The voting power approach : response to a philosophical reproach

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Response to a philosophical reproach

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1. Introduction

Despite its title – 'The voting power approach: Measurement without theory'– Albert's (2003) philosophical critique, which appeared in a previous issue of this journal, is actually directed against the *theory* of the measure of *a priori* voting power, based on the intuition of voting power as *I-power*. This theory, founded by Penrose (1946, 1952), is presented in detail in our book, (1998) and briefly outlined in our (2000). As we shall see, Albert has his own reasons for avoiding the terms 'theory' and 'measure' in this connection; but we have no such reason and we shall speak of the 'Penrose measure' (using this term to refer also to its derivatives and refinements)[1] and the 'Penrose theory'.

A priori voting power is that component of *actual* (or *a posteriori*) voting power that voters derive solely from the decision rule itself: computed without regard to (or in ignorance of) all information about the personality of the voters (their specific interests and preferences, relations of affinity or disaffinity between them) and the nature of the bills to be voted upon. Coleman (1971: 297) aptly describes it as 'formal power as given by the constitutional rules of a collectivity'.[2]

I-power is the notion of voting power as a voter's degree of influence over the outcome – under a specified decision rule – of a division of a decision-making body: whether a proposed bill is approved or rejected. Albert does not seem to have any philosophical objection to the alternative, *P-power* notion of voting power, which regards a decision rule as a simple cooperative game with transferable utility and conceptualizes voting power as a voter's relative share in a fixed total payoff.[3]

Albert has two fundamental philosophical objections to the Penrose theory. First, he claims that this theory is inapplicable to the real world because it cannot be used for purposes of prediction or explanation.[4] Second, he alleges that the *Principle of Insufficient Reason*, which underlies the Penrose measure, is unsound. We shall rebut these two objections in the next two sections.

2. Is the Penrose theory applicable?

Albert spends considerable space arguing that the Penrose theory (or, as he insists on calling it, 'the VP approach') is not empirical. He could have saved himself the trouble: the theory is avowedly about a priori voting power; and if 'a priori' means anything, it means 'prior to or independent of experience; contrasted with "a posteriori" (empirical).'[5] So much is uncontroversial. Nevertheless, as we shall show, this theory is applicable to the real world and does lead to empirically testable predictions.[6]

What is somewhat eyebrow-raising is Albert's rather extreme disparagement of, not to say hostility towards, non-empirical theories. This is made quite evident by his choice of terminology. On page 356 he tells us that '[a]mong the sciences, one must distinguish between *formal* sciences, like logic and mathematics, on the one hand, and *factual* sciences, like physics or economics, on the other.' This choice of terminology is very tendentious: is the proposition 2 + 2 = 4, or Fermat's last Theorem any less factual than Gresham's Law or the Law of Diminishing Marginal Utility? It seems that for Albert only empirical facts are really factual; the truths of mathematics, 'even truths about numbers' are, he asserts, 'empty', because they 'do not tell us anything about the physical or social world. … At least according to the prevailing view, mathematics provides only a language for (some of) the factual sciences.' (pp. 356–357). Perhaps this is the prevailing view in certain doctrinaire philosophical circles; but most scientists are aware that mathematics provides not merely a language but also, at the very least, an indispensable deductive apparatus for various sciences.

The status of mathematics is relevant to Albert's polemic, because of the following assertion he makes (in his Abstract) about the Penrose theory: 'Viewed as a scientific theory, it is a branch of probability theory and can safely be ignored by political scientists.' (p. 351).

The first half of this assertion is arguable. But the second half – if it has any connection at all to the first half – implies that probability theory as a whole, of which the Penrose theory is (allegedly) but a branch, can safely be ignored by political scientists. In our view, prudent political scientists should ignore this fundamentalist philosophical advice.

Because the Penrose theory is non-empirical, Albert not only prefers to refer to it as a mere 'approach' but would even deny it the right to speak of 'measuring': 'Felsenthal and Machover … talk as if they were using a positive theory.[7] For instance, … [they] repeatedly speak of "measuring" voting power. But this is a paradigmatic case of measurement without theory' (p. 359). Measuring is presumably a prerogative of empirical science; hence the title of his paper.

