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The Stochastic Concept of Economic Equilibrium: A Radical Alternative[1]

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1 Introduction

My aim in this article is to present the gist of some ideas first proposed in [2]: that the main economic quantities – such as the unit price of any type of commodity, and the rate of profit – at any time should be modelled not as determinate numerical magnitudes but as random variables; and that at equilibrium these quantities have characteristic *distributions* rather than determinate numerical values. I add some methodological remarks about mathematical models and about what economics can borrow from physics.

I am not an economist but a mathematician, whose knowledge of academic mainstream economics is quite patchy. I made my first acquaintance with it in the early 1960s, when I was assigned the task of teaching some courses of mathematics to students of economics. In order to find examples of applications and problems that would be close to the interests of these students, I decided to have a look at some books on mathematical economics.

My attention was attracted by a recent book [5], because it was by a well-known mathematician, Jacob T Schwartz, co-author (with Nelson Dunford) of an important monograph on linear operators. I found it very interesting and instructive. In particular, I was fascinated by Part A of the book, entitled 'The Leontief Model and the Technological Basis of Production'. I realized immediately that this was just the right framework for formalizing Marxian economics (with which I was familiar). Of course, this was no accident, as Leontief's input–output matrix formalism was a direct descendant of Marx's 'schemes of reproduction'.[1]

I played around with this formalism in order to analyse the so-called *transformation problem* of Marxian economics (of which more anon), unaware that I was duplicating other people's work. I didn't get very far, and let the matter rest.

[1] I found out later that before emigrating from the USSR in 1931, Wassily Leontief had worked in GOSPLAN, the Soviet Economic Planning Board, which used Marxian economic theory. Leontief's work, for which he was awarded the Nobel Prize in 1973, was one of the channels through which Marxian theory exercised its – largely unrecognized – influence on mainstream economics.

In the late 1970s, my interest was rekindled by my friend Emmanuel Farjoun. He got involved in the controversy between Sraffians and Marxists that flared up following some publications by the former (see, in particular, Steedman's book [6]). The Sraffians showed that the transformation problem – as commonly understood – is not solvable. Hence they concluded that Marx's labour theory of value is wrong and worthless. The Marxists sprang to the defence of this theory. (Some Marxist responses are collected in [4].)

At the heart of this controversy was a notion of equilibrium which was shared by both sides.

1.1 The equilibrium price–profit equation

Let me outline the simplest form of the linear equilibrium model of prices and profit in the Leontief formalism.

We consider a (closed) capitalist economy, in which n types of commodity (excluding labour power), say $C_1, \ldots, C_n$, are produced. In this simple model it is assumed that each type of commodity is produced by a unique technical process, and there are no by-products.[2] More importantly, it is assumed that each type of commodity has a determinate equilibrium unit price;[3] and that all types of production yield the same equilibrium rate of profit.

If p^i is the price of one unit of C_i and ρ is the rate of profit, then

$$p^i = \sum_{j=1}^{n} (a^i_j + \rho k^i_j) p^j, \quad i = 1, \ldots, n. \tag{1}$$

Here a^i_j is the amount of C_j consumed (ie, used up) as input in the production of one unit of C_i; and k^i_j is the amount of C_j employed (ie, used but not necessarily used up) in the production of one unit of C_i multiplied by the number of units of time during which it is so employed.[4,5]

[2]These two assumptions can be, and indeed have been, challenged as unrealistic. But this issue is irrelevant to the present discussion.

[3]These prices are determined only up to an arbitrary factor of proportionality; thus, it makes no difference if all unit prices are multiplied by the same positive number. Alternatively, we can put, arbitrarily, $p^1 = 1$, and then all unit prices are completely determined.

[4]Thus, if time is measured in years and x units of C_j are employed during a year in producing a total of y units of C_i, then $k^i_j = x/y$.

[5]Note that labour does not occur in (1); this is because it has been eliminated from the accounting by incorporating into the a^i_j the wage goods consumed by the workers who supply the labour power used in producing C_i. This elimination is possible due to the fact that the price of labour power does not contain any direct profit, but is just the total price of the wage goods consumed by the workers.

The system (1) of n equations can be written in matrix form:

$$\mathbf{p} = (\mathbf{A} + \rho\mathbf{K})\mathbf{p}, \tag{2}$$

where $\mathbf{p}$ is a column n-vector and $\mathbf{A}$ and $\mathbf{K}$ are $n \times n$ matrices with non-negative elements. The unknowns here are $\mathbf{p}$ and ρ. Since $\mathbf{p}$ is determined only up to proportionality,[6] the number of unknowns is n, the same as the number of equations. To make sense, a solution $\mathbf{p}$ and ρ should be positive.

