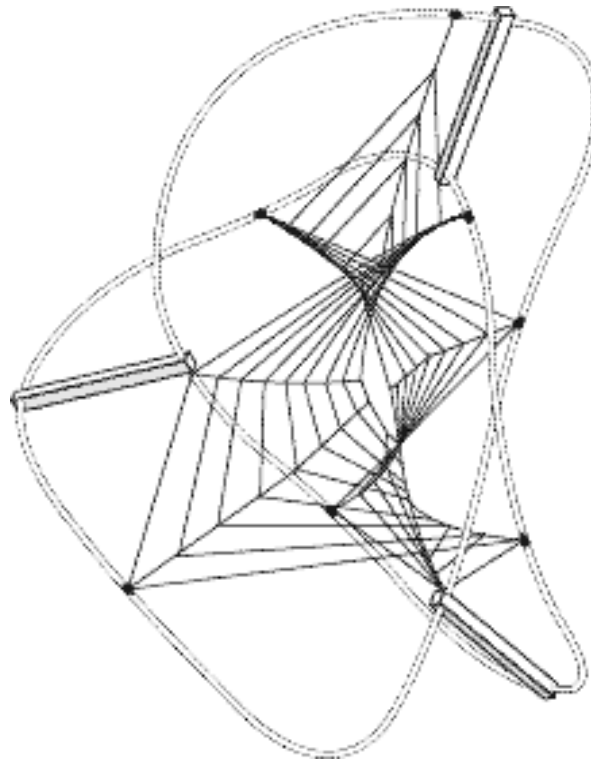


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Propensities in Quantum Mechanics

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Abstract: I review five explicit attempts throughout the history of quantum mechanics to invoke dispositional notions in order to solve the quantum paradoxes, namely: Margenau's *latencies*, Heisenberg's *potentialities*, Popper's *propensity* interpretation of probability, Nick Maxwell's *propensitons*, and the recent *selective propensities* interpretation of quantum mechanics. I raise difficulties and challenges for all of them, but conclude that the selective propensities approach nicely encompasses the virtues of its predecessors. I elaborate on some of the properties of the type of propensities that I claim to be successful for quantum mechanics, and finish by briefly sketching out ways in which similar notions can be read into some of the other well-known interpretations of quantum mechanics.

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1. Introduction

The history of dispositional properties in quantum mechanics is arguably as long as the history of quantum mechanics itself. A dispositional account of quantum properties is arguably implicit in the early quantum theory, for instance in Bohr's model of the atom, since transitions between quantum orbitals can be described as indeterministic processes that bring about certain values of quantum properties with certain probabilities. Similarly, on the orthodox Copenhagen interpretation measurements do not reveal pre-existent values of physical quantities, but rather bring about values that can not properly be said to exist independently of, or prior to, the measurement process – with some well-defined probability. Then, in addition, starting from the 1950's there has been a succession of attempts to explicitly employ dispositional notions in order to understand quantum mechanics. They include Henry Margenau's latency interpretation,¹ Werner Heisenberg's succinct appeal to Aristotelian potentialities,² Karl Popper's propensity interpretation of quantum probabilities,³ Nicholas Maxwell's propensiton theory,⁴ and my own recent defence of a dispositional reading of Arthur Fine's selective interactions solution to the measurement problem.⁵

In this paper I intend to describe and compare these five different interpretations of quantum mechanics that have *explicitly* appealed to dispositions and propensities in order to solve the paradoxes of quantum mechanics. I will distribute praise and blame liberally, and I will eventually contend that the virtues of these interpretations are appropriately subsumed under the latter account of selective interactions as interactions with dispositional properties that get displayed in experimental contexts as probability distributions. I will then briefly point out some reasons for thinking that this account is appropriate as well for other mainstream interpretations of quantum mechanics – even those that have not made explicit use of dispositional notions before, such as Bohr's version of the *Copenhagen* interpretation, Bohmian mechanics, and the Ghirardi-Rimini-Webber (GRW) *collapse* interpretation.

¹ Margenau (1954).

² Heisenberg (1966).

³ Popper (1957, 1959).

⁴ Maxwell (1988, 2004).

⁵ Fine (1987, 1992); Suárez (2004).

The paradigmatic interpretational question of quantum mechanics is a question about the general interpretation of superposed states: *What does it mean – with respect to the property represented by an observable Q – for a quantum system to be in state Ψ that is not an eigenstate of the observable Q ?* Different interpretations of quantum mechanics can be helpfully distinguished in terms of the different answers they provide to this question. The views described here vary greatly in their details, their complexity and their ontological assumptions, but their answer to the paradigmatic interpretational question is essentially the same and includes a reference to some among the nexus of dispositional notions. We may summarise the answer as follows: *It means that the system has the disposition, tendency, or propensity, to exhibit a particular value of Q if Q is measured on a system in state Ψ .* It is my purpose in this paper to argue that this answer to the question remains viable in spite of past failures to articulate it convincingly.⁶

2. Margenau's Latency Interpretation

In an excellent brief article published in 1954, Henry Margenau argued in favour of an interpretation of quantum observables as dispositional physical quantities. Margenau did not employ explicitly the terms “tendency” or “propensity” but he did use another dispositional term, namely “latency”. His argument proceeded in two stages, a negative one followed by a positive one. First, negatively, he argued against both Bohm’s theory and the Copenhagen interpretation. Margenau raised three fundamental objections against Bohm’s theory: (i) it postulated unnecessary structure in the form of particles’ trajectories, (ii) the wave-function in n -dimensional configuration space seemed implausible as a representation of physical reality, and (iii) the physical act of measurement was turned into a mystery as it involved the sudden infinitely rapid collapse of the quantum field.⁷

Margenau then went on to criticise the Copenhagen interpretation for its supposedly dualistic features – for asserting that particles have positions at all times, yet

⁶ It will also be important to characterise these notions precisely, and I will attempt to do so in section 7. In the meantime it suffices to say that I will take “disposition” to be an umbrella term that encompasses all the others, such as capacity, tendency or propensity.

⁷ Margenau (1954, p. 8).

we are unavoidably ignorant of what these are. In other words he assumed that the Copenhagen tradition takes a subjective reading of the quantum probabilities and the uncertainty relations, and that it postulates an essential role for consciousness and the observer. Margenau charged Bohr in particular with this view, and he criticised him for it.⁸ Few historians of physics would nowadays agree with Margenau's reading of the Copenhagen interpretation. The claim that particles have positions at all times yet we don't know which ones these are, is characteristic of a Bohm-type hidden variable interpretation; while the claim that the quantum probabilities are subjective seems to entail the long-discredited ignorance interpretation of superpositions. What is nowadays often taken to characterise the "Copenhagen interpretation" is the necessity of an observer, possibly endowed with consciousness, for there to be any definite elements of reality; or, alternatively, the impossibility of any meaningful discourse about the values of a non-measured quantity.⁹

By contrast Margenau proposed a "third-way" interpretation of quantum mechanics that treads an intermediate course, whereby the probabilities are given an objective reading, and they are understood as describing tendencies – more precisely: the tendencies of latent observables to take on different values in different experimental contexts. Here is an extensive quote:

"I propose a shift of attention. The contrast, or at any rate the difference, is now between [...] possessed and latent observables. Possessed are those, like mass and charge of an electron, whose values are "intrinsic", do not vary except in a continuous manner, as for examples the mass does with changing velocity. The others are quantized, have eigenvalues, are subject to the uncertainty principle, manifest themselves as clearly present only upon measurement. I believe that they are "not always there", that they take on values when an act of measurement, a

⁸ Margenau (1954, p. 9).

⁹ In addition few historians would agree that these claims are a part of Bohr's own interpretation of quantum mechanics. For instance, Don Howard (2004) makes a strong case that Bohr's complementarity principle is distinct and independent of what has passed as the radical (and flawed) emphasis on the role of the observer, measurement or consciousness in the so-called Copenhagen interpretation. Bohr only insisted on the necessity of using classical concepts in our descriptions of the phenomena. According to Howard the *Copenhagen interpretation* was invented by Heisenberg in the 1950s, and erroneously propagated as Bohr's doctrine by a number of influential philosophers of science, including Popper and Feyerabend. In the next section I argue that Heisenberg's 1950s writings can also be read as defending a more robust form of objective dispositional realism. And in section 8 I defend a dispositional reading of Bohr's own distinct interpretation.

perception, forces them out of indiscriminacy or latency. If this notion seems grotesque, let it be remembered that other sciences, indeed common sense, employ it widely. Happiness, equanimity, are observable quantities of man, but they are latent qualities which need not be present at all times; they too, can spring into being or be destroyed by an act of inquiry, a psychological measurement”.

