COSTLY COASIAN CONTRACTS*

Luca Anderlini
St John's College, Cambridge

and

Leonardo Felli
London School of Economics and Political Science

Paper presented at the STICERD 20th Anniversary Workshop in June 1998 on
Public Policy and the Theory of Institutions

Contents:

Discussion Paper
No. TE/98/362
October 1998

Theoretical Economics Workshop
The Suntory Centre
Suntory and Toyota International Centres
for Economics and Related Disciplines
London School of Economics and Political Science
Houghton Street
London WC2A 2AE
Tel.: 0171-405 7686

* This is a revised version of a working paper entitled 'Costly Contingent Contracts' (Anderlini and Felli, 1996). We are grateful to Sandeep Baliga, Jack Beatson, Tim Besley, Patrick Bolton, Jan Boone, Luis Cabral, Jacques Crϑmer, Thomas de Garidel, Antoine Faure-Grimaud, Oliver Hart, FranΗois Ortalo-Magnϑ, Stephen Matthews, John Moore, Klaus Schmidt, Emanuela Sciubba, Tomas Sj str m, Kevin Roberts, Ariel Rubinstein and Yoram Weiss for useful suggestions and comments. We thank seminar participants at a number of U.K., U.S. , European and Israeli institutions for their valuable feedback. The full list of institution names being too long to fit in this footnote. Financial support from St John's College, Cambridge, and the Suntory and Toyota International Centres for Economics and Related Disciplines at LSE is gratefully acknowledged. We are solely responsible for any remaining errors.

Abstract

We identify and investigate the basic ‘hold-up problem’ which arises whenever each party to a contract has to pay some *ex-ante cost* for the contract to become feasible. We then proceed to show that, under plausible circumstances, a ‘contractual solution’ to this hold-up problem is not available. This is because a contractual solution to the hold-up problem typically entails writing a ‘contract over a contract’ which generates a fresh set of *ex-ante costs*, and hence is associated with a new hold-up problem.

Keywords: Ex-ante contractual costs; hold-up problem; Coase theorem; contracts over contracts; incomplete contracts.

JEL Nos.: C70, D23, D60, D80

1. Introduction

1.1. Motivation

The Coase theorem (Coase 1960) has had a pervasive influence on the way economists think about situations in which there are *un-exploited gains from trade.* It tells us that, provided *property rights* are allocated, the potential parties to a bargain will draw-up an agreement which exhausts their potential mutual gains, and hence an efficient outcome is guaranteed.

In its strongest formulation, the Coase theorem is interpreted as guaranteeing an efficient outcome regardless of the "way in which property rights are assigned" (Nicholson (1989, p.725)) and whenever the potential mutual gains "exceed [the] necessary bargaining costs" (Nicholson (1989, p.726)).[1]

The predictions entailed by the stronger version of the Coase theorem are startling. In an advanced capitalist economy, whenever property rights are allocated, we should observe only outcomes which are *constrained efficient* in the sense that all potential gains from trade (net of 'bargaining' or 'contractual' costs) are exploited. This clearly contradicts even the most casual observation of empirical facts. Business interactions are often regulated by contracts which do not exploit all possible gains from trade. They are constantly re-negotiated, brought to court for trial, defaulted upon, or simply not written in the first place.

If we were to believe the predictions of the 'strong' Coase theorem, all these apparent inefficiencies would not be real inefficiencies at all. They should simply be viewed as the result of 'contractual costs' which are 'high' relative to the potential gains from trade. We take the view that this is not a satisfactory explanation of these observed facts.

Our aim in this paper is to seek an explanation of the impact of contractual costs over and above their size relative to the potential gains form trade. This stems from the *strategic role* which contractual costs might play in the emergence of contractual agreements. It turns out that a key factor in determining the strategic role of contractual costs is whether they are payable *before* or *after* the bargaining over the potential

[1] This stronger version of the Coase theorem does *not* correspond to what is claimed in Coase (1960), but it is an interpretation of it which is sufficiently common to have found its way into basic microeconomic text-books such as the one quoted above.

surplus takes place. Our analysis below highlights the impact of *ex-ante* contractual costs.

The primary effect of *ex-ante* contractual costs is that they may generate an inefficient outcome stemming from a contract which is *incomplete* in a well defined sense. In the most basic version of our model the agents may end up not exploiting any of the potential gains from trade, and hence writing no contract at all. When the choice of ex-ante costs is 'gradual', higher costs paid correspond to a more 'detailed' discipline of the agreement that allows the contracting parties to exploit the potential gains from trade and hence to higher surplus from the contract. In this case the agents will, in general, end up leaving part of the potential gains from trade unexploited. In other words, they will choose a contract which is less detailed than would be optimal, after the ex-ante costs are taken into account. In its simplest form, the strategic effect which drives our results below is not hard to outline.

Consider any 'Coasian' contractual situation with the following features.[2] Two agents contemplate entering a contract which yields a surplus of an arbitrary given size. Moreover, the two agents' shares of the surplus generated by the contract are exogenously given, say because the extensive form which they must use to negotiate the contract is itself exogenously given.

Suppose now that there are ex-ante costs associated with the contract which the agents contemplate drawing-up. In particular, suppose that the agents must each pay some cost before the contract-negotiating phase begins. Then, if the *distribution* of ex-ante costs is such that one (or both) agents will not be able to recoup the ex-ante cost given the distribution of surplus, the contract will not be drawn-up. This is possible even when the total of ex-ante costs across the two agents is *less* than the surplus which the contract generates so that it would be *socially efficient* for the agents to pay the ex-ante costs and draw-up the contract.

We model this situation both taking the distribution of surplus as exogenously

[2]By this we mean a situation in which the property rights of the agents are sufficiently well defined to allow them to enter into a negotiating phase, and that there are some un-exploited gains from trade. This obviously covers an extremely wide variety of possible situations, ranging from text-book like externalities to complex contingent contracts. In a previous version of this paper (Anderlini and Felli 1996), we concentrated on the case of two agents who negotiate a simple contingent contract — namely a *risk-sharing* agreement. All the results which we report here apply to such situation.

given, and considering a variety of extensive forms in which the agents are allowed to bargain over the distribution of surplus, provided of course that the ex-ante costs have been paid. We find that the problem we have described is 'pervasive' in the sense that in a whole variety of extensive forms, the agents will not draw-up a contract even though it would be socially efficient to do so.

What we have just described is a version of a source of inefficiencies well known in Contract Theory as the 'hold-up problem' (Grout 1984, Grossman and Hart 1986, Hart and Moore 1988, Hart and Moore 1990, among many others). The problem is particularly acute in our setting since it may be impossible for the contracting parties to find a 'contractual solution' to this hold-up problem for the following reasons.[3]

Imagine that the two agents in the contractual setting we have described attempt to resolve the inefficiency in the following (Coasian) way. Before the ex-ante costs are paid, they negotiate a transfer of money which will compensate the agent who is unable to recoup the ex-ante cost for his loss, of course contingent on his paying the ex-ante cost. Provided the sum of ex-ante costs does not exceed the surplus generated by the contract, such transfer can always be arranged so that both agents now benefit from paying the ex-ante costs and entering the contract. However, the problem which arises now is that the contingent compensating transfer can itself be viewed as a contract, which will involve a new set of ex-ante costs.

Suppose that the 'second tier' contract we have described does indeed involve a new set of strictly positive ex-ante costs. Suppose moreover that the second tier contract and ex-ante costs must be paid for and negotiated *before* the first order costs and contract are paid for and negotiated respectively.[4]

Then it is possible to show that the ex-ante costs associated with the second tier contract may not be paid. In particular there always exists an equilibrium such that the contingent compensating transfer is not negotiated and therefore does not take place. Hence, the ex-ante costs associated with the first tier contract will, in turn, not be paid, and the actual surplus-generating contract will not be drawn-up. This

[3]Chung (1991), Rubinstein and Wolinsky (1992), Rogerson (1992), Aghion, Dewatripont, and Rey (1994) and Nöldeke and Schmidt (1995), among others, analyse models of contractual solutions to the hold-up problem.

[4]This is obviously an *assumption* as such. However, we believe it to be plausible in a wide variety of cases.

outcome is of particular interest since it is the only one which survives when we restrict attention to 'renegotiation-proof' equilibria. In other words if, in a Coasian spirit, we insist that the outcome of negotiation at every stage must yield a (constrained) efficient outcome, the overall outcome is not efficient and therefore not Coasian at all.

As we show in Subsection 4.2, a whole hierarchy of 'higher order' contracts may not resolve the hold-up problem either.

The hold-up problem described above is less pervasive in an environment in which each party to the contract has an incentive to bestow a higher share of the surplus to his contracting partner for reasons that may have nothing to do with the partner's payment of the ex-ante contractual costs. In Section 7 below we consider one of these reasons. If the size of the surplus shared by the two parties depends on the distribution dictated by the contract, then each party may be willing to leave his counterpart more than his outside option in the attempt to increase the overall size of the surplus. This may result in a smaller range of parameter values for which the hold-up problem described above occurs.

1.2. Related Literature

Coase (1960) already suggested that the Coase Theorem might not hold in the presence of transaction costs. Here, we go further by identifying the crucial strategic role played by ex-ante contractual costs (as opposed, for instance, to those contractual costs which are payable ex-post) which may lead to an outcome that is *constrained inefficient*. In the models which we analyze below, such inefficient outcomes can be interpreted as contracts which are incomplete in a well defined sense. The only other paper we are aware of in which contractual costs play a strategic role is Lipman (1997), although the model he analyses is quite radically different from ours. Dixit and Olson (1997) analize a pure discrete public good problem in which agents must decide (costlessly) ex-ante whether to participate or not. They find both efficient and inefficient equilibria, and then proceed to highlight the inefficiency of the symmetric (mixed-strategy) equilibria of their model.

Starting from Williamson (1985) and Grossman and Hart (1986) a number of papers have focused on the effects of contract incompleteness. Most of these papers assume that contracts are incomplete and concentrate on the role of available mechanisms, and in particular institutions, to mitigate the inefficiencies generated by contract incompleteness. Some of the mechanisms considered are: vertical and lateral integration (Grossman and Hart 1986), the optimal allocation of ownership rights on physical capital (Hart and Moore 1990), and the delegation of authority within organizations (Aghion and Tirole 1997). We differ from these papers since we do not assume contractual incompleteness but rather derive it *endogenously* from the ex-ante costs associated with a contract.

A second strand of literature has concentrated on some of the possible causes of contractual incompleteness. Hart and Moore (1988) have asked whether contractual incompleteness might be due to the fact that the outcome that the parties want to implement may be, at least in part, un-observable to the enforcing agency (the court). They conclude that this un-observability might lead the parties to write a contract that will leave out some details that the court cannot observe. This will result in a basic 'hold-up problem'. Each party's final allocation of resources will be determined on the basis of an ex-post re-negotiation of the contract that cannot be specified, at least not fully, by means of the ex-ante contract. A number of subsequent papers have explored whether this basic hold-up problem might have a contractual solution. Chung (1991), Rubinstein and Wolinsky (1992), Rogerson (1992) Aghion, Dewatripont, and Rey (1994) Nöldeke and Schmidt (1995) analyse possible contractual solutions to the hold-up problem under a variety of different assumptions about the nature of informational asymmetries. By contrast, Maskin and Tirole (1997) show that it is *always* possible to devise an ex-ante contract (a mechanism) that implements the same outcome that would be implemented in the absence of these un-observabilities. This is achieved by asking the contracting parties — once the conditioning event has been realized — to report the event or, equivalently, to report the utility levels or payoffs accruing to each party, while truthful revelation is ensured through appropriate incentives.

The basic hold-up problem we are concerned with here is of the same variety as the problem identified by Holmström (1982) and analyzed in the context of the incomplete contracts literature by Hart and Moore (1988). However, causality is essentially reversed in this paper. In Hart and Moore (1988) the hold-up problem is induced by contractual incompleteness. Instead, in this paper it is the hold-up problem in the negotiation of a contract that yields contractual incompleteness. We also differ from the previous literature since we argue that in our setting a contractual solution to the hold-up problem is unlikely to be available for the reasons we outlined above.

Some recent papers have addressed the impact that *complexity* considerations may have on the form and shape of optimal contracts (Dye 1985, Segal 1995, Anderlini and Felli 1997, among others). Dye (1985) considers a model in which the contracting parties face fixed cost of 'adding more contingencies' to a contract. Segal (1995) focuses on a contracting problem in which the relevant event is not observable to the enforcing agency (the court). In such an environment he analyzes the parties' welfare gains from using ex-ante message-contingent mechanisms as in Maskin and Tirole (1997). He shows that such gains become negligible as the number of events on which the contract is contingent increases without bound. The implication is that, in a 'complex' environment, message-contingent mechanisms will not be used, even if they entail an arbitrarily small complexity cost. Aside from the basic differences in the formulation of the models, our analysis differs from Segal (1995) and Dye (1985) since the incompleteness they derive is *constrained efficient*; a central planner facing the same complex environment would choose the same incomplete contract. In our setting, the ex-ante contractual costs play a strategic role, and the incomplete contracts we derive are *constrained inefficient*.

In Anderlini and Felli (1997) we model a contract as an algorithmic procedure (a Turing Machine) and we explore whether any complexity measure in a general class associated with such a contract might induce the contracting parties to choose an incomplete contract. We conclude that this is indeed the case. However, in Anderlini and Felli (1997), while we model explicitly the complexity costs associated to a contract, we also assume away any strategic role they might play. As a consequence, we find that the impact of the complexity costs is directly related to their size, relative to the size of the surplus which the contract itself generates. This is not the case in the models we analyse in this paper.

Finally, starting with Rubinstein (1986) a whole strand of literature (Abreu and Rubinstein 1988, Piccione 1992, Piccione and Rubinstein 1993, among many others) has concentrated on the impact of complexity costs on the equilibrium set of a repeated game. Although we are concerned with a radically different environment, in our view, one similarity can be drawn between the two settings. In the repeated games literature, the impact of complexity costs is attributable to their *strategic* role. Similarly, in the models we analyse below the impact of ex-ante contractual costs (complexity or otherwise) is directly related to the strategic role they play.

1.3. Overview

We begin with an informal discussion of the possible interpretations of the ex-ante contractual costs in Section 2. In Section 3 we present the simplest possible model of the basic hold-up problem associated with the writing of a surplus-generating contract. This problem is analysed in the case in which the ex-ante costs associated with the contract are discrete and are either complements or substitutes. In Section 4 we address the question of whether a contractual solution to our basic hold-up problem is plausible. We do this by analysing the possibility of a 'contract over a contract' and a whole hierarchy of 'contracts over contracts'. We then turn (Section 5) to the analysis of a simple model in which the agents' choice of ex-ante costs is continuous. Section 6 characterizes the hold-up problem described in Section 3 in the case in which the distribution of surplus can be negotiated by the agents in a variety of extensive forms. In Section 7 we present a model in which the size of the surplus available to the parties depends on its distribution. Section 8 offers some concluding remarks. To ease the exposition, we have relegated all proofs to the Appendix.

2. Ex-Ante Contractual Costs

We are concerned with contractual situations in which the parties have to incur some costs ex-ante, before they reach the stage in which the actual contract is negotiated.

