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# Adjusting to Capital Liberalization

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Very Preliminary, Appendix to be Completed

## Abstract

We study theoretically how an economy adjusts to liberalization of international financial transaction. We consider an economy in which debtors do not repay unless the debts are secured by collateral, and collateralizable assets for international borrowing are more restricted than domestic borrowing. We examine how the adjustment to capital liberalization depends upon the domestic and international collateral constraints. We show that, with an intermediate level of domestic collateral constraint, capital liberalization leads to capital outflow, improvement of TFP, and transitional loss of wage and employment. Government policy can mitigate the loss of workers at the cost of prolonging the transition, but cannot eliminate the loss without halting the transition.

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# 1 Introduction

*"Capital account liberalization, it is fair to say, remains one of the most controversial and least understood policies of our day". (Eichengreen, 2002)*

This paper is a theoretical study into how an economy adjusts to the liberalization of international financial transaction – capital account liberalization. Although most economists agree that trade liberalization generally improves efficiency of resource allocation, they are sharply divided on the costs and benefits of capital account liberalization. According to standard microeconomic theory, the international financial transaction is international trade of goods of different dates (possibly contingent on the states of nature), and thus capital account liberalization should have similar benefits with trade liberalization. Why do economists disagree? We think that the intertemporal exchange of present goods and claims to future goods is fundamentally different from intra-temporal exchange of various goods at least in one respect: the intertemporal exchange requires commitment that those who made promises in the past keep their promises to deliver goods (or purchasing power for goods) at the promised date, while intra-temporal exchange does not require such commitment. If people's ability to keep their promises (commitment) is limited, then the equivalence of intertemporal trade and intra-temporal trade no longer holds, and thus we need to investigate the effects of capital account liberalization, taking into account the limitation of commitment.

In this paper, we consider an economy in which the debtor (who received present goods in the past in exchange of a promise of future repayment) does not keep his promise to repay unless the debt is secured by collateral assets – productive assets he loses if he defaults. Then, the creditor limits her loan to the debtor so that the debt repayment does not exceed the value of collateral. Moreover, we consider the

amount of collateralizable assets for foreign credits is restricted compared to domestic credits, because the foreign creditors have more difficulty in taking over control and utilizing the collateral assets in a different country. Thus, the constraint on international borrowing is tighter than the domestic borrowing. The extent of collateralizable asset depends upon both technology and the quality of institution of the economy, which affects the development of domestic financial system. The fraction of future output from the productive asset usable as collateral for domestic borrowing affects the overall financial depth of the economy. The gap between collateralizable assets for international borrowing and domestic borrowing – the relative tightness of international borrowing – affects how much the domestic economy is financially integrated into the international financial market. Our aim is to examine how the adjustment of the home economy to capital account liberalization depends upon the parameter of financial depth of the domestic economy and the relative tightness of the international borrowing constraint.

For this purpose, we construct a dynamic model of small open economy with two types of infinitely lived agents: entrepreneurs and workers. At each date, some entrepreneurs are productive and other are not. Entrepreneurs hire workers to produce output in the following period, and they can borrow domestically against only a fraction (domestic collateral factor) of future output. The fraction they can borrow from foreigners is smaller. The symptom of low domestic collateral factor is that the domestic financial system fails to transfer enough purchasing power from savers (typically unproductive entrepreneurs) to investing agents (productive entrepreneurs), so that unproductive entrepreneurs end up hiring workers. The domestic interest rate remains low – comparable to the rate of returns on production of the unproductive entrepreneurs, the productive entrepreneurs are credit constrained, and the total factor productivity (TFP) is low,

which leads to low a wage rate.<sup>1</sup>

The way the economy adjusts to the liberalization of international financial transaction depends upon the financial depth of the domestic economy and the relative tightness of the international borrowing constraint. If the domestic financial system is poor with a low financial depth, the wage rate is so low before liberalization that even unproductive entrepreneurs enjoy a higher rate of returns on production than the foreign real interest rate. After capital liberalization, both productive and unproductive entrepreneurs borrow from foreigners, causing capital inflow, which pushes up the wage rate. If the international borrowing constraint is tight relative to domestic borrowing constraint, then the unproductive entrepreneurs (who borrow from foreigners) extend domestic loans to the productive entrepreneurs, acting as financial intermediary. Although the workers gain from higher wages, TFP and accumulation of net wealth of the entrepreneurs stagnate after liberalization.

For the intermediate level of domestic financial development, the wage rate is not very low and the domestic real interest rate under financial autarky is below the foreign interest rate because of financial suppression. After capital liberalization, the unproductive entrepreneurs start lending to foreigners and cutting back their production. With this capital outflow, the workers suffer from loss of wage and employment, and entrepreneurs gain from a higher rate of return on their net worth. With the intermediate depth of domestic finance, capital liberalization serves as a catalyst to reduce inefficient production, by providing means to absorb the saving which was not productively used before. The catalyst effect is stronger over time, eliminating the inefficient production completely, which leads to eventual recovery of the wage rate and employment.

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<sup>1</sup>Kiyotaki and Moore (1997), Kiyotaki (1998), Aghion, Banerjee and Piketty (1999), and Aghion and Banerjee (2005), for examples, investigate these symptoms of the domestic collateral constraint.

If domestic financial system is more advanced than the rest of the world, the allocation of production is already efficient, and the large borrowing capacity of the productive entrepreneurs keeps the domestic interest rate above the foreign rate before liberalization. After liberalization, the productive domestic entrepreneurs will attract foreign fund, causing capital inflow, increasing the investment on wage bills. Worker gains from higher wage and employment, and the entrepreneurs gains from cheaper interest rate. With a superior financial institution, the domestic economy can take advantage of a cheaper fund because of financial suppression of the rest of the world.

What emerges from our analysis is that the adjustment of home economy to capital liberalization depends upon not only the absolute level of development of home financial system, but also on the relative level of development of home institution compared to the rest of the world.

Because production is inefficient when the domestic financial system is poor and there is painful period of adjustment for workers following capital liberalization for the economy with intermediate level of financial development, a natural step would be to examine the role of government policy. When people may not keep promises, the analysis of government policy has new aspects. People may not pay taxes, unless they loose the collateral asset by not paying tax. Even if people pay tax, the tax liability will crowd out their borrowing capacity, because the debtor may default if the total liabilities to government and the creditors exceed the value of collateral. Government may default on its debt too. In the economy with such limited commitment, we show that the government only reallocates the means of saving (liquidity) between public and private, instead of creating liquidity. For the economy with intermediate financial depth, a small subsidy to production of unproductive entrepreneurs mitigates the loss of the workers following the capital liberalization at the cost of prolonging the transition to efficient

production. But the large subsidy to completely prevent the temporal loss of workers will halt the transition, ending up hurting both entrepreneurs and workers in the long run.

There is an extensive literature that examines the relationship between the domestic financial development and capital account liberalization. Aghion, Bacchetta and Banerjee (2004) shows that the economy with the intermediate level of domestic collateral factor may become unstable following capital liberalization. Sakuragawa and Hamada (2001) analyzes the danger of capital flight from South to North following capital liberalization, by constructing an overlapping generations model with costly state verification. Caballero and Krishnamurthy (2004) emphasizes the interaction between domestic and international financial constraints to examine financial crisis. Although every entrepreneur anticipates the possibility of liquidity shock with some likelihood before the project yields the return, those who do not experience the shock will earn only meager return on saving because of limited domestic borrowing capacity and resulting financial repression in the interim period. Thus the entrepreneurs tend to under-save and over-invest in the first place, which makes financial crisis likely with small aggregate shock. A recent work of Caballero, Farhi and Gourinchas (2006) investigates how two economies with different levels of financial depth interact, by using a Blanchard and Yaari type overlapping generations model. Kim (2001) develops a two country model of adoption of vintages of technologies, and shows that, following capital liberalization, the country with better domestic financial system specializes in adopting more recent technology, while the country with poor financial system ends up with adopting older technologies, leading to a substantial gap in TFP between two. Our distinctive contributions to the literature would be that we systematically investigate the implications of limited commitment of both private and government agents against both domestic and foreign creditors, for the

entire adjustment process of the economy to capital account liberalization.

## 2 Model

We consider a small open economy with one homogeneous goods and two types of continua of agents: entrepreneurs and workers. Both of them live forever. Entrepreneurs hire workers to produce goods. Workers do not have production technology, simply supplying homogeneous labour in order to consume.

The preference of the entrepreneur is described by the expected discounted utility

$$E_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} \log c_s \right], \quad (1)$$

where  $c_s$  is the consumption at date  $s$ , and  $\beta \in (0, 1)$  is the subjective discount factor, and  $E_t$  is the expectations conditional on information at date  $t$ .

The entrepreneurs have a constant returns to scale production technology

$$y_{t+1} = a_t l_t, \quad (2)$$

where  $y_{t+1}$  is output of goods at date  $t + 1$ ,  $l_t$  is the labour input at date  $t$ , and  $a_t$  is a productivity parameter, which is known at date  $t$ . At each date some entrepreneurs are productive ( $a_t = \alpha$ ), and others are unproductive ( $a_t = \gamma \in (0, \alpha)$ ). Each entrepreneur shifts stochastically between productive and unproductive states following a Markov process. Specifically, if an entrepreneur is productive in this period he/she may become unproductive in the next period with probability  $\delta$ . Also, any unproductive entrepreneur in this period may become productive in the next period with probability  $n\delta$ . The shifts



of the productivity are exogenous and independent across entrepreneurs and over time. This transition matrix implies that the fraction of productive entrepreneurs is stationary over time and equal to  $n/(1+n)$ , given that the economy starts with such population distribution. We assume that the probability of the productivity shifts is not too large:

$$\delta + n\delta < 1. \quad (3)$$

This assumption is equivalent to the condition that the productivity of each agent is positively correlated between present and the next periods.

We assume that the production technology is specific to the entrepreneur, and that only the entrepreneur who started the production has the necessary skill to obtain full amount described by the production function. We also assume that the entrepreneur cannot precommit to work, always having freedom to withdraw its labour. (The entrepreneur's human capital is *inalienable*, following Hart and Moore (1994)). Besides this entrepreneur, a lead creditor who has been monitoring the production throughout has a specific skill to obtain  $\theta$  ( $< 1$ ) fraction of full amount of output, if she takes over the entrepreneur's production. Although production is divisible, we assume that there is only one lead creditor for each segment of production, and that only a home resident can become a lead creditor. All the other (non-lead) outside creditors, home or foreign, can obtain only  $\phi\theta$  fraction of full output, where  $\phi \in [0, 1)$ . If the entrepreneur who borrows to produce threatens to withdraw its labour immediately before output realizes in order to negotiate with the creditors, it is efficient for the entrepreneur to bribe the creditors into letting him to continue production. Assuming that the outside creditors (including foreign lenders) are weak in the bargaining against the entrepreneur and the lead creditor, the repayment to the outside creditors is negotiated down to  $\phi\theta$  fraction of

full output<sup>2</sup>. Knowing this possibility in advance, the foreign lenders restrict their loan of this period so that the repayment ( $b_{t+1}^*$ ) does not exceed  $\phi\theta$  fraction of output in the next period:

$$b_{t+1}^* \leq \phi\theta y_{t+1}. \quad (4)$$

Also, assuming that the creditors as a whole are weak against the entrepreneur in the bargaining, the domestic lead creditor restricts her loan ( $b_{t+1}$ ) so that the total sum of loans does not exceed  $\theta$  fraction of output,:

$$b_{t+1} + b_{t+1}^* \leq \theta y_{t+1}. \quad (5)$$

We take both  $\theta$  and  $\phi$  as exogenous parameters to represent the degrees of development of the country's financial institution. We consider the size of  $\theta$  as a domestic collateral factor, representing the overall financial depth of the home economy. The gap between  $\phi\theta$  and  $\theta$  reflects the difference between the outside creditors and the lead creditor in their skills of production and bargaining, (being influenced by legal protection of the outside creditors<sup>3</sup>), which explains why borrowing constraint from the foreigners is tighter than the borrowing constraint from the domestic lead creditor. We assume that the collateralized return from unit labour input of productive entrepreneurs is smaller than the return of unproductive entrepreneurs:

$$\theta\alpha < \gamma. \quad (6)$$

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<sup>2</sup>Diamond and Rajan (2001) called the lead creditor *relationship banker*. Unlike Diamond and Rajan, however, we consider an economy in which the lead creditor cannot precommit to bargain sequentially with one outsider creditor after another in order to give up her bargaining power.

<sup>3</sup>See La Porta, Shleifer, Lopez-de-Silanes and Vishny (1998, 2002).

Under this assumption, the productive entrepreneurs cannot borrow unlimited amount when the interest rate is at least as high as the rate of return on production of the unproductive entrepreneurs.

The flow-of-funds constraint of the entrepreneur is given by

$$c_t + w_t l_t = y_t - b_t - b_t^* + \frac{b_{t+1}}{r_t} + \frac{b_{t+1}^*}{r^*}, \quad (7)$$

where  $w_t$  is the real wage rate,  $r_t$  is the domestic real gross interest rate, and  $r^*$  is the foreign real gross interest rate. The left hand side (LHS) of the flow-of-fund constraint is expenditure; consumption ( $c_t$ ) and investment – wage bill ( $w_t l_t$ ) due to the time lag between labour input and output in production function. The right-hand-side (RHS) is financing; the internal finance from the net worth – output minus the debt repayment to domestic and foreign creditors –, and the external finance of the borrowings from home and foreign creditors. Throughout the analysis, we assume the home economy is small relative to the rest of the world, the real interest rate of the rest of the world is constant over time, and that there is no limitation on domestic lending to foreigners at this interest rate. We also assume the foreign interest rate is strictly less than the time preference rate:

$$r^* < 1/\beta. \quad (8)$$

The entrepreneur chooses consumption, labour input, output and domestic and foreign borrowing  $(c_t, l_t, y_{t+1}, b_{t+1}, b_{t+1}^*)$  to maximize the expected discounted utility (1) subject to the constraints of technology (2), the flow-of-funds (7), the borrowing from foreign and home creditors (4) and (5).

Next, we turn to workers. Unlike entrepreneurs, the workers do not have production technology, nor any collateralizable asset in order to borrow either domestically or in-

ternationally. They choose consumption  $c_t$ , labour supply  $l_t$ , and domestic and foreign net borrowings ( $b_{t+1}$  and  $b_{t+1}^*$ ) to maximize the expected discounted utility,

$$E_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} u(c_s - v(l_t)) \right], \quad (9)$$

subject to the flow of funds constraint

$$c_t = w_t l_t - b_t - b_t^* + \frac{b_{t+1}}{r_t} + \frac{b_{t+1}^*}{r^*}, \quad (10)$$

and the borrowing constraints

$$b_{t+1} \leq 0, \quad b_{t+1}^* \leq 0. \quad (11)$$

We assume decreasing marginal net utility of consumption,  $u'(c - v) > 0$ ,  $u''(c - v) < 0$ , and increasing marginal disutility of labour,  $v(0) = 0$ ,  $v'(l) > 0$ ,  $v''(l) > 0$ . The choice of labour supply implies  $w_t = v'(l_t)$ , or

$$l_t^s = L^s(w_t), \quad \text{where } L^s(w) = v'^{-1}(w). \quad (12)$$

Thus  $dL^s/dw_t > 0$ . We normalize the population size of workers to be unity.