Margenau’s “third” or “latency” interpretation is extraordinarily prescient and insightful in many respects. It ought to be a classic source, if not the classic reference, for all dispositional accounts of quantum mechanics. Yet it is often ignored, even by the proponents of dispositional accounts themselves. For instance, a reference to Margenau’s work is remarkably absent in Heisenberg’s late 1950’s writings (reviewed in section 3); Popper’s much better known writings on the propensity interpretation of quantum probabilities (which I describe in section 4) fail to discuss Margenau’s views; and a reference to Margenau’s writings is conspicuously absent in Nick Maxwell’s more recent writings.¹⁰ A notable exception is Michael Redhead, the distinguished British philosopher of physics, who refers to Margenau’s work in his book *Incompleteness, Non-Locality and Realism*. In this book, Redhead describes three different views on quantum mechanics (*A: hidden variables, B: propensities and potentialities, and C: complementarity*) and, although he does not develop the second view in any detail, he seems to favour it over the alternatives: “the conclusion is that view B is perfectly consistent with realism, and certainly gives no arguments at all in favour of idealism”.¹¹

Margenau’s *latency* interpretation provides a basic template for dispositional accounts. Suppose that state Ψ can be written as a linear combination $\psi = \sum_n c_n |v_n\rangle$ of the eigenstates v_n of the *latent* observable O with spectral decomposition given by $O = \sum_n a_n |v_n\rangle\langle v_n|$. Then we may answer the paradigmatic interpretational question as follows. We may say that *a system is in state ψ if and only if it has on a measurement of O the disposition to manifest eigenvalue a_i with probability $|c_i|^2$* . I will argue throughout this paper in favour of this basic template as the core of any appropriate dispositional

¹⁰ It is hard to believe that these authors, particularly Heisenberg and Popper, did not know of Margenau’s contributions. Henry Margenau was a well known figure in the post-war period: he was Professor of Physics and Natural Philosophy at Yale, a member of the American Academy of Arts and Sciences, President of the American Association for the Philosophy of Science, and a prominent defender of the use and need for philosophical reflection on physics.

¹¹ Redhead (1987, p. 49).

account of quantum mechanics. In particular it has the advantage that it turns quantum propensities into properties of the quantum systems themselves as opposed to relational properties of systems in their interaction with measurement devices, or with the environment. I claim that in this respect Margenau's latent interpretation is just right for quantum mechanics – and how right precisely we can only know *in retrospect*, in light of subsequent failed attempts to provide alternative dispositional accounts.

However, Margenau's third interpretation goes beyond the basic template in some ways that are unhelpful. He does not distinguish between the possession of a value of a physical property, and the possession of the property itself – a distinction that makes no sense for categorical properties, but is perfectly sensible for dispositional properties.¹² The distinction is in fact essential in order to understand how a dispositional property may be legitimately ascribed in the absence of its manifestation. A failure to draw this distinction leads Margenau to link inappropriately the actualisation of latent properties with their existence: “[latent properties] manifest themselves as clearly present only upon measurement”. And most clearly: “Hence I believe that they are not always there”. In other words, the act of measurement not only brings into existence the value of the latent property in question, but the latent property itself. So in the absence of a measurement of position, for instance, an electron has no value of position, and as a consequence it has no position at all.

Let me first provide a diagnosis of the motivating sources of Margenau's conflation. It seems to me that there are two reasons why Margenau is led this way. There is first a prior conceptual conflation of three terms (“property”, “physical quantity” and “observable”) that ought to be kept distinct from a contemporary point of view. There is then an unwarranted desire to navigate a middle course between Bohm's theory and Margenau's own reading of the Copenhagen interpretation.

As regards the first reason for the conflation we would nowadays approach Margenau's terms as follows. We would take “observable” to stand for a quantum property – some of these “properties” might be dispositional, others might be categorical, and we would only denote the latter as “physical quantities”. But failing to

¹² For the distinction categorical / dispositional see Mumford (1998, chapter 4).

distinguish them in this way, Margenau identifies all properties with physical quantities and is thus forced to conflate observables and physical quantities. It then follows that if an observable lacks a value (i.e. if it is not a physical quantity) then it fails to represent a real property. Hence the need to require that the property be actualised as a physical quantity if the property is to be real.

A second motivation for Margenau's unhelpful conflation can be found in his desire to build an interpretation that keeps neutral between Bohm's theory and what he identifies as the Copenhagen interpretation. Thus he claims that his interpretation is "less committal than the others. For clearly, if the electron did have a determinate position at all times and we could not possibly know it, this view would still stand aright. Likewise, it is compatible with, though again less committal than, the appeal to measurement as bringing about this latency".¹³ Margenau identifies the former option with Bohm's theory and the latter with the Copenhagen interpretation. His desire to avoid any realism *à la* Bohm leads him to discard what I would argue is the most natural dispositional account, namely: that systems possess their dispositional properties at all times – in a realist sense of the term, as applied to dispositions – without thereby implying that their values (physical quantities) are actualised, or manifested, at all times. And his peculiar reading of the Copenhagen interpretation does not help either – in his desire to avoid making physical quantities relative to measurement contexts Margenau ends up turning all properties ephemeral, including latent properties or dispositions: "there is an irreducible haziness in the very essence of perceived phenomena" (Margenau, 1954, p. 10). For the purposes of analysis, it will be more helpful here to consider a precise version of Margenau's view that links latent properties to measurement contexts (nothing that follows will cease to apply to the less precise version with arbitrarily ephemeral properties).

Margenau's conflation of properties and values has two pernicious consequences for his interpretation of quantum mechanics. First, any presumed advantages over other interpretations (including those discussed by Margenau himself) in solving the quantum paradoxes disappear. And second, new and additional problems related to the identity of quantum objects are imported into the picture. I will discuss them both in turn.

¹³ Margenau (1954, p. 10).

The first consequence becomes clearer in the context of the measurement problem. According to the model of measurement provided by the quantum formalism, if we let our initial quantum system interact with a macroscopic measurement device we obtain what is known as macroscopic superposition infection: the composite (system + device) goes into a superposition. Formally, the state of the composite at the end of the interaction looks like this: $\Psi = \sum_{n,m} c_{nm} v_n \otimes \mu_m$, where μ_m are the eigenstates of the pointer position observable with corresponding eigenvalues a'_m (and so $c_{nm} = a_n a'_m$). The challenge is then to predict theoretically that in this state the macroscopic measurement device pointer will point to some value or other (sometimes known as the problem of objectification of the pointer position). This is compounded by the fact that, on the standard interpretational rule for quantum states (the eigenstate / eigenvalue or e/e link), a system in a superposition of e-states of an operator has no value of the physical quantity represented by that operator. Hence the pointer takes no values in the final state of the composite. To resolve this a dispositional account will have to make the corresponding claim given by the basic template: *a system is in state Ψ if and only if it has on a measurement of O the disposition to manifest eigenvalue $a_i a'_j$ with probability $|c_{ij}|^2$* . This entails ascribing a property over and above those dictated by the (e/e link), since it entails ascribing a property without there being a value that it takes; and Margenau nowhere seems prepared to break the (e/e link) in this way.

On the contrary, on Margenau's interpretation the latent observable is not present ("manifested as clearly present") unless upon measurement. It follows that the pointer position observable is not "present" unless the measurement device is subject to its own measurement interaction – i.e. a second-order interaction – in order to find out the dispositions exhibited by the composite system in state Ψ ; and this way the problem just seems to recur indefinitely. There is no way to break the impasse simply by claiming that the system has some disposition that gets actualised at the conclusion of the measurement interaction – since (i) on Margenau's reading the system does not have a latent property unless the property is being measured, and (ii) no pointer position observable would get actualised in any case unless a third measurement apparatus is brought into the picture. Margenau does not – at least not explicitly – introduce an additional dispositional property in order to explain the manifestation of latent

properties on a corresponding measurement. So his account is left lacking in just the kind of respect needed for an appropriate solution to the quantum paradoxes.

But in fact the conflation of properties and values brings in added complications, which Margenau himself suspected. Issues of particle identity arise: Suppose that an electron has a number of possessed properties (“mass”, “charge”) and a number of latent ones (“spin”). For any particular electron the possessed properties remain constant but not so the latent ones. These jump in and out of existence in accordance with measurement situations (in the less precise version of Margenau’s interpretation that tries to eschew reference to measurement this problem gets even worse, since properties jump in and out of existence in a more or less arbitrary fashion). So what entity the electron really is, crucially depends on measurement contexts. As Margenau himself writes: “It may be that this latency affects even the identity of an electron, that the electron is not the same entity with equal intrinsic observables at different times”.¹⁴ Indeed on this view, given that dispositional properties are intrinsic and not relational properties of experimental set-ups, an electron subjected to a spin measurement has intrinsic properties that it strictly speaking fails to have prior to that measurement. We could say that the electron becomes a different entity at each instance of measurement. The measurement-dependent (or, worse, the arbitrarily ephemeral) nature of properties on Margenau’s reading turns entities themselves into measurement-dependent (or worse, arbitrarily ephemeral). This is not *prima facie* good news: the dissolution of system’s identities seems too high a price to pay for a coherent interpretation of quantum mechanics.

