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Spatial Costs in a Monocentric City (And Implications for Agglomeration)

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Abstract

Using water supply as a model for a wider range of infrastructure services, the effect of a negative exponential density gradient on distribution costs is investigated for four monocentric urban development scenarios: (a) *Densification*; (b) *Dispersion*; (c) *Suburbanisation*; and (d) *Constant density*. It is shown that economies of scale in production can be outweighed by diseconomies in distribution in cases (b) and (c), suggesting that the agglomeration benefits of infrastructure cannot be taken for granted. They depend as much on the effect of density on distribution costs as the effect of size on production costs.

JEL Classifications: R12, R32, D24, L95
Keywords: Urbanisation, spatial analysis, returns to scale, water utilities

SPATIAL COSTS IN A MONOCENTRIC CITY (AND IMPLICATIONS FOR AGGLOMERATION)

1. Introduction

It is common to assume that infrastructure is characterized by economies of scale (Fujita (1989), p.135; McDonald (1997), pp.40-41). However, except in the (rare) case of infrastructure that is a pure public good, this overlooks the cost of distributing goods and services or accessing facilities, i.e. spatial costs. By analogy with Arnott's (1979) analysis of commuting costs, it might be supposed that such costs are increasing in city size, but Arnott assumes constant density across the city, contrary to theory and observation. There are a variety of ways in which urban models of the Muth/Mills type can give rise to a negative exponential density gradient away from the central business district of a monocentric city (Brueckner (1982)); and there is empirical evidence that many cities broadly conform to this spatial pattern (DiPasquale & Wheaton (1996), pp.61-64).

The purpose of this article is to investigate the effect of this feature of cities on scale economies in the provision of basic infrastructure services, using data on water supply costs to illustrate the effects. There are two aspects to consider:

(a) How the spatial distribution of the population affects the cost of distributing goods and services or accessing facilities – as Schmalensee (1978, p.271) has remarked: "When services are delivered to customers located at many points, cost must in general depend on the entire distribution of demands over space."

(b) How the costs of distribution (or access) interact with the costs of production – in particular, whether economies of scale in production may need to be traded off against spatial diseconomies in distribution.

The structure of the article is as follows: In **section 2**, the basic algebra of a monocentric city is developed, and expressions derived for total population (N), total distance to customers (ψ) and average distance to customers (φ) in terms of density at the city centre (d_0), the density gradient (λ) and the radius of the urban area (R). Varying these parameters enables a rich array of urban development scenarios to be generated. Attention is then focused on four such scenarios characterized as (a) *Densification*, (b) *Dispersion*, (c) *Suburbanisation* and (d) *Constant density*. Distribution cost elasticities for these cases are derived. In **section 3**, data on water distribution costs for 35 “urban districts” in the supply area of one of the water companies in England & Wales is used to estimate the effect on these costs of variations in volume and distance to properties (measured as φ or ψ). Based on these relationships, the distribution cost elasticities are quantified for each of the four urban development scenarios. In **section 4** the interaction with water production costs is considered, showing how, in the case of *Densification*, scale economies in production are reinforced by density economies in distribution, whereas in the cases of *Dispersion*, *Suburbanisation* and *Constant density* they are offset to a greater or lesser extent by diseconomies in distribution, i.e. higher spatial costs. **Section 5** then considers how far these findings undermine the conventional wisdom that infrastructure services, such as water supply, are always characterized by economies of scale and therefore conducive to agglomeration. It concludes that scale effects in infrastructure may depend as much on density as on size *per se*. While high density settlement has the potential to permit both large scale production and low cost distribution, thereby favouring agglomeration, more dispersed settlement patterns lead to higher (per capita) costs of distribution or access.

2. Modelling spatial costs in a monocentric city

i. Population, density and distance

An exact representation of the location of each and every property in a city is generally impractical[1]. For the purposes of this paper, urban areas are modeled as monocentric settlements with density falling away smoothly from the centre, which, in the majority of cases, is a reasonable approximation to the actual situation. This enables an expression for the average distance to properties to be derived for each settlement, providing a compact summary measure of the spatial distribution of properties, which varies from place to place in line with its size and density gradient.

The basic algebra (and geometry) of the monocentric city can be summarized in a relationship between four parameters: d_0 (central density), N (population), λ (density gradient) and R (outer radius). **Figure 1** is a bird’s eye view of a monocentric city.

[1] Although the availability of postcodes and GIS software are improving matters.

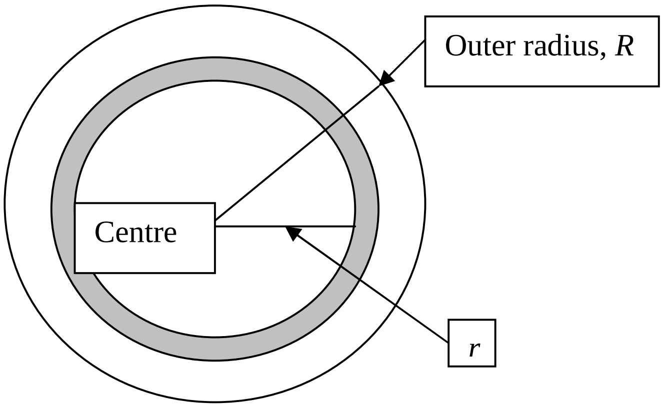

Figure 1: Monocentric city (top view)

In **Figure 1**, if density at radius *r* is *d(r)*, then total city population (*N*) is given by:

$$N = 2\pi \int_0^R d(r).r.dr \qquad (1)$$

If, further, $d(r) = d_0.e^{-\lambda r}$ (i.e. a negative exponential density gradient of λ away from the centre, where density is d_0), then (1) gives:

$$N = \frac{2\pi d_0}{\lambda^2}\left[1 - e^{-\lambda R}(1 + \lambda R)\right] \qquad (2)$$

This is the basic relationship between d_0 (central density), *N* (population), λ (density gradient) and *R* (outer radius) and shows them to be interdependent – given any three, the fourth is fixed.

Further useful relationships concern the total and average distance of people in this city from the centre. The distance from the centre to a person in the shaded ring, where density is *d(r)*, is *r*, and so the total distance (ψ) to everyone in the city is given by:

$$\psi = 2\pi \int_0^R d(r).r^2.dr = \frac{4\pi d_0}{\lambda^3}\left[1 - e^{-\lambda R}\left(1 + \lambda R + \frac{\lambda^2 R^2}{2}\right)\right] \qquad (3)$$

From (2) and (3), the average distance (φ) from the centre to a person in the city is then given by:

$$\varphi = \frac{\psi}{N} = \frac{2}{\lambda}\frac{\left[1 - e^{-\lambda R}\left(1 + \lambda R + \frac{\lambda^2 R^2}{2}\right)\right]}{\left[1 - e^{-\lambda R}(1 + \lambda R)\right]} \qquad (4)$$

The implications of this expression are sketched in **Figure 2** which shows a monocentric city in semi-profile and indicates how, for given *N*, higher values of λ will be associated with a larger settlement radius *R* if the central density d_0 is fixed.

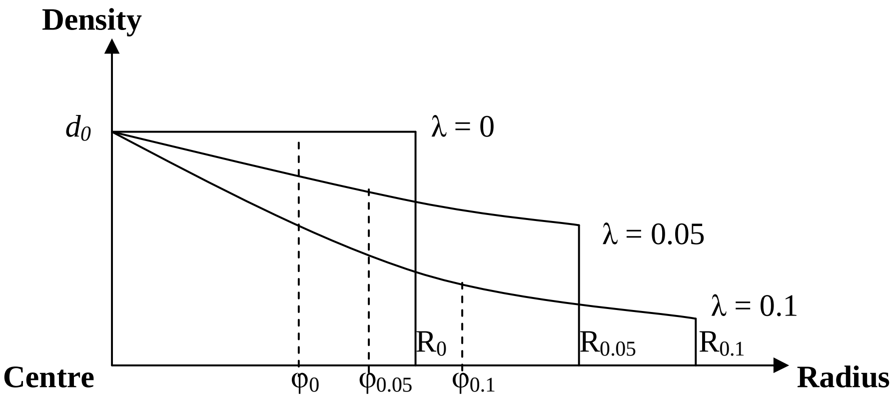

Figure 2: Monocentric city (semi-profile) – Relationship between density and settlement radius for different values of the density gradient λ (not to scale)

In **Figure 2,** the average distance to properties, φ, is indicated by the dotted lines: when the density gradient $\lambda = 0$, it is 2/3 *R*; with higher values of λ, it increases as determined by (4).

