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Consumption and Real Exchange Rates with Incomplete Markets and Non-Traded Goods

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Abstract
This paper addresses the consumption-real exchange rate anomaly. International real business cycle models based on complete financial markets predict a unitary correlation between the real exchange rate and the ratio of home to foreign consumption when subjected to supply side shocks. In the data, this correlation is usually small and often negative. This paper shows that this anomaly can be successfully addressed by models that have an incomplete financial market structure and a non-traded as well as traded goods production sector.

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Keywords: Consumption-real exchange rate anomaly, incomplete financial markets, nontraded goods

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1 Introduction

One of the well known puzzles in international finance is the so called consumption-real exchange rate anomaly (see Backus and Smith, 1993 for an early paper and Chari, Kehoe and McGrattan, 2002 for a recent contribution). Most international business cycle models predict that, under the assumption of perfect financial markets along with supply disturbances, consumption across countries should be higher in the country where its price, converted into a common currency is lower. This feature of the model is in sharp contrast with the empirical evidence which suggests that the consumption differential across countries does not move in any systematic pattern with its relative price (i.e. the real exchange rate). Chari *et al* (2002) refer to this discrepancy as the consumption-real exchange rate anomaly.

The removal of the assumption of perfect financial markets is not sufficient in replicating the observed evidence: indeed, in their study, Chari *et al* (2002) have shown that the same anomaly in the behavior of consumption and the real exchange rate does continue to hold. In this work we explore the extent to which the introduction of non-traded goods along with a limited international financial market structure might account for the aforementioned anomaly. Our results suggest that the combination of these two factors is a promising avenue for understanding the behavior of consumption across countries and the real exchange rate.

There are two key features that are important in accounting for our results. By assuming that international asset trade is limited to a risk-less bond we break the link between the real exchange rate and relative consumption that would arise under complete financial markets. While by introducing non-traded goods we allow for the possibility that, depending on the origin of the shock (i.e. traded versus non traded), the real exchange rate and relative consumption across countries can move in opposite directions.

In particular, following a positive shock to the traded goods sector in the home economy, home consumption increases with respect to consumption abroad. On the other hand, the real exchange rate appreciates if the effect coming from the relative price of non-traded to traded goods (the so-called Balassa-Samuelson effect) outweighs the terms of trade effect that would imply a depreciation of the real exchange rate. This effect will be stronger the more dominant the shocks to the traded goods sector relative to non-traded goods sector.

More generally, the structure of the disturbance and the specification of preferences determine the overall cross-correlation between real exchange rate and relative consumption.

Finally we check the performance of our baseline model in replicating standard international business cycle statistics. Our model overcomes the problem of an unrealistically high cross-correlation between relative consumption and the real exchange rate. Where our model departs from the data, is the volatility of other key variables like the real exchange rate and the terms of trade.

Our model follows closely the ones proposed by Backus and Smith (1993), Chari *et al* (2002) and Stockman and Tesar (1995): we construct a simple two-country stochastic dynamic open economy model in which we allow households to trade internationally in only one risk-less nominal bond, prices are flexible and households consume a final non-traded good produced with domestic as well as foreign-produced intermediate goods and a non-traded intermediate component. We allow for capital accumulation at the intermediate goods level and deviations from purchasing power parity are obtained by allowing for home-bias toward home-produced intermediate goods at the production level and because of the existence of non-traded intermediate inputs.

The remainder of the paper is structured as follows: in section two, we discuss the nature of the consumption-real exchange rate anomaly and survey related contributions in the literature. Section three presents the basic structure of the model. The model is calibrated in section four, and section five outlines the basic mechanism behind our results. The results of the calibrated model are discussed in sections six and seven, respectively. Section eight concludes.

2 Data and Related Literature

In their influential paper, Backus and Smith (1993) document the failure of international macroeconomic models based on the complete market assumption in replicating the features of international macroeconomic data: indeed, they show the lack of correlation between growth rates of relative consumption and the growth rate of the real exchange rate. Chari, Kehoe and McGrattan (2002) report the cross-correlations between consumption ratio and the real exchange rate for a subset of OECD economies from 1973 to 1994 at a quarterly frequency and find a median value of -0.07. In their work, they label the discrepancy between their model's prediction and the empirical evidence as the consumption-real exchange rate anomaly. Similarly, Corsetti, Dedola and Leduc (2004) show that the cross-correlations obtained from Hodrick-Prescott filtered as well as first-difference filtered data for a selection of OECD countries appear to be small and often negative. Their median estimate is between -0.30 and -0.2. We also report our estimates for the cross-correlation between logged and Hodrick-Prescott filtered relative consumption and the real exchange rate, in levels as well as in first differences, where the reference country is the US. The data for consumption and real exchange rates are annual series from 1970 to 2000. (See Table 1.)

These results can be used to question the assumption of financial market completeness, for that assumption would imply a cross-correlation between the real exchange rate and relative consumption of close to unity.[1]

Other empirical studies have similarly questioned the assumption of financial market completeness: in particular Ravn (2001) shows that there is no role for the real exchange rate in accounting for differences in marginal utilities of consumption in different countries. In his study, he rules out non-separabilities in the utility function as possible candidates in testing for risk-sharing. In another related study, Kollmann (1995) also rejects the complete market assumption.

Starting from these premises, recent theoretical papers assume an incomplete financial market structure as a necessary condition for explaining the observed evidence. In Chari *et al* (2002) domestic and foreign agents are only allowed to trade in a non state-contingent nominal bond. Their rich model with sticky prices is unable to break the link between real exchange rate and marginal utilities of consumption. Indeed, the cross correlation between relative consumption and the real exchange rate for the incomplete market case is still perfect as in the complete market case. They conclude by saying that "*the most widely used forms of asset market incompleteness does not eliminate - or even shrink- the anomaly*".

On the other hand, the papers by Corsetti, Dedola and Leduc (2004) and Selaive and Tuesta (2003)

[1] One would expect a cross-correlation equal to one only if utility is additively separable in consumption and leisure.

introduce other frictions along with asset market incompleteness and are able to get closer in replicating the empirical facts. Corsetti *et al* (2004) highlight the role of distributive trade along with market incompleteness. Assuming that bringing traded goods to the market requires non-traded distribution services can generate the low import elasticity crucial for explaining the observed patterns in the international transmission of productivity shocks and the high volatility of the real exchange rate. Their VAR analysis suggests that a positive productivity shock will improve the terms of trade, appreciate the real exchange rate and increase domestic consumption relative to the rest of the world: this pattern of transmission is compatible in their model with a relatively low price elasticity of imports. Selaive and Tuesta (2003) consider a richer structure in which prices are sticky and monetary policy is modelled through interest rate feedback rules. They emphasize the importance of financial frictions and the role of net foreign asset position in breaking the link between real exchange rate dynamics and relative consumption levels. Another related contribution is a recent work by Ghironi and Melitz (2004). In their work a non-traded sector arises endogenously because less productive firms decide not to export their products. They find that a Balassa-Samuelson effect and a real exchange rate appreciation is generated by aggregate productivity shocks rather than sector specific ones to the traded sector.

Our contribution differs from the aforementioned works in some important aspects: we follow Backus and Smith (1993) in constructing a model with a non-traded goods sector but we allow for international market incompleteness; differently from Corsetti *et al* (2004) there are no distribution costs and the law of one price always holds. In contrast to Selaive and Tuesta (2003) prices are perfectly flexible. As in Chari *et al* (2002), we assume that agents consume a final consumption good, which is not traded internationally. Unlike Chari *et al* we assume that this final good contains three types of intermediate inputs: home and foreign-produced traded intermediate inputs as well as non-traded domestically produced intermediate input.

We find that our model, calibrated in a canonical fashion, generates cross-correlations between the real exchange rate and relative consumption which are not at odds with the data. We attribute this to the combination of the presence of a non-traded production sector together with a simple form of incomplete financial markets.

3 A two-sector two-country model

The structure of the model follows closely Chari *et al.* (2002) and Stockman and Tesar (1995). There are two key modifications with respect to their baseline cases. Firstly we consider an incomplete market structure at the international level. Secondly, unlike Chari *et al.* (2002), but similar to Stockman and Tesar, we introduce non-tradeable intermediate inputs in the production process. Moreover, we focus on a perfectly competitive setting while Chari *et al* analyze an imperfectly competitive framework with staggered price setting behavior.

3.1 Consumer Behavior

We propose a two-country model with infinitely lived consumers. The world economy is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ belongs to the country

H (Home), while the segment $[n, 1]$ belongs to F (Foreign). Preferences for a generic Home-consumer are described by the following utility function:

$$U_t^j = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s^j, (1 - l_s^j)) \tag{1}$$

where E_t denotes the expectation conditional on the information set at date t, while β is the intertemporal discount factor, with $0 < \beta < 1$. The Home consumer obtains utility from consumption, C^j, and receive dis-utility from supplying labor, l^j.

The asset market structure in the model is relatively standard in the literature. We assume that Home individuals are assumed to be able to trade two nominal risk-less bonds denominated in the domestic and foreign currency. These bonds are issued by residents in both countries in order to finance their consumption expenditure. On the other hand, foreign residents can allocate their wealth only in bonds denominated in the foreign currency. [2] Home households face a cost (i.e. transaction cost) when they take a position in the foreign bond market. This cost depends on the net foreign asset position of the home economy as in Benigno (2001).[3] Domestic firms are assumed to be wholly owned by domestic residents, and profits are distributed equally across households. Consumer j faces the following budget constraint in each period t:

$$P_t C_t^j + \frac{B_{H,t}^j}{(1 + i_t)} + \frac{S_t B_{F,t}^j}{(1 + i_t^*)\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)} = B_{H,t-1}^j + S_t B_{F,t-1}^j + P_t w_t l_t^j + \Pi_t^j \tag{2}$$

where $B_{H,t}^j$ and $B_{F,t}^j$ are the individual's holdings of domestic and foreign nominal risk-less bonds denominated in the local currency. i_t is the Home country nominal interest rate and i_t^* is the Foreign country nominal interest rate. S_t is the nominal exchange rate expressed as units of domestic currency needed to buy one unit of foreign currency, P_t is the consumer price level and w_t is the real wage. Π_t^j are dividends from holding a share in the equity of domestic firms obtained by agent j. All domestic firms are wholly owned by domestic agents and equity holding within these firms is evenly divided between domestic agents.

The cost function $\Theta(.)$ drives a wedge between the return on foreign-currency denominated bonds received by domestic and by foreign residents. We follow Benigno, P. (2001) in rationalizing this cost by assuming the existence of foreign-owned intermediaries in the foreign asset market who apply a spread over the risk-free rate of interest when borrowing or lending to home agents in foreign currency. This spread depends on the net foreign asset position of the home economy. We assume that profits from this activity in the foreign asset market are distributed equally among foreign residents (see P. Benigno (2001)).[4]

[2] We want to highlight here the fact that this asymmetry in the financial market structure is made for simplicity. The results would not change if we allow home bonds to be traded internationally. We would need to consider a further arbitrage condition.