3. Heisenberg’s Aristotelian Potentialities

In 1958, soon after Margenau’s proposal, Werner Heisenberg published *Physics and Philosophy*, his best known philosophical reflection on quantum mechanics. The book is often celebrated as an exposition of a standard version of the Copenhagen interpretation. It is certainly explicit in its defence of that view – chapter 3 is even entitled “The Copenhagen Interpretation of Quantum Theory”. But a close reading of

¹⁴ Margenau (1954, p. 10).

the book reveals a very complex mixture of interpretational elements, only some compatible with what we nowadays would identify as a Copenhagen interpretation. A commitment to reading the quantum probabilities at least in part in terms of Aristotelian potentialities stands out among the elements apparently alien to the Copenhagen view: “The probability function combines objective and subjective elements. It contains statements about possibilities or better tendencies (“potentia” in Aristotelian philosophy), and these statements are completely objective, they do not depend on any observer; and it contains statements about our knowledge of the system, which of course are subjective in so far as they may be different for different observers”.¹⁵

Heisenberg is not very clear about how precisely these objective and subjective elements combine. The very locution that a probability function “contains statements” is puzzling from the standpoint of contemporary philosophical treatments of probability. Perhaps the most plausible interpretation of these cryptic passages in Heisenberg’s writings can be obtained by replacing “contains” with “implies”, since it does not seem implausible to claim that the probability function implies statements about possibilities or tendencies. But again Heisenberg is not very explicit about whether the quantum probability distributions represent subjective degrees of belief (and thus imply statements about our knowledge), or objective frequencies or propensities (thus implying statements about matters of fact independent of our knowledge).¹⁶

Sometimes Heisenberg seems to come close to asserting a version of David Lewis’ *Principal Principle*, or some other general rule whereby (rational) subjective degrees of belief must follow objective chances when these are known.¹⁷ Quantum probabilities may then just measure rational degrees of belief, while pertinently tracking objective chances. This would at least seem to give some substance to Heisenberg’s claim that the quantum probabilities imply both statements about our subjective knowledge of the system and statements about the objective potentialities of the system, and seems close to what Heisenberg aims for in the following paragraph, for example:¹⁸

¹⁵ Heisenberg (1958, p. 53).

¹⁶ The selective propensity account, by contrast, gives unambiguous answers to these questions (cf. the discussion in section 6).

¹⁷ Lewis (1980/6).

¹⁸ Heisenberg (1958, p. 54).

“Therefore, the transition from the ‘possible’ to the ‘actual’ takes place during the act of observation. If we want to describe what happens in an atomic event, we have to realise that the word ‘happens’ can apply only to the observation, not to the state of affairs between two observations. It applies to the physical, not the psychical act of observation, and we may say that the transition from the ‘possible’ to the ‘actual’ takes place as soon as the interaction of the object with the measuring device, and thereby with the rest of the world, has come into play; it is not connected with the act of registration of the result by the mind of the observer. The discontinuous change in the probability function, however, takes place with the act of registration, because it is the discontinuous change of our knowledge in the instant of registration that has its image in the discontinuous change of the probability function”.

Heisenberg does not provide a detailed model of these Aristotelian ‘potentia’. Rather he appeals to them as a brute explanation of the discontinuous change that measurements bring to the probability function: “[...] the probability function does not in itself represent a course of events in the course of time. It represents a tendency for events and our knowledge of events. The probability function can be connected with reality only if one essential condition is fulfilled: if a new measurement is made to determine a certain property of the system”.¹⁹ And, like Margenau, he is also unclear as to whether merely some of quantum systems’ properties are dispositional, or the systems themselves fully exist only “in potentia”.²⁰

The appeal to dispositional properties as grounding quantum measurements is one of the key two elements in Heisenberg’s otherwise vague discussion. I will argue in section 6 that the other key element, for an appropriate and detailed dispositional account of quantum mechanics, is the sharp distinction he draws between these dispositional properties and the quantum probabilities. For it is clear, at least, that for Heisenberg “potentia” are not merely an interpretation of quantum probabilities. On the contrary, it has been noted that the relationship between the quantum probabilities and these “potentia” is rather subtle on Heisenberg’s view. The selective-propensity view

¹⁹ Heisenberg (1958, pp. 47-8).

²⁰ For instance, when he writes (*ibid*, p. 160): “In the experiments about atomic events we have to do with things and facts, with phenomena that are just as real as any phenomena in daily life. But the atoms or the elementary particles themselves are not as real; they form a world of potentialities or possibilities rather than one of things or facts”.

that I will develop in section 6 will also essentially distinguish quantum probabilities from their underlying dispositions (although I will not follow Heisenberg in accepting a subjective interpretation of the quantum probabilities). This second key element in Heisenberg's discussion is particularly important in relation to historically misguided attempts to solve the quantum paradoxes by merely *interpreting* the quantum probabilities as propensities – among which Popper's attempt is possibly the paradigm. I turn to this interpretation in the next section.

4. Popper's Probabilistic Propensities

In a large number of publications, over a very large number of years, Popper famously defended a propensity account of quantum probabilities which he argued could solve the quantum paradoxes.²¹ The propensity interpretation of quantum probability was a philosophical milestone in Popper's system since he claimed that it (i) resolved the paradoxes of quantum mechanics, (ii) re-established the possibility of a thoroughly realist interpretation of the quantum theory, of physics and of science in general, and (iii) it provided strong empirical confirmation in favour of a propensity interpretation of the calculus of probability in general. For an illustration, the following is a nice quote from Popper that exemplifies these three theses: "The main argument in favour of the propensity interpretation is to be found in its power to eliminate from quantum theory certain disturbing elements of an irrational and subjectivist character ... it is by its success or failure in this field of application that the propensity interpretation will have to be judged".²²

I have provided elsewhere further textual evidence in favour of the claim that the following five theses are central to Popper's programme:²³

(Thesis 1): Propensities are real properties instantiated in the quantum world.

(Thesis 2): Propensities are not monadic, or intrinsic, properties of quantum systems, but relational properties of the entire experimental set-ups that test them. A one-electron

²¹ Popper (1957, 1959, 1967, 1982).

²² Popper (1959, p. 31)

²³ Suárez (2004a).

universe would lack any propensities: These can only be ascribed to particles in conjunction with whole experimental set-ups, including the measurement devices designed to test them.

(Thesis 3): Quantum theory is an essentially probabilistic theory, in the sense that it is a theory about the probabilities that certain outcomes obtain in certain experimental set-ups.

(Thesis 4): The quantum wave function, or quantum state, is a description of a propensity wave over the outcomes of an experimental set-up.

(Thesis 5): Providing an objective interpretation of the probabilities in quantum mechanics in terms of propensities is sufficient to solve the philosophical puzzles concerning quantum mechanics.

I have argued in addition that – with the exception of (Thesis 1), over which the right stand is possibly a neutral one – an appropriate dispositional interpretation of quantum mechanics should deny all the other theses (Theses 2-5). First a relational definition of “propensity” (Thesis 2) leads Popper into a hopeless set of problems that make (Thesis 5) impossible to satisfy. Relational propensities can not solve the paradoxes of quantum mechanics.²⁴ Then, another set of problems related to the ignorance interpretation of mixtures and super-positions make Theses 3 and 4 impossible to satisfy. Indeed if the function of the quantum theoretical state is merely to describe a probability distribution interpretable in all cases as a propensity wave then it becomes impossible to distinguish appropriately a superposition from a statistically indistinguishable mixture. Yet this distinction between a superposition and the corresponding mixture is absolutely essential to be able to solve any of the quantum paradoxes. Popper’s approach lacks the resources to draw this distinction appropriately as I will now show.

Consider the superposed state $\psi = \sum_n c_n |v_n\rangle$. As regards observable Q this state is indistinguishable from the mixture: $W = \sum_n |c_n|^2 |v_n\rangle \langle v_n|$. But while the latter can be given the ignorance interpretation (if it is a proper mixture – i.e. if it has not been mathematically derived from a larger composite simply by application of the axiom of reduction, but is instead the result of some preparation procedure), the former never can.

²⁴ A conclusive argument to this effect has been advanced by Peter Milne (1985).