Let us denote aggregate consumption of productive entrepreneurs, unproductive entrepreneurs, and workers as  $C_t$ ,  $C_t'$ , and  $C_t^w$ . (Similarly let  $B_t$ ,  $B_t'$ , and  $B_t^w$  be aggregate quantities of the other quantity  $b_t$  of productive entrepreneurs, unproductive entrepreneur, and workers, etc.). The market clearing condition for labour, goods, and domestic credit are written as

$$L_t + L_t' = L^s(w_t), \quad (13)$$

$$C_t + C'_t + C_t^w = Y_t + Y'_t + \frac{B_{t+1}^*}{r^*} - B_t^*, \quad (14)$$

$$B_{t+1} + B'_{t+1} + B_{t+1}^w = 0. \quad (15)$$

In the RHS of equation (14),  $B_t^*$  is defined as the aggregate net debt of all the home entrepreneurs and workers against foreigners matured at date  $t$ , and thus the last two terms are the net supply of goods by the foreigners to domestic agents. In equation (15), the debt of domestic agents to the other domestic agents should be net out in the aggregate, even though the total debts of the domestic agents need not because of the international borrowing and lending. (Remember that the domestic credit market may be segmented from the international credit market, because the home agents face the international borrowing constraint).

The competitive equilibrium is defined as a set of prices  $(r_t, w_t)$  and quantities  $(y_t, l_t, c_t, b_{t+1}, b_{t+1}^*, Y_t, Y'_t, L_t, L'_t, C_t, C'_t, C_t^w, B_{t+1}, B'_{t+1}, B_{t+1}^w, B_{t+1}^*)$ , which is consistent of the choice of all the individual entrepreneurs and workers as well as the clearing conditions of market for labour, goods and domestic credit. Because there is no shocks except for the idiosyncratic shock to the productivity of each entrepreneur, the agents have perfect foresight of future prices and aggregate quantities in the equilibrium. By Walras' Law, only two out of three market clearing conditions are independent.

Now, let us derive general property of the competitive equilibrium. In our small open economy, although home agents face the constraint on their borrowing from foreigners, there is no constraint on their lending to foreigners. Thus, in equilibrium, we learn that the domestic interest rate is at least as high as the foreign interest rate:

$$r_t \geq r^*. \quad (16)$$

(If the domestic interest rate were strictly lower than the foreign interest rate, there would be no domestic agent who provides domestic credit and there would be many producers who seek domestic credit, which contradicts the market equilibrium).

The entrepreneur has a few choices of accumulating net worth from this period to the next period subject to the flow-of-funds constraint (7). Let  $R_t(a_t)$  be the maximum rate of returns on the net worth from date  $t$  to date  $t+1$  for the entrepreneur with labour productivity  $a_t$ . Then, it is the maximum of all the options as:

$$R_t(a_t) = \text{Max} \left\{ r_t, \frac{a_t}{w_t}, \frac{a_t(1 - \phi\theta)}{w_t - (a_t\phi\theta/r^*)}, \frac{a_t(1 - \theta)}{w_t - (a_t\phi\theta/r^*) - [a_t(1 - \phi)\theta/r_t]} \right\}. \quad (17)$$

The first term in the bracket of RHS is the rate of return on domestic loan, which is at least as high as the return on making loan to the foreigners by (16). The second term is the rate of returns on production without borrowing, subject to the constraint of production technology (2). The third term is the rate of return on production with borrowing from foreigners by putting  $\phi\theta$  fraction of output as collateral ((4) holds with equality and  $l_t > 0$  in the flow-of-funds constraint). By borrowing from foreigners secured by  $\phi\theta$  fraction of output, the entrepreneur can finance  $a_t\phi\theta/r^*$  amount of unit labour cost externally. Thus the denominator is the required net worth (downpayment) for the unit labour input, and the numerator is the output after repaying the debt. The final term is the rate of returns on production with maximum borrowing from both foreigners and the domestic lead creditor. (Both (4) and (5) hold equality and  $l_t > 0$  in the flow-of-funds constraint). The denominator is downpayment for hiring unit labour, when the entrepreneur finances  $a_t\phi\theta/r^*$  of unit labour cost from borrowing from foreigners, and finances  $a_t(1 - \phi)\theta/r_t$  by borrowing additionally from the domestic lead creditor at interest rate  $r_t$ . (Note that the entrepreneur prefers to borrow maximum

first from foreigners at a lower interest rate<sup>4</sup>).

Given this optimal choice of accumulating net worth, the flow-of-funds constraint (7) can be written as

$$m_{t+1} = R_t(a_t)(m_t - c_t), \quad (18)$$

where  $m_t (= y_t - b_t - b_t^*)$  is net worth of the entrepreneur at date  $t$ . The entrepreneur chooses consumption and saving in order to maximize the expected discounted utility subject to the flow-of-fund constraint. The first order condition is given by the Euler equation:

$$\frac{1}{c_t} = \beta R_t(a_t) E_t \left( \frac{1}{c_{t+1}} \right).$$

Together with the flow-of-funds constraint, we have the explicit consumption function as

$$c_t = (1 - \beta)m_t = (1 - \beta)(y_t - b_t - b_t^*). \quad (19)$$

In the expression of maximum rate of returns on net worth (17), each of the last three rates of returns on production are strictly higher for the productive entrepreneur than the unproductive entrepreneur, while the rate of return on domestic loan is the same for both. Thus, the unproductive entrepreneur has comparative advantage in providing loan, while the productive entrepreneurs have comparative advantage in production with maximum borrowing. Later, we will show that the workers will not lend nor borrow in the equilibrium, so that the domestic debts of the productive and the unproductive entrepreneurs are net out in the domestic credit market. Thus we learn that unproductive entrepreneurs lend to productive entrepreneurs in the domestic credit market, and

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<sup>4</sup>The preference is strict if  $r^* < r_t$ . If  $r^* = r_t$ , then entrepreneur is indifferent, and any combination of domestic and foreign borrowing yields the same return as long as the sum of borrowing is at the maximum.

that

$$\begin{aligned} R_t(\gamma) &= r_t \geq \text{Max} \left[ \frac{\gamma}{w_t}, \frac{\gamma(1-\phi\theta)}{w_t - (\gamma\phi\theta/r^*)} \right], \\ 0 &= L'_t \left\{ r_t - \text{Max} \left[ \frac{\gamma}{w_t}, \frac{\gamma(1-\phi\theta)}{w_t - (\gamma\phi\theta/r^*)} \right] \right\}. \end{aligned} \quad (20)$$

The unproductive entrepreneurs produce if and only if the rate of return on production is equal to the domestic interest rate. Otherwise, the unproductive entrepreneurs specialize in providing loan.

We also learn that the productive entrepreneur always borrows to produce, and that their domestic and international borrowing constraints are binding if the rate of return on production with maximum leverage exceeds the domestic interest rate:

$$\begin{aligned} R_t(\alpha) &= \frac{\alpha(1-\theta)}{w_t - (\alpha\phi\theta/r^*) - [\alpha(1-\phi)\theta/r_t]} \geq r_t, \\ l_t &\leq \frac{\beta m_t}{w_t - (\alpha\phi\theta/r^*) - [\alpha(1-\phi)\theta/r_t]}, \text{ and the equality holds if } R_t(\alpha) > r_t. \end{aligned} \quad (21)$$

We derive the expression of upper bound of employment of the productive entrepreneur from the two borrowing constraints with equality, the flow-of-funds constraint and consumption function, (4) (5) (7), and (19). Let  $Z_t$  be aggregate net worth of all the entrepreneurs

$$Z_t = Y_t + Y'_t - B_t - B'_t - B_t^*,$$

and let  $s_t$  be the share of net worth of all the productive entrepreneurs, so that  $s_t Z_t$  is the aggregate net worth of the productive entrepreneurs. Because the labour demand function of the productive entrepreneur (21) is linear and common to all the productive entrepreneurs, we can derive the aggregate employment of the productive entrepreneurs



as

$$L_t \leq \frac{\beta s_t Z_t}{w_t - (\alpha \phi \theta / r^*) - [\alpha(1 - \phi)\theta / r_t]}, \text{ and the equality holds if } R_t(\alpha) > r_t. \quad (22)$$

Concerning the workers, they will decumulate their asset until they consume all, if the domestic real interest rate is strictly less than the time preference rate:

$$r_t < 1/\beta. \quad (23)$$

We will later verify this inequality holds in equilibrium. When workers consume all the asset, ideally they would like to borrow, but they cannot borrow because they do not have collateral. Thus, after the initial transition, the workers' asset stays zero with the borrowing constraint (11) binding, and the aggregate consumption of the workers is equal to the aggregate wages<sup>5</sup>:

$$B_t^w = 0, \text{ and } C_t^w = w_t L^s(w_t). \quad (24)$$

The workers do not save, not because the workers are impatient relative to the entrepreneurs, but because the real interest rate is lower than the time preference rate in equilibrium. The entrepreneurs nonetheless save because their rate of return on net worth exceeds the time preference rate when they are productive.

From the behavior of the workers, the domestic credit market equilibrium becomes  $B_t + B_t' = 0$ . Together with the consumption function of the entrepreneurs (19), the

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<sup>5</sup>If the workers expect sharp decline of wage in future, then they may save despite of the interest rate being lower than the time preference rate. We will later show that there is no such sharp and continual decline in wages in the equilibrium.

goods market clearing condition (14) can be written as

$$w_t L^s(w_t) + (1 - \beta)Z_t = Y_t + Y'_t - B_t^* + \frac{B_{t+1}^*}{r^*} = Z_t + \frac{B_{t+1}^*}{r^*}, \text{ or}$$

$$w_t L^s(w_t) = \beta Z_t + \frac{B_{t+1}^*}{r^*}. \quad (25)$$

The LHS is gross investment on wage bill by the entrepreneurs. The RHS is sum of gross saving and foreign borrowing of the entrepreneurs. The foreign borrowing of the entrepreneurs satisfies the international borrowing constraints, where the constraint is binding if the domestic interest rate exceeds the foreign interest rate:

$$B_{t+1}^* \leq \phi\theta(Y_{t+1} + Y'_{t+1}) = \phi\theta(\alpha L_t + \gamma L'_t), \text{ and the equality holds if } r_t > r^*. \quad (26)$$

We take the aggregate net worth of the entrepreneurs ( $Z_t$ ) and the share of the productive entrepreneurs' net worth ( $s_t$ ) as the state variables of the economy at date  $t$ . The equilibrium at date  $t$  is summarized by  $(r_t, w_t, L_t, L'_t, B_{t+1}^*)$  that satisfies the equilibrium conditions (13), (20), (22), (25) and (26).

In order to describe how the economy evolves over time, it is convenient to define the variable to measure how much extra rate of return the productive entrepreneur enjoys over the unproductive entrepreneur:

$$x_t \equiv \frac{R_t(\alpha) - R_t(\gamma)}{R_t(\gamma)} = \frac{(\alpha/r_t) + \phi\theta[(1/r^*) - (1/r_t)] - w_t}{w_t - (\alpha\phi\theta)/r^* - [\alpha(1 - \phi)\theta]/r_t}. \quad (27)$$

From the consumption function (19), we observe all the entrepreneurs save  $\beta$  fraction

of their net worth. Then, the aggregate wealth in the next period would be:

$$\begin{aligned} Z_{t+1} &= (1 + x_t)r_t\beta s_t Z_t + r_t\beta(1 - s_t)Z_t \\ &= (1 + s_t x_t)r_t\beta Z_t. \end{aligned} \quad (28)$$

Here, the aggregate saving of productive entrepreneurs ( $\beta s_t Z_t$ ) earns the rate of returns  $R_t(\alpha) = (1 + x_t)r_t$ , while the aggregate saving of the unproductive entrepreneurs ( $\beta(1 - s_t)Z_t$ ) only earns the rate of return  $R_t(\gamma) = r_t$ . We can also derive the law of motion of the share of productive entrepreneurs' net worth as:

$$\begin{aligned} s_{t+1} &= \frac{(1 - \delta)(1 + x_t)r_t\beta s_t Z_t + n\delta r_t\beta(1 - s_t)Z_t}{(1 + s_t x_t)r_t\beta Z_t} \\ &= \frac{(1 - \delta)s_t(1 + x_t) + n\delta(1 - s_t)}{1 + s_t x_t}. \end{aligned} \quad (29)$$

The denominator of RHS of the first equation is the aggregate net worth in the next period. The numerator is the aggregate net worth of productive agents in the next period, which is the sum of the net worth of those who continue to be productive,  $(1 - \delta)(1 + x_t)r_t\beta s_t Z_t$ , and those who shift from unproductive to be productive,  $n\delta r_t\beta(1 - s_t)Z_t$ . The dynamic evolution of the economy is characterized by the recursive equilibrium:  $(Z_{t+1}, s_{t+1}, x_t, r_t, w_t, L_t, L'_t, B_{t+1}^*)$  that satisfies (13), (20), (22), (25), (26), (27), (28) and (29), as functions of the state variables  $(s_t, Z_t)$ .

### 3 Steady State Autarky Equilibrium

Before looking into how the economy adjusts to the capital liberalization, we first analyze the steady state equilibrium of the economy with no financial transaction with foreigners.

Here, the home agents are not allowed to borrow nor lend, i.e.,  $\phi = B_t^* = 0$ . Because our economy has only one goods and labour is not tradeable, once there is no trade of financial assets, there would be no trade of goods in equilibrium: the economy becomes autarky. In the steady state, all the endogenous variables stay constant. Let us define  $X = sx$ , the product of the share of net worth and the extra rate of returns of the productive agents – the importance of extra returns of the productive entrepreneurs. Then, the equilibrium conditions (13), (20), (22),(25), (27), (28) and (29) can be written as

$$L + L' = L^s(w), \quad (30)$$

$$r \geq \frac{\gamma}{w}, \text{ and } L'(r - \frac{\gamma}{w}) = 0, \quad (31)$$

$$L \leq \frac{X}{(\alpha/r) - w} \beta Z, \text{ and the equality holds if } \frac{\alpha}{r} > r. \quad (32)$$

$$wL^s(w) = \beta Z, \quad (33)$$

$$x = \frac{(\alpha/r) - w}{w - (\alpha\theta/r)}, \quad (34)$$

$$1 = \beta(1 + X)r, \quad (35)$$

$$F(X, x) = X^2 + [\delta(1 + n) - (1 - \delta)x] X - n\delta x = 0, \text{ and } X \geq 0. \quad (36)$$

In the steady state equilibrium of the autarky economy, these seven equilibrium conditions determine  $(r, w, x, X, L, L', Z)$  endogenously. Then, we have the following proposition (See Figure 1. Proofs of all the Propositions are in Appendix):

**Proposition 1** *The steady state equilibrium of the autarky economy depends upon the financial depth of the economy  $\theta$  as:*

- (i) *If  $\theta < \bar{\theta} \equiv \frac{\delta}{\frac{\alpha-\gamma}{\gamma} + (1+n)\delta}$ , the unproductive entrepreneurs produce in equilibrium,*

and the productive entrepreneurs are credit constrained. The interest rate is lower than the time preference rate and is a decreasing function of  $\theta$ , while the wage rate is an increasing function of  $\theta$ . Specifically,  $r = 1/[\beta(1 + X)]$  and  $w = \gamma\beta(1 + X)$ , where  $X$  solves

$$F(X, \frac{\alpha - \gamma}{\gamma - \theta\alpha}) = X[X + \delta(1 + n)] - [(1 - \delta)X + n\delta] \frac{\alpha - \gamma}{\gamma - \theta\alpha} = 0 \text{ and } X \geq 0. \quad (37)$$

(ii) If  $\theta \in [\bar{\theta}, \frac{1}{1+n})$ , the unproductive entrepreneurs do not produce, while the productive entrepreneurs are credit constrained. The interest rate is lower than time preference rate and is an increasing function of  $\theta$ , satisfying:

$$r = \frac{1}{\beta[1 - \delta - n\delta + (\delta/\theta)]}. \quad (38)$$

The wage rate is equal to  $\beta\alpha$ .