[3] Further ways of rendering the wealth distribution stationary by eliminating the unit root in wealth dynamics are discussed in Schmitt-Grohé and Uribe (2003).

[4] Here we follow Benigno (2001) in assuming that the cost function $\Theta(.)$ assumes the value of 1 only when the net foreign asset position is at its steady state level, ie $B_{F,t} = \overline{B}$, and is a differentiable decreasing function in the neighbourhood of $\overline{B}$. This cost function is convenient because it allows us to log-linearise our economy properly since in steady state the desired amount of net foreign assets is always a constant $\overline{B}$. The expression for profits from financial intermediation is given by $K = \frac{B_{F,t}}{P_t^*(1+i_t^*)}\left[\frac{RS_t}{\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)} - 1\right]$.

As in P. Benigno (2001), we assume that all individual belonging to the same country have the same level of initial wealth. This assumption, along with the fact that all individuals face the same labour demand and own an equal share of all firms, implies that within the same country all individuals face the same budget constraint. Thus they will choose identical paths for consumption. As a result, we can drop the j superscript and focus on a representative individual for each country.

The maximisation problem of the Home individual consists of maximising (1) subject to (2) in determining the optimal profile of consumption and bond holdings and the labour supply schedule. Households' equilibrium conditions (Home and Foreign) are described by the following equations:

$$U_C\left(C_t,(1-l_t)\right)=(1+i_t)\beta E_t\left[U_C\left(C_{t+1},(1-l_{t+1})\right)\frac{P_t}{P_{t+1}}\right] \tag{3}$$

$$U_C\left(C_t^*,(1-l_t^*)\right)=(1+i_t^*)\beta E_t\left[U_C\left(C_{t+1}^*,(1-l_{t+1}^*)\right)\frac{P_t^*}{P_{t+1}^*}\right] \tag{4}$$

$$U_C\left(C_t,(1-l_t)\right)=(1+i_t^*)\Theta\left(\frac{S_tB_{F,t}}{P_t}\right)\beta E_t\left[U_C\left(C_{t+1},(1-l_{t+1})\right)\frac{S_{t+1}P_t}{S_tP_{t+1}}\right] \tag{5}$$

$$\frac{U_l(C_t,(1-l_t))}{U_C\left(C_t,(1-l_t)\right)}=w_t \qquad \frac{U_l(C_t^*,(1-l_t^*))}{U_C\left(C_t^*,(1-l_t^*)\right)}=w_t^* \tag{6}$$

3.2 Producer behavior

As in Chari *et al* (2002), in our economy final goods are obtained by combining intermediate goods produced in the Home and in the Foreign economy: final goods are used only for consumption. Differently from Chari *et al* (2002) we now also consider the possibility that non-traded intermediate inputs enter in the production process for the final goods. All the trade between the two countries is in intermediate goods.

We let Y be the output of final goods produced in the home country. Final goods producers combine home and foreign-produced intermediate goods to produced Y in the following manner:

$$Y\equiv C=\left[\omega^{\frac{1}{\kappa}}c_T^{\frac{\kappa-1}{\kappa}}+(1-\omega)^{\frac{1}{\kappa}}c_N^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}} \tag{7}$$

where c_T and c_N are the intermediate traded and non-traded inputs and κ is the elasticity of intratemporal substitution between traded and non-traded intermediate goods. The traded component is in turn produced using home and foreign-produced traded goods in the following manner:

$$c_T=\left[v^{\frac{1}{\theta}}c_H^{\frac{\theta-1}{\theta}}+(1-v)^{\frac{1}{\theta}}c_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{8}$$

where we denote with y_H and y_F are the intermediate goods produced in the Home and Foreign countries respectively. θ is the elasticity of intratemporal substitution between home and foreign-produced intermediate goods.

Final goods producers and producer of the composite traded goods are competitive and maximise their profits:

$$\max_{c_N, c_T} PC - P_T c_T - P_N c_N \tag{9}$$

$$\max_{c_H, c_F} P_T c_T - P_H c_H - P_F c_F \tag{10}$$

subject to (7) and (8) respectively. This maximisation yields the following input demand functions for the home economy (similar conditions hold for Foreign producers)

$$c_N = (1-\omega)\left(\frac{P_N}{P}\right)^{-\kappa} C, \tag{11}$$
$$c_H = \omega v \left(\frac{P_H}{P_T}\right)^{-\theta}\left(\frac{P_T}{P}\right)^{-\kappa} C \quad c_F = \omega(1-v)\left(\frac{P_F}{P_T}\right)^{-\theta}\left(\frac{P_T}{P}\right)^{-\kappa} C$$

Corresponding to the previous demand function we have the following prices indexes:

$$P_T^{1-\theta} = [vP_H^{1-\theta} + (1-v)P_F^{1-\theta}] \tag{12}$$

$$P^{1-\kappa} = [\omega P_T^{1-\kappa} + (1-\omega)P_N^{1-\kappa}] \tag{13}$$

3.2.1 Intermediate goods sectors

Firms in the traded intermediate goods sector produce goods using capital and labour services and domestic firms are owned by domestic households. The typical firm maximises the expected discounted value of profit:

$$\max_{k_{H,t+1}, l_{H,t}, x_{H,t}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_C(C_t, (1-l_t))}{U_C(C_0, (1-l_0))} \frac{P_0}{P_t} \left[P_{H_t} y_{H_t} - P_t w_t l_{H,t} - P_{H,t} x_{H,t}\right] \tag{14}$$

where $l_{H,t}$ is the total labour supply employed in the domestic traded intermediate sector, $x_{H,t}$ denotes investment in the traded domestic sector. Our maximization problem is constrained by the production function and the law of motion of capital:

$$y_{H_t} = F(k_{H,t-1}, l_{H,t}) = A_t {l_{H,t}}^{\alpha} k_{H,t-1}^{1-\alpha} \tag{15}$$
$$k_{H,t} = (1-\delta)k_{H,t-1} + \phi\left(\frac{x_{H,t}}{k_{H,t-1}}\right) k_{Ht-1}$$

where $\phi\,(.)$ is the cost for installing investment goods.[5] The first-order conditions at a generic time t are given by the following equations:

$$P_t w_t = \alpha P_{H,t} A_t \left(\frac{k_{H,t-1}}{l_{H,t}}\right)^{1-\alpha} \tag{16}$$

[5]The function $\phi(.)$ has the following properties: In the steady state, $\phi(.) = x/k$, $\phi'(.) = 1, \phi''(.) = b < 0$.

$$\beta \mathrm{E}_t U_C(C_{t+1},(1-l_{t+1}))\left\{\frac{P_{H,t+1}}{P_{t+1}}\frac{\partial F(k_t,l_{t+1})}{\partial k_t}\phi'\left(\frac{x_{H,t}}{k_{H,t-1}}\right)+\frac{P_{H,t+1}}{P_{t+1}}\frac{\phi'\left(\frac{x_{H,t}}{k_{H,t-1}}\right)}{\phi'\left(\frac{x_{H,t+1}}{k_{H,t}}\right)}(\Omega_{H,t+1})\right\}$$
$$= U_C(C_t,(1-l_t))\frac{P_{H,t}}{P_t}$$

where $\frac{\partial F(k_t,l_{t+1})}{\partial k_t}$ is the marginal product of capital and $\Omega_{H,t+1}=(1-\delta)+\phi\left(\frac{x_{H,t+1}}{k_{H,t}}\right)-\phi'\left(\frac{x_{H,t+1}}{k_{H,t}}\right)\left(\frac{x_{H,t+1}}{k_{H,t}}\right)$

A similar problem holds for the non-traded goods sector[6]:

$$\max_{k_{N,t+1},l_{N,t},x_{N,t}} \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\frac{U_C(C_t,(1-l_t))}{U_C(C_0,(1-l_0))}\frac{P_0}{P_t}\left[P_{N_t}y_{N_t}-P_t w_t l_{N,t}-P_{H,t}x_{N,t}\right] \tag{17}$$

$$y_{N_t}=F(k_{t-1},l_{N,t})=A_{N,t}{l_{N,t}}^{\alpha}k_{N,t-1}^{1-\alpha} \tag{18}$$

$$k_{N,t}=(1-\delta)k_{N,t-1}+\phi\left(\frac{x_{N,t}}{k_{N,t-1}}\right)k_{N,t-1} \tag{19}$$

And the corresponding first order conditions are given by:

$$P_t w_t=\alpha P_{N,t}{A_{N,t}}^{\alpha}\left(\frac{k_{N,t-1}}{l_{N,t}}\right)^{1-\alpha} \tag{20}$$

$$\beta \mathrm{E}_t U_C(C_{t+1},(1-l_{t+1}))\left\{\frac{P_{N,t+1}}{P_{t+1}}\frac{\partial F(k_t,l_{t+1})}{\partial k_t}\phi'\left(\frac{x_{N,t}}{k_{N,t-1}}\right)+\frac{P_{H,t+1}}{P_{t+1}}\frac{\phi'\left(\frac{x_{N,t}}{k_{N,t-1}}\right)}{\phi'\left(\frac{x_{N,t+1}}{k_{N,t}}\right)}(\Omega_{N,t+1})\right\}$$
$$= U_C(C_t,(1-l_t))\frac{P_{H,t}}{P_t}$$

where $\Omega_{N,t+1}=(1-\delta)+\phi\left(\frac{x_{N,t+1}}{k_{N,t}}\right)-\phi'\left(\frac{x_{N,t+1}}{k_{N,t}}\right)\left(\frac{x_{N,t+1}}{k_{N,t}}\right)$

3.3 Current account

One important implication of the incomplete market framework is that it allows us to characterise the dynamic of the current account. By aggregating the individual budget constraints in the home country, we obtain:

$$P_tC_t+\frac{S_tB_t^F}{(1+i_t^*)}\frac{1}{\Theta(\frac{S_tB_t^F}{P_t})}=S_tB_{t-1}^F+P_tw_l l_t+\Pi_t \tag{21}$$

[6] Note that we made the assumption that the investment goods is obtained out of the intermediate tradeable good.

where we have applied the assumption that home bonds are in zero net supply and only held by Home residents. The aggregate profits in the home economy are given by:

$$\Pi_t = P_{H_t} y_{H_t} - P_t w_t l_{H_t} - P_{H,t} x_{H_t} + \\ P_{N_t} y_{N_t} - P_t w_t l_{N_t} - P_{H,t} x_{N_t} \tag{22}$$

From which substituting the economy-wide constraint on labour and investment ($l = l_H + l_N$ and $x = x_H + x_N$) we obtain:

$$C_t + \frac{S_t B_t^F}{P_t(1+i_t^*)} \frac{1}{\Theta(\frac{S_t B_t^F}{P_t})} = \frac{S_t B_{t-1}^F}{P_t} + \frac{P_{H_t}}{P_t} y_{H_t} + \frac{P_{N_t}}{P_t} y_{N_t} - \frac{P_{H_t}}{P_t} x_t, \tag{23}$$

and after substituting in the goods' market equilibrium conditions ($y_N = c_N$ and $y_H = c_H + c_H^* + x_H + x_N$):

$$\frac{S_t B_t^F}{P_t(1+i_t^*)} \frac{1}{\Theta(\frac{S_t B_t^F}{P_t})} = \frac{S_t B_{t-1}^F}{P_t} + \frac{P_{H_t}}{P_t} c_{H,t}^* + \frac{P_{F_t}}{P_t} c_{F,t}. \tag{24}$$

A similar equation holds for the Foreign economy.