That is, there are some mixtures with the form of W that describe a system in a pure state $|v_i\rangle$ but we just don't know which pure state this is, and the probabilities $|c_i|^2$ just describe our ignorance. But the ignorance interpretation of super-positions has long been discredited, and it can not be applied to a pure state ψ .²⁵ However, were the only function of the quantum state to provide a probability distribution that could always be interpretable as a propensity wave, we would not be able to distinguish between these two states: ψ and W would essentially represent the same propensity.²⁶ In the standard cases of quantum paradoxes, such as the two-slit experiment and the Schrödinger cat paradox for the measurement problem we just can't solve the issues by appeal to Popper's propensities. In all these cases, it is essential to be able to derive the right ignorance interpretable mixture at the end of the interaction process – but the calculations that take us to that final state must be performed on the superposition, otherwise we lose the contribution that the interference terms make to that derivation, and our predictions will fail to be appropriate for those set-ups. In failing to distinguish between these two states Popper's propensity interpretation of probabilities is unable to give a proper account of quantum phenomena.

5. Maxwell's Propensitons

A more recent propensity-based version of quantum mechanics goes by the name *propensiton theory* and has been developed by Nicholas Maxwell.²⁷ It is a sophisticated version of Popper's propensity interpretation of probability, improving on it in certain important respects. The most important improvement, in my view, is that unlike Popper, Maxwell does not simply provide an interpretation of quantum probabilities, but goes on to make ontological commitments about the type of entities that may give rise to such probabilities, and their interactions. In this regard Maxwell makes two fundamental claims, one is a very general philosophical claim about entities and their structure in general; the other is a much more concrete claim specifically about quantum mechanical entities.

²⁵ The locus classicus of the argument against the ignorance interpretation of superpositions is Feyerabend (1957).

²⁶ A fully generalised version of this argument is due to Neal Grossman (1972).

²⁷ Maxwell (1988), (2004).

According to the first, the nature of an entity is inherently dependent upon the features of its dynamical laws. Maxwell writes:²⁸ “In speaking of the properties of fundamental physical entities (such as mass, charge, spin) we are in effect speaking of the dynamical laws obeyed by the entities – and *vice versa*. Thus, if we change our ideas about the nature of dynamical laws we thereby, if we are consistent, change our ideas about the nature of the properties and entities that obey the laws”. This statement seems *prima facie* misguided in light of the historical record. For example, there have been different models of the solar system endowed with their own dynamical laws (such as Tycho’s, Kepler’s, Newton’s or Einstein’s laws) but agreeing on the essential nature of the planets (size, density, mass, relative distances, etc). So it does not seem right on the face of it to say that the nature of the objects depends on the laws. However, Maxwell’s meaning is more subtle and is best brought out by his second claim:²⁹ “The quantum world is fundamentally probabilistic in character. That is, the dynamical laws governing the evolution and interaction of the physical objects of the quantum domain are probabilistic and not deterministic”. The second claim importantly qualifies the first: the distinction that matters is that between deterministic and probabilistic laws. Maxwell’s more subtle view is then that there are fundamentally only two kinds of entities: probabilistic and deterministic ones. Thus Maxwell would probably be committed to the view that in a model of the solar system with probabilistic laws the planets would just not be the kinds of entities that they are in our (supposedly deterministic) world, and that is regardless the actual form of the deterministic laws governing their dynamics.

So far, however, this remains all rather cryptic. We can unravel the claim by considering the difference between probabilistic and deterministic laws which seems quite clear on either a formal or a modal account. On the formal account, roughly, a law is deterministic if any future state of a system has conditional probability one or zero given the present state of the system: $\text{Prob}(S_f / S_p) = 1 \text{ or } 0$, for any $S_f > S_p$. On the modal account, roughly, a law is deterministic if there is only one possible world described by the law that is compatible with the history of the actual world so far.³⁰ Given this account of laws, what exactly is the ontological difference between essentially probabilistic and deterministic entities? For instance, in discussing the state

²⁸ Maxwell (1988, p. 10).

²⁹ Maxwell (1988, p. 10).

³⁰ Earman (1986) is the locus classicus for definitions of determinism. See particularly chapter 2.

of a quantum particle delocalised in space, Maxwell suggests that the spread-out wave-function in position space entails that quantum entities are not point-particles at all but rather take the form of expanding spheres:³¹ “A very elementary kind of spatially spreading intermittent propensiton is the following. It consists of a sphere, which expands at a steady rate (deterministic evolution) until it touches a second sphere, at which moment the sphere becomes instantaneously a minute sphere, of definite radius, somewhere within the space occupied by the large sphere, probabilistically determined”.

This suggests that, on the propensiton theory, the wave-function in position space literally represents the geometric shape of quantum entities, which develop deterministically in time and collapse probabilistically due to inelastic scattering. This is indeed a straightforward way to make true both of Maxwell’s fundamental claims. For it is now true – on both the formal and modal accounts of a probabilistic law – that the nature of the entity depends on the law – since its very shape now depends on the probabilistic character of the law. On either view the move from a deterministic to a probabilistic law has an effect on the very geometrical nature of the entity across time: On the formal account the probability that the future state of the sphere-particle be expanded with respect to its present state can no longer be one. And on the modal account there is more than one possible world with differently shaped spheres within them, all consistent with the history of the actual world so far.

The argument successfully avoids the criticisms to Popper’s propensity account of probability, but it brings its own problems. Two sets of difficulties stand out. The first one relates to the ontology invoked, and threatens the propensiton theory with incoherence; while the second problem has to do with the requirement that there be an inelastic creation event of a new particle every time there is a probabilistic collapse. The first problem is straightforward to see – the postulated process of contraction of the spheres breaks momentum and energy conservation principles, and invoking it in order to solve e.g. the problem of measurement generates as much of a paradox as the paradox that the process was intended to solve in the first place. For now the question becomes: what kind of internal mechanism and what sort of laws govern the sudden contraction of the spheres? The simplest way to get around this problem is to withdraw the claim that

³¹ Maxwell (2004, p. 327).

the quantum wavefunction literally represents quantum entities – and claim instead that it just represents the probabilistic propensities of point-particles. But such a move (a) fails to provide the desired rationale for claims (1) and (2), and (b) brings us back to the Popperian approach with all the problems that we have already reviewed.

The second set of difficulties is related to the notion of contraction under inelastic scattering – which lies at the heart of the proposal. These inelastic scattering events take place, according to Maxwell “whenever, as a result of inelastic interactions between quantum systems, new ‘particles’, new bound or stationary systems, are created”.³² Any measurement interaction is ultimately reducible to a measurement of position and, according to Maxwell, will generate some particle, since the localisation of any particle involves the ionisation of an atom, the dissociation of a molecule, etc. I see at least two objections to this proposal, which I will not have here time to explore in depth, but seem *prima facie* sufficiently robust to throw the proposal into doubt. First, it is unclear that there really are no measurement interactions that do not result in an inelastic scattering of a new particle; a particularly salient example could be destructive measurements. And second, whether there are or not such measurement interactions in practice, the measurement problem – as is often formulated ideally in the tensor product Hilbert space formalism – does not describe inelastic scattering creation events. Hence a solution to the paradoxes that demands that all measurement interactions result in inelastic scattering of particles does not solve the theoretical paradox presented by the measurement problem. To solve the problem one could give up on the requirement of inelastic scattering, and insist instead on some kind of law-like regularity in the collapse of the wave-function. But this would just assimilate the propensity theory to a kind of propensity version of the Ghirardi-Rimini-Weber spontaneous collapse theory. I look at the prospects of such a theory in the last section of this paper.

6. Selective Propensities

For a few years now I have been defending a new interpretation of quantum mechanics that appeals to propensities.³³ It employs Arthur Fine’s notion of selective

³² Maxwell (2004, p. 328).

³³ Suárez (2004a), (2004b).

interactions,³⁴ and gives a new account of them in terms of a particular kind of dispositional properties. We may call it the *selective-propensity* interpretation of quantum mechanics. I have expounded this view in detail elsewhere, so I will here just give a brief and schematic account. On this view a quantum system possesses a number of dispositional properties, among which are included those responsible for the values of position, momentum, spin and angular momentum. One could suppose that *all* quantum properties are irreducibly dispositional, although this is not in principle required. Later on in the paper I will distinguish between observables representing categorical properties and those representing dispositional properties; the distinction however is not meant to imply the existence of both types of observables, nor any particular account of the relation between dispositional and categorical properties. It is only meant to provide conceptual room for a large number of views on the coexistence of such two types of different properties.³⁵

We can represent quantum dispositional properties by means of what Fine calls the *standard representative*. Consider the following definition of the equivalence class of states relative to a particular observable O :

O-equivalence class: $W' \in [W]_O$ if and only if $\forall W' \in [W]_O$: $\text{Prob}(W, O) = \text{Prob}(W', O)$, where $\text{Prob}(W, O)$ stands for the probability distribution defined by W over all the eigenvalues of O .

