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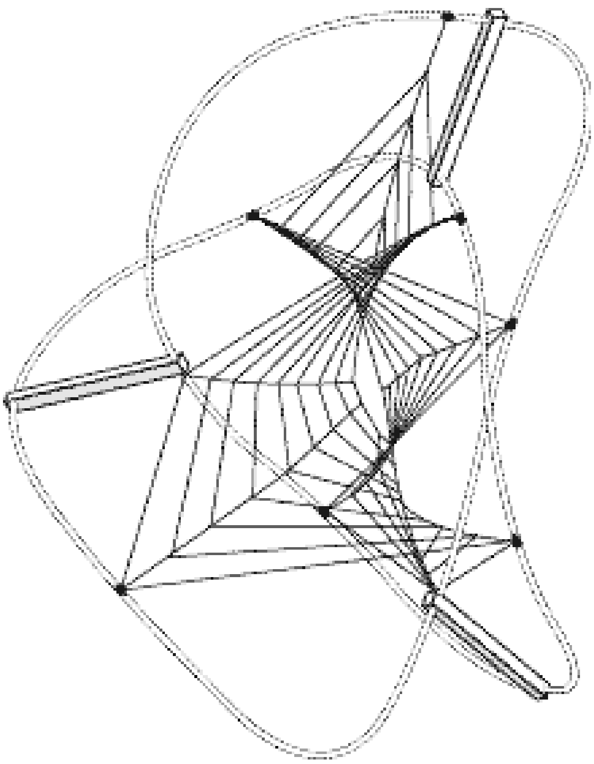
Selections, Dispositions and the Problem of Measurement

Mauricio Suárez
Universidad Complutense de Madrid



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Selections, Dispositions and the Problem of Measurement

Mauricio Suárez

Department of Philosophy
Bristol University
9 Woodland Road
Bristol BS8 1TB, UK

Abstract:

This paper expands on, and provides a qualified defence of, Arthur Fine's selective interactions solution to the measurement problem. Fine's approach must be understood against the background of the insolubility proof of the quantum measurement. I first defend the proof as an appropriate formal representation of the quantum measurement problem. Then I clarify the nature of selective interactions, and more generally *selections*, and I go on to offer three arguments in their favour. First, selections provide the only known solution to the measurement problem that does not relinquish any of the premises of the insolubility proofs. Second, unlike some no-collapse interpretations of quantum mechanics selections suffer no difficulties with non-ideal measurements. Third, unlike most collapse-interpretations selections can be independently motivated by an appeal to quantum dispositions.

1. Introduction.

In a series of papers in the late 1980's Arthur Fine proposed a novel solution to the quantum measurement problem, in terms of *selective interactions*, or as I shall call them, *selections*. The reception to Fine's approach has been nearly muted[1]. But in light of recent developments and difficulties with other proposals for solving the quantum measurement problem, it may be worth taking another look at Fine's proposal. In this paper I expand on Fine's original proposal by providing a general characterisation of selections that is independent of the measurement problem. I then defend selections as a valuable alternative to extant interpretations of quantum mechanics.

My defence of Fine's original proposal is thus a very qualified one. Unlike Fine I do not tie up the concept of a selection to a measurement interaction. I also reject Fine's own philosophical defence of selections as measuring "aspects". However, suitably re-interpreted as testing quantum dispositions, selections are coherent; and they have some definite advantages over other widely discussed options in the interpretation of quantum mechanics.

Although I shall argue that selections are not conceptually linked to the measurement problem, it is easier to introduce them formally in the context of the so-called insolubility proof of the measurement problem. The first part of the paper is devoted to a discussion of this proof. In section 2 of the paper I introduce some preliminary distinctions and notation, and I describe the basic intuition underlying the measurement problem. In section 3 I describe the premises of the insolubility proof and, in section 4, I defend it as an appropriate formal representation of the problem of measurement. In the second part of the paper I turn to the selections approach. Thus, in the fifth section I introduce the concept of a quantum selection, and I argue that selections are fully compatible with a) the unitary dynamics of the Schrödinger equation, and b) the denial of the ignorance interpretation of mixtures. I show that selections can solve the measurement problem without relinquishing any of the premises that generate the insolubility proof. In section 6, I show that selections have at least one advantage over several no-collapse interpretations: unlike these interpretations, selections can naturally accommodate non-ideal

[1] The exception is Stairs' [1991], whose reaction like mine was mixed. But my criticisms of Fine's approach are not Stairs'. On the contrary I believe that the characterization of selections provided in sections 2,4 and 5 dispenses with most of Stair's criticisms.

measurements. I then argue, in section 7, that the selections approach, unlike the collapse postulate, is not at all ad hoc, but can be motivated independently by an appeal to quantum dispositions.

PART I: THE INSOLUBILITY OF THE MEASUREMENT PROBLEM

2. The Problem of Quantum Measurement.

2.1. The Ignorance Interpretation of Mixtures

In the most general statistical operator formalism of quantum mechanics, systems can be in pure or in mixed states. A pure state is represented by an idempotent operator of trace one, that is, a projection operator $\boldsymbol{P}[\phi]$, upon a particular subspace ϕ of the Hilbert space. By contrast a mixed state, or a mixture, is a sum of such projectors upon pure states v_i with associated statistical weights (p_i, $0 \leq p_i \leq 1$, with $\Sigma p_i = 1$), represented by a non-idempotent operator of trace one: $W = \Sigma p_n \boldsymbol{P}[v_i]$.

A much-discussed interpretation of quantum mixtures is the ignorance interpretation. According to this interpretation, a quantum system is in a mixture $W = \Sigma p_n \boldsymbol{P}[v_i]$ if and only if the system is really in one of the pure states $\boldsymbol{P}[v_i]$, but we don't know which one. The probabilities $\{p_n\}$ are subjective and merely reflect our degree of ignorance. This interpretation, however, is not available for *improper* mixtures (roughly: those mixtures representing a subsystem of a larger system that result from the application of the axiom of reduction to the state of the larger system – notably the states of individual particles in an entangled EPR state).

The decisive argument against the ignorance interpretation of *improper* mixtures is as follows.[2] Consider a composite system S_{1+2} in a pure state $W_{1+2} = P_{[\psi]}$, where $\psi = \Sigma_{i,j}\, c_{ij}\, v_i \otimes w_j$, and where, as in previous sections, v_i, w_j are the eigenstates of A, B with corresponding eigenvalues a_i, b_j. The *reduced states* W_1, W_2 can be derived from the standard identifications (*, in appendix 1). We obtain: $W_1 = \Sigma_i\, c_{ii}\, c_{ii}^*\, v_i\, v_i^*$, and $W_2 = \Sigma_j\, c_{jj}\, c_{jj}^*\, w_j\, w_j^*$. W_1, W_2 are improper mixtures, found by derivation from the composite state W_{1+2}. Let us now assume that subsystem S_1 (S_2) is really in one of the states v_i (w_j) with probabilities $|c_{ii}|2$ ($|c_{jj}|2$). The state of the combined system can then be reconstructed, in the manner described in Appendix 1. We find that $W_{1+2} = \Sigma_i\, c_{ii}\, c_{ii}^*\, v_i\, v_i^* \otimes W_2$ (or $W_1 \otimes \Sigma_i\, c_{jj}\, c_{jj}^*\, w_j\, w_j^*$, or if both W_1 *and* W_2 are given the ignorance interpretation then: $W_{1+2} = \Sigma_i\, c_{ii}\, c_{ii}^*\, v_i\, v_i^* \otimes \Sigma_i\, c_{jj}\, c_{jj}^*\, w_j\, w_j^*$). Thus, on the assumption that W_1 (or W_2, or both) can be

[2] An incomplete version of this argument appears in Hughes (1989, pp. 149-151)

given the ignorance interpretation, we find that W_{1+2} is a mixture; but by hypothesis W_{1+2} is a pure state; therefore by *reductio*, neither W_1 nor W_2 can be given the ignorance interpretation.

The ignorance interpretation is nonetheless often thought to be required for a satisfactory solution of the problem of measurement. For it is often assumed that a solution to the problem of measurement requires that the final state of the measuring device be a pure state, namely an eigenstate of the pointer position observable. One aim of this paper is to show that this assumption is mistaken.

2.2. The Eigenstate-Eigenvalue link

We can express in this framework the orthodox interpretative principle of quantum mechanics, the *eigenstate-eigenvalue link*. The basic version of this principle is often formulated as follows:

> *(basic e/e link)*: A system has a value o_1 of a physical property *O* if and only if the system's state *v* is an eigenvector of the self-adjoint operator $\boldsymbol{O}$ that represents this physical property (i.e. if $\boldsymbol{O}v = o_1 v$).

Note that this is a necessary and sufficient condition. In other words, if the system is not in one of the eigenstates of an operator (if for instance the system is in a non-trivial superposition of eigenstates of $\boldsymbol{O}$ such as $c_1v_1 + c_2v_2$), then we are not entitled to say that the system has a value of the property represented by the operator in question.

However, in this paper I will, following (Fine, 1987), formulate the eigenstate-eigenvalue link in a more developed version, as follows:

> *(e/e link)*: A system has a value of a physical property if and only if the system's state is a) an eigenvector of the operator **O** that represents this physical property, or b) a mixture $W = \sum p_n \boldsymbol{W}n$, where **O** takes a value with certainty (i.e. with probability one), in every $\boldsymbol{W}_n$.