3.4 Monetary policy

Since we are characterizing a nominal model we need to specify a monetary policy rule. In what follows we assume that the monetary authorities in both countries follow a strategy of setting consumer price inflation equal to zero.

3.5 Solution technique

Before solving our model, we log-linearize around the steady state to obtain a set of equations describing the equilibrium fluctuations of the model. The log-linearization yields a system of linear difference equations which we list in the appendix and can be expressed as a singular dynamic system of the following form:

$$\mathbf{A}\mathrm{E}_t \mathbf{y}(t+1 \mid t) = \mathbf{B}\mathbf{y}(t) + \mathbf{C}\mathbf{x}(t)$$

where $\mathbf{y}(t)$ is ordered so that the non-predetermined variables appear first and the predetermined variables appear last, and $\mathbf{x}(t)$ is a martingale difference sequence. There are four shocks in $\mathbf{C}$: shocks to the Home traded and non-traded intermediate goods sectors' productivity and shocks to the Foreign traded and non-traded intermediate goods sectors' productivity. The variance-covariance as well as the autocorrelation matrices associated with these shocks are described in table 2. Given the parameters of the model, which we describe in the next section, we solve this system using the King and Watson (1998) solution algorithm.

4 Calibration

In this section, we outline our baseline calibration.

We assume that the Home and Foreign economy are of equal size and are calibrated in a symmetric fashion. Following Stockman and Tesar we choose the following functional form for the utility function:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{1}{1-\rho} (C_s^j)^{1-\rho} (1 - l_s^j)^{\eta} \right] \tag{25}$$

This functional form implies that consumption and leisure are non-separable. In choosing the parameters of utility function, we set β to match a 4% annual discount rate. As in Stockman and Tesar we set the coefficient of constant relative risk aversion, or the inverse of the intertemporal elasticity of substitution, ρ, is set to 2. The inverse of the intertemporal elasticity of substitution in leisure, η is set to -3.17. We assume along with most real business cycle studies that agents devote around 80% of their time endowment to leisure and the remaining 20% to work.

We calibrate the parameters pertaining to the final goods producing sector in the following way. The share of tradable intermediate goods in the final consumption good, ω is 0.55, while the share of home-produced intermediate inputs in the tradable intermediate input, v is 0.72. The calibration of this parameter is in line with other recent studies, such as Corsetti *et al* (2004). We assume an elasticity of substitution between home and foreign-produced traded goods, θ, of 2 and an elasticity of substitution between traded and non-traded goods, κ of 0.44 in the production of the final consumption good, as suggested by Stockman and Tesar.

We assume that the share of labour input in intermediate good production, α in our Cobb-Douglas production function, is the same across sectors. We choose a standard value from the real business cycle literature of $\alpha = 0.67$. We assume that the capital stocks depreciate at a rate of 10% per annum. We choose the adjustment cost parameter in investment, d, so as to ensure a volatility of investment relative that of GDP in excess of 3.

The two remaining parameters relate to our specification of incomplete markets. Following Benigno, P. (2001), we introduce a bond holding cost to eliminate the otherwise arising unit root in foreign bond holdings. We argue that this cost can be very small, and thus choose a 10 basis point spread (per annum) of the domestic interest rate on foreign assets over the foreign rate, such that $\varepsilon \equiv -\Theta'(\bar{b})\bar{Y} = 0.001$. Our parameter choice is conservative, Rabanal and Tuesta (2005) provide a Bayesian estimate of ε of 0.007 for quarterly data. The steady-state ratio of net foreign assets to GDP, $\bar{a} = \frac{\bar{b}}{\bar{Y}}$ is assumed to be equal to zero.[7]

We estimate Solow residuals for the home and foreign traded and non-traded goods sectors. We let the US be the 'home' country and assume that Japan plus the EU15 represent the 'foreign' economy. To estimate these shocks we use annual sectoral output and labour input from the Groningen Growth and Development Centre, 60-Industry Database which spans the years 1979 - 2002. We follow Backus, Kehoe and Kydland (1992) and impose cross-country symmetry on our estimated shock process. The shocks to sectoral technology are assumed to follow a first order autoregressive process:

$$\mathbf{A}_{t+1} = \Omega \mathbf{A}_t + \mu_t$$

[7] We set $\bar{a} = 0$ to maintain the symmetry of the model. Our sensitivity analysis, not reported, suggests that for reasonable values of $\bar{a}$ the acutal level of $\bar{a}$ does not affect the H-P flitered moments of the model in a significant way.

where $\mathbf{A}$ is a vector of shocks: $[A_H,\ A_F,\ A_N,\ A_{N^*}]$ and Ω is a 4×4 matrix describing the autoregressive components of the shocks. We set to zero those autoregressive components that proved not to be statistically significantly different from zero. The innovations to $\mathbf{A}$ are $[\mu_H,\ \mu_F,\ \mu_N,\ \mu_{N^*}]$ and the variance-covariance matrix is $V[\mu]$. The data appendix discusses our data in more detail.

5 Relative consumption and the real exchange rate: the role of incomplete markets and sectorial shocks

Before analysing the characteristics of our calibrated model in terms of second moments, this section looks at impulse responses for the real exchange rate and relative consumption following productivity shocks. Our impulse responses are derived under the assumption that all off-diagonal elements of the autocorrelation matrix Ω are set to zero and that the variance-covariance matrix $V[\mu]$ of the shocks takes the form of an identity matrix. This way shocks to the traded and non-traded sectors have the same size and we abstract from spill-overs effects from productivity shocks. For illustrative purposes, we also make the additional assumption that consumption and leisure are separable.

In this section we want to highlight the roles of market incompleteness, the importance of the non-traded goods sector as well as the sectorial origin of the disturbance. Our two-country, two-sectors model with no departures from the law of one price, implies that the real exchange rate can be expressed as a combination of the terms of trade and relative prices of traded versus non-traded goods in the home and the foreign economy. In log-linear terms we have:

$$\widehat{RS}_t = (v - v^*)\hat{T}_t + (\omega - 1)\hat{R}_t + (1 - \omega^*)\hat{R}_t^* \tag{26}$$

As in Benigno and Thoenissen (2003), we can decompose movements in the real exchange rate into two channels: the home-bias channel, $(v - v^*)\hat{T}_t$ where $\hat{T}$ represents the terms of trade (i.e. the relative price of foreign to home-produced traded goods) in deviation from its steady-state value and $(v - v^*)$ is the difference between the home and foreign share of home-produced intermediate input in the traded component of final consumption good; and what we call the internal real exchange rate $\left[(\omega - 1)\hat{R}_t + (1 - \omega^*)\hat{R}_t^*\right]$ where $\hat{R}$ and $\hat{R}^*$ are deviations from steady state of the relative price of non-traded to traded goods (P_N/P_T) at home and abroad, respectively. This expression shows that by allowing for home bias, $v > v^*$ the terms of trade affects directly into the dynamics of the real exchange rate via the home bias channel.

We start by considering a framework in which markets are complete as in Stockman and Tesar (1995) (see Figures 1 and 2). In the top panel we show the percentage deviation of the real exchange rate, the internal real exchange rate as well as the home-bias channel following a 1% positive productivity shock to the traded goods sector in the presence of Arrow-Debreu securities. The bottom panel shows the response of the relative consumption measured as a difference between the log-deviations of Home and Foreign consumption from their steady-state levels. Since markets are complete, the real exchange rate and relative consumption are linked by the following risk-sharing condition (here in log-linear terms):

$$\widehat{RS}_t = \hat{U}_C(C_t^*) - \hat{U}_C(C_t). \tag{27}$$

Risk-sharing equates the ratio of marginal utilities of consumption with the real exchange rate. For separable preferences, this risk-sharing relationship implies a unitary cross-correlation between the real exchange rate and relative consumption no matter what is the source of the disturbance. This theoretical result is illustrated in our figures 1 and 2. Figure 1, which corresponds to our baseline calibration except for the shock matrices, shows the response of our model to a 1% deviation to traded-sector productivity.

The real exchange appreciates in response to the increase in traded sector productivity, along the lines of the well known Balassa-Samuelson effect. If the real exchange rate appreciates, the complete risk-sharing condition (27) implies that home relative to foreign consumption must fall. Consumption in both countries rises, but Foreign consumption rises by more than Home consumption. One reason is the presence of state-contingent bonds, which transfer resources from Home to Foreign in the case of a Home productivity increase. Moreover, because the terms of trade depreciate in response to a positive domestic supply shock, purchasing power is further transferred from Home to Foreign agents. The net effect on relative consumption is that foreign consumption rises by more than home consumption, causing relative consumption to fall. Since relative consumption and the real exchange rate are linked through the above risk sharing condition, the real exchange rate appreciates as relative consumption falls.

In figure 2 we do the same experiment for a home productivity shock to the non-traded goods sector: as in the previous example, home consumption increases (because of the increase in the non-traded goods component) and risk-sharing operates via a depreciation of the real exchange rate that improves the purchasing power of foreign consumers. As in the previous case the dynamics of relative consumption are linked to that of the real exchange rate via the risk-sharing mechanism associated with Arrow-Debreu securities.

In both of these cases, the model with Arrow-Debreu securities generates a unitary cross-correlations between the real exchange rate and relative consumption (relative consumption and the real exchange rate move in the same direction). This behavior, which is at odds with the evidence reported in Section 2, is referred as the consumption-real exchange rate anomaly. In our next experiments we examine to what extent the removal of the assumption of market completeness will break the link between relative consumption and the real exchange rate.

One consequence of the incomplete financial market structure is that the risk-sharing condition (27) now only holds in terms of expected first differences, but not in levels:

$$\mathrm{E}_t\widehat{RS}_{t+1} - \widehat{RS}_t = (\mathrm{E}\hat{U}_C(C_{t+1}^*) - \hat{U}_C(C_t^*)) - (\mathrm{E}\hat{U}_C(C_{t+1}) - \hat{U}_C(C_t)) + \varepsilon\hat{b}_t \tag{28}$$

Because of our assumed bond-holding cost, the risk-sharing condition is further augmented by the term $\varepsilon\hat{b}$ which captures the deviation of foreign-currency denominated bond holdings from their steady state, relative to domestic GDP. Because ε is assumed to be very small and $\hat{b}$ not very volatile, for our calibration we find a near unitary cross-correlation between the expected first difference of the real exchange rate and that of relative consumption.