Under very reasonable conditions,[7] there is indeed such a solution, and it is unique.

1.2 The uniformity assumption

Like all mathematical models, the one just described makes various assumptions that simplify reality. Here I would like to draw attention to a very fundamental conceptual assumption: at equilibrium, the unit prices of all types of commodity produced by the economy have (up to proportionality) determinate numerical values; and the rates of profit accrued by capitals in all productive units are equal.

Clearly, this assumption does not purport to describe a real-life state of affairs in a capitalist economy. Everyone knows that if you shop around you will find that the same type of commodity is sold at the same time by different sellers at a variety of unit prices; and rates of profit vary greatly both within industries and between them. So the equilibrium that the input–output model describes is an ideal one. However, it is implied that the real economy is driven by market forces, the forces of competition, towards an ideal equilibrium of this sort, and are only prevented from actually reaching it by various *disequilibrating* forces, that act as 'noise'.

Schwartz [5, p. 9] provides the following justification for the assumption regarding the rate of profit:

> We have here taken an *essential* step in assuming the rate of profit, ρ, to be the same for all types of production. This corresponds to the *ordinary assumption*, in the theory of prices, of "free competition"; it can be justified *in the usual way* by arguing that a situation in which the production of different commodities

[6] See footnote 3.

[7] The economic meaning of these conditions is that the economy is capable of producing a physical surplus; and that it cannot be partitioned into two or more closed sub-economies. For details, see [5, Lecture 2]. The mathematical tool used here is the Frobenius theory of the eigenvalues and eigenvectors of non-negative matrices.

> yields different rates of profit cannot be stable, since investments would be made only in the industry yielding the highest rate of profit to the exclusion of other commodities yielding lower rates of profit. Long-term equilibrium, of which our simple theory is alone descriptive, would be reached only when all such rates of profit became equal. [My emphases]

Note that Schwartz refers to the uniformity assumption as ‘ordinary’ and to his justification of it as ‘the usual’ one. They are indeed both common and time-honoured: they go back at least to Adam Smith, and have been accepted and repeated (with some variations) by many authors of various schools, including Marx.[8]

Note also that Schwartz does not bother to justify the assumption that at equilibrium the unit prices are determinate, although a similar justification (in terms of competition) might surely be offered. Apparently he (like many others) regards this as more or less obvious.

1.3 The transformation problem

Marx had not one but two sets of ideal prices or price-like quantities. He starts off, as a first approximation, with the notion of *exchange value* (to which I shall refer here briefly as ‘value’). The value of a given commodity is, roughly speaking, the total amount of labour (measured, say, in labour hours) necessary to produce it, including not only the labour used directly in its production but also the labour used in producing its material inputs, as well as the inputs of these inputs, etc.[9]

But values cannot serve as equilibrium prices in an (ideal!) state of affairs in which the rate of profit is uniform. This is because if commodities were to exchange at their values, then a commodity whose production process has higher ‘organic composition’ (roughly speaking, smaller labour intensity)

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[8]See [2, pp. 14–16] for quotes from Smith and Marx, as well as from a later article by Schwartz.

[9]In the formalism of Subsection 1.1, let v^i be the value of one unit of C_i, and let l^i be the amount of labour used *directly* in producing it. Further, let c^i_j be defined like the a^i_j of (1), except that they *do not* incorporate the wage goods consumed by the workers who supply the labour power used directly in producing C_i (cf. footnote 5). Then the v^i are the unique solution of the system of n linear equations:

$$v^i = l^i + \sum_{j=1}^{n} c^i_j v^j, \quad i = 1, \ldots, n.$$

would yield a lower rate of profit than a commodity produced by a process with lower organic composition (greater labour intensity).

Since Marx – following the classical economists, especially Adam Smith and David Ricardo – subscribed to the uniformity assumption, he introduced what he called 'prices of production', which are ideal equilibrium prices corresponding quite closely to the p^i of Subsection 1.1.[10]

Marx's price–profit system of equations, connecting the ideal equilibrium rate of profit and prices of production was very similar to the Leontief equation (2), of which it was in fact a direct ancestor.[11] However, Marx postulated an additional constraint: the equilibrium ideal rate of profit must, according to him, equal the *average* rate of profit that would obtain if all commodities were priced at their values. Thus, if S is the total value of the surplus produced during a unit of time,[12] and K is the total value of the goods employed (but not necessarily used up) in production during this period, then the ideal rate of profit ought to be

$$\rho_{\mathrm{M}} := \frac{S}{K}. \tag{3}$$

Through this link, values are 'transformed' into prices of production.

