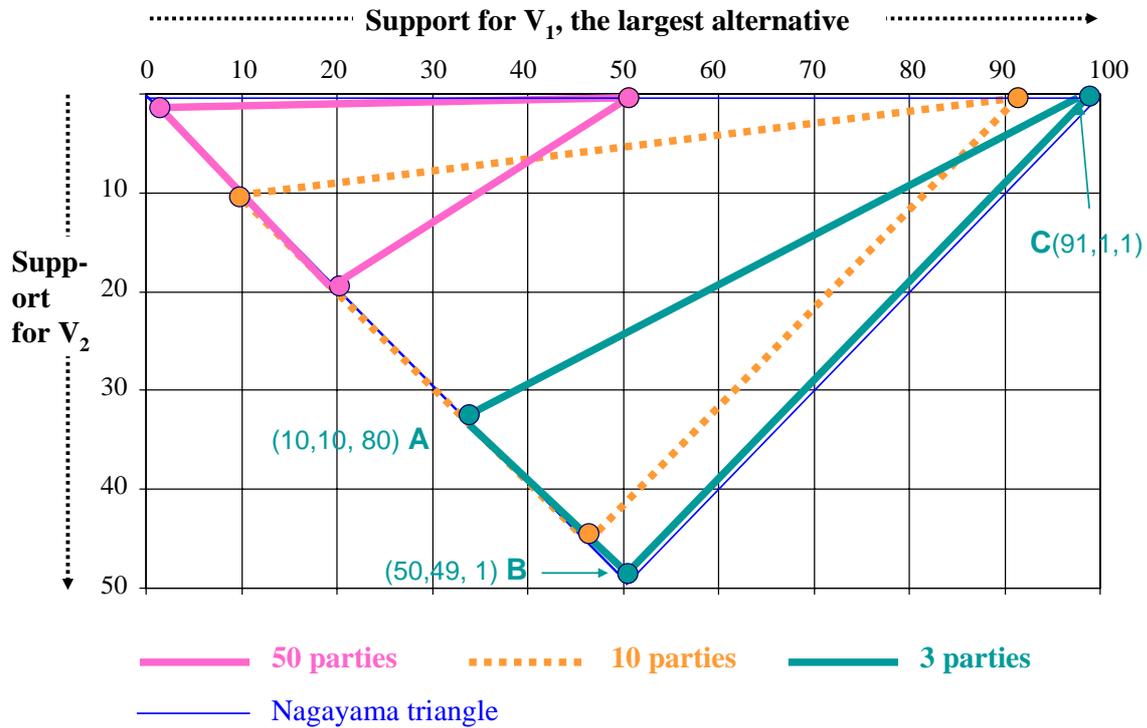


Figure 3: The percentage of district outcomes across categories for Number of Observable Parties (N_{op}) in four recent sets of elections

Number of observable parties is:	United States, Senate elections 2002, 2004, 2006	UK general election 2005	Indian general election 2004	Indian general election 1996
2	37	0	4.4	1.8
3	40	3.5	13.7	21.5
4	17	32.3	23.3	31.5
5	4	41.1	26.6	19.9
6	1	17.5	18.3	14.2
7	1	4.9	8.6	4.8
8+	0	0.6	5.1	6.3
Total	100%	100%	100%	100%
N of cases	100	628	546	543

Notes:

DL consistent rows

Sources: Author datasets – see endnote 4 for details.

To demonstrate that the N_{op} count connects to substantive politics we look below at the shaping the effective competition space. But even before this stage, it is worth establishing that the N_{op} variation is strongly consequential in other terms. Figure 4 shows that in all the cases examined here the mean levels of voting for all the V_3 to V_N parties (which we also term V_{rest} parties) rises appreciably as the number of observable parties increases. In the USA the variation is small and the initial floor mean is zero in districts where N_{op} is 2, with means rising only just over 4 per cent in states where N_{op} reaches 4. The standard deviation also shows a restricted scatter around means levels. By contrast, in the UK, the floor mean V_{rest} vote in districts with two observable parties is 17 per cent, rising to 29 per cent in districts with seven parties. In India the floor mean level in districts where N_{op} is either 2 or 3 is below 10 per cent in the heightened two-party competition conditions of 2004. But in districts with more parties competing at the 1 per cent level in 2004, and in all districts in 1996, the mean levels rise sharply reaching at least 23 to 39 per cent in areas with 6 or more observable parties. In two different but important ways, therefore, Figures 3 and 4 show DL-inconsistent patterns in the UK and India.

Turning next to competition space effects, in an integer universe with only two observable parties competing, all the possible contest outcomes lies along a straight line with 50 whole-number slots for the two largest party (V_1V_2) co-ordinates from (99, 1) to (50, 50). Figure 5 shows this outcome set situated within the Nagayama triangle, and also includes the actual constituency outcomes for US Senate elections for those states where $N_{op} = 2$ in elections across the complete cycle from 2002, 2004 and 2006. (There are too few cases in the UK and India). Because of rounding errors and the presence of non-observable parties (below the 1 per cent level), these actual outcomes drift slightly away from the pure line here. Finally, we also show the relevant ENP score lines, which vary between 1 at the top of the line and 2 at the bottom.

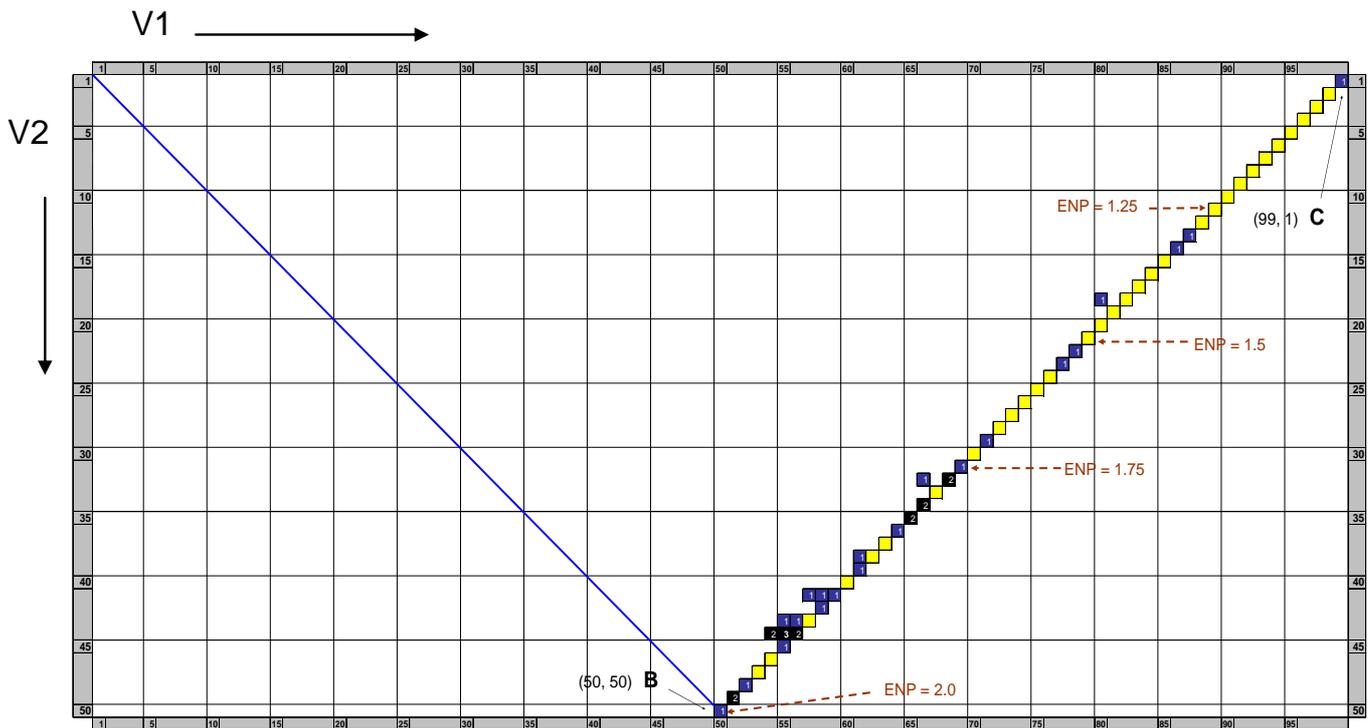
We noted above that modern empiricist DL studies considered an expectation that outcomes would bunch close to 2 (say above an ENP score of 1.75). But they rejected this restriction in favour of counting any result below any ENP of 2.5 as Duvergerian. However, from the ECS perspective no such claim can be sustainable on this criterion, since by definition ENP is less than 2 in Figure 5 – so we cannot distinguish any pattern of DL-consistent outcomes separate from the equiprobable outcome. Data from pure two-

