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The Impact of Trade on Aggregate Productivity and Welfare with Heterogeneous Firms and Business Cycle Uncertainty

Jang Ping Thia

THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

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Abstract
This paper presents a model with monopolistic competition, productively heterogeneous firms, and business cycle aggregate shocks. With firm-specific productive heterogeneity, weaker firms quit when faced with a negative aggregate shock. Consequently, trade does not always increase firm-level aggregate productivity as negative shocks on the home market can be compensated for by positive shocks elsewhere. Weaker firms, which would otherwise quit in autarky, can continue to operate by exporting. Despite this, trade can still improve welfare for risk-averse consumers by reducing aggregate price fluctuations.

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Jang Ping Thia is an Occasional Research Assistant at the Centre for Economic Performance, London School of Economics.

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1 Introduction

Business Cycles and Firm Heterogeneity The business cycle, which exerts a profound impact on many facets of the economy, has generally not been given much consideration in international trade models. Traditional trade theories highlight the gains from trade that arise from country level differences, either broadly due to technology (Ricardian) or endowment (Heckscher-Ohlin). Since these models describe the long run general equilibrium gains from trade, the business cycle is in some sense irrelevant. Perfect competition also renders the firm irrelevant in equilibrium trade considerations. On the other hand, suppose one introduces business cycle shocks to a 'new' trade model [Paul R. Krugman (1979, 1980)] with homogenous firms. Since firms are homogeneous, the business cycle affects all firms symmetrically and does not therefore have any reallocation effects. Any business-cycle driven reallocation of market shares can only be adequately described with a model of heterogeneous firms.

This paper therefore asks the following question: how do business cycle shocks affect heterogeneous firms? What are the reallocative and welfare implications? How do these shocks affect the production and exporting decisions of firms? These questions are interesting and important on several counts.

To begin, trade in the context of firm heterogeneity has received much theoretical research attention recently [Marc J. Melitz (2003); Andrew B. Bernard, Stephen J. Redding and Peter K. Schott (2007) - henceforth known as BRS; Melitz and Gianmarco I.P. Ottaviano (2005)] motivated by strong empirical evidence that points to the existence of persistent productivity differences between exporters and non-exporters. The key contribution of the firm heterogeneity literature is to formally model the reallocation of output and market shares between productively heterogeneous firms. Firms below the so-called productivity cutoff cease to operate, ceding market shares to more productive firms above the cutoff.

Economists are therefore able to formalise yet another source of welfare gains through trade liberalisation, which arises by increasing the productivity cutoffs and the transfer of market shares to more productive firms. But since these models set out to formalise the long-run equilibrium effects of trade liberalisation, the business cycle is mostly ignored. The notable exception is by Fabio Ghironi and Melitz (2004), which microfounds the Balassa-Samuelson effect through heterogeneous firms and productivity shocks. However, the authors consider only the exporting decisions of firms but not their production decisions.

With heterogeneous firms, it is also evident that business cycle shocks will affect different firms differently, even in autarky. There are potentially interesting reallocative effects of output and market shares. The macro consequence of the business cycle in an environment with heterogeneous firms is non-trivial since a firm's continued production through an adverse demand shock would depend on its productivity. The set of firms that quit production in the face of adverse demand is therefore not a random selection, and there exists a systematic relationship between productivity cutoffs and business cycle shocks[1].

Trade between economies with asymmetric shocks would therefore present another point of interest: how would heterogeneous firms behave and what would be the macroeconomic welfare consequences? The objective of this paper is to model the effect of business cycles and trade in an analytically tractable manner.

Model Outline The starting point of the paper is the introduction of productivity shocks into a Melitz type model. By altering the size of the market, these business cycle productivity shocks then translate into demand shocks for firms. This paper does not consider any nominal rigidities that affect firms' ability to adjust. However, firms have to invest in fixed assets first (due to production lags) before production takes place. The effect of this is that firms face an uncertain demand since they are investing before the shocks are realised[2]. Due to the heterogeneity in

[1] On the other hand, business cycle shocks with homogeneous monopolistically competitive firms do not yield much meaningful analysis. For example, suppose a Krugman type firm has to decide on market entry by making a fixed asset investment without knowing the level of demand entry under uncertainty will occur until ex-ante profit becomes zero for all firms. If demand turns out to be high, there will be insufficient entry and all firms will make a profit. Conversely, there will be too many entry firms if demand is low and all firms will be unable to recover fixed costs and thereby make losses. Depending on the realisation of the aggregate demand shock, either all firms make profits or all firms make losses since firms are homogenous. The equilibrium does not provide any richness in describing the reallocation effect that would occur with heterogeneous firms.

[2] In Ghironi and Melitz (2004), the aggregate shock in that model is introduced via firms' uncertainty over their future productivity. As there are no fixed production cost, production decisions are not affected by shocks - only exporting decisions are affected. The departure in this paper is the presence of fixed production cost, which then affects a firm's decision whether to continue through adverse shocks.

production costs, profit outcomes are no longer symmetric. For example, with a negative aggregate shock, weaker firms will make losses while stronger ones will still make profits. With a positive shock, all firms will make profits but again profits will be higher for more productive firms. Though aggregate profits shift up or down depending on the ex-post demand shocks, there is always a 'profit ranking' where a more productive firm always has a higher profit.

Furthermore, in a general equilibrium, productivity shocks also change the available aggregate resources in the market place. As Melitz (2003) notes, " . . . all the effects of trade on the distribution of firms are channelled through a second mechanism operating through the domestic factor market where firms compete for a common resource." The first mechanism - namely product market competition - is "not operative . . . due to CES preferences: the price elasticity of demand for any variety does not respond to changes in the number or prices of competing varieties."[3]

A similar mechanism of factor competition is at work in this paper. When faced with a negative aggregate shock, the aggregate savings (of consumers) fall. Since aggregate savings equal the gross investments into firms' fixed costs, fewer firms are able to invest and continue production. The upshot of this is that weaker firms will have to quit the market, fitting the stylised fact that recessions have a greater impact on weaker firms. In this paper however, an additional mechanism is introduced via demand uncertainty: Firms have to invest in fixed asset before demand is realised. The expected market size (in the next period) will change a firm's decision whether to continue in the market.

Trade and Capital Market Integration Away from autarky, two processes of integration occur. The first is capital market integration that allows capital to be shipped between countries. The second is goods market integration that allows consumption goods to be shipped. As this paper is focused on the effects of goods trade only, capital market integration is assumed to be as simple as possible. There is a perfectly

[3]Melitz and Ottaviano (2005) provide a model with quasi-linear preferences with firm heterogeneity that delivers reallocation of market shares through competition in the goods market. However in that model, any changes to income affect only the consumption of the competitive sector and have no impact on the monopolistic sector. The model is therefore less suitable in the context of modelling demand shocks to the monopolistic sector.

competitive international market for capital to be shipped between countries and returns to capital costlessly remitted back to capital owners for consumption. This is the key assumption of the 'Footloose Capital' class of models in Economic Geography. While this may not necessarily be a robust or realistic assumption, it nevertheless allows the paper to abstract from any capital market complications that might arise and focus on the goods market instead.