The crucial addition of clause b) allows us to ascribe values to the observables of a system in a mixed state, without requiring that the mixture in question be ignorance interpretable. Hence, this formulation allows us to ascribe values to observables of systems even when the systems are not in eigenstates of the corresponding operators. There are a number of important reasons why the new formulation of the (e/e link) is to be preferred which I will discuss in due course.

2.3. The Quantum Theory of Measurement

In order to make a measurement we must let the quantum object interact with a measuring device. The quantum theory of measurement, as first formulated by Von Neumann (1932), ascribes a quantum state to the measuring device, and treats the interaction as a quantum interaction, i.e. one that obeys the Schrödinger equation.

The theory further supposes that the observable of the system that we are interested in is represented by self-adjoint operator $\boldsymbol{O}$, with eigenvectors $\{\phi_n\}$ and eigenvalues $\{\lambda_n\}$. The pointer position observable A is represented by the self-adjoint operator $\boldsymbol{A}$, which has eigenvectors $\{\gamma_m\}$ with eigenvalues $\{\mu_m\}$. (And let us here further assume that $n=m$, without loss of generality.)

Suppose then that we have an object initially in state $W_o = \sum_n p_n\, \boldsymbol{P}[\nu_n]$, where each ν_n may be expressed as a linear combination of eigenstates of the observable O of the system that we are interested in (i.e. $\nu_n = \Sigma\, c_i\, \phi_i$); and a measuring device in $W_a = \sum_n w_n\, \boldsymbol{P}[\gamma_m]$. In what follows I shall often refer to the observable represented by the operator $\mathbf{I} \otimes \mathbf{A}$ in the tensor product Hilbert space as the *pointer position observable*. The eigenvalues of this observable are therefore given by the set $\{\mu_n\}$. As the interaction between the object system and the measuring device is governed by the Schrödinger equation, there must exist a unitary operator $\boldsymbol{U}$ that takes the initial state of the composite system (object system + measuring device) into its final state at the completion of the interaction, as follows: $W_o \otimes W_a \rightarrow \boldsymbol{U}\, (W_o \otimes W_a)\, \boldsymbol{U}^{-1}$. (For further details of the interaction formalism, see Appendix 1).

We can now state the basic intuition behind the problem of measurement. Take a system in an arbitrary superposition $\nu_n = \Sigma\, c_i\, \phi_i$. Then, due to the linearity of the Schrödinger equation, at the conclusion of an ideal measurement interaction with a measurement apparatus in any pure state, the composite (system+device) will be in a superposition of eigenstates of the pointer position observable. And in accordance with even the most basic description of the eigenstate-eigenvalue link (the *basic e/e link*), the pointer position observable can not have a value in this state. But surely quantum measurements do have some values – i.e. they have some value or other. Hence the quantum theory of measurement fails to describe quantum measurements completely!

3. The Insolubility Proof of the Quantum Measurement

The insolubility proofs are attempts to formally describe the measurement problem, in order to display precisely the set of premises that come into contradiction. The proofs go back to Wigner (1964), and include Earman and Shimony (1968), Fine (1970), Brown (1985), and Stein (1996).

3.1. Some Notation

First let me introduce some notation, following Fine (1970). Let us denote by *Prob (*W, Q) the probability distribution defined by $Prob_w\ (Q=q_n)$, for all eigenvalues q_n of Q. And let us denote *Q-indistinguishible* states *W, W'* as $W \equiv_Q W'$. Two states *W*, *W'* are *Q-indistinguishible* if and only if *Prob (W, Q) = Prob (W', Q)*.

We may now enunciate the following two conditions on measurement interactions. The insolubility proof (appendix 2) purports to show that these two conditions are inconsistent with the Schrödinger dynamics.

3.2. The *Transfer of Probability Condition* (TPC)

$$Prob\ (\mathbf{U}(W_o \otimes W_a)\mathbf{U}^{-1},\ I \otimes A) = Prob\ (W_o\ ,\ O)$$

This condition expresses the requirement that the probability distribution over the possible outcomes of the relevant observable *O* of the object system should be reproduced as the probability distribution over possible outcomes of the pointer position observable in the final state of the composite *object* + *apparatus* system. (TPC) entails the following minimal condition on measurements employed by Fine (1970) and Brown (1985): A unitary interaction on a (system + apparatus) composite is a W_a measurement if, provided that the initial apparatus state is W_a, any two initial states of the object system that are O-distinguishible are taken into corresponding final states of the composite that are ($I \otimes A$)- distinguishible. So we can use the pointer position of the measuring apparatus to tell apart two initial states of the object system that differ with respect to the relevant property.

But is (TPC) really a necessary condition on measurements? It could be argued that an interaction that transfers only part of the probability distribution of the object observable to the pointer observable is nonetheless a measurement, albeit only an approximate one. For *some* information is thereby transferred. This worry about (TPC) seems to me deep and legitimate. However I will argue in section 4.3 that the measurement problem arises in the highly idealised conditions imposed by the formal quantum theory of measurement; and in the context of such idealisations (TPC) is justified.[3]

3.3. The *Occurrence of Outcomes Condition* (OOC)

$$\boldsymbol{U}(W_o \otimes W_a)\boldsymbol{U}^{-1} = \Sigma\, c_n\, W_n \text{ where } \forall W_n\ \exists\, \mu_n\text{: } Prob_{Wn}\ (\boldsymbol{I} \otimes \boldsymbol{A} = \mu_n) = 1$$

This condition is often taken to express the *eigenstate-eigenvalue link*-inspired requirement that the final state of the composite be a mixture over eigenstates of the pointer position observable. But to be precise, it expresses the more general idea that the final state of the composite must be a mixture over states in each of which the pointer position observable takes one particular value or other with probability one.

I can now provide the reasons for my formulation of the *(e/e link)* in the previous section. It is conventional wisdom that a solution to the measurement problem can always be provided if the eigenstate-eigenvalue link is denied, in particular its necessary part.[4] But now note that (OOC) follows from *(e/e link)*, together with the fact that quantum measurements have outcomes (i.e. that they have one particular outcome or other). A stronger condition would follow from *(basic e/e link)*. However (OOC) is strong enough for the insolubility proof: It is possible to escape it, and hence solve the measurement problem, merely by denying *(e/e link)*, and thus (OOC). Hence the adoption of *(e/e link)* preserves conventional wisdom. An additional reason in favour of *(e/e link)* is its already mentioned consistency with the standard understanding of quantum states.

4. A Defence of the Insolubility Proof

4.1. Stein's critique

In a recent paper, Howard Stein has provided an interesting critique of the insolubility proof. He begins by deriving a lemma in the theory of Hilbert spaces (appendix 3) that has as a direct application a version of the insolubility proof. This lemma, he argues, is true given the ignorance

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[3] It is in addition important to emphasise that i) that selections are not generally committed to (TPC), and ii) even those selections that obey (TPC) are able to account for a very large class of approximate *non-ideal* measurements. See the discussion in section 6.

[4] That is, at any rate, how modal interpretations solve the measurement problem. See, for illustration, the essays in (Dieks and Vermaas, 1998).

interpretation of mixtures, but does not necessarily follow without that interpretation. And, he continues, the ignorance interpretation of mixtures equivocates on the nature of quantum states, as expressed by means of the statistical operator formalism: A quantum mixed state so expressed should not be thought of as an ensemble of pure states, but as a set of probability distributions, each one defined over the eigenvalues of each observable of the system. Thus, Stein concludes, the insolubility proof cannot constitute an accurate representation of the measurement problem.

Throughout this paper I will adopt Stein's understanding of quantum states as defining probability distributions over the possible values of a system's dynamic quantity. In fact, I will refer to it as the *standard understanding of quantum states*, as I believe it to be established in the literature. There are two reasons, however, why I want to resist Stein's conclusion. The first is that the ignorance interpretation of mixtures is not strictly required for the formulation of the insolubility proof: the proof may be a valid representation of the measurement problem even if the ignorance interpretation is not appropriate. The second is that the type of idealisations that go into the formulation of the insolubility proof, which Stein's critique may be taken to question implicitly, are also part and parcel of the quantum theory of measurement, within which the measurement problem arises. The insolubility proof captures *as much of the measurement problem as there is to be captured*.

4.2. Ignorance is not required

First, note, as a preliminary observation, that the insolubility proof can be stated in a manner that respects the standard understanding of quantum states. For the statements of conditions (OOC) and (TPC) given in the previous section are *prima facie* perfectly consistent with that understanding of statistical operators.

I claim that the ignorance interpretation is not required for the insolubility proof. (OOC) is a strictly weaker condition on the final state of the composite than the ignorance interpretation. For suppose that the final state of the composite is degenerate; then it possesses no unique representation in terms of pure states. (OOC) is happy to accept this plurality of representations. By contrast, the ignorance interpretation insists that only one among these representations is physically meaningful (namely the one which contains the pure state that the system *really* is in, with the corresponding epistemic probability). But that means that the ignorance interpretation does not and can not be used to motivate (OOC). Rather, as I already emphasised, (OOC) is motivated by the *(e/e link)*, together with the requirement that quantum measurements have outcomes and the standard understanding of quantum states. Thus rejecting the ignorance interpretation can not by itself suffice to explain why (OOC) may fail. And it is (OOC), not the ignorance interpretation, that figures as a premise in the insolubility proof.

