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THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

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Abstract

This note explores the difference between QALYs gained and DALYs averted in estimates of health benefits from interventions, where DALYs are estimated using local life expectancy tables. I assume that disability weights in the DALYs framework correspond to quality adjustment weights in QALYs, that there is no age weighting and that both frameworks use the same discounting methodology. I find that for the same intervention, health benefit measured as a reduction in DALYs is always smaller than the same benefit measured as a gain in QALYs. The higher the age of deaths prevented by the intervention, and the lower the quality of life in the years of life gained, the bigger the difference between DALYs and QALYs. The difference is reduced when benefits are discounted. I show that the difference can lead to a different ranking in cost-effectiveness league tables based on DALYs averted compared to gains in QALYs. I conclude that the use of the DALY framework based on local life expectancy tables might be appropriate for estimating the total burden of disease, but leads to troubling results if used for cost-effectiveness analysis. The use of a fixed reference age would avoid those implications, but might not be a reasonable assumption for estimating the total burden of disease.

Acknowledgments

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Contents

1 Background

Quality-adjusted life years (QALYs) and Disability-adjusted life years (DALYs) are currently widely used in the economic evaluation of health. Although in their original formulations the methodological differences were important (Drummond et al., 2005, Gold et al., 1996, Gold et al., 2002, Fox-Rushby, 2002). In the light of subsequent developments, Morton has asked whether these differences are mainly semantic (Morton, 2007). Sassi analysed the difference between estimates of gains in QALYs and DALYs averted maintaining age weighting as in the original formulation of DALYs but using continuous discounting for both approaches (despite the fact that the original QALYs formula uses discrete time intervals) and assuming that the loss in quality of life associated with a disease in the QALY framework corresponded to the disability weight in the DALY one (and later relaxed this assumption) (Sassi, 2006). As Morton pointed out, however, the use of age weighting is currently considered discretionary among users of the DALY approach and it seems that once age weighting, differences in discounting and differences between loss in quality of life and disability weighting are set aside, the two frameworks do not differ substantially (Morton, 2007). In this technical note I explore a comment made by Aki Tsuchiya on Morton's 2007 paper[1]. Tsuchiya pointed out that although developments have aligned the two approaches, as DALYs are usually estimated using residual life expectancy at the time of death, there is a difference between gains in QALYs and DALYs averted by the same health intervention.

In this paper I explore how fundamental this difference is. The paper begins with an illustrative example and then considers three questions:

1. In general how large is the difference in benefits when measured in DALYs and QALYs and what are its main determinants?
2. What happens to the difference if we include discounting of future outcomes?
3. Would the difference lead to contrasting ranking in league tables based on DALYs averted compared to QALYs gained?

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[1] Aki Tsuchiya discussed Morton's paper at the Health Economists' Study Group at the University of Birmingham in January 2007.

The paper is not a general discussion of the most appropriate measure of health outcomes, nor an analysis of the limitation of these techniques. Rather, it analyses the two most commonly used measures of health outcomes to test whether there is a substantial difference between them after having set aside any dissimilarity in discounting method (which is stylistic), age weighting (which is currently discretionary) and divergences between the sets of weights representing quality of life in QALYs and disability weights in DALYs (which belong to the separate issue of measuring preferences). The intervention used in carrying out the comparison is a single year intervention, a context in which the use of measures such as DALYs with local life tables is recommended (Tan-Torres Edejer et al., 2003, Preston, 1993).

Consider a person who lives until the age of 35 with a disease and then dies. We can represent her health over time using the QALY approach (Drummond et al., 2005, Gold et al., 1996) on a graph with time (age) on the horizontal axis and quality of life on the vertical axis, where quality of life varies between 0 (i.e. death) and 1 (i.e. full health) (see Figure 1). If we assume that the disease is associated with a quality of life lower than full health - say 0.8 - then the health profile of that person is h_0 in the figure. The QALYs of this health profile are the number of years lived, multiplied by their quality and are graphically represented by the area below h_0, which is 28 QALYs (35*0.8). Imagine now an intervention that could save her from death and extend her life by 30 years, but would leaver her with a more severe disability with a quality of life of 0.5. The new health profile can be drawn and is represented by h_1 in the figure, which corresponds to 43 QALYs (35*0.8+30*.5). The health outcome of the intervention can then be thought as the difference between the QALYs of h_1 and those of $h_{0;}$ that is, 15 or the years of life gained weighted by their quality (30*0.5).

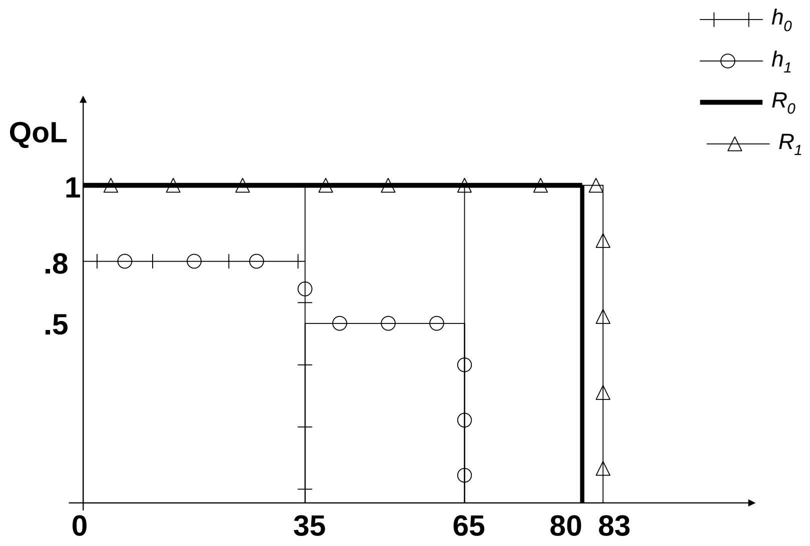

Figure 1. QALYs gained vs. DALYs averted. A numerical example.