It may be worth emphasizing here the differences between this approach and the analysis of commuting costs by Arnott (1979). Arnott shows average commuting cost to be an increasing function of city size by considering a circular city of uniform population density, where all commuting is to a central business district and transport cost is proportional to distance. Aggregate commuting costs are then given by:

$$ACC = \int_0^R t.r.2\pi r.dr = t\frac{2}{3}\pi.R^3 = t\frac{2}{3\sqrt{\pi}}N^{3/2} \qquad (5)$$

Where *R* is the radius of the city, *N* is its population and *t* is unit transport cost, i.e. aggregate commuting costs increase more than proportionately with population, and average commuting cost (*ACC/N*) is an increasing function of *N*. This result depends on the assumption of uniform density ($\lambda = 0$) and each commuter travelling individually and radially to the CBD with linear transport costs. In contrast, the set-up in this paper allows density to vary while ψ and φ are simply consequential distance measures, whose relationship to costs is a matter for empirical investigation.

ii. Urban development scenarios

By varying the four parameters in (1), a rich array of urban development scenarios can be generated. Here, d_0 is taken to be fixed and attention is focused on four contrasting cases that can arise as one or more of N, λ and R vary:

(a) ***Densification***[2]: Number of properties (N) varies, while settlement radius (R) is held constant (density gradient λ also therefore varying);

(b) ***Dispersion***: Density gradient (λ) varies, holding number of properties (N) constant (R also therefore varying);

(c) ***Suburbanisation***[3]: Number of properties (N) varies, holding λ constant (R also therefore varying);

(d) ***Constant density***: Number of properties (N) varies, holding average density (N/A) constant, where $A = \pi R^2$ (when both λ and R vary).

These cases encapsulate the characteristics of urban development most likely to be of policy interest.

The resulting city configurations are portrayed in cross section in **Figures 3 (a)-(d).**

[2] It is recognised that this term has acquired particular policy connotations in the urban planning context; here it is simply adopted as a convenient descriptive label.

[3] The term "suburbanisation" is applied here to the case where the density gradient (λ) does not change as the city expands, as this seems a good descriptor for what is portrayed in **Figure 3(c)**. However, some authors have used *changes* in density gradient as a measure of suburbanization (e.g. Kopecky & Suen (2009)).

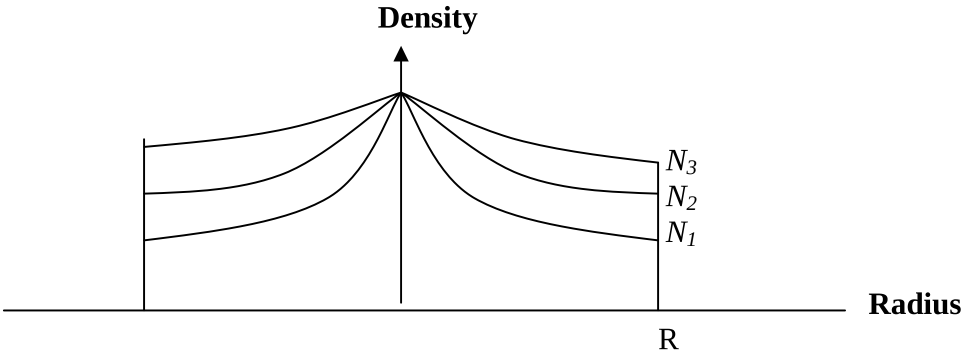

Figure 3: (a) City cross-sections: *R* constant, *N* varies (‘*densification*’)

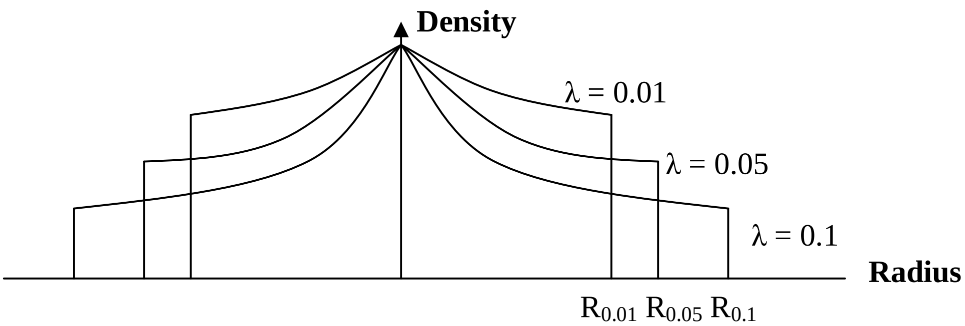

Figure 3: (b) City cross-sections: *N* constant, λ varies (‘*dispersion*’)

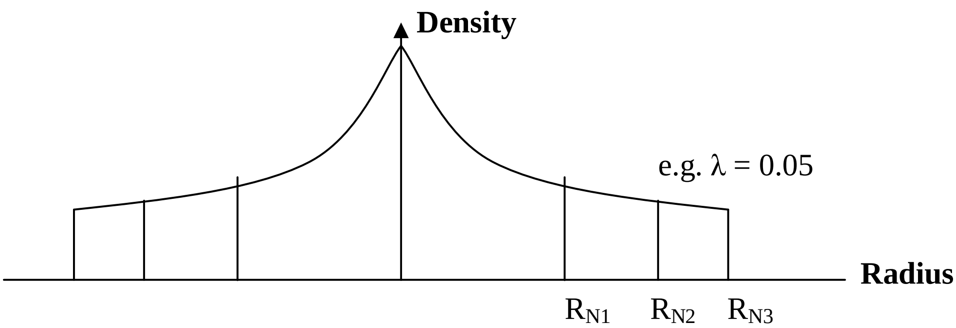

Figure 3: (c) City cross-sections: λ constant, *N* varies (‘*suburbanisation*’)

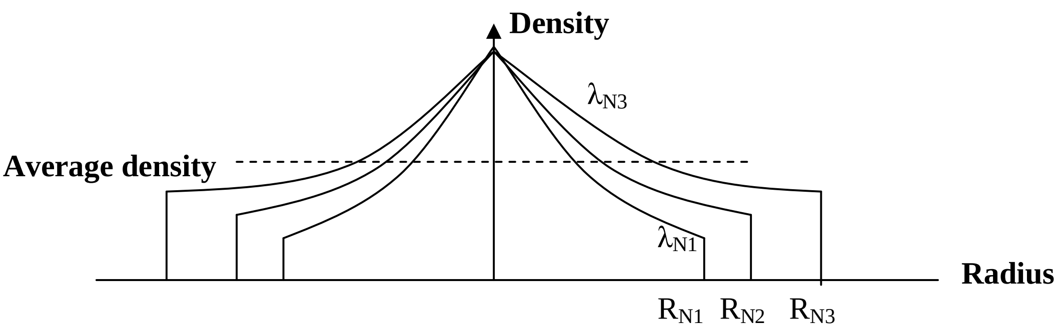

Figure 3: (d) City cross-sections: Density constant, *N* varies (‘*constant density*’)

iii. Distribution costs and elasticities

In utility studies, output is usually measured simply as the amount consumed, so missing the spatial aspect of the distribution stage. Here, which is an innovation in this context[4], the output of the water distribution system (*DO*) is measured as the product of the amount consumed (*QC*) and the average distance to properties (φ)[5]. *QC* in turn is the product of consumption per property (w)[6] and the number of properties (N)[7]. Thus:

$$DO = QC.\varphi = w.N.\varphi \qquad (6)$$

A simple cost function for water distribution[8] can now be estimated as:

$$\ln VCD = \alpha + \beta_1 \ln QC + \beta_2 \ln \phi \qquad (7)$$

Where *VCD* is the variable costs of distribution.