In a bond economy there are only limited opportunities for sharing risk between countries. Non state-contingent bonds offer one avenue for risk diversification. The other way to share risk is through changes in the terms of trade. Indeed these two ways of sharing risk are inter-connected. To transfer purchasing power from Home to Foreign through changes in bond holdings requires an improvement in the net foreign assets position of the Home economy. In other words, Home requires an improvement in the trade balance (or current account). To achieve this trade balance improvement, the terms of trade must depreciate, which by itself contributes to risk-sharing. We can illustrate this by rewriting our log-linearised current account equation for the case in which there are no investment dynamics and the steady-state net foreign asset position is zero:

$$\begin{gathered}\beta\hat{b}_t = \hat{b}_{t-1} + (\theta-1)(1-v)\omega\hat{T}_t \\ +\theta\omega\,(1-v)\,\widehat{RS}_t + \omega(v-1)\left(\hat{C}_t - \hat{C}_t^*\right) + (\theta-\kappa)(\omega-1)\omega(v-1)\hat{R}_t \\ +(\theta-\kappa)\,(\omega^*-1)\,\omega\,(1-v)\,\hat{R}_t^*,\end{gathered} \tag{29}$$

where $\hat{b}$ is the deviation of foreign currency denominated bond holdings from their steady state, relative to domestic GDP. Note that, as long as $\theta > 1$, *ceteris paribus*, a depreciation (rise) in the terms of trade improves the current account.

Figures 3 and 4 show the response of our key variables following a productivity shock to the Home traded (figure 3) and Home non-traded (figure 4) sectors. In our model and for our calibration, the terms of trade depreciate (rise) following a positive productivity shock to home-produced traded goods. Whereas an increase in productivity raises domestic output and consumption, part of the increase in consumption is shared with foreign agents via the terms of trade depreciation. In our model and for our calibration, this effect on the real exchange rate is outweighed by an increase in the relative price of non-traded to traded goods, so that overall the real exchange rate appreciates in response to an increase in home traded sector productivity. Following a positive supply-side shock to the home economy's traded goods sector, home agents become wealthier and demand more goods of all types. As a risk-sharing mechanism the terms of trade depreciate, improving the purchasing power of foreign consumers. However, since risk is not shared completely, the terms of trade do not have to transfer as much purchasing power from Home to Foreign (the required improvement of the trade balance is smaller), so that the terms of trade do not depreciate by as much as in the complete market case. Because there is less risk-sharing and therefore less of a terms of trade depreciation, foreign consumption does not increase by as much as home consumption so that relative consumption increases. Thus the real exchange rate and relative consumption move in opposite directions indicating a negative cross-correlation.

When the productivity shock occurs in the non-traded sector, as depicted in figure 4, the real exchange rate depreciates and relative consumption rises, just as in the model with state-contingent assets.

Figures 3 and 4 suggest that the size and sign of the cross-correlation between the real exchange rate and relative consumption depends, at least to some extent, on the relative size of the shocks hitting the traded and non-traded goods sectors. When the source of the disturbance arises in the non-traded goods sector, the real exchange rate and relative consumption move in the same direction, whereas the real exchange rate and relative consumption move in opposite directions when the shock originates in the traded goods sector. Figures 3 and 4 suggests that predominance of traded sector productivity shocks will result in large and negative cross-correlation, whereas predominance of non-traded productivity shocks will result in large and positive cross-correlations between relative consumption and the real exchange rate.

6 Characteristics of the calibrated model

In this section, we analyze the second moments generated by our model using the calibration in table 2 for model parameters as well as shock processes. Table 3 summarizes a selection of second moments from the data and compares these with moments generated by the artificial model economies under different calibrations. Both the actual data (authors' own calculations), as well as the artificial model economy data are of annual frequency, logged and Hodrick-Prescott filtered.[8]

The column headed *Data* contains selected second moments. Moments for the domestic economy refer to US data, whereas foreign variables refer to weighted aggregates of EU15 and Japanese data. The moments are calculated on annual data from 1970 - 2000. The column headed *Baseline model* in table 3 shows a selection of second moments generated by our model under the calibration proposed in table 2. The numbers in the bottom rows of table 3 show that for our baseline calibration our model generates a negative cross-correlation between the real exchange rate and relative consumption.

Given our estimated shock processes, our model comes reasonably close to matching the standard deviation of GDP. Our baseline model yields a standard deviation of 1.86, whereas in the data the figure is 1.57. A clear shortcoming of our modelling approach is the extremely low volatility of the real exchange rate. With flexible prices and without allowing for deviations from the law of one price for traded goods, the real exchange rate in our model is mainly driven by deviations in the relative price of non-traded goods and to some extent through changes in the terms of trade through the home-bias channel. Neither of these channels is capable of generating a realistically volatile series for the real exchange rate, at least when the model is driven only by supply-side shocks. Consumption and investment are also somewhat less volatile than in the data. The relative volatility of hours worked is reasonably close to the observed value.

The real exchange rate in our model is not just not volatile enough, it is also only about 2/3 as persistent as in the data. Our baseline model is, however, able to match the persistence of GDP and over 2/3 of the persistence of real consumption. In terms of correlations between home and foreign variables, our model captures the fact that output is more highly correlated across countries than is investment, but not the fact that the correlation between home and foreign GDP exceed that of home and foreign consumption. In our baseline model, consumption is more highly correlated across countries than is GDP.

Where our baseline calibration departs from the data is the correlation between the real exchange rate and the terms of trade. We saw from our analysis of impulse responses that the real exchange rate and terms of trade move in opposite directions following a shock to the traded goods sector, and in the same direction in response to non-traded goods shocks. The fact that in our baseline model the terms of trade and the

[8] We have chosen a smoothing parameter, $\lambda = 100$ for the Hodrick-Prescott filter in all filtered data. This value has been suggested by Backus and Kehoe (1992) and is the default setting in Eviews which we have used for our calculations. Cooley and Ohanian (1991) suggest a value of 400, whereas Baxter and King (1999) and Ravn and Uhlig (2002) suggest values of 10 and 6.25, respectively. Our main result is robust to these alternative values for the smoothing parameters.

real exchange rate are negatively correlated reflects the predominance of traded sector productivity shocks, which also helps us account for the negative cross-correlation between the real exchange rate and relative consumption.

So far, we have shown that our baseline model succeeds in showing that a simple two-country model with incomplete markets, as well as traded and non-traded goods, driven only by shocks to sectoral TFP can generate realistically low values of the cross-correlation between the real exchange rate and relative consumption, measured in levels. The original analysis of Backus and Smith (1993) looks at the correlation between the real exchange rate and relative consumption, not in levels, but in *ex post* growth rates. In the final row of table 3, we report the correlation between the change in the real exchange rate and the change in relative consumption: $Corr(\widehat{RS}_t - \widehat{RS}_{t-1}, [\hat{C}_t - \hat{C}_t^*] - [\hat{C}_{t-1} - \hat{C}_{t-1}^*])$. For our model, we find that this correlation is positive, but close to zero. By construction, our model generates a correlation between the real exchange rate and relative consumption in terms of expected first differences close to unity.

Our baseline model fails to match the volatility as well as the persistence of the real exchange rate, this is no doubt due to our parsimonious model and shock structure. For example, Kollmann (2005) shows that, even under flexible prices, a good proportion of the volatility of the real exchange rate can be accounted for by allowing shocks to the uncovered interest rate parity (UIP) condition. Finally, we want to point out why our model is not able to generate volatile real exchange rates as in the data: one reason is that we assume that the law of one price holds and that deviation of the real exchange rate from the purchasing power parity level are caused by the existence of non-tradable goods and asymmetric consumption preferences. Indeed, Engel (1999) has documented that deviations from the law of one price are the main source of real exchange rate volatility. In the next section, we analyse the extent to which our model's ability to address the Backus-Smith anomaly is dependent on our parameter choices and on our estimated shock processes.

7 Sensitivity analysis

In this section, we perform sensitivity analysis on some of the key parameters of our model that have a baring on on the cross-correlation between the real exchange rate and relative consumption in order to understand the determinants of our results and to check the robustness of our results to parameters for which there is uncertainty on their calibration in the literature. Figure 5, as well as columns 3 and 4 in table 3, analyses the effects of varying θ, the intratemporal elasticity of substitution between home and foreign-produced traded goods. In our model, θ determines the magnitude of the terms of trade response to a change in the supply of home relative to foreign goods-produced traded goods. The larger is the elasticity of substitution between two goods, the smaller will the relative price response to a relative supply change. For $\theta = 0.5$ the cross-correlation between the real exchange rate and relative consumption, as well as the volatility of the real exchange rate relative to the volatility of GDP are closer to the data than in our baseline model where we set $\theta = 2$. When $\theta = 0.5$ home and foreign produced goods are complements in consumption. As a result, relative consumption falls in response to a rise in traded sector TFP, while the consumption-based real exchange rate depreciates. For this calibration, the model is however quite sensitive to changes in θ around a value of 0.5. The cross-correlation approaches unity as θ rises to about 0.8 and falls to below zero for values of θ in excess of 1.5. As θ increases the response of the terms of trade to a productivity shock becomes smaller, so that the appreciation of the real exchange rate following a traded sector TFP shock becomes more and more pronounced. This lowers the cross-correlation between the real exchange rate and relative consumption and raises the relative volatility of the real exchange rate. Table 3 also reports a set of second moments for both $\theta = 0.5$ and $\theta = 10$.

Figure 6 analyses the effects of varying κ, the the intratemporal elasticity of substitution between traded and non-traded goods. The higher is κ, the less the relative price of non-traded goods has to adjust to changes in the relative supply of non-traded goods. Subsequently our cross-correlation increases and the relative volatility of the real exchange rate decreases as κ rises. Evidence on this parameter suggests a value below unity. Stockman and Tesar suggest a value of κ of 0.44, whereas Mendoza (1991) suggest a value of 0.74. Figure 6 suggests that for values of κ in this range, there is no significant difference in either the cross-correlation or in the relative volatility.

Figure 7 analyses the effects of varying the degree of consumption home-bias. Relative volatility seems not to be affected in a material way by altering the degree of home-bias. The cross-correlation on the other hand, rises with the degree of home-bias. Between no and near-complete home-bias ($v = 0.5$ to $v = 0.99$), the cross-correlation varies between close to -0.25 and 0.4, which is still significantly below zero for most relevant values. Our baseline calibration assumes a value of $v = 0.72$, as suggested by Corsetti *et al.*

In figure 8, we analyse the effects of varying ω, the share of traded goods in the final consumption goods basket. Figure 8 shows that for very low values of ω the real exchange rate is volatile, but the cross-correlation between the real exchange rate and relative consumption is quite high. This is because most of the goods consumed are non-traded. As a result, changes in the relative supply of non-traded goods have a large effect the relative price of non-traded goods. From our definition of the real exchange rate we can see that the lower ω the larger is the effect of changes in the relative price of non-traded goods on the real exchange rate. However, the lower the share of traded goods is in consumption, the smaller will be the effect of shocks to the traded goods sector's TFP on the real exchange rate. Recall that it is this effect that can cause the cross-correlation to become negative. As ω exceeds 0.65, the cross-correlation begins to increase as the Balassa-Samuelson effect on the real exchange rate diminishes. In the limit as ω approaches unity, so that the model converges towards a one sector model, as in Chari *et al* (2002) the cross-correlation tends towards 1, as the results of Chari *et al* (2002) suggest.