There are however two important caveats to the above argument. The first one: it is anyway the case that when the final state of the composite is non-degenerate, (OOC) coincides with the ignorance interpretation. In that particular case there is only one decomposition of the system's mixed state, and only for a particular (pure) eigenstate of the observable will the probability of any particular eigenvalue be one. Perhaps this coincidence underlies Stein's thought that the ignorance interpretation is somehow involved. But it does not seem to provide an argument in favour of Stein's conclusion, because (OOC) *in general* can not be justified merely by an appeal to the ignorance interpretation.

The second caveat to my argument against Stein's conclusion can not be dismissed so lightly. It concerns the use in Fine and Brown's insolubility proof of a condition called *Real Unitary Evolution* (Brown, 1986). According to this condition the unitary evolution of a mixed state is given by the unitary evolution of its component pure states. Suppose that W_o, W_a are the statistical operators representing the initial states of the object system and measuring device respectively. And suppose that $W_o = \Sigma_n c_n P[\phi_n]$, and $W_a = \Sigma_m d_m P[\gamma_m]$. Brown states the principle of real unitary evolution as follows:

Real Unitary Evolution (RUE)

$$\hat{U}_t (W_o \otimes W_a) \hat{U}_t^{-1} = \hat{U}_t (\Sigma_n c_n P[\phi_n] \otimes \Sigma_m d_m P[\gamma_m]) \hat{U}_t^{-1} =$$

$$= \Sigma_{n,m} c_n d_m \hat{U}_t (P[\phi_n] \otimes P[\gamma_m]) \hat{U}_t^{-1} = \Sigma_{n,m} w_{n,m} P[\hat{U}_t (\phi_n \otimes \gamma_m)],$$

where $w_{n,m} = c_n \times d_m$ for all values of n,m.

The status of (RUE) has been a matter of some debate, but I think everyone would agree that it is motivated by the ignorance interpretation. In introducing it explicitly, Brown wrote: "It should be clear, moreover, that the principle is an extremely natural extension of the ignorance interpretation of mixtures, which as a rule is postulated for instantaneous ensembles, to the case of ensembles of systems whose states are evolving over time according to the Schrödinger equation" (Brown, p 860). To be more precise (RUE) is equivalent to the dynamical extension of the ignorance interpretation. For if, as stated by the ignorance interpretation, a mixed state represents our subjective degree of ignorance of the (pure) state of a system, then any dynamical evolution of the system that fails to provide us with additional information about the initial state of the system must result in a final state that reflects our initial uncertainty. In other words, the pure states evolve unitarily and independently with coefficients c_n, d_n that are invariant under this evolution – and this is indeed what (RUE) asserts. Conversely (RUE) entails this dynamical extension of the ignorance interpretation, for it imposes exactly the same condition on the time evolution of states that would be expected if the ignorance interpretation were true.

However, it does not seem to have been noticed that the insolubility proof does not employ as strong a condition as (RUE), but rather:

Quasi-Real Unitary Evolution (QRUE)

$$\hat{U}_t (W_o \otimes W_a) \hat{U}_t^{-1} = \hat{U}_t (\Sigma_n c_n P [\phi_n] \otimes \Sigma_m d_m P [\gamma_m]) \hat{U}_t^{-1} =$$

$$= \Sigma_{n,m} w_{nm} \hat{U}_t (P [\phi_n] \otimes P [\gamma_m]) \hat{U}_t^{-1} = \Sigma_{n,m} w_{nm} P [\hat{U}_t (\phi_n \otimes \gamma_m)]$$

where $1 \leq w_{nm} \leq 0$ and $\Sigma w_{nm} = 1$; but w_{nm} need not equal $c_n d_m$.

This condition is strong enough to generate the inconsistency between (OOC), (TPC) and the Schrödinger equation.[5] Crucially it is not equivalent to the dynamic extension of the ignorance interpretation. The latter entails (RUE), which is a special case of (QRUE); but does not entail (QRUE) in general. Conversely, the dynamic extension of the ignorance interpretation is entailed by (RUE) but not by (QRUE). Unitary interactions are possible for which (QRUE) holds even when the ignorance interpretation (and RUE) is plainly false. For notice that there are possible choices of n,m for which (QRUE) is true while the ignorance interpretation and (RUE) are plainly false. Thus (QRUE) is neither a natural extension of the ignorance interpretation, nor is it motivated by it. What motivates (QRUE) instead is its natural compatibility with the usual rule for the evolution of the spectral decomposition of mixed states, namely:

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[5] I have made the use of (QRUE) explicit in my presentation of the insolubility proof in appendix 2. Brown (1986) made (RUE) explicit, but it is (QRUE) which is implicitly employed in Fine (1970). Stein (1996) invokes the commutativity between $(I \otimes A)$ and $\hat{U} (W_o \otimes W_a) \hat{U}^{-1}$ which he seems to think is logically equivalent to the ignorance interpretation of $\hat{U} (W_o \otimes W_a) \hat{U}^{-1}$. Stein's condition is indeed necessary and sufficient for (QRUE), and his proof is the closest to the one in appendix 2. But Stein's condition is not sufficient, only necessary, for (RUE); and hence, in my view, it is not logically equivalent to the ignorance interpretation.

$$\hat{U}_t (W_0) \hat{U}_t^{-1} = \hat{U}_t (\Sigma_n w_n(0) P_n) \hat{U}_t^{-1} = \Sigma\, w_n(t) \hat{U}_t P_n \hat{U}_t^{-1} = W_t$$

Hence, I conclude that the ignorance interpretation of mixtures is neither an explicit premise of the insolubility proof, nor is it logically entailed by any of its premises ((TPC), (OOC), (QRUE), or the Schrödinger dynamics).

4.3. The Problem of Quantum Measurement is an Idealisation

There is a further question about how appropriate the assumptions made by the insolubility proof are for measurement interactions in general. I have already expressed doubts that (TPC) is an appropriate necessary condition for realistic models of actual measurement interactions. I now want to argue that in the context of the usual tensor-product Hilbert space formalism these assumptions are reasonable. As outside this context the question of a measurement problem does not even arise, the measurement problem is reasonably captured by the insolubility proof.

I will take here an idealisation to be a description of a system that, for the sake of presentation or ease of calculation, involves some assumptions that are known to be false. Thus, what I need to show is i) that any false assumptions that may be involved in (TPC), (OOC), (QRUE) or the use of the Schrödinger equation, also affect the quantum theory of measurement; and ii) that without those assumptions the theoretically based intuition of a measurement problem disappears.

(TCP) is idealised on at least two counts. First, it assumes that whether interactions are measurements is an all-or-nothing affair that does not depend on the actual initial state of the system to be measured at a particular time, but on all the possible states that the object may have had in accordance with the theory. This is hardly satisfied by any real measurement we know. For instance, in setting up a localisation measurement of the position of an electron in the laboratory, we do not assume that the device should be able to discern a position outside the laboratory walls, even if it is theoretically possible that the particle's position be infinitely far away from us. All real measurement devices are built in accordance to similar assumptions about the *physically* possible, as opposed to merely theoretically possible, states of the object system, on account of the particular conditions at hand. So real measurement devices do not strictly speaking fulfil (TPC).[6] However, this idealisation has been a part of the quantum theory of measurement from its inception; and it would

[6] To my knowledge this concern was first voiced by Stein (1973).

be very difficult to see how the measurement problem would arise at all in its absence. For if we do not expect quantum theory to completely describe the physically possible initial states of a system, we should hardly expect it to describe completely the physically possible *outcomes* of a measurement; and that expectation is at the heart of the measurement problem.

The second count of idealisation against (TPC) is that it appears to require measurements to be ideal in the technical sense of correlating one-to-one the initial states of the object system with states of the composite at the end of the interaction. However, many real measurements are not ideal in this sense. Most measurement apparatuses make mistakes, and no matter how much we may try to fine-tune our interaction Hamiltonian, we are likely in reality to depart from perfect correlation.[7] In section 6 I argue that, contrary to this appearance, (TPC) is not committed to all measurements being ideal. On the contrary it is possible to capture a large variety of approximate non-ideal measurements by means of (TPC). In fact (TPC) turns out to be as good a theoretical guide as any for distinguishing those interactions for which a *measurement* problem can arise from those interactions that it makes no sense even to describe as *measurements*.

Let us now turn to (OOC). This is also idealised, in that it assumes that the measuring device can only "point" to the eigenvalue of the pointer position observable that has probability one in the final state that results at the end of the interaction. The same idealisation is built into the quantum theory of measurement in the form of the *(e/e link)*, which was anticipated by Von Neumann's original statement of *(basic e/e link)*. It can of course be relaxed, but only at the expense of introducing new rules for value-ascription into the quantum theory of measurement. In addition it is clear that without (OOC) there is no measurement problem; for (OOC) captures precisely the intuition that is at the heart of the problem, namely that any quantum measurement ought to yield an outcome, that is, some outcome or other. Without that intuition, and without the *(e/e link*) to back it up, there is no problem of measurement.