Although specification (7) provides an indication of the different effect on distribution costs of changes in volume and changes in average distance to properties, the estimated coefficients do not provide direct measures of distribution elasticities. This is because N (which is a component of *QC*) and φ are both functions of λ and R and so are not independent of each other. Three elasticities are of particular interest:

(i) ε_w, measuring the response of distribution costs to changes in water consumption per property;

(ii) ε_A, measuring the response of distribution costs to changes in distribution area;

(iii) ε_N, measuring the response of distribution costs to changes in the number of properties.

To evaluate these elasticities, it is necessary to start from a variant of (7).

We can re-write *DO* as:

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[4] It is however common in transport studies to measure output using ton-miles, passenger-km, etc.

[5] This implies that water is distributed from a central point whereas water treatment works are generally on the outskirts of towns. But if water is delivered in bulk to the distribution system, the effect on costs is not very material.

[6] For simplicity, w is taken to be uniform within each urban district in the subsequent analysis (although varying between districts).

[7] Note that N here is numbers of properties rather than population.

[8] It can be assumed that capital in water distribution is to all intents and purposes fixed so that the production function is of the Leontief type – hence the absence of terms for capital or prices in (7).

$$DO = w.\psi \text{, where } \psi = N.\varphi \text{ is total distance to properties (see (3))} \qquad (8)$$

(7) can then be re-stated as:

$$\ln VCD = \alpha + \beta_1 \ln w + \beta_2 \ln \psi \qquad (9)$$

Evaluating ε_w is now straightforward:

$$\varepsilon_w = \frac{\partial(\ln VCD)}{\partial(\ln w)} = \beta_1 \qquad (10)$$

This can be viewed as a pure quantity effect, measuring the response of distribution costs to changes in water consumption per property, numbers of properties and other distribution area characteristics held constant.

The complex form of equations (2) and (3) makes the derivation of expressions for the other elasticities for the scenarios of **Figure 3** rather tricky[9] (they are not constants but vary with scale). The least mathematically awkward case is (c) *Suburbanization*. In this case λ is constant, say $\bar{\lambda}$. An expression for $\varepsilon_{R/\bar{\lambda}}$ (the elasticity of distribution cost with respect to variations in *R*, conditional on $\bar{\lambda}$) can then be derived as follows:

$$\varepsilon_{R/\bar{\lambda}} = \frac{\partial(\ln VCD)}{\partial(\ln \psi)}.\frac{\partial(\ln \psi)}{\partial(\ln R)} = \beta_2.\frac{R}{\psi}.\frac{\partial \psi}{\partial R}$$

$$= \beta_2.\frac{R}{\psi}\left\{\frac{4\pi d_0}{\lambda^3}\left[-e^{-\lambda R}\left(\lambda + \lambda^2 R\right) + \left(1 + \lambda R + \frac{\lambda^2 R^2}{2}\right)\lambda.e^{-\lambda R}\right]\right\}$$

$$= \beta_2.\frac{R}{\psi}.2\pi R^2 d_0.e^{-\lambda R} \qquad (11)$$

Which can alternatively be expressed in area form, using $\frac{d(\ln R)}{d(\ln A)} = \frac{1}{2}$, as

$$\varepsilon_{A/\bar{\lambda}} = \beta_2.\frac{R}{\psi}.\pi R^2 d_0.e^{-\lambda R} \qquad (12)$$

This is the elasticity of distribution cost with respect to area served, conditional on $\bar{\lambda}$. Evidently, it is a (rather complex) function of *R* and λ but is clearly positive. Discussion of the interpretation of this elasticity is deferred to **Section 3** below.

[9] I am grateful to George Fane (Australian National University, Canberra) for helping me to come to grips with this point.

From (2), number of properties (*N*) varies with *R* (and *A*), so that there is a related elasticity $\varepsilon_{N/\bar{\lambda}}$, the elasticity of cost with respect to variations in *N*, conditional on $\bar{\lambda}$. It can be derived as follows:

$$\varepsilon_{N/\bar{\lambda}} = \varepsilon_{R/\bar{\lambda}.} \frac{N}{R}.\frac{\partial R}{\partial N} = \varepsilon_{R/\bar{\lambda}}.\frac{N}{R}.\frac{1}{\left\{\frac{2\pi d_0}{\lambda^2}\left[-e^{-\lambda R}.\lambda + (1+\lambda R).\lambda.e^{-\lambda R}\right]\right\}}$$

$$= \beta_2.\frac{N}{\psi}.2\pi R^2.d_0.e^{-\lambda R}.\frac{\lambda^2}{2\pi.d_0\lambda^2 R.e^{-\lambda R}} = \beta_2.\frac{N}{\psi}.R = \beta_2.\frac{R}{\varphi} \qquad (13)$$

This elasticity simplifies quite nicely but it also is a function of *R* and *λ*. Since volume rises in line with *N* (if *w* is constant), a value for $\varepsilon_{N/\bar{\lambda}}$ = 1 would indicate constant returns to scale. However, higher values are to be expected because of diseconomies associated with expansion into lower density suburbs.

The algebra involved in deriving elasticities corresponding to cases (a) ("*densification*"), (b) ("*dispersion*") and (d) ("*constant density*") proved intractable (the last two involving simultaneous variation in both *λ* and *R*). Evaluation for these cases is therefore carried out by means of illustrative calculations for hypothetical urban areas using average data values, as described in **Section 3**. In case (a), a value of 1 for $\varepsilon_{N/\bar{R}}$ would indicate constant returns to scale, if *w* is held constant. However, the expectation is of a value between 0 and 1, as more properties in a given area should give rise to density economies. In case (b) *N* is fixed, so a positive value for $\varepsilon_{A/\bar{N}}$ would indicate diseconomies (higher unit distribution costs), if *w* is also held constant. In case (d), *N*, *λ* and *R* move in tandem and while a value of 1 for $\varepsilon_{N/\bar{D}}$ would indicate constant returns to scale, there is no *a priori* reason why observed values should not be greater or less than 1.

3. Estimated spatial costs and spatial elasticities under different urban development scenarios

i. Data used

To illustrate the effect of urban configuration on spatial costs, I use water distribution costs. Information provided by one of the larger water companies in England & Wales enabled me to put together data for 35 "urban districts", each comprising one urban area (as defined in ONS (2004)[10]) and its surrounding area of non-urban land. These cases therefore approximate monocentric cities. The processing of the data is described in **Appendix I,** with summary statistics in **Appendix II**.

ii. Econometric estimates

Implementing (7) for the 35 "urban districts" produced:

$$\ln VCD = 2.047 + 0.393^{***}\ln QC + 1.095^{***}\ln\varphi \quad (14)$$

(S.E. 0.161) *(S.E. 0.329)* *(R^2 = 0.9474)*

These results indicate significant economies of scale with respect to volume ($\beta_1 < 1$) and significant diseconomies with respect to the average distance measure ($\beta_2 > 0$). The interpretation of the coefficient on ln*QC* in (14) is that higher consumption in a district, whether due to greater usage per property or more properties on the existing network has a less than proportionate effect on costs (e.g. a 10% increase in *QC* would increase operating costs by about 4%). The interpretation of the coefficient on $\ln\phi$ is less obvious. ϕ is a measure of the average distance to properties. Therefore a higher value for ϕ, if *QC* is fixed, indicates that properties are more dispersed, implying a higher value for λ and hence also for *R*, as shown in the "dispersion" case in **Figure 3 (b)**[11]. Any positive value for the coefficient on φ indicates that greater dispersion adds to the cost of distributing a given volume of water and is therefore a diseconomy. In fact this effect appears to be rather large here with (e.g.) a 10% increase in φ increasing operating costs by about 11%)[12]. This can be interpreted as a form of density effect, with lower density adding to distribution costs and higher density reducing costs.

Re-estimating (14) in the (8) form gave:

$$\ln VCD = -4.572 + 0.432^{**}\ln w + 0.617^{***}\ln\psi \quad (15)$$

(S.E 0.219) *(S.E. 0.037)* *(R^2 = 0.9455)*

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[10] In ONS (2004) "urban areas" are defined as areas of built up land of at least 20 Ha, with a population of 1,500 or more.

[11] If *N* is fixed, λ and *R* cannot vary independently of each other as they are linked through the relationship (3).