Figure 9 as well as columns headed *Low* ε and *High* ε in table 3 analyse the effects of changing ε, the cost of holding foreign currency-denominated bonds faced by home agents. In figure 9, our baseline calibration corresponds to the first observation, where $\varepsilon = 0.001$. Figure 9 illustrates that raising ε lowers the cross-correlation between the real exchange rate and relative consumption. In the limit as ε becomes very large, the economy tends towards autarky. The column headed *High* ε in table 3 corresponds to a calibration where $\varepsilon = 0.028$, which corresponds to the estimate of ε by Rabanal and Tuesta (2005). We find that there is almost nothing to be gained by choosing a cost that is lower than our baseline calibration. This is confirmed by comparing the baseline calibration with the column headed *Low* ε which corresponds to $\varepsilon = 0.0001$ in table 3. Our sensitivity analysis suggests that for reasonably small values of ε, the effects on our model are minimal. There is much uncertainty about the true value of ε. As a result of this uncertainty, we have chosen a value for ε that does not enhance our results should our chosen value prove to be too high. Should the true value of ε prove to be higher than our chosen parameter value, then our results will be enhanced. In summary, the bond holding cost can affect the cross-correlation between the real exchange rate and relative consumption, but the level we choose for the bond holding cost is so low as to not affect our results in a meaningful way.

As we argue above, the sign of the cross-correlation between the real exchange rate and relative consumption depends to some extent on the relative size of the productivity shocks hitting the traded and non-traded goods sectors. Our baseline model is driven by our estimated shock processes. In estimating these shocks we have to make a series of potentially unrealistic assumptions: (i) since we assume a symmetric calibration we impose equal factor shares on each of the economies, and (ii) we impose symmetry on the shock processes, essentially assuming that the EU15 plus Japan are symmetric to the US. To check if our results go through under an alternative shock process, we re-solve our model using the shocks from Corsetti *et al* (2004) and Stockman and Tesar (1995). Table 3 shows that the cross-correlation is close to zero for our model calibrated on Corsetti *et al* (2004) shocks, but takes a value of 0.59 when we shock the model with Stockman and Tesar's shock processes.[9]

8 Conclusion

In this paper, we address the consumption-real exchange rate anomaly. This anomaly refers to the property of international business cycle models based on complete financial markets to generate cross-correlations between the real exchange rate and relative consumption close to unity. In the data, this correlation is close to zero or even negative. We show that if a canonical international business cycle model, similar to the one proposed by Chari *et al* (2002) includes both an incomplete financial markets structure as well as a non-traded goods sector, then such a model, calibrated in a standard way will generate cross-correlations between the real exchange rate and relative consumption close to those in the data.

The presence of a non-traded goods sector allows the real exchange rate to appreciate (decrease) in response to a productivity shock to the domestic traded goods sector - the familiar Balassa-Samuelson effect - while limited risk-sharing opportunities cause consumption in the domestic economy to increases by more than consumption in the foreign economy following such a shock. The result is a negative cross-correlation between the real exchange rate and relative consumption.

[9] Note that Stockman and Tesar's Solow residuals are detrended using the HP filter, which accounts for the low AR(1) coefficients generated by the model.

Figure 1

The real exchange rate and its components (top panel) and relative consumption (bottom panel) following a positive productivity shock to the domestic traded goods sector, when financial markets are complete

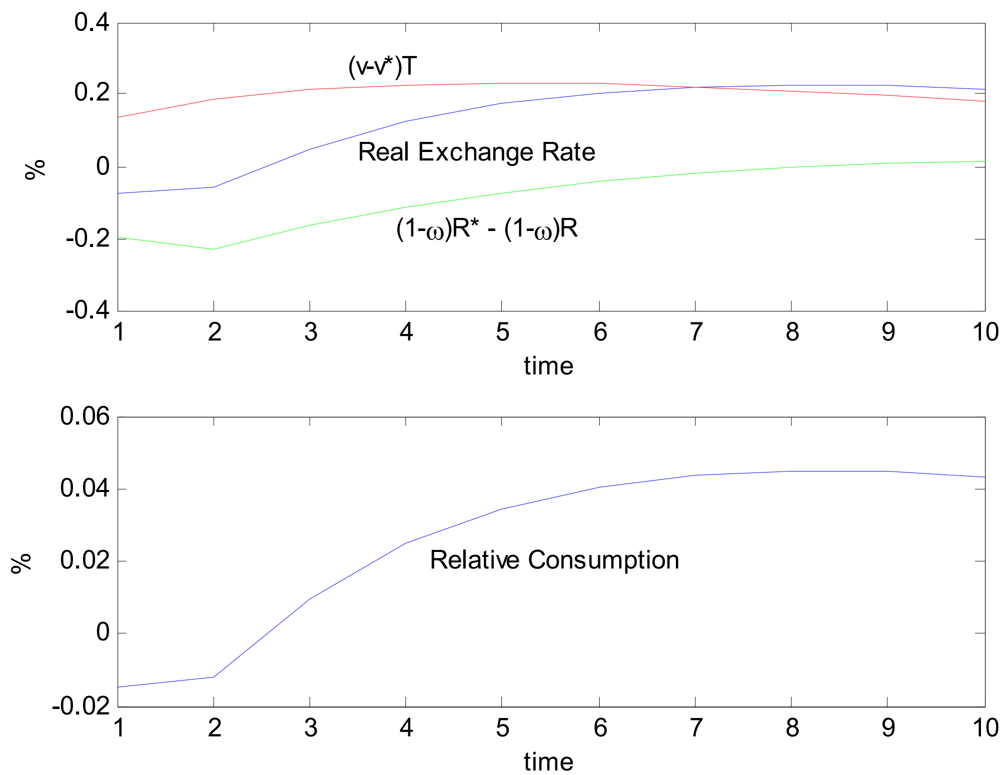

Figure 2

The real exchange rate and its components (top panel) and relative consumption (bottom panel) following a positive productivity shock to the domestic non-traded goods sector, when financial markets are complete

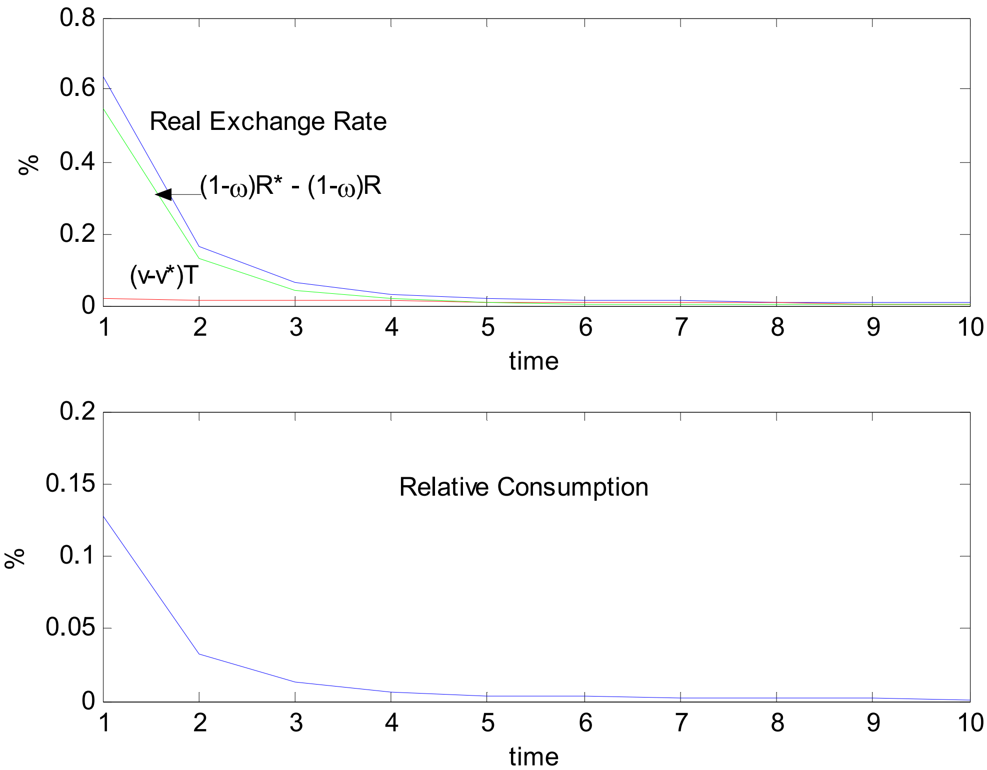

Figure 3

The real exchange rate and its components (top panel) and relative consumption (bottom panel) following a positive productivity shock to the domestic traded goods sector, when financial markets are incomplete

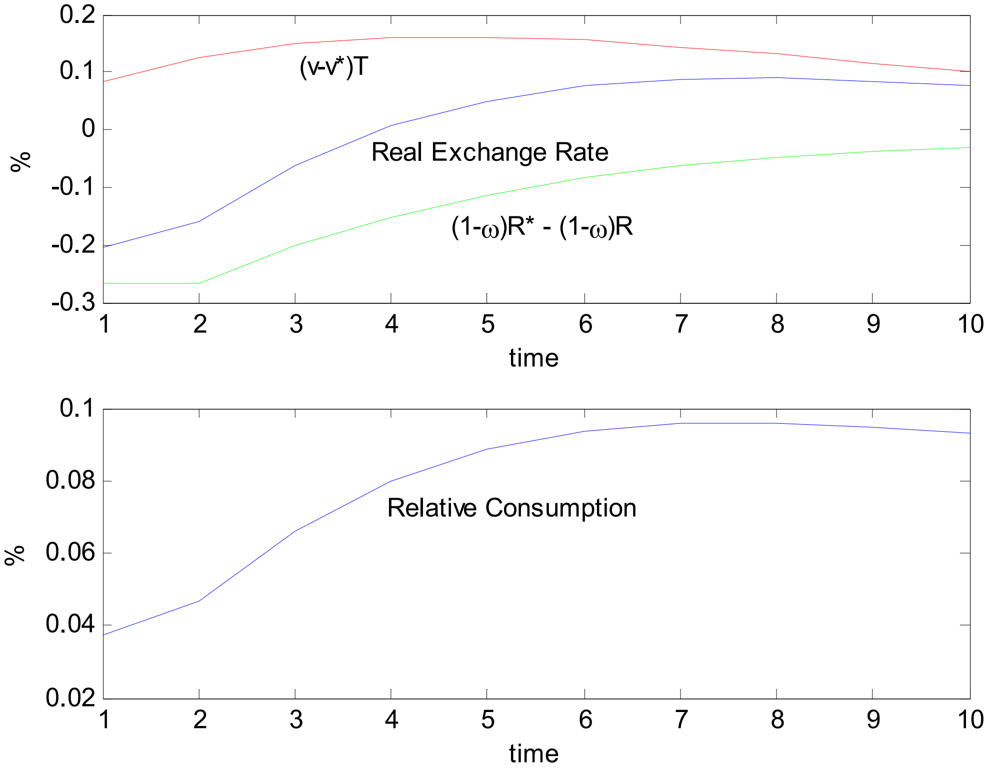

Figure 4

The real exchange rate and its components (top panel) and relative consumption (bottom panel) following a positive productivity shock to the domestic non-traded goods sector, when financial markets are incomplete

