Book Review: Numbers Rule: The Vexing Mathematics of Democracy, from Plato to the Present

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Patrick Dunleavy reviews a fascinating, but flawed, history of democratic thinking from an American perspective. It throws often unexpected light on democratic innovations through the ages; and if the government's project to slice the UK electorate up into equal constituencies is your bag, you can get stuck in here.


What is it about Americans and democracy? OK we know that the US is today's Rome, spending half the world's military budget and still bestriding international policy like a colossus. And OK too, many of today's 300 million Americans have much the same appalling ethno-centrism as the people of any imperial power in full bloom (like Edwardian England). But does George G. Szpiro have to be quite so transparently Victorian and revisionist in producing a book whose hidden but deeply felt sub-title should be The United States as the Pinnacle of Democracy?

A book that starts out as an elegant, well-written and thought-provoking discussion of the problems of representing democratically a mass of voters' diverse and complex views becomes sidetracked into a tiny backwater about 'apportionment' problems, so boring that your toes curl. Szpiro begins with the ancient Greeks and a full-spectrum discussion of democracy, yet he ends up for much of the last third of the book obsessed with largely irrelevant maths cake-cutting problems.

Let's start with what works well first though, which is the kind of Plato to NATO thread that runs for much of the book, starting with the Greek sage himself and ending up with Kenneth Arrow at the height of the cold war. Szpiro has a nice approach to shedding new and interesting light on what might otherwise have seemed a familiar story about innovations in democratic thinking and solution-finding. His first key trick is to personalise issues and breakthroughs in his sub-title theme of 'The vexing mathematics of democracy', by fixing a clear spotlight on some key thinkers.

Giving a clear narrative about each of his key figures, Szpiro successfully shows how knotty many problems of representation actually are until they are solved. He gives readers valuable biographical notes and context about each person discussed, full of details and incidents that sweeten the pill of absorbing some useful maths thinking and technical knowledge. The personalisation is also summed up in an often useful chapter heading that synoptically characterizes the author involved.

Szpiro's second key contribution is to introduce some new and surprising characters into the over-time story. Plato himself (summarised as 'The Anti-Democrat') is an interesting choice to start from, since his Republic is chiefly known as the denial of any democracy in favour of (allegedly) enlightened technocratic rule. In fact Szpiro shows that Plato tackled interesting problems about how many guardians to have, and how they would make collective decisions, as well as requiring elections for choosing many less important office-holders in city life.

With a quick diversion discussing Pliny in Roman times (not sure why), Szpiro skips forward to Ramon Llul ('The Mystic'), a thirteenth century Spanish intellectual who had a major influence on Liebniz and Newton but whose name will be unfamiliar to most British readers. Llul was obsessed with binary numbers and choices and he devised two interesting methods for choosing office holders from a large field of candidates through what we would now call 'pair-wise comparison'.

Szpiro’s next port of call is the early fifteenth century cardinal, Niklaus Causanus, who discussed in forensic detail the key election of his times and of many previous centuries, namely how to choose the Pope in the College of Cardinals – which uses exhaustive balloting and the ultimate voting threshold, i.e., complete
unanimity. Not surprisingly one of the key drivers for getting to a decision was in fact exhausting the voters, plus organising side-payments of offices and benefits to persuade the recalcitrant.

Szpiro does not bring out fully the broader message behind the Llul and Causanus stories, namely that it was largely in the unlikely setting of the Catholic church that the knowledge and practice of elections as means for choosing leaders in abbeys and convents survived for many hundreds of years. This story was recently reconstructed by two scholars, Ian McLean in Britain (whom Szpiro has read) and Josep Colomer in Spain (whom he hasn’t).

From just before the Reformation the book skips across the era of European absolute monarchies to the late eighteenth century to cover the more familiar story of Charles Borda and Nicolas Condorcet and the founding fathers of the American republic. Szpiro adds in a discussion of the eminent mathematician, Pierre de Simon Laplace, who made big contributions in probability but whose input discussed here seems fairly small. From there by way of Charles Dodgson (aka Lewis Carroll)’s discussion of multiple preferences the book links forward to Kenneth Arrow whose ‘impossibility theorem’ defined the height of cold war thinking about democracy.