Suppose that O is a discrete and not maximally degenerate observable of the system with spectral decomposition given by $\sum_n \lambda_n P_n$, where $P_n = P_{[\phi_n]} = |\phi_n\rangle\langle\phi_n|$. And consider a system in a state ψ , a linear superposition of eigenstates of the system. We can construct the *standard representative* $W(O)$ of the equivalence class $[W]_O$ as follows:

Standard representative: $W(O) = \sum_n (\text{Tr } \psi P_n) W_n$, where $W_n = P_n / \text{Tr}(P_n)$.

³⁴ Fine (1987).

³⁵ See my distinction between observables representing categorical and dispositional properties below. For an argument that not all fundamental properties can be irreducibly dispositional see Psillos (forthcoming).

Now, the *selective-propensity* interpretation claims that each standard representative of the state ψ , corresponding to each observable defined over the Hilbert space of the system, is a representation of the dispositional property O of the system. Thus the only *categorical* properties that a quantum system in state ψ can be said to have are those represented by operators that have ψ as an eigenstate. All other observables correspond to *dispositional* properties of the system. It is thus possible to make the following claim: *For a given system in a state ψ , if ψ is not an eigenstate of a given observable O of the system, then $W(O)$ represents precisely the dispositional property O of the system.*

The *selective-propensity* interpretation embodies the main virtues of its predecessors in the history of dispositional accounts of quantum mechanics, while avoiding their defects. My argument for this conclusion will have four stages. First I point out that the selective-propensity interpretation, unlike Margenau's latency interpretation and perhaps Heisenberg's "potential", distinguishes neatly between systems and properties. Secondly, I point out that unlike Maxwell's propensity theory, the selective-propensity view does not entail that the nature of systems and their properties depends essentially upon their laws. Then I explain how this interpretation draws a sharp distinction between dispositional properties and their manifestations. The former are quantum propensities and they both explain and underlie the latter, which are the objective probability distributions characteristic of quantum mechanics – under no particular interpretation of "objective probability". Finally I show that the selective-propensity interpretation, unlike its competitors, solves the measurement problem effortlessly.³⁶

The *selective-propensity* account introduces no new metaphysics. Systems are conceived in the traditional classical way, as physical objects endowed with certain properties with changing values over time. The state specifies both the set of well defined properties of a system and their values at any particular time. The dynamical laws specify the evolution of the state over time, i.e. the evolution of the set of well defined properties and of their values over time. The selective-propensity account departs from the traditional classical view, if at all, in postulating that some of these

³⁶ The analysis of the two-slit experiment that I provide in Suárez (2004a) completes the empirical part of the argument in favour of the selective-propensity interpretation.

properties are dispositional – i.e. even though they are always possessed by the systems, their values are not always manifested.³⁷ But the distinction between systems and their properties is never blurred, and consequently no issues of identity arise out of the ascription of propensities.

Neither is the distinction blurred between systems and their dynamical laws. On the selective propensity view systems only undergo probabilistic transitions, thus actualising their “propensities”, when they interact with other systems in particular ways that test such propensities (measurement interactions are a salient case). Closed quantum systems, by contrast, evolve entirely in accordance with the Schrödinger equation, so their propensities remain non-actualised. Hence the selective-propensity view explains the emergence of the classical regime by assuming that quantum systems are typically open systems, constantly interacting with the environment. This is the standard assumption in decoherence accounts too, but it is questionable whether these accounts actually bring about the classical realm, since they can not transform a pure state into a mixture in the way required for definite values – this is another way to say that decoherence approaches can not solve the problem of measurement even in their own terms.³⁸ The effect of the selective-propensity view is in this regard closer to the more successful treatments of measurement within the quantum state diffusion, or continuous stochastic collapse approaches, since it effectively provides the right mixed state at the end of the interaction. It is just that on the selective-propensity view, this is achieved without having to replace the Schrödinger equation with a non-linear version.

On the *selective-propensity* view the systems’ possession of its dispositional properties does not depend upon the character of the laws. A system has exactly the same propensities whether it is open (and hence subject to probabilistic ‘actualisation’ or ‘collapse’), or closed (and hence evolving always in accordance to the deterministic Schrödinger equation). It is not the possession of the propensity but its manifestation that turns on the character of the interaction. The type of entity that is endowed with these properties does not itself depend upon the type of interaction that takes place. Thus the selective-propensity view rejects the idea defended by Maxwell that the shape

³⁷ I say “if at all” since I am not convinced that there are no legitimate dispositional readings of the properties of classical mechanics, electromagnetism, thermodynamics, etc. For discussion see Lange (2002, chapter 3)

³⁸ Maudlin (1995, pp. 9-10).

of the quantum system is literally as represented by the wave-function – e.g. an expanding sphere. Instead on the selective-propensity view the quantum state is an economical representation of the system’s dispositional properties, including its position. There is no need to picture the particle in any particular way in between measurements of position; and there is concomitantly no need to avoid the point-particle representation of quantum systems.

Finally, the *selective-propensity* view solves the measurement problem in a very elegant and natural way. It does so by supposing that every measurement of a propensity O of a system is an interaction of a measurement device with the system that tests that particular property O of the system. Since each of the system’s propensities is represented by the corresponding standard representative $W(O)$, we can represent the measurement interaction as the Schrödinger evolution of the composite: $W(O) \otimes W(A) \rightarrow U W(O) \otimes W(A) U^\dagger$. The result of this interaction is a mixture over the appropriate eigenspaces of the pointer position observable $(I \otimes A)$: $W_{o+a}^f = U \sum_n (Tr \psi P_n) W_n \otimes W_a U^\dagger = \sum_{nm} \eta_{nm}(t) P_{[\beta_{nm}]}$, which is a mixture over pure states, namely projectors onto the eigenspaces of $(I \otimes A)$. Hence the interaction represents the actualisation of the propensity under test, and the resulting state prescribes the probability distribution over the eigenvalues of the pointer position observable that displays the propensity, since each $P_{[\beta_{nm}]}$ ascribes some value to $(I \otimes A)$ with probability one.³⁹ Hence the selective-propensity view can ascribe values to the pointer position at the end of the interaction, thus solving the measurement problem.

7. The Properties of Selective-Propensities

I would like to end the exposition of the virtues of the *selective-propensity* view with three remarks regarding the nature of the notion of propensity that I have employed here. The first remark concerns the distinction between dispositions and propensities. Throughout the paper I have been assuming that the former is a more general notion that encompasses the latter: a propensity is always a *kind* of disposition, but not vice-versa (see footnote 6). But as a matter of fact there is a more specific use of the term

³⁹ For the details see Suárez (2004b, pp. 233-8).

‘disposition’ that is (unfortunately in my view) entrenched in the literature. According to this use a disposition is a sure-fire property that is always manifested if the testing circumstances are right. My use of the term in this paper is different – since I reserve the term ‘disposition’ for the umbrella notion that covers all the others: tendencies, capacities and propensities are all dispositions on this view. Instead I employ the term ‘deterministic propensity’ for a sure-fire disposition. Typically dispositional notions have been analysed in terms of conditionals. In those terms my use of these notions is roughly as follows:

Full Conditional Analysis of Dispositions: Object O possesses disposition D with manifestation M if and only if were O to be tested (under the appropriate circumstances C_1, C_2, \dots etc) it might M.

I believe that this nicely encompasses all the other uses of the terms including tendencies, latencies, capacities and propensities. But it is clearly distinct from an entrenched use of “disposition” which is best rendered as “deterministic propensity” in my terminology, as follows:

Full Conditional Analysis of Deterministic Propensities: Object O possesses the deterministic propensity D with manifestation M if and only if were O to be tested (under the appropriate circumstances C_1, C_2, \dots etc) it would definitely M with probability one.

It must be noted that a fully fledged conditional analysis of sure-fire dispositions along the lines of this definition is controversial in any case. Martin (1994) and Bird (1998) in particular have advanced a number of arguments that make it suspect. I do not believe these arguments to be conclusive in the case of fundamental or irreducible dispositions,⁴⁰ but I need not broach the dispute here, since for my purposes in this paper it is only necessary to assert the left-to-right part of the bi-conditional analysis. My claim is thus not that the conditional statement provides a complete analysis of any dispositional notion, but merely that the ascription of a deterministic propensity entails the following conditional:

⁴⁰ Neither does Bird – see his (2004).

Conditional Entailment of Deterministic Propensities: If object O possesses the deterministic propensity D with manifestation M *then:* were O to be tested (under the appropriate circumstances C₁, C₂, ... etc) it would definitely M with probability one.