What about (QRUE)? Is it also idealised, and in what respect? (QRUE) assumes that a mixture of pure states of the composite (system + apparatus) evolves into a mixture of the unitarily evolved pure states of the

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[7] The claim that real measurements are (almost) never ideal in this sense has become common lore in recent philosophy of quantum mechanics, following Albert (1992) and Albert and Loewer (1993). There are, however, surprisingly few sound arguments offered in favour of this common lore; but it is certainly the case that at least some real measurements (destructive measurements) are not ideal in this technical sense. See Suárez (1996) and Del Seta and Suárez (1999) for a discussion.

composite. In order to find out whether and how this assumption is idealised we need to ask the following question: Under what real-life conditions do we expect (QRUE) to fail? We do, without doubt, in cases of environmentally induced decoherence. For in such cases, the environment induces a non-unitary evolution on the states of the measuring device that is inconsistent with (QRUE). This phenomenon is well known to be ubiquitous in practice; so (QRUE) is indeed strongly idealised. More precisely (QRUE) assumes that the "composite" system formed by the quantum objects and the measuring device is isolated from the rest of the universe, which is almost always false in the real world. Yet, notice that the same idealisation is also present in the quantum theory of measurement, which takes the interaction between the object and the apparatus to be unitary, at least prior to the occurrence of an outcome. This assumption has in the past been contested, and is often rightly repudiated in some realistic accounts of measurement, for instance those offered by decoherence and quantum state diffusion approaches. And although it isn't agreed by everyone that the measurement problem is solved completely in these approaches, it is generally agreed that describing the further interaction of the measuring device with its environment takes us closer to a solution of the problem.

Finally, the Schrödinger equation is idealised because it assumes that all quantum systems, not only composite systems involving measuring devices, are closed systems. It assumes that the quantum Hamiltonian can transform pure states into pure states, or mixtures into mixtures, but never a pure state into a mixture or viceversa. But this again is a pre-requisite for a problem of measurement. For there would be no problem at all if we assumed, as for instance Von Neumann was forced to assume, that at some point in the measurement process a pure state quantum mechanically evolves into a mixture.

I conclude that the idealising assumptions that pervade the premises of the insolubility proof are concomitant to the quantum theory of measurement itself. The insolubility proof does not trade in a description of the measurement process that is any more idealised, or any less realistic, than the one offered by the quantum theory of measurement. And it is precisely these idealising assumptions that account for our theoretically-driven intuition that there is something problematic about quantum measurements. Without them the insolubility proof would be empty; but so would the measurement problem itself.

PART II: SELECTIONS AND DISPOSITIONS

5. Selections.

I have been arguing that the insolubility proof, in particular Stein's version, succeeds in capturing the essence of the measurement problem. And in one particular respect Stein's version succeeds admirably. His lemma brings out very explicitly the fact that the measurement problem would not arise if the initial states of the system were suitably restricted. For, as Stein writes, the lemma is valid "if for every nonzero $u \in \nu$, the commutativity condition (...) holds", where ν is a vector subspace of H, and thus includes all linear combinations of vectors already in ν. In particular if the superpositions of eigenstates of the object included in ν were discounted, the insolubility proof could not be formulated. The proof does not apply to a space of possible states that excludes arbitrary linear combinations of states already in the space, in other words a space of states that is not a vector subspace. This fact conspicuously points to an appropriate solution to the problem in terms of selections. The rest of this paper is devoted to a discussion of the selections approach.

5.1. Selections.

A selection is an interaction designed to test a particular disposition of a quantum system between the measuring device and that dispositional property of the system. Among the dispositional properties I include position, momentum, spin and angular momentum. In a selection, typically, the pointer position interacts only with the property of the system that is under test.[8] There is an analogue of selections in everyday observation: For instance, we observe the colour of a table by interacting directly with those properties of the table that are responsible for its colour, and only those properties. Similarly, a selection can in principle test "selectively" for the position of a quantum object without in any way testing for its momentum, and so on. Quantum measurements, I claim, are selections; they are designed to test for a particular property of a quantum system. We seem perfectly able to measure the position of quantum systems in the laboratory without measuring their charge, or their momentum.

However, the possibility of selections is not reflected in the formalism of the quantum theory of measurement, which insists in modelling any

[8] I say "typically" because I want to leave open the possibility that a measurement may test a particular property of a system without interacting directly with *that* property of *that* system.

interaction process by feeding in the full initial quantum state of the object system. On the standard understanding a quantum state is an array of probability distributions over the eigenvalues of *all* the observables of the system. Thus according to the quantum theory of measurement, any interaction whatsoever with a quantum object is, *ipso facto*, an interaction with all the properties of the object – and hence, on my definition, not a selection.

Something must be added to the formalism to represent selections. We may begin by noting that the quantum state ψ defines a distinct probability distribution for each observable. Hence ψ is an economical representation of *all* the properties of the system. We may thus wonder if there is a more precise representation, for any quantum system, of each of its properties, individually taken. Suppose that there is a representation W(O) of precisely the property O of a system in state ψ. The least that we would expect W(O) to satisfy is the following consistency condition: W(O) must define exactly the same probability distribution over the eigenvalues of O as does ψ.

It is indeed possible in general to find a more precise representation of each property of a quantum system in state ψ. Following Fine (1987), consider the following definition of equivalence of states relative to a particular observable:

<u>Q-equivalence</u>: Two states W and W' are Q-equivalent if and only if Prob (W, Q) = Prob (W', Q)

That is, two states are equivalent with respect to an observable if and only if they define the same probability distribution over the observable's eigenvalues. Thus if two states are Q-equivalent, they are statistically indistinguishible with respect to Q, and we call those states Q-indistinguishible. Hence every observable Q effectively determines, for each state W, an equivalence class formed by all of W's Q-indistinguishible states. We can define the equivalence class $[W]_Q$ as follows:

<u>Q-equivalence class</u>: $W \in [W]_Q$ if and only if $\forall\, W' \in [W]_Q$: $W \equiv_Q W'$.

Now we see that our desideratum on any more precise representation W(O) of the property O of a system in state ψ amounts to the claim that W(O) be O-indistinguishible from ψ.

Suppose that O is (a discrete and not maximally degenerate) observable of the system with spectral decomposition given by $\Sigma_n \lambda_n P_n$, where $P_n = P_{[\phi n]}$

= $|\phi_n\rangle \langle\phi_n|$. We can construct the *standard representative* W(O) of the equivalence class $[W]_O$ as follows:

$$W(O) = \Sigma_n \text{Tr}(\psi P_n) W_n, \text{ where } W_n = P_n / \text{Tr}(P_n).$$

It is now possible to make the following claim: for a given system in a state ψ, and a given observable O of this system, if ψ belongs to the equivalence class $[W]_O$, then W(O) represents precisely the property O of the system.[9]

A selection (of observable O of a specific quantum system in state ψ) is then a quantum mechanical interaction of a measuring device with the specific property of the system represented by W(O).

5.2. Selections Solve the Measurement Problem

All proposed solutions to the measurement problem so far have tried to tinker formally with the final state of the composite, by replacing the superposition predicted by the Schrödinger equation with an appropriate mixture that will obey OCC. Collapse interpretations do this more or less explicitly, either by introducing an additional dynamics that will yield the appropriate mixture, or (as is the case, for instance, in quantum state diffusion) by replacing the Schrödinger dynamics altogether. No-collapse interpretations do this implicitly. Thus, Everett's "relative state" is just the mixture that corresponds to a system in an entangled composite when the state of the rest of the universe is a particular eigenstate. The modal interpretation (in its Kochen-Healey-Dieks version) takes the final state of the composite yielded by the Schrodinger equation (the "dynamical state") to be equivalent for the purposes of ascription of values to observables to a mixture (the "value state"). And Bohmian mechanics advices us to regard every superposition as essentially reducible to an ignorance interpretable mixture of eigenstates of position.[10]

In a selection by contrast, the full quantum state of a system is to be replaced by a mixture. From the formal point of view of the quantum theory of measurement, this amounts to "tinkering" with the initial state. Fine[11]

[9] In section 7 I ask the question: what kind of properties must these be to be so representable? There is no analogue of this type of representation in classical mechanics. In the classical case, W(O) is simply the value of a particular dynamical quantity of a system, as extracted from its state; and such extraction is a completely trivial operation. But as has been emphasised before, quantum states are not to be interpreted á la classical mechanics, as catalogues of actually possessed properties and their values.
[10] For a description of these, and other interpretations of quantum mechanics, see for instance, Albert (1992) or Dickson (1998).
[11] Fine (1987), (1993).

employed this fact to solve the measurement problem: If the initial state of the object system is an appropriate mixture over the eigenstates of the object observable, the final state of the composite resulting from Schrödinger evolution satisfies (TPC) and (OOC).[12]

To see this, let us return to the discussion of measurement interactions with the definitions of Q-equivalence and the standard representative in mind. A quantum object in state W_o interacts with an apparatus initially in state W_a. We are interested in the property O of the object, represented by the Hermitian operator O with eigenvalues λ_i and eigenvectors ϕ_i. The pointer position observable of the apparatus is represented by the hermitian operator $I \otimes A$, with eigenvalues μ_{ni} and eigenvectors β_{ni} (corresponding to the eigenvalues μ_n and eigenvectors γ_m of A). The insolubility proof of the measurement problem shows that no unitary interaction can be set up where the probability distribution laid out by W_o over the λ_i – eigenvalues of O is matched by that defined by the final state of the composite over the μ_{in} eigenstates of the pointer position observable, *as long as we allow that the initial state of the system may be any arbitrary state*, including crucially superpositions of the ϕ_i.