It turns out that in equilibrium, even the perfect mobility of capital cannot replicate the outcome of free goods trade[4]. Why might this be so? The presence of trade costs alter the perceived expected market size faced by monopolistically competitive firms. In the presence of trade costs, the productivity cutoffs of two countries cannot be equalised in some circumstances, leading to different selection effects in both countries. The fact that two economies have different productivity cutoffs is not trivial. First, it implies that capital is not optimally invested as some less efficient firms (in the country with lower productivity cutoff) can continue production when they otherwise cannot with free trade. Secondly, it implies that the reallocative effect is not maximised since some weaker firms continue producing behind trade barriers. In the presence of trade costs, market shares are therefore not allocated in the most efficient manner across economies.

What then are the benefits of free trade? As firms in each country operate in a larger fully integrated market, the productivity cutoffs in both countries are completely equalised (but may still change with different demand states). This represents the optimum deployment of capital and allocation of market shares between heterogeneous firms and across economies. More significantly, this leads to a diversification effect which in equilibrium reduces price-output fluctuations faced by each economy in autarky. This result stems from the fact that only free trade equalises the productivity cutoffs between countries, and allows for an equally productive set of producers to operate. The increased macroeconomic stability presents yet another source of welfare gains for the risk-averse consumer.

Limitations In this paper, business cycle shocks are introduced by way of a two-state (good or bad) Markov process. While this is

[4]This is a different result from Robert Mundell (1957) that shows that free trade in factors is equivalent to free trade in goods in a neoclassical setting.

more limiting compared to where productivity shocks (innovations) are normally distributed with mean zero [see Ghironi and Melitz (2004)][5], the Markov process allows the paper to solve various variables in a stationary equilibrium. When analysing the effects of international trade, this paper considers only a two-country setting to highlight the diversification effect clearly. Nevertheless, the insights can be extended to a multi-country setup. Finally, to derive analytical solutions explicitly, the paper assumes the productivity distributions to be Pareto [Ghironi and Melitz (2004); BRS (2007)].

2 The Model Setup

2.1 Endowments

There are L identical consumers (who are also workers) in the economy. The consumers have infinite lives, and each is endowed with some mean level of human capital denoted by H, thereby providing a mean level of effective labour force of LH.

2.2 States of the World

There are two possible states of the world, bad and good, denoted by subscript $S = B$ or G. There is high H_G in the good state and low H_B in the bad state. This is the characterisation of the aggregate shock. The transition from period to period is given by a simple Markov process

$$\Pr(H_{G,t+1} \mid H_{G,t}) = \Pr(H_{B,t+1} \mid H_{B,t}) = \rho$$

where $1 > \rho > \frac{1}{2}$ reflects the persistence of the shocks. To abstract from growth dynamics, the model assumes the shocks to be symmetric around the mean level

$$H_G = (1 + \gamma)H \qquad\qquad H_B = (1 - \gamma)H$$

[5] For example, it will not be possible to generate variable moments to fit the data, greatly reducing the testable implications on parameters.

where $\gamma < 1$ is the size of the shock. Naturally, the average size of this economy over many periods will be LH[6]. Workers sell their labour services to the market inelastically.

2.3 Preferences

In each period t, the j consumer's utility is given by

$$u_{jt} = x_0^{1-\lambda} x_1^{\lambda} \qquad \text{where } x_1 = \left[\int_{\Omega_t} c_{it}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

and $\sigma > 1$. Good x_0 is the homogenous good, produced competitively with unit labour, costlessly traded, and used as the numeraire ($P_0 = 1$). Good x_1 is the differentiated good, where Ω_t is the set of varieties available to the consumers at discrete period t. Furthermore, each consumer's discounted lifetime utility is given by

$$U_j = \sum_{t=0}^{\infty} \beta^t \ln u_{jt}$$

where $\beta < 1$ is the subjective discount factor in each period.

This preference specification thus exhibits the 'double-diminishing' property. There is diminishing marginal utility to the consumption of each variety in any time period and also diminishing marginal utility to the number of varieties in each period. The log utility also implies that the consumers are strictly risk-averse, preferring a stable level of utility (or varieties) over time. This will be the key property that gives rise to welfare gains when countries trade since aggregate price (or output) stability is welfare enhancing.

2.4 Technology and Firms

The homogeneous good is competitively produced. Even after the opening of economies to trade, this paper assumes that the homogeneous good will be produced everywhere (incomplete specialisation), thereby pinning down the price of homogeneous good and wages everywhere to $w = P_0 = 1$ [see Elhanan Helpman, Melitz and Stephen R. Yeaple (2004)].

For the differentiated industry, there is an exogenous mass M of exist-

[6]There is no long-run growth, and the model abstracts from growth effects considered in Richard E. Baldwin and Frederic Robert-Nicoud (2006).

ing firms with heterogeneous productivity characterised with a productivity distribution that has a cumulative distribution $G(\varphi)$ and density function $g(\varphi)$[7]. Each atomistic firm has a constant productivity φ specific to itself on this distribution[8].

In every period, each firm has a per period fixed cost f. The key requirement is that f has to be in place one period before production takes place due to production lags. If a firm does not invest in f during this period, it will not be able to produce in the period after that. After the fixed cost is incurred, a firm can begin production in the next period with the production function given as

$$l = \frac{q}{\varphi}$$

where l is the labour requirement to produce q units of output.

2.5 Capital Goods

Consumers save by investing in a perfectly competitive mutual fund, which then supplies f to the firms in return for next period's operating profits as dividends to the fund. The fund then channels the dividends back to the consumers. This approach is seen in Ghironi and Melitz (2004) and it greatly simplifies the saving-investment process of consumers. With heterogeneous firms, each existing firm will have a different firm value. Considering the investment into a mutual fund this way allows one to ignore the investment choices of individual consumers. This simplification means that the consumers effectively own the entire portfolio of heterogeneous firms through the mutual fund (in equal shares), and receive the same stream of dividend. In this way, one can also characterise the economy with a representative consumer who owns all the firms in the economy.

[7] In the Melitz (2003) model, steady state firm mass is pinned down by the size of the market. There is constant steady state entry to replace the exogenous steady state exit (subject to paying sunk cost f_e and drawing a productivity level above the cut-off). This paper has elected to keep the number of firms fixed to simplify the exposition. This can be motivated by the fact the economy has a long-run size of LH, despite period to period γ shocks. To be explicit, the paper is making the assumption that the γ shocks are small enough relative to a large sunk cost f_e such that no firms will find it profitable ex-ante to enter on the basis of business cycle shocks alone. Coupled with the assumption of no exogenous destruction, the number of firms becomes fixed.