[12] However, the dispersion variable φ is relatively insensitive to changes in area served, as can be seen in **Table A**.

From (15), the distribution elasticities identified at (10), (12) and (13) above can be evaluated for the 35 "urban districts" as:

From (10): $\varepsilon_w = \beta_1 = 0.432$

This is significantly less than 1 (at 5% level), indicating quite large increasing returns to this dimension of scale, although with a relatively high standard error.

From (12): $\varepsilon_{A/\bar{\lambda}} = \beta_2.\frac{R}{\psi}.\pi R^2 d_0.e^{-\lambda R}$

Taking β_2 = 0.617 from (15), values for this elasticity calculated using the 35 urban districts data range from about 0.8 to about 0.2, with a tendency for higher values of $\varepsilon_{A/\bar{\lambda}}$ to be associated with lower values of λ (See **Figure 4**).

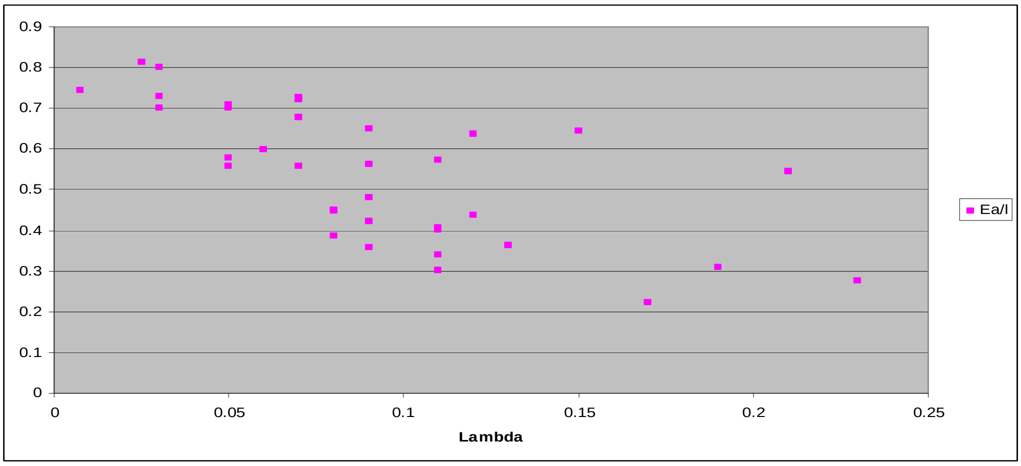

Figure 4: Relationship between $\varepsilon_{A/\bar{\lambda}}$ and λ for 35 "urban districts"

In all cases this elasticity is < 1, so that with suburbanisation the proportionate increase in costs is generally less than the proportionate increase in area at the margin. Whether this implies scale economies in the usual sense (higher unit cost) will depend on the relationship between increase in area and increase in numbers of properties. This is best assessed by considering $\varepsilon_{N/\bar{\lambda}}$, as is done next.

From (13): $\varepsilon_{N/\bar{\lambda}} = \beta_2.\frac{R}{\varphi}$

The values for R/φ observed in the 35 urban districts' data range between about 1.6 and 2.4[13]. In conjunction with the estimated value for β_2 of 0.617 from (15) above, this gives

[13] The minimum value for R/φ is 1.5 as $\varphi = 2R/3$ when $\lambda = 0$.

values for $\varepsilon_{N/\bar{\lambda}}$ in the range 0.99 to 1.48, indicating roughly constant returns to scale for less dispersed districts but decreasing returns to scale for the more dispersed districts (See **Figure 5**).

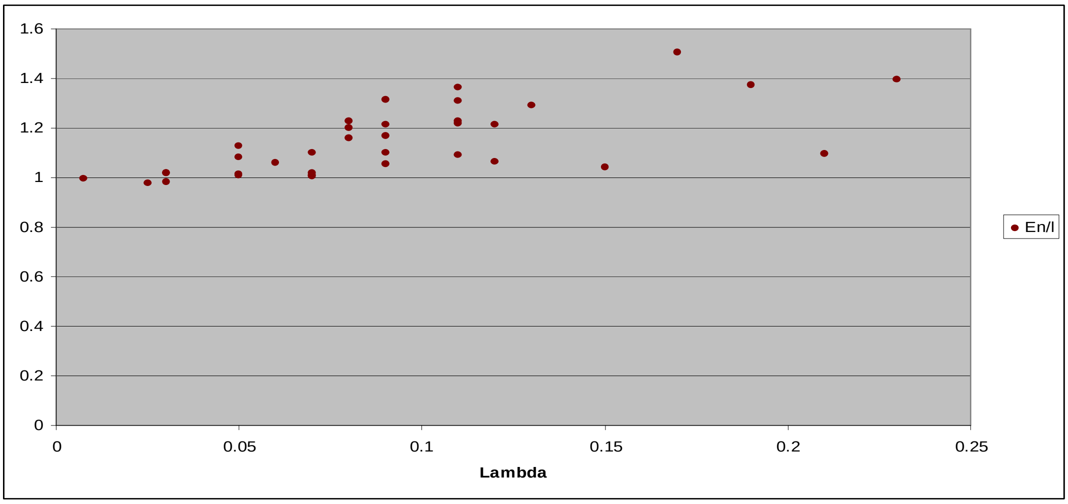

Figure 5: Relationship between $\varepsilon_{N/\bar{\lambda}}$ and λ for 35 "urban districts"

iii. Additional calculations

To further explore the implications for distribution costs of the four scenarios in **Figure 3**, illustrative calculations were carried out for these cases[14]. In summary, the calculations show that distribution costs depend strongly on the spatial configuration of the distribution area. With a monocentric structure, *densification* reduces unit distribution costs whereas greater *dispersion* of properties (higher λ) raises them. The calculations also suggest that more properties (higher *N*) with λ held constant (*suburbanisation*) also raises distribution costs but to a much smaller extent (because higher *N* with λ held constant means lower density and a larger settlement area). On the other hand, with density rather than λ held constant (*constant density*), more properties lead to lower unit distribution costs.

The detail of the calculations for the 35 urban districts can be seen in **Table A**, expressed as unit costs as the implications are most easily appreciated in this form. The numbers to focus on are in the last 5 columns, where *VCD* and *CCD* are respectively the annual variable and

[14] The method is to start with a figure for *N*, then use the interpolation table in **Appendix III** to infer either λ or *R* given the other variable, enabling a value for ψ to be calculated. The estimated relationship (15) can then be used to obtain a value for *VCD*, taking an average value of 420 litres/property/day for *w*. For capital costs, the relevant costs were allocated to areas by length of mains and a regression similar to (15) carried out.

capital costs of distribution, *UVCD* and *UCCD* are the related unit costs and *UTCD* is the total unit cost.

- ***Densification***: Section (a) of **Table A** shows how adding properties within a fixed urban boundary substantially reduces unit distribution costs. This is because volume economies of scale in distribution outweigh the effect of a small increase in dispersion as measured by φ.
- ***Dispersion***: Section (b), on the other hand, shows that for a settlement of a given size in terms of numbers of properties, greater dispersion leads to diseconomies in distribution. In this case, although the number of properties (and hence total consumption) does not change, higher λ leads to a larger service area with distribution costs rising by 50% as λ rises from zero to 0.1.

(These first two cases provide good illustrations of density economies in distribution, as in both cases higher density leads to lower distribution costs.)

- ***Suburbanisation***: In section (c) of **Table A**, increasing the number of properties with λ constant results at first in economies of scale with respect to volume more or less offsetting the effect of greater dispersion, although above 10,000 properties, the latter effect increasingly dominates, leading again to diseconomies in distribution.
- ***Constant density***: In contrast, section (d), which compares settlements of similar density but different size, shows scale economies, particularly in capital costs. In this case, although more properties result in a larger radius settlement, this is accompanied by reduction in λ and hence less dispersion, leading to savings in the unit cost of distribution.