Fine's proposal is then to formally restrict the class of initial states of the object, in accordance with the possible selections. If the initial state of the object is W_o, and if we are interested in the observable O, Fine suggests that we focus only on the standard representative of the equivalence class $[W_o]_O$, namely $W_o(O)$, and that we ignore (1) all O-equivalence classes formed by attending to states other than W_o, and (2) all X-equivalence classes of W_o for observables of the object system other than O.

We are then able to model the interaction of a system in a state W_o by a measuring device in state W_a as a selection of the property of the system represented by $W_o(O)$, as follows:

$$W_o(O) \otimes W_a \rightarrow \quad \hat{U}_t\, (W_o(O) \otimes W_a)\, \hat{U}_t^{-1} =$$

$$= \hat{U}_t\, (\Sigma_n \operatorname{Tr}\, (W_o\, P_n)\, W_n \otimes \Sigma_m\, w_m\, P_{[\gamma m]})\, \hat{U}_t^{-1} =$$

$$= \Sigma_{nm}\, \eta_{nm}\, (t)\, \hat{U}_t\, (W_n \otimes P_{[\gamma m]})\, \hat{U}_t^{-1} =$$

$$= \Sigma_{nm}\, \eta_{nm}\, (t)\, \hat{U}_t\, (P_{[\phi n]} \otimes P_{[\gamma m]})\, \hat{U}_t^{-1},$$

[12] This presentation may suggest that the existence of selections is a *logical* consequence of the insolubility proof because they are the only interpretation of quantum mechanics that can get around the proof without relinquishing any of the proof's premises. In section 5.4. I argue against this suggestion.

where $\eta_{nm}(0) = \Sigma_{nm} \mathrm{Tr}(W_o P_n) w_m$.

It is now easy to see that as long as this selection satisfies (QRUE), the pointer position observable will take values in the final state of the composite, in accordance with *(e/e link)*. For simplicity consider the ideal, non-disturbing, (QRUE)-obeying interaction U_t:

$$\hat{U}_t (\phi_n \otimes \gamma_m) \hat{U}_t^{-1} = \phi_n \otimes \gamma_n.$$

This interaction has the following effect:

$$\hat{U}_t (P_n \otimes P_{[\gamma m]}) \hat{U}_t^{-1} = \hat{U}_t (P_{[\phi n \otimes \gamma m]}) \hat{U}_t^{-1} = P_{[Ut\, (\phi n \otimes \gamma m)\, Ut]} = P_{[\phi n \otimes \gamma n]} = P_{[\beta nn]},$$

where β_{nn} is an eigenvector of $(I \otimes A)$ with eigenvalue μ_{nn}.

The final state of the composite resulting from this selection is then:

$$W_{o+a} = \Sigma_{nm} \eta_{nm}(t) P_{[\beta nn]} \qquad \text{(F)}$$

This is a mixture over pure states: projectors associated with the eigenspaces of $(I \otimes A)$, and according to *(e/e link)* the pointer position observable takes a value in this state.

5.3. Selections and Ignorance

Does the ignorance interpretation play a role in the solution to the measurement problem offered by selections? Perhaps contrary to appearances, it plays no role.

I begin by drawing a distinction between selective interactions and selections. Fine defined selective interactions as unitary interactions with the standard representative of a system that obeyed (TPC) and (QRUE). He was then able to show that such selective interactions solved the measurement problem, for he was able to show that the final state of the composite resulting from any such selection obeys (OOC). I have defined *selections*, more generally, to be unitary interactions designed to test a particular property of a system, as represented by a standard representative. There is no reason in principle why a selection should obey (TPC), or (QRUE). And thus there is no reason in principle why a selection should yield a final state of the composite that satisfies (OOC). In my view any unitary interaction whatever between a pointer position observable of a measuring device and the standard representative mixed state of a system may represent a selection.

The ignorance interpretation is not involved in selections generally. But even in the case of selective interactions, which obey (QRUE), there is no entitlement to the ignorance interpretation. For recall from section 4 that (QRUE) is not sufficient for the ignorance interpretation.

This result has two important consequences. First, it shows why it is mistaken to think of selections in general either as an artifact of the insolubility proof, or as a logical consequence of this proof. Selections turn out to be a more general class of interactions, which include selective interactions as a subset. And although the insolubility proof shows that selections can solve the measurement problem, and thus provides one reason in favour of selections, nothing like a logical demonstration of selections from the premises of the insolubility proof is forthcoming. There is no reason in principle why all selections should obey (QRUE). Even if *some* selections (selective interactions) obey (QRUE) and get around the insolubility proof, this is hardly the basis for a deduction of this particular set of selections because, as argued in section 4, (QRUE) is itself highly idealised and empirically weakly motivated. Additional empirical reasons in favour of the existence of selections must be sought, and that is what I do in sections 6 and 7 of this paper.

The second consequence requires some preliminary discussion. One may be tempted by the following argument to claim that selections make the mistake of ascribing the wrong state to quantum systems that enter into interaction with measuring devices.[13] Consider the final state of a selective interaction, (F):

$$W_{o+a} = \Sigma_{nm}\ \eta_{nm}(t)\ P_{[\beta nn]}.$$

The probabilities η_{nm} are the time-evolved of the product of the probabilities of the eigenvalues λ_n in the initial state $W_o(O)$ of the object system and the probabilities of μ_m in the initial state W_a of the apparatus. Now, *suppose* that in a selection we were required to give the ignorance interpretation to the final state of the composite, and to understand the probabilities η_{nm} as subjective probabilities describing our incomplete knowledge of the "true" state. And *suppose* in addition that η_{nm} is constant

[13] I thank some of the participants at the VI Foundations of Physics and BSPS conferences in Nottingham, in 1998, for emphasising this worry.

in time, i.e. $\eta_{nm}(t) = \eta_{nm}(0) = \Sigma_{nm} Tr (W_oP_n) w_m$. This would commit us to understanding $Tr (W_oP_n)$, and w_m as subjective probabilities; it follows that we are required to give the ignorance interpretation to the initial mixed state of the apparatus W_a, and to the standard representative of the object system $W_o(O)$.

It is possible to do so in spite of the argument against the ignorance interpretation of improper mixtures in section 2 of this paper because neither W_a nor W_o is in general improper. But giving the ignorance interpretation to W_o (O) raises a puzzle. Recall that $W_o(O)$ is a mixture over W_n states. In giving an ignorance interpretation to it, we are claiming that the true state of the object system at the beginning of the interaction is *really* one of the states W_n with the prescribed probabilities. But the *true* initial state of the system is W_o! This may not even be a mixed state, and it will generally be very different to any of the W_n. Moreover although the mixture $W_o(O)$ is, by construction, in W_o's equivalence class, neither one among the pure states W_n that appear in the decomposition of W_o (O) is.

The point can be made more poignantly by considering formally the simple case of a Schrödinger cat-like measurement. We are invited to consider a two-dimensional observable O with eigenstates ϕ_1 and ϕ_2 and corresponding eigenvalues λ_1 and λ_2 respectively. We are then asked to consider three O-distinguishible states, ϕ_1, ϕ_2 and ϕ_3, where ϕ_3 is the linear combination: $a_1\phi_1 + a_2\phi_2$. Given ϕ_3 and the spectral decomposition of $O = \lambda_1 P_{[\phi 1]} + \lambda_2 P_{[\phi 2]}$, we can construct the standard representative of ϕ_3's O-equivalence class, namely the mixed state: $W_o(O) = |a_1|2 P_{[\phi 1]} + |a_2|2 P_{[\phi 2]}$. The argument above entails that in order to solve the Schrödinger cat paradox by means of a selection, we need to give the ignorance interpretation to $W_o(O)$. This amounts to the claim that the system really is in state ϕ_1 or ϕ_2, although we ignore which one exactly. And this contradicts our prior knowledge that the state of the system is ϕ_3 instead. Surely we are not here being asked to entertain the long-discredited ignorance interpretation of superpositions!

The argument is fallacious. It incorrectly assumes that the ignorance interpretation of the final state of the composite is required to solve the measurement problem; and that selections are in the business of providing this by advancing a subjective interpretation of the probabilities. But in light of the previous discussion, i) the measurement problem does not call for the ignorance interpretation of mixtures, ii) the concept of a selection in no way involves the ignorance interpretation; and iii) even those selections that obey (QRUE) and defeat the insolubility proof do not require the ignorance interpretation – in fact they may be inconsistent with it, for the probabalities η_{nm} may evolve in time.

To suggest that quantum measurements are quantum selections is *not* to suggest that there are no systems in superpositions; nor is it to suggest that the actual initial state of a system that is just about to measured is the mixture $W_o(O)$ instead of the superposition ψ. That would not agree with experience as it is always possible to run an interference experiment on the system which can only be modelled correctly by means of the superposition. The suggestion is simply that measuring devices interact with only one property of a system at a time.

6. Non-Ideal Selections.

An important kind of no-collapse interpretation is the so-called Kochen-Healey-Dieks interpretation. A well-known objection to this interpretation is that it cannot account for non-ideal measurements.[14] In this section I show that selections can account for non-ideal measurements naturally.

6.1. No-collapse interpretations and non-ideal measurements.

In their most elementary version these interpretations ascribe values to the O property of the object system and to the pointer position observable of the measuring device if and only if the final state of the composite is in a biorthonormal decomposition form:

$$|\psi\rangle = \Sigma_i\, c_i\, |\phi_i\rangle \otimes |\gamma_i\rangle.$$

However, note that this is a small subset of the set of all possible final states:

$$|\psi\rangle = \Sigma_{ij}\, c_{ij}\, |\phi_i\rangle \otimes |\gamma_j\rangle,$$

in which it is typically not possible to predict perfect correlations between the values of the pointer position observable and the object observable. Ideal interaction Hamiltonians yield final states in the biorthonormal decomposition form; but for the larger class of Hamiltonians that govern non-ideal interactions, the modal interpretation cannot ascribe values in the final state to the pointer position observable.