(iii) If  $\theta > \frac{1}{1+n}$ , no entrepreneurs are credit constrained. The interest rate is equal to the time preference rate ( $r = 1/\beta$ ), and the wage rate is equal to  $\beta\alpha$ .

If the financial depth of the economy is below the threshold  $\bar{\theta}$ , the allocation of labour is inefficient, because unproductive entrepreneurs (who have strictly lower productivity than productive entrepreneurs) employ labour. Intuitively, if the domestic financial system is underdeveloped (so that the domestic credit constraint is tight), then it fails to transfer enough purchasing power from the unproductive entrepreneurs (savers) to the productive entrepreneurs (investing agents), so that the unproductive entrepreneurs end up employing workers with the inferior technology. Because the productive entrepreneurs have limited borrowing capacity secured by the collateral, the interest rate is suppressed below the time preference rate in the equilibrium. (This verifies our pre-

vious conjecture (23)). Since production allocation is inefficient, the aggregate wealth and the wage rate are remained to be low in the steady state. The threshold level  $\bar{\theta}$  of domestic collateral factor to induce the productive inefficiency is an increasing function of the transition probability from the productive to unproductive state  $\delta$ , because the higher the transition probability is, the lower is the share of net worth of the productive entrepreneurs, and the smaller is the aggregate borrowing capacity of the productive entrepreneurs relative to the aggregate saving.<sup>6</sup>

In the second type of equilibrium,  $\theta$  is at least as large as  $\bar{\theta}$  but smaller than the share of population of the unproductive entrepreneurs,  $1/(1+n)$ . Here, the domestic financial system is developed enough to transfer necessary purchasing power to productive entrepreneurs to achieve the efficiency in production. (The efficiency in production means aggregate output is at the maximum for a given total employment. It does not mean the allocation is the first best, because the credit constraint is binding). Then, the larger the borrowing capacity of the productive entrepreneurs is, the larger is the demand for domestic credit relative to the supply, and the higher is the equilibrium interest rate in the domestic credit market. This explains why the interest rate is an increasing function of  $\theta$ .

If the domestic financial system is very well developed  $\theta > \frac{1}{1+n}$  in the third type equilibrium, both the productive and unproductive entrepreneurs enjoy the same rate of return on saving, behaving similarly, and thus the entrepreneurs as a whole behave like the representative entrepreneur.<sup>7</sup> The economy achieves the first best allocation with

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<sup>6</sup>A higher  $\bar{\theta}$  implies the economy is more likely to be inefficient in production for the same financial depth. The threshold  $\bar{\theta}$  is also a decreasing function of the productivity gap between productive and unproductive entrepreneurs  $(\alpha - \gamma)/\gamma$ , because the larger the gap is, the larger is the share of net worth of the productive entrepreneurs, and the larger is their borrowing capacity relative to aggregate saving.

<sup>7</sup>For  $\theta < 1/(1+n)$ , Proposition 1 verifies our conjecture (23). Because (23) no longer holds for  $\theta \geq 1/(1+n)$ , workers may not be credit constrained. Also, we must use (22) instead of (32) in order

the advanced financial system.

It would be of interest to see the total factor productivity (TFP) of the economy. Since labour is the only input, the TFP is defined as the average productivity of labour

$$A = \frac{\alpha L + \gamma L'}{L^s} = (\alpha - \gamma) \frac{L}{L^s} + \gamma. \quad (39)$$

For  $\theta < \bar{\theta}$ , from (31), (32) and (33), we observe:

$$A = (1 + X)\gamma, \text{ and } w = \beta A.$$

Thus, from (37), we learn both TFP and the wage rate are increasing functions of the financial depth of the economy  $\theta$ . Intuitively, the more the domestic financial system develops in the relatively underdeveloped stage (an increase of  $\theta$  for  $\theta < \bar{\theta}$ ), the more purchasing power is transferred from the unproductive entrepreneurs to the productive entrepreneurs, which leads to a larger share of employment by the productive entrepreneurs and higher TFP and wage rate in the equilibrium. Once the domestic financial system develops enough to achieve the efficiency in production for  $\theta \geq \bar{\theta}$ , the TFP is equal to  $\alpha$ , which is the labour productivity of the productive entrepreneurs, and wage rate is equal to  $\beta\alpha$ .<sup>8</sup>

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to characterize the equilibrium, because both the denominator and the numerator become zero in the latter equation. If we redefine  $Z$  as the total wealth of the economy, instead of the aggregate net worth of the entrepreneurs, then the remaining equilibrium conditions are unchanged.

<sup>8</sup>Kiyotaki (1998) and Casselli and Gennaioli (2003) made similar observation on why TFP depends upon the financial depth of the economy.

## 4 Adjusting to Capital Liberalization

We now study how the economy is going to adjust to the liberalization of financial transactions with foreigners, starting from the steady state autarky equilibrium. We are going to focus on the situation in which the productive entrepreneurs are credit constrained, (i.e., equation (22) holds with equality). Later, we will derive the condition for this to be true, (and will discuss how our results are extended for the case in which the productive entrepreneurs are not credit constrained). Combining (13) with (20) and (22), we can summarize the labour market equilibrium condition as<sup>9</sup>:

$$L^s(w_t) \geq \frac{\beta s_t Z_t}{w_t - (\alpha \phi \theta / r^*) - [\alpha(1 - \phi)\theta / r_t]}, \quad (40)$$

$$r_t \geq \frac{\gamma(1 - \phi\theta)}{w_t - (\gamma\phi\theta / r^*)}, \quad (41)$$

$$\left\{ L^s(w_t) - \frac{\beta s_t Z_t}{w_t - (\alpha \phi \theta / r^*) - [\alpha(1 - \phi)\theta / r_t]} \right\} \left[ r_t - \frac{\gamma(1 - \phi\theta)}{w_t - (\gamma\phi\theta / r^*)} \right] = 0. \quad (42)$$

For given state variables  $(s_t, Z_t)$ , the interest rate  $r_t$  that satisfies (40) with equality is a decreasing function of the wage rate  $w_t$ , so is the interest rate that satisfies (41) with equality. Because we know

$$\left\{ L^s(w_t) - \frac{\beta s_t Z_t}{w_t - (\alpha \phi \theta / r^*) - [\alpha(1 - \phi)\theta / r_t]} \right\}_{r_t = \frac{\gamma(1 - \phi\theta)}{w_t - (\gamma\phi\theta / r^*)}}$$

is an increasing function of  $w_t$ , when (41) holds with equality, the inequality (40) is satisfied for  $w_t \geq w^*$  and is violated for  $w_t < w^*$  for some  $w^*$ . Thus, the labour market equilibrium locus of  $(w_t, r_t)$  is described by ABC in *Figure 2*.

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<sup>9</sup>From  $r_t \geq r^*$ , we know the second term in the bracket of RHS of (20) is at least as large as the first term.



From equations (25) (26) and (22), the goods market equilibrium condition is summarized by inequality (16) and:

$$\beta Z_t \left\{ 1 + \frac{(\alpha - \gamma)(\phi\theta/r^*)s_t}{w_t - (\alpha\phi\theta/r^*) - [\alpha(1 - \phi)\theta/r_t]} \right\} \geq \left( w_t - \frac{\gamma\phi\theta}{r^*} \right) L^s(w_t), \quad (43)$$

$$\left\{ \beta Z_t \left[ 1 + \frac{(\alpha - \gamma)(\phi\theta/r^*)s_t}{w_t - (\alpha\phi\theta/r^*) - [\alpha(1 - \phi)\theta/r_t]} \right] - \left( w_t - \frac{\gamma\phi\theta}{r^*} \right) L^s(w_t) \right\} (r_t - r^*) = 0. \quad (44)$$

The goods market equilibrium is described by the DEF in *Figure 3*.

Given the state variables  $(s_t, Z_t)$ , the equilibrium of date  $t$  is described by  $(w_t, r_t)$  that clears both labour and goods markets, i.e., the intersection of curves ABC of *Figure 2* and DEF of *Figure 3*. When (40) holds with equality, the inequality (43) can be rewritten as

$$\beta Z_t \geq L^s(w_t) \left( w_t - \frac{\alpha\phi\theta}{r^*} \right).$$

Thus, if (40) holds with equality, the inequality (43) is satisfied for  $w_t \leq w^{**}$ , and it is violated if  $w_t > w^{**}$  for some  $w^{**}$ . Then, the locus of  $(w_t, r_t)$  that satisfies (43) with equality is steeper than the locus that satisfies (40) with equality, which is in turn steeper than the locus that satisfies (41) with equality. Therefore, there exists a unique intersection of the labour market equilibrium locus and the goods market equilibrium locus, and there are four possible types of equilibrium (See *Figure 4*).

**Proposition 2** *For a given state variables  $(s_t, Z_t)$ , there exists a unique date  $t$  equilibrium of one of the four types:*

(Type A :  $L'_t > 0$  and  $r_t > r^*$ ) (41) and (43) hold with equality, and (40) with strict inequality;

(Type B :  $L'_t > 0$  and  $r_t = r^*$ ) (41) holds with equality, (40) with strict inequality,

and (43) with inequality;

(Type C :  $L'_t = 0$  and  $r_t = r^*$ ) (40) holds with equality, and (41) (43) with inequality;

(Type D :  $L'_t = 0$  and  $r_t > r^*$ ) (40) (43) hold with equality, and (41) with strict inequality.

In equilibrium of Type A and Type B, the production allocation is inefficient as the unproductive entrepreneurs employ labour. In contrast, the production is efficient in equilibrium of Type C and Type D. In equilibrium of Type B and Type C, the domestic credit market is integrated into the international credit market, so that the domestic and foreign interest rates are equal. In equilibrium of Type A and Type D, on the other hand, the domestic interest rate exceeds the foreign interest rate, so that the domestic credit market is segmented from the international credit market.

Using Propositions 1 and 2, we can describe the economy immediately after the capital liberalization. Let  $r^A(\theta)$ ,  $w^A(\theta)$ ,  $s^A(\theta)$  and  $Z^A(\theta)$  be the domestic interest rate, wage rate, share of net worth of the productive entrepreneurs and the aggregate net worth of the entrepreneurs in the steady state equilibrium of autarky economy with financial depth  $\theta$ . From Proposition 1, we know  $r^A(\theta)$  is a decreasing function of  $\theta$  for  $\theta < \bar{\theta}$ , and is an increasing function of  $\theta$  for  $\theta \geq \bar{\theta}$ . Let us assume that

$$r^A(\bar{\theta}) = \frac{\gamma}{\beta\alpha} < r^*. \quad (45)$$

The inequality implies that the foreign interest rate is higher than the minimum value of the domestic interest rate in the steady state autarky equilibrium for all possible financial depth of the home economy. (If the foreign economy has the same environment as the home economy except for the financial depth, then this assumption holds except for an exceptional case that foreign financial depth is exactly equal to  $\bar{\theta}$ ). Let us define two

critical values of the financial depth of the domestic economy:

$$\theta_2 = \theta \text{ that solves } r^A(\theta) = r^* \text{ in } (0, \bar{\theta}) \text{ if such } \theta \text{ exists, and } \theta_2 = 0 \text{ otherwise,} \quad (46)$$

$$\theta_4 = \theta \text{ that solves } r^A(\theta) = r^* \text{ in } \left( \bar{\theta}, \frac{1}{1+n} \right). \quad (47)$$

Then we know from Proposition 1 that:

$$\begin{aligned} r^A(\theta) &> r^*, \text{ for } \theta \in [0, \theta_2) \text{ and for } \theta > \theta_4, \text{ and} \\ r^A(\theta) &< r^*, \text{ for } \theta \in (\theta_2, \theta_4). \end{aligned} \quad (48)$$

In the following, we continue our discussion by assuming that:

$$r^A(0) > r^*, \quad (49)$$

so that  $\theta_2 > 0$ . We will later discuss briefly how our analysis will be simplified when this assumption is not satisfied.

Proposition 3 summarizes the equilibrium immediately after capital liberalization: (See *Figure 5*)

**Proposition 3** *Suppose that the home economy liberalizes the international financial transaction at date  $t_0$  when the economy was in the steady state equilibrium of autarky at date  $t_0 - 1$ . Let us define  $\theta_1$ ,  $\theta_3$  and  $\theta_5$  as*

$$\beta Z^A(\theta_1) \left\{ 1 + \frac{(\alpha - \gamma)\phi\theta_1 s^A(\theta_1)}{\gamma - \alpha\theta_1} \right\} = \frac{\gamma}{r^*} (1 - \phi\theta_1) L^s\left(\frac{\gamma}{r^*}\right), \quad (50)$$

$$L^s\left(\frac{\gamma}{r^*}\right) = \frac{\beta r^* s^A(\theta_3) Z^A(\theta_3)}{\gamma - \alpha\theta_3}, \quad (51)$$

$$\begin{aligned} \theta_5 &= \text{Min}(1 - \frac{n\beta r^*}{1+n} \frac{L^s(\alpha\beta)}{L^s(\alpha/r^*)}, \theta'_5), \text{ where} \\ \left[ w(\theta'_5) - \alpha \frac{\phi\theta'_5}{r^*} \right] L^s(w(\theta'_5)) &= \beta Z^A(\theta'_5), \text{ and } w(\theta) \text{ solves } \left[ w(\theta) - \frac{\alpha\theta}{r^*} \right] L^s(w(\theta)) = \beta s^A(\theta) Z^A(\theta). \end{aligned} \quad (52)$$

We learn

$$0 < \theta_1 < \theta_2 < \theta_3 < \bar{\theta} < \theta_4 < \theta_5,$$

and that the equilibrium at date  $t_0$  is:

- (Type A:  $L'_{t_0} > 0, r_{t_0} > r^*$ ), if  $\theta \in [0, \theta_1)$ ,
- (Type B:  $L'_{t_0} > 0, r_{t_0} = r^*$ ), if  $\theta \in [\theta_1, \theta_3)$ ,
- (Type C:  $L'_{t_0} = 0, r_{t_0} = r^*$ ), if  $\theta \in [\theta_3, \theta_5]$ ,
- (Type D:  $L'_{t_0} = 0, r_{t_0} > r^*$ ), if  $\theta \in (\theta_5, 1)$ .