On the same graph we could also represent the DALYs (Murray, 1996) associated with the two health profiles and evaluate the intervention in terms of reduction in the burden of disease. Although the DALY approach is a population-based method, in this paper we consider an individual as a special case of a population or as a summand in the estimate of the health outcome at the population level (Gold et al., 2002, Morton, 2007, Sassi, 2006). The burden of disease is the difference between an ideal health profile, which typically corresponds to living in full health for the rest of one's life, and the actual health profile. In studies that evaluate a one-year intervention, the use of local life tables and of residual life expectancy at the age of death to quantify the 'rest of one's life' is recommended (Murray, 1996, Preston, 1993). In the example of Figure 1, the ideal health profile in the absence of the intervention is R_0, that is living in full health for the residual life expectancy at age 35 in the population (45 years according to recent life tables for England (GAD, 2006); that is, until the age of 80). The burden of disease associated with health profile h_0 is then 52 DALYs (35*(1-0.8)+45). The selected intervention, extending the person's life until age 65, shifts the ideal health profile to R_1, that is living in full health for the residual life expectancy at age 65 in the population (18 years according to the life table used above, that is until 83 years old); the burden of

h_1 is 40 (35*(1-0.8)+30*(1-0.5)+18) and the DALYs averted through the intervention are therefore 12, which is lower than the QALYs gained which we estimated to be 15.

I begin the analysis with a simple model of mortality and a life-extending intervention, initially without discounting and then including a positive discount rate r. I then briefly consider a model with changes in morbidity only and move to the more complete model of an intervention that affects both morbidity and mortality.

2 Measuring changes in mortality

2.1 Without discounting (r=0)

Consider an individual who will live in full health and die at age x; now imagine an intervention that could save her from death and extend her life by k years, maintaining her in full health. The gains in QALYs associated with the intervention are graphically represented by area K in Figure 2; that is, k as shown by (1). The same health outcome measured as a reduction in DALYs is the difference between area K and area G in Figure 2 and formulated in (2). Graphically it is easy to see that the use of QALYs or DALYs gives a different measure of the health benefit from the intervention and that this difference is area ,G which is measured by (3). G is the difference between the age a person is expected to die if she is ($x+k$) years old and the age she is expected to die when she is x years old. As we discuss in the Appendix, $G \geq 0$ always.

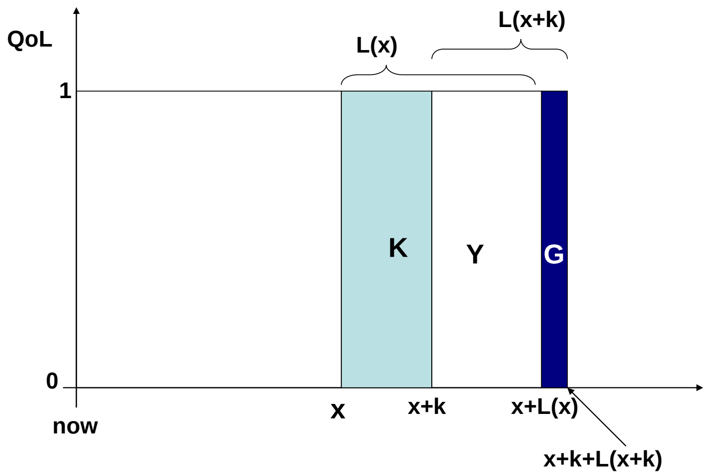

Figure 2. Health profiles with change in mortality only and no discounting.

(1) $\Delta QALYs = (x+k) - x = k$;

(2) $\Delta DALYs = L(x) - L(x+k)$;

where $L(x)$ is the residual life expectancy of a person aged x in the local general population.

(3) $G(x, k) = \Delta QALYs - \Delta DALYs = k - L(x) + L(x+k)$;

The discrepancy, *G,* can be estimated numerically by varying parameters x and k. For $L(\cdot)$ I used the life expectancy data for the male English population (GAD, 2006), varied the age of death in the absence of the intervention, x, between 0 and 100, and varied the years of life added by the health intervention, k, between 1 and the residual life expectancy at age x, assuming that an intervention able to extend life beyond the life expectancy in the population is not feasible, on average. Using life tables for England, where infant mortality is relatively low, we also expect $G(x, k) \leq k$ (see discussion in the Appendix).

Results for a small selection of k values are shown in Figure 3, where the x-axis is the age of death without the intervention. As expected, $0 \leq G(x, k) \leq k$. The discontinuity between age 0 and 1 is due to the massive impact of mortality rates between age 0 and 1 on the calculation of residual life expectancy.