[14] See Albert, 1993; Albert and Loewer 1990, 1993.

6.2. Exact and Approximate Measurements

I will here adopt the following terminology: An exact measurement may only result from an ideal interaction while an approximate measurement may result from a non-ideal one. Let us begin by characterising these two forms of interaction:

Ideal Interaction:

$\Sigma_i \, c_i \, \phi i \otimes \gamma_o \;\rightarrow\; \Sigma_i \, d_i \, \phi_i \otimes \gamma_i$

Non-ideal Interaction:

$\Sigma_i \, c_i \, \phi i \otimes \gamma_o \;\rightarrow\; \Sigma_{ij} \, d_{ij} \, \phi_i \otimes \gamma_j$

On the account of measurement adopted in this paper, an ideal interaction is a measurement of O by I⊗A only if it obeys (TPC): $d_i = c_i$. We may then define an exact measurement as an ideal interaction that obeys (TPC) and correlates possible values of the relevant property of the *object* system with possible values of the *pointer*. We may also define the notions of ∈-measurement and approximate measurement as follows:

∈-measurement: A non-ideal interaction is an ∈-measurement if $|cij|^2 < \in$, if $i \neq j$, where $0 < \in < ½$.

Approximate measurement (Shimony, 1974): An ∈-measurement is an approximate measurement if $|cij|^2 \approx 0$, if $i \neq j$.

In general ∈-measurements are not proper measurements of the state of the object system. Most ∈-measurements are not (TPC)-obeying, and can not be used to reliably infer the state of the object system from the experimental outcome. Instead these measurements generally test for the probabilities of states of the object system given the measurement outcome, and may be used to reliably infer conditional probabilities of states on outcomes.

Approximate measurements are a special kind of ∈-measurements which approximate ideal measurements, and are thus approximately (TPC)-obeying. These *are* proper measurements of the states of the object, as they allow us to infer the states of the object system to a high approximation.

6.3. Selections for Non-Ideal Interactions

I claim that the selections approach accounts for precisely that subset of $\in$-measurements that *are* proper measurements of the initial state of the object system (as opposed to measurements of conditional probabilities) as well as all exact and approximate measurements. In other words, selections are not only able to account for non-ideal measurements in general; they also provide a useful wedge to classify very precisely which non-ideal interactions *are* actually measurements.

In the previous section I already showed how any exact measurement may be modelled as an exact selection; here I show how selections may model i) $\in$-measurements of the initial state of the object system that obey (TPC) and ii) approximate measurements.

A non-ideal selection of a disposition O of a quantum system is a non-ideal interaction of the pointer position property of a measuring device with the O disposition of the system as represented by the standard representative Wo(O):

$$\Sigma_i |c_i|^2 P[\phi_i] \otimes P[\gamma_o] \quad \rightarrow \Sigma_{i,j} |d_{ij}|^2 P[\phi_i \otimes \gamma_j]$$

Now it is easy to show that any non-ideal selection obeys (TPC) if and only if it obeys the following general condition:

$$\forall j \; \Sigma_i |d_{ij}|^2 = |c_j|^2.$$

But this general condition is also required for $\in$-measurements to obey (TPC). We conclude that all $\in$-measurements that obey (TPC) can be modelled as non-ideal selections that obey the general condition.

As an illustration, a two dimensional selection that constitutes an $\in$-measurement is given by the following three expressions:

1. $(|c_1|^2 P[\phi_1] + |c_2|^2 P[\phi_2]) \otimes P[\gamma_0] \quad \rightarrow$

$$|d_{11}|^2 P[\phi_1 \otimes \gamma_1] + |d_{12}|^2 P[\phi_1 \otimes \gamma_2] + |d_{21}|^2 P[\phi_2 \otimes \gamma_1] + |d_{22}|^2 P[\phi_2 \otimes \gamma_2].$$

2. $|d_{11}|^2 \quad + |d_{21}|^2 = |c_1|^2.$

3. $|d_{12}|^2 \quad + |d_{22}|^2 = |c_2|^2.$

6.4. Approximate Selections

Let us now turn to approximate measurements. These may be characterised as selections by means of the general condition:

$\forall j\ \Sigma_i\ |d_{ij}|^2 \approx |c_j|^2$.

However, these selections do not strictly obey (TPC) so we may question whether they are measurements at all. We may address the worry by independently developing a fully-fledged account of approximate selections, as follows:[15]

<u>Approximate Selection</u>

An approximate selection of the O property of a system in a pure state ϕ_n is a selection of the O property of a system in the mixed state ρ_n that approximates ϕ_n, where ρ_n approximates ϕ_n if $\rho_n = \Sigma_m\ w^n{}_m\ P[\phi_m]$, and $w^n{}_n \approx 1$.

An approximate measurement of observable $O = \Sigma_n\ c_n\ \phi_n$ on a system in the state $W_o = \Sigma_n\ c_n\ \phi_n$ can be modelled as an approximate selection: Substitute W_o with the standard representative of its O-equivalence class, namely $W_o(O) = \Sigma_n\ |c_n|^2\ P[\phi_n]$. We may substitute each $P[\phi_n]$ with the mixed state ρ_n which approximates it to yield: $\Sigma_n\ |c_n|^2\ \Sigma_m\ w^n{}_m\ P[\phi_m]$. We may now run an ideal selection of the O property of this state:

$\Sigma_{n,m}\ |c_n|^2\ w^n{}_m\ P[\phi_m] \otimes P[\gamma_o] \rightarrow$
$\Sigma_{n,m}\ |c_n|^2\ w^n{}_m\ P[U(\phi_m \otimes \gamma_o)] =$
$\Sigma_{n,m}\ |c_n|^2\ w^n{}_m\ P[\phi_m \otimes \gamma_m]$.

It is easy to see that $\Sigma_{n,m}\ |c_n|^2\ w^n{}_m\ P[U(\phi_m \otimes \gamma_m)] \approx \Sigma_n\ |c_n|^2\ P[U(\phi_n \otimes \gamma_n)]$. In words, the state that results from an approximate selection approximates the final state of the exact measurement given by an ideal, (TPC)-obeying selection. This shows that it is legitimate to model approximate measurements by means of approximate selections.

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[15] The account that follows was developed in conversations with Arthur Fine.

6.5. Implications for Ignorance.

In the second part of this paper I have been arguing that selections do not in general need to obey (TPC) or (QRUE). And in section 4 I showed that (RUE) – and the ignorance interpretation – may fail even when (QRUE) holds. So selections are not just one but two steps away from the ignorance interpretation. And indeed it is easy to show that (RUE) fails in i) non-ideal selections that obey (TPC), such as proper $\in$-measurements; and ii) non (TPC) obeying selections, such as approximate measurements. On the other hand it is also easy to show that the special kind of *ideal* selections that *do* obey (TPC) and (QRUE), such as exact measurements, automatically obey (RUE). The requirement that the probability distribution be matched is, in the case of exact measurements, enough to keep fixed the values of the probabilistic coefficients. This result strengthens the case for the dispensability of (RUE) in the insolubility proofs that I made in section 4 of the paper, for it shows that (TPC) and (QRUE) together already do some of the work that (RUE) has been thought to be necessary for.[16]

7. Selective Interactions Test Quantum Dispositions.

In the final section I turn to interpretational issues. How can we understand selections? And why are measurements selections? I first critically address the answer to these questions given by Fine himself, and then provide my own account in terms of dispositions.

7.1. Equivalence Classes as Physical "Aspects": A Critique

Fine's thought was that some interactions are "selective" in the sense that they respond only to a certain aspect of the system. For every property of a quantum system originally in a superposition, there is a corresponding mixed state that is probabilistically equivalent (for that property) to the superposition. For instance, a system in a superposition of E-eigenstates $\psi = \Sigma\, c_i v_i$ is probabilistically indistinguishible, as regards E, from a system in the mixed state $W = \Sigma\ |c_i|^2\, \mathbf{P}[v_i]$. An interaction is selective if it has been set up in order to find out about this particular E aspect of the system and no other. In modeling this selective interaction the mixed state may be used, for the superposition is not a precise enough representation of this and only this aspect of the system. Thus Fine writes:

[16] A different argument for the dispensability of (RUE) is given by Del Seta (1998).

“The basic proposal, then, is to regard the measurement of an observable E on a system *in state ψ* as a measurement interaction that selects the aspect of the system corresponding to the probability distribution for E that is determined by state ψ.” [Fine (1992), p. 126, my italics].

Although I agree with Fine’s contention that selections can solve the measurement problem, I disagree with Fine’s interpretation of selections as measurements of *aspects* of physical systems. (In section 7.2 I develop my own interpretation of selections as measuring dispositional properties of quantum systems.) Fine’s interpretation contains counterintuitive elements, and provides a weak motivation for the existence of selections.

Fine’s suggestion is that we interpret quantum systems in superpositions (regardless of whether individual particles or entangled sets of particles) as *made up* of smaller subsystems. He writes:

“My exploration starts out from the idea that some interactions are selective. They do not actually involve the whole system, only some physical subsystem. Thus the interaction formalism ought not be applied to the state of the whole system, only a representative of the subsystem engaged in the interaction.” [Fine (1987), page 502].