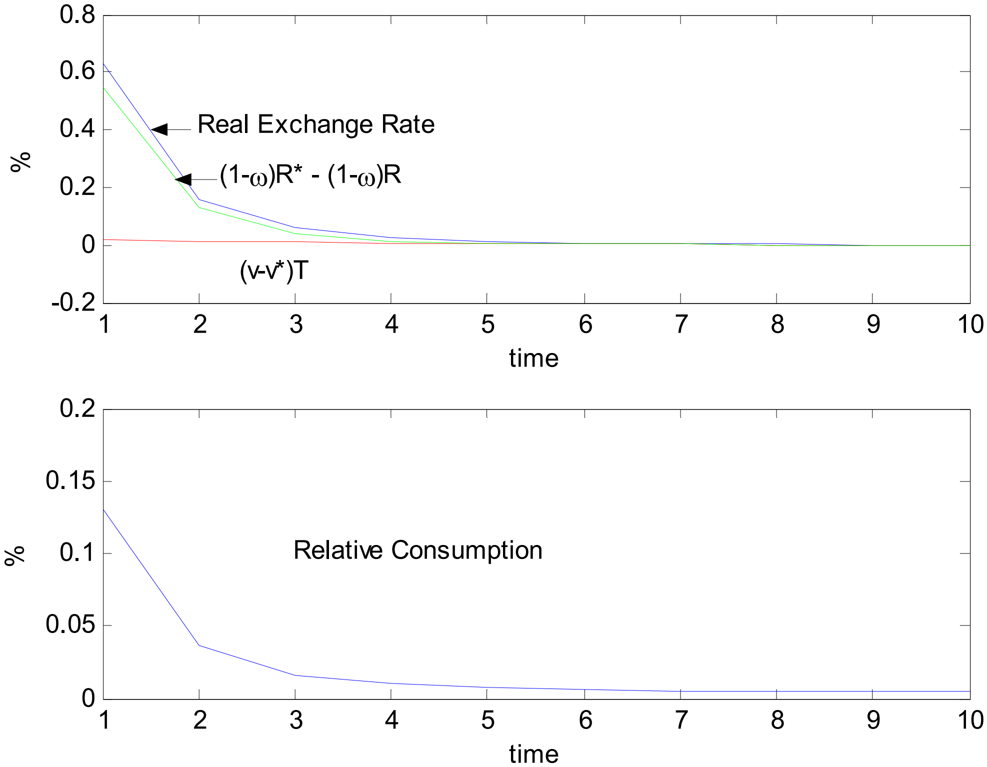

Figure 5

Sensitivity analysis. The cross-correlation between the real exchange rate and relative consumption and the standard deviation of the real exchange rate relative to the standard deviation of GDP for various values of intra-temporal elasticity of substitution between traded goods, θ

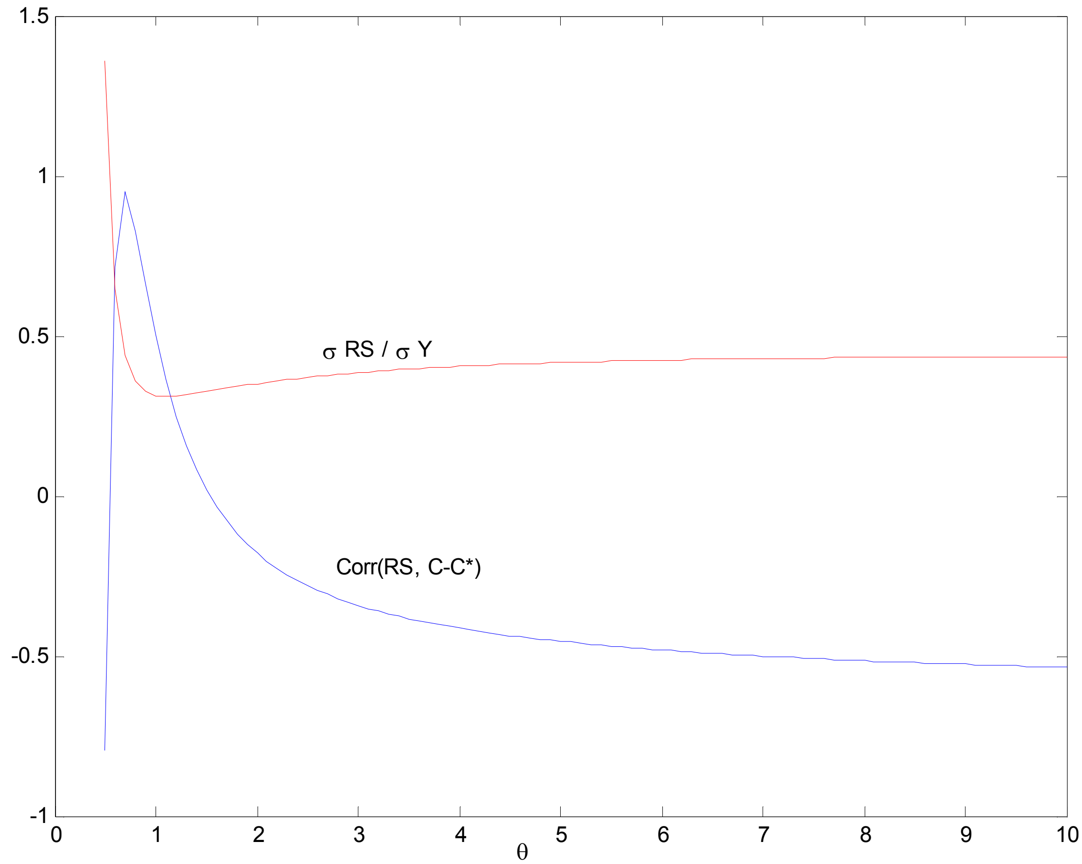

Figure 6

Sensitivity analysis. The cross-correlation between the real exchange rate and relative consumption and the standard deviation of the real exchange rate relative to the standard deviation of GDP for various values of intra-temporal elasticity of substitution between traded and non-traded goods, κ

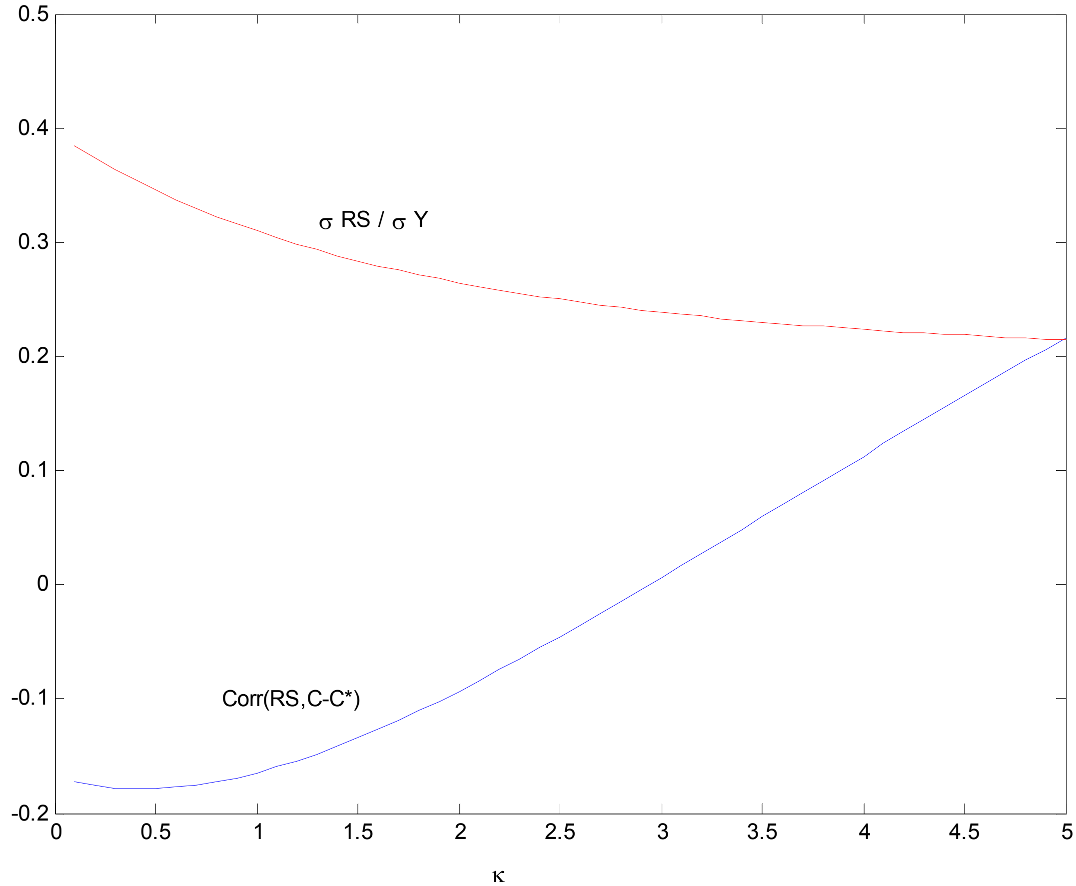

Figure 7

Sensitivity analysis. The cross-correlation between the real exchange rate and relative consumption and the standard deviation of the real exchange rate relative to the standard deviation of GDP for various values of consumption home bias (v=0.5 no home bias, v=1 complete home bias)

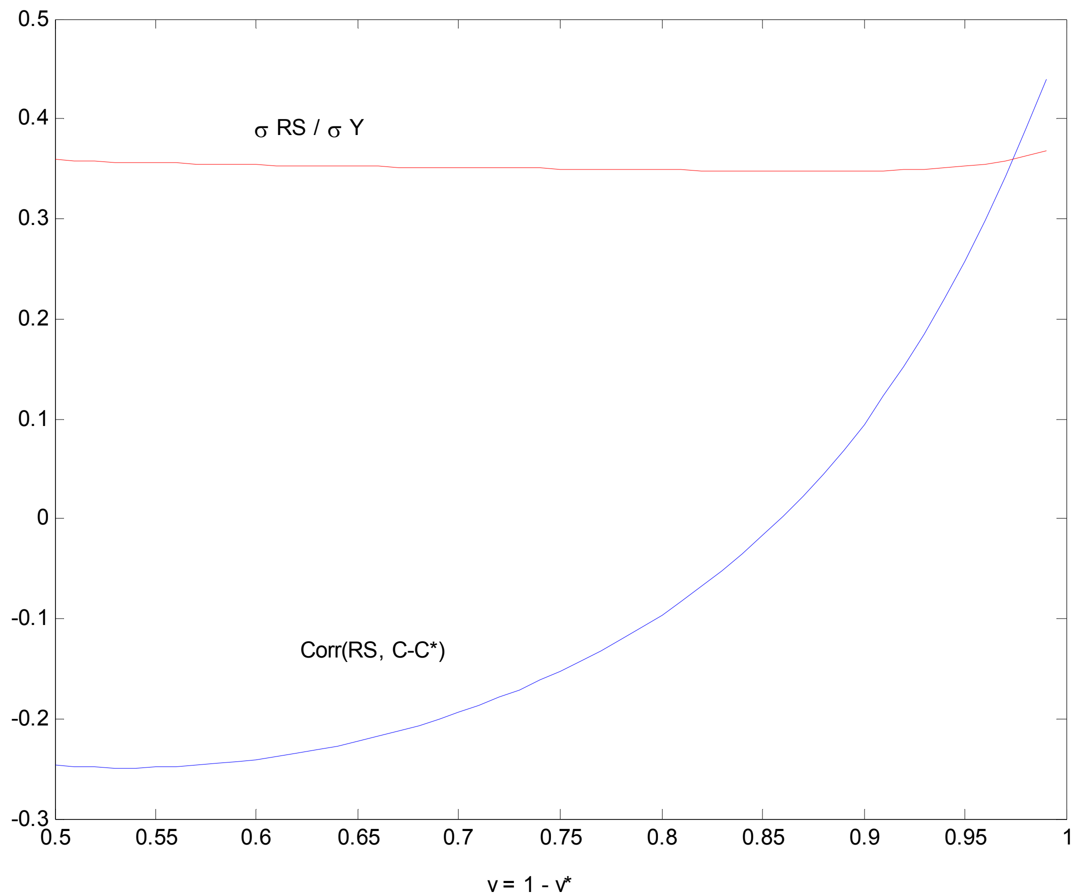

Figure 8

Sensitivity analysis. The cross-correlation between the real exchange rate and relative consumption and the standard deviation of the real exchange rate relative to the standard deviation of GDP for various values of the share of traded goods in final consumption good, ω