The interpretation of these ex-ante contractual costs which we favour is that of time spent 'preparing' for the negotiation of the contract. Typically, a variety of tasks need to be carried out by the contracting parties before the actual negotiation begins.

In those cases in which a contract contingent on a state of nature is concerned, both parties need to conceive of, and agree upon, a suitable contractual language to describe precisely the possible realizations of the state of nature. The contracting parties also need to collect and analyse information about the 'legal environment' in which the contract will be embedded. For instance, in different countries the same contract will need to be drawn-up differently to make it legally binding (enforceable).

In virtually all contractual settings the parties need to spend time arranging a way to 'meet', and they need to 'earmark' some of their time schedules for the actual meeting.

In many cases, before a meaningful negotiation can start, the parties will need to collect and analyse background information which may be relevant to their understanding of the actual trading opportunities. These activities may range from collecting information about (for instance the credit-worthiness of) the other party, to actual 'thinking' or 'complexity' costs incurred to understand the contractual problem. We view this type of ex-ante contractual costs as both relevant and important for the type of effects which we identify in our analysis below. However, it should be emphasized that our model does not *directly* apply to this type of costs. This is because in our model the size of the surplus is fixed and known to the parties. On the other hand, the lack of information and/or understanding of the contractual setting which we have just described, would clearly make the size of the surplus uncertain for the contracting parties. We have not considered the case of uncertain surplus for reasons of space and analytical convenience. However, we conjecture that the general 'flavour' of our results generalizes to this case.

We conclude this section with an observation. In many cases the parties to a contract will have the opportunity to delegate to outsiders many of the tasks which we have mentioned as sources of ex-ante contractual costs. The most common example of this is the hiring of lawyers. In these cases, the time costs which we have just discussed will be *monetized* at an appropriate rate. Abstracting from agency problems (between the contracting party/principal and the lawyer/agent), which are likely to increase the ex-ante costs anyway, our analysis applies, unchanged, to the case in which the ex-ante contractual costs are payable in money.

3. A 'Reduced Form' Model with Discrete Ex-Ante Costs

3.1. Perfect Complements

Our model consists of two agents, called A and B, who face a 'Coasian' opportunity to realize some gains from trade. With little loss of generality we normalize the size of the surplus which is realized if an agreement is reached to be one and we set the parties' payoffs in the case disagreement to be equal to zero.[5]

In the simple model we analyse in this section, once the contract-negotiating phase is reached the division of the surplus between the two agents is exogenously given and cannot be changed by the agents. This should be thought of as the result of the agents having exogenously given bargaining power in the contract-negotiating phase (the *extensive form* of the bargaining game they play to divide the surplus is exogenously given), and/or the possibility that the *size* of the surplus may depend on its distribution across the agents as is the case in the model analyzed below in Section 7.

For the time being, we simply let λ in $[0, 1]$ be the share of the surplus which accrues to agent A if a contract is drawn-up, and $1 - \lambda$ the share of the surplus which accrues to B.

For the contract to become feasible, each agent has to pay a given *ex-ante cost*. In other words, the agents reach the contract-negotiating phase only if they both pay a certain amount before the negotiation of the contract begins.[6] These costs should be thought of as representing a combination of the activities necessary for a contract to become feasible which we discussed in some detail in Section 2 above.

Let $c_A > 0$ and $c_B > 0$ be the two agents' ex-ante costs. Clearly, if $c_A + c_B > 1$

[5]If the agreement is interpreted as a *risk-sharing* contract there is some loss of generality in working with a surplus of fixed size as we do here. In Anderlini and Felli (1996) we spell out in detail the assumptions which are needed to ensure that the model we use here can be though of as a model of a risk-sharing contract. In short, we would need to assume that the agents have constant absolute risk-aversion and that there is no *aggregate* risk.

[6]Notice therefore that we are implicitly assuming that the agents have some endowments of resources out of which the ex-ante costs can be paid. In Anderlini and Felli (1996) we model explicitly the agents's endowments from which the ex-ante costs are paid, and examine the bounds which they have to satisfy. This becomes inessential in the simplified model which we use in this version of the paper. For our purposes, it is enough to assume that the agents both start off with a unit endowment of resources which is available to them when the ex-ante costs are payable.

	pay c_B	not pay c_B
pay c_A	$\lambda - c_A,\ 1 - \lambda - c_B$	$-c_A,\ 0$
not pay c_A	$0,\ -c_B$	$0,\ 0$

Figure 1: Normal form of the two-stage game with ex-ante costs.

then the two agents will never draw-up a contract yielding the unit surplus, but then neither would a social planner since the total cost of the contract exceeds the surplus which it yields. We are interested in the case in which it would be socially efficient for the two agents to draw-up a contract. Our first assumption guarantees that this is the case.

ASSUMPTION 1: *The surplus which the contract yields exceeds the total ex-ante costs which are payable for the contract to become feasible. In other words* $c_A + c_B < 1$.

Our two agents play a two-stage game. In period $t = 0$ they both simultaneously and independently decide whether to pay their ex-ante cost. Only if both agents pay their ex-ante cost at $t = 0$, do they have the possibility of negotiating a contract yielding a surplus of size one at $t = 1$.[7] In this case the game at $t = 1$ is, for the time being, a 'black box' yielding payoffs of λ to A and $1 - \lambda$ to B. If one or both agents do not pay their ex-ante costs at $t = 0$, the game at $t = 1$ is trivial: the contract which yields the unit surplus is not feasible; the agents have no actions to take and they both receive a payoff of zero.

Throughout the paper, unless otherwise stated, by equilibrium we mean a *subgame perfect* equilibrium of the game at hand.

The normal form which corresponds to the two-stage game we have just described is depicted in Figure 1. From this it is immediate to derive our first two propositions, which therefore are stated without proof.

[7]Notice that we are therefore assuming that the two agents' ex-ante costs are *perfect complements* in the 'technology' which determines whether the surplus-generating contract is feasible or not. We examine the cases of perfect substitutes, and of strategic complements in Subsections 3.2 and 3.3 below respectively.

PROPOSITION 1: *Let a pair of ex-ante costs* $c_A > 0$ *and* $c_B > 0$ *satisfying Assumption 1 be given. Then there exists a range of values — namely* $\Lambda = [0, c_A) \cup (1 - c_B, 1]$ *— for the distribution parameter* λ *such that the only equilibrium of the two-stage game represented in Figure 1 has neither agent paying the ex-ante cost, and therefore yields the no-contract outcome.*

PROPOSITION 2: *Let any value of the distribution parameter* $\lambda \in [0, 1]$ *be given. Then there exists a set* $\mathcal{C} = \{c_A, c_B \mid \text{either } c_A > \lambda \text{ or } c_B > 1 - \lambda \text{ and } c_A + c_B < 1\}$ *of pairs of ex-ante costs which satisfy Assumption 1, and such that the only equilibrium of the two-stage game represented in Figure 1 has neither agent paying the ex-ante cost, and therefore yields the no-contract outcome.*

We view Propositions 1 and 2 together as implying that in the presence of ex-ante contractual costs, if the *distribution* of ex-ante costs across the parties is sufficiently 'mis-matched' with the given distribution of surplus, then the ex-ante costs will generate a version of the hold-up problem which will induce the agents not to draw-up a contract even though it would be socially efficient to do so. In this case the agents will end up in a situation which can be interpreted as an 'incomplete' contract in a very strong sense: no contract at all.

The intuition behind our results above is simple enough. If entering a contract involves some costs which are payable ex-ante, the share of the surplus accruing to each party will not depend, in equilibrium, on whether the ex-ante costs are paid. Therefore, the parties will pay the costs only if the distribution of the surplus generated by the contract will allow them to recoup the cost ex-post. If the distribution of surplus and that of ex-ante costs are sufficiently 'mis-matched', then one of the agents will not be able to recoup the ex-ante cost. In this case, a contract will not be drawn-up, even though it would generate a total surplus large enough to cover the ex-ante costs of both agents.

We conclude this subsection with three observations. First of all, the analogues of Propositions 1 and 2 hold when the model is modified so that it is enough that some portion of the ex-ante costs (c_A, c_B) has to be paid by each of the contracting parties for the contract to become feasible, while the rest of the total ex-ante costs can be paid by either agent.[8]

Secondly, the simultaneity in the payment of the ex-ante costs is not essential to Propositions 1 and 2. Both results apply to the case in which the ex-ante costs are payable sequentially by the two agents.

Thirdly, while the model has a unique equilibrium for the parameter configurations identified in Propositions 1 and 2, it has multiple equilibria whenever these two propositions do not apply. It is clear that, whenever both $\lambda > c_A$ and $(1-\lambda) > c_B$, the model has two equilibria. One in which the ex-ante costs are paid and a contract is drawn-up, and another in which neither agent pays the ex-ante costs simply because he expects the other agent not to pay his cost either. The equilibrium in which the contract is drawn-up strictly Pareto-dominates the no-contract equilibrium. Clearly, the multiplicity of equilibria disappears if the costs are payable sequentially as above. Our third observation will become relevant again in Section 4 below.

3.2. Perfect Substitutes

So far, we have assumed that the agents' ex-ante costs are 'perfect complements' in determining whether a contract is feasible or not. The next proposition tells us that when the agents' ex-ante costs are *perfect substitutes* our constrained inefficiency results of Subsection 3.1 still hold although the inefficiency may take a different form.

The intuition behind these new results is straightforward. In an environment in which the ex-ante costs may be paid by *either* agent a contract is constrained efficient if *at least one* of the two ex-ante costs is smaller than the size of the surplus. It is then easy to envisage a situation in which the share of the surplus accruing to each agent is strictly smaller than his ex-ante costs although there is enough surplus to cover the smallest of these costs. In this case, in equilibrium, the parties will not draw-up the contract although it would be socially efficient to do so.

When the ex-ante costs are perfect substitutes, a new type of inefficiency can also arise in equilibrium. In particular, it is possible that the agents draw-up a contract, but the equilibrium involves the highest of the two ex-ante costs being paid.

[8]In Anderlini and Felli (1996) we state and prove the analogues of Propositions 1 and 2, which apply to this case.

	pay c_B	not pay c_B
pay c_A	$\lambda - c_A,\ 1 - \lambda - c_B$	$\lambda - c_A,\ 1 - \lambda$
not pay c_A	$\lambda,\ 1 - \lambda - c_B$	$0,\ 0$

Figure 2: Normal form when the ex-ante costs are perfect substitutes.

Formally, when the ex-ante costs are perfect substitutes Assumption 1 needs to be modified. Assumption 2 below identifies the range of ex-ante costs which guarantees that drawing-up a contract is socially efficient in this case.

Assumption 2: *The surplus which the contract yields exceeds the minimum ex-ante cost payable for the contract to become feasible. In other words* $\min\{c_A, c_B\} < 1$. *Without loss of generality let* $c_A < c_B$. *Hence* $c_A < 1$.

Consider now the reduced form model with ex-ante costs that are perfect substitutes and let Assumption 2 above hold. The normal form of our new two-stage game is depicted in Figure 2.

We start with the case in which the value of the distribution parameter $\lambda \in [0, 1]$ is given. In such a case by varying the values of the contractual costs (c_A, c_B) it is always possible to generate two different types of inefficiencies.

First of all, it is always possible that the ex-ante costs be such that the no-contract outcome obtains. This is immediate from the payoffs in Figure 2 and it is stated without proof in our next proposition.

Proposition 3: *For any given* $\lambda \in [0, 1]$ *there exists a set* $\hat{\mathcal{C}}_1 = \{c_A, c_B \mid 1 > c_A > \lambda$ *and* $c_B > (1 - \lambda)\}$ *of pairs of ex-ante costs satisfying Assumption 2 such that the only equilibrium of the two-stage game represented in Figure 2 has neither agent paying the ex-ante cost, and therefore yields the no-contract outcome.*

Secondly, given any $\lambda \in [0, 1]$, it is possible that the ex-ante costs are such that the second type of inefficiency we have mentioned above obtains in equilibrium. A contract is drawn-up, but it is the agent with the highest ex-ante cost who pays in equilibrium. Once again our claim follows immediately from the payoff matrix in Figure 2 and it is stated without proof in the following proposition.

PROPOSITION 4: *For any given* $\lambda \in [0, 1/2)$ *there exists a set* $\hat{\mathcal{C}}_2 = \{c_A, c_B \mid 1 > c_A > \lambda \text{ and } c_B < (1-\lambda)\}$ *of pairs of ex-ante costs satisfying Assumption 2 such that the only equilibrium of the two-stage game represented in Figure 2 has agent* A *not paying the ex-ante cost* c_A*, and* B *paying the ex-ante cost* $c_B > c_A$.[9]

Next, we consider the case in which we take as given the ex-ante costs (c_A, c_B). In this case it is always possible that the value of the distribution parameter λ be such that one of the two types of inefficiencies we have identified obtains in equilibrium. Again, we state our next proposition without proof since it is an immediate consequence of the payoff matrix in Figure 2.

PROPOSITION 5: *Let a pair of ex-ante costs* $c_A > 0$ *and* $c_B > 0$ *satisfying Assumption 2 be given.*

If it is the case that $c_A + c_B \leq 1$*, then for any* λ *in* $[0, c_A)$ *the unique equilibrium of the model is constrained inefficient in the sense that a contract is drawn up, but it is* B *who pays the ex-ante cost* $c_B > c_A$.

If it is the case that $c_A + c_B > 1$*, then for any* λ *in* $[0, \min\{0, 1 - c_B\}]$ *the (unique if* $\lambda \neq 1 - c_B$*) equilibrium of the model is, again, that a contract is drawn up, but it is* B *who pays the ex-ante cost. Moreover, for any* λ *in* $(1 - c_B, c_A)$ *the unique equilibrium of the model involves neither agent paying his ex-ante cost, and hence yields the no-contract outcome.*

3.3. Strategic Complements

Our goal in this subsection is to show that the analogues of Propositions 1 and 2 hold when the ex-ante costs are technologically perfect substitutes, but are 'strategic complements' in the game-theoretic sense. We conjecture that this is true 'in general', but of course this cannot be shown in a general result since we would need to consider formally all the extensive forms which guarantee strategic complementarity of the agents' ex-ante costs.

We limit our formal analysis to an example which is a modification of the model of the previous subsection.

[9] Recall that we are assuming that $c_A < c_B$. To deal with the range $\lambda \in (1/2, 1]$ it is sufficient to modify Assumption 2 to read $c_B < c_A$. Proposition 4 does not hold when $\lambda = 1/2$.

The description of our next model is as follows. At $t = 0$ both agents decide simultaneously and independently whether to pay their ex-ante costs. If both agents decide not to pay the ex-ante costs, then a contract is not feasible and both receive a payoff of zero. If either agent $i \in \{A, B\}$ pays the ex-ante cost c_i at $t = 0$ the contract which yields one unit of surplus becomes feasible. If both pay their ex-ante costs at $t = 0$ the distribution parameter λ determines the contract which is drawn-up and the A's and B's payoffs are $\lambda - c_A$ and $1 - \lambda - c_B$ respectively.