Notice that the minimum level of the financial depth of the home economy to achieve the productive efficiency falls from  $\bar{\theta}$  to  $\theta_3$  immediately after capital liberalization. Thus, the economy is more likely to achieve the efficiency in production after capital liberalization than the autarky economy for the same financial depth. Intuitively, the international capital market provides additional means to absorb the saving of the unproductive entrepreneurs and to reduce inefficient production, especially when the domestic financial system is underdeveloped relatively to the rest of the world but it is not extremely underdeveloped. Also, we observe that capital liberalization does not necessarily leads to the complete financial integration of the home economy with the rest of the world immediately after the liberalization. If the financial depth of the economy is very different from the rest of the world, either extremely low  $\theta < \theta_1$  or extremely high  $\theta > \theta_5$ , the domestic interest rate stays higher than the foreign interest rate because the international

borrowing constraint is binding.<sup>10</sup>

How does the new steady state after capital liberalization depends upon the financial depth of the domestic economy? From the generic equilibrium conditions, (13), (20), (22),(25), (26) (27), (28) and (29), we learn the steady state equilibrium of the open economy is characterized by  $(r, w, x, X, L, L', Z)$  that satisfies the conditions (30), (35), (36) and

$$r \geq \frac{\gamma(1 - \phi\theta)}{w - (\gamma\phi\theta/r^*)}, \text{ and } \left[ r - \frac{\gamma(1 - \phi\theta)}{w - (\gamma\phi\theta/r^*)} \right] L' = 0, \quad (53)$$

$$L = \frac{\beta X Z}{(\alpha/r) - w + \alpha\phi\theta[(1/r^*) - (1/r)]}, \quad (54)$$

$$\beta Z + \frac{\phi\theta}{r^*} [\gamma L^s(w) + (\alpha - \gamma)L] \geq w L^s(w), \text{ and } \quad (55)$$

$$(r - r^*) \left\{ \beta Z + \frac{\phi\theta}{r^*} [\gamma L^s(w) + (\alpha - \gamma)L] - w L^s(w) \right\} = 0,$$

$$x = \frac{(\alpha/r) - w + \alpha\phi\theta[(1/r^*) - (1/r)]}{w - (\alpha\theta/r) - \alpha\phi\theta[(1/r^*) - (1/r)]}. \quad (56)$$

From these conditions, we have the following proposition (See *Figure 6*):

**Proposition 4** *Let  $r(\theta)$ ,  $w(\theta)$  and  $Z(\theta)$  be the domestic interest rate, the wage rate and the aggregate net worth of the entrepreneurs in the steady state equilibrium of the open economy with financial depth of  $\theta$ . The equilibrium depends upon the financial depth of the home economy as:*

(i) *If  $\theta < \theta_2$ , the unproductive entrepreneurs produce.  $r^* < r(\theta) < r^A(\theta)$ ,  $w(\theta) > w^A(\theta)$ ,  $r'(\theta) < 0$  and  $w'(\theta) > 0$ .*

(ii) *If  $\theta \in [\theta_2, \tilde{\theta}]$ , where  $\tilde{\theta} \equiv \frac{\delta + \phi\theta[n\delta + (1/\beta r^*) - 1]}{(1+n)\delta + (1/\beta r^*) - 1} \in (\theta_4, 1)$ , the unproductive entrepreneurs*

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<sup>10</sup>For the economy with  $\theta > 1 - \frac{n\beta r^*}{1+n} \frac{L^s(\alpha\beta)}{L^s(\alpha/r^*)}$ , the productive entrepreneurs are not credit constrained. However, the equilibrium is still *Type D*.

do not produce,  $r(\theta) = r^*$  and  $w'(\theta) > 0$ .

(iii) If  $\theta > \tilde{\theta}$ , the unproductive entrepreneurs do not produce,  $r(\theta) > r^*$ ,  $w(\theta) > w^A(\theta)$ ,  $r'(\theta) \geq 0$  and  $w'(\theta) > 0$ .

(iv)  $Z(\theta)$  is an increasing function of  $\theta$ , if labour supply is relatively elastic:

$$\frac{w}{L^s} L^{s'}(w) > \frac{(n\delta/X^*) + 1 + [(1+n)\delta + X^*](\theta - \theta_2)}{n\delta + X^* - [(1+n)\delta + X^*](\theta - \theta_2)}, \text{ where } X^* \equiv \frac{1}{\beta r} - 1 > 0. \quad (57)$$

Note that the unproductive entrepreneurs do not produce if the financial depth is at least as high as  $\theta_2$ . Thus the economy is more likely to achieve efficiency in production for the same financial depth in the steady state than the equilibrium immediately after the liberalization, (because  $\theta_2 < \theta_3$  of Proposition 3). Also, we see that *Type B* equilibrium with inefficient production and complete financial integration no longer exists in the steady state. Intuitively, the international capital market has a stronger catalyst effect of eliminating the inefficiency in production in the long run than the short run, so that the inefficiency of production remains in the long run only if the domestic financial market is not perfectly integrated with the international financial market.<sup>11</sup>

From Proposition 4, we learn the wage rate is an increasing function of the financial depth of the economy  $\theta$ , so is the total employment of the economy (which is equal to labour supply). The aggregate net worth of the entrepreneurs depends upon both aggregate output and income distribution between entrepreneurs, workers and foreigners. The more developed the domestic financial system is, the larger is the aggregate output, and the less favorable is the income distribution of the entrepreneurs after achieving the efficiency in production. Proposition 4(iv) says the aggregate net worth of the entrepreneurs is an increasing function of the financial depth if labour supply is relatively

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<sup>11</sup>If  $\theta$  is close enough to 1, then the productive entrepreneurs are not credit constrained, but the steady state equilibrium is still characterized by (iii).

elastic. Although the threshold elasticity of labour supply is higher than 1, we are focusing on relatively underdeveloped economy which tends to have elastic labour supply, and thus we are going to continue the analysis under the assumption of (57).<sup>12</sup>

In order to understand the transition from the economy immediately after liberalization (in Proposition 3) to the new steady state (in Proposition 4), we examine the dynamics of the aggregate net worth of the entrepreneurs ( $Z_t$ ) and the share of net worth of the productive entrepreneurs ( $s_t$ ). The following proposition summarize the results on the transition (See *Figure 7*)

**Proposition 5** *For Type A equilibrium ( $L'_t > 0, r_t > r^*$ ),  $Z_t$  decreases over time unless  $\theta$  is close to zero, and  $s_t$  increases over time.*

*For Type B equilibrium ( $L'_t > 0, r_t = r^*$ ),  $Z_t$  decreases over time if  $\theta < \theta_2$  and increases over time if  $\theta > \theta_2$ , while  $s_t$  stays constant at the autarky steady state level.*

*For Type C equilibrium ( $L'_t = 0, r_t = r^*$ ),  $Z_t$  decreases over time for  $\theta \in (\theta'_3, \theta_4)$  for some  $\theta'_3 \in (\theta_3, \bar{\theta})$  and increases for  $\theta < \theta'_3$  and for  $\theta > \theta_4$ , and  $s_t$  increases over time.*

*For Type C equilibrium ( $L'_t = 0, r_t > r^*$ ),  $Z_t$  increases over time, and  $s_t$  increases over time.*

From Propositions 1, 3, 4 and 5, we can now describe the dynamic adjustment of the economy following capital liberalization. From Proposition 1, in the steady state equilibrium of the autarky economy with poor domestic financial system, the domestic interest rate is below the time preference rate if  $\theta < \frac{1}{1+n}$ , and the wage rate stays low due to the inefficiency in production for  $\theta < \bar{\theta}$ . For the very poor domestic financial system,  $[0, \theta_2)$ , the wage rate is so low that even the unproductive entrepreneurs enjoy a higher rate of return on production under autarky than the foreign interest rate.

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<sup>12</sup>Lewis ( ) characterized the underdeveloped economy as an economy with unlimited surplus labour. We will later discuss briefly how our results are modified, if Assumption (57) does not hold.

Thus, the capital liberalization causes capital inflow, which pushes up the wage rate. For the extremely poor financial system  $\theta < \theta_1 (< \theta_2)$ , the wage rate continues to be low and the domestic real interest rate continues to be higher than the foreign interest rate as in *Type A* equilibrium, (despite of the wage hike after liberalization). Because the foreign interest rate is lower than the domestic interest rate, both productive and unproductive entrepreneurs borrow from foreigners up to the international borrowing constraint. In addition, the productive entrepreneurs borrow from the unproductive entrepreneurs who become their lead creditors in the domestic credit market. Here, the unproductive entrepreneurs serve as *financial intermediary*: they borrow from the foreigners secured by the fraction of their output, and, at the same time, extend loan to the productive entrepreneurs in the domestic credit market as the lead creditors.<sup>13</sup> The dynamics of employment for the economy with very poor financial system  $\theta \in [0, \theta_2)$  is illustrated by *Figure 8*. Immediately after liberalization, total employment expands with capital inflow and the wage hike, employment of unproductive entrepreneurs (who serve as financial intermediary) expands, while employment of the productive entrepreneurs shrinks (who suffers from the wage hike). Then TFP falls, and the aggregate net worth of the entrepreneurs decumulate over time, which will partially offset the initial expansionary effect on total employment. Thus the adjustment of the economy with poor financial system is characterized by expansion of total employment induced by capital inflow, which is diluted later by the TFP deterioration. Possibly, countries like India and China (at least in the early stage) may experience this type of adjustment.

For the intermediate level of domestic financial development  $\theta \in (\theta_2, \theta_4)$ , the wage rate is not very low and the domestic interest rate under autarky is suppressed below

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<sup>13</sup>Caballero and Krishnamurty (2001) has a similar feature. During the rapid economic growth era after WWII, Japanese general trading companies play such role of financial intermediary, borrowing from abroad against their international collateral and lending to domestic businesses.



the foreign interest rate due to relatively limited borrowing capacity of the productive entrepreneurs. Then, following the capital liberalization, the unproductive domestic entrepreneurs will lend to foreigners, earning a higher interest rate than the autarky domestic interest rate, and shrink their production. The domestic interest rate is equalized with the foreign interest rate, while the domestic investment (wage bill) and the wage rate fall after the liberalization. Both the productive and unproductive entrepreneurs gain from the capital liberalization, while the workers suffer, at least temporally. *Figure 9* illustrates the typical dynamics of employment. Because of the capital outflow and wage loss immediately after liberalization, total employment and employment of unproductive entrepreneurs fall, while the employment of productive entrepreneurs and TFP rise. (The equilibrium immediately after liberalization is *Type B* ( $L'_t > 0, r_t = r^*$ ) if  $\theta < \theta_3$ , and it is *Type C* ( $L'_t = 0, r_t = r^*$ ) if  $\theta \geq \theta_3$ ). Over time, employment of productive entrepreneurs increases together with their accumulation of net worth, until it absorbs the entire employment (so that there is no longer employment of the unproductive entrepreneurs). Thereafter, the wage rate and employment start recovering. Perhaps, some Latin American countries experience this type of adjustment, which is characterized by capital outflow and the loss of employment of the unproductive sector, which may cultivate the anti-globalization sentiment.

When home economy has more advanced financial system than the rest of the world  $\theta > \theta_4$ , the production is efficient with large borrowing capacity of the productive entrepreneurs, which pulls up the domestic interest rate above the foreign interest rate under autarky. Thus, after liberalization, the productive domestic entrepreneurs will attract foreign fund, causing capital inflow. The investment on wage bills increases and the wage rate rises after liberalization. (If For  $\theta \in [\theta_3, \theta_5]$ , the domestic financial market is perfectly integrated with international financial market (after the adjustment of wage)

and there is no inefficiency in production as the economy is in *Type C* equilibrium. If the financial depth of the domestic economy is much higher than the rest of the world ( $\theta > \theta_5$ ), then the domestic interest rate continues to be higher than the foreign interest rate after the capital inflow as in *Type D* equilibrium). *Figure 10* illustrates the path of employment. The total employment (which is equal to employment of the productive entrepreneurs) continues to expand over time with the accumulation of the aggregate net worth of the entrepreneurs, if labour supply is relatively elastic to satisfy Assumption (57).<sup>14</sup> When the home economy has more advance financial system than the rest of the world, the home economy can take advantage of the low interest rate (and the saving glut) of the rest of the world. Both workers and entrepreneurs will gain from capital liberalization. If current US belongs to this type, then it may explain why some people are optimistic about the current account deficit, (even though such optimism is rationalized only if the capital inflow helps financing investment of the productive entrepreneurs rather than government consumption according to our model).

## 5 Government Policy

We observed that, when the domestic financial system is underdeveloped, the allocation of production is inefficient as the unproductive entrepreneurs produce using the inferior technology. Also, we observe capital liberalization can eventually eliminate the inefficiency of production in the economy with intermediate stage of financial development, but that such process can be painful to the workers who suffer from the loss of wage and employment. Is there any role for government to complement the domestic financial system in order to reduce the inefficiency of production? Can the government miti-

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<sup>14</sup>If labour supply is relatively inelastic so that Assumption (57) is violated, then the aggregate net worth of the entrepreneurs decumulate over time because of unfavorable income distribution.

gate the loss of workers during the adjustment to the capital liberalization? Woodford (1999) considers a model with heterogeneous entrepreneurs who cannot borrow, in order to argue that government can issue public debt to absorb the saving of the unproductive entrepreneurs and improve the efficiency. Here, we would like to examine the effects of such policy as well as tax and subsidy policy.

Our economy is characterized by limited commitment: possibility of default of the borrowers. Many countries with poor financial system tend to have poor tax system as well as underdeveloped market for government bond. One possible explanation is that limited commitment and shortage of collateral have implications for both private finance and public finance. When creditors have difficulty to enforce the debtors to repay debt, government may have difficulty to enforce private agents to pay their tax liability. Even if private agents pay tax, they may not be able to borrow as much as before without tax, because they may default if their total liabilities to government and the creditors exceed the value of their collateral. Government may default on their debt too. Such possibility limits the amount of government debt private agents are willing to buy. In this section, we are going to explore systematically the implication of the limited commitment, by considering the economy in which both private agents and government may default.

Government taxes on the entrepreneurs and issues one period discount bond in order to finance government consumption, subsidy to workers and some entrepreneurs, and repayment of the debt. The budget constraint is given by:

$$G_t + B_t^G = \tau_{t-1}Y_t + \tau'_{t-1}Y'_{t-1} + \frac{B_{t+1}^G}{r^*}, \quad (58)$$

where  $G_t$  is government consumption and subsidy to workers,  $B_t^G$  is government debt

matured at date  $t$ ,  $\tau_{t-1}$  and  $\tau'_{t-1}$  are tax rates on production of the productive entrepreneurs and the unproductive entrepreneurs from date  $t-1$  to date  $t$ . (If the tax rates are negative, they are subsidy to production). Because workers consume all the disposable income and there is no income effect on labour supply for a given wage rate, the subsidy to workers and government consumption have the same effect on aggregate production.