One way of viewing the *suburbanization* figures in section (c) of **Table A** is as showing the effect of extending water supply from an urban core first to the suburbs and then to a rural fringe. The first 10,000 properties (the urban core) occupy only about 556 Ha at an average density of 18.0 properties/Ha. The next 15,000 properties (the suburbs) occupy about 1700 Ha (average density 8.8 properties/Ha). The next 15,000 properties (the rural fringe) occupy about 4450 Ha (average density 3.4 properties/Ha); and another 10,000 properties would add about 14,000 Ha at an average density of 0.7 properties/Ha. The effect on distribution costs is plotted in **Figure 6** below. Compared with the total unit cost of distribution in the urban core, £407/Ml, adding the suburbs raises this cost by about 4% to

£422/Ml; adding the rural fringe adds another 7% bringing the cost to £453/Ml; and then with the outer fringe (bringing the total number of properties to 50,000) the cost rises further to £495/Ml, over 20% above the figure for the urban core alone. Clearly, the marginal cost of distribution to these more remote and highly dispersed properties is high[15].

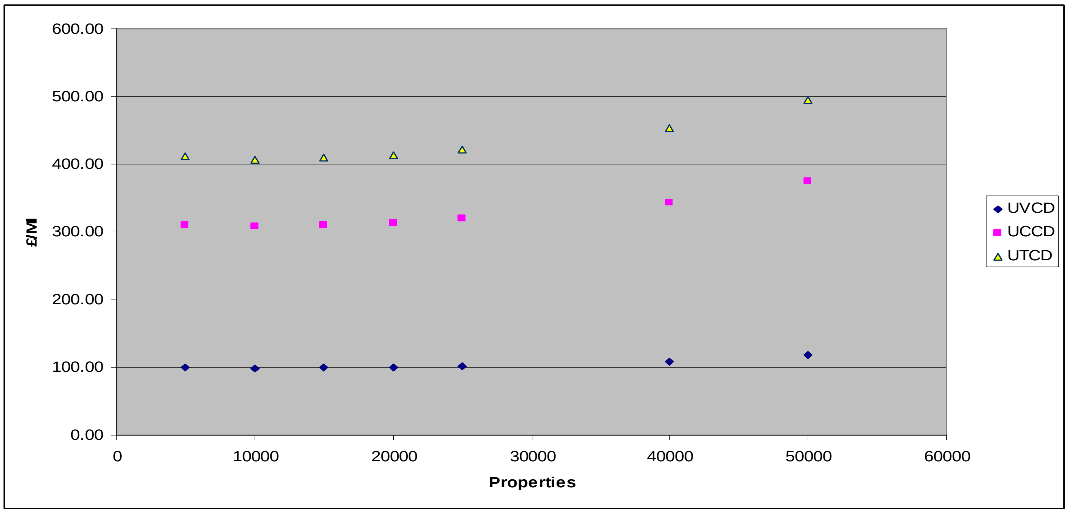

Figure 6: Effect of increasing settlement size with constant density gradient λ (“*suburbanisation*”) (from section (c) of Table A)

The calculated results in **Table A** can be used to derive estimated elasticities corresponding to those discussed earlier in this section. Being estimated from intervals rather than by continuous variation, these values are approximations with uncertain confidence intervals. The values in **Table 1** below are for an average sized urban district of 18,000 properties, using variable costs (*VCD*)[16]. The elasticities shown are:

(a) ***Densification***: $\varepsilon_{N/\bar{R}}$, the elasticity of costs as the number of properties (*N*) varies, while settlement radius is held constant at 2680m. If $\varepsilon_{N/\bar{R}} < 1$, there are scale economies;

(b) ***Dispersion***: $\varepsilon_{A/\bar{N}}$, the elasticity of costs as the density gradient (λ) varies, holding number of properties constant at 18,000. If $\varepsilon_{A/\bar{N}} > 0$, there are scale diseconomies;

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[15] For the last 10,000 properties, the unit cost is £660/Ml, some 60% higher than the £407/Ml unit cost for the 10,000 properties in the urban core.

[16] Similar values would be obtained using capital costs (*CCD*) or total costs (*TCD*) because of the similarity of the values for the coefficient on $\ln\psi$.

(c) ***Suburbanisation***: $\varepsilon_{N/\bar{\lambda}}$, the elasticity of costs as the number of properties (*N*) varies, holding *λ* constant at 0.06; and the related elasticity $\varepsilon_{A/\bar{\lambda}}$. If $\varepsilon_{N/\bar{\lambda}} > 1$, there are scale diseconomies (the value of 1.03 is consistent with what was found earlier, as shown in **Figure 5**);

(d) ***Constant density***: $\varepsilon_{N/\bar{D}}$ (which is equal in value to $\varepsilon_{A/\bar{D}}$), the elasticity of costs as the number of properties (*N*) varies, holding density (*N/A*) constant at 10 properties/Ha. If $\varepsilon_{N/\bar{D}} < 1$, there are scale economies.

	Typical "urban district"	**Returns to scale (1/ε)**
No. of properties	18,000	
(a) Densification		
$\varepsilon_{N/\bar{R}}$ *(range)*	0.73 *(0.80 – 0.70)*	1.34
(b) Dispersion		
$\varepsilon_{A/\bar{N}}$ *(range)*	0.18 *(0.21 – 0.07)*	
(c) Suburbanisation		
$\varepsilon_{N/\bar{\lambda}}$ *(range)*	1.03 *(0.97 – 1.45)*	0.97
$\varepsilon_{A/\bar{\lambda}}$ *(range)*	0.63 *(0.70 – 0.17)*	
(d) Constant density		
$\varepsilon_{N/\bar{D}} = \varepsilon_{A/\bar{D}}$ *(range)*	0.91 *(0.92 – 0.90)*	1.10

Table 1: Spatial effect distribution cost elasticities derived from calculated values in Table A

4. Interaction of spatial costs with production economies

The 35 "urban districts" were selected for analysis because they seemed to provide a reasonable approximation to the kind of monocentric settlement envisaged in our distribution model. Ideally, to assess the effect of bringing together water production and water distribution, one would use direct information about the relevant costs for each of the 35 districts. However, the supply arrangements were found mostly not to be self-contained within these districts. Instead, to calculate water production costs, it is assumed that in each case water production is from a single water treatment works (WTW) of the appropriate size, using the parameters obtained from another part of my research[17]. Illustrative cost calculations for hypothetical settlements of varying sizes and densities can then be carried out for the same four scenarios ("*densification*", "*dispersion*", "*suburbanization*" and "*constant density*"), with distribution costs taken directly from **Table A**.

Thus, for water production, the average (or unit) cost (£/Ml) of production for a WTW producing *QP* Ml/day is calculated as:

$$UCT = 2^{0.31}.474.QP^{-0.24} \qquad (16)$$

If, in addition, for the purposes of these illustrative calculations, a leakage rate of 20% is assumed, then:

$$QP = QC / 0.8 \qquad (17)$$

The calculations in this section thus give a somewhat stylized view of the effect on production costs of different settlement characteristics. They do however help to show up such trade-offs as there are between economies of scale in production and diseconomies in distribution, without too many extraneous factors complicating the comparisons.

Now, the distribution costs shown in **Table A** can be brought together with the production costs obtained using (16) to give illustrative total costs of water supply for the four scenarios, leading to the results shown in **Table B**. In this table, *TCP* is the total cost of water production, *TCD* is the total cost of water distribution and *TC(P+D)* is the total cost of water supply, comprising production and distribution. *UTCP*, *UTCD* and *UTC(P+D)* are the related unit costs, obtained by dividing by *QC* converted to an annual rate.

- ***Densification*:** Section (a) of **Table B** shows the two-fold advantage of densification, leading to lower unit costs for both production and distribution. The unit cost of supply for a settlement of 50,000 properties is about 40% lower than for a settlement of 5,000 properties covering the same area. Returns to scale estimated from the last column are about 1.5.
- ***Dispersion*:** In section (b) of **Table B**, the unit cost of water production does not vary between cases so that this cost (about £428/Ml) is simply added to distribution

[17] See Wenban-Smith (2009), Ch. IV. These parameters are for total production costs, including capital costs.

costs. As in **Table A**, greater dispersion (higher λ) leads to higher distribution costs (the increase in the unit cost of distribution is about 52% as λ increases from $\lambda = 0$ to $\lambda = 0.1$) and hence a total unit cost which also rises, from about £778/Ml when $\lambda = 0$ to about £959/Ml when $\lambda = 0.1$.

