Paul Williams

Stanley Skewes and the Skewes number

Article (Accepted version)
(Refereed)

Original citation:
Williams, H. Paul (2007) *Stanley Skewes and the Skewes number*. Journal of the Royal Institution of Cornwall . pp. 70-75. ISSN 0968-5396

Stanley Skewes and the Skewes Number

H.P.Williams

The Skewes family (with variant spellings) is well documented in the book Cornish Heritage[1] by Keith Skues. Many originate from Cury on the Lizard peninsula. A number of farms on the Lizard, and close by, bear the name 'Skewes' and many members of the family still live in Cornwall as well as there being descendants overseas whose ancestors emigrated to metal mining regions in South and North America, Mexico, Australia and South Africa. Keith Skues remarks " The family may not have produced any kings, queens, prime ministers, statesmen or internationally known figures. It is a typical middle/working class family.". There are many interesting aspects to the places and family, from the famous siege of Skewes which took place at the Skewes farmhouse near Nancegollan in the 18th century to 'Froggie' Skewes who, within the author's memory, bred frogs for commercial purposes on Carn Brea.

What is not widely known in Cornwall is that there was a famous Skewes who is remembered for his discovery of a number which is named after him. A search of the World Wide Web for the name 'Skewes' will deliver a wealth of information about the Skewes Number.

It was discovered by Stanley Skewes who was the son of Henry and Emily Skewes. They emigrated to the Transvaal in South Africa in 1894 shortly before Stanley was born in 1899. Henry ('Harry') was born in 1873 in Cury where his ancestors were also born. He was a tin miner and went to Camborne School of Mines where he learned to be an assayer. He lived with his parents Samuel and Mary Skewes in Ford's Row, Redruth. Emily (nee Moyle) came from Redruth, living in East End, although she was born in the USA. Henry and Emily married in Redruth in 1893. (He was presented with a bible by the United Methodist Free Church ('Flowerpot Chapel') in Redruth on his departure to South Africa). Henry died in 1902 of silicosis.

After a degree in Civil Engineering at the University of Cape Town Stanley returned to England to study Mathematics at Cambridge graduating with a BA, MA and finally a PhD. He discovered what became known as the Skewes Number in 1932 and published the results in 1933 [2] and 1955 [3]. Although he returned to South Africa he revisited Cambridge frequently and must have also visited Cornwall. In his youth he was a keen rugby player. Also , although not born in Cornwall, his accent would often revert to a strong West Country one.

Among his contemporaries at King's College, Cambridge was Alan Turing who became famous for his notion of the Turing Machine (one of the first theoretical conceptions of a computer) as well as helping to break the Enigma Code and therefore helping to defeat Hitler. Skewes rowed with Turing at Cambridge. Stanley married Ena Allen, in Cambridge, who was the daughter of the head chef at Kings. She was a talented opera singer. Skewes studied for his PhD under the mathematician Littlewood. According to the *Penguin Dictionary of Curious and Interesting Numbers* the famous mathematician G.H.Hardy described the Skewes number as 'the largest number that has ever served any definite purpose in mathematics'. It is $10^{10^{10^{34}}}$. This number is truly huge. It could not be written down, in usual notation, using all the books in the world. By comparison, the number of particles in the universe is estimated at only 10^{80}, a minute fraction of the size of Skewes' Number. The number is also discussed in the book by the science fiction writer Isaac Asimov, *Skewered!, Of Matters Great and Small* [1]. It and Stanley Skewes are also cited in the 20th edition of the Guinness Book of Records in 1973. Among other honours he was elected a Fellow of the Royal Astronomical Society as a result of his other interest in Astronomy.