Government may default. The reason people nonetheless buy government debt is that they can sell it to the agents who can use the government debt to pay their tax liabilities. Thus the private agents are willing to buy the government debt as long as the debt at the maturity date does not exceed the total tax liability:

$$B_{t+1}^G \leq (\tau_t)^+ Y_{t+1} + (\tau'_t)^+ Y'_{t+1}, \quad (59)$$

where  $(\tau_t)^+$  is the shorthand notation for  $\text{Max}(\tau_t, 0)$ . As long as this constraint is satisfied, the foreigners are also happy to buy the government debt (which they can sell to home tax payers). Thus the market interest rate for government debt is the foreign interest rate (which is no higher than the domestic interest rate).<sup>15</sup>

The private agent will not pay tax, unless the tax liability is secured by the collateral. If the entrepreneur defaults on tax liability, the government can threaten to take over his production project as the most senior outside creditor. If the government does not have

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<sup>15</sup>We ignore reputation of government here. The government debt can include government paper money, which is subject to the same constraint. (There is a tradition of monetary theory, which argues that people are willing to accept government paper money because they can ultimately use them to pay tax). Our analysis does not change by taking into accounts government paper money, because the paper money and real government bond must be perfect substitutes in order to coexists in our economy. In this paper, however, we are not going to consider circulation of intrinsically useless non-government money (such as seashells). If the foreign interest rate is positive ( $r^* > 1$ ), then there is no equilibrium in which such money circulates.

The readers should not confuse this constraint (59) with recent fiscal theory of price level, because the latter is related to equilibrium selection by using infinite horizon government budget constraint, instead of the commitment constraint of government.

bargaining power during the negotiation, then government cannot enforce tax liability more than the collateral value for the outside creditors:

$$\tau_t y_{t+1} \leq \phi \theta y_{t+1}, \quad (60)$$

where  $\tau_t$  represents a generic tax rate on the individual entrepreneur here<sup>16</sup>. Because workers do not have collateral assets, the government cannot enforce workers to pay tax<sup>17</sup>.

Because the tax liability to the government is assumed to be the most senior debt of the entrepreneur, it affects his domestic and international borrowing constraints as

$$(\tau_t)^+ y_{t+1} + b_{t+1}^* \leq \phi \theta y_{t+1} \quad (61)$$

$$(\tau_t)^+ y_{t+1} + b_{t+1}^* + b_{t+1} \leq \theta y_{t+1} \quad (62)$$

The first constraint implies that the foreign creditors will limit their loans so that the sum of the tax liability and the foreign debt repayment does not exceeds the value of collateral for the outside creditors. The second constraint says the domestic lead creditor restricts her loan so that sum of all liabilities of the entrepreneur to government and home and foreign creditors does not exceed the collateral value of the project to the lead creditor.<sup>18</sup>

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<sup>16</sup>If, instead, government has bargaining power, we can think of this condition as the assumption that the government tax is not very predatory.

<sup>17</sup>The entrepreneurs are responsible to pay payroll tax on wage income. Because of our constant returns to scale technology, the payroll tax is equivalent to the tax on production as in text.

<sup>18</sup>Here the entrepreneur cannot borrow against the future production subsidy, because the creditor who take over the project may not receive the production subsidy from the government.

The flow-of-fund constraint of each entrepreneur becomes

$$c_t + w_t l_t = (1 - \tau_{t-1})y_t - b_t - b_t^* + \frac{b_{t+1}}{r_t} + \frac{b_{t+1}^*}{r^*} \quad (63)$$

Each entrepreneur chooses quantities  $(c_t, l_t, y_{t+1}, b_{t+1}, b_{t+1}^*)$  to maximize the expected discounted utility subject to the constraints on technology, the flow-of-funds, and the domestic and international borrowing. As the results, behavior of the unproductive entrepreneurs is modified from (20) to:

$$R_t(\gamma) = r_t \geq \frac{\gamma [1 - \phi\theta + (-\tau_t')^+]}{w_t - \gamma [\phi\theta - (\tau_t')^+] / r^*}, \text{ and } L_t' \left\{ r_t - \frac{\gamma [1 - \phi\theta + (-\tau_t')^+]}{w_t - \gamma [\phi\theta - (\tau_t')^+] / r^*} \right\} = 0. \quad (64)$$

The denominator of the RHS is the downpayment for unit labour input, because, if there is tax on production (i.e.,  $\tau_t' > 0$ ), the unproductive entrepreneur can finance  $\gamma [\phi\theta - (\tau_t')^+] / r^*$  of unit labour cost by borrowing from foreigners. The numerator is the return from unit labour input after repaying debt and receiving the production subsidy (if  $\tau_t' < 0$ ). The employment of the productive entrepreneurs is modified from (22) to

$$L_t \leq \frac{\beta s_t Z_t}{w_t - \alpha [\phi\theta - (\tau_t)^+] / r^* - [\alpha(1 - \phi)\theta / r_t]}, \text{ and } \quad (65)$$

equality holds if  $R(\alpha) = \frac{\alpha [1 - \theta + (-\tau_t)^+]}{w_t - \alpha [\phi\theta - (\tau_t)^+] / r^* - [\alpha(1 - \phi)\theta / r_t]} > r_t$ .

The denominator of RHS is downpayment for unit labour input, when the productive entrepreneur borrows  $\alpha [\phi\theta - (\tau_t)^+] / r^*$  from foreigners and  $\alpha(1 - \phi)\theta / r_t$  from domestic lead creditor for unit labour cost.

The goods market clearing condition (14) can be written as

$$w_t L^s(w_t) + G_t + (1 - \beta) Z_t = Y_t + Y'_t + \frac{B_{t+1}^*}{r^*} - B_t^*, \text{ or}$$

$$w_t L^s(w_t) + \frac{B_{t+1}^G}{r^*} = \beta Z_t + \frac{B_{t+1}^*}{r^*}, \quad (66)$$

where the aggregate net worth of the entrepreneurs is defined as  $Z_t = (1 - \tau_{t-1})Y_t + (1 - \tau'_{t-1})Y'_t + B_t^G - B_t^*$ , and  $B_t^*$  is the aggregate debt repayment to foreigners by all the domestic agents (both private *and* government) at date  $t$ . Equation (66) implies the gross investment on wage bill and purchase of government bond are financed by gross saving of the entrepreneurs and gross borrowing from foreigners. The aggregate foreign borrowing of the entrepreneurs satisfies the international borrowing constraints, where the constraint is binding if the domestic interest rate exceeds the foreign interest rate as

$$B_{t+1}^* - B_{t+1}^G \leq [\phi\theta - (\tau_t)^+] \alpha L_t + [\phi\theta - (\tau'_t)^+] \gamma L'_t, \text{ equality holds if } r_t > r^*. \quad (67)$$

The extra rate of returns by the productive entrepreneur is now

$$x_t = \frac{R(\alpha) - R(\gamma)}{R(\gamma)} = \frac{\alpha [1 - \phi\theta + (-\tau_t)^+] / r_t + \alpha [\phi\theta - (\tau_t)^+] / r^* - w_t}{w_t - \alpha [\phi\theta - (\tau_t)^+] / r^* - [\alpha(1 - \phi)\theta / r_t]} \quad (68)$$

The competitive equilibrium with government policy is defined recursively by  $(w_t, r_t, x_t, L_t, L'_t, s_{t+1}, Z_{t+1}, B_{t+1}^G, B_{t+1}^*)$  as functions of the state variables  $(s_t, Z_t, B_t^G)$  that satisfy (13), (28), (29), (58), (64), (65), (66), (67) and (68), together with inequality (59) for a given policy  $(\tau_t, \tau'_t, G_t)$ .

Using the above framework, we first examine the role of government in providing liquidity. The idea is that government may be able to improve the efficiency of production,

by providing means of saving – government debt – for the unproductive entrepreneurs to save and cut down their unproductive investment. In our economy, however, the government has to tax in order to issue debt as in (59) and the borrowing capacity of the private agents are crowded out by the government tax as in (62) and (67). Thus, government role of providing liquidity may be offset by the distortion of reducing borrowing of the entrepreneurs (especially the productive ones). More specifically, concerning the steady state equilibrium of the autarky economy, we have the following proposition:

**Proposition 6** *Consider a steady state autarky economy in which the financial depth is less than threshold  $\bar{\theta}$  so that the unproductive entrepreneurs produce.*

*(i) Suppose government taxes uniformly on all the entrepreneurs in order to provide the maximum liquidity subject to the constraint of commitment (59), and that government uses the surplus to finance government consumption and lump-sum subsidy to workers. Then TFP is lower than the laissez faire economy in the autarky steady state;*

*(ii) Suppose government taxes uniformly on all the entrepreneurs in order to provide the maximum liquidity, and that government uses the surplus to finance lump-sum subsidy to all the entrepreneurs. Then, TFP is lower than the laissez faire economy in the autarky steady state.*

*(iii) Suppose government taxes only on unproductive entrepreneurs in order to provide the maximum liquidity, and that the government uses the surplus to finance government consumption and lump-sum subsidy to workers. Then, TFP is higher than the laissez faire economy in the autarky steady state.*

Proposition 6 says that government fails to improve the efficiency in production by issuing debt and uniform tax. The main reason why TFP falls by uniform tax is that the uniform tax on production hurts the productive entrepreneurs more than the



unproductive entrepreneurs, because the taxation reduces the leverage of the productive entrepreneurs. If the tax is only on production of the unproductive entrepreneurs, then the combination of such tax and the government bond issue is beneficial to TFP as in (iii) of Proposition 6. ...From the borrowing constraints of the government and entrepreneurs (59) and (62), we learn the aggregate value of gross debts of government and private agents does not exceeds the aggregate value of the collateral of the entrepreneurs. Thus, the government policy reallocates liquidity between public and private debts, instead of creating liquidity in our economy. (In contrast, in Woodford's model, there is no limited commitment of government nor the crowding out effect of tax on private borrowing, and thus the government creates liquidity which can improve the efficiency).<sup>19</sup> Roughly speaking, it is better for the small country to use the means of saving provided by foreigners (foreign liquidity) than the government debt in order to absorb saving of the unproductive entrepreneurs, when the domestic government and private agents have problems of limited commitment

If government has not much beneficial role in providing liquidity, can government mitigate the loss of the workers during the adjustment to capital liberalization? In order to explore such role, we are focusing on the economy in the intermediate level of financial depth so that the capital liberalization cause the capital outflow but the unproductive entrepreneurs still produce immediately after liberalization. In such economy, government may be able to mitigate the loss of wage and employment by subsidizing the production of the unproductive entrepreneurs.

**Proposition 7** *Consider home economy with intermediate level of financial depth,  $\theta \in$*

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<sup>19</sup>Tirole (2006) takes into account the crowding out effect of tax on private borrowing, but derives the limited commitment of the government from political economy, (rather than outright default of our economy). Perhaps Tirole's model is more applicable to a matured democratic government, while our framework is applicable to a simply opportunistic government.

$(\theta_2, \theta_3)$ , that was at the steady state equilibrium under autarky at date  $t_0 - 1$ . Suppose that the home economy liberalizes the international financial transaction at date  $t_0$ . Simultaneously, the government provides subsidy to production of the unproductive entrepreneurs, which is financed by the tax on the productive entrepreneurs without relying on debt finance.

(i) If the subsidy is small enough, then the loss of wage and total employment are smaller during the transition periods in which the unproductive entrepreneurs continue to produce, while the transition periods last longer than the laissez faire economy. Eventually, the unproductive entrepreneurs stop producing, and thereafter the adjustment is identical to the laissez faire economy except for the time lag of the adjustment.

(ii) If the subsidy is large enough to prevent completely the temporal loss of wage and employment, then the economy fails to achieve the transition to the equilibrium with efficient production.

Proposition 7(i) shows the trade-off of the government subsidy. When the unproductive entrepreneurs receive subsidy, they will pay a higher wage and employ more workers in equilibrium than the laissez faire economy. But because the productive entrepreneurs are taxed, the accumulation of the net worth and expansion of employment of the productive entrepreneurs are slower. Thus the transition to the equilibrium efficient production takes longer period than the laissez faire economy, even though the economy with small subsidy will eventually complete the transition. Proposition 7(ii) says the economy cannot achieve the transition to the efficient production, if the government tries to avoid completely the temporal loss of wage and employment by large subsidy.<sup>20</sup>

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<sup>20</sup> In fact, with such a large subsidy, the employment of the productive entrepreneurs starts shrinking as their net worth decumulates. Then the tax rate on output of the productive agents have to be higher in order to balance the budget, which leads to further decumulation of their net worth. Thus, there is a possibility that the large subsidy program may not be sustainable in the long run.

## 6 Appendix

### 6.1 Proof of Proposition 1:

From (31) and (32), we learn there are three possible types of the equilibrium:

- (i) Unproductive entrepreneurs produce ( $L' > 0$ ,  $r = \frac{\gamma}{w}$ )
- (ii) Unproductive entrepreneurs do not produce and productive entrepreneurs are credit constrained ( $r \in (\frac{\gamma}{w}, \frac{\alpha}{w})$ )
- (iii) Unproductive entrepreneurs do not produce and none is credit constrained: ( $r = \frac{\alpha}{w}$ )

Let us now examine each type of equilibrium in turn in order to derive the necessary and sufficient condition on the parameters for such equilibrium to exist.

#### 6.1.1 (i) Autarky equilibrium with inefficient production:

Because the interest rate is less than the rate of return of production on productive entrepreneurs:

$$r = \frac{\gamma}{w} < \frac{\alpha}{w}, \quad (69)$$

the productive entrepreneurs are credit constrained from (32) as:

$$L = \frac{X}{(\alpha/r) - w} \beta Z = \frac{rX}{\alpha - \gamma} \beta Z. \quad (70)$$

For employment of unproductive entrepreneurs to be positive, we need from goods market equilibrium condition (33) that:

$$wL = \frac{\gamma X}{\alpha - \gamma} \beta Z < wL^s(w) = \beta Z, \text{ or}$$

$$X < \frac{\alpha - \gamma}{\gamma}. \quad (71)$$

From (34) and (69), we learn  $x = (\alpha - \gamma)/(\gamma - \theta\alpha)$ . Thus, from (36),  $X$  solves equation (37) in the text. Because  $F(0, \frac{\alpha - \gamma}{\gamma - \theta\alpha}) < 0$ , we know  $X > 0$ , which implies from (35) that

$$r = \frac{1}{\beta(1 + X)} < \frac{1}{\beta}.$$

Thus, we verify the condition (23) that guarantee that workers do not save in the neighborhood of the steady state equilibrium. Also, we learn the condition for inefficient production (71) holds if and only if,  $F(\frac{\alpha - \gamma}{\gamma}, \frac{\alpha - \gamma}{\gamma - \theta\alpha}) > 0$ , or

$$\theta < \frac{\delta}{\frac{\alpha - \gamma}{\gamma} + (1 + n)\delta} \equiv \bar{\theta}. \quad (72)$$

From (37), both  $X$  and  $w$  are increasing functions of  $\theta$ .