- ***Suburbanisation***: Section (c) of the table is more interesting: here the higher volumes produced as N increases result in savings in unit production costs, which fall by about 40% from £583/Ml when N = 5,000 to £335/Ml when N = 50,000, thus offsetting the increase in distribution costs associated with serving less dense suburbs and rural areas. The effect is shown in **Figure 7**. Whereas distribution cost alone is minimized at about 10,000 properties, the minimum for production and distribution costs together in this case occurs at about 35,000 properties.

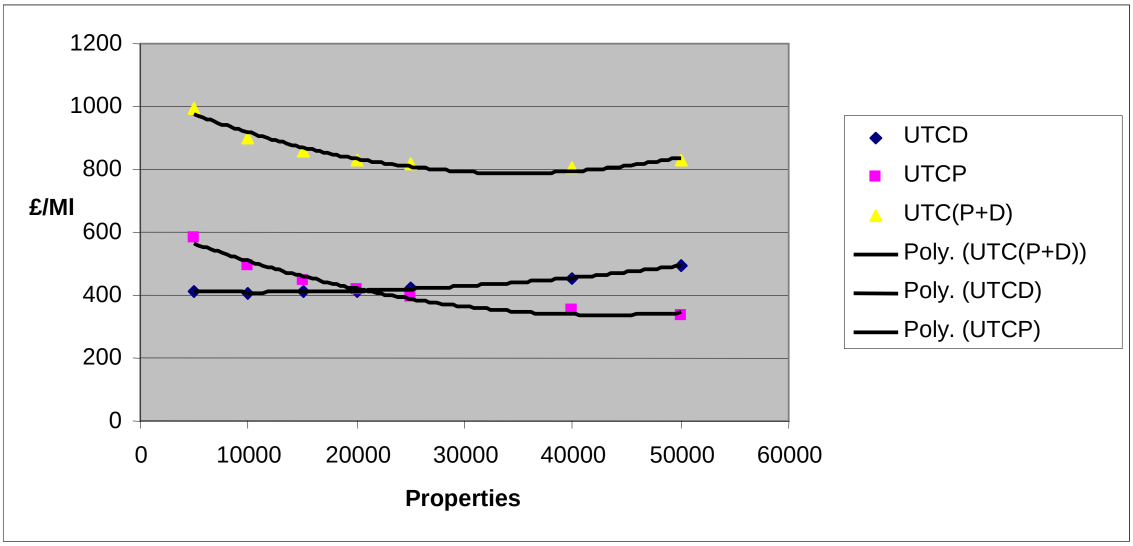

Figure 7: Unit production cost (UTCP), unit distribution cost (UTCD) and unit total cost (UTC(P+D)) from section (c) of Table B

- ***Constant density***: Section (d) of **Table B** then shows how economies of scale in production reinforce the decline in distribution costs when property numbers increase but density remains constant, so that the combined unit cost falls by about 30% from £1059/Ml when N = 5,000 to £745/Ml when N = 50,000. Returns to scale, estimated from the last column are about 1.25 (compared with about 1.10 for distribution alone).

These results indicate that the benefits of more compact settlement will be clearest when comparing cities of similar area or similar population but differing in density, as in sections (a) and (b) of **Table B**. Adding population by expanding into peripheral areas (suburbanization) introduces a trade-off between volume economies (in both production and distribution) and diseconomies of average distance, which may on balance be favourable, despite lower average density, at least for moderate expansion, as shown in **Figure 7**. Constant density expansion, on the other hand, is unequivocally favourable so that in comparing towns of similar density but different populations, the larger towns should benefit from scale economies in both production and distribution, as in section (d) of **Table B**

5. Conclusions and implications for agglomeration

Infrastructure is the Cinderella of urban economics. The accumulated investment in urban infrastructure is absolutely massive; yet it does not feature prominently in the literature. While the part played in urban agglomeration by thick labour markets, economies of scale in manufacturing, specialisation, technological spill-overs and consumption externalities have all recently attracted considerable attention, infrastructure has rather been taken for granted, providing a backdrop to the urban drama but not, seemingly, playing an active part.

Insofar as infrastructure has attracted attention, the predominant proposition is that it is characterised by economies of scale. Thus McDonald (1997), discussing urbanisation economies in his standard text remarks (pp.40-41): "Economies of scale exist in the provision of inputs that are not specific to a particular industry. An important example is the general urban infrastructure." Similarly, Fujita (1989, p.135) observes that "… the provision of many *public services and facilities* (such as schools, hospitals, utilities, and highways) typically exhibits the characteristic of economies of scale." If this is the case, one would expect infrastructure to make a large positive contribution to urban agglomeration economies. However, the evidence for such an effect is not strong. Although some studies of urbanisation economies have found a positive effect, others have not (Eberts & McMillen (1999, pp.1460-1491) provide a review of the evidence) and there is a tendency in the theoretical literature to downplay the role of scale economies in agglomeration (Duranton & Puga (2004)).

While my research into water supply costs has confirmed that there are economies of scale in water treatment works (WTWs), it is important to recognize that these are *plant level* findings. When two or more works are operated by a company (for example, because the size of a given works is limited by the capacity of the water sources; or because the communities it serves are small and/or widely separated), these scale economies will be less evident. The benefits of large scale production can generally only be reaped where circumstances permit the operation of large WTWs, typically where there is a large population and access to high capacity water resources. Birmingham, for example, which has a population of over 1 million and access to water from the Elan Valley, is mostly supplied by a single large WTW (the Frankley works) leading to relatively low water supply costs for that city. In other cases, the distribution cost effects discussed earlier are likely to be dominant.

In studies of agglomeration, it is common to use population as the measure of size[18]. One implication of the work reported here is that it may not be sufficient to look at numbers alone. Whereas increase in size through *densification* would, it seems, bring economies of scale (in water supply at least), with a positive influence on agglomeration, as would (to a lesser extent) *constant density* increase, increase in size through *suburbanization* would be roughly neutral in cost terms once distribution costs are taken into account. To get the full picture, it would appear necessary to take density into account as well as size. Moreover, it would be misleading to regard urban areas of similar size, as measured by population, as equivalent from an agglomeration perspective, if they have very different densities. As the *dispersion* case suggests, lower density towns or cities are likely to have higher distribution costs. Put differently, agglomeration by densification would have real cost advantages (at

[18] "The urban area population is the standard measure of urban size in studies of urbanisation economies." Eberts & McMillen (1999, p.1481) Although urban areas will by definition probably have relatively high densities, there can still be considerable variation in density between one urban area and another.

least up to the point where congestion costs become appreciable[19]) whereas suburbanization would not. Of course, infrastructure costs are not the only consideration but if, for example, people have a preference for suburban living, these calculations indicate that there is likely to be a cost penalty (whether or not this is visited on suburbanites through tariffs and connection charges).

Another way to look at the matter is to compare water supply costs as between a small town and a large one. Even if they have the same density, the '*constant density*' calculations point to lower costs in the larger town. If this effect generalizes to other types of infrastructure, it suggests an important reason why large settlements might over time prosper more than small ones; and if the larger one is also denser, the advantage becomes greater still. A related point arises when an area is occupied by several small settlements rather than one large one. If each settlement operates its own water production facilities, it risks a double cost penalty, on the production side from smaller plant size and on the distribution side from the density effect.

So, what about other types of infrastructure? Without carrying out further studies, it is only possible to offer some pointers to the relevance of these water supply findings to a wider range of urban infrastructure. Much of the man-made urban infrastructure can be seen as belonging to one of two broad types:

- **Area-type**: Provides services within a defined area (e.g. water supply, other utilities, postal services, fire protection, transport systems). In such cases, getting the service to users involves distribution costs;
- **Point-type**: Provides services at a specific point (e.g. hospitals, schools, offices, shops, museums, theatres, etc). In such cases, the equivalent consideration is the cost to users of accessing the facility.

Water supply was chosen for study as an example of Area-type urban infrastructure because the technology is relatively simple and distribution costs are high so that the effects of interest should be particularly evident.