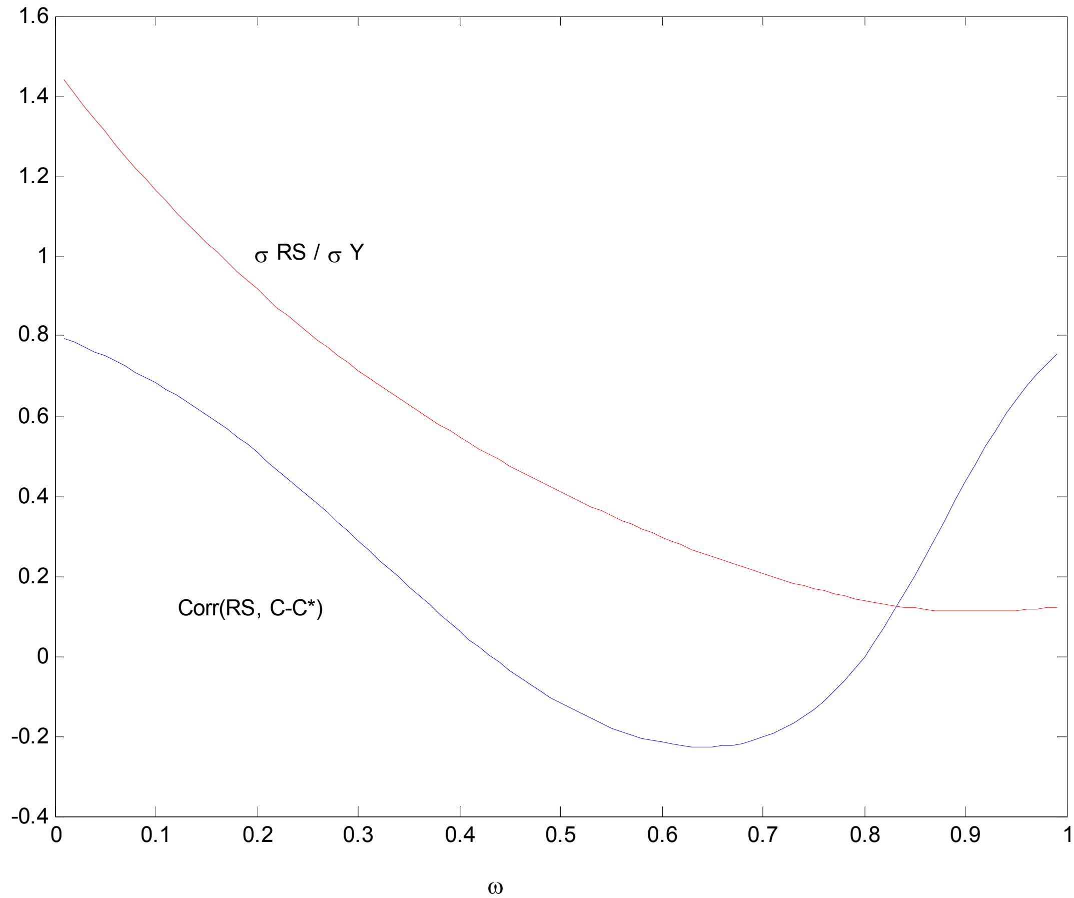

Figure 9

Sensitivity analysis. The cross-correlation between the real exchange rate and relative consumption and the standard deviation of the real exchange rate relative to the standard deviation of GDP for various values of the cost of financial intermediation. For ε=0.001 (calibration) to 0.1

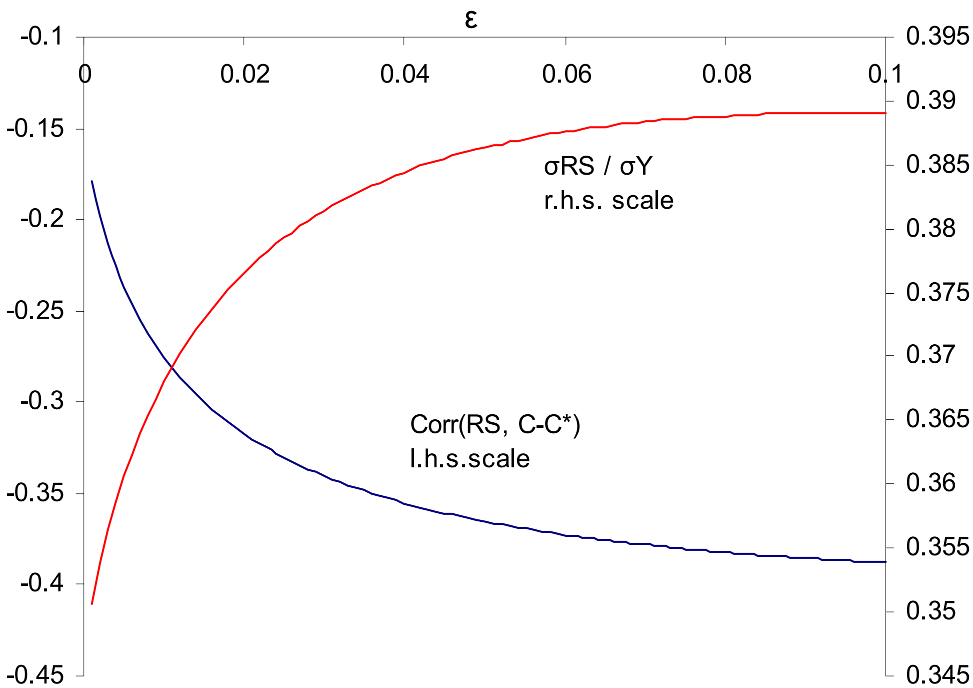

Table 1: Selected cross-correlations between real exchange rate and relative consumption. Authors' calculations.

	Corr(RS,C-C*)	Corr(ΔRS,Δ(C-C*))
Australia	-0.386	-0.196
Austria	-0.153	0.071
Canada	-0.474	-0.214
France	-0.254	-0.168
Germany	-0.288	0.032
Italy	-0.313	-0.272
Japan	0.000	0.260
Netherlands	-0.435	-0.258
New Zealand	0.515	0.550
Spain	-0.654	-0.377
Sweden	0.634	0.464
Switzerland	0.030	0.091
UK	-0.587	-0.529
Median	-0.288	-0.168

Table 2: Parameter values

Preferences	$\beta = 1/1.04$, $\rho = 2$, $\eta = -3.17$
Final goods technology	$\omega = \omega^* = 0.55$, $v = 1 - v^* = 0.72$, $\theta = 2$, $\kappa = 0.44$
Intermediate goods technology	$\alpha = 0.67$, $\delta = 0.1$, d adjusted
Financial markets	$\varepsilon = 0.001$, $\bar{a} = 0$
Shocks	$\Omega = \begin{bmatrix} 0.84 & 0 & 0.22 & 0 \\ 0 & 0.84 & 0 & 0.22 \\ 0 & 0 & 0.30 & 0 \\ 0 & 0 & 0 & 0.30 \end{bmatrix}$
	$V[\mu] = \begin{bmatrix} 3.76 & 1.59 & 0.72 & 0.44 \\ 1.59 & 3.76 & 0.44 & 0.72 \\ 0.72 & 0.44 & 0.51 & 0.21 \\ 0.44 & 0.72 & 0.21 & 0.51 \end{bmatrix}$

Table 3: Data and Models - Baseline model and sensitivity analysis

	Data	Baseline Model	Low θ	High θ	Low ε	High ε	Corsetti shocks	Stockman Tesar shocks
Standard deviation of GDP	1.57	1.86	1.80	1.96	1.86	1.84	0.67	2.13
Standard deviations relative to GDP								
Real exchange rate	6.16	0.35	1.36	0.43	0.35	0.38	0.49	0.49
Terms of trade	2.12	0.36	2.43	0.20	0.37	0.31	0.37	0.25
Relative price of non-traded	1.46	0.87	0.89	0.85	0.87	0.89	0.87	0.72
Consumption	0.76	0.39	0.40	0.38	0.39	0.40	0.48	0.34
Investment	4.33	3.04	2.99	3.56	3.04	3.05	2.83	3.27
Hours worked	0.31	0.44	0.42	0.47	0.44	0.43	0.40	0.55
AR(1) coefficients								
Real exchange rate	0.67	0.40	0.44	0.44	0.40	0.40	0.44	0.31
GDP	0.50	0.48	0.47	0.52	0.48	0.48	0.47	0.03
Consumption	0.66	0.46	0.48	0.47	0.46	0.46	0.54	0.20
Cross correlation between home and foreign								
GDP	0.35	0.38	0.48	0.24	0.38	0.41	0.36	0.35
Consumption	0.06	0.66	0.73	0.58	0.67	0.59	0.78	0.39
Investment	0.07	0.20	0.20	0.17	0.20	0.19	0.19	0.27
Cross-correlation between GDP and								
Net exports	-0.26	0.01	-0.48	-0.12	0.04	-0.20	0.07	0.04
Real exchange rate	-0.09	-0.30	0.49	-0.48	-0.29	-0.35	-0.13	0.12
Cross-correlation between real exchange rate								
and relative consumption	-0.45	-0.18	-0.79	-0.54	-0.16	-0.34	0.07	0.59
and terms of trade	0.32	-0.11	0.98	0.23	-0.11	-0.14	0.29	0.34
Corr($\Delta\widehat{RS}_t, \Delta(\hat{C}_t - \hat{C}_t^*)$)	-0.28	0.10	0.05	0.08	0.10	0.10	0.17	0.16

A Summary of equations

In this appendix, we list the linearised equations of the model pertaining to Home country variables.