To illustrate these distinctions consider the paradigmatic case of fragility as a deterministic propensity. The full analysis would imply the following: Object O possesses the deterministic propensity of fragility F if and only if were O to be thrown (with sufficient strength, against an appropriately tough surface, etc) it would definitely break. While the conditional entailment would merely imply that: *If* object O possesses the dispositional property of fragility *then:* were O to be thrown (with sufficient strength, against an appropriately tough surface, etc) it would definitely break.⁴¹

It follows on either view that the ascription of fragility to a glass, for instance, entails that were the glass smashed (with sufficient strength, against an appropriately tough surface, etc) it would break. Or to be even more precise, the statement “this glass is fragile” is true only if a series of conditional statements of the form: “if the glass is thrown (under each of a set of conditions C₁, C₂, etc) it would break” are all true. Note that the ascription of fragility does not depend on the truth of the antecedents of these conditional statements (it does not require the actual throwing or smashing of the glass), but on the truth of the conditional itself. The glass is fragile even if it is never smashed; since the possession of fragility does not imply the breakage. The breakage of the glass is rather a *contingent manifestation* of the fragility of the glass, caused by, or at least explained by, its fragility in the appropriate circumstances.

Let us now turn to propensities in general. A propensity can now be generally defined as a probabilistic disposition. In other words a propensity is a dispositional property whose ascription does not imply a deterministic clause (“with probability one”) in the consequent of the corresponding conditional statements, but a general

⁴¹ I am ignoring for the purposes of analysis the important distinction between measure zero and physical impossibility. A further notion would have to be introduced to account for that – perhaps “sure-fire disposition” could be made to correspond with definite manifestations, while “deterministic propensities” could be reserved for manifestations with probability one, which are not physically necessary. But the distinction, however important and cogent, is not relevant to my discussion here.

probabilistic clause instead (“with probability p ”). We may then replace the conditional entailment for deterministic propensities by the following necessary condition on the ascription of propensities:

Conditional Entailment of Propensities: If object O possesses propensity P with manifestation M then: were O to be tested (under the appropriate circumstances C_1, C_2, \dots etc) it would break with probability p ($0 \leq p \leq 1$).

It then follows that a “deterministic propensity” is just a limiting case of the more general notion of “propensity”. For an illustration, consider the often used example of the medical evidence that links the use of tobacco with lung cancer. And suppose, for the sake of argument, that there is indeed a real tendency, with diverse strength in each of us, to contract lung cancer. Such a property would be a propensity since its ascription notoriously does not require the truth of any conditional statement of the type: “if individual X continues smoking 20 cigarettes a day, X will definitely contract lung cancer”, but rather a set of statements of the sort: “if X continues to smoke at this rate, the probability that X will contract lung cancer is p ”. The crucial difference then, between a propensity and a sure-fire dispositional ascription, is that the sure-fire disposition (or deterministic propensity) logically implies its manifestation if the circumstances of the testing are appropriately carried out, while the propensity only implies logically a certain probability p of manifestation, *even if* the circumstances of the testing are right for the manifestation. Under the appropriate circumstances the manifestation of a sure-fire disposition is necessary, while the manifestation of a propensity might only be probable.

The second remark is related to the distinction between *single-case* and *long-run* varieties of propensity.⁴² According to the *long-run* theory a propensity is a feature of a very large sequence of events generated by identical experimental conditions. The advantage of the long run theory is that it turns a propensity ascription into an empirical claim testable by means of a repeated experiment: the observed relative frequency must then gradually approximate the propensity ascription. (It is instructive here to think of the case of loaded die, where the relative frequency observed in a very long trial

⁴² For some excellent reviews of different notions of propensity, as well as a balanced and considerate defence of the long-run theory, see Gillies (2000a, and 2000b, chapter 6).

progressively approximates the propensity). Its disadvantage is that it fails to provide objective single case probabilities. On this view it makes no sense to speak of the propensity of a single isolated event, in the absence of a sequence that contains it: all single case probabilities on this account are subjective probabilities.

Donald Gillies defends the long run theory as the correct interpretation of objective probability in general, and quantum probabilities in particular.⁴³ But his defence of the long run theory in the quantum case turns out to depend on a long run account of the experimental probabilities, and so seems circular as an analysis of the theoretical probabilities provided by quantum mechanics. Gillies thinks that the fact that it is extraordinarily difficult to ever repeat exactly the same scientific experiment means that no single case probabilities ever obtain in quantum mechanics. But even if Gillies were right that no objective singular *experimental* probabilities can be introduced for any real laboratory experiment performed on quantum entities, this need not mean that the probabilities as predicted by the theory can not be objective *and* singular. On most interpretations of quantum mechanics – with the exception of the largely discredited ensemble interpretation – the quantum state allows us to calculate the probabilities for the different outcomes of a *single* measurement performed just *once* on a *individual* quantum system prepared in that state. Hence the single-case propensity theory is, in my view, the most likely objective interpretation of quantum probabilities in light of the inadequacies of the ensemble interpretation of quantum mechanics. (There are in turn a number of different versions of the single-case propensity view⁴⁴ but, given what follows I do not here need to opt for either).

However it should be clear that I am not advocating a single-case interpretation of objective probabilities in general, nor of quantum probabilities in particular. It has already been noted (particularly in section 1) that the selective-propensity view is not an interpretation of quantum probabilities, but an interpretation of quantum *mechanics*. It does not address the question “what is the nature of the quantum probabilities” in any way, but instead the paradigmatic interpretational question of quantum mechanics, namely: “*What does it mean – with respect to the property represented by an*

⁴³ Gillies (2000a, pp. 819-820).

⁴⁴ Such as the relevant-conditions theory of Fetzer (1981) and the state of the universe theory of Miller (1994); they differ on the type of conditions that they take to be necessary in order to define a propensity.

observable Q – for a quantum system to be in state Ψ that is not an eigenstate of the observable Q ? In addressing this question the selective-propensity view postulates the existence of propensities as an explanation of the observed probability distributions, *but it does not interpret these distributions in any particular way.*⁴⁵

This leads me to the final comment regarding the nature of the propensities involved in the selective-propensity view. A rightly influential argument against the propensity interpretation of objective probability is known as *Humphrey's Paradox*. It was first noted by Paul Humphreys that conditional probabilities are symmetric but propensities are not, in the following sense.⁴⁶ For a well-defined conditional probability $P(A/B)$, the event B that we are conditionalising upon need not be temporally prior to the event A . But if B is the propensity of a system to exhibit A , then B must necessarily precede A in time; the propensity ascription seems to make no sense otherwise. Hence *Humphrey's paradox* shows that not all objective probabilities can be propensities. But the paradox is only a problem for propensity interpretations of probability, and I have already made it clear that on the selective-propensity view, quantum probabilities are not to be interpreted in any particular way. The point of introducing selective-propensities is not to interpret quantum probabilities but to explain them.⁴⁷

8. Propensities in other Interpretations of Quantum Mechanics

I have been arguing for the essential explanatory role of a particular notion of propensity in an appropriate interpretation of quantum mechanics. In this final section I would like to sketch out ways in which this notion can be profitably applied to a proper understanding of other interpretations of quantum mechanics. In particular I would like to briefly point out some reasons why selective propensities can be fruitfully applied to (a) Bohr's version of the Copenhagen interpretation, (b) the Ghirardi-Rimini-Weber (GRW) collapse theory, and (c) Bohmian mechanics. It should be clear that I am not claiming that these interpretations have made use of the particular notion of

⁴⁵ Other than in insisting that they are not subjective, for which the kind of no-theory theory of objective probability recently defended by Sober would seem to suffice. See Sober (2005, p. 18).

⁴⁶ Salmon (1979), Humphreys (1985).

⁴⁷ The probability distributions of quantum mechanics are explained as the typical displays of the underlying propensities in the appropriate experimental circumstances. In this respect the selective-propensity view is closer to Hugh Mellor's account of "propensities" (see Mellor, 1971, chapter 4).

propensity that I advocate. I am not even claiming that they have made any use of any dispositional notions – unlike the five views that I have described so far in this paper. My claim is merely that they *could* make use of propensities, and that they are consistent with them. I even conjecture, more generally, that a notion of propensity very similar to the one I defend here can be applied to *all* interpretations of quantum mechanics (without being logically required by any); but I must leave a full analysis of this stronger claim for another paper.

a. Bohr's Copenhagen Interpretation

According to contemporary commentators, Bohr's actual interpretation of quantum mechanics was really the combination of two basic interrelated principles: a) the principle of complementarity according to which at any given time a system can only be conceived as particle or wave; and b) the principle that all macroscopic phenomena and systems (including measurement devices) can only properly be described in classical mechanical terms.⁴⁸ He did not endorse the claim that measurement inevitably disturbs the state of the system measured that is often attributed to him but is more properly Heisenberg's. Instead Bohr thought that macroscopic superposition infection (the entanglement of quantum system and measuring device) was the essential quantum contribution; and that a classical description of the measurement interaction had to be somehow extracted from it. An appeal to selective propensities is natural here. Don Howard describes Bohr's thinking as follows:⁴⁹

“What I think Bohr meant is this: Given a pure state correctly describing any system, including a joint system consisting of an entangled instrument-object pair, and given an experimental context, in the form of a maximal set of commensurable observables, one can write down a mixture that gives for all observables in that context exactly the same statistical predictions as are given by the pure state. But then, with respect to the observables measurable in that context, one proceeds as if the instrument and object were not entangled. One can speak as if the measurement reveals a property of the object alone, and one

⁴⁸ The account that follows is indebted to Howard (2004) and Dickson (2004), although they might subscribe the propensity reading of it.