Figure 4: The variations in the means and standard deviations for the percentage support of all parties $V_3...V_N$ (the V_{rest} parties) in district with different Number of Observable Parties (N_{op}) scores, in four recent sets of elections

Cell entries show V_{rest} per cent	United States, Senate elections 2002, 2004, 2006		UK general election 2005		Indian general election 2004		Indian general election 1996	
	Means	StDev	Mean	StDev	Mean	StDev	Means	StDev
$N_{op} = 2$	0.4	0.6			2.3	1.2		
3	2.7	2.1	17.1	3.7	8.3	7.1	13.0	7.5
4	4.3	2.5	20.9	4.5	11.7	9.0	19.9	10.3
5			23.1	5.7	18.8	12.0	24.8	10.9
6			26.5	4.8	22.7	11.5	29.8	10.7
7			29.2	4.9	26.9	12.3	29.7	9.6
8+					30.8	11.3	39.5	10.5
<i>All districts</i>	2.6	3.3	23.1	5.7	17.1	12.6	22.2	12.3
<i>N of cases</i>	100		628		543		543	

Sources: Author databases. See endnote 4. We show data only for cells with 15 or more cases.

Figure 5: District outcomes where $N_{op} = 2$, for the US Senate elections of 2002, 2004 and 2006



Notes: The blue line shows the Nagayama triangle boundary. Within this, for pure $N_{op} = 2$ outcomes only the shaded cells along the BC diagonal are feasible, with empty cells in yellow; and empirical outcomes in dark blue (1 case) or black (2+ cases). Shaded empirical outcomes *inside* the BC diagonal are created by the presence of smaller parties in those districts, all below the 1 per cent level for ‘observable’ parties, but still slightly restricting the votes for one or both of the top two parties.

party systems are hence necessarily ‘inadmissible evidence’ in debates about Duverger’s Law – because it is only when N_{op} reaches 3 or more that a DL effect can begin to be detected or not detected.

This argument will be disturbing for many political scientists because it turns on its head some long-lived conventional wisdom. A possible counter-response is to try and respecify a DL expectation, perhaps in terms of predicting a bunching of outcomes in $N_{op} = 2$ districts towards point B in Figure 5 (where ENP = 2), or reverting to some lower ad hoc cut-off level, like 1.75 parties. But such evasive tactics have no theoretical basis and they are easily countered by (for instance) being more discriminating in specifying an equi-probability outcome.

An obvious candidate to consider here is the bi-nomial distribution, which says that outcomes across districts should scatter in a manner that is set simply by changes in the mean level of support for V1 (and hence V2). The binomial distribution is the discrete distribution of the number of successes in a sequence of n independent trials with the probability p . The probability that a random variable X with binomial distribution $b(n, p)$ is equal to k , where $k = 0, 1, 2, \dots, n$ is given by

$$P(X = k) = {}_n C_k * p^k * (1 - p)^{n - k}$$

where p refers to probability of k successes. In our case, n represents the number of districts where $N_{op} = 2$, k or the success rate is the hypothesised number of districts in a single slot in the feasible ECS for two observable parties, and p or the probability of success represents the number of hypothesised districts in a single slot divided by the total number of feasible slots. Because we have too few districts with only two observable parties in datasets available to us, we are not yet in a position to assess how commonly outcomes in contests with only two observable parties conform to or deviate significantly from a bi-nomial distribution. But this could be a fruitful area for some comparative research. We conclude that unless large-scale and systematic deviations can be established, the evidence from pure two-party contests inherently cannot be interpreting as supporting Duverger’s Law.

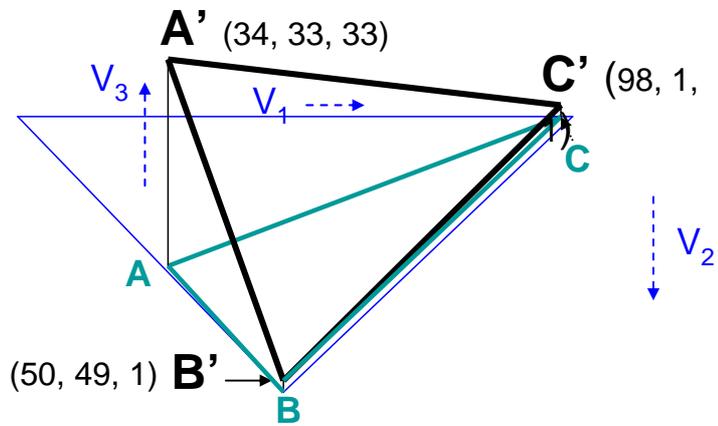
Turning to districts with three observable parties, Figure 6a shows that the actual space of all possible competition outcomes may be thought of as an angled, flat triangular plane with a thickness of one slot, shown here as A’B’C’, here rising at an angle out of the surface of the paper. Because this plane is tricky to use for displaying empirical results, and in systems with four or more parties it actually becomes a complex volume in

multi-dimensional space (see below), in the rest of the paper we focus on the ‘floorplate’ ABC shape here. This can be thought of as a projection downward from the A’B’C’ plane onto the horizontal plane, showing all available V_1 and V_2 ‘slots’. With only three observable parties, these two values in turn specify a unique V_3 level (which we also calibrate in the triangle in order that they are not lost sight of). In an integer universe when N_{op} is 3 there are 834 slots, each defined as unique V_1 , V_2 and V_3 combinations, or ‘non-equivalent distributions’. The effective competition space is then the yellow-shaded triangle area shown in Figure 6b. While the feasible range for V_2 is almost unchanged here on 1-49, that for V_1 increases from 50-99 with two observable parties to 34-98. and we also show the patterning of ENP lines within the effective space.

With $N_{op} = 3$ and where all logically possible results are equally likely to occur, it should be obvious that outcomes with strong V_1V_2 support (spread across the lower regions of the triangle) occur far more often than those showing an equitable three-way distribution of party support towards point A. In an integer world, there are 50 possible outcome slots along the BC boundary here (with the third party on its minimum score of 1 per cent), compared with only one chance of a perfect three-party outcome at point A. For empiricist approaches using an ENP level of 2.5 to assess consistency with Duverger’s Law, the boundary lines demarcating ENP levels for 1.5, 2.0, 2.5 are shown here as curved brown lines. At any point on these curves, the same ENP score is generated, albeit in very different ways. But at least the scores curves here are still definite lines - at higher N_{op} levels the curves first become fuzzy and then later extensively overlap. Three quarters (74.8 per cent) of all possible V_1V_2 slots here fall below the empiricist cut-off score of 2.5 effective parties. So if district outcomes are *randomly* distributed across the competition space, we should expect the same proportion to be DL-consistent. Unlike $N_{op} = 2$ situations (where any random outcome is definitionally DL consistent), in principle non-DL consistent results can occur in Figure 6b. But they are unlikely. Here again, the empiricists are playing with heavily loaded dice, pre-programmed to produce results that they will find favourable.