[8] The minimum support of the pareto distribution is given as $\bar{\varphi}$, while the shape is given by parameter k.

As countries move away from autarky, this paper makes a departure from Ghironi and Melitz (2004), which assumes that consumers in each country invest in a mutual fund that owns only the portfolio of domestic firms. Here, the paper allows a country's savings to be invested into the fixed cost of firms in another country, and the operating profits to be costlessly remitted back to owners for consumption. This is in a sense the assumption of perfect capital mobility widely used in Footloose Capital models in New Economic Geography.

3 Equilibrium in Autarky

As the paper deals with a Markov type uncertainty with only two states, the equilibrium is in fact stationary - the economy switches between good or bad state equilibrium instantly once the shocks are realised, and there are no further transitional dynamics.

3.1 Consumer's Problem

Since all consumers are identical, one can deal with the model with a representative consumer (normalising L to 1). The consumer faces a decision on how much to spend (and save) in each period given the state of the world and how much to spend on each good. In each period, the consumer simply solves the following Bellman equation with value function χ

$$\chi_t(\omega_t, H_t) = \max \ln u_t + \beta\chi_{t+1}(\omega_{t+1}, H_{t+1}|S_t)$$

subject to the inter-temporal budget constraint

$$E\left[\frac{\omega_{t+1}}{\iota_t}|S_t\right] = H_t + \omega_t - E_t$$

The representative consumer holds the entire market portfolio of shares of all firms. The first source of income is wage income H_t. The second is the net revenue of firms ω_t returned to the consumer as a dividend. His expenditure is E_t and he saves by again investing in the market portfolio of firms with an expected return ω_{t+1}, suitably discounted by interest rate ι_t.

The optimisation of the Bellman equation with the log utility and

Markov process give the following Euler equations

$$\frac{1}{E_G} = \beta\iota_G\left[\rho\frac{1}{E_G} + (1-\rho)\frac{1}{E_B}\right] \qquad\qquad \frac{1}{E_B} = \beta\iota_B\left[\rho\frac{1}{E_B} + (1-\rho)\frac{1}{E_G}\right] \tag{1}$$

where E_G and E_B are the expenditures of the good and bad states, ι_G and ι_B are the real interest rates. From equation (1), as expenditure is higher in the good state $E_G > E_B$, the real interest rate is also higher in the bad state $\iota_B > \iota_G$ (the real interest rates will be solved explicitly in later sections). This is a standard result - a higher real interest rate is necessary in the bad state for the consumer to be indifferent between current and future consumption. Since there are only two levels of aggregate expenditure, of which a constant λ is spent on the differentiated sector, there are also only two levels of aggregate revenue in the differentiated sector given by

$$R_G = \lambda E_G \qquad\qquad R_B = \lambda E_B$$

Indirect Utility The indirect utility of the consumer in each period can be written as

$$V = \frac{\lambda^\lambda(1-\lambda)^{1-\lambda}E_{S(t)}}{\left[P_{S(t)}\right]^\lambda} \tag{2}$$

where S denotes the state. The consumer's indirect utility depends on two factors - his current state-contingent expenditure $E_{S(t)}$ and the aggregate price level $P_{S(t)}$ of the differentiated sector[9]. However, since the number of firms that are producing in period t is determined in period $t-1$ given the production lag, the CES aggregate price level $P_{S(t)}$ in fact depends on the investment decisions in the previous period. For example, if today is a good state while the previous state is bad, the indirect utility is in fact given as $V_B^G = \frac{\lambda^\lambda(1-\lambda)^{1-\lambda}E_G}{P_B}$, where $P_G < P_B$. Though today's income is high, welfare is lower due to the higher CES aggregate price. V_G^G therefore gives the highest indirect utility and V_B^B the lowest.

3.2 Profit Conditions

The productivity cutoff is defined as a productivity level φ^* that allows a firm to break even in expectation with the investment into fixed cost.

[9] The price of the homogenous good is normalised to 1, and therefore does not appear in the indirect utility equation.

A firm with this cutoff productivity level is labelled as the marginal firm. Any firm with a productivity level below this cutoff will not invest in fixed assets and not produce in the next period [see Melitz (2003); BRS (2007)].

Proposition 1 *There exists a 'Profit Condition' for the good state PC_G that provides the relationship between the expected profitability of a firm and its productivity φ, when the economy is hit with a positive shock.*

Proof. In equilibrium, the marginal firm with productivity φ^* will have expected revenue $r^e(\varphi^*)$ that recovers investment cost with interest in expectation only. This can be written as

$$r^e(\varphi^*) = \frac{p(\varphi_G^*)^{1-\sigma}}{P_G^{1-\sigma}} \left[\rho R_G + (1-\rho) R_B\right] = \sigma \iota_G f \tag{3}$$

where $p(\varphi_G^*)^{1-\sigma} = \frac{\sigma}{\sigma-1}\frac{1}{\varphi_G^*}$ is the CES optimal price, and $[\rho R_G + (1-\rho)R_B]$ is the expected aggregate market size in the next period given the Markov process. Because of the CES function, the ratio of (expected) revenues between two firms with productivity φ and φ^* is given as $\frac{r^e(\varphi)}{r^e(\varphi^*)} = \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1}$. This allows the expected revenue of any firm with productivity φ to be expressed as $r^e(\varphi) = \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} r^e(\varphi^*)$. The expected profit becomes $\pi(\varphi) = \frac{1}{\sigma}\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} \sigma\iota_G f - \iota_G f$. This is simplified to be a function of the productivity cutoff only

$$\pi_G(\varphi) = \left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right] \iota_G f \tag{4}$$

■

Proposition 2 *There exists a bad state Profit Condition PC_B that provides the relationship between the expected profitability of a firm and its productivity φ, when the economy is hit with a negative shock.*

Proof. From the previous proposition, the marginal firm condition becomes

$$r^e(\varphi^*) = \frac{p(\varphi_B^*)^{1-\sigma}}{P_B^{1-\sigma}} \left[\rho R_B + (1-\rho) R_G\right] = \sigma \iota_B f \tag{5}$$

The expected profit, characterised by PC_B, can be written as the function of the marginal firm with productivity φ^* only. As the real interest rate is now ι_B, PC_B can be written as

$$\pi_B(\varphi) = \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] \iota_B f \tag{6}$$

Since $\iota_B > \iota_G$, the cost of capital is higher in the bad state, shifting the profit function downwards. The real interest rates will be solved explicitly in later subsections. ■

3.3 The Impact of Uncertainty and Shocks

Before the paper proceeds to provide the analytical solution to the equilibrium, it is useful to highlight several key facts of this equilibrium. Four realisations of ex-post profits can occur even though there are only two levels of average productivity (since there are only two cutoffs φ_G^* and φ_B^*), because firms have to make investment decisions before the shocks are realised. Actual profitability is therefore not only a function of productivity but is also affected by ex-post demand. Measuring productivity using ex-post realisations of profit can therefore be misleading. Because of the lag structure, high profits can be due to a positive demand shock without any change in the underlying productivity of firms.