⁴⁹ Howard (2004, p. 675).

can regard the statistics as ordinary ignorance statistics, the experiment being taken to reveal a definite, though previously unknown value of the parameter in question”.

On the face of it this proposal is formally indistinguishable from the selective-propensities proposal that I have defended in section 6. The only difference is that Bohr does not explicitly invoke a notion of propensity in order to justify the replacement of the full quantum superposed state ψ with the corresponding mixture as represented by the standard representative $W(O)$. But he could have done. Similarly Michael Dickson has convincingly argued that Bohr made implicit use of the notion of a reference frame as provided by the measurement device at rest in the laboratory:⁵⁰

“Bohr (on one reading) concluded that it is up to us to stipulate some object (normally a measuring apparatus) as defining a reference frame, and that this stipulation requires us to treat the object classically, because the stipulation requires the object to be well defined in position and momentum”.

Dickson has demonstrated that reference frames fix a classical context and he has suggested that this gives rise to the uncertainty relations. Once again no dispositional notions are explicitly employed in order to explain the appearance of the classical realm, but the selective-propensity view is consistent with the notion of a reference frame picking up an experimental context that permits the manifestation of an underlying quantum propensity in the form of a probability distribution. Once we fix a reference frame we can justify the re-description of the system’s state as the indistinguishable mixture with respect to the particular propensity under test.

b. Ghirardi-Rimini-Weber collapse interpretations

There is of course a very long history to collapse interpretations of quantum mechanics, going as far back as Von Neumann⁵¹ who famously invoked collapse mechanisms in order to explain the appearance of definite-valued observables as the

⁵⁰ Dickson (2004).

⁵¹ Von Neumann (1932/1955, esp. chapter 6).

outcome of measurement procedures – which would be impossible to predict on a standard Schrödinger evolution. Contemporary collapse approaches to quantum mechanics are not exactly interpretations of the quantum theory, since they replace the Schrödinger equation evolution with a non-linear stochastic evolution equation. In this sense they are competitor theories to quantum mechanics, like Bohm’s theory. The GRW theory is the best known collapse interpretation of quantum mechanics. It was developed by Giancarlo Ghirardi, Alberto Rimini and Tullio Weber in a number of papers over the 1980’s.⁵² The GRW theory supposes that systems in quantum states governed by the Schrödinger equation, also undergo sudden and spontaneous state-transitions that instantaneously localise them in physical space.

In this section I will only comment briefly on the original GRW theory. But I believe my conclusions extrapolate rather well to the more sophisticated and plausible models that GRW has given rise to in the last decade or so. In particular further work by Gisin, Pearle and Percival has been crucial in the development of a series of continuous localisation models where the “jumps” in the original GRW are replaced by smoother continuous stochastic evolutions that achieve the desired localisation of the state over a relatively brief period of time. In the more recent and sophisticated localisation models provided by quantum state diffusion theory this process of localisation corresponds to a version of Brownian drift on the Bloch sphere that represents the quantum state of the system.⁵³

On the GRW collapse theory an isolated, closed, quantum system will undergo a spontaneous transition that localises it within a region of space of dimension $d = 10^{-5}$ cm *very infrequently*, more precisely with a frequency $f = 10^{-16}$ seconds⁻¹. In other words such a system gets spontaneously localised on average every one hundred million years on average, and for most practical purposes we can assume its evolution to be indefinitely quantum-mechanical. Yet, when such a system is part of much larger macroscopic composite, its spontaneous transition will trigger the collapse of the whole composite, previously entangled in a massive superposition state. Since macroscopic objects are composed of the order of 10^{23} such particles, it turns out that such triggering processes will take place on average every 10^{-7} seconds. This is shorter than the time

⁵² Ghirardi, Rimini, Weber (1986) is a landmark. Ghirardi (2002) provides a good overview.

⁵³ See Percival (1998) for a complete overview.

required to complete a measurement interaction with the composite, which explains why we never experience macroscopic objects in super-positions, and always observe them highly localised in space.

The GRW theory models such spontaneous collapse processes as “hits”, with the relevant frequency, of the quantum state by a Gaussian function appropriately normalised: $G(q_i, x) = Ke^{-\frac{1}{2}d^2(q_i-x^2)}$, where d represents the localisation accuracy, and q_i represents the position of the i particle. The wavefunction of a n -particle system, denoted as $F(q_1, q_2, \dots, q_n)$, undergoes a transition with each hit that results in the new wavefunction: $L_i(q_1, q_2, \dots, q_n; x) = F(q_1, q_2, \dots, q_n)G(q_i, x)$. For my purposes in this essay it is sufficient to note that such a localisation procedure is at the very least compatible with the assumption that each quantum particle has an irreducible disposition to localise in an area given by d with frequency f . But moreover, on the GRW theory, the particle has a certain probability to localise in each area d in the its position space given by the appropriate quantum probability as calculated by the standard application of the Born rule on its wavefunction. That is, the probability that it localises on a particular region x of space is given by $|P_x\Psi|^2$ in accordance with Born’s probabilistic postulate, where P_x is the projector upon that region,. In other words the dispositions that according to GRW each particle has to spontaneously reduce upon a region x of area d are propensities, in the sense that I elaborated in section 7.

It is clear on the other hand that the propensities that can be usefully employed in the interpretation of GRW are not exactly selective-propensities since the GRW transitions are spontaneous and not in any way the result of any selective interaction with a measurement device. This is an obvious and important difference with the selective-propensities account of quantum mechanics that I elaborated in section 6, which presupposes that a closed quantum system always evolves in accordance with the Schrödinger equation. Nonetheless, it is remarkable that the type of spontaneous localisation propensity that systems are endowed with on this reading of GRW is on all other respects just like selective propensities.⁵⁴

⁵⁴ The account of propensity required to make sense of the more sophisticated localisation processes of QSD and Pearle’s continuous localisation theory are even closer to selective-propensities, since on those theories localisation is supposed to emerge in the interaction of the systems with the environment, if not a measurement device.

c. Bohmian mechanics

As is well known, Bohmian mechanics is an alternative hidden variable theory that is provably empirically equivalent to quantum mechanics, while preserving many ontological features of a classical theory. Most notably Bohmian mechanics conceives of quantum particles as point-particles, always endowed with a particular location in space and time, and it ascribes to them fully continuous classical trajectories. A minimal version of the theory can best be summarised in four distinct postulates:⁵⁵

(i) The state description of an n-particle system is given by (Ψ, Q) , where $\Psi(q, t)$ is the quantum state with $q = (q_1, q_2, \dots, q_n) \in \mathfrak{R}^{3N}$ and $Q = (Q_1, Q_2, \dots, Q_n)$, where $Q_k \in \mathfrak{R}^3$ is the actual position of the k^{th} particle.

(ii) The quantum state Ψ evolves according to the Schrödinger equation:

$$i\hbar \frac{d|\Psi\rangle}{dt} = \hat{H}|\Psi\rangle, \text{ where } H \text{ is the Hamiltonian:}$$

$$H = -\sum_{k=1}^N \frac{\hbar^2}{2m_k} \nabla_k^2 + V(q), \text{ with } \nabla_k^2 = \frac{\partial^2}{\partial q_k^2}, \text{ and where } V(q) \text{ is the classical potential}$$

for the system and m_k is the mass of the k^{th} particle.

(iii) The velocity of the N-particle system is defined as:

$$v^w(Q) \equiv \frac{dQ}{dt}, \text{ where } v^w(Q) = (v_1^w, v_2^w, \dots, v_N^w) \text{ is a velocity field on the configuration}$$

space \mathfrak{R}^{3N} that evolves as a function of Q according to:

$$v_k^w = \frac{\hbar}{m_k} \text{Im} \frac{\nabla_k \Psi}{\Psi} = \frac{dQ_k}{dt}, \text{ where } \nabla_k = \frac{\partial}{\partial q_k}.$$

(iv) The ‘quantum equilibrium’ configuration probability distribution for an ensemble of systems each having quantum state Ψ is given by $\rho = |\Psi|^2$.

⁵⁵Belousek (2003, pp. 113-4).

Postulates (i) and (ii) are the extension of quantum mechanics to an n-particle system. Postulate (iii) is unique to Bohmian mechanics: it guarantees that each particle has a classical continuous trajectory in physical 3-d space, and an n-particle system has a corresponding velocity field in the n-dimensional configuration space. Finally, the quantum equilibrium postulate (iv) guarantees the empirical equivalence between Bohmian and quantum mechanics. The postulates make it very clear that Bohmian mechanics, even in this minimal version, is not merely an interpretation of quantum mechanics – it is rather a distinct theory in its own right. It does not just provide an interpretation of quantum mechanics, but advances a whole theoretical machinery of its own, while making sure to account for all of the successful predictive content of quantum mechanics.