Figure 6: The effective space of competition with three observable alternatives

(a) Illustrative view



(b) 'Floorplate' view, with data from India 2004 general election

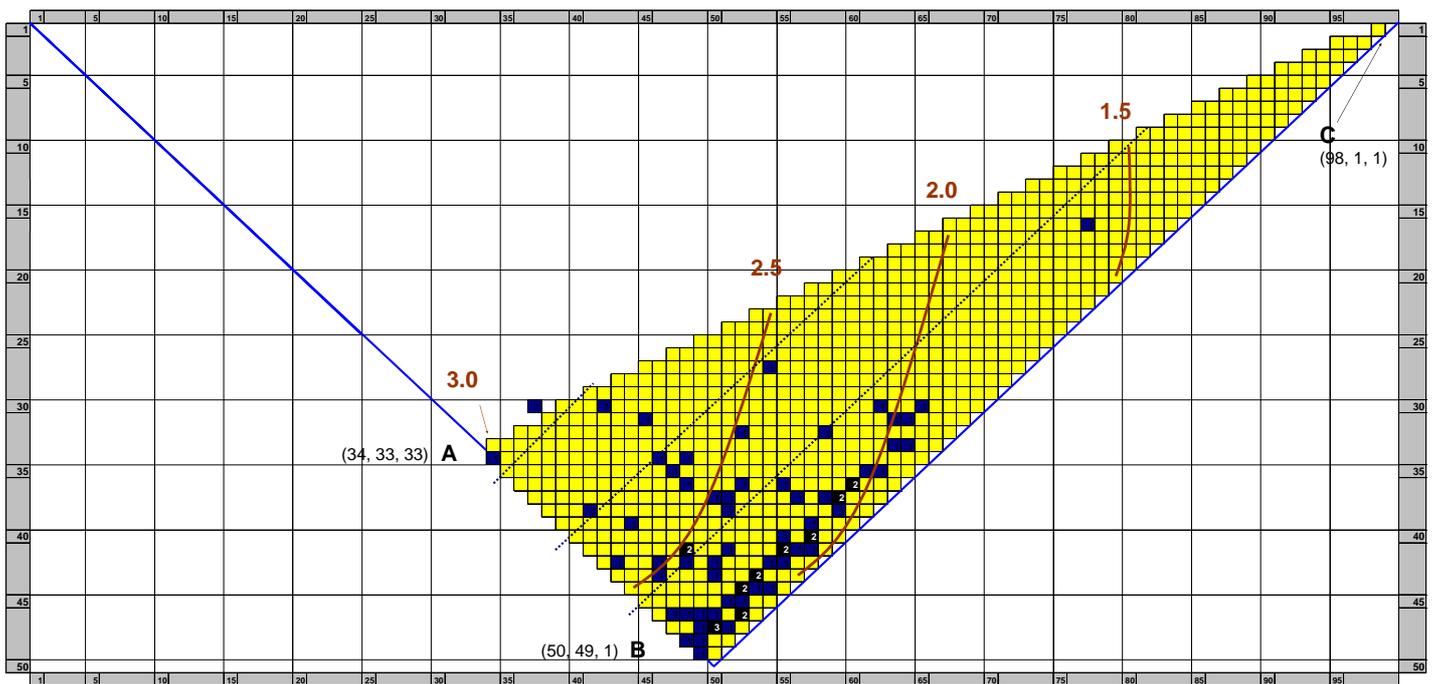


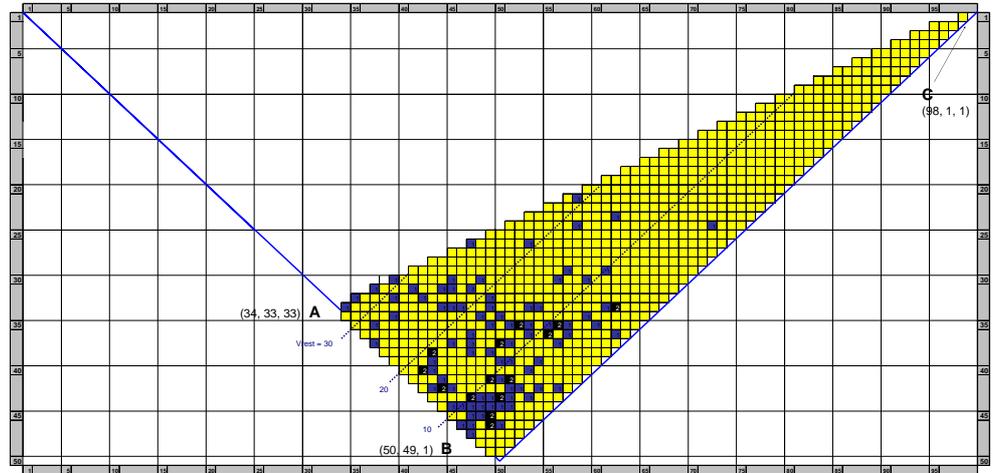
Figure 6b also shows the empirical outcomes in the 75 districts where $N_{op} = 3$ in the 2004 Indian election, which scatter considerably to populate far more of the feasible space (compared with the two observable parties situation in Figure 5). Around two thirds of the results are at low levels of V_3 , below 10 per cent and often very close to the BC boundary. In line with DL predictions, the minimum V_3 level in India is very low and a bloc of results with a two-party configuration looks prominent. In ENP terms the bulk of results are right of the 2.5 line, and many of them straddle the $ENP = 2.0$ line where the top two parties monopolize most of the votes. How should we interpret these results? Just eyeballing the outcomes pattern, or counting scores above a cut-off level, are unlikely to be useful approaches. Yet on the most basic ('blank') equi-probability grounds alone (without taking into account mean vote share levels) we might expect a quarter of the results to be above an ENP level of 2.5 - and in fact almost this many results (18 out of 75) are on or above this line.

To illustrate the difficulties here a little further, consider Figure 7 where the top picture shows the empirical results for constituencies with only three observable parties from the 1996 Indian general election. Clearly this set of outcomes is more scattered across the whole feasible space and has a much lesser concentration of outcomes close to the BC line than in 2004. But simply eye-balling the results it might still be tempting to see a two-party concentration in Figure 7a. Yet consider Figure 7b where we have generated a similar size random set of outcomes, constrained only by a requirement that V_1 is less than 64. This particular random outcomes distribution is somewhat lighter on the bottom left than the actual pattern, but the two sets of results are not markedly different. Of course, nothing can be inferred from any one random distribution either. Our point is just visual checks cannot show what is or is not a distinctive pattern. What is clear is that in the 1996 election, far more than in 2004, the Indian outcomes clearly scatter so as to fill the whole of the outcomes space bounded by V_1 's maximum size.