But Marx was unable to solve [his rudimentary form of] (2) with the added constraint $\rho = \rho_{\mathrm{M}}$. In fact, we now know that this so-called 'transformation problem' is in general not solvable – at least not in the sense in which it has commonly been understood. As pointed out by the Sraffians, the unique ρ solving (2) need not in general equal ρ_{M} as defined by (3): there are reasonable counter-examples, cases of the model outlined in Subsection 1.1, whose solution fails to satisfy $\rho = \rho_{\mathrm{M}}$.

Emmanuel Farjoun, who got involved in the controversy around the transformation problem (and who contributed to [4]), eventually came up with a radical idea: the point was not whether the Sraffians were right or wrong about the Leontief model, or whether the model could be tweaked in some way so as to satisfy Marx's postulate; rather, it was whether that model – or indeed anything like it – was a reasonable way of theorizing equilibrium. An analogy with statistical mechanics suggested very strongly that it was not: the uniformity assumption (which both sides in the transformation

[10] Both values and prices of production must be distinguished from what Marx called 'market prices': these are real-life prices, whose relationship to the prices of production is as described in Subsection 1.2.

[11] Cf. footnote 1.

[12] This is obtained from the value of the whole product by deducting from it the value of the non-labour inputs consumed in its production and the value of the goods consumed by the workers engaged in this production.

controversy subscribed to) was all wrong. We elaborated this idea together in [2].

2 Stochastic equilibrium

The main aim of [2] was to amend and reconstruct the Marxian labour theory of value, preserving what we regard as its invaluable core, while ditching the concept of prices of production (which we regard as unnecessary and mistaken) and avoiding altogether the transformation problem as commonly understood (which we regard as a non-problem).

But the book's basic methodological message – which is my present topic here – is much more general. It concerns the notion of economic equilibrium used in several economic theories of various school. In this connection, Marx's theory and the Leontief model discussed in the Introduction serve as a mere illustration.

The methodological message is, briefly, that the concept of equilibrium in which unit prices and 'the' rate of profit are determinate quantities is fundamentally erroneous. This is suggested by analogy with statistical mechanics, a theory underlying thermodynamics, founded by Boltzmann and Maxwell in the 1870s. This analogy is explained in some detail in [2], and here I shall only highlight a few points.

2.1 Equilibrium in statistical mechanics

In a volume of gas enclosed in a closed container, the molecules are in constant motion (the total kinetic energy of this motion is what constitutes the heat energy stored in the gas). In this motion, the molecules collide with one another and as a result the more energetic molecules tend to slow down (by imparting some energy to slower molecules with which they collide); conversely, the less energetic molecules tend to speed up.

However, this does not mean that at equilibrium all molecules reach an equal level of kinetic energy.[13] The point is not merely that such a state of uniformity does not actually occur; but that if it ever did it could not last for a split second. If it were miraculously brought about through the mechanism of incessant collision, then *this very same mechanism* would instantly disturb it. Note also the use of the word 'tend' in the description of this mechanism:

[13]This false assumption – analogous to the uniformity assumption discussed in Subsection 1.2 – was actually made by J P Joule before the advent of statistical mechanics. This is discussed by us in [3].

what is meant by it is that a more energetic molecule is *more likely* to slow down than to speed up, not that it will *always* do so.

Another important feature of this physical system is the distinction between macro-state and micro-state. A macro-state can be described by some global data, such as the total energy of the gas, its volume, as well as some statistical data that will be mentioned below. A micro-state is described by an enormous number of data: the position of each molecule (given by its three space coordinates) and its momentum (given by three independent numerical components). The number of these data is called the *number of degrees of freedom* of the system. And the system in question has a great many. Thus, to each macro-state there corresponds a very large set of micro-states.

The notion of equilibrium of such a system refers to the macro level. It does not imply that all the molecules are motionless – they never can be, for this would happen only at 0°K, which is unattainable – but that the macro-state remains unchanged unless perturbed by external forces.