However, if only one agent, say A, pays the ex-ante cost at $t = 0$, he is allowed to make a take-it-or-leave-it offer ℓ to B at $t = 1$. The value of ℓ is interpreted as an offer to make A's and B's payoffs equal ℓ and $1 - \ell$ respectively, minus any costs paid. This can be thought of as a crude way to say that if only one agent pays the ex-ante cost then the bargaining power shifts dramatically in his favour.

Moreover, we assume that A, if he alone has paid the ex-ante cost, can, in principle, make some offers which would push agent B below his individual rationality constraint. In other words we assume that the take-it-or-leave-it offer ℓ must be in the interval $[-\epsilon, 1 + \xi]$ with ϵ and ξ some (possibly small) positive numbers.

At $t = 2$, B has two choices. He can either pay an ex-ante cost $c'_B > 0$ or pay nothing.[10] If he does not pay he does not observe A's offer, but is still allowed to accept or reject it blind. If B decides to pay his ex-ante cost, he can then observe A's offer and subsequently decide to accept or reject it.

The description of the extensive form which is played if it is B alone who pays the ex-ante cost at $t = 0$ is exactly symmetric to the case we have just described.

Notice that the strategic complementarity of the two agents' ex-ante costs is built into the extensive form game we have described precisely via the shift in bargaining power which obtains when one agent alone pays the ex-ante costs at $t = 0$.

Suppose now that the parameters λ, c_A and c_B are such that the agents would not draw-up a contract in the model described in Subsection 3.1. Our next proposition then tells us that, in the model with strategic complementarities we have just described, they will not draw-up a contract either.

[10]Notice that while Proposition 6 below restricts the values of c_A and c_B to be in an appropriate range, c'_B can take any positive (small) value.

Proposition 6: *Consider the model with ex-ante costs which are strategic complements described above in this subsection. Assume that either $\lambda < c_A$ or $1 - \lambda < c_B$. Then the unique equilibrium outcome of the model has neither agent paying the ex-ante cost at $t = 0$, and hence the no-contract outcome obtains.*

4. Contracts Over Contracts, ... Over Contracts

4.1. Simple Ex-Ante Compensating Transfers

In Section 3 we have argued that ex-ante contractual costs may give rise to a version of the hold-up problem which in turn generates an inefficient (no-contract) outcome. As we mentioned in the Introduction, in all the previous literature of which we are aware (Grossman and Hart 1986, Hart and Moore 1988, Hart and Moore 1990, among others) the *reason* a hold-up problem might arise in the first place is that the parties are *constrained* in their ability to write contracts: given that certain variables are not negotiable ex-ante (or that only limited ex-ante negotiation is feasible because of the constraints imposed by the possibility of renegotiation ex-post (Hart and Moore 1988)) the parties' 'relationship specific' investment(s) will be inefficiently low.

In a model with 'relationship specific' investments and incomplete contracting, the hold-up problem typically has a 'contractual solution'. If either the assumption that the parties are constrained to write incomplete contracts is removed (for example by increasing the information which the enforcing agency can verify as in Nöldeke and Schmidt (1995)), or if a contracting stage is added to the model in which the parties can write a 'grand' ex-ante contract in which either the amounts of relationship specific investment are specified or alternatively reported by the parties to the enforcing agency as in Maskin and Tirole (1997), or finally if the parties can commit to a given renegotiation procedure as in Aghion, Dewatripont, and Rey (1994), then the hold-up problem is resolved, and an efficient outcome is guaranteed.

The next natural question to ask is then whether a contractual solution to the hold-up problem is generally available in the present set up. In other words: is it possible to add another stage to our model (say $t = -1$), prior to the stage in which the ex-ante costs are incurred, in which the agents can negotiate a 'grand contract', which will resolve the hold-up problem and hence restore efficiency?

The answer to the above question is trivially 'yes', if at $t = -1$ a *truly grand* contract can be negotiated *costlessly*, which specifies everything, including the payment of the ex-ante costs, *and* the division of the actual surplus at time $t = 1$. The answer, however, changes dramatically if the 'grand contract' is itself costly.

We specify two crucial details of the grand contract stage. First of all we assume, as seems plausible in the present context, that in order to be able to negotiate a contract at $t = -1$ a fresh set of ex-ante costs must be incurred by the parties before $t = -1$, say at $t = -2$. Secondly, we restrict the agents to negotiate a *compensating transfer* at $t = -1$. In other words, we take a specific view on the agreements which the agents can enter at $t = -1$. Indeed, we restrict them to be transfers *contingent* on the payment of ex-ante costs at $t = 0$. This seems to be in the spirit of our reduced form model of Section 3, in that, in principle, it allows the agents to effectively transfer surplus between them, but it keeps the distribution of surplus in the last stage of the contracting process, $t = 1$, exogenously fixed, as before.[11]

It is worth emphasizing at this point that we find that the presence of any *strictly positive* 'second tier' ex-ante costs might be sufficient to keep the addition of a grand contract stage from resolving the hold-up problem of Section 3. We view this as a strength of the results we present in this section. Indeed, in many situations it would be sensible to assume that the second tier ex-ante costs are in fact at least as large as the 'first tier' ex-ante costs, on the grounds that a 'contract over a contract', in an intuitive sense, is a more complex object than the contract itself.

Formally, we modify the model of Section 3 as follows. There are now four time periods, $t \in \{-2, -1, 0, 1\}$. The sequence of decisions and events for the two agents (depicted schematically in Figure 3) is as follows. In period $t = -2$, the two agents decide simultaneously and independently whether to pay the second tier ex-ante costs (c_A^2, c_B^2). If either or both agents decide not to pay these ex-ante costs, the period $t = -1$ compensating transfers to be described shortly are automatically set equal to 0, and the agents effectively move directly to time $t = 0$. If, on the other hand both

[11] While the assumption that a fresh set of ex-ante costs arises at $t = -2$ is crucial for our result (Proposition 7), we conjecture that the restriction to the negotiation of *compensating transfers* is not. The parties could, for instance, negotiate which extensive form to use in the following stage of the game (limited to those extensive forms which guarantee a unique set of equilibrium payoffs). This would be 'payoff equivalent' to a compensating transfer of the type we analyze here.

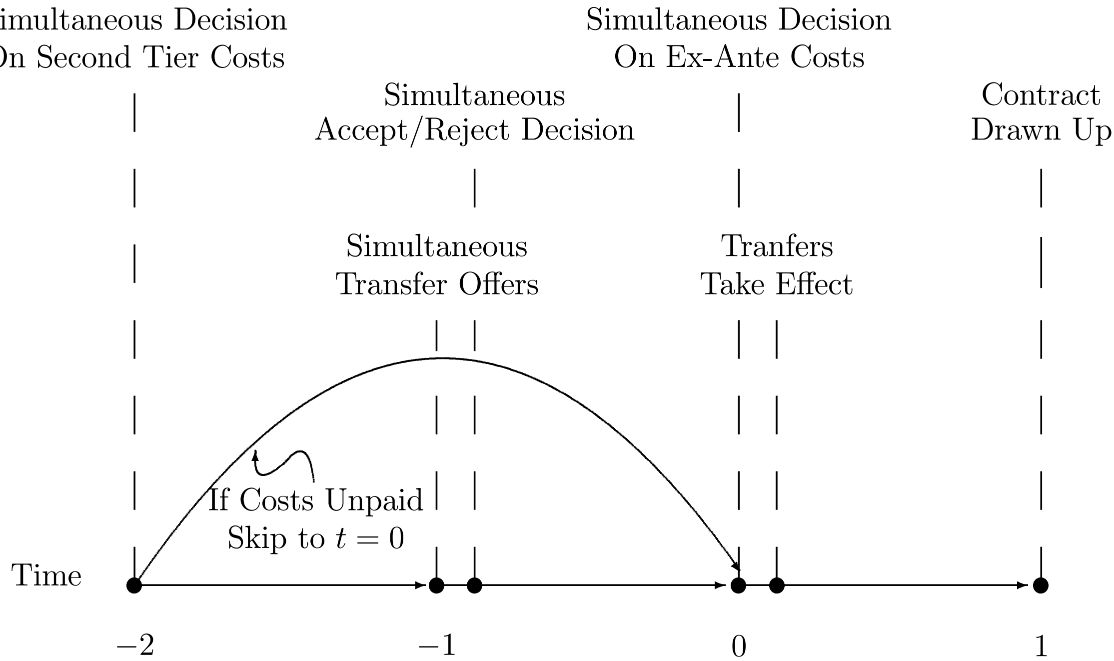

Figure 3: Timing in the two tier contracting model.

agents pay the second tier ex-ante costs, then period $t = -1$ compensating transfers can be negotiated.

For simplicity, we assume that (provided that both pay the second tier costs) at $t = -1$, both agents make simultaneous offers of contingent compensating transfers to each other.[12] Formally, each agent $i \in \{A, B\}$ chooses a real number $\sigma_i \geq 0$, which is interpreted as a commitment to transfer the amount of wealth σ_i to the other agent, $j \neq i$, if and only if j pays the first tier ex-ante cost c_j^0 in period $t = 0$. Immediately after choosing σ_i, still in period $t = -1$, A and B simultaneously choose whether to accept or reject the other agent's offer of compensating transfer. Those offers which are accepted at this stage are binding in period $t = 0$.

The decisions and events in periods $t = 0$ and $t = 1$ are analogous to those described in Subsection 3.1. At $t = 0$, both agents choose simultaneously and independently whether to pay the first tier ex-ante costs (c_A^0, c_B^0). Each agent $i \in \{A, B\}$

[12] In Anderlini and Felli (1996) we also explore a variety of different extensive forms for the negotiation of compensating transfers. In essence, our results of this subsection generalize to the analogues of the extensive forms described in Cases 1 to 5 of Section 6 below, modified so that each offer of a value for the distribution parameter is changed to be an offer of compensating transfer.

then incurs an ex-ante cost of c_i^0 at this time, and subsequently receives a compensating transfer of σ_j from agent $j \neq i$. Only if both agents have paid the first tier ex-ante costs the $t = 1$ surplus-generating contract becomes possible.

Provided both agents have paid their first tier ex-ante costs their payoffs are $\lambda - \gamma_A$ and $1 - \lambda - \gamma_B$ respectively, where γ_i denotes the total ex-ante costs paid by agent $i \in \{A, B\}$ during the entire game, minus any compensating transfer received from agent $j \neq i$, and plus any compensating transfers paid by i to j. If the surplus-generating contract is not drawn-up, then the two agents payoffs are simply $-\gamma_A$ and $-\gamma_B$ respectively.

The assumption that the total (for both tiers) of ex-ante costs must be low enough so that it is socially efficient for the parties to draw-up a contract is easy to state for this version of our model.

ASSUMPTION 3: *Let $c_i = c_i^2 + c_i^0$ for $i \in \{A, B\}$. Then $c_A + c_B < 1$.*

It is apparent from the description of our reduced form model with simple compensating transfers above (cf. Figure 3) that this model, viewed from $t = 0$, is in fact identical to the simple reduced form model of Subsection 3.1, whenever both agents have chosen not to pay the second tier ex-ante costs. We can therefore ask whether the parameters of our model with compensating transfers are such that either Proposition 1 or Proposition 2 guarantee that, in the absence of compensating transfers, the no-contract outcome is the unique equilibrium of the model. This motivates our next definition.

DEFINITION 1: *Assume that either $c_A^0 < \lambda$ or $c_B^0 < 1 - \lambda$ so that, provided that neither agent has paid the second tier ex-ante cost then the only equilibrium outcome of the model is the no-contract outcome (see Propositions 1 and 2 above). Then we say that the reduced form model with simple compensating transfers 'yields the no-contract outcome in the final stage'.*

We are now ready to state our next proposition. It tells us that, if the parameters of the reduced form model of Subsection 3.1 yield the no-contract outcome, then adding a new stage to the model, with a second tier of positive ex-ante costs and compensating transfers may not solve the hold-up problem generated by the first tier ex-ante costs. In particular, the reduced form model with compensating transfers has multiple equilibria, and at least one equilibrium that yields the no-contract outcome.

PROPOSITION 7: *Consider the reduced form model with simple compensating transfers. Suppose that c_A^0, c_B^0 and λ yield the no-contract outcome in the final stage (cf. Definition 1), and assume that the second tier ex-ante costs are strictly positive for both agents ($c_i^2 > 0$ for $i \in \{A, B\}$). Then the model has multiple equilibria. In particular, there always exists an equilibrium in which neither agent pays either tier of ex-ante costs, and hence yields the no contract outcome. Moreover, there is also an equilibrium in which both agents pay both tiers of ex-ante costs and a contract is drawn-up.*

The reason why our reduced form model with compensating transfers always has one equilibrium in which none of the costs are paid is an obvious one. Recall that at each stage the two agents decide simultaneously and independently whether to pay their ex-ante costs. Moreover a contract (or compensating transfers) are feasible only if both agents pay. It is then immediately clear that if one agent expects the other not to pay his ex-ante cost he should not pay either. The cost would be wasted since it has no effect on the remainder of the game. Therefore it is an equilibrium for both agents to pay none of the costs.

The intuition behind the existence of a subgame perfect equilibrium in which the parties do pay both tiers of ex-ante costs and draw-up the contract is less straightforward.[13]

Imagine that some compensating transfers have been agreed. If the transfers are such that the first tier ex-ante costs are 'covered' for both agents (which is always possible in principle because of Assumption 3), then the terminal subgame of the model has two equilibria. One in which a contract is drawn-up, and another one in which neither agent pays the first tier ex-ante cost and the no-contract outcome obtains. Note that these equilibria are strictly Pareto-ranked.

[13] We are indebted with Stephen Matthews for suggesting to us the existence of this type of equilibrium.

It is then possible to construct an equilibrium in which the agents switch (off-the-equilibrium-path) between equilibria of the terminal subgame, according to what transfers have been offered and agreed in the first stage of the game. The 'switching point' can always be constructed in such a way that it is in the interest of the agent whose share of the surplus exceeds his costs to compensate the other for the shortfall between his share of the surplus and both tiers of ex-ante costs. The 'threat' of switching to the inefficient equilibrium is 'credible' because the no-contract outcome is always one of the possible equilibrium outcomes of the terminal subgame.

Two observations come to mind with respect to the equilibrium just described. First, even if the contract is drawn-up this is done by paying two tiers of ex-ante costs rather than one. Therefore, even when a contract is drawn-up the equilibrium of the model is constrained inefficient.

Remark 1: *All equilibria of the reduced form model with compensating transfers are constrained inefficient. In particular, the inefficiency takes the form of the no-contract outcome for some of the equilibria, while for others it takes the form of a contract that is drawn-up paying two tiers ex-ante costs rather than one.*

Secondly, and in our view more importantly, the equilibria of our reduced form model with compensating transfers in which a contract is drawn-up rely on the agents playing (off-the-equilibrium-path) an equilibrium in the terminal subgame which is strictly Pareto-dominated by another equilibrium of the same subgame. This runs against the intuition that the parties to a contract will be able to re-negotiate ex-post to an equilibrium which makes them both better off when one is available.