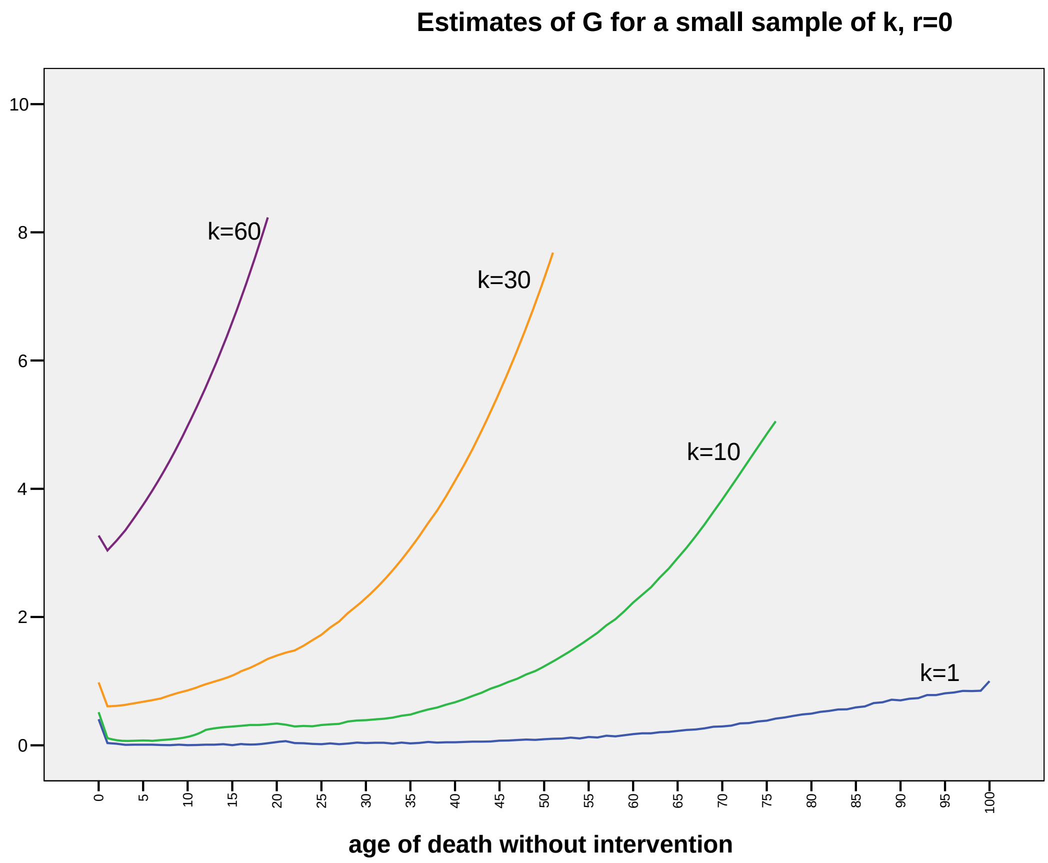

Figure 3. Estimates of G for a small selection of *k* values, no discounting. The age of death without the intervention (x) is reported on the x-axis. The age of death with the intervention is *x*+*k*.

The relative underestimate of health benefits as measured in DALYs compared to QALYs can be expressed by a ratio, α, as in (4). The smaller the difference, the closer α will be to 1. The bigger the difference, the smaller the value of α.

$$(4) \qquad \alpha = \frac{\Delta DALYs}{\Delta QALYs} = \frac{L(x) - L(x+k)}{k}; \qquad 0 \le \alpha \le 1$$

Figure 4 reports the results for α, where each line represents a different *k* (from 1 to 76, i.e. the maximum length of life added by an intervention is equal to life expectancy at birth) and where higher lines correspond to smaller *k* (with the exception of deaths avoided at birth). Figure 4 shows that α decreases both with x and k, with discontinuities for death at birth or in the first year of life.

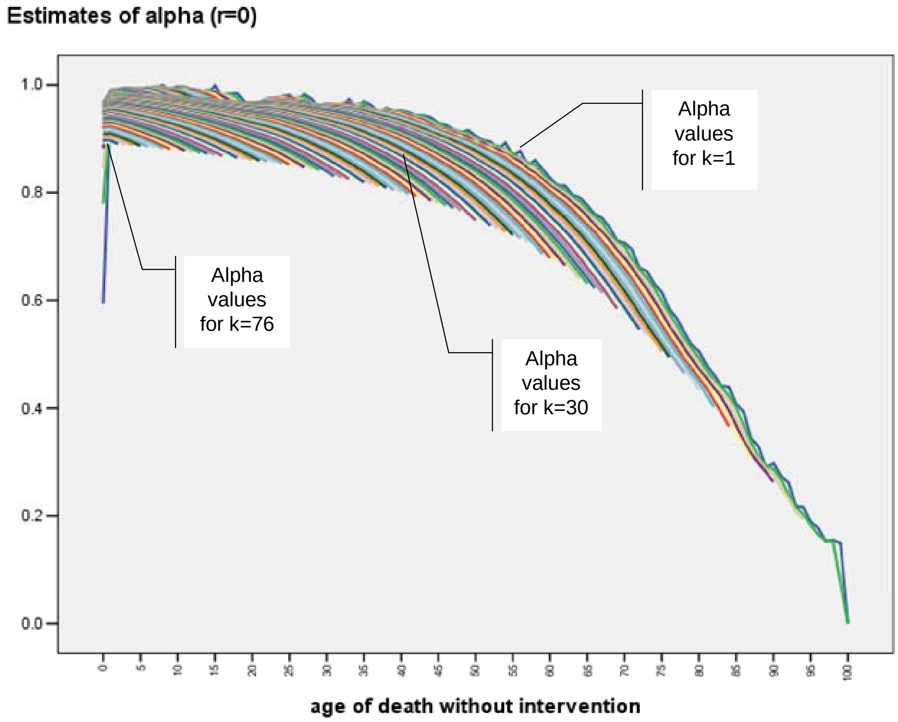

Figure 4. Estimates of alpha, no discounting.

α is the ratio between DALYs averted and QALYs gained by virtue of the intervention and is on the vertical axis. Each line corresponds to a different value of k, the years of life added by the intervention.

2.2 With discounting

Discounting affects the results in two ways. First, it reduces the life expectancy of young people proportionally more than that of older people (because a larger proportion of their residual life expectancy is in a more distant future). Secondly, the more the life years gained (k), the later the death under the intervention scenario, i.e. the later its burden will manifest and the smaller its discounted value (Figure 5).