### 6.1.2 (ii) Autarky equilibrium with efficient production and credit constrained productive entrepreneurs:

Here, because there is no employment of the unproductive entrepreneurs ( $L' = 0$ ) and the productive entrepreneurs are credit constrained, the equilibrium conditions (30), (32) and (33) imply

$$L^s = \frac{\beta Z}{w} = L = \frac{X\beta Z}{(\alpha/r) - w}, \text{ or}$$

$$w = \frac{\alpha}{(1 + X)r} = \alpha\beta.$$

Together with (34) and (36), we learn

$$X = \delta \frac{1 - (1 + n)\theta}{\theta}.$$

Then, we learn the productive entrepreneurs earns extra returns ( $X > 0$ , so that they are credit constrained), if and only if

$$\theta < \frac{1}{1+n}.$$

### 6.1.3 (iii) Autarky equilibrium with representative entrepreneurs:

If,  $\theta \geq 1/(1+n)$ , then we learn

$$X = 0, \quad r = 1/\beta, \quad w = \beta\alpha, \quad \text{and} \quad s = \frac{n}{1+n}$$

satisfy all the equilibrium conditions of the steady state autarky equilibrium in which none of the entrepreneurs are credit constrained. (See footnote 7). Concerning the quantities, we have  $L' = 0$  and:

$$L = L^s(\beta\alpha) = Z/\alpha.$$

*Q.E.D.*

## 6.2 Proof of Proposition 2

If (40) holds with equality, (43) can be written as:

$$\frac{\alpha(1-\phi)\theta}{r_t} \leq (1-s_t) \left( w_t - \frac{\alpha\phi\theta}{r^*} \right).$$

Along the locus of  $(w_t, r_t)$  that satisfies (40) with equality, this inequality holds for sufficiently large  $r_t$ . Also the labour market equilibrium locus is downward sloping as in *Figure 2*, and the locus of (41) with equality intersects with the locus of  $r_t = r^*$

at  $(w_t, r_t) = ((\gamma/r^*), r^*)$ . Thus there always exists an intersection between the labour market equilibrium locus and the goods market equilibrium locus *Figure 3*.

Moreover, we already showed that the labour market equilibrium locus (both the locus that satisfies (40) with equality and the locus that satisfies (41) with equality) are flatter than the locus that satisfies (43) with equality. Thus if the labour market equilibrium locus intersects with the locus of (43) with equality, it intersects only once from the northwest to the southeast. If the labour market equilibrium locus intersects with the locus of  $r_t = r^*$ , then it also intersects once. Thus, the date t equilibrium exists uniquely for a given  $(s_t, Z_t)$ .

From the labour market clearing conditions ((40), (41), (42)), and the goods market clearing conditions ((43),(44),(16)), we learn that the equilibrium of date t is one of four types described in Proposition 2, depending upon the state variables  $(s_t, Z_t)$  and the parameters. *Q.E.D.*

### 6.3 Proof of Proposition 3

We learn from Proposition 1 that  $s^A(\theta)$  and  $Z^A(\theta)$  are both increasing function of  $\theta$  for  $\theta \leq \bar{\theta}$ . Thus

$$\gamma(1 - \phi\theta)L^s\left(\frac{\gamma}{r^*}\right) < \beta r^* Z^A(\theta) \left[ 1 + \frac{(\alpha - \gamma)\phi\theta s^A(\theta)}{\gamma - \alpha\theta} \right], \text{ iff } \theta > \theta_1.$$

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Figure 1 (Autarky Steady State)

Degree of development of financial system

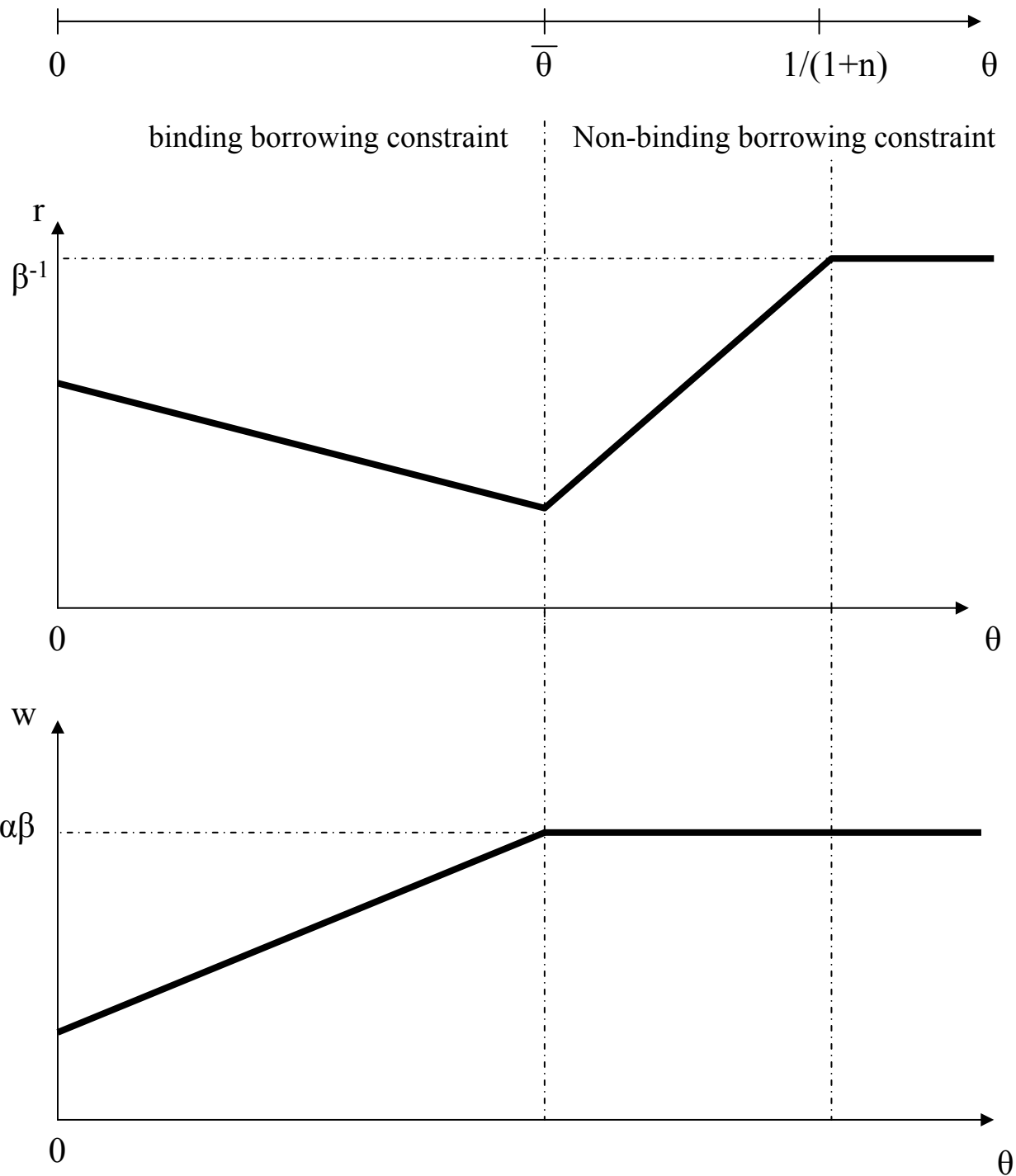


Figure 2 (Labor Market Equilibrium)

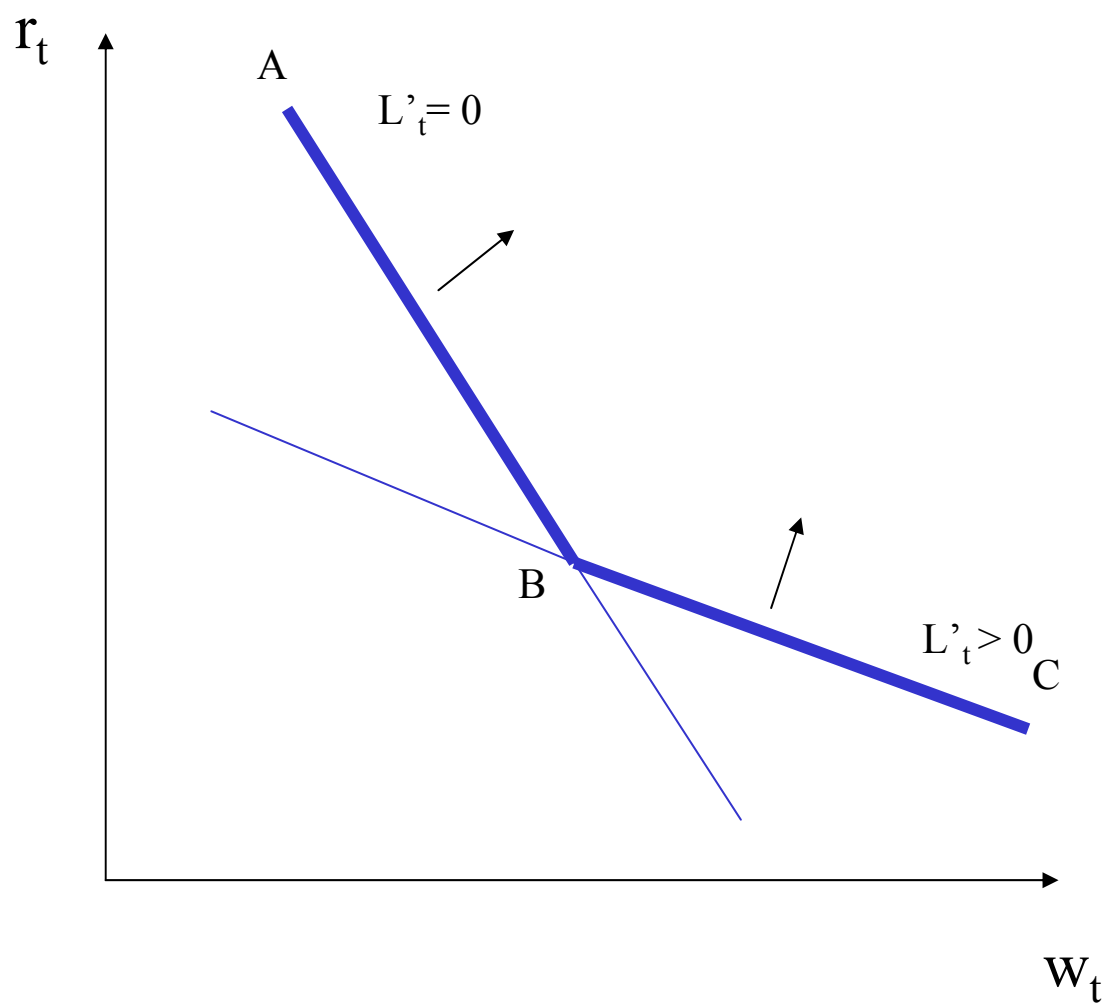


Figure 3 (Goods Market Equilibrium)

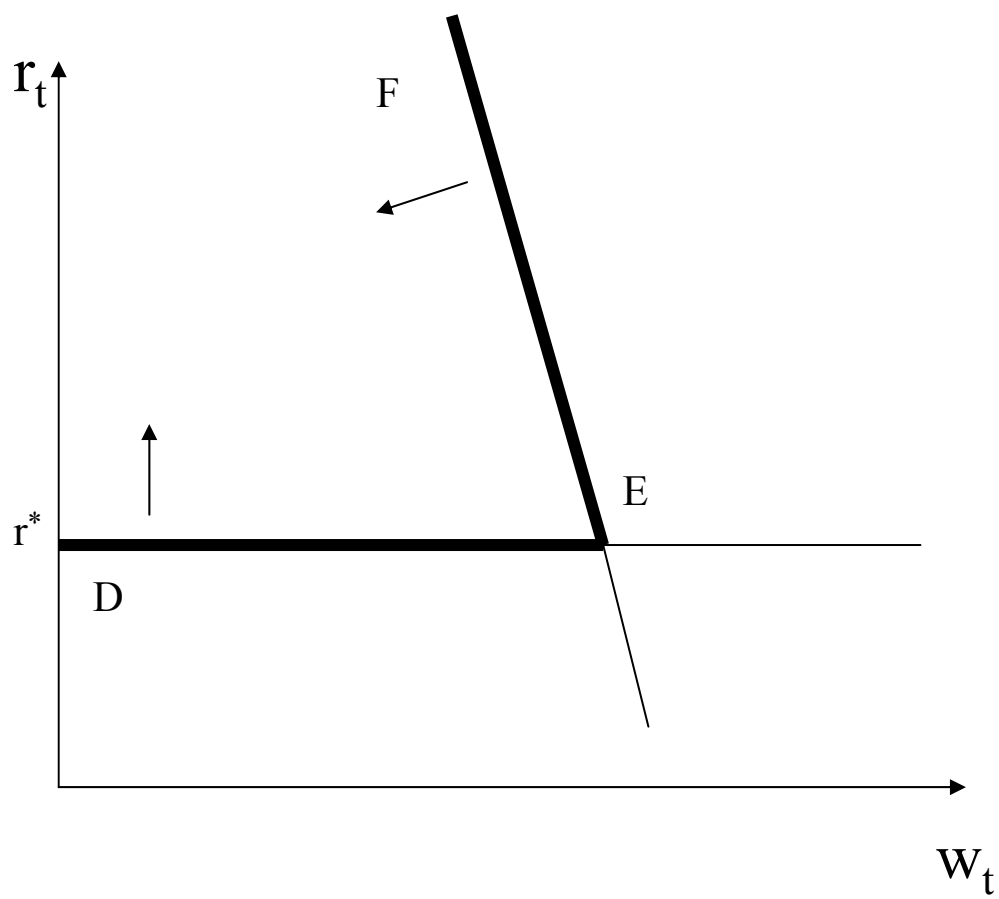


Figure 4 (Short Run Equilibrium)