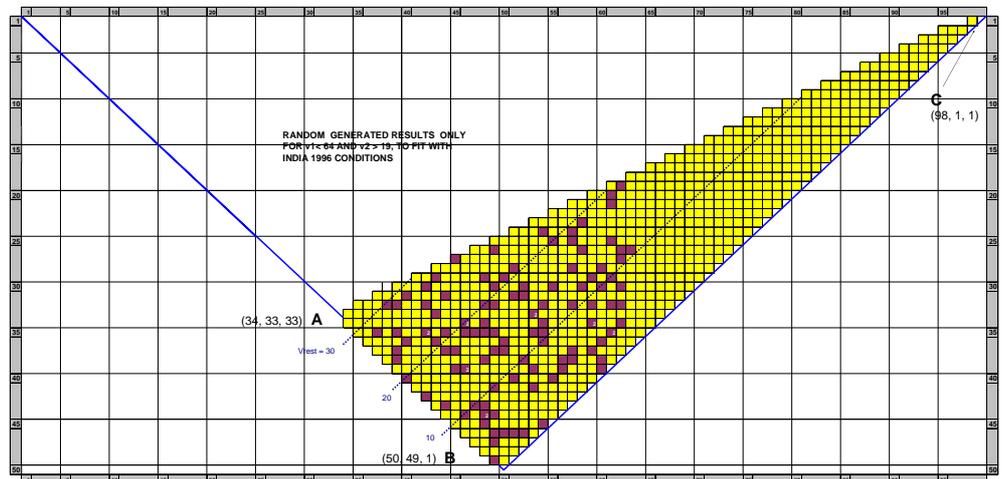
Contrast this also with the results for the 2005 British general election shown in Figure 7c. The pattern here is clearly bounded empirically, first by a restrictive V_1 range (from 43 to 65 per cent) and then by outcomes being compressed into a narrow band of V_3 range (from 11 to 22 per cent). The minimum V_3 level here is much higher than in either of the Indian cases, contrary to the DL prediction, and in ENP terms

Figure 7: Comparing outcomes across Nop = 3 constituencies in other elections

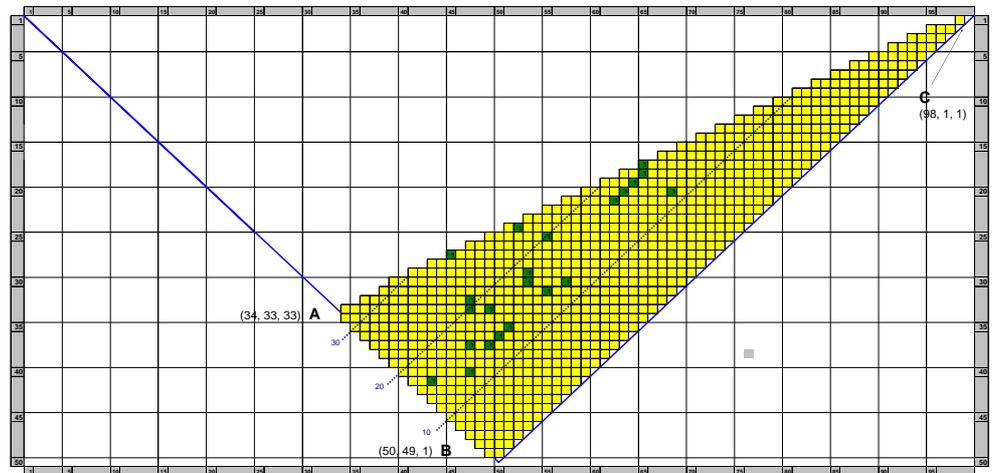
(a) Indian
1996 general
Election result



(b) Random outcome
set for the same
parameters
($v1 < 64$)



(c) Great Britain
2005 general
Election result



very few points lie below or right of the 2.5 line. However, unlike India again, the results are also not scattered widely. Instead the narrow V_3 range gives a strong ‘linear slot’ shape (which in fact recurs in all the Britain 2005 plots for districts with higher levels of N_{op}). This strong top two-party patterning does not necessarily specify a nation-wide two party pattern, because different parties constitute V_1 and V_2 in different kinds of seats and various parts of the country. None the less, this is a strongly DL consistent patterning of V_1 and V_2 at district level. Yet again, within the V_1 and V_3 limits, the outcomes could easily be seen as randomly distributed, and they certainly do not show any bunching towards the lower end of the angled box.

Clearly the next stage for analysis should be to analyse systematically how far the actual outcomes distributions differ from those that might arise randomly. Several specification strategies might be relevant here. A minimally restrictive approach might look at all random distributions within the V_1 , V_2 and V_3 ranges, and from this basis compare how likely it is that outcomes arose by chance. A more constraining approach would take account of the actual mean levels of V_1 to V_3 scores in any given election. The multi-nomial distribution is a generalised version of the binomial distribution, showing the patterning of the number of successes in n independent trials. Each trial can lead to one out of finite k possible outcomes, with corresponding probabilities p_1, \dots, p_k . Then, the probability of an outcome following a multinomial distribution is given by

$$P = [n! / (n_1! * n_2! * \dots * n_k!)] * (p_1^{n_1} * p_2^{n_2} * \dots * p_k^{n_k})$$

On this basis the set of outcomes across districts should cluster in response to observed mean levels across the parties. Results that are significantly more concentrated or patterned on two-party predominance than predicted by the multi-nomial distribution could be taken as distinctively supporting Duverger’s Law. But those that are broadly consistent with predicted distributions are inherently ambiguous. A detailed assessment here would involve significant methods issues and take us beyond the scope of this paper, and so we merely note that in principle the multi-nomial distribution looks like a valuable additional test.

