Inference on epidemic models with time-varying parameters: methodology and preliminary applications
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1 Why have time-varying parameters?

Inference on epidemic models is an active topic of research. Motivations are multiple: exploring mechanisms, testing theories, monitoring control interventions, surveilling upcoming epidemics.

A very typical compartmental model would be:

\[
\begin{align*}
S(t) &\quad \text{Susceptibles} \\
I(t) &\quad \text{Infectives} \\
R(t) &\quad \text{Resistant}
\end{align*}
\]

\[
\beta = \text{transmission rate} \\
\gamma = \text{recovery rate}
\]

With the following definitions:

\( \mathcal{S} \): proportion of the population that is susceptible, that can be infected

\( \mathcal{I} \): proportion of the population that is infective

\( \mathcal{R} \): proportion of the population that is resistant, that is not infective anymore and cannot be infected again

\( \beta \): transmission rate

\( \gamma \): recovery rate

Usually, inference is made for constant values of \( \beta \) and \( \gamma \). However, there are many reasons for \( \beta \) to be time-varying:

- Climate forcing is likely to have an impact on immunity and virus transmission
- Contact patterns evolve according to holidays, school/week periods, seasonal migrations
- Individual awareness to an epidemic can spontaneously decrease, or at the contrary increase under the influence of preventive measures
- etc...

2 How to model these time variations?

Fully parametric models for time-varying transmission rates have been explored:

- sinusoidal
- low-dimensional polynomials, splines...
- tractable inference with classic MCMC algorithms

"Semi-parametric" models, on the other hand, have been used:

- random walk diffusion (Cazelles and Chau 1997, Mathematical Biosciences)
- very flexible model

Inference implied gaussian approximations (Extended Kalman Filter)

Our proposition

- use a diffusion process for \( \beta \)’s trajectory, typically a geometric Brownian motion to preserve positivity
- apply novel MCMC algorithms to solve the inference problem, with less-informative priors on the diffusion coefficients

Classic SIR model:

\[
\begin{align*}
\frac{dS}{dt} &= -\beta S I \\
\frac{dI}{dt} &= (\beta S - \gamma I) dt \\
\frac{dR}{dt} &= \gamma I dt
\end{align*}
\]

Time-varying \( \beta \) model

\[
\begin{align*}
\frac{dS}{dt} &= -\beta S I dt \\
\frac{dI}{dt} &= (\beta S - \gamma I) dt \\
\frac{dR}{dt} &= \gamma I dt \\
\frac{d\log \beta}{dt} &= \sigma_d dB_t \\
\end{align*}
\]

Going further...

- Try other diffusion processes (Ornstein-Uhlenbeck processes, integrated random walks...)
- Chose model from expert knowledge and/or indicators as the Bayes factor and the DIC

3 A challenging inference problem

Objective

We want, under the following notations,

\( \mathcal{X}_t \): dynamic vector of compartments populations

\( \theta \): static parameters

\( \beta \): dynamic parameters

\( y(t) \): observation process model

\n \): number of observations \( (y_1, \ldots, y_n) \)

\N : number of particles

...to explore the posterior density \( p(\mathcal{X}_t, \beta, t \in [0, T], \theta | y_n) \).

Difficulties:

- it is a high-dimensional density
- the posterior density and the Kolmogorov forward equation are intractable

Estimating time-varying parameters with a Particle MCMC algorithm

\( f(y(t)) = \text{likelihoods for } y(t) \) from \( \mathcal{Q}(\theta) \)

\( \theta^{(i)} \) from \( \mathcal{Q}(\theta) \) for \( i = 1 \) to \( N \) do

Sample \( \mathcal{X}_{(t+1)}^{(i)}, \beta^{(i)} | \mathcal{X}_{(t)}^{(i)} \), \( y(t), \theta^{(i)} \)

Noting \( Y_{(t+1)}^{(i)} = h(\mathcal{X}_{(t+1)}^{(i)} | t \in [0, T_{(t+1)})] \)

\( \omega^{(i)} = \frac{g(Y_{(t+1)}^{(i)}, | y_{t+1}^{(i)})}{g(Y_{(t+1)}^{(i)}, | y_{t+1}^{(i)})} \)

end for

Estimating time-varying parameters with a Particle MCMC algorithm

4 Preliminary application: surveilling Influenza outbreaks from Google’s FluTrend data

Google FluTrend Data:

Estimates of Influenza-Like Illnesses cases

(Ginsberg et al. 2008, Nature)

A simple model for Influenza:

\[
\begin{align*}
\frac{dS}{dt} &= -\beta S I dt \\
\frac{dI}{dt} &= (\beta S - \gamma I) dt \\
\frac{dR}{dt} &= \gamma I dt \\
\frac{d\log \beta}{dt} &= \sigma_d dB_t \\
\end{align*}
\]

Note: \( E \) is the group of individuals who were infected but are not infectious yet. \( k^{-1} \) is the referred to as the latency period.

Informative priors were taken for \( k \) and \( \gamma \) based on bibliography.

Questions:

- How transmittable is the upcoming strain of influenza?
- Does the effective reproduction rate \( R_t = \frac{dS}{dI} \) vary along time?
- What is the population immunity to the upcoming strain of influenza?

a) Validating the algorithm on simulated data

b) 2008-2009 epidemic in France, a "classic" seasonal epidemic

c) 2009-2010 epidemic in France, the H1N1 pandemic