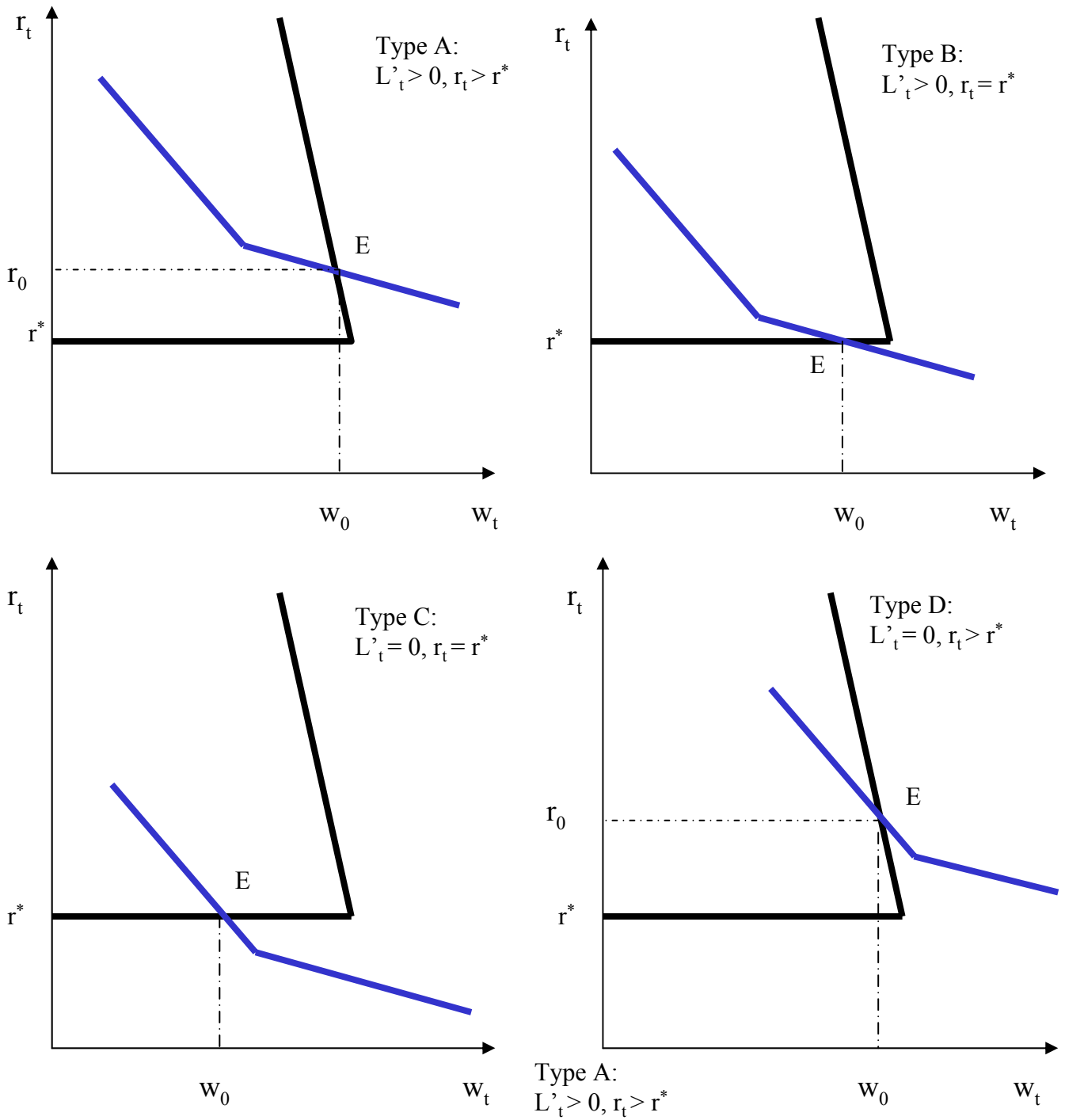


Figure 5

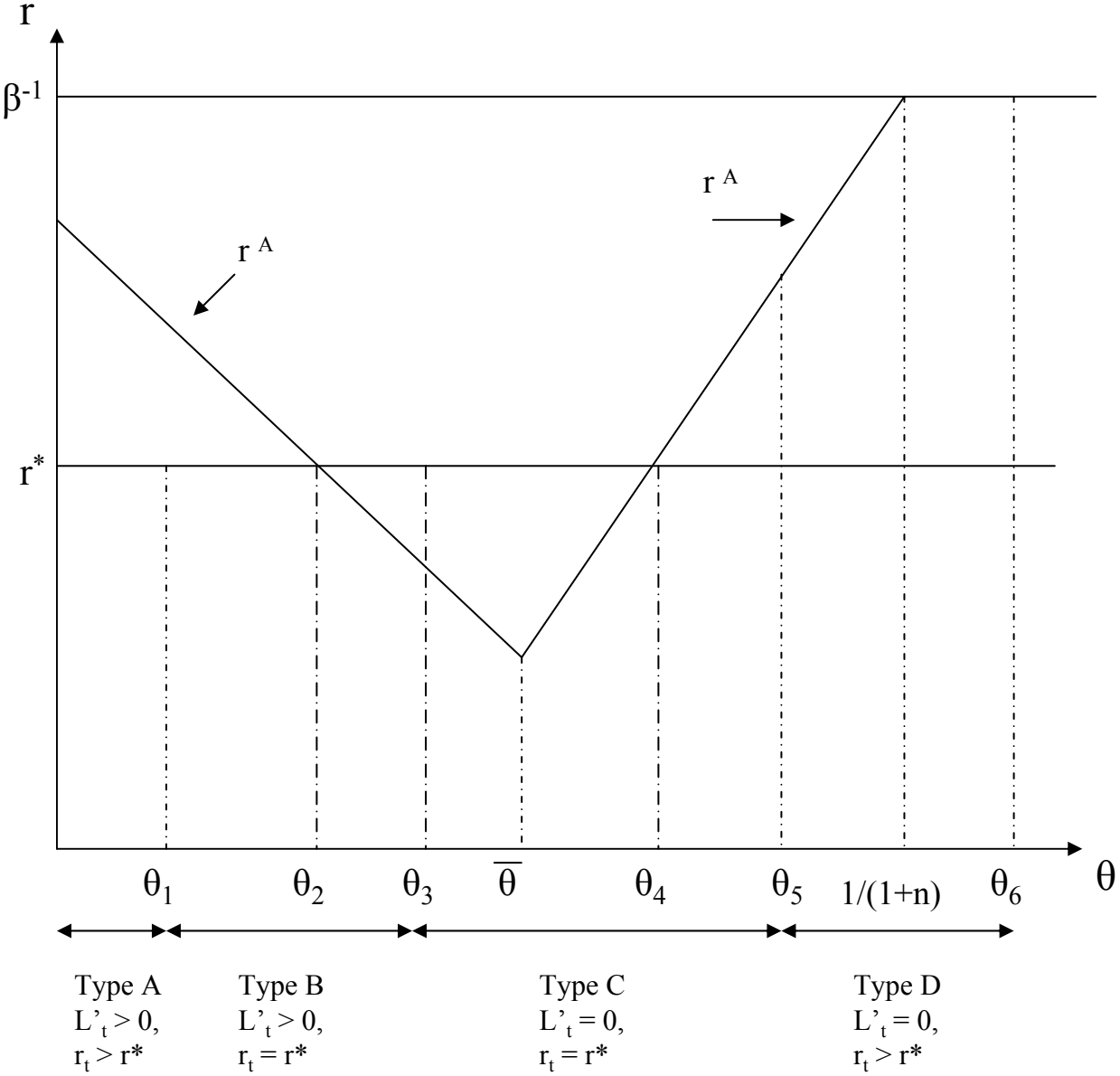


Figure 6

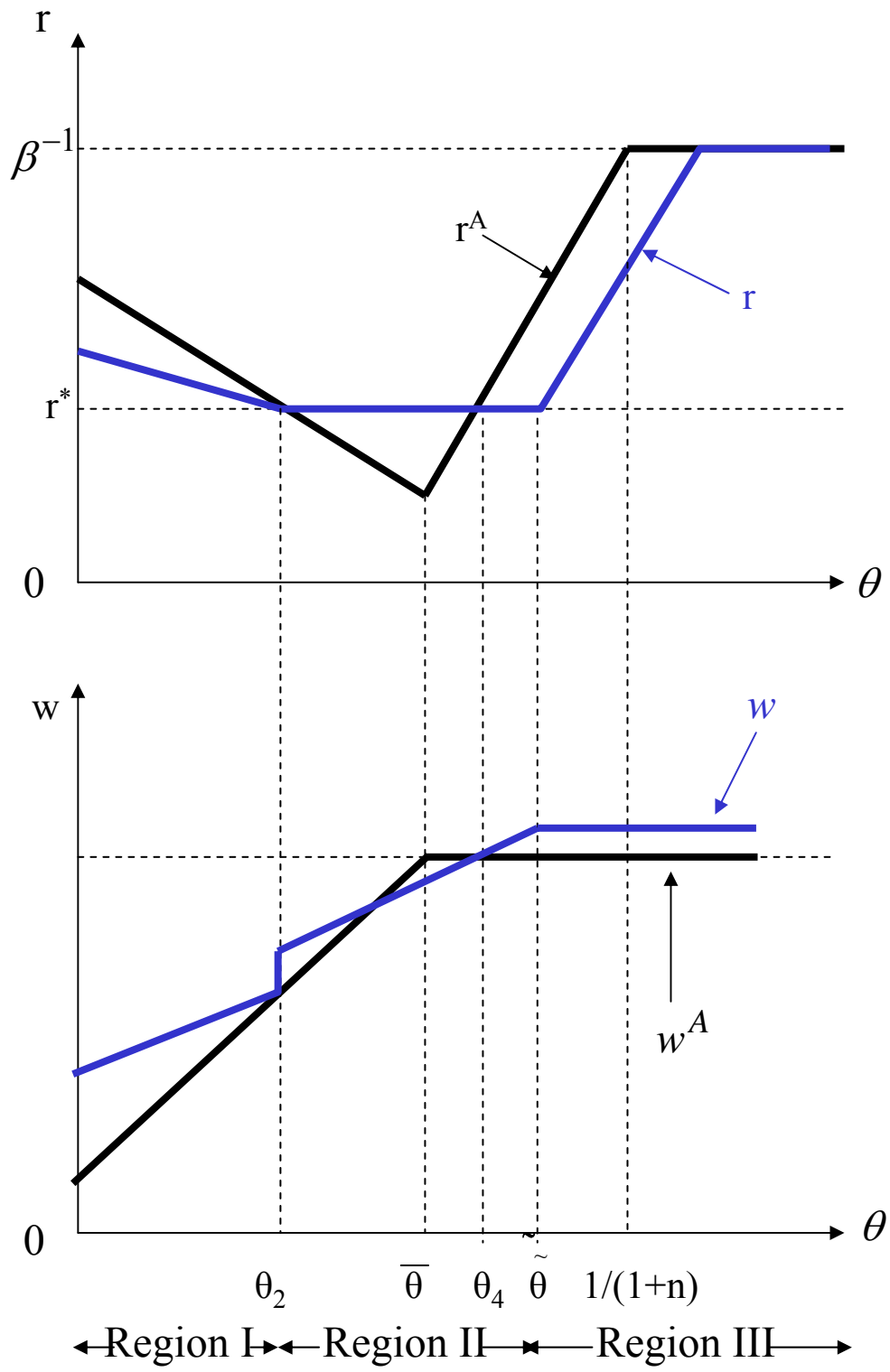
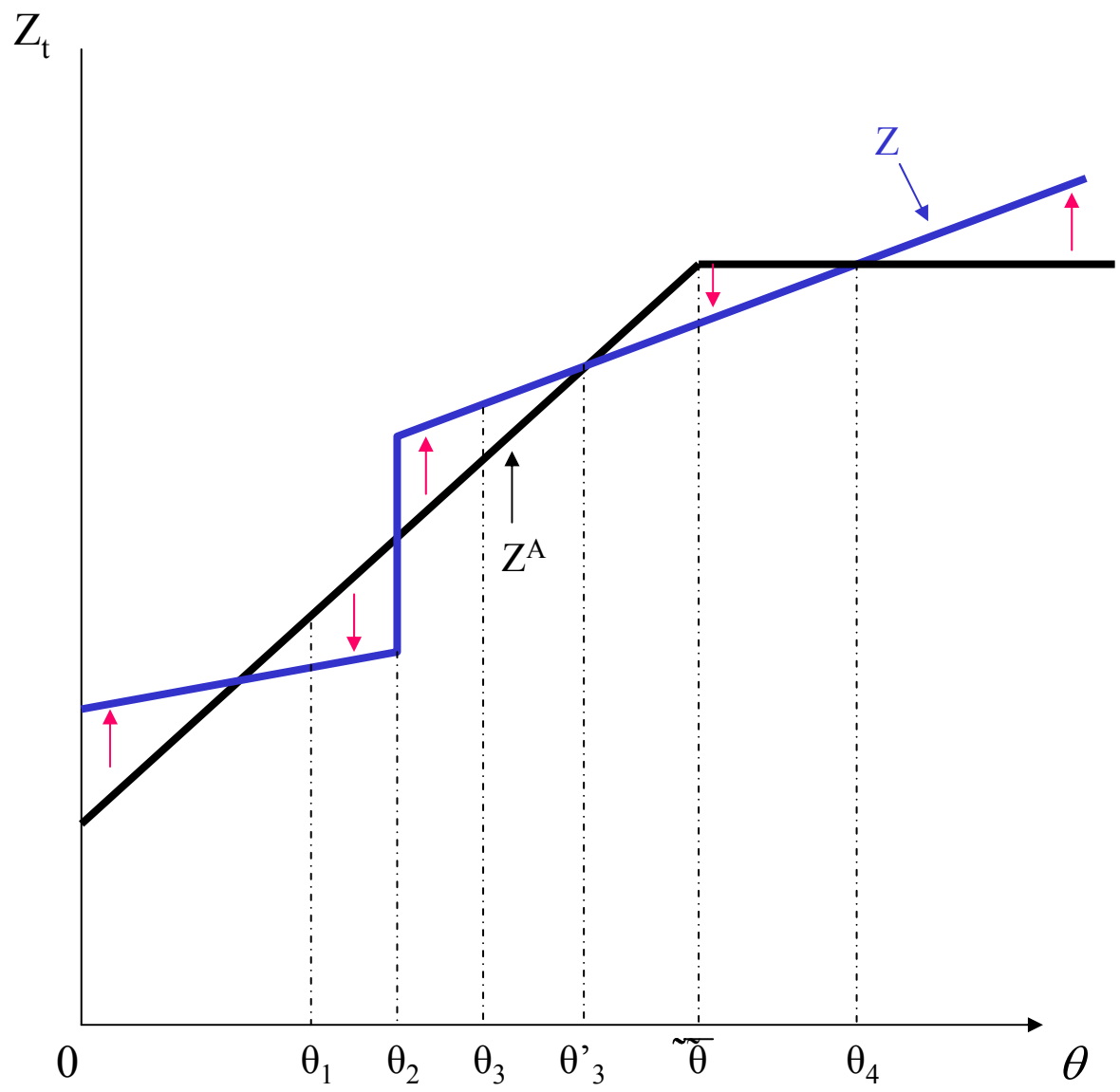
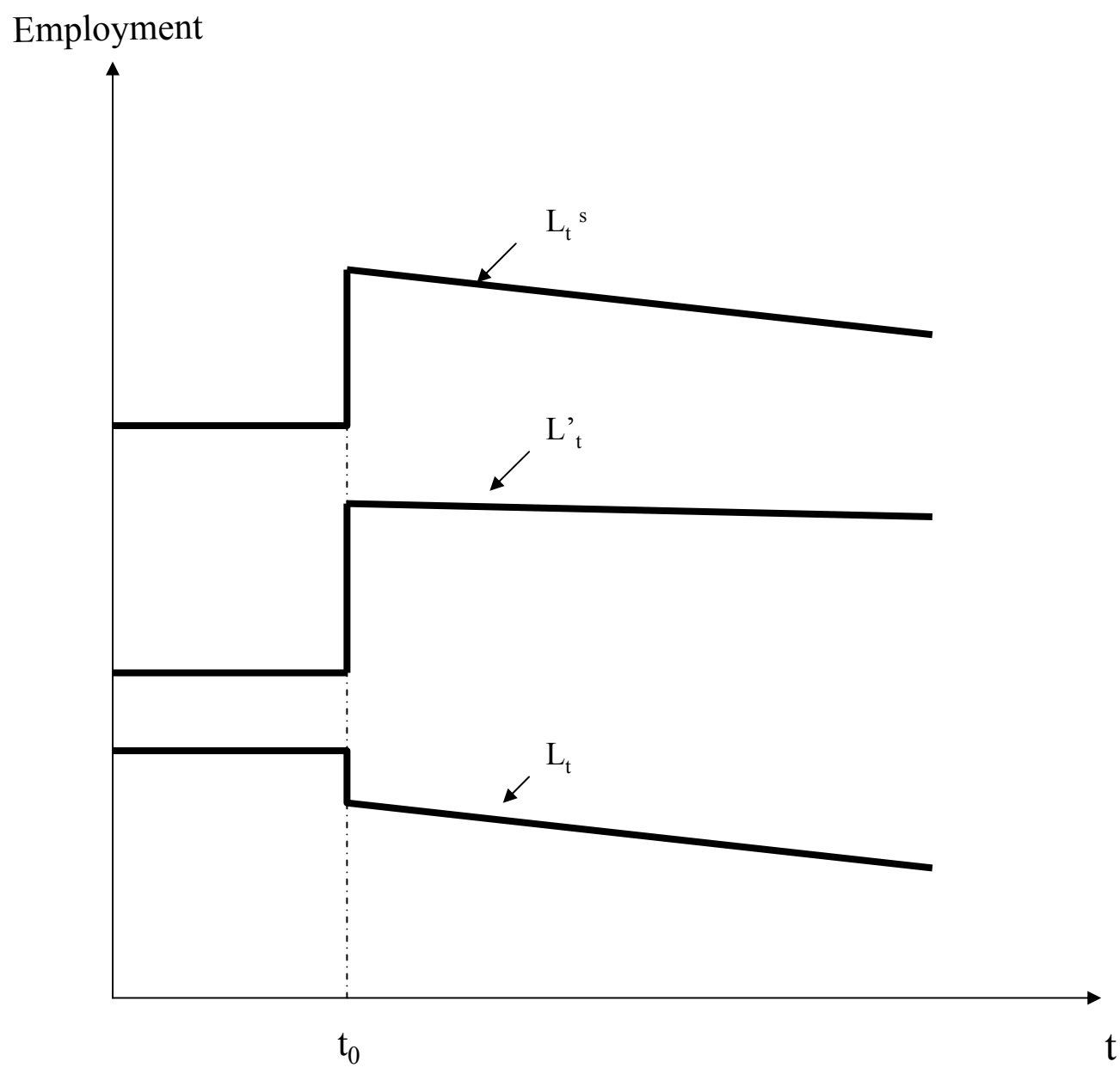