Fine is here reasoning as follows: a system in a mixture has no “subsystems”. Hence in interacting with it, a measurement device interacts with the whole system. But, as the system is in a mixture, some outcome will result. By contrast, a system (even if a single particle) in a superposition is made up of several “subsystems”. In an interference experiment, such as a two-slit experiment, the device interacts with the entire system, or with all the subsytems at once, and this explains why interference terms occur. In a measurement interaction, however, the measuring device will interact only with an individual subsystem. A “selective interaction” then takes place, and this explains why a precise outcome results with a certain probability.

However, the suggestion that any system in a superposition is *made up* of several “subsystems” is deeply counterintuitive from an ontological point of view. For suppose that the system is a single particle. The claim that the particle is composed of further “subsystems” corresponding to each standard representative, is essentially nothing but the claim that the particle is composed of further (smaller?) particles, each of them in a particular quantum state. This brings about a bizarre ontology and leaves us lacking in any explanation for the curious fact that in an interference experiment all the subsystems are interacted with, but not in a measurement.

Suppose on the other hand, that the initial superposition is a representation of the entangled state of two or more particles. For illustration, consider an EPR pair of particles (1 and 2) in a singlet state of spin "up" and spin "down" along the x direction:

$\psi = (1/\sqrt{2})\ |up_x\rangle_1\ |down_x\rangle_2 - (1/\sqrt{2})\ |down_x\rangle_1\ |up_x\rangle_2.$

The suggestion that this superposition represents a system *made up* of further subsystems is even more counterintuitive. For while there is now an unambiguous ontological prescription for individuating these subsystems, it disagrees with Fine's prescription. Fine prescribes the standard representatives for each of the "subsystems":

$W(x) = ½\ P_{[up,\ down]}(x) + ½\ P_{[down,\ up]}(x)$, $W(y) = ½\ P_{[up,\ down]}(y) + ½\ P_{[down,\ up]}(y)$, etc.

But W(x), W(y) represent distinct properties of the composite system of particles 1 and 2, and cannot be interpreted as states of each of the particles, individually taken.

Even if these problems could be solved, it is difficult to see how Fine's prescription could possibly constitute a physical motivation for selections. There is no independent reason why interacting with a "subsystem" will yield an outcome while interacting with a whole system won't. We certainly do not have an analogue of this in classical mechanics, or in any other physical theory that I know of. (In classical mechanics, for instance, we typically assume that a gravitational interaction with a massive object designed to measure its weight will result in an outcome even if the object is constituted by smaller particles. In electrodynamics, measurements of the charge of large conductors give outcomes, even if conductors are made up of smaller, equally charged, parts.) Fine's use of the system-subsystem distinction is *sui generis*, and specifically tailored for quantum mechanics; his proposal can provide neither physical understanding nor motivation for selections.

I believe that these are definite objections against Fine's interpretation of selections. The basic problem, in my view, is that Fine's interpretation constitutes a return to an unacceptable understanding of a quantum state as describing a complete set of actually possessed properties of a quantum system. On this understanding, each standard representative must represent a complete set of actually possessed properties of *something*, which (mis)leads Fine into "subsystem" speech. A better alternative, consistent with the standard understanding of quantum states, is that there is only one system (which may well be a composite) with each standard representative representing a different dispositional property of *that* system.

7.2. Dispositions.

I defend the view that a selection is an interaction of the pointer position observable of a measurement device with a dispositional property of a quantum object as represented by the standard representative.

On this view quantum entities do not have further constituent parts or "subsystems", but they possess irreducibly dispositional properties. An electron, for instance, possesses a disposition to manifest a value of momentum when tested in the appropriate way, even if in the absence of such test it typically lacks a specific value of momentum (for its wavefunction will rarely be sufficiently peaked in momentum space). The possession of this dispositional property by the electron would be unconditional: that is, the electron possesses this property in the actual world, just like any ordinary object possesses any of its categorical properties. However, a disposition is only manifested in the context of an appropriate test; in the absence of such test, the property may never be manifested –throughout the lifetime of the particle. (Just as a glass possesses its fragility throughout its lifetime even though it may never manifest it.) Hence I am adopting a sufficiently robust sense of dispositions that takes them to be genuine properties actually possessed by real systems in the world.

This view of quantum entities as endowed with irreducible dispositions, provides us with an extremely natural way to understand selections, and their solution to the measurement problem. A measurement is a (QRUE), (OOC), (TPC) obeying selection with one of a particle's dispositional properties. In order to measure the momentum of an electron, we set up a selective interaction of the measuring device with the momentum disposition. In order to measure the position we need to set up a different selective interaction --one that is responsive to position, but not momentum. And so on.

Notice that there is nothing particularly shocking or revolutionary in this procedure for measuring physical quantities. Measurements in classical mechanics are no different. Willis Lamb has a concrete model to measure the position of a classical particle (Lamb [3]). He doesn't set up an interaction that correlates every single one of the particle's dynamical variables to a dynamical variable of the apparatus. That would be unnecessarily cumbersome. He sets up instead an interaction that correlates the particle's position to one of the dynamical variables of the measuring apparatus (momentum). The measuring process is no different in classical or quantum mechanics: but quantum properties (unlike classical properties, in the conventional wisdom) are irreducibly dispositional. The difference is not in how we measure things, but in the kinds of things that we measure.

So I suggest to interpret selections as follows: Each disposition of a quantum particle in a superposition ψ gets represented by a mixed state that is probabilistically indistinguishable from the superposition, as far as the property in question is concerned. The complete state of the system is, on this account, implicit or explicitly given by the superposed state ψ together with the full and complete set of mixtures representing its dispositional properties. If we set up a measurement interaction designed to measure a dispositional property of a system, and no other property, we must take seriously the fact that only that property is being interacted with, and model the initial state appropriately.

Otherwise there would be relevant physical facts about the interaction that we would not be representing. For the interaction Hamiltonian is the same whatever disposition of the particle we measure. If there are genuinely physical differences (and not for instance merely differences in the experimenter's intentions) between different experimental set ups designed to measure different dispositions of a quantum system, then the formalism of the quantum measurement theory is incomplete, for it does not distinguish between such interactions. This is where selections step in: in providing a separate representation of each of the dispositional properties of a system, selections provide us with a representation of relevant physical facts.

A question remains regarding the use of the superposed ψ state. Why do we need it? Given that all the information about the dispositional properties of the particle is encoded in the set of mixed states that represent the particle's dispositions, it may seem that ψ is not needed, but this is not the case at all. The superposition has two main functions. First, it is an economical way to represent all the relevant information at once. Instead of writing down a long collection of mixtures to fully characterise a quantum system, I may just write down ψ, from which it is always possible to derive the set of mixtures by means of Fine's algorithm for the standard representative. A second function of ψ, which explains why it is not possible to dispense with it even in principle is related to the fact that dispositions may interact with each other. In quantum mechanics, unusually perhaps, they typically do: testing for a particular disposition of an object precludes us from testing another. No test for the position disposition of a quantum system can be carried out simultaneously with a test for its momentum disposition. This type of information (about which interactions preclude, or condition, which others) is not encoded in the corresponding mixtures. Only the state ψ of the system contains this type of information. Hence if the experiment is set up to test the interactive character of the dispositions of some quantum particle (such as a two slit experiment) we must represent the state of the particle by means of the superposition, which fully represents the interference aspect of the physical interaction.

7.4. Dispositions and propensities

The account that I have been developing takes dispositions to be central to the interpretation of quantum mechanics, and to solving the measurement problem. Appeal to quantum dispositions is not new, and has a considerable pedigree.[17] In philosophy of science perhaps the best known proposal in this direction has been Karl Popper's. In this final section I would like to briefly distinguish my interpretation of selections from Popper's propensity interpretation of the wave function.

Popper's interpretation[18] subscribes, among others, to the following five tenets, roughly described:

1. Propensities are real quantum properties instantiated in nature.
2. Propensities are not monadic properties of any isolated quantum system, but relational properties of quantum entities and experimental set-ups. A one-electron universe would lack any propensities.
3. Quantum theory is essentially a probabilistic theory, in the sense that it is a theory about the probabilities that certain outcomes obtain in certain experimental set-ups
4. The quantum wavefunction, or state, is a description of a propensity wave over the outcomes of an experimental set-up.

[17] Among the founding fathers of quantum mechanics, Heisenberg was particularly keen to understand quantum mechanics in terms of "potentialities". See his (19??).

[18] See Popper, REF.

5. Providing an objective interpretation of the probabilities in quantum mechanics in terms of propensities is sufficient to solve the philosophical puzzles concerning quantum mechanics.

My account of selections shares with Popper's interpretation an emphasis on the quantum probability distribution as the basis for the ascription of dispositions. To the extent that propensities can be defined as probabilistically quantified dispositional ascriptions, my account is also a propensity-based one. However, the similarities end there. My interpretation either denies or is non-committal about Popper's thesis 1-5.

The dispositional account of selections remains neutral about Popper's thesis 1. It is only required that dispositions may be ascribed even in the absence of any actual (past, present or future) test. Beyond this requirement the account neither denies realism about disposition ascriptions nor requires it. In particular a conditional analysis of dispositions is acceptable as long as it accommodates this requirement.