Yet, pure mathematics – that non-factual science of empty truths – abounds with talk of 'measuring': one of Archimedes' best-known works is *On the Measurement of the Circle*; and modern pure mathematics has an important branch (which, as it happens, encompasses probability theory) called 'measure theory'.

Albert is much occupied with categorizing the Penrose theory: is it part of political science (he thinks it isn't), or a branch of probability theory (he thinks it is), or perhaps political philosophy (he thinks that under an 'alternative interpretation' it may be). We think that it may partake of all three branches of knowledge – depending of course on how their boundaries are defined. But we are not really worried about this kind of demarcation dispute, beloved of certain taxonomically-minded philosophers of science. What we do wish to argue is that the Penrose theory is applicable and useful in a political and constitutional context.

Contrary to the impression Albert wishes to create – the Penrose theory can and does lead to empirically testable predictions. Here is an example. One of the concepts of the theory is the a priori probability A that a decision-making body acting under a given decision rule will adopt a bill rather than blocking it.[8] A is not directly observable, because in the real world decision making is mediated by voters' preferences, and other behavioural factors. However, A does have a definite effect on the *actual* propensity of the decision-making body to adopt proposed bills. In our paper (2001a) we showed that the decision rule (known as 'qualified majority vote' or QMV) prescribed by the Treaty of Nice for an enlarged 27-member EU Council of Ministers (CM) has reduced the value of A – in other words, lengthened the a priori odds against a bill being adopted – to such drastic extent, compared to its past and current values, that the engine of diplomacy will have great difficulty overcoming this hidden but very real obstacle. On these grounds we predict in that paper that if the quota of the QMV rule for the enlarged CM will not be considerably lowered, that body will tend to get bogged down in immobilism. This is a definite prediction of an observable phenomenon, made on the basis of the Penrose theory.[9]

However, the main application of the Penrose theory – certainly its intended aim – is not as a predictive or descriptive tool but as a prescriptive normative one.

Here it may be noted, by the way, that the main aim and intended application of game theory – a theory that Albert holds up for praise and emulation as truly scientific, in contrast to the 'VP approach' – is also normative. Although game-theoretic models are now used for explaining various empirical phenomena (such as evolutionary equilibria), the main purpose, for which game theory was invented, is as a normative guide for 'rational behaviour' in certain situations of conflict.[10]

Although the Penrose theory can also be used to prescribe rational behaviour in precisely the game-theoretic sense,[11] its main prescriptive application is in the analysis and design of decision rules, especially as part of the constitutional design of a decision-making body. In this connection it is vital to focus on 'formal [voting] power, as given by the constitutional rules of a collectivity' Coleman (1971: 297), rather than on actual voting power. Thus, when the designers of QMV for the CM of the EU assigned equal voting weights to member states with roughly equal population size – for example, France and Italy – they could not thereby equalize these members' *actual* voting power; nor was this their intention. What they did – and evidently intended to do – was to equalize that component of these members' voting power that derives solely from the decision rule itself: their a priori voting power.[12] Similarly, when they assigned Italy greater voting weight than Spain (on the grounds that the former is more populous) they could not thereby guarantee that Italy would have greater actual power than Spain; nor could this be their intention. What they were doing was to give Italy greater a priori voting power than Spain. But *how much* greater? Another important question is how to allocate weights under QMV so as to equalize the *indirect* a priori voting powers of all citizens of the EU (exercised through the political representatives, whom the citizens elect). To answer questions such as these, one needs a sophisticated mathematical theory of a priori voting power. This is where the Penrose theory comes in.

Here we must correct a grossly mistaken allegation made by Albert. For some reason which is not clear to us he attributes to the Penrose theory (generally or as expounded by us) the 'premise [that] … fairness requires the adoption of voting rules that equalize the probability of each voter to be decisive'. Against this he protests that in joint-stock companies, '[i]t is usually not considered unfair if each stockholder has one vote per share, although this means that, under simple random voting, the voting power of shareholders with different shares in the company is different.' Moreover, '[o]ne might argue … that … different stakes of voters must be taken into account. But this is problematic, because in a political context it could be argued that, for instance, land owners, tax payers, younger people, or parents have higher stakes than others' (p. 362).