Secondly, firm level aggregates are now affected by the relevant state. In the good state, firms with productivity levels higher than φ_G^* will invest f to produce in the next period. With a negative shock, the cutoff level increases to φ_B^* as market conditions go from easy to tough. The result is that firms between $G(\varphi_B^*)$ and $G(\varphi_G^*)$ will have negative expected profits if they choose to stay in the market.

Since the parameters are constant, the model in fact has stationary equilibrium properties. The equilibrium shifts to the good state or the bad state without any further dynamics. This allows the relationship between the numbers of firms to be written as

$$M_B = \left[\frac{1 - G(\varphi_B^*)}{1 - G(\varphi_G^*)} \right] M_G \tag{7}$$

When a bad state comes after a good one, a proportion of firms $\left[\frac{1-G(\varphi_B^*)}{1-G(\varphi_G^*)} \right]$ will not invest in f and quit the market. The business cycle therefore introduces a selection effect where only a stronger and smaller subset of firms is productive enough to continue investing through the bad state.

The final point to make here is that firms below φ_G^* will never invest since they can never recover the fixed cost.

3.4 Aggregate Resource Constraints

The aggregate resource constraint for the good state can be written as

$$H(1+\gamma) + \frac{\lambda E_G}{\sigma} = E_G + M_G f \tag{8}$$

The terms on the left hand side are total wage income $H(1+\gamma)$ where γ is the size of the aggregate shock, and dividend $\frac{\lambda E_G}{\sigma}$ which is the operating profits of firms producing in the current period (they invested f previously)[10]. The left hand side thus represents total income flow to the representative worker. The corresponding expression for the bad state can be written as

$$H(1-\gamma) + \frac{\lambda E_B}{\sigma} = E_B + M_B f \tag{9}$$

This paper has done away with the Melitz exit mechanism by assuming a fixed number of firms M on the distribution $G(\varphi)$ [see Thomas Chaney (2006)]. This allows one to write M_G and M_B explicitly as a function of M and the respective cutoffs only

$$M_G = [1 - G(\varphi_G^*)]M \qquad\qquad M_B = [1 - G(\varphi_B^*)]M \tag{10}$$

This is consistent with equation (7) provided earlier.

Consider the good state aggregate constraint in equation (8). It can be re-written as

$$H(1+\gamma) + E_G\left(\frac{\lambda - \sigma}{\sigma}\right) = M_G f$$

By writing the equation this way, the left hand side of the equation is simply the aggregate savings (net of expenditure). By making use of

[10]See Appendix A:2.8.1 for the distribution of revenues and profits across firms. The proofs show that aggregate operating profits (which flow back to consumers as dividend) are functions of aggregate expenditures only, independent of the number of firms. In other words, the distribution of market shares across firms does not affect the aggregate resource constraints.

equation (10), the mass of firms investing in the good state becomes

$$[1 - G(\varphi_G^*)]M = \frac{H(1+\gamma) + E_G\left(\frac{\lambda-\sigma}{\sigma}\right)}{f} \tag{11}$$

Similarly, the mass of firms investing in the bad state becomes

$$[1 - G(\varphi_B^*)]M = \frac{H(1-\gamma) + E_B\left(\frac{\lambda-\sigma}{\sigma}\right)}{f} \tag{12}$$

In short, the mass of firms that can carry on investing is a function of the net available resource saved in the economy in each period divided by the per firm capital requirement. These two equations therefore allow the productivity cutoffs to be pinned down once the aggregate expenditure (and hence savings) in each state is known. Since aggregate savings are smaller in a bad state, the productivity cutoff φ_B^* must be higher.

3.5 Equilibrium Characterisation

The equilibrium is a set of variables $\{\varphi_G^*, \varphi_B^*, E_G, E_B, \iota_G, \iota_B\}$ that satisfy the pair of Euler equations in (1), resource constraints (11) and (12), and the marginal firm conditions (3) and (5).

Making use of the two Euler equations in (1), the ratio of expenditures can be written as

$$\frac{E_B}{E_G} = \frac{\iota_G}{\iota_B}\left[\frac{\rho\frac{1}{E_G} + (1-\rho)\frac{1}{E_B}}{\rho\frac{1}{E_B} + (1-\rho)\frac{1}{E_G}}\right]$$

Let $\frac{E_G}{E_B} = \theta$, where $\theta > 1$ is the ratio of good to bad state expenditure (θ will be solved later). The above equation can be written as

$$\frac{1}{\theta} = \frac{\iota_G}{\iota_B}\left[\frac{\rho + (1-\rho)\theta}{\rho\theta + (1-\rho)}\right]$$

This gives the ratio of interest rates as

$$\frac{\iota_B}{\iota_G} = \left[\frac{\rho + (1-\rho)\theta}{\rho\theta + (1-\rho)}\right]\theta \tag{13}$$

which is greater than one.

Dividing equation (5) by (3) gives the following relationship

$$\frac{\frac{p(\varphi_B^*)^{1-\sigma}}{P_B^{1-\sigma}}}{\frac{p(\varphi_G^*)^{1-\sigma}}{P_G^{1-\sigma}}}\left[\frac{\rho R_B+(1-\rho)R_G}{\rho R_G+(1-\rho)R_B}\right]=\frac{\iota_B}{\iota_G}$$

This relationship can be simplified in two steps. Firstly, the definition of aggregate prices - which is a function of firm mass and average productivity - can be substituted into the above equation. Secondly, one can make use of the convenient relationship that arise from the pareto distribution - that the ratio of average to cutoff productivity is a constant[11]. This constant is therefore cancelled out on the left hand side of the above equation. Together, these simplify the relationship to

$$\frac{M_G}{M_B}\left[\frac{\rho+(1-\rho)\theta}{\rho\theta+(1-\rho)}\right]=\frac{\iota_B}{\iota_G}$$

By substituting the ratio of interest rates from equation (13), the ratio of firm mass can be solved as

$$\frac{M_G}{M_B}=\theta \tag{14}$$

Dividing equation (11) by (12) gives

$$\frac{M_G}{M_B}=\frac{H(1+\gamma)+\theta E_B\left(\frac{\lambda-\sigma}{\sigma}\right)}{H(1-\gamma)+E_B\left(\frac{\lambda-\sigma}{\sigma}\right)}$$

With the left hand side to be exactly θ from equation (14), one can simplify the above relationship and solve for

$$\theta=\frac{1+\gamma}{1-\gamma} \tag{15}$$

as a function of shock parameter γ only. Therefore, the ratio of expenditures $\frac{E_G}{E_B}$ and ratio of firm mass $\frac{M_G}{M_B}$ are exactly the ratio of productivity shocks θ.