Another difference concerns the nature of the quantum dispositions or propensities. Popper's thesis 2 is false in my dispositional account.[19] Although the dispositions that I take quantum mechanics to ascribe to systems can only be revealed by means of interactions with measuring devices designed to carry out measurements of the appropriate observables, their ascription is fully independent of the existence of such interactions. On my account an electron in a one-electron universe may be in state ψ, and thus possess all the dispositional properties described by the appropriate standard representatives.

Popper's thesis 3 is also false in my account: Quantum mechanics is a theory about quantum entities (including certainly, subatomic particles) and their properties, not about probabilities. It just happens that the properties of quantum entities are dispositional.

My interpretation is not committed to Popper's thesis 4. On my account the quantum wave function does not directly describe a "propensity wave": instead the wave function is an economic tool to derive the mixed standard representative states which describe probabilities of outcomes. There is no need for a literal interpretation of the wavefunction as representing a real "wave. [20]

[19] Thesis 2 has at any rate been to my mind convincingly refuted by Peter Milne (1985) who shows that it leads to incorrect predictions in the case of the two-slit experiment.
[20] In addition, Neal Grossman (1982?) showed that Popper's interpretation fails to distinguish appropriately between mixtures and superpositions; a problem that does not affect my dispositional account of selections.

My account also denies the spirit if not the letter, of Popper's thesis 5. Let us leave aside other paradoxical issues of quantum mechanics: the measurement problem at least cannot be solved merely by providing an interpretation of the calculus of probabilities, whether objective or subjective. One needs instead to work hard on the formal representation of the physics. In particular one has to i) introduce the notion of a selection and represent it formally; ii) provide an interpretation of selections that supports the claims that all measurements are selections; iii) show that the measurement problem only arises in the context of assumptions (TPC), (QRUE), (OOC) and the Schroedinger equation, and iv) show that there is no measurement problem for those selections that obey (TPC), (QRUE), (OOC) and the Schroedinger equation. I take myself to have achieved all these in this paper.

Appendix 1: The Interaction Formalism

In this appendix I describe the tensor-product space formalism provided by the quantum theory of measurement to represent the interaction between an object system and a measuring device. Given two Hilbert spaces, H_1 and H_2, we can always form the tensor-product Hilbert space $H_{1+2} = H_1 \otimes H_2$, with $dim (H_1 \otimes H_2) = dim (H_1) \times dim (H_2)$. If $\{v_i\}$ is a basis for H_1 and $\{w_j\}$ is a basis for H_2, then $\{v_i \otimes w_j\}$ is a basis for H_{1+2}. Similarly if A is an observable defined on H_1 with eigenvectors $\{v_i\}$ and eigenvalues a_i, and B an observable on H_2 with eigenvectors $\{w_i\}$ and eigenvalues b_j then $A \otimes B$ is an observable on H_{1+2} with eigenvectors $v_i \otimes w_j$, and corresponding eigenvalues a_ib_j.

Consider two systems S_1 and S_2. If S_1's state is W_1 on H_1, and S_2's state is W_2 on H_2, we can represent the state of the combined system S_{1+2} as the statistical operator $W_{1+2} = W_1 \otimes W_2$ acting on the tensor-product Hilbert space H_{1+2}. If either W_1, W_2 is a mixture, then W_{1+2} is also a mixture. If, on the other hand, *both* W_1, W_2 are pure states then W_{1+2} is pure. Suppose that $W_1 = P_{[\psi]}$, and $W_2 = P_{[\phi]}$, where $\psi = \Sigma_i c_i v_i$ and $\phi = \Sigma_j d_j w_j$. Then $W_{1+2} = \Sigma_{i,j} c_i d_j v_i \otimes w_j$, which is a superposition of eigenstates of $A \otimes B$ in H_{1+2}. More specifically, if S_1, S_2 are in eigenstates of A,B, the combined system S_{1+2} is in an eigenstate of $A \otimes B$. If $W_1 = v_i$ and $W_2 = w_j$, then $W_{1+2} = v_i \otimes w_j$, a so-*called product state*.

For an arbitrary (pure or mixed) state W_{1+2} of the combined system, and arbitrary observable $A \otimes B$ the Generalised Born Rule applies. The probability that $A \otimes B$ takes a particular a_ib_j value is given by:

$$Prob_{W1+2} (A \otimes B = a_ib_j) = Tr (W_{1+2} P_{ij}),$$

And the expectation value of the "total" $A \otimes B$ observable in state W_{1+2} is:

$$Exp_{W1+2} (A \otimes B) = Tr ((A\otimes B)W_{1+2}).$$

We will sometimes be given the state W_{1+2} of a composite system, and then asked to figure out what the reduced states W_1, W_2 of the separated subsystems must be. Given a couple of observables A and B on H_1, H_2, there are some relatively straightforward identifications that help to work out the reduced states, namely:

$$Tr ((A \otimes I)W_{1+2}) = Tr (AW_1)$$

$$Tr((I \otimes B)W_{1+2}) = Tr(BW_2), \qquad (*)$$

where I is the identity observable. This amounts to the demand that the probability distribution over the eigenspaces of observable A (B) defined by the reduced state W_1 (W_2) be the same as that laid out over $A \otimes I$ ($I \otimes B$) by the composite state W_{1+2}; thus effectively ensuring that the choice of description (either in the larger or smaller Hilbert space) of a subsystem in a larger composite system, has no measurable consequences as regards the monadic properties of the individual subsystems.

Appendix 2: The Insolubility Proof

Consider three O-distinguishible initial states of the object system:

$P[\phi_1]$, $P[\phi_2]$, $P[\phi_3]$,

where ϕ_1, ϕ_2 are eigenvectors of O with eigenvalues λ_1, λ_2, and ϕ_3 is a non-trivial superposition $\phi_3 = a_1 \phi_1 + a_2 \phi_2$.

Set up a Schrödinger interaction, in accordance with (QRUE) and (OOC):

$\hat{U}_t (P[\phi_i] \otimes W_a) \hat{U}_t^{-1} = \Sigma\, w_n P[\beta_{ni}]$, where

(QRUE) $\beta_{ni} = \hat{U}_t (\phi_i \otimes \gamma_n)$,
(OOC) $\forall n \forall i = 1,2,3$: $\hat{I} \otimes \hat{A} (\beta_{ni}) = \mu_{ni}\, \beta_{ni}$

By the linearity of $\hat{U}_t$:

$\hat{U}_t (\phi_3 \otimes \gamma_n) = a_1 \hat{U}_t (\phi_1 \otimes \gamma_n) + a_2 \hat{U}_t (\phi_2 \otimes \gamma_n)$.

Hence $\beta_{n3} = a_1 \beta_{n1} + a_2 \beta_{n2}$.

Now we can calculate:

(A) $\hat{I} \otimes \hat{A} (\beta_{n3}) = \hat{I} \otimes \hat{A} (a_1 \beta_{n1} + a_2 \beta_{n2}) =$
$a_1 (\hat{I} \otimes \hat{A}) \beta_{n1} + a_2 (\hat{I} \otimes \hat{A}) \beta_{n2} =$
$a_1 \mu_{n1} \beta_{n1} + a_2 \mu_{n2} \beta_{n2}$,

and

(B) $\hat{I} \otimes \hat{A} (\beta_{n3}) = \mu_{n3} \beta_{n3} =$
$\mu_{n3} (a_1\beta_{n1} + a_2 \beta_{n2}) =$
$a_1 \mu_{n3} \beta_{n1} + a_2 \mu_{n3} \beta_{n2}$.

However, (A) and (B) are equal if and only if $\mu_{n1} = \mu_{n2} = \mu_{n3}$, in which case β_{n1}, β_{n2}, β_{n3} are not $(\hat{I} \otimes \hat{A})$ – distinguishible. Thus (TPC) fails for this choice of initial states of the system. QED.

Appendix 3: Stein's Lemma and its implications

Stein's lemma: If Q and R are bounded linear operators on the Hilbert spaces H_2 and $H_1 \otimes H_2$ respectively; if ν is a vector subspace of H_1; and if for every non-zero $u \in \nu$ the commutativity condition $S_u = (P_u \otimes Q) R = R (P_u \otimes Q)$ holds; then there is a uniquely determined bounded linear operator T on H_2 such that:

$S_u = P_u \otimes T$, for every nonzero $u \in \nu$.

Application to the Measurement Problem: Take Q to be the initial state of the apparatus, , i.e. $Q = W_a$, and R to be the inverse time-evolved pointer position observable, i.e. $R = U^{-1} (I \otimes A)U$. It is straightforward that $U (P_u \otimes Q) U^{-1}$ commutes with $(I \otimes A)$ if and only if $P_u \otimes Q$ commutes with R. In addition, according to the results in section 4 of the paper, this commutativity condition holds if and only if (QRUE) and (OOC) hold for $P_u \otimes W_a$.

Stein's lemma then shows that there is a uniquely determined bounded linear operator T on H_2 such that $S_u = P_u \otimes T$. However the quantum statistical algorithm predicts that the expectation of the pointer position observable when the system is in the initial state $P_u \otimes W_a$ is: $Tr (U(P_u \otimes W_a)U^{-1}\ I \otimes A) = Tr (P_u \otimes T)$, which is equal to Tr (T) because the trace of P_u is one. So the expectation of the pointer position observable is independent of the initial state of the system, and no measurement at all has been carried out.

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