Imagine now that we impose the restriction that in the terminal subgame the agents must play the Pareto-efficient equilibrium when the subgame has two equilibria. Then, after the second tier ex-ante costs have been paid, they are *sunk* in a strategic sense. This means that the agent who has a 'deficit' in the last stage of the game, by subgame perfection, will accept any offer of compensating transfer which leaves him with a positive continuation payoff. Therefore in any equilibrium which obeys this new restriction, the compensating transfers will *not* take into account the second tier ex-ante costs. Therefore, one of the two agents will find it profitable not to pay the second tier ex-ante cost for which he would not possibly be compensated.

This, in turn, means that compensating transfers will not be observed in equilibrium, and therefore yields the no-contract outcome. This is the focus of Proposition 8 below.

The idea that some type of renegotiation-proofness is an appealing additional restriction to impose on the set of subgame perfect equilibria is not new, both in contract theory (Grossman and Hart 1986, Hart and Moore 1988, Hart and Moore 1990, Rubinstein and Wolinsky 1992, Aghion, Dewatripont, and Rey 1994, Nöldeke and Schmidt 1995, among others), and in game theory (Farrell and Maskin 1989, Benoît and Krishna 1993, among many others).

Below, we give an informal definition of a *renegotiation-proof* equilibrium which applies to our model of this section. A more formal definition can be found in the Appendix (Definition A.1). It coincides with the definition commonly accepted in game theory (see, for instance, Benoît and Krishna (1993)).

Definition 2: *A subgame perfect equilibrium of the reduced form model with simple compensating transfers is renegotiation-proof if and only if the equilibria played in every proper subgame are not strictly Pareto-dominated by any other equilibrium of the same subgame.*[14]

Our next result says the if we restrict attention to renegotiation-proof equilibria, then the possibility of compensating transfers does not resolve the hold-up problem identified in Section 3. It is true that the model always has an equilibrium in which transfers take effect and a contract is drawn-up. But this equilibrium is not renegotiation-proof. Thus, although it may be tempting to select (in a Coasian 'spirit') the contract equilibrium among the two mentioned in Proposition 7 simply because it Pareto-dominates the no-contract equilibrium, this type of selection is open to an objection which is, in our view, fatal.

Surely, if we are willing to select among Pareto-ranked equilibria in favour of the dominating one, we should also be willing to apply the same logic to every subgame.

[14]Notice that our informal definition is made particularly simple by the fact that our reduced form model with compensating transfers only has one 'level' of proper subgames. In a game with many stages, the definition needs to take into account that the set of equilibria of each subgame is constrained by the renegotiation-proofness of the subsequent subgames. See Definition A.1 below.

After all, once entered, every subgame is just like a game. However, if we apply this selection criterion to every subgame (in a recursively consistent way, of course), the only equilibrium of the entire game which survives is the constrained inefficient one, in which the no-contract outcome obtains.

PROPOSITION 8: *Consider the reduced form model with simple compensating transfers. Suppose that* c_A^0, c_B^0 *and* λ *yield the no-contract outcome in the final stage and that* $c_A^2 > 0$ *and* $c_B^2 > 0$. *Then every renegotiation-proof subgame perfect equilibrium of the model involves neither agent paying either tier of ex-ante costs and therefore yields the no-contract outcome.*

We view Proposition 8 as saying that the possibility of compensating transfers does not resolve the hold-up problem identified in Section 3 in the following sense. Either, we are willing to accept the multiple equilibria identified in Proposition 7, and therefore to accept the no contract equilibrium as being just as plausible as the one in which a contract is drawn-up. Or, we attempt to select among Pareto-ranked equilibria in favour of the efficient ones. However in this case, we should apply this logic consistently to every subgame, and hence single out those equilibria which are renegotiation-proof. In this case only the no-contract outcome survives.

Note that the multiplicity of equilibria in the terminal subgame of our model is crucially dependent on the fact that the ex-ante costs are payable simultaneously by the agents. Therefore if the game is modified so that the costs are payable sequentially, all subgames have a unique equilibrium and the no-contract outcome is certain to prevail.

In particular, consider the following modification of the extensive form depicted in Figure 3. At $t = 0$ A decides whether to pay the ex-ante cost c_A^0, then B observes A's choice and decides whether to pay the ex-ante cost c_B^0. The rest of the extensive form is identical to the one in Figure 3. The following proposition characterises the equilibria of this modification of the reduced form model with simple compensating transfers.[15]

[15] The alternative game in which B decides whether to pay the ex-ante cost before A is just a relabelling of the one we just described. Proposition 9 below obviously applies to this extensive form as well.

Proposition 9: *Consider the modified reduced form model with simple compensating transfers which we have just described. Suppose that c_A^0, c_B^0 and λ yield the no-contract outcome in the final stage and that $c_A^2 > 0$ and $c_B^2 > 0$. Then, the unique subgame perfect equilibrium of the model is for neither agent to pay either tier of ex-ante costs, and hence yields the no-contract outcome.*

4.2. A Hierarchy of Contracts Over Contracts

Given the intuition behind our results of Subsection 4.1, it is natural to ask whether they generalize to a model which includes a whole hierarchy of 'contracts over contracts'. In this subsection, we examine this question in two separate, but essentially 'nested', models, and find that our results of Subsection 4.1 generalize in both cases. We begin with the simpler of the two set-ups.

Consider adding N time periods prior to period 0 to the model of Section 3, rather than 2 as we did in Subsection 4.1. There are now $N + 2$ time periods $t \in \{-N, -N + 1, \ldots, 0, 1\}$.

For ease of exposition, we divide each period from $-N$ to -1 into three consecutive *stages*, called I, II and III respectively. In stage I of period $-N$ both agents decide whether to pay the $(N + 1)$-th tier ex-ante costs. Formally, each agent i decides whether to pay the cost c_i^N, where c_i^N is a given, strictly positive constant. If one or both agents do not pay c_i^N, the agents effectively move directly to period $-N + 1$, skipping stages II and III of period $-N$. Only if both agents have paid the $N+1$-th tier ex-ante costs, in stage II of period $-N$, they can make compensating transfer offers (σ_A^N, σ_B^N). In stage III of period $-N$, exactly as in Subsection 4.1 each agent can then accept or reject the other's offer. Those offers which are accepted in stage III of period $-N$ become binding.

The agents then move to period $-N + 1$, and must decide whether to pay the N-th tier ex-ante costs. In stage I of period $-N + 1$, each agent decides whether to pay the cost c_i^{N-1}, where c_i^{N-1} is a given, positive constant. Immediately after, still in stage I of period $-N + 1$, each agent i receives a compensating transfer of σ_j^N, if such transfer was agreed on at time $t = -N$. Only if both agents pay c_i^{N-1}, in stages II and III of period $-N + 1$ a fresh set of compensating transfers can be offered and accepted or rejected respectively. Otherwise, the agents move directly to period $-N+2$.

The same structure of moves is then repeated up to period $t=-1$. In periods $t=0$ and $t=1$, the model is identical to the one in Subsection 4.1 (cf. Figure 3). Below, we refer to the model we have just described as the 'reduced form model with N tiers of simple compensating transfers'.

The second model we consider in this subsection is a generalization of the reduced form model with N tiers of simple compensating transfers which we have just described. The structure of time periods and stages is unchanged. We assume that, provided the ex-ante costs have been paid by both agents in stage II of period $-n$, then each agent i can make compensating transfer offers to $j \neq i$ for *all subsequent* tiers ($m=1,\ldots,n$) of ex-ante costs; all offers are made at the same time and the two agents make offers simultaneously in stage II of period $t=-n$. In stage III of period $-n$, the agents simultaneously decide which offers to accept and which ones to reject. Those offers which are accepted then become binding. We denote the offer of compensating transfer which $i \in \{A,B\}$ makes at time t relative to the m-th tier of ex-ante costs by $\sigma_i^{t,m}$. Whenever i actually pays a tier of ex-ante costs, he receives, immediately after the payment, the *sum* of the compensating transfer offers previously agreed for that tier of ex-ante costs. Below, we refer to the model we have just described as the 'reduced form model with N tiers of multiple compensating transfers'.

The equivalent of Assumption 3, stipulating that it is socially efficient for the parties to draw-up a contract even when all tiers of ex-ante costs are payable, is easy to state for the two models we have just described.

Assumption 4: *Let* $c_i = \sum_{n=0}^{N} c_i^n$ *for* $i \in \{A,B\}$. *Then* c_A *and* c_B *satisfy* $c_A + c_B < 1$.

The purpose of exploring the two models with N tiers of compensating transfers described above is to show that the possibility of such transfers will not necessarily resolve the hold-up problem of Section 3, if the parameters of the model are such that a hold-up problem in fact exists. Therefore, as in Subsection 4.1, we assume that c_A^0, c_B^0 and λ are such that the model 'yields the no-contract outcome in the final stage' (cf. Definition 1).

The following proposition tells us that adding N tiers of compensating transfers to the model of Section 3 does not solve the hold-up problem identified there.

PROPOSITION 10: *Consider the reduced form model with N tiers of simple or multiple compensating transfers. Suppose that c_A^0, c_B^0 and λ are such that this model yields the no contract outcome in the final stage, and assume that all tiers of ex-ante costs are strictly positive for both agents ($c_i^n > 0$ for $i \in \{A, B\}$ and $n = 0, \ldots, N$). Then the unique renegotiation-proof (cf. Definition A.1) subgame perfect equilibrium outcome of the model involves both agents not paying any of the ex-ante costs, and hence yields the no-contract outcome.*

Furthermore, using a construction similar to the one presented in the proof of Proposition 7 it is possible to prove the following, which we state for the record and without proof.

PROPOSITION 11: *Consider the reduced form model with N tiers of simple or multiple compensating transfers. Suppose that c_A^0, c_B^0 and λ yield the no-contract outcome in the final stage, and assume that all tiers of ex-ante costs are strictly positive for both agents ($c_i^n > 0$ for $i \in \{A, B\}$ and $n = 0, \ldots, N$). Then the model has multiple equilibria. In particular, in addition to the equilibrium singled out in Proposition 10, there also always exists an equilibrium in which compensating transfers are agreed and a contract is drawn-up.*

The results of this subsection are a generalization of the findings of Subsection 4.1. Adding N tiers of possible compensating transfers still does not resolve the hold-up problem identified in Section 3, if each tier of compensating transfers carries a positive ex-ante cost for each agent. The reason is, again, that once a tier of ex-ante costs are paid these costs are *sunk*. Therefore, in any renegotiation-proof equilibrium the compensating transfers will *not* take into account the previous set of ex-ante costs. This, in turn, means that the ex-ante costs will not be paid and therefore yields the no-contract outcome.

5. A 'Reduced Form' Model with Continuous Costs

So far, we have assumed that the agents' decision regarding the ex-ante costs is 'lumpy'; a contingent contract is not possible unless both agents sink a minimum, strictly positive, ex-ante cost. This is the reason why Propositions 1 and 2 above refer to a *range* of the distribution parameter λ for any given ex-ante costs, and to a *range* of ex-ante costs given any value of the distribution parameter λ.

In this section we consider a model in which the agents have a continuous choice of ex-ante costs. Our model is still a 'reduced form' one, in that we do not model explicitly the effects of increased ex-ante costs paid by the agents.[16] We simply postulate that the *size of the surplus* yielded by the contract which the agents draw-up is an increasing function of the magnitude of the ex-ante costs paid by the two agents.

The interpretation of our reduced form increasing relationship between the ex-ante costs and the surplus generated by the contract, we believe, is a natural one. We imagine a situation in which, as the agents pay larger amounts of ex-ante costs, more *detailed contracts* become feasible between them. The meaning of the word detail here can range from a more accurate description of the 'contractual variables', to a more detailed description of the possible states of nature (and therefore, in a dynamic model, to contracts with a longer time horizon), to a contract which is better specified in legal terms, which as a consequence is more easily enforced, and therefore yields a higher level of surplus 'net of enforcement costs'.

The results which we derive in this section are the analogues in our set-up of the general under-investment results stemming from a hold-up problem (Hart and Moore 1988). Formally, the model which we analyze is close to Holmström (1982), and can be described as follows.

The two agents, A and B, play a two stage game. At $t = 0$, both agents decide, simultaneously and independently, how much ex-ante contractual cost to pay. Agent $i \in \{A, B\}$ chooses a number $c_i \in [0, \overline{c}_i]$ with $\overline{c}_i \in (0, 1)$.[17] At $t = 1$ the agents do

[16] In Anderlini and Felli (1996) we present a model (Section 8) in which the agents have a 'gradual' choice of ex-ante costs, and the effect of increased ex-ante costs paid is modelled explicitly as affording the agents a more detailed contract which extends further into the future.

[17] Recall that we are assuming throughout that the agents pay their ex-ante costs out of unit

not in fact have any choices to make; the pair of ex-ante costs (c_A, c_B) paid at $t = 0$, determines the size of the surplus that the contract yields to the agents. This is then divided among them according to the exogenously given distribution parameter λ. We denote with $x(c_A, c_B)$ the surplus corresponding to the pair (c_A, c_B), with $x_A(c_A, c_B)$ and $x_B(c_A, c_B)$ its partial derivatives with respect to the first and second argument respectively, and with $x_{A,A}(c_A, c_B)$, $x_{B,B}(c_A, c_B)$ and $x_{A,B}(c_A, c_B)$ the second and cross partial derivatives with respect to the same arguments. We assume that x is a (twice-differentiable) strictly increasing and strictly concave function which satisfies the Inada conditions $\lim_{c_A \to 0} x_A(c_A, c_B) = \infty$, $\lim_{c_B \to 0} x_B(c_A, c_B) = \infty$, $x_A(c_A, c_B) = 0$ for all $c_A \geq \overline{c}_A$ and $x_B(c_A, c_B) = 0$ for all $c_B \geq \overline{c}_B$.[18] We also assume that the ex-ante costs are complements in the sense that the cross partial derivative $x_{A,B}(c_A, c_B)$ is always positive.

Given a pair of ex-ante costs (c_A, c_B), the payoffs accruing to A and B are given by $\lambda x(c_A, c_B) - c_A$ and $(1 - \lambda)x(c_A, c_B) - c_B$ respectively. We denote by c_A^* and c_B^* the (unique) equilibrium ex-ante costs which the agents pay in the game we have just described. Given that our assumptions on the function x guarantee an interior solution, the equilibrium is easy to characterize.

Remark 2: *The model with continuous ex-ante costs we have described above yields a unique equilibrium pair (c_A^*, c_B^*), which can be characterized as follows by the corresponding first order conditions.*

$$x_A(c_A^*, c_B^*) = \frac{1}{\lambda} \quad \text{and} \quad x_B(c_A^*, c_B^*) = \frac{1}{1 - \lambda} \tag{1}$$

The efficiency benchmark with which to compare the equilibrium identified in Remark 2 is straightforward to define and to characterize.

Definition 3: *The socially efficient levels of ex-ante costs in the model with continuous costs are denoted by c_A^E and c_B^E. They are defined as the pair of ex-ante costs*

endowments which are available to them at $t = 0$. The constraint that c_i should be in $[0, \overline{c}_i]$ with $\overline{c}_i \in (0, 1)$ is simply designed to ensure that any choice of ex-ante cost is 'affordable' for both agents.

[18] Therefore, we are assuming that there are decreasing returns to scale in the relationship between the ex-ante costs paid and the size of the surplus which the contract generates.

which maximize the difference between the surplus given by the contract and the sum of ex-ante costs $x(c_A, c_B) - c_A - c_B$.