From the bad state Euler equation

$$\frac{1}{E_B}=\beta\iota_B\left[\rho\frac{1}{E_B}+(1-\rho)\frac{1}{E_G}\right]$$

[11] With the pareto distribution, $\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1}=\left[\frac{k}{k+1-\sigma}\right]$.

Multiplying across by E_G gives $\theta = \beta\iota_B\left[\rho\theta + (1-\rho)\right]$. This allows the real interest rate to be solved as a function of parameters only

$$\iota_B = \frac{\theta}{\beta\left[\rho\theta + (1-\rho)\right]} \tag{16}$$

Similarly, the good state interest rate can solved as

$$\iota_G = \frac{1}{\beta\left[\rho + (1-\rho)\theta\right]} \tag{17}$$

With the solution to the interest rates, one can solve for E_G and E_B by plugging ι_B and ι_G into the marginal firm equations in equations (5) and (3), and then making use of the firm constraint conditions in equations (9) and (8). These will provide four equations to solve for the remaining endogenous variables E_G, E_B, M_G, and M_B. Nevertheless, because of the complexity of the equations, this method is algebraically cumbersome.

There is a quicker way to solve for the variables. Suppose that $\gamma = 0$ (no shocks). In equilibrium, there will only be one interest rate since $\iota_B = \iota_G = \frac{1}{\beta}$, there will only be one level of expenditure $E = E_G = E_B$, and one constant firm mass $M = M_G = M_B$. The marginal firm condition from equations (5) and (3) collapse to one single equation

$$\frac{1}{M}\psi E = \frac{1}{\beta} f\sigma$$

where $\psi = \left(\frac{\varphi^*}{\tilde{\varphi}}\right)^{\sigma-1} = \left[\frac{k+1-\sigma}{k}\right]$ simply reflects the nice property of the pareto distribution where the ratio of average to cutoff productivity is a constant. Without aggregate shocks, there is also only one aggregate constraint

$$Mf = H + E\left(\frac{\lambda - \sigma}{\sigma}\right)$$

By making the substitution of Mf into the previous relationship, one can solve for

$$E = \frac{\sigma H}{\lambda\beta\psi - \lambda + \sigma} \tag{18}$$

This is the solution to the expenditure level in the absence of shocks ($\gamma = 0$).

Since any shocks are symmetric around the mean level of H, and that $\frac{E_G}{E_B} = \theta = \frac{1+\gamma}{1-\gamma}$ in equilibrium, the exact level of expenditures in the presence of shocks are simply solved as

$$E_G = \frac{\sigma H(1+\gamma)}{\lambda\beta\psi - \lambda + \sigma} \qquad\qquad E_B = \frac{\sigma H(1-\gamma)}{\lambda\beta\psi - \lambda + \sigma} \tag{19}$$

Note that the levels of expenditures depend on parameters only. Firm level variables such as productivity average or cutoff productivities, or aggregate variables such as interest rates, have no bearing at all on the level of expenditures. Fluctuation in expenditures is purely a result of γ with no other influences. With the solutions to the level of aggregate expenditure, one can easily solve for the mass of firms using the aggregate constraints in equations (9) and (8)

$$M_G = \frac{H(1+\gamma)}{f}\left[\frac{\lambda\beta\psi}{\lambda\beta\psi - \lambda + \sigma}\right] \qquad\qquad M_B = \frac{H(1-\gamma)}{f}\left[\frac{\lambda\beta\psi}{\lambda\beta\psi - \lambda + \sigma}\right] \tag{20}$$

Aggregate Prices and Welfare Implication The expression of aggregate price is

$$P = M_S^{\frac{1}{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\frac{1}{\tilde{\varphi}_S}\right)$$

where M_S is the number of producing firms with state $S = G$ or B, and $\tilde{\varphi}_S$ is the average productivity defined as

$$\tilde{\varphi}_S = \left[\frac{1}{1-G(\varphi_S^*)}\int_{\varphi_S^*}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$$

With the pareto distribution, the average productivity becomes a function of the cutoff only

$$\tilde{\varphi}_S = \left[\frac{k}{k+1-\sigma}\right]^{\frac{1}{\sigma-1}}\varphi_S^*$$

where k is the parameter that characterises the shape of the distribution.

Using the definition of the aggregate prices, the ratio of bad to good CES prices is given as

$$\frac{P_B}{P_G} = \left(\frac{M_G}{M_B}\right)^{\frac{1}{\sigma-1}}\frac{\varphi_G^*}{\varphi_B^*} \tag{21}$$

Following a bad state (due to the lag structure), there are fewer firms and the effect of this is to increase the CES aggregate price. This effect is seen in the term $\left(\frac{M_G}{M_B}\right)^{\frac{1}{\sigma-1}}$ which is greater than 1. However, the average productivity following a bad state rises since only a smaller subset of firms above φ_B^* survive. With firm heterogeneity, there are fewer firms but they are of higher productivity, thereby resulting in an opposite effect on the aggregate price level. This is seen by the ratio $\frac{\varphi_G^*}{\varphi_B^*}$ which is less than 1. Another way of seeing this is to realise that firm heterogeneity softens the effect of underlying shocks because the firms that stop investing f in a bad state are the least productive ones.

Despite the opposing effects, there is no ambiguity on the price level with the pareto distribution. Using the fact that $M_G = \left(\frac{\bar{\varphi}}{\varphi_G^*}\right)^k M$ and $M_B = \left(\frac{\bar{\varphi}}{\varphi_B^*}\right)^k M$ from equation (10) given the pareto distribution, the productivity cutoffs are explicitly solved as

$$\varphi_G^* = \bar{\varphi}\left(\frac{M}{M_G}\right)^{\frac{1}{k}} \qquad \varphi_B^* = \bar{\varphi}\left(\frac{M}{M_B}\right)^{\frac{1}{k}} \tag{22}$$