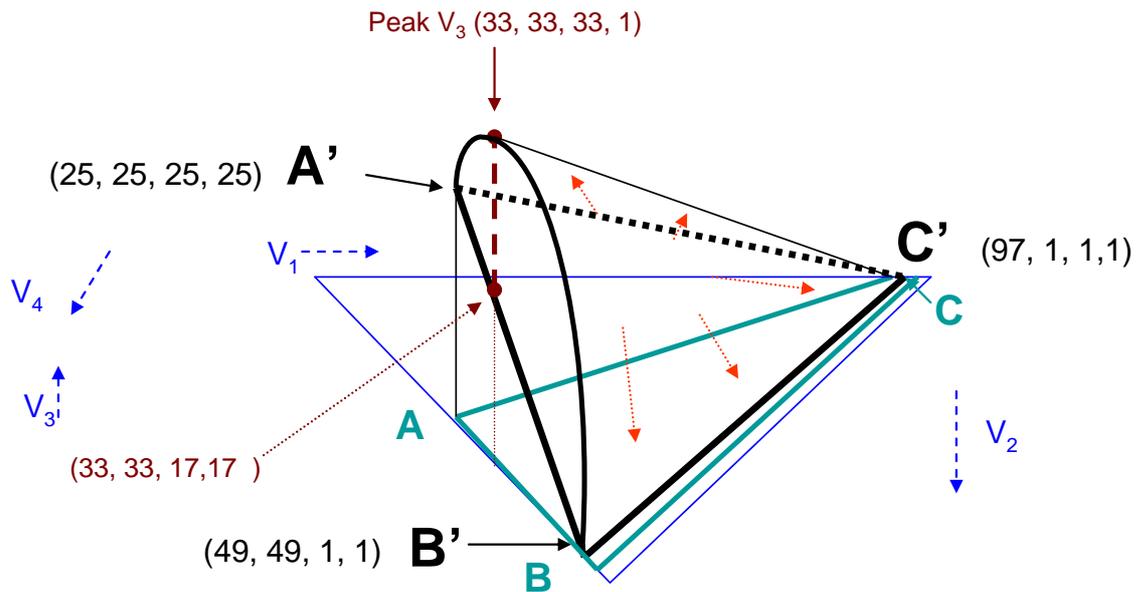
3. COMPETITION SPACE IN ELECTIONS WITH FOUR TO SIX OBSERVABLE PARTIES

When the number of observable parties grows to 4 or more, the effective competition space becomes a multi-dimensional volume, in which every point represents a unique combination of outcomes scores for all the parties competing. We can still (just) illustrate the inter-relationships of the scores for all the parties graphically, as in Figure 8. All the possible score combinations here lie in the volume created by fitting a dome shape to the inclined A'B'C' triangle shown in Figure 6a. As in that previous view, the third dimension here (rising out of the page surface) shows the V_3 outcome, while the range of V_4 's scores is shown by the thickness of the 'tri-dome' volume here. Notice that here A' is the peak score for V_4 at 25, where all parties are equal. But V_3 's peak score is further along the top of the tri-dome surface on the upper A'B' boundary, where the top three parties have 33 per cent each and V_4 has 1. In fact V_4 's score is 1 across the whole top of the tri-dome surface connecting A'B'C'. For any given combination of V_1 and V_2 scores, V_4 's score reaches a maximum at the bottom plane surface connecting the A'B'C' triangle plane. So the range of V_4 's score is measured *downwards* by the distance from the top surface to the A'B'C' plane. For instance, with V_1V_2 scores of 33 each, then following the brown line down shows that V_4 does best when it splits the remainder evenly with V_3 , each getting 17. V_4 's range is at its maximum on the A'B' boundary.

Yet even with just four parties Figure 8 is no more than an intuitive illustration of what is going on, and with five or more parties the multi-dimensional outcomes spaces cannot be graphically represented. A partial but very economical and illuminating way of summarizing the structure of the resulting outcome space is to compile density plots of how many logically possible combinations of the smaller parties' votes shares there are for each feasible V_1V_2 slot on the 'floorplate' diagram. These unique vote combinations are technically called NEDs, or 'non-equivalent distributions', because we are not interested in which particular party occupies the V_3 or V_4 (collectively the V_{rest}) slots, but only in the share of votes going to each rank.

Figure 9 shows that the density distribution for four parties follows a sloping escarpment shape, with scores highest on the AB boundary and then falling away from it, with a descending 'ridge line' of higher scores also falling down from there towards C.

Figure 8: An illustrative view of the competition space when there are four observable alternatives



Note: In this diagram *only* the co-ordinates show V₁ to V₄ scores

The upper (or 'scarp') slope is steeper, while the lower ('back') slope falls gently towards the BC boundary, where all V_{rest} parties are on their minimum score of 1 and hence only a single V_{rest} distribution is feasible. This pattern also recurs with any higher number of observable parties, but the peak density levels rises very sharply with N_{op} levels, because the number of NED vote combinations that are feasible shoots up explosively.

This effect transforms the problem of differentiating between equi-probability outcomes and DL predictions for all situations with 4 or more observable parties. In line with Figure 9, strict equi-probability predicts that district outcomes should cluster close to the AB boundary line, and should be pulled towards the peak zone where NEDs cluster most intensively. Outcomes should particularly be pulled away from the very low probability zone close to the BC boundary, exactly the area where DL predicts they should concentrate. Furthermore all these strict equiprobability effects should very strongly and automatically increase as N_{op} levels go up, because the difference between the BC line outcomes (on 1) and the peak concentrations of NEDs grows thirty-fold between 4 and 8 observable parties , as Figure 10 below shows.

Figure 9: The number of non-equivalent distributions of V_{rest} support per V_1V_2 slot with four observable parties

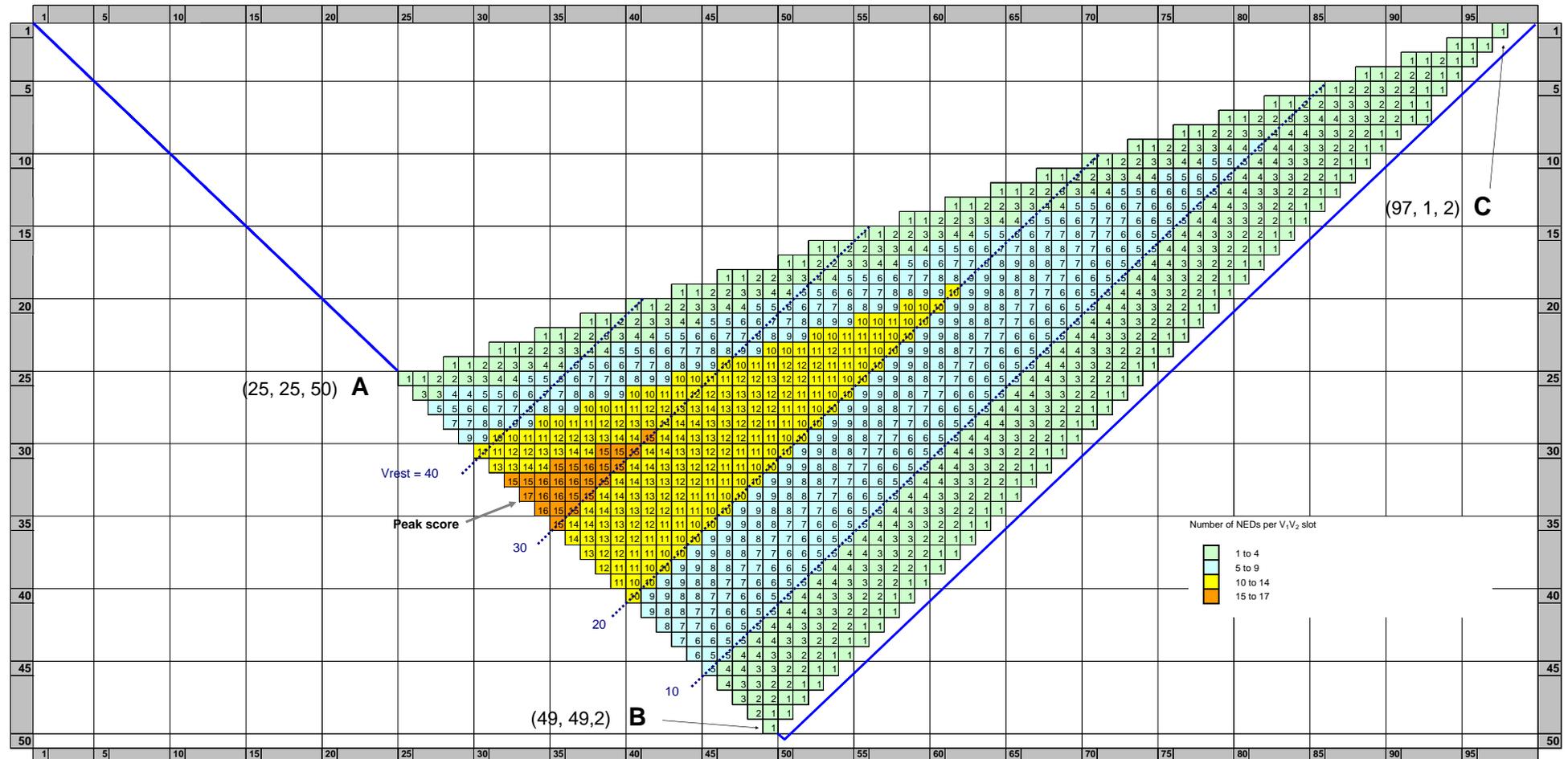


Figure 10: How the number of non-equivalent distributions (NEDs) with different numbers of observable parties