[19] While our data has not shown evidence of higher water distribution costs in larger, denser urban areas, such an effect does not seem *a priori* unlikely due to more difficult access, high rise buildings and higher wages.

It is likely that distribution costs are less significant in the case of other utilities, such as electricity supply and telecommunications, although capital investment in distribution systems is still important. While in general lower distribution costs can be expected to favour agglomeration by extending the area that can be economically served, high capital costs nevertheless require that settlements be dense as well as relatively large if the necessary investments are to be viable[20]. At the same time, there have been some recent developments, such as small types of sewage treatment works and local forms of power generation, which may help small settlements.

The scope for application to Point-type infrastructure, such as hospitals, appears good. Access costs, although often neglected, are relatively high while the extent of economies of scale in the production unit (e.g. hospital) is somewhat under researched. There would appear to be good potential to apply the methods developed here for water distribution costs to the access costs to hospitals (and other similar infrastructure), perhaps moderating enthusiasm for very large facilities.

Application to transport is less obvious. While there are some suggestive similarities, notably when the spatial aspect of transport networks is under consideration, transport also raises issues which go beyond those arising with water supply. An important instance is congestion, which is not a major consideration in the case of water supply[21] but is of considerable importance in transport. At the same time, the role of density in facilitating the provision of low cost, high capacity transit has parallels in water supply, as does the difficulty of maintaining viable public transport where density is low, for reasons entirely analogous to those applying to water distribution, i.e. higher infrastructure requirements and longer distances per unit of output.

What is clear is that economies of scale in production are not the only factor at work. The spatial aspect with its impact on distribution and access costs is also important. In my research, I have tried to bring this aspect into focus by considering four contrasting urban

[20] As the case of high capacity optical fibre cable perhaps demonstrates.

[21] The drop in pressure which can occur at times of peak demand for water is perhaps the nearest equivalent.

growth scenarios, characterised as (a) *densification*, (b) *dispersion*, (c) *suburbanisation*, and (d) *constant density*. The general conclusion emerging from this work is that scale effects in infrastructure may depend as much on density as on size *per se*. While high density settlement has the potential to permit both large scale production and low cost distribution, thereby favouring agglomeration, more dispersed settlement patterns lead to higher (per capita) costs of distribution or access. It follows that the general presumption in urban economics that infrastructure services are always characterised by economies of scale and therefore conducive to agglomeration may not be correct, because economies of scale in production may be offset by higher distribution (or access) costs. This suggests that there should be more direct consideration of density effects in studies of urbanisation economies (by including density as an independent variable, or area *as well as* population, or by using some measure of sprawl[22] as a proxy for density).

[22] Note however that the density gradient (λ) does not provide an unambiguous measure of "sprawl". In **Figure 3**, although cases (b), (c) or (d) might all loosely be described as sprawl, in (b) λ increases, in (c) λ is constant and in (d) λ decreases in value.

N	**λ**	***R* ('00m)**	***φ(λ,R)***	***VCD* (£m)**	***UVCD* (£/Ml)**	***CCD* (£m)**	***UCCD* (£/Ml)**	***UTCD* (£/Ml)**
a. Varying *N*, *R* constant ('densification')								
5,000	0.19	26.8	9.7	0.109	142.23	0.339	441.90	584.13
10,000	0.12	26.8	12.5	0.196	127.94	0.612	399.40	527.34
15,000	0.095	26.8	13.6	0.266	115.59	0.832	361.73	477.32
20,000	0.075	26.8	14.6	0.331	107.81	1.036	337.99	445.79
25,000	0.06	26.8	15.3	0.390	101.86	1.225	319.76	421.62
40,000	0.03	26.8	16.6	0.550	89.64	1.730	282.17	371.81
50,000	0.015	26.8	17.3	0.646	84.24	2.035	265.54	349.78
b. Varying λ, *N* constant ('dispersion')								
18,000	0	13.8	9.2	0.233	84.60	0.730	264.48	349.08
18,000	0.02	15.3	9.9	0.245	88.63	0.765	277.17	365.80
18,000	0.04	17.3	10.8	0.258	93.53	0.807	292.62	386.15
18,000	0.06	20.3	12.1	0.276	99.91	0.863	312.74	412.65
18,000	0.08	25.8	13.9	0.301	109.22	0.944	342.13	451.35
18,000	0.10	48.7	18.1	0.354	128.30	1.111	402.45	530.75
c. Varying *N*, λ constant ('suburbanisation')								
5,000	0.06	8.6	5.5	0.077	100.27	0.238	310.64	410.91
10,000	0.06	13.3	8.2	0.152	98.97	0.473	308.31	407.28
15,000	0.06	17.7	10.7	0.229	99.44	0.715	310.80	410.24
20,000	0.06	22.0	12.9	0.307	100.16	0.962	313.82	413.98
25,000	0.06	26.8	15.3	0.390	101.86	1.225	319.76	421.62
40,000	0.06	46.2	22.9	0.669	109.14	2.110	344.10	453.24
50,000	0.06	81.1	30.1	0.911	118.88	2.880	375.76	494.64
d. Varying *N*, density=10 ('constant density')								
5,000	0.15	12.6	7.0	0.089	116.17	0.276	360.33	476.50
10,000	0.1	17.8	10.0	0.170	111.18	0.531	346.67	457.85
15,000	0.08	21.9	12.2	0.249	108.15	0.778	338.25	446.40
20,000	0.07	25.2	14.1	0.324	105.72	1.016	331.36	437.08
25,000	0.065	28.2	15.7	0.397	103.48	1.245	324.87	428.35
40,000	0.05	35.7	19.9	0.615	100.27	1.937	315.92	416.19
50,000	0.045	39.9	22.2	0.755	98.54	2.384	310.99	409.53

Table A: Illustrative calculations to show the effect of different values of *λ* and *N* on unit distribution costs (using relationships estimated for 35 "urban districts")

Illustrative values			Unit costs (£/Ml)			Total costs (£m pa)		
N	**λ**	**QC=w.N (Ml/d)**	**UTCP**	**UTCD**	**UTC(P+D)**	**TCP**	**TCD**	**TC(P+D)**
a. Varying *N*, *R* constant ('densification')								
5,000	0.19	2.1	582.66	584.13	1166.79	0.447	0.448	0.895
10,000	0.12	4.2	493.37	527.34	1020.71	0.756	0.808	1.564
15,000	0.095	6.3	447.62	477.32	924.94	1.029	1.098	2.127
20,000	0.075	8.4	417.76	445.79	863.55	1.281	1.367	2.648
25,000	0.06	10.5	395.97	421.62	817.59	1.518	1.615	3.133
40,000	0.03	16.8	353.73	371.81	725.54	2.169	2.280	4.449
50,000	0.015	21.0	335.29	349.78	685.07	2.570	2.681	5.251
b. Varying λ, *N* constant ('dispersion')								
18,000	0	7.56	428.45	349.08	777.53	1.182	0.963	2.145
18,000	0.02	7.56	428.45	365.80	794.26	1.182	1.010	2.192
18,000	0.04	7.56	428.45	386.15	814.60	1.182	1.065	2.247
18,000	0.06	7.56	428.45	412.65	841.10	1.182	1.139	2.321
18,000	0.08	7.56	428.45	451.35	879.80	1.182	1.245	2.427
18,000	0.10	7.56	428.45	530.75	959.21	1.182	1.465	2.647
c. Varying *N*, λ constant ('suburbanisation')								
5,000	0.06	2.1	582.66	410.91	993.58	0.447	0.315	0.762
10,000	0.06	4.2	493.37	407.28	900.65	0.756	0.625	1.381
15,000	0.06	6.3	447.62	410.24	857.85	1.029	0.944	1.973
20,000	0.06	8.4	417.78	413.98	831.74	1.281	1.269	2.55
25,000	0.06	10.5	395.97	421.62	817.59	1.518	1.615	3.133
40,000	0.06	16.8	353.73	453.24	806.97	2.169	2.779	4.948
50,000	0.06	21.0	335.29	494.64	829.93	2.570	3.791	6.361
d. Varying *N*, density=10 ('constant density')								
5,000	0.15	2.1	582.66	476.50	1059.15	0.447	0.365	0.812
10,000	0.1	4.2	493.37	457.85	951.22	0.756	0.701	1.457
15,000	0.08	6.3	447.62	446.40	894.01	1.029	1.027	2.056
20,000	0.07	8.4	417.76	437.08	854.84	1.281	1.34	2.621
25,000	0.065	10.5	395.97	428.35	824.32	1.518	1.642	3.160
40,000	0.05	16.8	353.73	416.19	769.92	2.169	2.552	4.721
50,000	0.045	21.0	335.29	409.53	744.81	2.570	3.139	5.709