Substituting these into equation (21), the aggregate price ratio can be solved as

$$\frac{P_B}{P_G} = \theta^{\frac{k+1-\sigma}{k(\sigma-1)}} \tag{23}$$

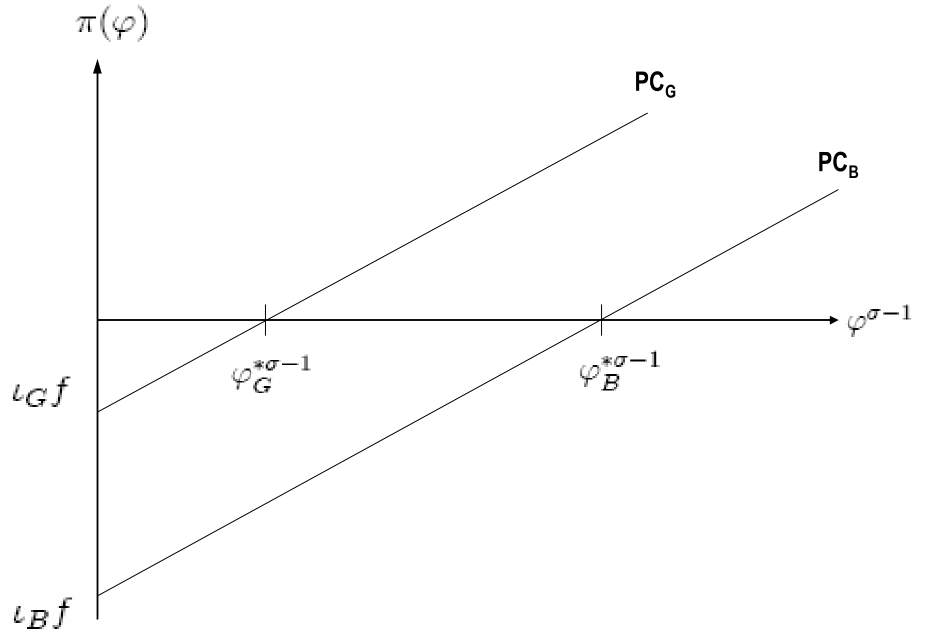
which is strictly greater than 1 (in other words $P_B > P_G$). Aggregate CES prices are always counter-cyclical. A good state leads to lower prices while a bad state leads to higher prices, amplifying the effect of the business cycle shocks. The larger the γ shock, the larger the fluctuation in aggregate prices and welfare.

Diagrammatic Representation Diagrammatically, the equilibrium can be illustrated in Figure 1[12]. The profit conditions here are forward looking. Once a firm has invested fixed cost f in the last period, it will definitely produce in the current period because of the CES demands; it does not care about cutoffs. However, the firm has to decide whether to quit or to continue investing f. The Y-axis therefore represents not realised average profits firms earn but expected profits. The X-axis represents the cutoff level of productivity below which firms will choose not to invest in f and quit the market.

[12] Note that by putting φ in the X-axis raised to the power of $\sigma - 1$, the profit conditions become straight lines. The level of capital costs becomes the Y-axis intercepts [see Helpman, Melitz and Yeaple (2004)].

Therefore, while there is an exogenous mass of M heterogeneous firms along the entire distribution of $G(\varphi)$, the number of firms that stay in the market is endogenous. Not all are sufficiently productive to stay in the market given the cost of capital. Since aggregate savings are higher in the good state, there will be a larger mass of firms that will invest in f as compared to the bad state $M_G > M_B$. The larger the γ shock, the greater is the firm mass ratio θ, which results in a larger aggregate price fluctuation.

Figure 1: Profit Conditions In Autarky



A Simple Numerical Example This subsection provides a simple numerical example to the equilibrium just characterised. The parameters used here are not meant to be realistic as the purpose of this exercise is simply to demonstrate the equilibrium effects in the presence of shocks. The productivity distribution $G(\varphi)$ is assumed to be pareto with support at 0.1 and shape of $k = 4$. The rest of the parameters are provided in Table 1.

The equilibrium at two levels of γ shocks are given in Table 2.

Table 2 illustrates a clear point. The larger the aggregate shock γ, the larger the differences between good and bad state variables. There is

Table 1: Parameters for Firms and Business Cycles

Parameter	H	σ	λ	ρ	β	M	f
Value	1000	4	0.5	0.75	0.9	100	1

Table 2: Equilibrium of Business Cycles

γ	φ_G^*	φ_B^*	ι_G	ι_B	M_G	M_B	E_G	E_B	P_G	P_B
0.05	0.1322	0.1355	1.083	1.138	32.76	29.64	1162.6	1051.8	1.986	2.003
0.10	0.1305	0.1372	1.053	1.164	34.51	28.23	1217.7	996.3	1.977	2.011

greater fluctuation of the price level between P_B and P_G. Note that the aggregate prices are counter cyclical - a good state leads to lower prices while a bad state leads to higher prices. Given the per period indirect utility in equation (2), the counter-cyclical price fluctuations therefore amplify the effect of expenditures E_G and E_B, resulting in welfare loss for the risk-averse consumer.

4 Opening to Trade

Despite firm heterogeneity softening the impact of fewer firms investing in a bad state, there continues to be fluctuation in the aggregate prices caused by business cycle shocks. The important welfare question is: can trade integration between two economies reduce the fluctuation?

In answering this question, a few simplifying assumptions should be made. Firstly, the consumers' expenditures in both economies continue to be uncorrelated after opening to trade. There is no insurance or risk-sharing between consumers of both economies[13]. The implied assumption here is that the international capital market exists for firms only, it does not facilitate borrowing or lending for consumption smoothing. This assumption greatly simplifies the characterisation of the trade equilibrium since it ignores the potential interactions between consumers of two different countries. For the firms, the effect of this assumption is that aggregate demands are uncorrelated across countries. This is not a wholly realistic assumption, but is nevertheless well supported empirically. Indeed, the lack of correlation between consumption of countries

[13] This could be due to incentives issues such as moral hazard, or costly monitoring and high transaction costs. Because of these reasons, income insurance between countries is not widely observed. Therefore, the trading of international bonds is ruled out.

is just one of the six major puzzles of international macroeconomics [see Maurice Obstfeld and Kennedy Rogoff (2000)].

Secondly, there is a perfectly competitive international capital market that allows savings in one economy to be invested towards fixed cost f in another, and net revenue costless remitted back to capital owners for consumption. Consumers (savers) in one economy can invest into and become owners of firms in the other economy in return for next period's profits.

Thirdly, the paper considers only two-country trade for the ease of exposition and to bring out the analytical results more clearly. Nevertheless, as the reader shall see, the insights can be easily extended to multi-country trade.

5 Two Country Model

Two economies are identical in every way - labour size L, average productivity H, preferences, production technology and productivity distribution. They also have the same mass of firms M on the same productivity distribution $G(\varphi)$. However, both have independent aggregate shocks even after they are open to trade.