There is a unique socially efficient pair of ex-ante costs (c_A^E, c_B^E), *which can be characterized as follows using the corresponding first order conditions.*

$$x_A(c_A^E, c_B^E) = 1 \qquad \text{and} \qquad x_B(c_A^E, c_B^E) = 1 \tag{2}$$

Using the concavity of x and the fact that the cross partial derivative of x is positive, it is easy to show that (1) together with (2) imply that $c_i^* < c_i^E$ for all $i \in \{A, B\}$. This is the content of our next proposition.

PROPOSITION 12: *Let any value of the distribution parameter* $\lambda \in (0, 1)$ *for the model with continuous choice of ex-ante costs be given.*[19] *Then, in equilibrium, both agents pay an inefficiently low level of ex-ante contractual costs in the sense that* $c_i^* < c_i^E$ *for all* $i \in \{A, B\}$. *This obviously implies that* $x(c_A^*, c_B^*) < x(c_A^E, c_B^E)$.

Thus, the agents under-invest in the ex-ante costs that determine the degree of completeness of the contract which they choose in equilibrium. Therefore, the agents choose a contract which is incomplete in the sense that it is less 'detailed' than would be socially efficient, even after contractual costs are taken into account.

The intuitive reason why the agents under-invest in their ex-ante costs according to Proposition 12 is simple to outline. Each party's share of the surplus generated is fixed, the total surplus is an increasing concave function of both ex-ante costs and these costs are complements. Each agent invests in his ex-ante costs only up to the point at which *his own* marginal net return is zero. Such point is therefore below the point at which the marginal *social* (across both agents) net return on his investment in the ex-ante cost is equated to zero.

We conclude this section with an observation. Proposition 12 describes a stronger inefficiency result than Propositions 1 and 2 above since it yields an inefficient outcome *regardless* of the value of λ. Notice that our model of Subsection 3.1 can be viewed as

[19] Notice that if we allow for λ to take the values 0 and 1 as well, we still obtain under-investment. For instance if $\lambda = 0$, A will choose $c_A = 0$, and B will set $0 < c_B < c_B^E$. If $\lambda = 1$, then $0 < c_A < c_A^E$ and $c_B = 0$.

a 'special case' (in which the assumption of concavity of x is violated) of our model with a continuous choice of ex-ante costs in which the size of the surplus yielded by the contract is a *discontinuous* function of the ex-ante costs paid by the two agents. Intuitively, the 'marginal' conditions for efficiency are therefore easier to satisfy in the model of Subsection 3.1 than in our present set-up.

6. Negotiation of the Distribution Parameter

6.1. Preamble

In our analysis so far, we have assumed that the value of the distribution parameter λ is exogenously given and cannot be changed by the agents.

Notice that since Proposition 12 tells us that the agents will pay inefficient levels of ex-ante costs *whatever* the valued of the distribution parameter, this is not a reason for concern in the case of continuous ex-ante costs. Whatever the extensive form which decides the value of the distribution parameter, given the equilibrium value of λ, the agents will draw-up a contract which is inefficiently incomplete.

In this section, we relax the assumption of a given value of λ for the case of discrete ex-ante costs. The hold-up problem which we have identified in Section 3 turns out to be 'pervasive' even when the agents, negotiate the value of the distribution parameter λ.

6.2. Discrete Costs

Propositions 1 and 2 of Section 3 can be paraphrased as saying that, for any given pair c_A and c_B there exists a range of values of λ (Proposition 1), and for any value of λ there exists a range of values of c_A and c_B (Proposition 2), such that the agents will not draw-up a contract, even though it would be socially efficient to do so.

As we mentioned above, it is natural to think of the value of λ as determined by an extensive form game in which the agents negotiate the distribution of surplus if a contract is drawn-up. Therefore, another way to paraphrase Propositions 1 and 2 is as follows.

Proposition 1 tells us that, for any given pair c_A and c_B, there exist a 'set of extensive forms' such that the agents will not draw-up a contract if the distribution parameter is determined as an equilibrium of the given extensive form, even though it would be socially efficient to do so.[20]

Proposition 2, on the other hand, says that, for any given extensive form which determines the value of the distribution parameter λ, there exists a pair of ex-ante costs c_A and c_B such that, it would be socially efficient for the agents to pay the ex-ante costs, but in equilibrium this will not be the case and a contract will not be drawn-up.

The question which we pursue in this subsection is the following. Fix some positive values for c_A and c_B. Is it then the case that for any extensive form in some interesting set the agents will not draw-up a contract in equilibrium? The interpretation of the word 'interesting' is of course open to disagreement, but we believe the answer to our question to be yes. In Cases 1 through to 4 below we find that the agents will not draw-up a contract in equilibrium even for arbitrarily small contractual costs. In Case 5 below we characterize the range of contractual costs for which the only equilibrium outcome is that no contract is drawn-up.

The set of extensive forms described in Cases 1 through to 5 below can be described intuitively as follows. We start with a set of extensive form bargaining games which can be described as 'canonical' in that it includes a full range of bargaining games ranging from take-it-or-leave-it offers to infinite alternating offer games à la Rubinstein (1982). We then modify these models assuming that they generate ex-ante costs for the agents as they are played. In particular, the key assumption is that a set of ex-ante costs must be payable by the agents *immediately before* they choose the actions prescribed by the extensive form (for example, before making an offer or a counter-offer, or before deciding whether to accept or reject a given offer).[21]

[20] Here and in the next paragraph we refer to 'sets of extensive forms' without making this notion precise. This is simply to save space and new notation. Intuitively, we think of the set of all extensive forms as a set of games the (unique) equilibrium payoffs of which determine the distribution parameter λ. Moreover, we think of this set as being sufficiently rich so as to ensure that given any value of $\lambda \in [0, 1]$, there exists some extensive form game in the set considered which yields precisely λ as the equilibrium value of the distribution parameter. Examples of what such extensive form games might be are given in the descriptions of Cases 1 through to 4 below.

[21] This of course does not preclude the consideration of extensive forms in which some contractual costs are paid at other points in the game as well. What matters is that *some* of the contractual costs must be payable immediately before making offers and counter-offers, and accept/reject decisions. However, to keep matters relatively simple, we abstract from all ex-ante costs which are not payable immediately before any such actions are decided.

We start by describing the set of extensive forms as Cases 1 to 5. Proposition 13 below summarizes our findings for Cases 1 through to 4, while Proposition 14 deals with Case 5. All ex-ante costs in all the extensive forms considered below are arbitrary strictly positive numbers. In Cases 1 through 4 below we assume that the sum of all ex-ante costs (over all stages of the game and across the two agents) is strictly less than one. The assumption we make in Case 5 differs because of the presence of a potentially infinite number of stages. We discuss this assumption when we present Case 5 below.

We begin with the simple case in which A makes a take-it-or-leave-it offer to B.

Case 1: At time $t = 0$ both agents decide simultaneously and independently whether to sink the ex-ante costs (c_A, c_B). If either agent decides not to sink his cost then the game moves directly to period 2 and both parties receive a payoff of zero (minus any costs paid) since a contract is not feasible in this case.

If instead both agents pay the ex-ante costs at $t = 0$, the contract which yields one unit of surplus becomes feasible, and A makes an offer to B at $t = 1$. This offer specifies a value of $\lambda \in [0, 1]$. Agent B then has the possibility to accept or reject such offer.

In either case the agents move to period $t = 2$. If B accepted A's offer at $t = 1$ the agents' payoffs are $\lambda - c_A$ and $1 - \lambda - c_B$ respectively. If B rejected A's offer at $t = 1$, then the agents have not reached an agreement on how to draw-up the contract which yields one unit of surplus. Therefore the surplus is not available and they receive payoffs of $-c_A$ and $-c_B$ respectively.[22]

The next extensive form we consider is a modification of the one described in Case 1 above. It is designed to show that simultaneity of the agents' decisions to pay the ex-ante costs can be abandoned, provided that the party which receives the offer is unable to observe it, unless he pays his ex-ante cost. This observation generalizes to extensive forms considered in Cases 3, 4 and 5 below, although we do not provide the details for reasons of space.

[22] The case in which B makes a take-it-or-leave-it offer to A is just a relabelling of Case 1. Proposition 13 below obviously applies to this case as well.

Case 2: Consider the extensive form described in Case 1 modified as follows. At $t = 0$, A chooses whether to sink the ex-ante cost c_A. If A does not pay c_A at $t = 0$ the surplus-generating contract is not feasible. Therefore, in this case the game ends and both agents receive a payoff of zero. If A sinks c_A, he can then make an offer to B specifying a value $\lambda \in [0, 1]$ for the distribution parameter if a contract is drawn-up. The game then moves on to period $t = 1$. At $t = 1$, agent B can observe whether A has decided to make an offer of λ, but he cannot observe the value of λ, unless he pays his own ex-ante cost c_B. If B does not pay c_B the surplus-generating contract is not feasible.[23] Therefore, in this case the game ends and the agents' payoffs are $-c_A$ and zero respectively. If B decides to pay c_B at $t = 1$, then he observes λ, and can subsequently decide to accept or reject A's offer. If B rejects A's offer the game ends and the agents receive payoffs of $-c_A$ and $-c_B$ respectively. If, on the other hand, B accepts A's offer, the game moves on to $t = 2$, when the surplus generating contract is drawn-up. In this case the agents's payoffs are $\lambda - c_A$ and $1 - \lambda - c_B$ respectively.[24]

Our next case is that of an extensive form obtained from a randomization between the extensive form in which A makes a take-it-or-leave-it offer to B (Case 1) and the symmetric case in which B makes a take-it-or-leave-it offer to A.

Case 3: The two agents observe the outcome of a public randomization device — a coin toss for example — which has outcomes $\mathcal{A}$ with probability $\psi \in [0, 1]$ and $\mathcal{B}$ with probability $1 - \psi$. If the outcome of the public randomizing device is $\mathcal{A}$, the negotiation proceeds according to the extensive form described in Case 1 above. Conversely, if the outcome is $\mathcal{B}$ the negotiation proceeds according to the extensive form obtained swapping the names of A and B in the description of Case 1.

Notice that for Proposition 13 below to hold in Case 3 above, it is of critical importance that the parties cannot sink the ex-ante costs (c_A, c_B) *before* the outcome

[23] As in Subsection 3.3 above the technological complementarity of the ex-ante costs (c_A, c_B) is not crucial. It is enough that the two costs are strategic complements. This is the case if, for example, B can accept or reject the offer even if he has not paid his ex-ante cost provided that in this case B has to decide without seeing the offer, and the offer λ can exceed 1 thus pushing agent B below his individual rationality constraint.

[24] The case in which costs are paid sequentially and B makes a take-it-or-leave-it offer to A is just a relabelling of Case 2. Proposition 13 below obviously applies to this case as well.

of the public randomization is known. This is in keeping with our discussion above in which we emphasized that the ex-ante costs must be payable *immediately before* any offers, counter-offers and decisions to accept or reject are taken by the agents.[25] Notice however, that we could imagine an *additional* tier of ex-ante costs, payable before the outcome of the randomization is known, which would be interpreted as the cost of setting up the randomization we have just described. In this case, Proposition 13, which tells us that a contract will not be drawn-up in equilibrium would still apply.

We now turn to two dynamic extensive forms for the negotiation of the distribution parameter. One involves a finite number, N, of 'alternating offers', and the other a potentially infinite number of them.[26] The two extensive forms described in Cases 4 and 5 below are designed to embody the fact that each party has the ability to make a counter-offer after rejecting an offer of λ by the other agent. For the sake of clarity, in the description of the next two cases we divide again each time period in three consecutive stages: stage I in which costs are paid, stage II in which offers are made, and stage III in which offers can be accepted or rejected. We denote by c_i^n, with $i \in \{A, B\}$, the ex-ante costs that the agents have to pay in order to make and accept/reject offers in period $t = n$.

We are now ready to describe the extensive form for the compensating transfers negotiation game with N rounds of alternating offers.[27]

Case 4: We only deal with the case in which N is even, and agent A makes the first offer. Proposition 13 below is valid for the other three cases as well, in which B makes the first offer and/or N is odd. The details of the other three cases can be obtained in the obvious way from the particular case we describe.

The game starts in period 0. In stage I of period 0 both parties decide, simultaneously and independently, whether to sink the ex-ante costs (c_A^0, c_B^0). If either agent

[25] Of course, if either $\psi < c_A$ or $1 - \psi < c_B$ a contract will not be drawn-up even if the ex-ante costs are payable before the outcome of the randomization is known. Our assumption that they are paid after the randomization guarantees that Proposition 13 below holds for *any* value of $\psi \in [0, 1]$.

[26] Both the formulation and the analysis of Case 4 and 5 below are closely related to a vast literature on alternating offers models of bargaining sparked off by Rubinstein (1982) and subsequently enriched by the contribution of Shaked and Sutton (1984) among *many* others.

[27] It is easy to generalise Case 4 to allow for discounting of future payoffs. We do not provide the details for reasons of space.

(or both) decides not to sink this cost then the game moves to period $t = 1$. If, on the other hand, both agents pay the costs then they enter stage II of period 0. At this point A makes an offer $\lambda^0 \in [0, 1]$ to B, specifying a value for the distribution parameter. The game then moves to stage III, when B has the possibility to accept or reject A's offer. If B accepts the offer, the surplus-generating contract is drawn-up and the game ends with the agents receiving payoffs of $\lambda^0 - c_A$ and $1 - \lambda^0 - c_B$ respectively. If, instead, B rejects the offer the negotiation moves to period $t = 1$.

The description of the game in period $t = 1$ is the same as in period 0, except that the agents' roles are exchanged. It is now B who can make an offer λ^1 to A (if the ex-ante costs have been paid by both), and then A who has the chance to accept or reject.

The following periods up to and including period $t = N - 1$ are the same as the first two, with the agents making offers in turn. If the game ends with an offer being accepted at time $t = 0, \ldots, N - 1$, the agents' payoffs are $\lambda^t - \gamma_A$ and $1 - \lambda^t - \gamma_B$, where γ_A and γ_B are the total ex-ante costs paid by the two agents respectively. If the game ends in period $N - 1$ with no offer being accepted, the agents' payoffs are $-\gamma_A$ and $-\gamma_B$ respectively.

We are now ready to state our next proposition. Whenever the negotiation of the distribution parameter is carried out according to one of the extensive forms in Cases 1 through to 4, the two agents face an extreme version of the hold-up problem which we identified in Propositions 1 and 2. The surplus-generating contract will not be drawn-up in equilibrium, whatever the size of the ex-ante costs, provided they are positive.

Proposition 13: *Consider any of the extensive forms described in Cases 1 through 4 above. Assume that all 'tiers' of ex-ante costs are strictly positive. Then the unique equilibrium outcome of the model involves both agents not paying any of the ex-ante costs, and hence yields the no-contract outcome.*

The intuition behind the proof of Proposition 13 for all the cases considered is very much the same. After the ex-ante costs have been paid, they are *sunk*. Any offer which comes after these costs have been paid will therefore not take these costs into account, and will leave one of the two agents with a 'deficit' which will not be covered in future stages of the game. Thus, the ex-ante costs will not be paid, and the no-contract outcome will obtain.