Arrow took five allegedly neutral or non-controversial sparse conditions and proved mathematically that in a society with multiple preferences there was no institutional choice mechanism that would allow us to arrive at an agreed outcome consistent with all the five conditions. Szpiro does not tell this story very well, but rather slides into the conventional public choice view (especially in America) of it, as a story of the necessary imperfections of democracy.

This was a message that the US right has always been keen to stress and that went down a storm in the Eisenhower years. Every democracy is imperfect, so we don’t need to improve ours – it’s the free market and the American way that’s really valuable. Every democracy is imperfect – and so we are justified in imposing flawed or questionable ‘democracies’ on subject peoples in our neo-colonies.

But what Arrow really says, if you read it carefully, is that every social choice mechanism (including allocation by markets) is imperfect in realising social welfare or consensus, not just every voting mechanism. And what are the five Arrow conditions that are allegedly so neutral? A key one is Pareto optimality which says (in effect) that we can only improve the lot of the poorest person in society if we can find a way of doing so that leaves Bill Gates or Rupert Murdoch with every one of their ill-gained, monopoly profit dollars. Can you spot the bias in this condition? Should we be terribly concerned if we have to break it and make some well-off folks a bit worse off in the cause of remedying egregious harms to others? But in Arrow’s formulation this is as serious a breach as dictatorship.

Apart from this goofy maths view of Arrow, Szpiro’s account goes off course badly with the US founding fathers who obsessed quite a bit about how to get exactly fair-sized constituencies for representatives, especially for allocating seats in the House of Representatives fairly across the US states. (The Senate was easy – everyone gets two seats and proportionality can go hang!) The story here is that you cannot do this apportionment task completely fairly, and that some high-end minor paradoxes and wrinkles remain in any solution that you choose. Worth perhaps one chapter in this kind of book, Szpiro in fact gives it three long ones, also introducing into his story some very low key maths/stats people – cake-splitters who have at best footnote status in any sensible history of voting systems.

Meanwhile the obvious continuation of the story beyond Borda and Condorcet into west European proportional representation systems and wider multi-preference systems for choosing representatives and chief executives gets almost no coverage. You can see that some reviewer from an east coast US university read the book for Princeton UP and said, “Great, but haven’t you missed out the Europeans?” So Szpiro adds a rushed little chapter at the end (‘The Post-Moderns’) that is just terribly inadequate.

Thomas Hare gets a brief mention (but not John Stuart Mill or votes for women) and the single transferable vote (STV) is terribly explained in a few lines. In general even here Szpiro’s focus is on one-vote, list PR systems. It is almost as if (despite reviewing Borda, Condorcet, Dodgson and Arrow) he has never heard of multi-preference voting systems, such as the Alternative Vote (AV) or Supplementary Vote (SV).

And this, of course, is because multi-preference voting is not now used in public elections in the USA, which all the coverage of ‘apportionment’ problems seems designed to tell us is the most perfect democracy feasible. Yet American professional associations do use sophisticated voting systems like STV. And many Americans have agonised for years now about their increasingly three-way Presidential races that may easily end up choosing the ‘wrong’ candidate in terms of multiple preferences.

At a city level too, some US mayors are elected on very low vote shares in multi-candidate fields. So the main pro-reform movement in the USA focuses on what is there called ‘instant run-off’ elections (that is, AV or SV)
for the presidency, for state governors and city mayors, and for members of Congress and state legislatures. Here are the issues that Borda and Condorcet (and before them many Catholic monks and nuns) discussed so brilliantly, and their lively relevance for contemporary American politics is recently attested by discussions of Szpiro’s book in the New Yorker.

What should be the central theme of Szpiro’s story is alive and is clearly just as relevant today as ever, not least because the imperial power of our times still struggles along using a mediaeval vote-tallying method for all its elections. So it is a great shame that after a burst of unjustified Arrovian pessimism from the cold war, his initially brilliant story of democratic advances peters out in the mathematics of perfect cake-splitting.

Read the book with this caveat in mind though, and for two thirds of its length readers who are interested in democracy, without being political scientists or mathematicians themselves, will get an acute introduction to some fundamentals about how voting problems occur, and how some of them might be solved. It won’t tell you anything at all about the Alternative Vote referendum in May 2011 under that name, but the Borda and Condorcet chapters are highly relevant. Szpiro also will make you think apportionment problems are much more important and chronic that they really are – for an antidote just watch the British state define 600 exactly equal constituencies in the UK in record time for 2014 or 2015.

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