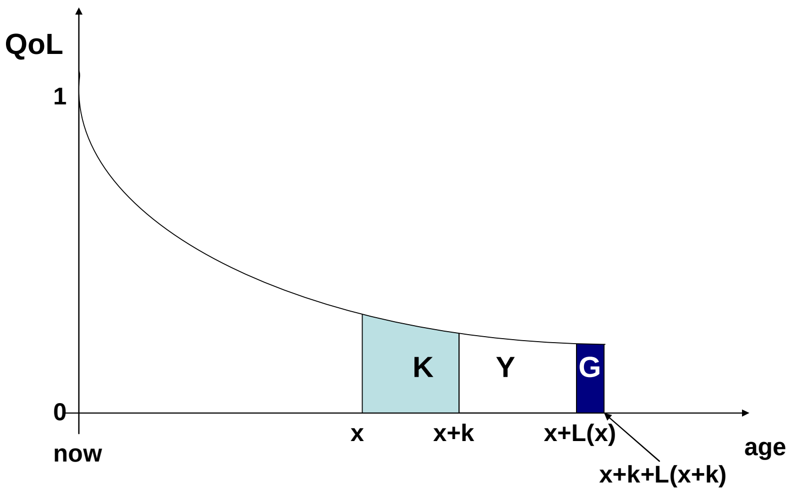

Figure 5. Health profiles with change in mortality only and with discounting.

For notational simplicity, and without any loss of generality, we may assume that in the absence of the intervention, the individual affected by the disease will die in the current year at age *x,* whereas under the intervention scenario he will die *k* years from now at age *x*+ *k*.

I estimated the gains in DALYs and reduction in DALYs varying the parameters *k* and *x* as before and I used exponential discounting for both DALYs and QALYs. The ratio between the two measures with exponential discounting is given in (5). Note that this is a generalisation and corresponds to (4) for *r*=0.

$$(5) \qquad \alpha = \frac{\Delta DALYs}{\Delta QALYs} = \frac{\int_0^{L(x)} e^{-rt}dt - \int_k^{k+L(x+k)} e^{-rt}dt}{\int_0^k e^{-rt}dt}$$

The results are shown in Figure 6, with a discount rate of 3.5% (the rate recommended for economic evaluations in the United Kingdom (NICE, 2006)). As expected, discounting reduces the difference between the two measures; that is, for any *x* and *k*, α is now closer to 1 (in terms of Figure 5 this is the ratio between area *K*-*G* and area *K*:

discounting reduces *G* proportionally more than *K*, and hence increases the ratio of *K*-*G* to *K*).

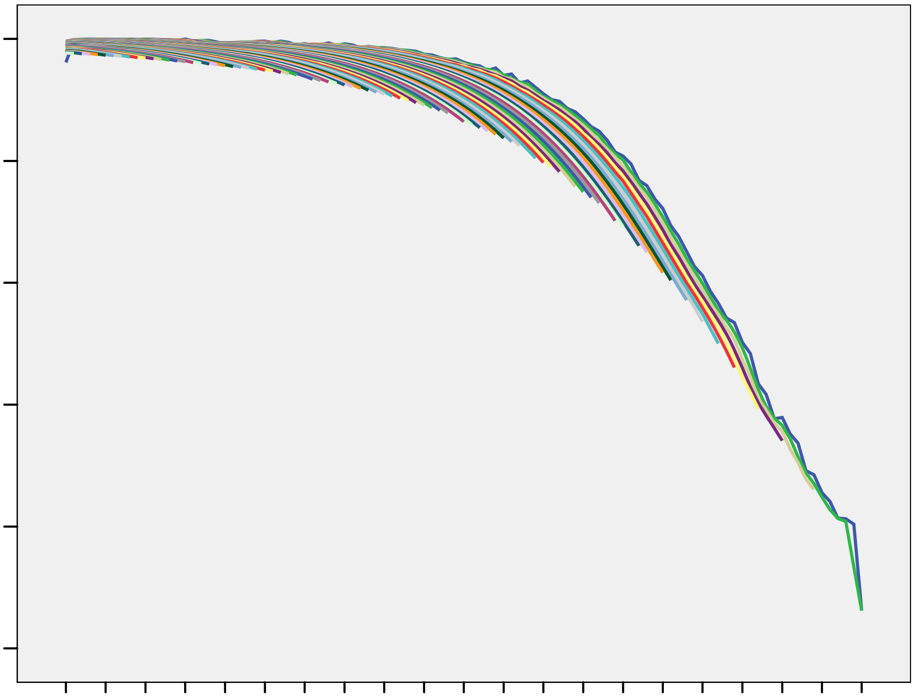

Figure 6. Estimates of alpha discounting at 3.5%.

Table 1 gives values of α for different values of *x* and *k*, with and without discounting. The table shows that, in general, the difference between DALYs averted and QALYs gained:

- increases (i.e. α becomes smaller) with the age of death without the intervention;
- increases with the length of life added by the intervention (*k*); and
- decreases (i.e. α is closer to 1) if future outcomes are discounted.

Table 1. Values of α for a selection of ages at death without the intervention (*x*), length of life added by the intervention (*k*) and discounting.

x	No discounting (*r*=0)			With discounting (*r*=3.5%)		
	k=1	*k*=10	*k*=30	*k*=1	*k*=10	*k*=30
5	.99	.99	.97	.999	.999	.996
30	.96	.95	.91	.99	.99	.975
65	.74	.67	n/a	.85	.80	n/a
90	.25	n/a	n/a	.34	n/a	n/a

3 Measuring changes in morbidity

The quality of life of an individual is denoted by $\lambda\in[0,1]$ where λ=0 corresponds to death and λ=1 to full health and we assume in the DALY framework the corresponding disability weight is 1-λ.

Consider an individual of age *x* who will live for *k* years with a quality of life $\lambda_0\in(0,1)$ and die at age $x+k$. Figure 7 illustrates his health profiles for an intervention that improves his quality of life to $\lambda_1\in(\lambda_0,1)> \lambda_0$ but has no effect on his life expectancy. Both the gain in QALYs and the reduction in DALYs corresponds to area *Q;* that is, there is no difference between the two measures as also shown in (6) and (7).