Number of observable parties (N_{op})	Total NEDs feasible	Number of 'floorplate' V_1V_2 slots	Mean NEDs per 'floorplate' V_1V_2 slot	Peak Total NEDs per V_1V_2 slot	Ratio of peak total to mean NEDs per V_1V_2 slot
2	50	50	1	1	1
3	834	834	1	1	1
4	7,153	1,213	5.7	17	2.9
5	38,225	1,404	27.2	113	4.2
6	143,247	1,520	94.2	546	5.8
7	407,254	1,591	256.0	1,911	7.5
8	930,912	1,637	568.7	5,170	9.1
9	1,786,252	1,656	1,078.7		
10	2,977,876	1,666	1,787.4		

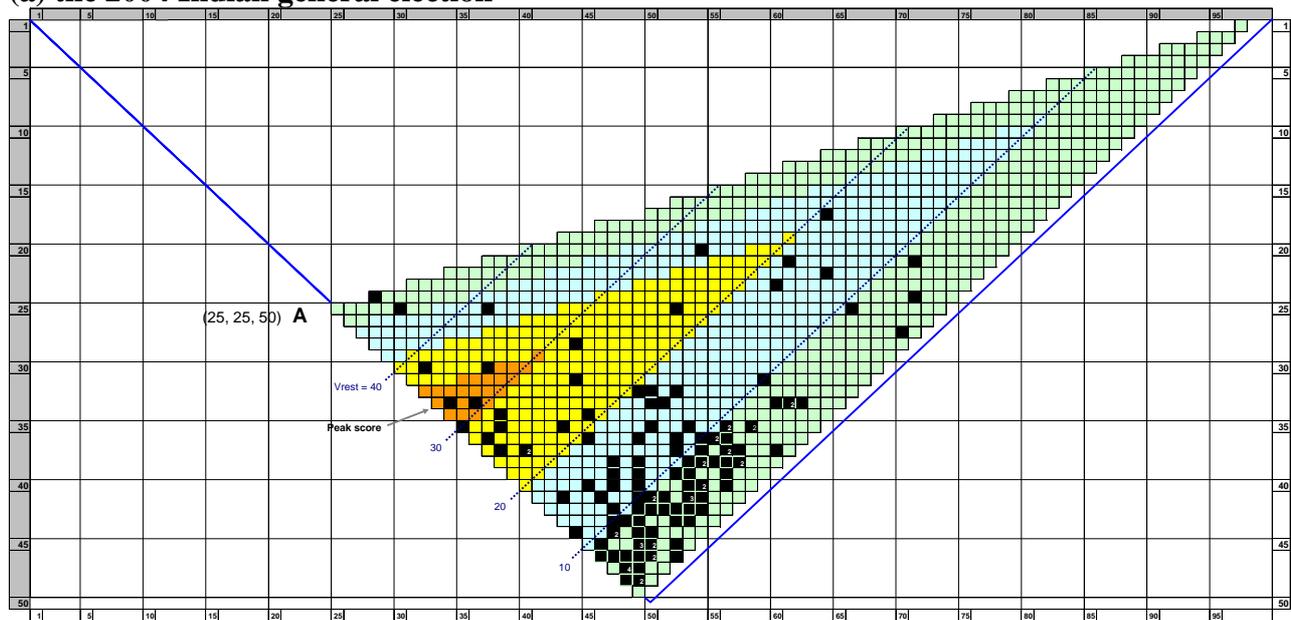
Sources: Authors' computations, except column 2 'Total NEDs feasible', rows 2 to 8, which is an authors' replication of results in Benoit and Laver (2005); in column 2 the data for N_{op} rows 9 and 10 are from Benoit and Laver alone.

However, note that strict equi-probability could *only* apply here in a highly unlikely situation where voters choose directly and independently across all the myriad of different NED combinations, however small parties may be. So in fact divergences from strict equi-probability are to be expected not just by DL, but by almost any realistic theory of voting. For instance, a common sense model might say that voters choose first or most directly whether to support 'major' parties and come later to other choices – they also care less about tiny differences in minor party vote shares than the vote shares of the biggest parties.

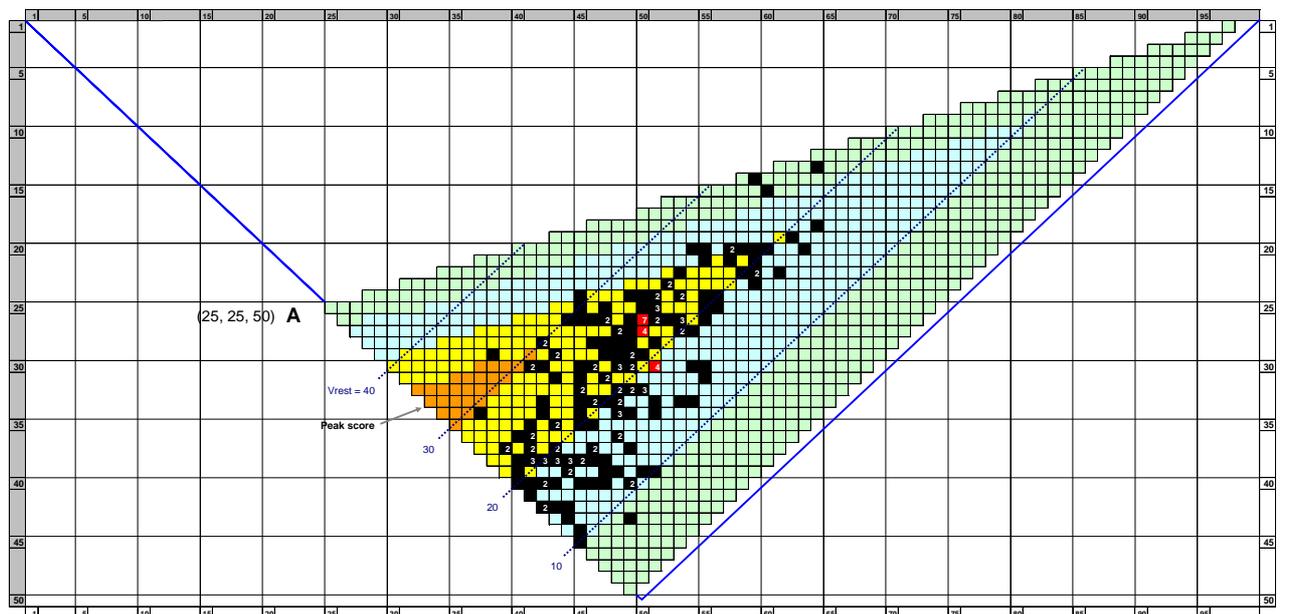
We can shed some preliminary light on the impact of changing competition space s at higher levels of N_{op} by plotting the empirical outcomes for districts with four observable parties against the contours for the density of NEDs across V_1V_2 slots, shown in Figure 11 for the 2004 Indian election and the UK 2005. In India there is a marked concentration of outcomes in low probability areas near to point B and close to the joint means for V_1 and V_2 . The minimum size of V_{rest} (the combined third and fourth party vote shares) is also very low. Both these effects are potentially consistent with a strong DL effect operating. But at this low N_{op} level we would still need to check against the patterning suggested by the multi-nomial distribution. Figure 11a also shows no clustering of India outcomes in the zone with the highest density of NEDs, contradicting the equi-probability prediction. However, there is a very wide scatter of results and no particularly clear