All this is a red herring. The Penrose theory is not concerned with laying down principles of fairness (nor are we, *qua* its exponents). Such value judgements are a matter for legal and political ethics. For example, it is generally accepted in parliamentary democracies that the a priori voting power – the voting power derived from the constitution – of all citizens ought to be as nearly equal as possible. This is what is normally meant by the slogan 'One person, One vote'. (Note, by the way, that this does not imply equality of actual voting powers!) What the Penrose theory can prescribe is *how to implement* this equality. But it can also prescribe how to implement as precisely as possible a given *unequal* distribution of a priori voting powers, if that is desired.

3. The Principle of Insufficient Reason

The Penrose measure of voter *v*'s voting power (under a given decision rule) is the a priori probability of *v* being *decisive*: that is, of the event that the other voters are so divided, that *v* is in a position to determine the outcome of the division.[13]

Since we are concerned with a priori rather than actual voting power, we must go 'behind a veil of ignorance' and disregard any information about the personality of the voters (their specific interests and preferences, relations of affinity or disaffinity between them) and the nature of the bills to be voted upon.

The normal practice in the absence of all such behavioural and substantive information is to assign equal a priori probability to all possible 'atomic events' – in the present case, to all possible divisions of the set of voters into 'yes' and 'no' camps. (If there are n voters, the number of possible divisions is 2^n.) This is an application of the *Principle of Insufficient Reason* (PIR) of classical probability theory, which many authors (including Albert) attribute to Laplace (1749–1827), but in fact goes back to Jacob Bernoulli's (1654–1705) posthumous classic, *Ars conjectandi* (1713).[14]

Citing Howson and Urbach (1993, Ch. 4), Albert claims that PIR 'has been devastatingly criticized since the nineteenth century' and must be rejected. This would of course undermine the Penrose measure and with it the whole of the Penrose theory. But Albert's claim is, at the very least, quite misleading. PIR is indeed incoherent and may lead to contradiction when applied to *infinite* probability spaces.[15] But it is quite safe and unobjectionable when applied to a *finite* probability space consisting of finitely many clearly distinguished indivisible 'atomic' events. In this special case, PIR is not rejected by the best critical authorities on the subject, including Keynes (1921), and the one cited by Albert himself, Howson and Urbach (1993).[16] It is precisely such a safe application of PIR that is required for justifying the Penrose measure.

Apart from his misguided appeal to authority, Albert has one substantive argument against this use of PIR:

> 'The example of voting nicely illustrates the inherent difficulties of the principle. The basic VP approach assumes simple random voting, which leads, for large constituencies, to an approximately normal distribution of yes-votes. However, one could apply the principle of insufficient reason directly to the distribution of yes-votes, assuming that this distribution is uniform. Which application of the principle is correct? If nothing is known about voter behavior, this is a matter of taste.' (2003: 361).

This argument is fallacious, because it stands the whole issue upside down, on its head. In order to discredit an application of PIR to a *finite* probability space (of 2^n atomic events) Albert first takes us to the *infinite* limit, where he is faced with the normal limit distribution. Then he invites us to wonder: why use PIR to choose the normal distribution? Why not choose a uniform distribution?

But in the Penrose theory PIR is applied not to the infinite limit distribution, but to the finite case. Here the choice is quite clear. Suppose there are 20 voters. A uniform limit distribution would result if the event that exactly *k* of them vote 'yes' had a priori probability 1/21, independently of *k*. For example, the probability that all 20 vote 'yes' would be the same as the probability that only half vote 'yes' and the other half 'no'. But the former event can happen in only one way, while the latter can occur in 184,756 different ways, because the 10 'yes' voters can be chosen in 184,756 different ways from among the 20. Suppose we had no behavioural information whatsoever about the 20 voters and knew nothing about the issue on which they are going to divide. If we had to bet, would it be rational to assign the same a priori odds to the event that they all vote 'yes' as to the event that exactly half vote 'yes'? Of course it wouldn't. The most rational choice would be to assign equal probability to all 2^{20} (= 1,048,576) possible divisions (atomic events). This is what PIR would prescribe, and is also the position taken by modern information theory.