We can now proceed to the description of the last case we consider, in which the alternating offers negotiation of the distribution parameter may last indefinitely. Case 5 which follows is the extension to a potentially infinite number of rounds of the extensive form we have just described in Case 4 above. However, we make a specific assumption on the ex-ante costs c_i^n. We assume them to be constant across periods so that $c_i^n = c_i$ for every n. The purpose of this assumption is to preserve the 'stationarity' of the game. This, in turn, greatly simplifies the proof of Proposition 14 below.

This assumption, of course, raises a problem if we require that drawing-up a contract must yield an amount of surplus which is greater than the sum of *all* ex-ante costs (cf. Assumption 4 above). Notice however, that provided that the sum of ex-ante costs for *one* period is less than one, then drawing-up a contract is socially efficient.

CASE 5: We describe the game for the case in which A makes the first offer. Proposition 14 also applies to the symmetric case in which the first offer is made by B.

The game starts in period $t = 0$. In stage I of period 0, both agents decide, simultaneously and independently, whether to sink the ex-ante costs (c_A, c_B). If either party (or both) decides not to pay this cost, the game moves directly to period $t = 1$. If both A and B pay these ex-ante costs, the game moves to stage II of period 0. Now A can make an offer to B of a value $\lambda^0 \in [0, 1]$ for the distribution parameter. In stage III of period 0, B can decide to accept or reject A's offer. If B accepts A's offer, the surplus-generating contract is drawn-up and the game ends with the agents receiving a payoff of $\lambda^0 - c_A$ and $1 - \lambda^0 - c_B$ respectively. However, if B rejects A's offer, the game moves on to period 1.

The description of period 1 is essentially the same as period 0, save for the fact that the roles of the two agents are exchanged. It is now B who (provided the ex-ante costs (c_A, c_B) are paid by both agents) can make an offer λ^1 to A, who then has the chance to accept or reject.

All odd periods $3, 5, 7, \ldots$ are essentially the same as period 1, and all even periods $2, 4, 6, \ldots$ are essentially the same as period 0. However, recall that if at any time t an offer of λ^t is accepted, the surplus-generating contract is drawn-up and the game ends with payoffs $\lambda^t - \gamma_A$ and $1 - \lambda^t - \gamma_B$ where γ_A and γ_B are the total ex-ante costs paid during the entire game by A and B respectively.

To complete the description of Case 5, we stipulate that if the agents never reach an agreement on a value for the distribution parameter, and therefore the game does not terminate in finite time, then the agents receive payoffs of $-\gamma_A$ and $-\gamma_B$ respectively.

PROPOSITION 14: *Consider the extensive form described in Case 5. Then the unique equilibrium outcome of the model involves both agents not paying any of the ex-ante costs and hence yields the no-contract outcome if and only if*

$$\max\{c_A, c_B\} > 1 - c_A - c_B \tag{3}$$

In particular it follows that that in equilibrium the agents will not draw up a contract whenever $1 > c_A + c_B > 2/3$.

Thus Proposition 14 tells us that when c_A and c_B are in a certain range the only equilibrium outcome of the model is constrained inefficient even when potentially infinite many rounds of negotiation are allowed. The difference between the behaviour of the model in Cases 4 and 5 is mainly due to the fact that in Case 5, since infinitely many rounds of negotiation are allowed, the logic of backwards induction does not apply.

7. Surplus Size as a Function of Its Distribution

In our entire analysis so far we have assumed that the size of the potential surplus generated by the contract is determined independently of its distribution across the two agents. Whether this is a good assumption or a bad one, depends on the details of the contractual situation at hand.

It is easy to think of contractual situations in which the 'expected quality' of the object which the contract concerns is determined (at least in part) by the action(s) of one (or both) agents, after the contract has been drawn-up. This is, for instance, the case if the contract to be drawn-up concerns a 'principal' and an 'agent'. The action of the agent in this case affects the amount of surplus available, and is in turn affected by how much surplus the agent is able to appropriate. In Anderlini and Felli (1996), we analyse a simple principal-agent model in which the interplay of incentive compatibility and limited liability is the mechanism through which the size of the surplus depends on the share left to the agent.

For reasons of space, in our analysis below we use, again, a 'reduced form' model in which we simply assume that the size of the potential surplus depends on its distribution between the two agents.

Intuitively, when the size of the surplus depends on its distribution, the hold-up problem we have analysed so far becomes less acute for the following reasons. When the distribution of surplus is negotiated between the two agents, one (or both) agent(s) may have an incentive to propose a 'fairer' distribution in order to increase surplus size. It is then possible that, as a result, the distribution of surplus which is agreed by the agents is 'less mis-matched' with the distribution of ex-ante costs than would be the case otherwise. As we know from Section 3 this may resolve the hold-up problem, and yield an equilibrium outcome in which the surplus-generating contract is drawn-up.

We make our next point in a simple setting. We take the simplest extensive form for the negotiation of the distribution parameter (Case 1) which in Proposition 13 — when the size of the surplus was assumed to be independent of its distribution — yielded the no contract outcome *regardless* of the size and distribution of the ex-ante costs. We then couple this extensive form with a simple functional form describing how the surplus size depends on its distribution, and we show that for *some* possible configurations of (positive) ex-ante costs, in equilibrium a contract will be drawn-up by the agents. Thus the hold-up problem identified above becomes less acute when the size of the potential surplus generated by the contract depends on its distribution across the two agents.

As before, x denotes the size of the surplus, while λ is the distribution parameter. The function $x(\lambda)$ embodies the dependence of surplus size on its distribution. We assume $x(\lambda)$ to be non-negative for every λ in $[0, 1]$, and that for some λ in $[0, 1]$,

$x(\lambda)$ is strictly positive. The function $x(\cdot)$ is also assumed to be differentiable on its domain.

At $t = 0$, A and B decide simultaneously and independently whether to pay the given pair of ex-ante costs (c_A, c_B) which we assume to be affordable and strictly positive. If either agent does not pay the ex-ante costs the surplus is not available and the agents' payoffs are equal to minus any ex-ante costs paid. If, on the other hand, both pay the ex-ante costs, then A makes an offer λ to B, who may accept or reject it. If B accepts the offer, a contract is drawn-up and A's payoff is $\lambda x(\lambda) - c_A$, while B's payoff is $(1 - \lambda)x(\lambda) - c_B$.

We start by identifying the socially efficient level of the distribution parameter λ.

Definition 4: *The socially efficient level of λ in the model we have just described is denoted λ^E, and is given by*

$$\lambda^E = \arg \max_{\lambda \in [0,1]} x(\lambda) \tag{4}$$

Notice that, since we are assuming that the surplus is strictly positive for at least some λ, we also know that $x(\lambda^E) > 0$.

The analogue of Assumption 1 above which guarantees that it is socially efficient for the agents to draw-up a contract is easy to state in our new model.

Assumption 5: *Let $x(\lambda^E) > c_A + c_B$.*

The value of the distribution parameter in any equilibrium in which a contract is drawn-up is easy to characterise. Consider the subgame which begins after both agents have paid the ex-ante costs (c_A, c_B). Then A will make an offer to B which maximizes his continuation payoff, subject to the constraint that it should be in B's interest to accept the offer. In other words, A will offer to B a λ^* which solves

$$\max_{\lambda \in [0,1]} \lambda x(\lambda) \qquad \text{s.t.} \quad (1 - \lambda)x(\lambda) \geq 0 \tag{5}$$

The solution to (5) is easy to find once we notice that the shadow price of the constraint in (5) is always zero because $x(\cdot)$ is non-negative. Moreover, since $x(\cdot)$ is strictly positive for some λ in $[0, 1]$, it must be that $\lambda^* x(\lambda^*) > 0$. Using these facts and the first order conditions for (5) it is immediate that

REMARK 3: *If* $x'(1) + x(1) > 0$ *then* $\lambda^* = 1$. *Conversely, if* $x'(1) + x(1) < 0$ *then* $\lambda^* \in (0, 1)$. *Moreover, if* $x'(1) + x(1) < 0$ *then* $x(\lambda^*) < x(\lambda^E)$.

We are now in a position to identify the range of ex-ante costs for which the fact that the size of the surplus depends on its distribution is sufficient to resolve the hold up problem described in Section 6.

We state Proposition 15 below without proof since it is an immediate consequence of Remark 3.

PROPOSITION 15: *Assume that* $x'(1)+x(1) < 0$. *Then there exists a range of ex-ante costs — namely* $\mathcal{C}_\lambda = \{c_A, c_B \mid 0 < c_A < \lambda^*$ *and* $0 < c_B < (1 - \lambda^*)\}$ *— which satisfy Assumption 5 and such that the model has an equilibrium in which both agents pay the ex-ante costs and a contract is drawn-up.*

If A finds it profitable to 'bribe' B in order to increase the overall size of the surplus, it is possible that, in equilibrium, both agents will pay strictly positive ex-ante costs and draw-up a contract. This is in contrast to Case 1 of Proposition 13 above in which a contract is never drawn-up, regardless of the magnitude and distribution of the ex-ante costs.

A final observation is in order. The possible equilibria identified in Proposition 15 in which a contract is drawn-up are still inefficient in the sense that $x(\lambda^*) < x(\lambda^E)$. Since A only appropriates a fraction of the surplus, it is not in his interest to offer B a λ which guarantees a global maximum of x.

8. CONCLUDING REMARKS

If the parties to a contract need to sink some ex-ante contractual costs before they can reach the contract-negotiating phase of their interaction, the ex-ante costs may generate a version of the hold-up problem. If the distribution of ex-ante costs and the distribution of the surplus generated by the contract are sufficiently 'mis-matched', one of the two parties to the contract will not find it to his advantage to pay the ex-ante contractual cost, even though the surplus generated by the contract would be sufficient to cover the total ex-ante costs associated with it. Therefore, in equilibrium the contract will not be written. We have verified this claim in a variety of simple models, including a number of extensive forms for the negotiation of the distribution of surplus among the agents. The hold-up problem we have identified is most acute when the agents have a continuous choice of ex-ante costs.

Unlike many other versions of this problem, under appropriate conditions, the hold-up problem generated by ex-ante contractual costs is unlikely to have a 'contractual solution'. This is because a 'contract over a contract' is likely to generate a fresh set of ex-ante contractual costs and hence a new hold-up problem. We have found this to be true in a variety of settings which include a whole hierarchy of 'contracts over contracts'.

Lastly, we have explored a reduced form model in which the size of the surplus depends on its distribution. In this case it is apparent that the hold-up problem we identify is less pervasive than in the case in which the size of the surplus is not affected by its distribution.

APPENDIX

Proof of Proposition 6: Since either $\lambda < c_A$ or $1-\lambda < c_B$, it is clear that there is no equilibrium in which both agents pay the ex-ante cost at $t=0$.

We only show that it is not possible that in any pure strategy equilibrium A alone pays the ex-ante cost at $t=0$. Any equilibrium in which B alone pays the ex-ante cost at $t=0$ can be ruled out in a symmetric way and we omit the details. Mixed strategy equilibria can be ruled out using standard arguments and we omit the details.

Suppose then that there is an equilibrium in which only A pays the ex-ante cost at $t=0$. There are two cases to consider. Either B pays his cost to see A's offer or he does not.

Suppose next that there is an equilibrium in which A only pays the ex-ante costs at $t=0$ and subsequently B either accepts or rejects A's offer without seeing it. Note that in this case B cannot condition his decision to accept or reject on the value of ℓ since he does not pay to see it. If B accepts in equilibrium, clearly A will set $\ell = -\epsilon$. But this would give an equilibrium payoff of $-\epsilon$ to B, and therefore yields a contradiction since B can always guarantee himself a payoff of zero by not paying any costs and rejecting any offer. If B rejects A's offer blind in equilibrium, then A's equilibrium payoff is $-c_A$ since no contract is drawn-up and A pays his ex-ante cost at $t = 0$. This is again a contradiction since A can guarantee himself a payoff of zero by not paying the ex-ante cost at $t = 0$ (and rejecting any offers made by B if he pays his ex-ante cost).

Lastly, consider the possibility of an equilibrium in which A alone pays the ex-ante costs at $t = 0$ and subsequently B pays his ex-ante cost c'_B to see the value of ℓ, and then accepts or rejects A's offer. Notice that now B can condition his decision to accept or reject A's offer on the actual value of ℓ. Using subgame perfection, it is immediate to see that, in equilibrium, it must be the case that B accepts all offers which guarantee that $1 - \ell > 0$ (his ex-ante cost is *sunk* when the accept/reject decision is made). Therefore, in equilibrium, A will offer precisely $\ell = 1$. It follows that in any equilibrium in which A alone pays the ex-ante cost at $t = 0$ and subsequently B pays to see A's offer, B's payoff is at most $-c'_B$. But this is a contradiction since B, as before, can guarantee himself a payoff of zero by not paying any costs and rejecting any offer. ■

DEFINITION A.1: *Let any two players finite-stage extensive form game Γ be given. Denote by $\Gamma_0^1, \ldots, \Gamma_0^{m_0}$ all the terminal subgames (subgames that do not have any proper subgame) and by $\Gamma_1^1, \ldots, \Gamma_1^{m_1}$ all the subgames which have only one proper subgame. Recursively we can then denote $\Gamma_i^1, \ldots, \Gamma_i^{m_i}$ all the i-level subgames.*

A strategy profile (s_A, s_B) constitutes a renegotiation-proof subgame perfect equilibrium of Γ if and only if it satisfies the following set of recursive conditions.

0. In every terminal subgame $\Gamma_0^1, \ldots, \Gamma_0^{m_0}$ the strategy profile (s_A, s_B) prescribes the play of a Nash equilibrium which is not strongly Pareto dominated by any other Nash equilibrium of the same subgame.

1. In every subgame $\Gamma_1^1, \ldots, \Gamma_1^{m_1}$ the strategy profile (s_A, s_B) prescribes the play of a subgame perfect equilibrium which satisfies condition 0 and which is not strongly Pareto dominated by any other subgame perfect equilibrium of the same subgame which also satisfies condition 0.

....

i. In every subgame $\Gamma_i^1, \ldots, \Gamma_i^{m_i}$ the strategy profile (s_A, s_B) prescribes the play of a subgame perfect equilibrium which satisfies conditions 0 through $i - 1$ and which is not strongly Pareto dominated by any other subgame perfect equilibrium of the same subgame which also satisfies conditions 0 through $i - 1$.