Table B: Illustrative calculations to show the effect of different values of λ and *N* on water supply costs for 35 urban districts, assuming a single Water Treatment Works

Appendix I: Processing the data for 35 "urban districts"

One company's information on numbers of properties, length of mains, water consumption, leakage and geographical area for some 3000 District Metering Areas (DMAs) was aggregated and combined with information on operating costs to enable the relationships developed in **Section 3** of the paper to be estimated, first for 184 Water Quality Zones (WQZs) and then for 35 "Urban Districts" (the term "urban district" is adopted here as the areas concerned, being assembled from water company metering areas do not match standard administrative or statistical boundaries). For the purposes of this research, DMAs are too small, having little relationship to urban areas; WQZs are better but large urban areas may still comprise several WQZs, while in other cases more than one urban area is included in a WQZ. The 35 urban districts (omitting polycentric and wholly rural districts) have been selected to try to overcome these difficulties.

To obtain a measure of distribution output (*DO*) for these urban districts, some simplifying assumptions are required:

1. First, it is supposed that each district can be treated as if it were a monocentric settlement;
2. Next, a measure of area is needed. Actual areas include unoccupied or unserviced areas; but only areas having access to water mains can be serviced. The area of accessible land in each zone (A_o) can be estimated as $M/0.15$, where M is length of mains. This is because M/A is observed to be approximately 0.15 in fully urban zones; the argument then is that a similar ratio of mains to land with access to a supply will prevail in less urbanized zones (density of properties in terms of properties per km of mains is however generally much lower outside urban areas);
3. Now the effective radius (R) for each zone can be estimated as $R = \sqrt{A_o / \pi}$, where A_o is the area of accessible land;
4. The density gradient λ can then be estimated from the observed property density N/A_o by interpolation in a table which calculates density in properties/Ha for different values of R and λ (see **Appendix III** for an extract from this table);
5. Density at the centre of each zone (d_0) is taken to be 30 properties/Ha (a little above the highest value observed for any WQZ in the data);

6. Finally, by using water consumed, i.e. $w.N = QC$, in (8) that part of distribution costs attributable to leakage will be reflected in a higher unit distribution cost (the cost of producing the water lost to leakage is a separate matter, not relevant to this part of the analysis).

With these assumptions, distribution output (*DO*) for each urban district can be calculated as:

$$DO = QC.\phi(\lambda, R) \qquad \text{where } \varphi(\lambda,R) \text{ is given by (6) in the main text.}$$

Summary statistics for the 35 urban districts is shown in **Appendix II** (full data available on request from the author).

Appendix II: Summary statistics for 35 urban districts

Variable	Units	Average	Max	Min
No of properties (*N*)	Nos	35,535	639,307	2,277
Household water consumption (*w*)	Litres/prop/day	423	738	335
Urban district water consumption (*QC*)	Megalitres[23]/day	14.3	257.5	0.9
Gross area (*A*)	Hectares	20,550	123,988	1,173
Accessible area (A_0)	Hectares	2,826	35,336	211
Effective radius (*R*)	`00 metres	25.4	106.1	8.2
Average distance to properties (φ)	`00 metres	14.0	65.8	4.6
Density gradient (λ)	% per `00m	0.092	0.23	0.0075
Length of mains (*M*)	km	424	5,300	32
Distribution variable costs (*VCD*)	£'000	548	9,930	41
Distribution capital costs (*CCD*)	£'000	1,347	16,839	100

[23] 1 Megalitre = 1,000,000 litres

Appendix III: **Average density of a monocentric settlement with radius *R* whose density declines at a rate λ from the centre, where density is 30 properties/Ha (Extracted from full table, approx 4 times as large)**

URBAN AREAS: AREA/LAMBDA/DENSITY TABLE

Radius 100m	Area Ha	Do Prop/Ha	λ = 0.01	λ = 0.02	λ = 0.03	λ = 0.04	λ = 0.05	λ = 0.06	λ = 0.07	λ = 0.08	λ = 0.09	λ = 0.1
5	78.54	30	29.02	28.07	27.16	26.28	25.44	24.62	23.84	23.08	22.35	21.65
6	113.10	30	28.83	27.70	26.63	25.61	24.62	23.69	22.79	21.93	21.10	20.32
7	153.94	30	28.64	27.34	26.11	24.95	23.84	22.79	21.79	20.84	19.94	19.08
8	201.06	30	28.45	26.98	25.61	24.31	23.08	21.93	20.84	19.81	18.84	17.93
9	254.47	30	28.26	26.63	25.11	23.69	22.35	21.10	19.94	18.84	17.81	16.85
10	314.16	30	28.07	26.28	24.62	23.08	21.65	20.32	19.08	17.93	16.85	15.85
11	380.13	30	27.89	25.94	24.15	22.50	20.97	19.56	18.26	17.06	15.95	14.92
12	452.39	30	27.70	25.61	23.69	21.93	20.32	18.84	17.49	16.25	15.11	14.06
13	530.93	30	27.52	25.27	23.23	21.37	19.69	18.15	16.75	15.47	14.31	13.25
14	615.75	30	27.34	24.95	22.79	20.84	19.08	17.49	16.05	14.75	13.57	12.49
15	706.86	30	27.16	24.62	22.35	20.32	18.49	16.85	15.38	14.06	12.87	11.79
16	804.25	30	26.98	24.31	21.93	19.81	17.93	16.25	14.75	13.41	12.21	11.13
17	907.92	30	26.81	23.99	21.51	19.32	17.38	15.66	14.14	12.79	11.59	10.52
18	1017.88	30	26.63	23.69	21.10	18.84	16.85	15.11	13.57	12.21	11.01	9.95
19	1134.12	30	26.46	23.38	20.71	18.38	16.34	14.57	13.02	11.66	10.46	9.41
20	1256.64	30	26.28	23.08	20.32	17.93	15.85	14.06	12.49	11.13	9.95	8.91
21	1385.44	30	26.11	22.79	19.94	17.49	15.38	13.57	12.00	10.64	9.46	8.44
22	1520.53	30	25.94	22.50	19.56	17.06	14.92	13.09	11.52	10.17	9.01	8.00
23	1661.90	30	25.77	22.21	19.20	16.65	14.48	12.64	11.07	9.73	8.58	7.59
24	1809.56	30	25.61	21.93	18.84	16.25	14.06	12.21	10.64	9.31	8.17	7.20
25	1963.50	30	25.44	21.65	18.49	15.85	13.65	11.79	10.23	8.91	7.79	6.84
26	2123.72	30	25.27	21.37	18.15	15.47	13.25	11.39	9.84	8.53	7.43	6.50
27	2290.22	30	25.11	21.10	17.81	15.11	12.87	11.01	9.46	8.17	7.09	6.18
28	2463.01	30	24.95	20.84	17.49	14.75	12.49	10.64	9.11	7.83	6.77	5.88
29	2642.08	30	24.78	20.58	17.17	14.40	12.14	10.29	8.77	7.51	6.47	5.60
30	2827.43	30	24.62	20.32	16.85	14.06	11.79	9.95	8.44	7.20	6.18	5.34
31	3019.07	30	24.46	20.06	16.55	13.73	11.46	9.62	8.13	6.91	5.91	5.09
32	3216.99	30	24.31	19.81	16.25	13.41	11.13	9.31	7.83	6.64	5.66	4.86
33	3421.19	30	24.15	19.56	15.95	13.09	10.82	9.01	7.55	6.37	5.42	4.64
34	3631.68	30	23.99	19.32	15.66	12.79	10.52	8.72	7.28	6.12	5.19	4.43
35	3848.45	30	23.84	19.08	15.38	12.49	10.23	8.44	7.02	5.88	4.97	4.23

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