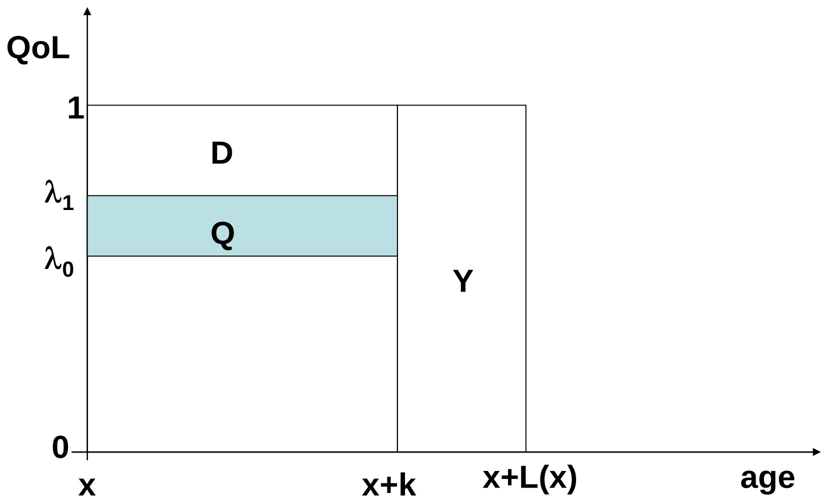

Figure 7. Health profiles with change in morbidity only, no discounting.

(6) $$\Delta QALYs = k*(\lambda_1 - \lambda_0)$$

(7) $$\Delta DALYs = (1-\lambda_0)*k + L(x+k) - (1-\lambda_1)*k - L(x+k) = k*(\lambda_1 - \lambda_0)$$

Including a positive discount rate will have the same effect on the two measures, which will therefore remain the same.

The next section considers the ratio α for interventions that have an impact on both life expectancy and morbidity.

4 Measuring changes in mortality and morbidity

4.1 Without discounting

Figure 8 illustrates the health profile of a person who will live until age *x* with a quality of life of λ_0 and an intervention that could extend her life by *k* years, during which her quality of life will be $\lambda_1<\lambda_0$. The gains in QALYs graphically correspond to area *A* in Figure 8 and are equal to (8). The same health outcome measured with the DALYs approach corresponds to the difference between area *A* and area *G* in the figure as shown by (9).

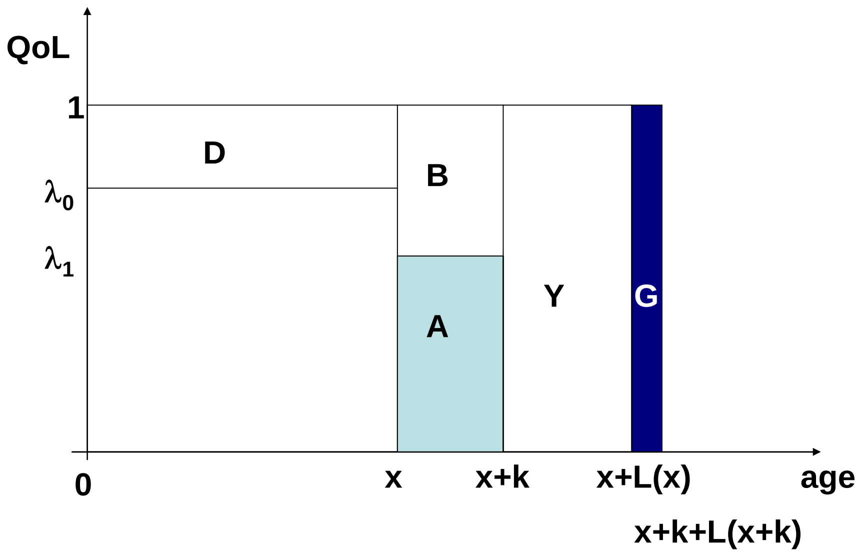

Figure 8. Health profiles with changes in mortality and morbidity. No discounting.

(8) $$\Delta QALYs = k * \lambda_1;$$

(9) $$\Delta DALYs = L(x) - L(x+k) - k*(1-\lambda_1);$$

(10) $$\Delta QALYs - \Delta DALYs = k + L(x+k) - L(x).$$

As in our initial model of changes in mortality only, the difference between the two measures is due to the increase in life expectancy that accrues with age (area *G* in the Figure) as shown in (10), which is identical to (3). This is as expected because, as shown in the previous section, changes in morbidity are identically measured in the two frameworks, once we assume identical evaluation of health states, same discounting technique and no age-weighting. But this makes it more complex to estimate the effect on α, the ratio between the two measures.

Let α' denote the ratio between the reduction in DALYs and the gain in QALYs in this model (11). The difference between the ratio α' and ratio α in the model with mortality only (5) is given in (12). By inspecting equation (12) it is easy to see that it is positive for any $\lambda_1 \in (0,1)$ and that the higher the quality of life during the years of life gained, λ_1, the closer the two ratios. (Note that the model analysed here is a generalisation of the model with mortality only and reduces to that model when λ_1=1).

(11) $$\alpha' = \frac{\Delta DALYs}{\Delta QALYs} = \frac{L(x) - L(x+k) - k*(1-\lambda_1)}{k*\lambda_1};$$

(12) $$\alpha - \alpha' = \frac{(1-\lambda_1)*[k - L(x) + L(x+k)]}{k*\lambda_1}.$$

I estimated α', varying parameters x, k as before and λ_1 between 0.1 and 1. Results for k=1 and k=30 are reported below. Note that these graphs are different from the previous ones and the series now correspond to different values of λ_1. In both Figure 9 and Figure 10 the highest line represents λ_1=1 and corresponds, respectively, to the series k=1 and k=30 in Figure 4. It can be easily seen that for λ_1 equal to 1, α' equals α. As expected, the higher the disability, the lower α'.