Among the data characterizing a macro-state are the statistical distributions of the individual molecules' energies, speeds and positions. In particular, an equilibrium macro-state is characterized by specific distributions, which are supposed to stay unchanged in the absence of external perturbation.[14]

Even such a macro-equilibrium does not exist in actual reality, because no system can in practice be perfectly isolated from external interactions. Nevertheless, fairly close approximations to it – good enough for practical and even for many theoretical purposes – do actually exist: this is what insulation is all about.

2.2 Economic equilibrium

The analogy between the kind of system just described and an economy need hardly be spelt out.

The fundamental error of the uniformity assumption is that it conceptualizes the market forces driving the economy towards an ideal equilibrium as endogenous, while the disequilibrating forces perturbing the system are conceptualized as exogenous. But the latter surely include also market forces, the forces of competition. This is an untenable logical inconsistency.

The trouble with the kind of ideal equilibrium assumed in Subsection 1.1 is not that it is purely ideal but that it is prevented from occurring by the

[14]Note that this is quite another matter from the micro-state staying unchanged. Similarly, if the age distribution of a population is unchanged, it does not follow that each member of it remains of constant age.

very forces that are supposed to drive the economy towards it.

The distinction between macro-state and micro-state is surely valid in economics no less than in statistical mechanics. And if a model of a capitalist economy is to have any verisimilitude, it must possess a large number of degrees of freedom: a micro-description must include detailed data on the simultaneous states of a great multitude of agents and the transactions between them.

The concept of economic equilibrium surely makes sense only as a macro concept, which is compatible – indeed presupposes – great mobility at the micro level. Basic economic quantities such as the rate of profit of an enterprise and unit prices must be conceptualized as random variables, which at equilibrium have specific distributions rather than determinate values.

Such a representation is needed even if in the end one is only interested in relations between the equilibrium mean values of these quantities. These mean values cannot, generally speaking, be taken in advance as reasonable approximations for the random variables themselves. The reason for this is that a given functional relation that holds between random variables does not, in general, hold between their means.[15]

Finally, simple observation suggests that a real economy of a country is most of the time not all that far from what common sense would regard as macro-equilibrium. Macro quantities, such as annual GNP, the level of employment, or the statistical distribution of incomes, do not change very rapidly except at some critical moments, when instability becomes evident. It is therefore unreasonable to use a theoretical concept of equilibrium that is not even approached, let alone attained, by a real economy.

3 Methodological comments

Let me end with a couple of methodological comments.

First, a mathematical model of a complex real system need not – indeed cannot – be realistic in every way. It is not a duplicate of reality but a simplified simulacrum of it. But the vital question is what simplifications are acceptable.

A model can work very well in simulating real behaviour even if it makes quite drastic simplifications of some aspects of reality. But it must fail if it simplifies away essential aspects of the reality under investigation. Of course, what counts as an essential aspect depends on the specific phenomena being investigated and on the purpose of the investigation.

[15] For example, if X, Y and Z are random variables such that $\mathsf{XY} = \mathsf{Z}$, it is not in general true that $\mathrm{E}\,\mathsf{X} \cdot \mathrm{E}\,\mathsf{Y} = \mathrm{E}\,\mathsf{Z}$, where 'E' denote the mean operator.

The thesis that I have argued in this paper is that a non-stochastic concept of equilibrium is inappropriate for modelling the behaviour of prices and profits in a capitalist economy.

On the other hand, Ian Wright's paper [7] illustrates how a model that is in many ways very simplistic can nevertheless display in a surprisingly realistic way various phenomena of a capitalist economy. Wright's model does not impose a deterministic concept of micro-equilibrium; rather, the macro-equilibrium that emerges from it is stochastic.[16]

Second, it is quite fruitful for economic theory to look for concepts it can usefully borrow from natural science, particularly physics. Of course, by no means all physical concepts can be borrowed, or have useful analogues in economics.

Some specifically physical concepts – such as mass, force field, three-dimensional physical space, four-dimensional space-time, mechanical micro-equilibrium, and perhaps even energy (in the sense in which it features in the law of preservation of energy) – need not have useful applications or analogues in economic theory. On the other hand, more general concepts that originated in physics and engineering – such as feedback, degrees of freedom, steady state, macro-equilibrium, and perhaps entropy – seem to be quite usefully applicable.

[16] I am grateful to Ian Wright for the following comment on a draft of the present paper: 'You may want to note that Econophysics is a relatively new field that has extensively used statistical equilibrium concepts with some success (particularly in providing very simple, abstract and satisfying explanations of the detailed income distribution). Your book [2] presaged this development by some years.'

Wright's work is presented and amplified in a forthcoming book by Cottrell et al. [1].

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