Figure 11: The outcomes in districts with four observable parties

(a) the 2004 Indian general election



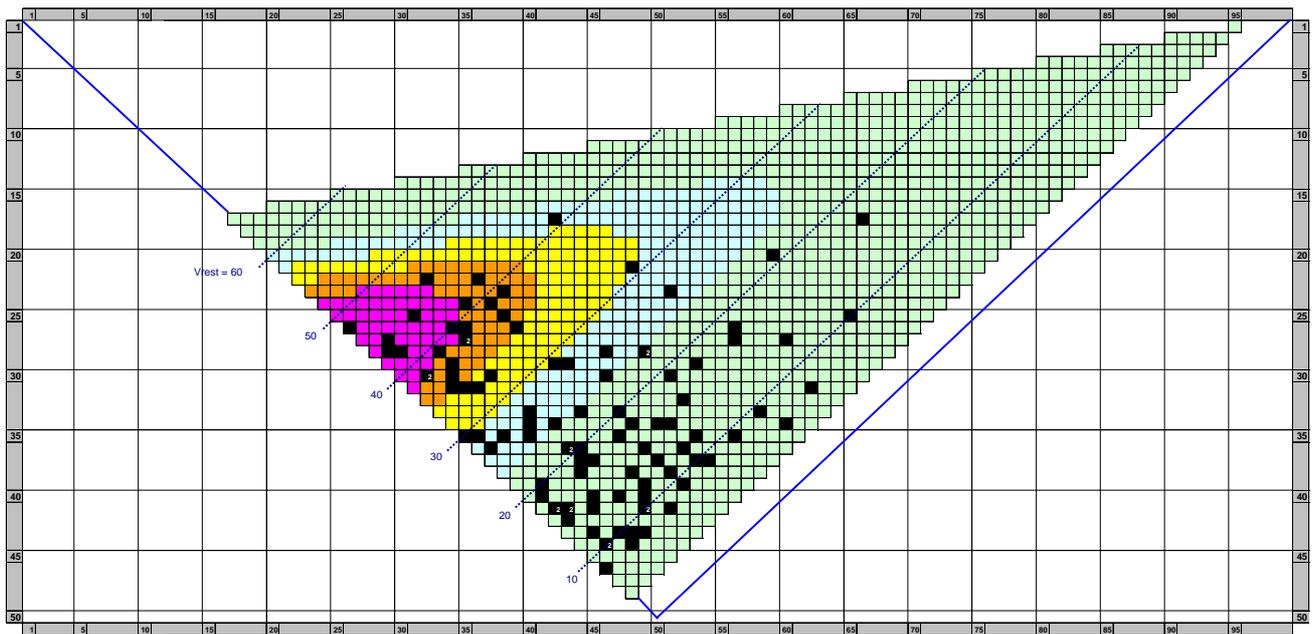
(b) the 2005 general election, Great Britain



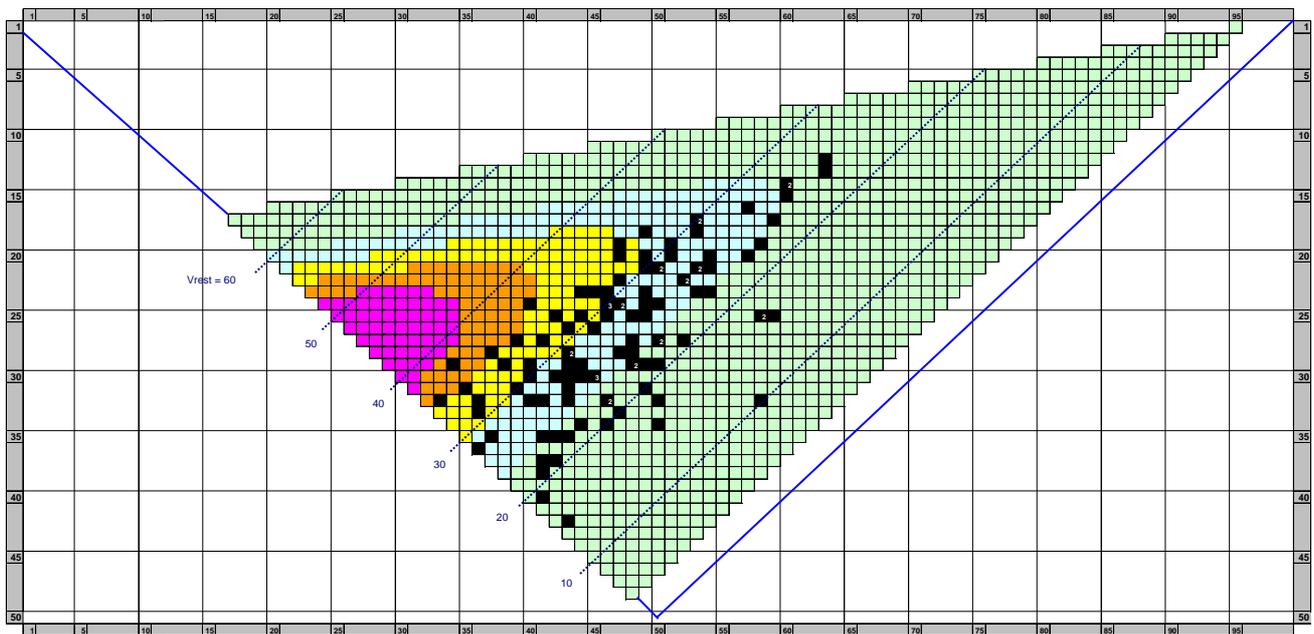
Number of NEDs per V_1V_2 slot



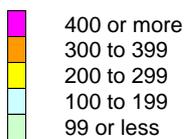
Figure 12: The outcomes in districts with six observable parties
(a) 2004 general election, India



(b) 2005 general election, Great Britain



Number of NEDs per slot



‘two-party’ patterning. By contrast, the Great Britain results in Figure 11b show a strong two-party relationship, narrowly constrained by levels of V_{rest} from 10 to 30, which is much more DL consistent. However, the minimum V_{rest} level is high here, so that results are well away from the BC boundary, in the middle of the feasible space – not what DL predicts at all. In fact the bulk of results is concentrated on medium to high probability areas on the ‘back’ slope and close to the ridge line for the tri-dome shape – close to where equiprobability predicts that they should be, but not in the peak zone for NEDs itself.

Turning to the outcomes in districts with six observable parties, shown in Figure 12, it is important to note that the scaling of the NED contours has changed markedly here. The peak density is 519 different V_{rest} combinations at the centre of the pink zone (compared with a peak of 17 in Figure 9). In both India and Great Britain with 6 observable parties there are far fewer outcomes close to the BC boundary line and more broadly in the low probability (green) zones of the overall feasible area. In both India and the UK there is more bunching of outcomes in the high density zone and a greater scattering around the V_1 and V_2 means. But there the common differences end. Figure 12a shows that in India outcomes mainly occur close to the AB boundary (as equi-probability predicts). Relatively few are close to the BC boundary and the denser scatter of results spreads right into the peak zone for equi-probability. Statistical testing may or may not show some minor clustering here different from a random multi-nomial pattern. In the UK the results in Figure 12b show a strong V_1V_2 linear slot patterning, clustering away from both the AB and the BC boundaries. There are no results at all in the peak equiprobability zone, but on the other hand there are also very few results in the green (low probability) zones. Instead the bulk of outcomes occur moderately high up on the ‘back’ slope of the NEDs density plot, in zones of medium to high equiprobability.