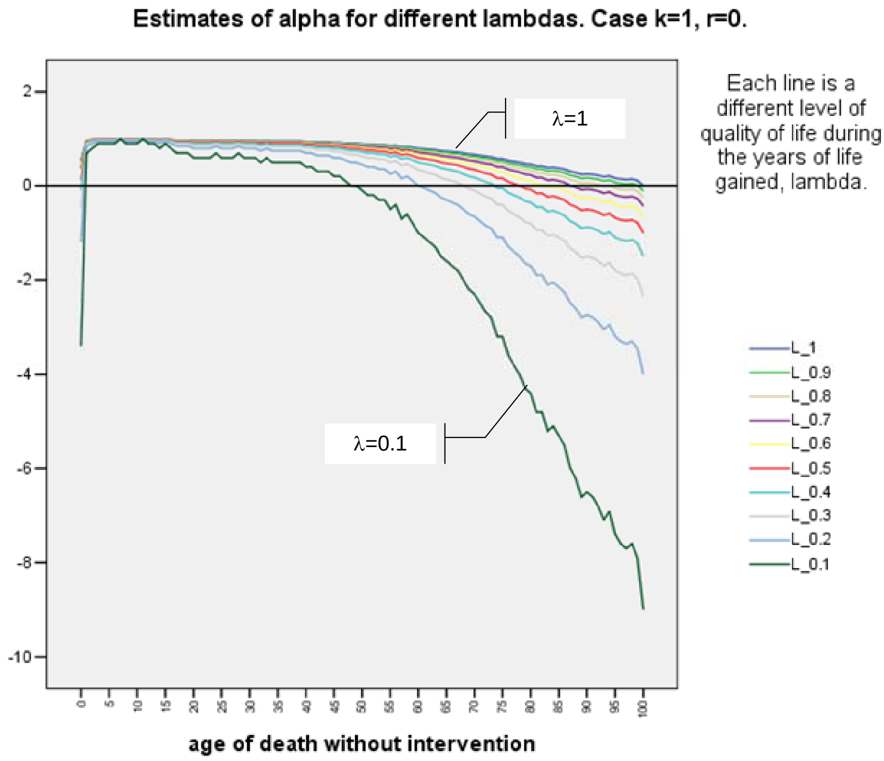

Figure 9. Estimates of α, varying λ. Case k=1, r=0.

One troubling characteristic of the DALY methodology is that for deaths avoided in a relatively older age group, the reduction in DALYs becomes negative. This happens when the years added to life, adjusted for their quality, are less then the gains in life expectancy associated with surviving until an older age (in Figure 8, this means that area *G* is bigger than area *A*).

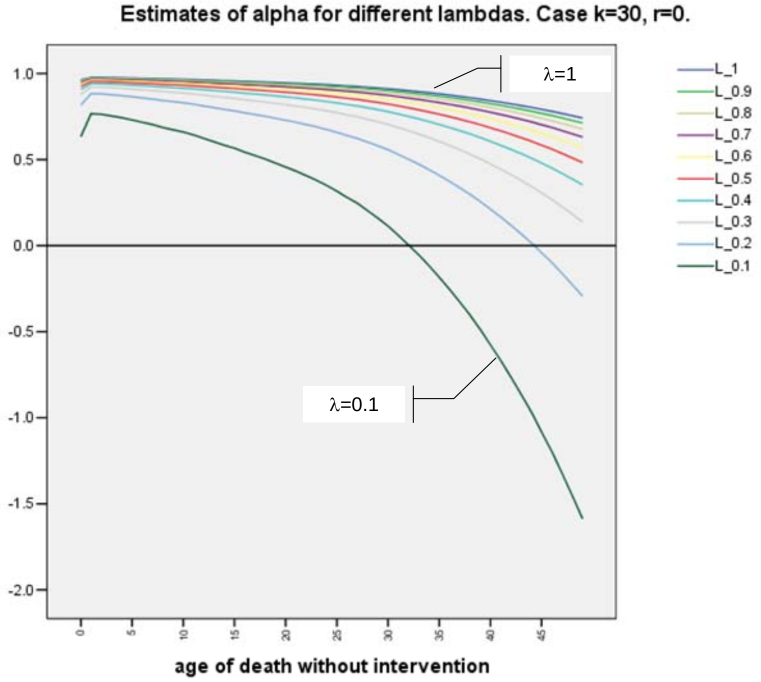

Figure 10. Estimates of α, varying λ. Case k=30, r=0.

4.2 With discounting

Using exponential discounting at 3.5% as before, I estimated the gains in QALYs and the DALYs averted through the intervention, varying parameters *x, k* and *λ.* The graphical representation of the discounted health profiles is given in Figure 11 and the ratio α in (13).

$$\text{(13)} \qquad \alpha' = \frac{\Delta DALYs}{\Delta QALYs} = \frac{\lambda * \int_0^k e^{-rt} dt - \int_{L(x)}^{k+L(x+k)} e^{-rt} dt}{\lambda_1 * \int_0^k e^{-rt} dt}$$

Sample results for *k*=1 and *k*=30 are reported in Figure 11 and Figure 12. Results are qualitatively similar to the case without discounting, but the difference between the two measures is now lower (α is closer to 1). This is as we expect, because discounting affects *G* more than A.