Since Bohmian mechanics is a theory in its own right, it makes sense that it should have multiple interpretations, just as quantum mechanics has a number of competing interpretations itself. Here I will mention just two, rather extreme versions of the so-called guidance and causal views. There are a number of further views that lie somewhere in between these two in terms of their ontological commitment – each of these views is underdetermined by Bohmian mechanics itself.⁵⁶ My aim in this section is just to show that propensities are *compatible* with Bohmian mechanics; for this purpose it is enough to show that they are compatible with one interpretation of Bohmian mechanics (as a matter of fact I conjecture that propensities can be applied to nearly all of them, but I will not show this, stronger, claim here).

A minimal formal interpretation of Bohmian mechanics has been advanced by Dürr, Goldstein and Zanghi (the DGZ minimal guidance view).⁵⁷ According to these authors, postulates (i-iv) characterise the theory entirely; no other postulates are needed. On this interpretation Bohmian mechanics is a first-order theory, formulated entirely in kinematical terms: no dynamic concepts are required. In particular the DGZ interpretation rejects the need for the ontology of quantum sub-fields, or quantum potentials that is often thought to characterise Bohmian mechanics: all that is needed over and above quantum mechanics is the guidance equation as described in postulate

⁵⁶ Belousek (2003) is a good description and review of many of them, including the DGZ version of the guidance view, and Holland's version of the causal view that I discuss in the text; but also others such as David Albert's radically dualistic guidance view, Antony Valentini's pilot wave guidance view, etc.

⁵⁷ Dürr, Goldstein and Zanghi (1992).

(iii). This interpretation is an equivalent of the bare theory for Everett relative states,⁵⁸ since it sticks to the conception of the phenomena in accordance to the theory, and refrains from making any additional suppositions regarding the causal or explanatory structure that might underpin and give rise to such phenomena. Hence the application of selective-propensities to this interpretation of Bohmian mechanics is not likely, but only because no causal or explanatory concept whatever is demanded. It is worth noting that the interpretation has been contested precisely on account of its minimalism; hence Belousek, for instance, writes:⁵⁹

“So we find DGZ’s arguments for the elimination of the quantum potential unconvincing and their monistic particle guidance view to be inadequate with regard to explanation of quantum phenomena and, hence, unsatisfactory qua physical theory. Again, we emphasise that this by itself is no argument in favor of the quantum potential or causal view. Rather, it points out only that the first-order concepts of their guidance view are by themselves explanatorily inadequate, which suggests that second-order concepts be introduced or, at least, that some physically real basic entity over and above particles and their positions be admitted into the ontology of Bohmian mechanics for the purposes of explanation.”

The family of interpretations of Bohmian mechanics that falls under the “causal view” rubric adopt an additional postulate regarding the quantum potential; this postulate describes the second order dynamical concepts that according to this view are indispensable for a proper causal and explanatory physical theory:⁶⁰

(v) The quantum state $\Psi = R \cdot e^{iS/\hbar}$ gives rise to a quantum potential:

$$U = -\frac{\hbar^2}{2m} \frac{\nabla^2 R}{R},$$

so that the total force (classical plus quantum) influencing the

trajectory of a particle is (the particle’s equation of motion):

$$\vec{F} = -\nabla(V + U) = \frac{d\vec{p}}{dt}.$$

⁵⁸ Barrett (1999, chapter 4)

⁵⁹ Belousek (2003, p. 140).

⁶⁰ Bohm (1952, pp.170); Bohm and Hiley (1993, pp.29-30).

This postulate is introduced in analogy with classical mechanics, but it introduces an additional element in the form of the quantum potential U , a dynamical second-order entity, responsible for the quantum force $-\nabla U$ that appears in the particle's equation of motion. These terms, the quantum potential U and the related quantum force field $-\nabla U$ are essential to explain particle trajectories, on any of the views often referred to as “causal” – but they demand an interpretation of their own, which differs on each of the causal views.

Peter Holland's is perhaps the best known “causal view” today – it is also possibly the closest to Bohm's original interpretation and remains most committed ontologically. For an n -particle system, Holland assumes that each of the particles and their properties in physical 3-dimensional space are real – which guidance views accept – but in addition he postulates the existence the wavefunction and its associated quantum potential and force field in n -dimensional configuration space.⁶¹ This forces him to give an account of the causal interaction whereby the potential and force fields in configuration space affect the trajectories of the particles in 3-d space; an account that turns out to be enormously complicated and fraught with conceptual difficulties.⁶²

In response Belousek has proposed that the quantum potential and force field are “real” – thus justifying postulate (v) – but only if interpreted as a catalogue of all possible interactions between the n -particles in physical space, not as distinct entities in a distinct configuration space. As he writes:⁶³

“Configuration space itself would be taken to be merely an abstract space representing possible histories and interactions in physical space of actually existing particles, the realization of such possibilities being contingent upon actual initial conditions.”

My suggestion would be to reinterpret the quantum potential and force field along similar lines – except the modalities described by the quantum wave-function in configuration space would now describe a full catalogue of the dispositional properties

⁶¹ Holland (1993, esp. pp. 75-78).

⁶² For an account of some of these difficulties, see Belousek (2003, pp. 155-161)

⁶³ Belousek (2003, p. 162). The view is in some ways similar to Valentini's version of the pilot wave theory – see Valentini (1996).

of the system – its selective-propensities. So there is a sense in which the quantum potential and the force field are “real” on this view too – since selective propensities are real properties of quantum systems – but the existence of a distinct space (configuration space) over and above physical 3 dimensional space would not be required, thus avoiding the need to describe the causal interaction between these two spaces. In the case of the two particle system formed by a quantum object subject to a measurement interaction with a macroscopic apparatus, this boils down to writing down each and every possible interaction between the measurement device and the propensities $\{O_1, O_2, \dots O_n\}$ of the quantum system described by its corresponding standard representatives $\{W(O_1), W(O_2), \dots W(O_n)\}$.⁶⁴

Interesting complications will arise in the case of *n-particle* systems subject to measurements. In these cases the trajectories of each of the particles (the only observable consequences of the theory according to Bohm) would be the result of not just of the selective-propensities of each particle, but also the selective propensities of all the other particles as described through the quantum potential; and the resulting force field would be the result of all the selective-propensities and their mutual interactions. Hence the well-known non-locality of the quantum potential in Bohm’s theory transforms itself – on this view – into a non-locality of propensities.⁶⁵

Thus the application of the selective-propensities to Bohmian mechanics would require the acceptance of postulate (v) as part of the core of the theory – in line with “causal” interpretations of Bohm’s theory – but would then go on to interpret this postulate as a description of the highly non-local nature of each of the particles’ selective-propensities, and their effect on particles’ trajectories through the force field. Thus the selective-propensities view of Bohmian mechanics has all the advantages of associated to the “causal” views of Bohmian mechanics, in particular its superior explanatory power in comparison with “guidance” views; but it purchases these advantages at a lesser ontological cost – since it refrains from postulating the existence of a complex *n-particle* system in an equally real *n-configuration* space.

⁶⁴ Pagonis and Clifton (1995) have provided a model for measurements of spin in Bohmian mechanics, which they have argued – rightly in my view—leads to a dispositional account of quantum properties.

⁶⁵ I have explored some of the consequences and features of non-local propensities in the case of EPR experiments, in Suárez (2004c)

9. Conclusions

Let me recapitulate what I believe has been achieved in this paper. In the first five sections of this paper different accounts of quantum mechanics (QM) that employ dispositional properties have been reviewed. I have raised objections to each of them – which I consider conclusive in every case. So far, the history of dispositional accounts of QM may be considered a failure. But in section 6 I have presented the elements of an account of QM in terms of selective-propensities that I regard as essentially appropriate and likely to be successful. Section 7 explores some of the philosophical properties of these selective-propensities, and provides the bare bones of a philosophical defence. In section 8 I have sketched out ways in which selective-propensities could be read into at least three prominent and established contemporary interpretations of QM: Bohr’s version of the Copenhagen interpretation, the GRW collapse theory, and Bohmian mechanics. There is no doubt that more detailed work is needed to provide a fully comprehensive and convincing treatment of each of these interpretations of QM in terms of selective-propensities. But at least I hope to have provided in this paper enough in terms of a description of the barebones of such treatments to have made a compelling case that propensities are alive and well as a helpful notion to understand and interpret QM. Past failures notwithstanding, propensities afford, now as ever, an intriguing and progressive research programme in the philosophy of quantum physics; it is a programme that demands yet more philosophical work and attention.

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