Contrary to Albert's claim, the modern successor, and generalization, of PIR (in its legitimate uses) is not *subjective* Bayesianism. It is in fact the *Principle of Maximal Information Entropy* of modern information theory.[17] There is nothing subjective at all about it. Subjective Bayesianism (which is rejected by many authors on the subject) depends on individual beliefs, not on objectively specified information. The Principle of Maximal Information Entropy is also widely used in the science of statistical mechanics.[18]

4. Concluding remark

We found two of Albert's criticisms of the Penrose theory to be worthy of detailed answers. First, we have refuted his charge that this theory is inapplicable to real-life situations: albeit an a priori theory, it does have both predictive power and prescriptive value. Second, we have argued that his wholesale rejection of the Principle of Insufficient Reason is unwarranted: its use for certain finite spaces, as needed to justify the Penrose measure, is perfectly legitimate. All the rest of Albert's critique consists of pedantic arguments about demarcation and the scientific status of non-empirical theories, which may be of interest to some philosophers but not to one seriously interested in studying and applying voting power.

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Notes

[1] The Penrose measure has been mistakenly attributed to Banzhaf (1965) and is often referred to as the `absolute Banzhaf index'. The most important derivative of the Penrose measure is the *relative* (or *normalized*) Banzhaf index. An important refinement is the pair of measures – of a voter's *power to prevent action* and *to initiate action*, respectively – defined by Coleman (1971).

[2] For a more detailed discussion, see our (1998: 19ff, 105ff, 112); Felsenthal and Machover (2004).

[3] For detailed explanation of the I-power/P-power distinction, see our (1998: 35ff, 171ff). For a summary see, for example, our (2001) or Machover (2000).

[4] He also mentions more pragmatic objections to the Penrose measure, made by other authors; but brushes them aside as playing into the hands of proponents of the Penrose theory by taking the latter too seriously.

[5] *The Cambridge Dictionary of Philosophy*, Cambridge University Press, 1995; p. 29.

[6] Non-empirical truths can lead to empirically testable predictions. Consider, as a very simple example, the theorem that 2 + 2 = 4. It asserts an a priori truth about abstract ideal entities (numbers). But it leads to the prediction that if there are two philosophers ensconced in an ivory tower, and two other philosophers join them, then – unless they bore one another to death or procreate – there will be four philosophers in the tower.

[7] Albert prefers the term 'positive' to the more usual 'empirical': it sounds so much more … well, positive!

[8] This is *the power of the collectivity to act*, defined by Coleman (1971).

[9] Other theories may also lead to a similar prediction; but the Penrose theory is arguably the simplest to do so. In any case, the issue here is not how to choose between the Penrose theory and rivals but whether it leads to any testable predictions.

[10] Thus Morgenstern (1949: 303 in the 1968 reprint): 'The initial problem of the theory of games was to give precision to the notion of "rational behavior".'

[11] See our (1998: 45) for a 'vote-buying game' in which an outsider stands to gain or lose a unit of transferable utility, depending on whether a given bill is adopted or blocked by a decision-making body. The outsider knows the decision rule, but has no behavioural information about the voters. The vote of one of the members is offered for sale. How much would it be rational for the outsider to pay for it? The answer is provided by the Penrose power of the seller.

[12] Equality of a priori voting power is thus analogous to equality of rights or equality before the law: it is a formal rather than empirical equality, which does not produce equality of actual attainment, nor is intended to do so.

[13] This probability is closely and simply connected with the a priori probability of v being *successful*: that is, of the outcome going the way v votes. For details see our (1998) or (2000).

[14] For a detailed critical discussion see Keynes (1921).

[15] According to PIR, each point of an infinite probability space should be assigned probability 0. If the space is denumerably infinite, this would imply that the whole space must also have probability 0, which is absurd. In the case of a continuous random variable X, PIR is usually interpreted as assigning to X a uniform distribution in some finite interval. But if we transform X

to $Y = f(X)$, where f is a one-to-one continuous function, then by the same token Y should presumably also have a uniform distribution. If f is not linear, this leads to contradiction.

[16] See also the more recent Howson (2000: 81–6).

[17] See Shannon (1948).

[18] See , for example, Jaynes (1968).