Proposition 3 *With free trade, both economies will always have a common productivity cutoff.*

Proof. The proof can be made by contradiction. With free trade, every firm has complete market access into both markets wherever they are located. With free trade, the levels of competitive intensity (characterised by the trade weighted CES price aggregates) are also the same in both locations. Therefore, a firm has to be indifferent between the two locations. Suppose one location (labelled as Home) has a productivity cutoff of φ_H^* while the other (labelled as Foreign) has a cutoff of φ_F^* such that $\varphi_H^* \neq \varphi_F^*$, a firm that lies between φ_H^* and φ_F^* is above one cutoff (profitable) and below the other cutoff (unprofitable). There exists a mass of firms between φ_H^* and φ_F^* that will not be indifferent since they can invest f in one of the market with positive expected profits. This violates the definition of productivity cutoffs (this proposition will be given a further formal proof later). ■

5.1 Open Economy with Trade Costs

5.1.1 Iceberg Trade Cost

Variable trade costs are introduced as the standard iceberg trading cost of $\tau > 1$ for every unit of good shipped across the economies. With only variable trade cost, the price of export is simply a mark-up over the price of domestic sales $p_X = \tau p$.

Proposition 4 *In the presence of iceberg trade costs, the productivity cutoffs between countries cannot be equalised when they are faced with asymmetric shocks.*

Proof. The paper first sketch a intuitive proof, with the formal proof provided later in the next sub-section. Suppose Home and Foreign economies have asymmetric shocks (Home in a bad state and Foreign in a good state with no loss of generality) and that cutoffs are equalised $\varphi_H^* = \varphi_F^*$. If cutoffs are equalised, the mass of firms investing f is the same in both locations given the assumption of a fixed number (or density) of firms along the same productivity distribution. If the cutoffs are the same at both locations, the aggregate price indices will be equal at both locations whatever the level of trade costs. Since Home is in a bad state, the expected aggregate expenditure, taking into account both domestic and export revenue subjected to trade cost, is

$$\{\rho R_B + (1-\rho)R_G + \phi[\rho R_G + (1-\rho)R_B]\}$$

This is strictly smaller than the expected aggregate expenditure of the Foreign economy

$$\{\phi[\rho R_B + (1-\rho)R_G] + \rho R_G + (1-\rho)R_B\}$$

since it is in a good state and $\phi < 1$ because of trade costs. If φ_H^* defines the firm having zero expected profit if it invests f at Home, a firm with this productivity must have positive expected profits in Foreign given the larger expected market size there. This violates the definition of φ_F^* as the productivity cutoff. ■

5.1.2 Equilibrium Characterisation With Iceberg Cost

This subsection proceeds to characterise the productivity cutoffs in the presence of iceberg trade cost. The impact of fixed export costs f_X will be briefly discussed in Appendix A.

Symmetric Shocks

Proposition 5 *The productivity cutoff, common to both economies, is φ_G^* when they are in the good state; and is φ_B^* when both economies are in the bad state.*

Proof. Consider the case when both economies are in the bad state. Whatever the level of τ, both economies have the expected aggregate revenues since they are hit with symmetric shocks. Furthermore, both economies will have low aggregate savings, with the same aggregate resource constraint in equation (12). Hence, there is no capital flow between the economies. This pins down a common productivity cutoff φ_B^*. The same reasoning applies when both economies are in the good state. ■

Asymmetric Shocks The only case where iceberg cost results in different cutoffs is when Home and Foreign are hit with asymmetric shocks. In this case, aggregate savings in both economies are different and there is the possibility of capital flows affecting the productivity cutoffs in each economy.

Without a loss of generality, suppose Home economy has the bad state while Foreign has the good state, and that trade cost is positive $\tau > 1$. Given $E_H = E_B$ and $E_F = E_G$[14], the trade equilibrium is a set of variables $\{\varphi_H^*, \varphi_F^*, \iota_M\}$ that satisfy the following conditions, where ι_M is the cost of capital faced by the firms in the Home and Foreign economy respectively.

First, the marginal firms with productivities φ_H^* and φ_F^* must have zero profits in their respective locations. This gives the pair of equations

$$\frac{p(\varphi_H^*)^{1-\sigma}}{P_H^{1-\sigma}}[\rho R_B + (1-\rho)R_G] + \frac{\phi p(\varphi_H^*)^{1-\sigma}}{P_F^{1-\sigma}}[\rho R_G + (1-\rho)R_B] = \iota_M \sigma f$$

[14]Note that from equation (19), since there is no insurance across consumers in the different countries, their levels of expenditures are affected by their domestic shocks only.

$$\frac{\phi p(\varphi_F^*)^{1-\sigma}}{P_H^{1-\sigma}}[\rho R_B+(1-\rho)R_G]+\frac{p(\varphi_F^*)^{1-\sigma}}{P_F^{1-\sigma}}[\rho R_G+(1-\rho)R_B]=\iota_M\sigma f$$

where $p(\varphi_H^*)=\frac{\sigma}{\sigma-1}\frac{1}{\varphi_H^*}$ and $p(\varphi_F^*)=\frac{\sigma}{\sigma-1}\frac{1}{\varphi_F^*}$ are the optimal prices charged by the marginal firms, P_H and P_F are the trade weighted CES price aggregates. By substituting the expressions for the CES price aggregates and cancelling out some terms, the above equations become simplified to

$$\frac{\varphi_H^{*\sigma-1}[\rho R_B+(1-\rho)R_G]}{M_H\tilde{\varphi}_H^{\sigma-1}+\phi M_F\tilde{\varphi}_F^{\sigma-1}}+\frac{\varphi_H^{*\sigma-1}\phi[\rho R_G+(1-\rho)R_B]}{\phi M_H\tilde{\varphi}_H^{\sigma-1}+M_F\tilde{\varphi}_F^{\sigma-1}}=\iota_M\sigma f \quad (24)$$

$$\frac{\varphi_F^{*\sigma-1}\phi[\rho R_B+(1-\rho)R_G]}{\phi M_H\tilde{\varphi}_H^{\sigma-1}+M_F\tilde{\varphi}_F^{\sigma-1}}+\frac{\varphi_F^{*\sigma-1}[\rho R_G+(1-\rho)R_B]}{M_H\tilde{\varphi}_H^{\sigma-1}+\phi M_F\tilde{\varphi}_F^{\sigma-1}}=\iota_M\sigma f \quad (25)$$

Secondly, given the global pool of savings which is the resource constraint, the total number of firms is given as

$$[1-G(\varphi_H^*)]M+[1-G(\varphi_F^*)]M=\frac{H(1+\gamma)+E_G\left(\frac{\lambda-\sigma}{\sigma}\right)}{f}+\frac{H(1-\gamma)+E_B\left(\frac{\lambda-\sigma}{\sigma}\right)}{f} \quad (26)$$

Together, these provide three conditions to solve for $\{\varphi_H^*, \varphi_F^*, \iota_M\}$ given E_G and E_B. Note that it must be the case that $\varphi_H^* \geqslant \varphi_F^*$ in equilibrium since there is a smaller set of producers for the Home country (which is in a bad state)[15].