Figure 7

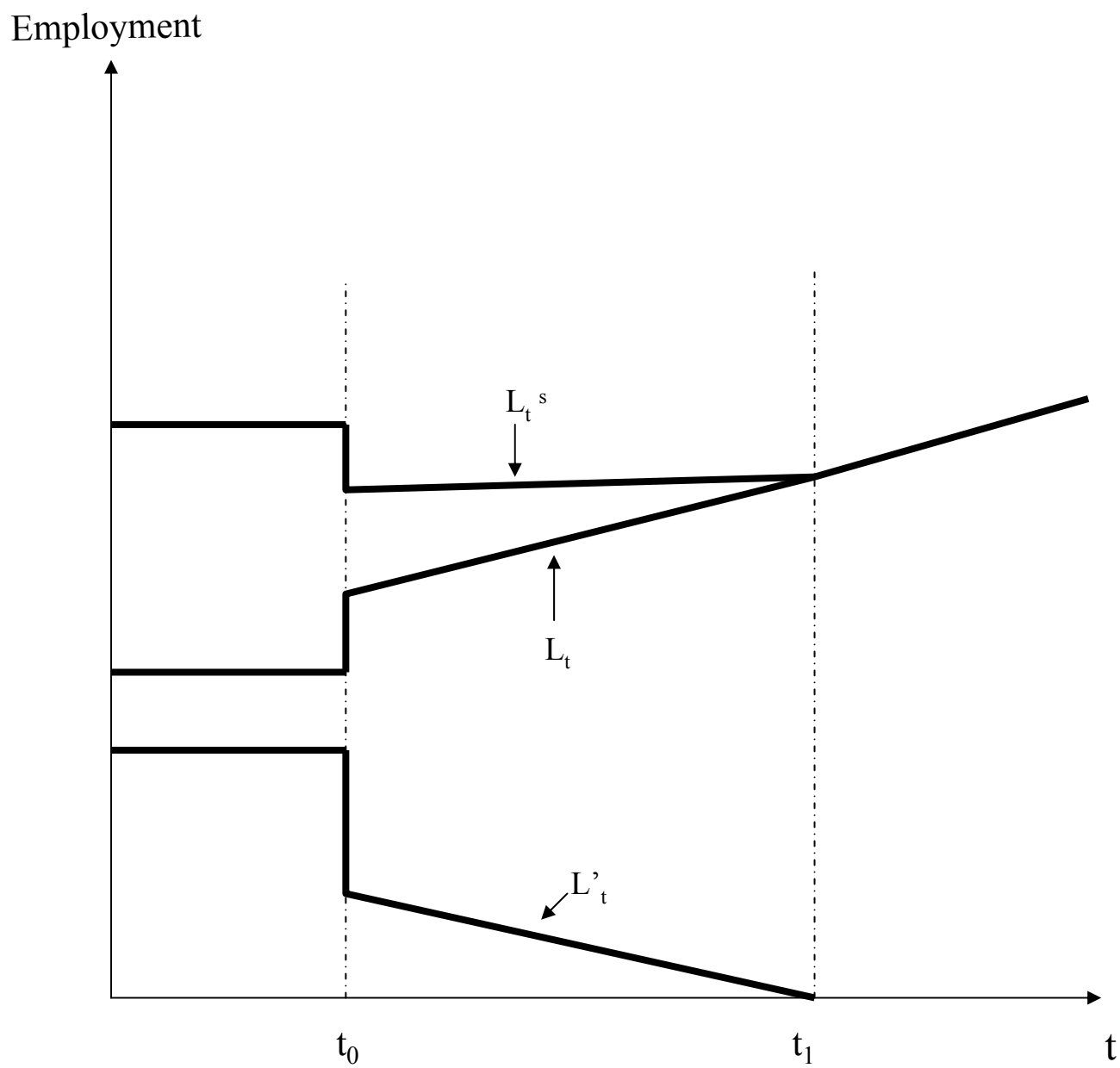


**Figure 8 (Dynamics of Employment -  $\theta < \theta_2$ )**

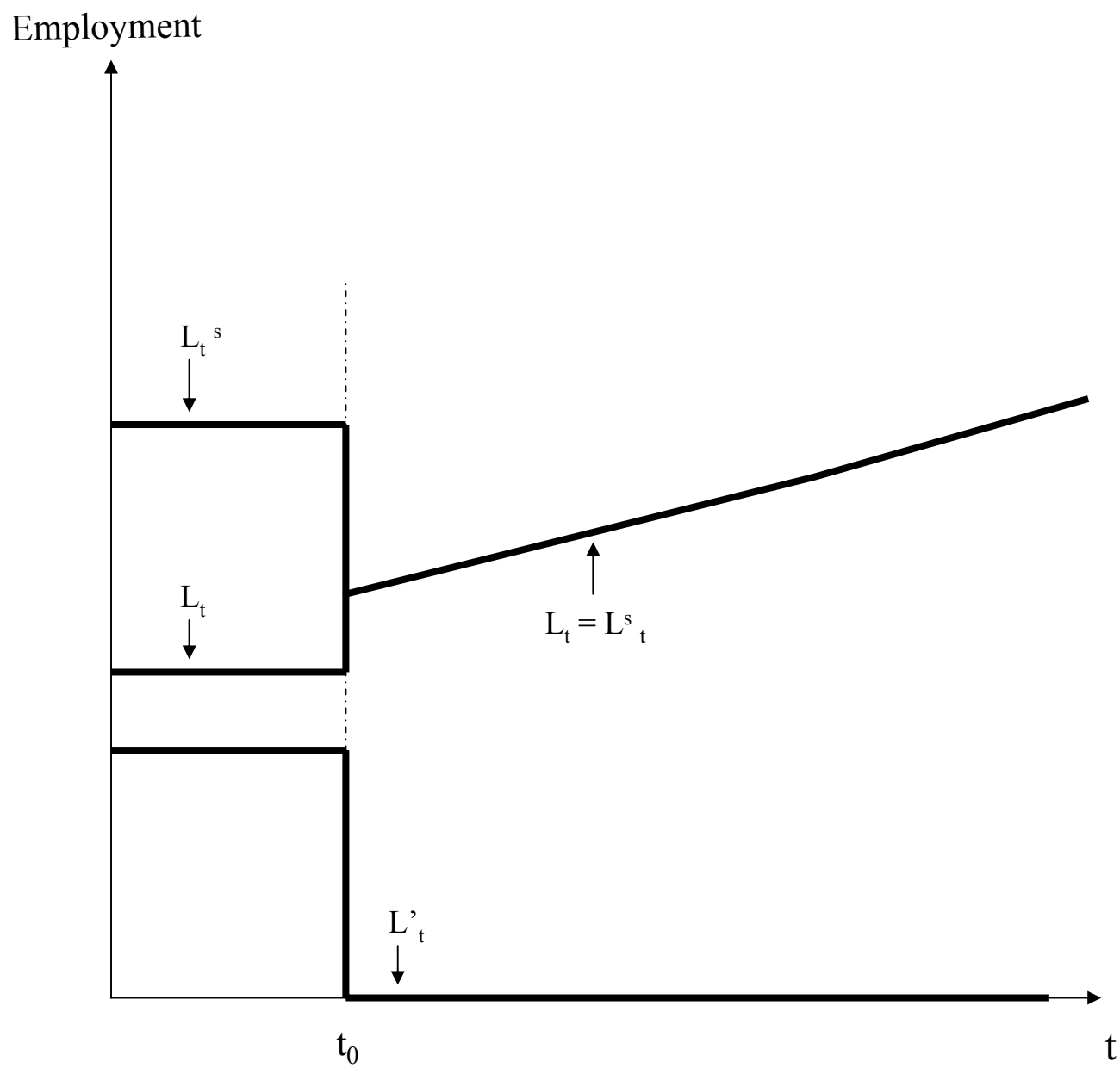




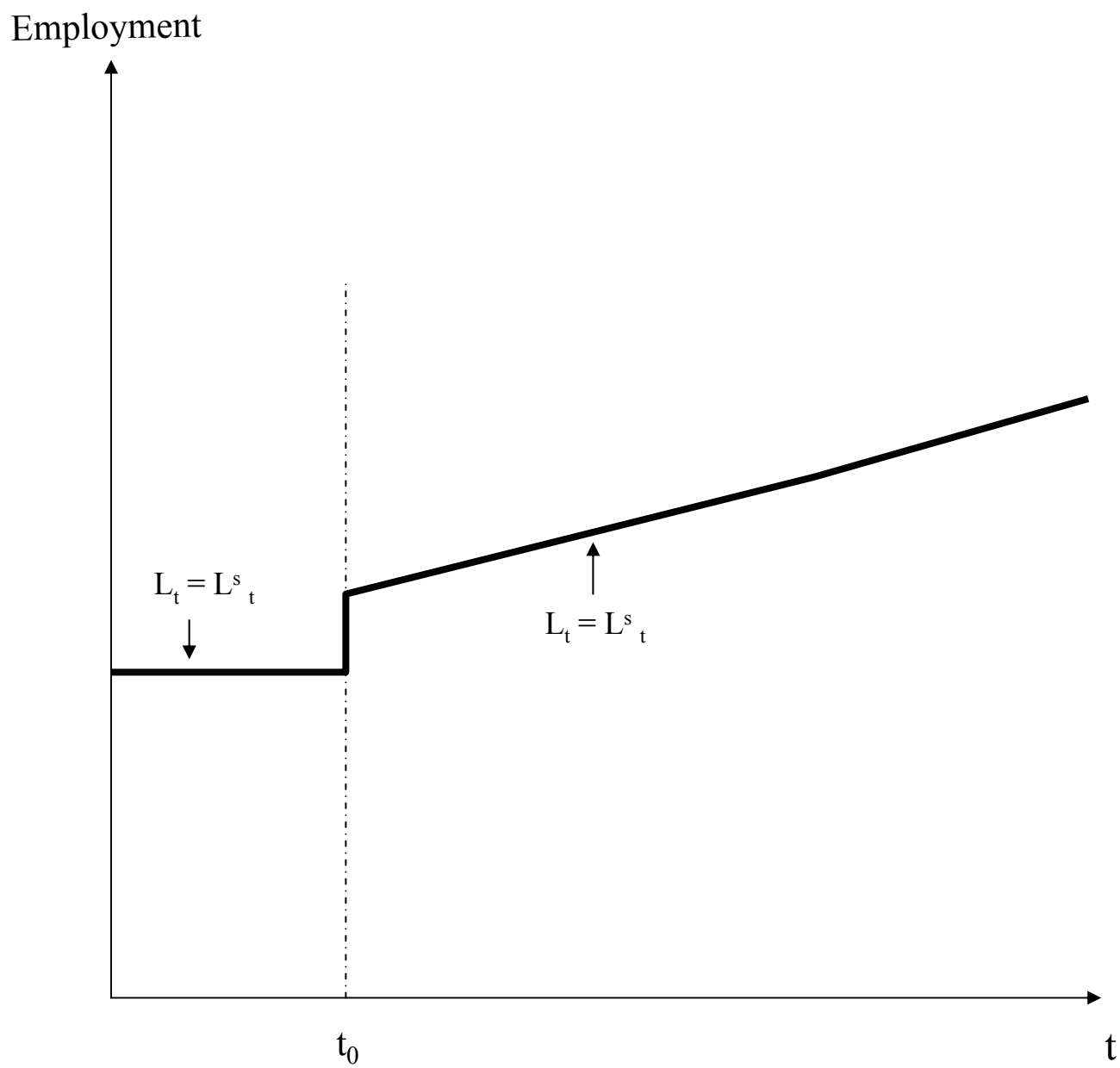
**Figure 9a (Dynamics of Employment –  $\theta_2 < \theta < \theta_3$ )**



**Figure 9b (Dynamics of Employment –  $\theta_3 < \theta < \bar{\theta}$  )**



**Figure 10 (Dynamics of Employment –  $\theta_4 < \theta$  )**



**Figure 10 (Dynamics of Employment **with** and without government policy–  $\theta_2 < \theta < \theta_3$ )**