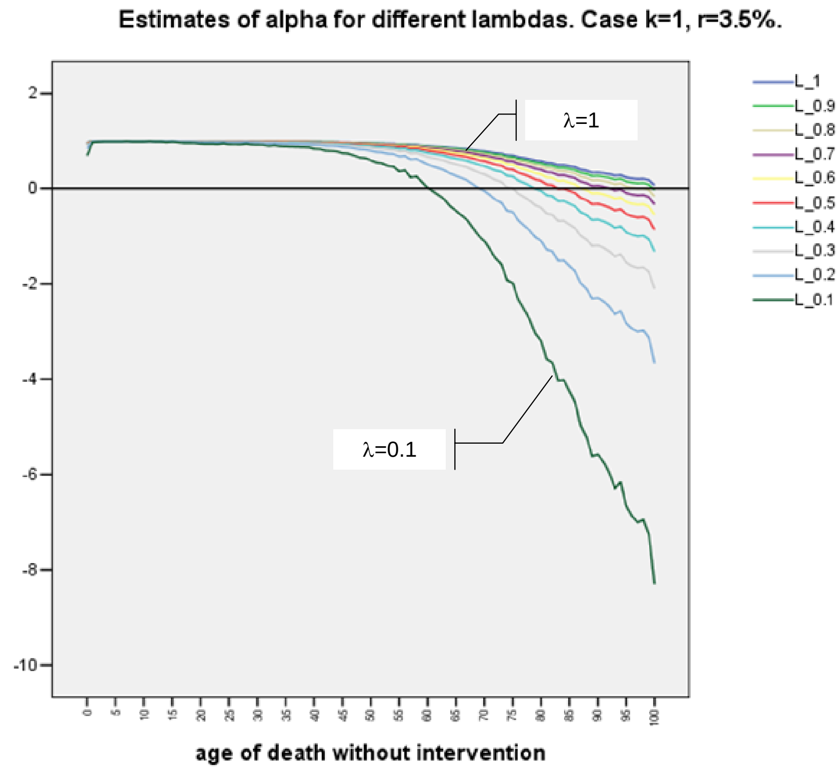

Figure 11. Estimates of α, varying λ. Case k=1, r=3.5%.

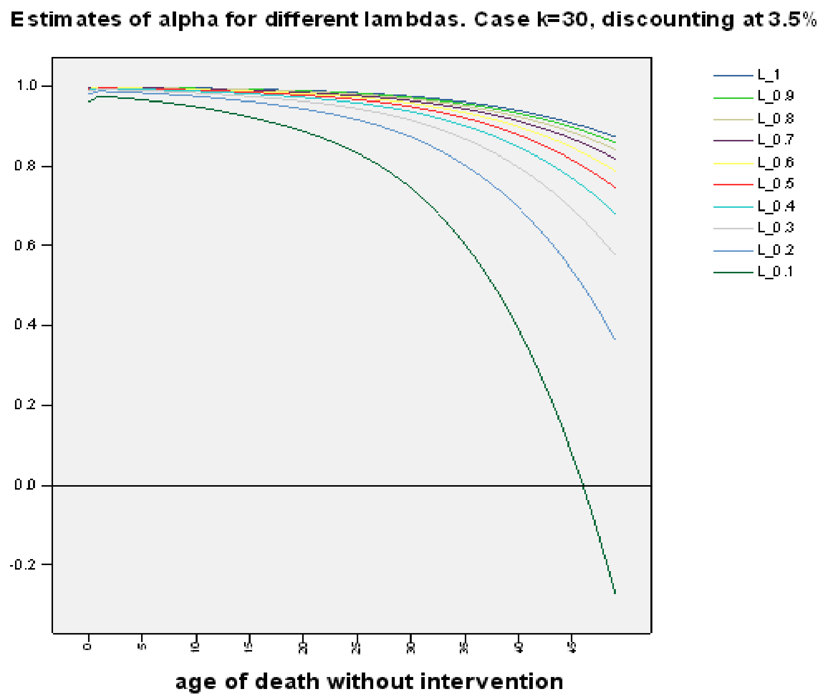

Figure 12. Estimates of α, varying λ. Case k=30, r=3.5%.

5 Policy implications

The existence of a systematic difference between health benefits measured as gains in QALYs or DALYs averted for the same intervention might be considered a mere theoretical issue if it made no difference to policy decisions. I will discuss the policy implications of this systematic difference using the following example.

Consider a decision maker who can fund treatment for one and only one of the following people:

- a 65 year old man who has entered the end stage of an illness and is assumed to die today.
- a 45 year old man who has entered the end stage of an illness and is assumed to die today.

The person receiving the treatment will live for an extra year with quality of life valued at 0.1 on a 0 to 1 scale where the quality of life associated with being dead is 0 and that associated with perfect health is 1. Let us assume that the decision maker wants to maximise the health benefit, measured as gains in QALYs or reduction in DALYs.

If the decision maker measures the health benefit with a QALY metric, funding any of the two interventions would lead to a gain of 0.1 QALYs and the decision maker might set up a lottery to determine who will receive the intervention or invoke further decision criteria, e.g. to favour younger over older patients on a *fair-innings* argument. On the other hand, if the decision maker measures the health benefit using a DALY metric, funding the first intervention would lead to an *increase* in the burden of disease of 0.16 DALYs, but funding the second intervention to a slight reduction in the burden of disease of 0.02 DALYs and would then offer the treatment to the 45 year old man. In fact, s/he would not provide the intervention to the 65 year old man even if resources were available to fund it, because his death today is associated with a lower burden of disease than his death in a year’s time, which is at variance with the original assumption that a quality of life of 0.1 is better than death.

The size of the QALY gain depends solely on the years of life gained, that is one year in both cases, and the quality of life during those years, that is 0.1 in both cases. In this sense it endorses an egalitarian judgment, that is, QALYs are equal, no matter who receives them. This is a controversial issue in the literature and some argue that a QALY gained by a younger person should be valued more than a QALY gained by an older one (Williams, 1997). Arguments in favour of age weighting have not yet been successful in shifting the standard QALY framework. On the contrary, the debate has moved the DALY framework somewhat away from its original formulation which advocated the use of an age-weighting function (Anand and Hanson, 1997, Tan-Torres Edejer et al., 2003). The DALY framework, however, still systematically favours younger over older patients on the basis of a statistical artefact, as shown in this paper. In the example discussed in this section, for instance, the decision maker would favour the 45 year old not because she explicitly uses an age-weighting function, nor because the younger patient has a greater ability to benefit. Being alive for an additional year shifts the reference age used in the DALY calculation relatively more for the older patient than for the younger one. Hence, for an identical gain in health, the burden of disease remaining would be higher after treating the old patient than the young one and the latter will be favoured.

Similarly, this proportionally different shift in the reference age between younger and older patients determines the internal inconsistency mentioned above. That is, although a state of poor health is valued more than death, an intervention leading to immediate death might be preferred to one leading to poor health.