LEMMA A.1: *Consider the terminal subgame of the model with simple compensating transfers described in Subsection 4.1 which occurs after the pair of compensating transfers (σ_A, σ_B) has been agreed, as a function of the pair (σ_A, σ_B). If the following inequalities are satisfied*

$$\lambda - \sigma_A + \sigma_B - c_A^0 \geq 0 \quad \text{(A.1)}$$

$$1 - \lambda + \sigma_A - \sigma_B - c_B^0 \geq 0 \quad \text{(A.2)}$$

the subgame has two equilibria. If both inequalities are strict one equilibrium strictly Pareto-dominates the other. The Pareto-superior equilibrium is such that both parties pay the ex-ante costs (c_A^0, c_B^0) *and the contract is drawn-up leaving the parties with continuation payoffs* $(\lambda-\sigma_A+\sigma_B-c_A^0)$ *and* $(1-\lambda+\sigma_A-\sigma_B-c_B^0)$. *The inferior equilibrium is such that both parties do not pay the ex-ante costs* (c_A^0, c_B^0) *and yields the no-contract outcome. If either or both inequalities (A.1) and (A.2) are violated the terminal subgame has a unique equilibrium in which neither agent pays the ex-ante costs* (c_A, c_B), *and hence yields the no-contract outcome.*

PROOF: The claim follows immediately from the fact that a contract is feasible only if both A and B pay the ex-ante costs (c_A^0, c_B^0), and from the observation that either agent i can guarantee a continuation payoff of zero by not paying his ex-ante cost c_i^0. ■

LEMMA A.2: *Consider the reduced form model with simple compensating transfers described in Subsection 4.1. If there exists an equilibrium of the model in which both* $\sigma_A > 0$ *and* $\sigma_B > 0$, *then there exists another, payoff equivalent, equilibrium of the model in which the transfers take the values* $\tilde{\sigma}_A = \sigma_A - \sigma_B$ *and* $\tilde{\sigma}_B = 0$ *if* $\sigma_A \geq \sigma_B$, *and* $\tilde{\sigma}_A = 0$ *and* $\tilde{\sigma}_B = \sigma_B - \sigma_A$ *if* $\sigma_B \geq \sigma_A$.

PROOF: We examine only the case in which $\sigma_A \geq \sigma_B$. The other case is a simple re-labelling of this one. To construct the new equilibrium, let the strategies of both agents be identical to the strategies in the original equilibrium, except for the way actions are conditioned on the other agents' compensating transfer offer. In the new equilibrium, each agent $i \in \{A, B\}$ responds to any offer $\tilde{\sigma}_j$ (with $j \neq i$) exactly as he would respond to the offer $\tilde{\sigma}_j + \sigma_i$ in the original equilibrium. ■

PROOF OF PROPOSITION 7:

Recall that both tiers of ex-ante costs are payable simultaneously by the agents and that a contract (compensating transfer) is feasible only if both agents have paid the first (second) tier of ex-ante costs. Therefore it is obvious that a pair of strategies which prescribe not to pay any ex-ante costs for both agents (and some equilibrium behaviour off-the-equilibrium-path) constitutes an equilibrium. This proves our first claim.

We now move to the construction of a subgame perfect equilibrium of the model with simple compensating transfers in which the parties do draw-up a contract.

We only deal with the case in which $1-\lambda < c_B^0$. The case in which $\lambda < c_A^0$ is a simple re-labelling of this one and we omit the details.

Consider the subgame occurring after the transfers (σ_A, σ_B) have been agreed. If $\sigma_B \geq \sigma_A$ the only equilibrium of this subgame is such that both parties do not pay the ex-ante costs (c_A^2, c_B^2). If instead $\sigma_A \geq \sigma_B$ then by Lemma A.2 we can restrict attention to transfers which satisfy $\sigma_A > 0$ and $\sigma_B = 0$.

If σ_A is such that inequalities (A.1) and

$$1-\lambda+\sigma_A \geq c_B^0+c_B^2 \tag{A.3}$$

are satisfied, we assume that the agents play the Pareto-superior of the two equilibria described in Lemma A.1 in which the contract is drawn-up. If instead σ_A is such that inequality (A.3) is *not* satisfied while (A.1) and (A.2) are satisfied we assume that the agents play the Pareto-inferior of the two equilibria described in Lemma A.1 that yields the no-contract outcome. In case either or both (A.1) and (A.2) are violated then the agents play the unique subgame perfect equilibrium of the subgame.

Proceeding backwards, it is then a best reply for B to accept any offer $\sigma_A > 0$ such that inequality (A.3) is satisfied. Indeed, if B rejects the offer his continuation payoff is zero while by accepting the offer his continuation payoff is non-negative.

It is then optimal for A to make an offer σ_A such that

$$\sigma_A = c_B^2+c_B^0-(1-\lambda) \tag{A.4}$$

This offer is associated with a positive continuation payoff for A. A higher offer is associated with a smaller continuation payoff while a lower offer is associated with a continuation payoff of zero, since the parties expect to play the inefficient equilibrium whenever (A.3) is violated.

Therefore, in equilibrium both parties pay the second tier ex-ante costs (c_A^2, c_B^2). Paying the cost, B obtains the payoff of zero which coincides with the payoff he gets by not paying. By paying, A gets a strictly positive payoff while he gets a payoff of zero by not paying. This concludes the proof. ■

Proof of Proposition 8: We only deal with the case in which $1-\lambda < c_B^0$. The case in which $\lambda < c_A^0$ is a simple re-labelling of this one and we omit the details.

Since we are assuming that the parameters of the model yield the no contract outcome in the final stage, any renegotiation-proof subgame perfect equilibrium which yields a contingent contract as an outcome must have both agents paying both tiers of ex-ante costs.

Assume by way of contradiction that such an equilibrium exists and denote by a superscript '$*$' the equilibrium values of all variables in this equilibrium.

Notice first of all that if $\sigma_B^* \geq \sigma_A^*$ we have an immediate contradiction since in this case $\gamma_B^* > c_B$ and therefore B's equilibrium payoff must be negative. Since B can guarantee a payoff of zero by not paying any of the ex-ante costs this is a contradiction.

By Lemma A.2, we can then assume without loss of generality that $\sigma_A^* > 0$ and $\sigma_B^* = 0$.

Next, consider the subgame that starts after the transfers (σ_A^*, σ_B^*) have been agreed. We now claim that every renegotiation-proof subgame perfect equilibrium must be such that

$$1 - \lambda + \sigma_A^* - c_B^0 = 0 \tag{A.5}$$

To see this notice that Definition A.1 and Lemma A.1 imply that every renegotiation-proof subgame perfect equilibrium must prescribe that in this subgame when

$$\lambda - \sigma_A - c_A^0 \quad > \quad 0 \tag{A.6}$$

$$1 - \lambda + \sigma_A - c_B^0 \quad > \quad 0 \tag{A.7}$$

are satisfied the parties play the Pareto superior equilibrium. This equilibrium involves both agents paying the costs (c_A^0, c_B^0), drawing-up the contract and obtaining the strictly positive continuation payoffs: $1 - \lambda + \sigma_A - c_B^0$ and $\lambda - \sigma_A - c_A^0$. However, any offer σ_A which satisfies (A.7) cannot be payoff maximizing for A. Therefore the only renegotiation-proof subgame perfect equilibrium offer has to satisfy (A.5).

It follows directly from (A.5) that B's payoff in this renegotiation-proof subgame perfect equilibrium would be $-c_B^2$. But this is a contradiction since B can guarantee a payoff of zero by not paying any ex-ante costs. This is enough to prove the proposition. ■

Proof of Proposition 9: The proof can be constructed in a way which is completely analogous to the one of Proposition 8 once we observe that when inequalities (A.6) and (A.7) are satisfied the unique subgame perfect equilibrium of the terminal subgame starting at $t = 0$ is for both parties to pay the costs (c_A^0, c_B^0), draw-up a contract and obtain payoffs: $1 - \lambda + \sigma_A - c_B^0$ and $\lambda - \sigma_A - c_A^0$.

Indeed, if A has paid his cost c_A^0 it is optimal for B to pay the cost c_B^0 as well given that B obtains a strictly positive continuation payoff by doing so, while he gets a continuation payoff of zero by not paying. On the other hand, if A does not pay his ex-ante cost c_A^0 then it is optimal for B not to pay his cost c_B^0 either. If B does not pay he gets a payoff of zero while if he does pay he gets a negative payoff. Therefore the unique subgame perfect equilibrium of this subgame is for both parties to pay their costs (c_A^0, c_B^0) and draw-up a contract.

We omit the details of the remaining part of the proof. ■

Lemma A.3: *Consider the reduced form model with N tiers of simple compensating transfers described in Subsection 4.2. Given any equilibrium of the model, let $\ell_n = \arg\min_{i \in \{A,B\}} \sigma_i^n$, for $n = 1, \ldots, N$. Then there exists another, payoff equivalent, equilibrium of the model in which the compensating transfers are $\tilde{\sigma}_i^n = \sigma_i^n - \sigma_{\ell_n}^n$.*

Proof: The new equilibrium can be constructed in a way which is completely analogous to the one in the proof of Lemma A.2. We omit the details. ■

Proof of Proposition 10 (Simple Transfers): We only deal with the case in which $1 - \lambda < c_B^0$. The case in which $\lambda < c_A$ is a simple re-labelling of this one and we omit the details.

Notice that when $N = 1$, the reduced form model with N tiers of simple compensating transfers of Subsection 4.2 is identical to the reduced form model with simple compensating transfers of Subsection 4.1. It follows that Proposition 7 implies that Proposition 10 is true when $N = 1$. To prove the proposition it then remains to show that, if it is true for a given value of N, then it is also true for $N + 1$.

Suppose then that the proposition is true for a given value of N, and assume, by way of contradiction, that it is false for $N+1$. Since the proposition is true for N, if there exists a renegotiation-proof subgame perfect equilibrium of the reduced form model with $N + 1$ tiers of simple compensating transfers in which some ex-ante costs are paid, it must be that in such equilibrium both agents pay the time $t = -N - 1$ ex-ante costs. Let the values of all choice variables in such equilibrium be denoted by superscript '$*$'.

In a way analogous to the proof of Proposition 8, using Lemma A.3 we can now assume without loss of generality that $\sigma_A^{N+1*} > 0$ and $\sigma_B^{N+1*} = 0$. Again as in the proof of Proposition 8 this implies that in any renegotiation-proof subgame perfect equilibrium B's payoff in this equilibrium is $-c_B^{N+1}$. But this is clearly a contradiction since B can guarantee himself a payoff of zero by not paying any tier of ex-ante costs. The proof of Proposition 10 for the case of N tiers of simple compensating transfers is therefore complete. ■

Lemma A.4: *Consider the reduced form model with N tiers of multiple compensating transfers described in Subsection 4.2. Given any equilibrium of the model, let $\ell_{n,m} = \arg\min_{i\in\{A,B\}} \sigma_i^{n,m}$, for $n = 1, \ldots, N$ and $m = 0, \ldots, n-1$. Then there exists another, payoff equivalent, equilibrium of the model in which the compensating transfers are $\tilde{\sigma}_i^{n,m} = \sigma_i^{n,m} - \sigma_{\ell_{n,m}}^{n,m}$.*

Proof: The new equilibrium can be constructed in a way which is completely analogous to the one in the proof of Lemma A.2. We omit the details. ■

Proof of Proposition 10 (Multiple Transfers): We only deal with the case in which $1 - \lambda < c_B^0$. The case in which $\lambda < c_A$ is a simple re-labelling of this one and we omit the details.

Notice that when $N = 1$, the reduced form model with N tiers of multiple compensating transfers of Subsection 4.2 is identical to the reduced form model with simple compensating transfers of Subsection 4.1. It follows that Proposition 7 implies that Proposition 10 is true when $N = 1$. To prove the proposition it then remains to show that, if it is true for a given value of N, then it is also true for $N + 1$.

Suppose then that the proposition is true for a given value of N, and assume, by way of contradiction, that it is false for $N+1$. Since the proposition is true for N, if there is a renegotiation-proof subgame perfect equilibrium of the reduced form model with $N + 1$ tiers of multiple compensating transfers in which some ex-ante costs are paid, it must be that in such equilibrium both agents pay the time $t = -N - 1$ ex-ante costs. Let the values of all choice variables in such equilibrium be denoted by superscript '$*$'.

In a way analogous to the proof of Proposition 8, using Lemma A.4 we can now assume without loss of generality that $\sigma_A^{N+1,m*} > 0$ for some $m = 0, \ldots, N$ and $\sigma_B^{N+1,m*} = 0$ for all $m = 0, \ldots, N$. Again as in the proof of Proposition 8 this implies that B's payoff in any renegotiation-proof subgame perfect equilibrium is $-c_B^{N+1}$. But this is clearly a contradiction since i can guarantee himself a payoff of zero by not paying any tier of ex-ante costs. The proof of Proposition 10 for the case of N tiers of multiple compensating transfers is therefore complete. ■

LEMMA A.5: *Let any value of the distribution parameter $\lambda \in [0,1]$ be given. In equilibrium, in the model with a continuous choice of ex-ante costs, it is not possible for both agents to pay an inefficiently high level of ex-ante contractual costs in the sense that $c_i^* > c_i^E$ for all $i \in \{A, B\}$. This is true irrespective of the sign of the cross partial derivative $x_{A,B}(c_A, c_B)$.*

PROOF: Let $\beta = c_A^* - c_A^E$ and $\delta = c_B^* - c_B^E$. Therefore, as k varies in $[0,1]$, $(c_A^E + k\,\beta, c_B^E + k\,\delta)$ describes the whole segment between (c_A^E, c_B^E) and (c_A^*, c_B^*). Assume, by way of contradiction, that $\beta > 0$ and $\delta > 0$. The first order conditions (1) and (2) imply that $x_A(c_A^E + \beta, c_B^E + \delta) > x_A(c_A^E, c_B^E)$. Therefore for some k it must be the case that the derivative of $x_A(c_A^E + k\,\beta, c_B^E + k\,\delta)$ with respect to k is strictly greater than 0. In other words

$$x_{A,A}\,\beta + x_{A,B}\,\delta > 0 \tag{A.8}$$

Since $\beta > 0$, (A.8) implies

$$x_{A,A}\beta^2 + x_{A,B}\delta\beta > 0. \tag{A.9}$$

Proceeding in a completely symmetric way we also obtain that

$$x_{B,B}\delta + x_{B,A}\beta > 0 \tag{A.10}$$

Since $\delta > 0$, (A.10) implies

$$x_{B,B}\delta^2 + x_{B,A}\delta\beta > 0 \tag{A.11}$$

Inequalities (A.9) and (A.11) imply that

$$x_{A,A}\beta^2 + x_{B,B}\delta^2 + 2x_{A,B}\delta\beta > 0 \tag{A.12}$$

which contradicts the fact that, by strict concavity the Jacobian of $x(\cdot,\cdot)$ must be negative definite. ■

Proof of Proposition 12: From Lemma A.5 we know that either $\beta \leq 0$ or $\delta \leq 0$. We only consider the case in which $\beta \leq 0$. The case in which $\delta \leq 0$ can be treated in a symmetric way, and we omit the details.

Assume that the proposition is false. Then $\beta \leq 0$ and $\delta \geq 0$. By concavity $x_{BB} < 0$, and by assumption $x_{AB} > 0$. Therefore

$$x_{BB}\delta + x_{AB}\beta \leq 0 \tag{A.13}$$

Since (A.13) contradicts (A.10) this is enough to prove the claim. ■

Proof of Proposition 13 in Case 1: Assume that the proposition is false and hence that there is an equilibrium in which both agents pay their ex-ante costs.