The Skewes Number

The number arises from studying the prime numbers. These are numbers which are not divisible by any number other than 1 and themselves (0 and 1 are not regarded as prime numbers). They are sometimes referred to as the 'building blocks of arithmetic' since any number can be factored uniquely into primes.. The first few prime numbers are:

2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,…

Many attempts have been made to find a 'formula' which will generate them but one has never been found. Although they are irregular it has long been recognised that they get less dense as one gets to larger and larger numbers. A famous result (the 'Prime Number Theorem' due to Gauss) shows that around any number x the density is about 1/log x where log x is the natural logarithm of x (the logarithm to the base e = 2.718...). So at 100 one would expect about 1 in 5 numbers to be prime but at 1000 one would expect only about 1 in 7 to be prime. This estimate allows one to work out roughly how many prime numbers there are less than any given number. One has to sum up all the densities up to the given number n. Technically this is done by what is known as an integral $\int_2^n \frac{dx}{\log x}$. The value of this expression can be calculated for different values of n and is given in the table below together with the actual number of prime numbers up to n.

n	Number of Primes less than n	$\int_2^n \frac{dx}{\log x}$
1000	168	178
10000	1229	1246
50000	5133	5167
100000	9592	9630
500000	41538	41606
1000000	78498	78628
2000000	148933	149055
5000000	348513	348638
10000000	664579	664918
20000000	1270607	1270905
90000000	5216954	5217810
100000000	5761455	5762209
1000000000	50847534	50849235
10000000000	455052511	455055614

It can be seen that the integral gives a remarkably close approximation to the number of primes as n gets large. In fact, the ratio of the integral to the actual number approaches 1 as n approaches infinity.

Notice, however, that the estimate is always an overestimate. It remains an overestimate for astronomically large values of n. It was thought for a long time that this would always be the case until it was proved by Littlewood that it would eventually become an underestimate (but then switch back again and alternate between an underestimate and overestimate an infinite number of times). Skewes showed that it must switch to being an underestimate before n reaches $10^{10^{10^{34}}}$. This is the

Skewes Number. In fact, his proof depends on assuming that one of the most famous conjectures of mathematics, the Riemann Hypothesis, is true. If the Riemann Hypothesis were not true then it would still become an underestimate before some even larger number, sometimes known as Skewes Second Number. This is $10^{10^{10^{1000}}}$ **.**

This demonstrates, among other things, the fallacy of assuming that because something is true a very large (in this case astronomical) number of times it will always be true!

Since Skewes proved these results his number has been reduced in size but $10^{10^{10^{34}}}$ remains known as the Skewes Number.

A memorandum, written by Skewes on his retirement, which discusses the Skewes Number and further developments is kept in a glass case in the Mathematics Department of Capetown University where he worked as a lecturer then a professor. He died in 1988 at the age of 89 .

References

1.Isaac Asimov, *Skewered!, Of Matters Great and Small*, Ace Books, New York 1976.

2.Stanley Skewes, On the Difference π (x) – Li (x), *Journal of the London Mathematical Society,* **8**, 227-283,1933.

3.Stanley Skewes, On the Difference π (x) – Li (x) II, *Proceedings of the London Mathematical Society,* **5**, 48-70, 1955.

4.Keith Skues, Cornish Heritage, Werner Shaw, London 1983.

5.David Welles, The *Penguin Dictionary of Curious and Interesting Numbers,* Penguin Books, London, 1997.

Acknowledgements

The author would like to acknowledge Professor Brian Griffiths who first brought the Skewes Number to his attention, and the help of Dr Kenneth Hughes of Capetown University. Mr Stephen Skewes, his son, has provided much useful information including a copy of Skewes original paper and a copy of the front page of the ‘family bible’ from Redruth now owned by Stephen.

The Author

Paul Williams is a professor at The London School of Economics. He was born and brought up in Cornwall and went to Redruth Grammar School, then read mathematics at Cambridge University. He has a house near Helston and is a member of The Royal Institution of Cornwall. His great grandmother Edith was a Skewes. He believes that Stanley Skewes’ great great great grandparents were also his great great great great great grandparents. They were farmers at Treloskan in Cury.