These two troubling implications would disappear if a fixed reference age were used. In this case the DALYs averted by an intervention that would prolong life from age x to age $x+k$ with quality of life $\lambda_1 \in (0,1)$ would be equal to the QALY gained (having set aside: the use of age-weighting in DALYs, the difference between quality of life and disability weights and the different discounting techniques). Equation (14) shows the DALYs averted assuming a fixed reference age of T years, which corresponds to gains in QALYs (equation (8)).

$$(14) \qquad \Delta DALYs = [T - (x + k) + k * \lambda_1] - (T - x) = k * (1 - \lambda_1)$$

The choice of a reference age higher than the oldest person in the population would be sufficient to avoid the implications discussed above.

6 Conclusions

This note has explored the difference between estimating the benefits of a health intervention in DALYs averted and in QALYs gained. I assumed that the health intervention adds *k* years of life with quality of life equal to λ_1; I also assumed that disability weights to estimate DALYs correspond to quality adjustment weights from the QALY framework. I did not use any age weighting and I used local period life tables in the DALY calculation as recommended for evaluating the benefits of a one-year intervention that does not affect age-specific mortality rates in the population (Murray, 1996, Preston, 1993). I found that, first, there is a systematic difference in the benefits from an intervention estimated as DALYs averted versus QALYs gained. DALYs averted are always less than QALYs gained and the higher the age of avoided deaths and the lower the quality of life in the years of life gained, the bigger the difference. Second, discounting reduces the difference between gains in QALYs and DALYs averted. Third, the ranking of interventions according to DALYs averted can be different from the ranking of the same interventions measured by gains in QALYs. For instance, for deaths avoided in a relatively older age group, the reduction in DALYs can be negative; that is, the intervention increases the burden of disease, whereas the gains in QALYs are always positive.

The difference between gains in QALYs and DALYs averted, *G(x, k)*, is determined by the change in residual life expectancy at different ages, which affects the DALY calculation but not the QALY one, as shown in Figure 2, Figure 5,
Figure 8 and in the Appendix. There is an historical justification for this. QALYs were originally developed to estimate the benefits of interventions for the average patient. The gain in life years resulting from the intervention, weighted for quality, is the natural

way to measure them and they are unaffected by considerations of the average length and quality of life of people that do not need the intervention. On the contrary, DALYs were originally developed to measure the current burden of disease for a population where the burden is the gap between the health of a specific, real population and an ideal, healthier population. The natural way to measure the burden associated with premature mortality at age x in the actual population is the residual life expectancy of a person who is x years old in the ideal population and will live another $L(x)$ years. The use of the DALY framework to estimate the change in the burden of disease associated with an intervention is a subsequent development (Murray and Lopez, 1994). As this note shows, however, the use of residual life expectancy generates troubling results when the DALYs framework is used to estimate the health benefits of an intervention. The use of a fixed reference age would remove these differences in measuring the benefits of an intervention, but might not be a reasonable assumption for the estimate of the current burden of disease.

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Appendix

In this appendix I discuss when $0 \le G(x, k) \le k$. I do this in two steps. First, I discuss the shape of the residual life expectancy *L(x),* identifying the conditions under which its first derivative lies between -1 and 0. Then, I will show that *G(x ,k)* is always positive and discuss when $G(x, k) \le k$.

Let us define the following three functions (Lindsey, 2004, Keyfitz, 1968):

(1) the *survivor function*, *S(x)*, that is the probability of living until age *x:*

(15) $$S(x) = \Pr[T > x] = 1 - F(x) = \int_x^{\infty} f(t)dt\ ;$$

where *F(x)* is the cumulative distribution and *f(x)* is the corresponding density function;

(2) the *mortality rate*, *λ(x),* that is the instantaneous probability that death will occur at age *x:*

(16) $$\lambda(x) = \frac{f(x)}{S(x)};$$

(3) the *residual life expectancy, L(x),* that is the average prospective lifetime of an individual aged *x:*

(17) $$L(x) = \frac{\int_x^{\infty} S(t)dt}{S(x)};$$

Differentiating *L(x)* with respect to *x*:

(18) $$\frac{dL}{dx} = \frac{-S^2 - (-f(x))\int_x^{\infty} S(t)dt}{S^2} = -1 + \lambda(x) \cdot L(x);$$

It can be easily seen that $dL/dx \ge -1$ always, because both *λ(x)* and *L(x)* are non negative; and $dL/dx \le 0$ if and only if $\lambda(x) \cdot L(x) \le 1$, that is residual life expectancy is a decreasing function in correspondence of ages *x* where $L(x) \le \frac{1}{\lambda(x)}$.

Empirical analysis of life tables shows that *L(x)* may increase during early years of life, when there is a high risk of infant mortality. In developed countries, where the life expectancy at birth is above 70 years, this usually happens only for the first year of life or even just for the first few months, and *L(x)* is a decreasing function of age *x* thereafter (Goldman and Lord, 1986, Coale and Demeny, 1983, Shrestha, 2005).

Let us now discuss when $0 \leq G(x, k) \leq k$. We recall from (3) that $G(x, k) =$ $k - L(x) + L(x+k)$.

We can re-write *G(x, k)* as

$$G(x,k) = k \cdot \left[1 + \frac{L(x+k) - L(x)}{k}\right] =$$

$$= k \cdot \left[1 + \frac{L(x+k) - L(x+k-1)}{k} + \frac{L(x+k-1) - L(x+k-2)}{k} + ... + \frac{L(x+1) - L(x)}{k}\right].$$

The years gained with the intervention, *k*, are non-negative. As we discussed above, the first derivative of *L(x)* is greater than -1 for any *x*, hence the term in square brackets is non negative, that is $G(x, k) \geq 0$ always. Similarly, for values of *x* where the first derivative $dL/dx \leq 0$, that is when $\lambda(x) \cdot L(x) \leq 1$, the term in square brackets is less than one, hence $G(x, k) \leq k$.

List of Tables and Figures

Page

List of Tables

List of Figures

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