Consider now the subgame in which both parties have already sunk the ex-ante costs (c_A, c_B). Clearly B will accept any offer from A which guarantees that

$$1 - \lambda > 0 \tag{A.14}$$

It now follows that the highest offer which A will possibly make in equilibrium guarantees that

$$1 - \lambda = 0 \tag{A.15}$$

But (A.15) implies that B's payoff in any equilibrium in which a contract is drawn-up is at most $-c_B$. This is a contradiction since B can guarantee himself a payoff of 0 by simply not paying his ex-ante cost. This is enough to prove the proposition. ■

Proof of Proposition 13 in Case 2: The argument is a simple modification of the proof of Proposition 13 for Case 1. Observe that B cannot condition his paying c_B on the actual offer λ (since this is un-observable before c_B is paid). In the subgame which starts after both agents have paid the ex-ante costs, B will accept any offer which satisfies (A.14). Therefore, the rest of the proof of Proposition 13 for Case 1 applies. ■

Proof of Proposition 13 in Case 3: Using sub-game perfection, the proof follows immediately from the proofs of Proposition 13 in Case 1 and from the proof obtained swapping the names A and B in the proof of Proposition 13 in Case 1. We omit the details. ■

Proof of Proposition 13 in Case 4: We prove the result by backward induction. Note that the extensive form in Case 1 is equivalent to Case 4 when $N = 1$ and A makes the first offer. Therefore, from Proposition 13 in Case 1 we know that the claim is true for $N = 1$. In the case in which B makes the first offer and $N = 1$ then the proof is easily obtained by swapping the names of A and

B in the proof of Proposition 13 in Case 1. It is therefore enough to show that if the claim is true for N then it is also true for $N+1$.

We prove the backward induction step for the case in which it is A who makes an offer to B in the $N+1$-th period, $t=N$. The details for the case in which B makes an offer to A at $t=N$ are symmetric and therefore omitted.

Assume, by way of contradiction, that the claim is true for N, but false for $N+1$. Consider any equilibrium of the game with $N+1$ rounds of negotiation in which either A or B does not sink his ex-ante cost c_A^0 or c_B^0. Since the claim is true for N, clearly this equilibrium yields the no-contract outcome. Therefore if the claim is true for N and false for $N+1$, then there must be an equilibrium for the model with $N+1$ rounds of negotiation in which in the last period both agents sink the ex-ante costs c_A^0 and c_B^0. Consider now the subgame which starts in stage II of period 0 in which both agents have already sunk these costs. Clearly, at this point B will accept any offer from A which guarantees that

$$1-\lambda^0>0$$

It therefore follows that the highest offer which A will possibly make in this equilibrium guarantees that

$$1-\lambda^0=0 \tag{A.16}$$

Notice next that (A.16) implies that in any equilibrium in which the statement is true for N but false for $N+1$, B's payoff in the entire game is at most $-c_B^0$. But this is clearly a contradiction since B can guarantee himself a payoff of 0 by not paying any of his ex-ante costs. ■

LEMMA A.6: *Consider the extensive form described in Case 5. Whatever the values of c_A and c_B, there exists an equilibrium of the model in which neither player pays his participation cost in any period, and therefore a contract is not drawn-up.*

PROOF: We simply display a pair of strategies (s_A^0, s_B^0) which constitute an equilibrium of the model and which yield the desired outcome.

For all $i \in \{A, B\}$, the strategy s_i^0 is described as follows. In stage I of any period s_i^0 prescribes that i does not pay his participation cost, regardless of the previous history of play. In stage II of any period t in which i makes an offer, s_i^0 prescribes that i demands the entire surplus for himself ($\lambda^t=1$ if $i=A$ and $\lambda^t=0$ if $i=B$), regardless of the previous history of play. In stage III of any period t in which i must accept or reject, s_i^0 prescribes that i accepts any offer $\lambda^t\in[0,1]$, regardless of the previous history of play. It is easy to check that these strategies constitute an equilibrium of the model, and therefore this is enough to prove the claim. ■

LEMMA A.7: *Consider the extensive form described in Case 5. Assume that c_i for $i \in \{A, B\}$ are such that the model has an equilibrium in which a contract is drawn-up.*

Let λ^L be the infimum and λ^H the supremum of all possible equilibrium distribution parameters over the entire set of equilibria of the model in which a contract is drawn-up. Both λ^H and λ^L are undefined if the set of equilibria of the model in which a contract is drawn-up is empty. Let also λ_i^L be the infimum and λ_i^H the supremum of all possible equilibrium distribution parameters over the set of equilibria in which a contract is drawn-up in a period in which i is the proposer (the set of i equilibria). Leave both λ_i^L and λ_i^H undefined if the set of i equilibria is empty.

Then λ_i^L and λ_i^H are defined for all $i \in \{A, B\}$, and they satisfy $\lambda_A^H = \lambda^H \leq 1 - c_B$, $\lambda_B^L = \lambda^L \geq c_A$ as well as

$$\lambda_B^H \leq \lambda_A^H - c_A \tag{A.17}$$

and

$$\lambda_A^L \geq \lambda_B^L + c_B \tag{A.18}$$

PROOF: Clearly in any equilibria of the model the payoffs to both agents must be non-negative. The fact that it must be that $\lambda^H \leq 1 - c_B$ and $\lambda^L \geq c_A$ is now obvious since if either inequality is violated one of the two agents would get a negative payoff in equilibrium.

By hypothesis, the set of equilibria in which a contract is drawn-up is not empty. Therefore, either the set of A equilibria is not empty, or the set of B equilibria is not empty, or both are not empty.

If the set of B equilibria is not empty we must have that

$$\lambda_B^H \leq \lambda^H - c_A \tag{A.19}$$

To see this, consider the subgame which starts in stage III of the period in which a contract is drawn-up. If A rejects B's offer at this stage, he will get a continuation payoff which is bounded above by $\lambda^H - c_A$. Therefore, it must be that A's equilibrium strategy prescribes to accept *any* offer above $\lambda^H - c_A$. Therefore, in stage II of this period, B's equilibrium strategy cannot be one that offers any $\lambda > \lambda^H - c_A$, since otherwise he could reduce his offer by a small amount and A would still respond by accepting the offer. Therefore B's offer must be some $\lambda \leq \lambda^H - c_A$, and this is clearly enough to prove that (A.19) must hold in this case.

Notice next that (A.19) also implies the following. If the set of B equilibria is not empty, then the set of A equilibria must also be not empty. This is because (A.19) says that $\lambda_B^H < \lambda^H$ so that it must be that case that $\lambda_A^H = \lambda^H$.

Using a completely symmetric argument to the one above, we can now argue that if the set of A equilibria is not empty then

$$\lambda_A^L \geq \lambda_B^L + c_B \tag{A.20}$$

must hold.

Using a symmetric argument again, we can then see that (A.20) implies that if the set of A equilibria is not empty than it must be the case that the set of B equilibria is not empty either. Indeed, it must be the case that $\lambda_B^L = \lambda^L$.

Since we have just argued that either the sets of A and B equilibria are both empty or both not empty, and by hypothesis at least one is in fact not empty, (A.19) and (A.20) are enough to prove the claim. ■

Proof of the 'If' Part of Proposition 14: Using Lemma A.7, we know that if the set of equilibria in which a contract is drawn-up is not empty we must have that

$$\begin{array}{ll} \lambda_A^H \leq 1 - c_B & \lambda_A^L \geq c_A + c_B \\ \lambda_B^H \leq 1 - c_A - c_B & \lambda_B^L \geq c_A \end{array} \tag{A.21}$$

Recalling that, by definition, $\lambda_i^H \geq \lambda_i^L$ for $i \in \{A, B\}$, (A.21) directly implies $1 - c_A - c_B \geq \max\{c_A, c_B\}$. This is clearly enough to prove the claim. ■

Proof of the 'Only If' part of Proposition 14: We proceed by construction. Assume that $1 - c_A - c_B \geq \max\{c_A, c_B\}$. We then display a pair of strategies (s_A^*, s_B^*) which prescribe that a contract is drawn-up in period $t = 0$, and which constitute an equilibrium of the model.

To describe (s_A^*, s_B^*), it will be useful to distinguish between two types of possible histories of past play. We say that a history of past play h^t is of type α if and only if at some point along h^t one of the following three things has taken place. Either one of the two agents (or both) has not paid his ex-ante cost, or A has rejected an offer $\lambda \geq c_A$, or B has rejected an offer $\lambda \leq 1 - c_B$. Any history of play h^t which is not of type α (including the 'empty history') is called a history of type β.

The description of (s_A^*, s_B^*) can now be made explicit. Following any history of type α, (s_A^*, s_B^*) are exactly the same as the strategies (s_A^0, s_B^0) described in the proof of Lemma A.6.

Following any history of type β, if t is such that it is A's turn to make an offer to B, the strategies (s_A^*, s_B^*) prescribe that A offers $\lambda^t = 1 - c_B$ to B and subsequently that B accepts any offer $\lambda^t \leq 1 - c_B$. Following any history of type β, if t is such that it is B's turn to make an offer to A, the strategies (s_A^*, s_B^*) prescribe that B offers $\lambda^t = c_A$ to A and subsequently that A accepts any offer $\lambda^t \geq c_A$.

Clearly, when the strategies (s_A^*, s_B^*) are played, a contract is drawn-up in period $t = 0$ with distribution parameter $\lambda^0 = 1 - c_B$. Therefore, it only remains to check that (s_A^*, s_B^*) constitute an equilibrium of the model. We do this by verifying that neither agent can profit from a deviation in any period t in which it is A's turn to make an offer to B. The argument for the periods in which it is B's turn to make an offer to A is completely symmetric, and therefore the details are omitted.

If the previous history of play is of type α, it is clear that neither can profit from a deviation at any stage since (s_A^0, s_B^0) constitute an equilibrium of the model as in Lemma A.6. Assume therefore that the previous history of play is of type β. In stage I of period t, if either player does not pay his ex-ante cost as prescribed, his continuation payoff is zero. Adhering to the strategies (s_A^*, s_B^*) yields a continuation payoff of $1 - c_A - c_B > 0$ to A and of 0 to B. Therefore neither agent can profit from a deviation in stage I of period t.

Consider now stage II of period t. If A offers $\lambda^t > 1 - c_B$, B will reject, the negotiation will proceed to the next period with a type β history of play, and therefore A's payoff from this deviation is 0. Playing according to s_A^* in stage II yields a continuation payoff of $1 - c_B > 0$ to A. Therefore he cannot profit from deviating and offering $\lambda^t > 1 - c_B$. If in stage II of period t A offers $\lambda^t < 1 - c_B$, B will accept the offer. Therefore in this case A's continuation payoff is $\lambda^t < 1 - c_B$. It follows that this is also not a profitable deviation for A.

Finally, consider stage III of period t. Suppose that A has made an offer $\lambda^t \leq 1 - c_B$. If B rejects this offer the history of play becomes of type α and therefore B's continuation payoff is zero. However, if B plays according to s_B^* and accepts the offer his continuation payoff is $1 - \lambda^t \geq c_B$. Therefore B cannot profit from deviating and rejecting any offer $\lambda^t \leq 1 - c_B$ in stage III of period t. Lastly, suppose that A has made an offer $\lambda^t > 1 - c_B$. If B accepts the offer, his continuation payoff is $1 - \lambda^t < c_B$. On the other hand, if B plays according to s_B^* and rejects the offer, the negotiation moves on to period $t + 1$, with a history of type β. It follows that B's continuation payoff in this case is $1 - c_A - c_B \geq \max\{c_A, c_B\}$. Therefore accepting $\lambda^t > 1 - c_B$ is not a profitable deviation for B. This is clearly enough to prove our claim. ■

References

Abreu, D., and A. Rubinstein (1988): "The Structure of Nash Equilibrium in Repeated Games with Finite Automata," *Econometrica*, 56, 1259–81.

Aghion, P., M. Dewatripont, and P. Rey (1994): "Renegotiation Design with Unverifiable Information," *Econometrica*, 62, 257–82.

Aghion, P., and J. Tirole (1997): "Formal and Real Authority in Organizations," *Journal of Political Economy*, forthcoming.

Anderlini, L., and L. Felli (1996): "Costly Contingent Contracts," Theoretical Economics Discussion Paper TE/96/313, STICERD, London School of Economics.

——— (1997): "Incomplete Contracts and Complexity Costs," *Theory and Decision*, forthcoming.

Benoît, J.-P., and V. Krishna (1993): "Renegotiation in Finitely Repeated Games," *Econometrica*, 61, 303–323.

Chung, T. Y. (1991): "Incomplete Contracts, Specific Investments, and Risk Sharing," *Review of Economic Studies*, 58, 1031–42.

Coase, R. H. (1960): "The Problem of Social Cost," *The Journal of Law and Economics*, 3, 1–44.

Dixit, A., and M. Olson (1997): "Does Voluntary Participation Undermine the Coase Theorem?," Princeton University, mimeo.

Dye, R. A. (1985): "Costly Contract Contingencies," *International Economic Review*, 26, 233–50.

Farrell, J., and E. Maskin (1989): "Renegotiation in Repeated Games," *Games and Economic Behavior*, 1, 327–60.

Grossman, S. J., and O. D. Hart (1986): "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 94, 691–719.

Grout, P. (1984): "Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Solution," *Econometrica*, 52, 449–460.

Hart, O. D., and J. Moore (1988): "Incomplete Contracts and Renegotiation," *Econometrica*, 56, 755–85.

——— (1990): "Property Rights and the Nature of the Firm," *Journal of Political Economy*, 98, 1119–58.

Holmström, B. (1982): "Moral Hazard in Teams," *Bell Journal of Economics*, 13, 324–340.

Lipman, B. L. (1997): "Limited Rationality and Endogenously Incomplete Contracts," University of Western Ontario, mimeo.

Maskin, E., and J. Tirole (1997): "Unforseen Contingencies, Property Rights, and Incomplete Contracts," Harvard University, mimeo.

Nicholson, W. (1989): *Microeconomic Theory: Basic Principles and Extensions.* Chicago: The Dryden Press. Fourth International Edition.

Nöldeke, G., and K. M. Schmidt (1995): "Option Contracts and Renegotiation: A Solution to the Hold-Up Problem," *RAND Journal of Economics*, 26, 163–179.

Piccione, M. (1992): "Finite Automata Equilibria with Discounting," *Journal of Economic Theory*, 56, 180–193.

Piccione, M., and A. Rubinstein (1993): "Finite Automata Play a Repeated Extensive Game," *Journal of Economic Theory*, 61, 160–168.

Rogerson, W. P. (1992): "Contractual Solutions to the Hold-Up Problem," *Review of Economic Studies*, 59, 777–793.

Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50, 97–110.

——— (1986): "Finite Automata Play the Repeated Prisoner's Dilemma," *Journal of Economic Theory*, 39, 83–96.

Rubinstein, A., and A. Wolinsky (1992): "Renegotiation-Proof Implementation and Time Preferences," *American Economic Review*, 82, 600–614.

Segal, I. (1995): "Complexity and Renegotiation: A Foundation for Incomplete Contracts," Harvard University, mimeo.

Shaked, A., and J. Sutton (1984): "Involuntary unemployment as a Perfect Equilibrium in a Bargaining Game," *Econometrica*, 52, 1351–1364.

Williamson, O. (1985): *The Economic Institutions of Capitalism.* New York: Free Press.