- Euler equation, the UIP condition, home and foreign consumption-labor effort trade-off and current account:

$$\rho E_t \hat{C}_{t+1} + E_t \eta \frac{l}{1-l} \hat{l}_{t+1} = \rho \hat{C}_t + \eta \frac{l}{1-l} \hat{l}_t + \hat{\imath}_t - E_t \pi_{t+1} \tag{A1}$$

$$E_t \Delta \hat{s}_{t+1} = \hat{\imath}_t - \hat{\imath}_t^* + \varepsilon \hat{b}_t \tag{A3}$$

$$-\hat{C}_t + \hat{w}_t = \frac{l}{1-l} \hat{l}_t \tag{A4}$$

$$\begin{aligned} \beta \hat{b}_t \quad = \quad & \hat{b}_{t-1} + (1-\theta)(v-1)\omega \hat{T}_t \\ & + (\omega - 1)(1-v)\,\omega(\kappa - \theta)\hat{R}_t \\ & + (\theta - \kappa)(\omega^* - 1)(1-v)\,\omega \hat{R}_t^* \\ & + \theta(1-v)\,\omega \widehat{RS}_t + \omega(1-v)\,\hat{C}^* - \omega(1-v)\hat{C} \end{aligned} \tag{A5}$$

- The firms' optimality conditions for investment, capital and labor input in traded and non-traded

$$\begin{gathered} \rho E_t \hat{C}_{t+1} + E_t \eta \frac{l}{1-l} \hat{l}_{t+1} = \rho \hat{C}_t + \eta \frac{l}{1-l} \hat{l}_t \\ + (1 + \beta(\delta - 1)) E_t \left[\widehat{mpk}_{H_{t+1}} \right] \\ - b\delta \left[\hat{x}_{H_t} - \hat{k}_{H_{t-1}} \right] + b\delta\beta \left[E_t \hat{x}_{H_{t+1}} - \hat{k}_{H_t} \right] \\ + (1-v)\hat{T}_t + (1-\omega)R_t - (1-v)E_t \hat{T}_{t+1} - (1-\omega)E_t R_{t+1} \end{gathered} \tag{A6}$$

$$\begin{gathered} \rho E_t \hat{C}_{t+1} + E_t \eta \frac{l}{1-l} \hat{l}_{t+1} = \rho \hat{C}_t + \eta \frac{l}{1-l} \hat{l}_t \\ + (1 + \beta(\delta - 1)) \left[\widehat{mpk}_{N_{t+1}} \right] \\ - b\delta \left[\hat{x}_{N_t} - \hat{k}_{N_{t-1}} \right] + b\delta\beta \left[\hat{x}_{N_{t+1}} - \hat{k}_{N_t} \right] \\ + (1-v)\hat{T}_t + (1-\omega)R_t \\ - (1-v)\beta(\delta - 1)E_t \hat{T}_{t+1} - (1-\omega)(1 + \beta(\delta - 1))E_t R_{t+1} \end{gathered} \tag{A7}$$

$$\widehat{k}_{H_t} = (1-\delta)\widehat{k}_{H_{t-1}} + \delta \widehat{x}_{H_t} \tag{A8}$$

$$\widehat{k}_{N_t} = (1-\delta)\widehat{k}_{N_{t-1}} + \delta\widehat{x}_{N_t} \tag{A9}$$

$$\hat{w}_t = (v-1)\hat{T}_t + (\omega-1)\hat{R}_t + \hat{A}_{H_t} + (\alpha-1)\hat{l}_{H_t} + (1-\alpha)k_{H_{t-1}} \tag{A10}$$

$$\hat{w}_t = \omega\hat{R}_t + \hat{A}_{N_t} + (\alpha-1)\hat{l}_{N_t} + (1-\alpha)k_{N_{t-1}} \tag{A11}$$

- Production constraints

$$\hat{A}_{N,t} + \alpha\hat{l}_{N,t} + (1-\alpha)\hat{k}_{N,t-1} = -\kappa\omega\hat{R}_t + \hat{C}_t \tag{A12}$$

$$\begin{aligned}
\hat{y}_{H,t} \quad = \quad & \theta(1-v)\left(\frac{c_H}{\bar{y}_H} + \frac{c_H^*}{\bar{y}_H}\right)\hat{T}_t + (1-\omega)\left(\kappa\frac{c_H}{\bar{y}_H} + \theta\frac{c_H^*}{\bar{y}_H}\right)\hat{R}_t \qquad \text{(A13)} \\
& +\theta\frac{c_H^*}{\bar{y}_H}\widehat{RS} \ + (\theta-\kappa)\left(\omega^*-1\right)\frac{c_H^*}{\bar{y}_H}\hat{R}_t^* \\
& +\hat{C}\frac{c_H}{\bar{y}_H} + \hat{C}^*\frac{c_H^*}{y_H} + \hat{x}_H\frac{x_H}{\bar{y}_H} + \hat{x}_N\frac{x_N}{\bar{y}_H}
\end{aligned}$$

$$\hat{y}_{H,t} = \Big((1-\alpha)\widehat{k}_{H_{t-1}} + \alpha\widehat{l}_{H_t} + \widehat{A}_{H_t}\Big) \tag{A14}$$

- Labour market constraint

$$\hat{l}_t = \frac{\bar{l}_H}{\bar{l}}\hat{l}_{H_t} + \frac{\bar{l}_N}{\bar{l}}\hat{l}_{N_t} \tag{A15}$$

- Steady-state ratios

$$\begin{aligned}
\frac{x_H}{y_H} \quad &= \quad \frac{x_H}{k_H}\left(\frac{k_H}{y_H}\right) = \delta\left(\frac{k_H}{y_H}\right) = \delta\left(A_H\left(l_H\right)^{\alpha}\left(k_H\right)^{-\alpha}\right)^{-1} = \delta\left(\frac{1-\alpha}{1/\beta-1+\delta}\right) \\
\frac{x_N}{y_N} \quad &= \quad \frac{x_N}{k_N}\left(\frac{k_N}{y_N}\right) = \delta\left(\frac{k_N}{y_N}\right) = \delta\left(A_N\left(l_N\right)^{\alpha}\left(k_N\right)^{-\alpha}\right)^{-1} = \delta\left(\frac{1-\alpha}{1/\beta-1+\delta}\right)
\end{aligned}$$

$$\frac{y_H}{c_H} = \left(\frac{\omega v + (1-v)\,\omega + (\beta-1)\bar{a}}{\omega v} + \frac{\frac{x_N}{y_N}(1-\omega)}{\omega v}\right)\left[1 - \frac{x_H}{y_H}\right]^{-1}$$

$$\frac{y_H}{c_H^*} = \frac{\omega + (\beta-1)\bar{a}}{(1-v)\,\omega + (\beta-1)\bar{a}} + \frac{x_H}{y_H}\frac{y_H}{c_H}\frac{\omega v}{(1-v)\,\omega + (\beta-1)\bar{a}} + \frac{x_N}{y_N}\frac{1-\omega}{(1-v)\,\omega + (\beta-1)\bar{a}}$$

$$\frac{x_N}{y_H} = 1 - \frac{c_H}{y_H} - \frac{c_H^*}{y_H} - \frac{x_H}{y_H}$$

$$\frac{l}{l_N} = 1 + \frac{\omega + (\beta - 1)\bar{a}}{(1-\omega)} + \frac{x_H}{y_H}\frac{y_H}{c_H}\frac{\omega v}{(1-\omega)} + \frac{x_N}{y_N}$$

$$\frac{l_H}{l} = 1 - \frac{l_N}{l}$$

- The real exchange rate and the terms of trade

$$\widehat{RS}_t = (v - v^*)\hat{T}_t + (\omega - 1)\hat{R}_t + (1-\omega^*)\hat{R}_t^* \tag{A16}$$

$$\hat{T}_t = \hat{T}_{t-1} + \Delta s_t + \pi_t^{F^*} - \pi_t^H \tag{A17}$$

- CPI implies setting

$$\pi_t = \omega v \pi_t^H + \omega(1-v)\pi_t^{F^*} + \omega(1-v)\Delta s_t + (1-\omega)\pi_t^N = 0$$

take $\omega v \pi_t^H + \omega(1-v)\pi_t^{F^*} + \omega(1-v)\Delta s_t + (1-\omega)\pi_t^N = 0$ and solve for π^N

$$\pi^N = -\frac{\omega v}{1-\omega}\pi_t^H - \frac{\omega(1-v)}{1-\omega}\pi_t^{F^*} - \frac{\omega(1-v)}{1-\omega}\Delta s_t$$

define:

$$\hat{q}_t = \hat{R}_t + (1-v)\hat{T}_t$$

$$\begin{aligned}
\hat{q}_t - \hat{q}_{t-1} &= -\left(\frac{\omega v + 1 - \omega}{1-\omega}\right)\pi^H - \frac{\omega(1-v)}{1-\omega}\pi_t^{F^*} - \frac{\omega(1-v)}{1-\omega}\Delta s_t \\
\pi_t^H &= -\left(\frac{1-\omega}{\omega v + 1 - \omega}\right)[\hat{q}_t - \hat{q}_{t-1}] - \frac{\omega(1-v)}{1-\omega}\left[\pi_t^{F^*} + \Delta s_t\right]
\end{aligned}$$

now use the expression for the terms of trade to eliminate $\pi_t^{F^*} + \Delta s_t$

$$\pi_t^H = -\left(\frac{1-\omega}{\omega v + 1 - \omega}\right)[\hat{q}_t - \hat{q}_{t-1}] - \frac{\omega(1-v)}{1-\omega}\left[T_t - T_{t-1} + \pi_t^H\right]$$

solve for π_t^H

$$\pi_t^H = (1-\omega)[\hat{q}_t - \hat{q}_{t-1}] - \omega(1-v)[T_t - T_{t-1}] \tag{A18}$$

doing the same for $\pi_t^{F^*}$

$$\pi_t^{F^*} = (1-\omega^*)\left[\hat{q}_t^* - \hat{q}_{t-1}^*\right] + \omega v^*[T_t - T_{t-1}] \tag{A19}$$

B Data appendix

Data appendix

1) The series for GDP, Consumption Investment and net exports in table 3 are of annual frequency from 1970 to 2000 and are taken from the Penn World Tables.

2) Consumption deflators and exchange rates used to construct the data in table 1 as well as the terms of trade in table 3 are taken from Datasteam, are of annual frequency and span from 1970 - 2000.

4) The relative price of non-traded to traded goods in table 3 is defined as PPI/CPI taken from the IFS data base. Annual frequency from 1970 - 2000.

3) Data to construct the Solow residual are taken from the Groningen Growth and Development Centre, 60-Industry Database. This data is annual from 1979 to 2002. We construct the industry specific Solow residuals by taking a linear detrended of $\ln A_t^i = \ln y_t^i - \alpha \ln n_t^i$ where i denotes the sectors. y_t^i value added in sector i, n_t^i is hours worked in sector i and $\alpha = 0.67$ as in the calibration of the model. Further details on which sectors we classed as traded and which as non-traded are available from the authors' by request.

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