At still higher levels of observable parties our plurality rule countries yield too few instances to present more systematic data. However, on theoretical grounds plus indications from India and the UK data we can predict that three contradictory effects are likely to operate. First, the overall scatter of results will increase at $N_{op}=7$ and above, an effect visible in the ‘hollowing out’ of the plots for India and the UK (but not showing up in the standard deviations in Figure 4). Second, if strict equi-probability were to apply the explosive growth in the number of non-equivalent distributions should *prima facie* create an ever stronger pull of outcomes towards the NEDs peak zone. In fact, the small amount of relevant data we have from India and Britain shows no evidence of such a pull – instead with 7+ observable parties the results seem to form a hollow centre within the feasible area. So if the greater and greater NEDs peaks do have any attractive influence it must be more and more heavily discounted. Third, the NEDs peak zone moves further and further up the Nagayama triangle area as the number of observable parties increases. In Britain one interpretation

of Figure 12b is that at high levels of N_{op} a maximum constraint on the size of V_{rest} prevents outcomes occurring in the peak NEDs zone. At six observable parties V_{rest} levels would have to be over 40 per cent to access the peak zone, and at $N_{op} = 8$ the requisite V_{rest} level would be 50 per cent, a very high level indeed. So as N_{op} levels rise, equi-probability increasingly predicts the pattern actually shown in Figure 12b, that is a clustering of outcomes high up on the back slope of the NEDs escarpment and along the ridge line (wherever this is reachable with prevailing levels of V_{rest}).

We will need to accumulate many more empirical examples in the format of Figures 11 and 12, and across plurality rule and proportional representation systems, to begin to appreciate how the interaction of the density zones, the maximum V_{rest} constraint and the multi-nomial distribution operates with higher levels of observable parties. But it is already clear that we have a good chance of being able to account for increased multi-party patterning of the vote at higher N_{op} levels *simply in terms of competition space effects*, without having to resort to the distinctive electoral system effects posited by Duverger's Hypothesis. At the least, any effect for PR systems to increase party fragmentation must be seen as strictly supplementary to competition space effects, triggered by the decisions of very small parties to enter competition.

CONCLUSIONS

The original Duverger's Law (and Hypothesis) were historically important propositions and the subsequent debate and research about DL effects has added considerably to our knowledge. Yet there are also many signs that the Duverger's Law literature has become what Lakatos terms a 'degenerating research programme'. The core DL propositions have been extensively re-specified (by successive efforts to protect them against falsifying evidence) and poorly operationalized (by trying to fit a later, more empirical-looking superstructure onto the original, vague proposition). The near-religious determination of some adherents to making the DL propositions unfalsifiable is well exemplified in Gordon Tullock's argument that: 'Duverger's Law is true, but it may take 200 years to work itself out'. By contrast we have specified a clear, three-part null hypothesis, all parts of which would need to be rejected for a DL effect to be established. After more than six decades of work, very little in the existing literature seems to bear convincingly on substantiating these propositions.

While the DL literature undeniably opened the way for us to explore some fundamental underlying regularities in the operation of all elections, we have argued here that it extensively misattributes competition space effects to electoral system differences. Using the key concepts of the number of observable parties and 'effective competition space', we developed a series of precisely

measurable tests for Duvergerian two-party drift, and also contrasting predictions of how district-level performance should be structured if results are random or reflect only equi-probability influences (that is, assuming that all logically possible vote combinations across parties are equally likely to occur).

When the ‘number of observable parties’ is just 2 or 3 we can predict the district-level results by assuming equi-probability. With only two observable parties, outcomes should follow the bi-nomial probability distribution, defined essentially by the mean vote for the largest party (V_1). For three observable parties, outcomes should reflect the multi-nomial probability distribution, defined by the mean support for the two largest parties (V_1 and V_2). This approach means that most of the ‘strong’ cases conventionally cited as evidence of Duverger’s Law (such as perfect 2 party district-level outcomes in many US Congressional districts) can be fully and more parsimoniously explained in ECS terms, rendering them inadmissible as support for DL effects.

When the N_{op} level reaches 4 or more, predicting equi-probability outcomes becomes a complex task. We charted possible outcomes as plots of the two largest parties’ vote shares on to which density contours are mapped to show the number of all possible distributions of smaller parties’ support per V_1V_2 slot. In four or more party competition equi-probable outcomes are defined by the complex interaction of two pressures. The first is the multi-nomial distribution set by the mean party vote-shares. But the second is the bunching of logically possible outcomes in particular parts of the density plot, discounted in some way by voters. This second effect is initially weak when $N_{op}=4$, because the maximum number of possible $V_3..V_N$ permutations is small, but *ceteris paribus* it should strengthen as the number of non-equivalent distributions grows explosively with increases in N_{op} levels. However, with 7 or more observable parties the peak NEDs zone typically becomes inaccessible because of constraints on the maximum size of the combined $V_3..V_N$ vote.

Both the theoretical argument and the preliminary data from India, the UK and USA reviewed here, suggest that as more and more parties enter competition so strong automatic pressures come into play that make it less and less feasible for two-party drift to occur. Again it is not clear that any reference to electoral system effects is needed to explain multi-party vote outcomes – the driving force here is simply the increase in the number of parties getting a tiny 1 per cent of the vote each and the impacts of such changes on the shaping of competition space. In fact we might turn around Sartori’s famous (and loaded) question (2005, p. 107): ‘How much feebleness makes a party irrelevant?’ A clear implication of our approach (and of the data reviewed here) is that every small party getting enough votes to enlarge the competition space can be very relevant indeed for the evolution of party systems.

We conclude that clarifying *separate* effects from changes in the number of observable party and effective competition space on the one hand, and changes from the electoral system on the other, opens up a large agenda of new research questions, and (no doubt) debates. Once many different Duvergerian effects can be accurately specified in defensible and well-operationalized ways, and contrasted more precisely with alternative predictions, then the prospects should improve that (if they exist) genuine DL differences across plurality rule and proportional representation systems can be better identified and measured in future. This is a large agenda for research, and one paper can only make a small start on it. But with some fresh concepts and tools in place, we hope it will be possible to more quickly accumulate the comparative evidence from district-level outcomes that is needed to finish the job.

Endnotes

1. For Shugart (2005, p.xx): ‘The comparative study of electoral systems is a mature field... [T]he core concerns of the field can be summed up by the words “Duverger’s Law”’. For Bowler (2006, p. 578): ‘Duverger’s Law can be taken to be the canonical statement of what electoral systems as institutions do and why the choice of electoral institutions matters so much’. For William Riker (1982) the ‘Law’ provided a (relatively lonely) core example of how political science can develop cumulatively on ‘normal science’ lines.

2. It is hard to know quite why an empirically contingent average pattern (of V_3 being larger than V_1-V_2) can justify this methodological decision. There are many counter-examples where ENP is 2.5 or more but V_3 is far less than V_1-V_2 . For instance, consider a seven party system where $V_1 = 60$, $V_2 = 18$ and $V_3 = 6$, with V_4 to V_7 on 4 each.

3. Data sources are the author’s own corrected, cleaned and checked datasets. For India the data is derived from Indian Electoral Commission data - see also Diwakar (2006 and 2007). For the UK, the sources are Election Commission, ‘UK Parliamentary general election 2005 – A-Z of constituency results’ at

<http://www.electoralcommission.org.uk/templates/search/document.cfm/15241> cross-checked individually against a valuable data set by Pippa Norris (obtainable from

<http://ksghome.harvard.edu/~pnorris/Data/Data.htm>). The Norris dataset contains a few district

errors corrected here, but for our purposes its chief limitation is that it aggregates the votes of many smaller parties into one ‘Other’ category. For the USA Senate elections the sources are at Federal Election Commission, <http://www.fec.gov/pubrec/fe2004/federalections2004.shtml> and the Clerk of the US House of Representatives, at

http://clerk.house.gov/member_info/electionInfo/index.html . The specific tables used are for 2006 the *Recapitulation of Votes Cast for United States Senators (Terms Beginning Jan. 4, 2007)*, *Election of Nov. 7, 2006*

at http://clerk.house.gov/member_info/electionInfo/2006election.pdf and earlier equivalent tables for 2004 and 2002.

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