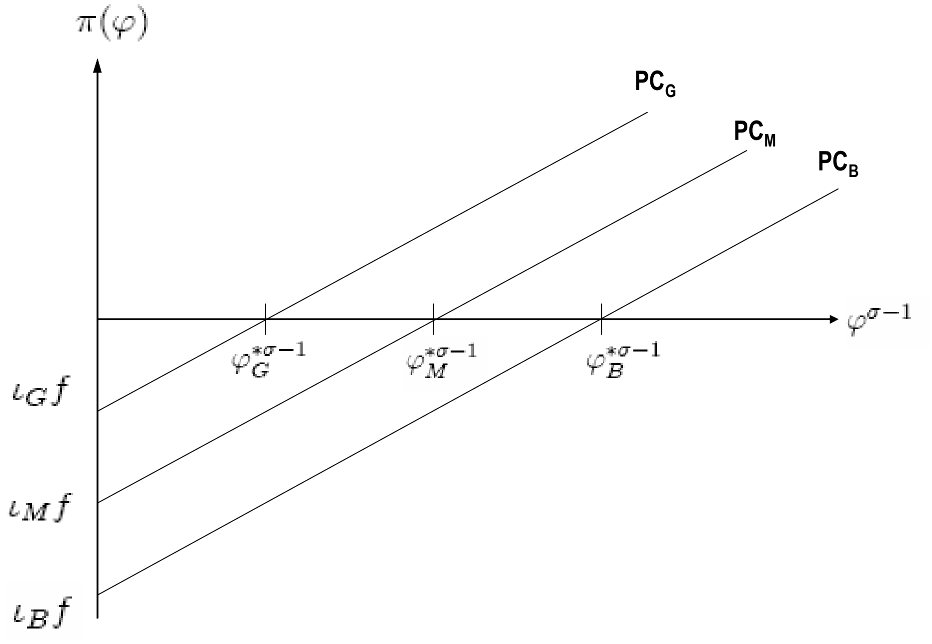
5.1.3 The Diversification Effect with Free Trade

Given the characterisation of the equilibrium with costly trade (positive iceberg costs), it is easy to show the equilibrium effects under free trade. With equations (24) and (25), one arrives at the following equality

$$\begin{aligned}&\frac{\varphi_H^{*\sigma-1}[\rho R_B+(1-\rho)R_G]}{M_H\tilde{\varphi}_H^{\sigma-1}+\phi M_F\tilde{\varphi}_F^{\sigma-1}}+\frac{\varphi_H^{*\sigma-1}\phi[\rho R_G+(1-\rho)R_B]}{\phi M_H\tilde{\varphi}_H^{\sigma-1}+M_F\tilde{\varphi}_F^{\sigma-1}}\\ =\;&\frac{\varphi_F^{*\sigma-1}\phi[\rho R_B+(1-\rho)R_G]}{\phi M_H\tilde{\varphi}_H^{\sigma-1}+M_F\tilde{\varphi}_F^{\sigma-1}}+\frac{\varphi_F^{*\sigma-1}[\rho R_G+(1-\rho)R_B]}{M_H\tilde{\varphi}_H^{\sigma-1}+\phi M_F\tilde{\varphi}_F^{\sigma-1}}\end{aligned}$$

[15] Note that even though trade cost is positive, the two economies continue to have a common cost of capital ι_M because capital is completely mobile. That is, the last unit of capital f invested must recover the same expected amount $\iota_M f$ in both economies even though they have asymmetric shocks. Therefore, $\iota_H=\iota_F=\iota_M$ in equilibrium even in the presence of positive trade cost. The fact that productivity cutoffs are not equalised is due to trade cost altering the degree of capital flows between the two economies when they have asymmetric shocks.

Figure 2: Profit Conditions with Free Trade



With free trade ($\phi = 1$), the aggregate prices - given by the denominators - are equal. The expected revenues in brackets also become equal. Together, these imply that $\varphi_H^* = \varphi_F^*$ even in the presence of asymmetric shocks. A Home firm will be a perfect substitute for the foreign firm. In other words, free trade will result in a common cutoff φ_M^* even with asymmetric shocks to both economies. This is the formal proof to Proposition 3. The effects of free trade can be seen in Figure 2.

This result can also be inferred from the firm mass equations in (20). These give the firm masses in equilibrium with the good and bad state, which is a function of γ shocks and other parameters only. In a fully integrated economy with free trade, it simply means that the γ shocks cancel out. With free trade, the firm mass that is common to both economies becomes

$$M_M^{FT} = \frac{H}{f}\left[\frac{\lambda\beta\psi}{\lambda\beta\psi - \lambda + \sigma}\right] \tag{27}$$

Since there is a common firm mass, there is a common cutoff φ_M^* [analogous to the relationships specified in equation (10)][16].

From equation (23), the ratio of aggregate prices is a function of productivity cutoffs only. Denoting free trade variables with superscript FT, the price ratios become $\frac{P_B^{FT}}{P_G^{FT}} = \theta^{\frac{k+1-\sigma}{k(\sigma-1)}}$, where P_B^{FT} denotes the aggregate price when both economies are faced with a negative shock (analogous definition for P_G^{FT})[17]. With asymmetric shocks, the productivity cutoff becomes φ_M^* for both economies (where $\varphi_B^* > \varphi_M^* > \varphi_G^*$) given free trade. As the γ shocks are cancelled out, the aggregate price level with asymmetric shocks P_M^{FT} therefore lies between P_B^{FT} and P_G^{FT}.

There are now two sources of gains from trade. Firstly there is an expansion of varieties leading to lower aggregate prices and higher welfare. Secondly, there is a reduction in the probability that extreme price levels are reached. This reduces the variance in the aggregate price level and the fluctuation in real income, thereby representing a welfare gain from diversification for the risk-averse consumer. This is a gain from trade above and beyond the expansion of variety effect.

Free trade therefore results in the optimal allocation of market shares for there will always be an equally productive subset of firms producing in each economy and selling across markets. The result is that productivity cutoffs are completely equalised even as countries face asymmetric shocks.

When economies are hit with asymmetric shocks, there is essentially a diversification equilibrium. For example, suppose the Home economy is in a bad state. Under autarky, the cutoff productivity would have been φ_B^*. However, if the trading partner Foreign is in a good state, Home's cutoff productivity falls to φ_M^* with free trade. In other words, some firms that would have quit a domestic negative shock at Home in autarky will now continue to produce, as expected profits from exporting more than compensate for the expected domestic loss. Aggregate firm-level productivity therefore does not always increase with trade.

Finally, there is a subtle implication from the price ratio $\frac{P_B^{FT}}{P_G^{FT}} = \theta^{\frac{k+1-\sigma}{k(\sigma-1)}}$. The parameter k characterises the level of firm heterogeneity.

[16] In the presence of trade cost, the firm mass is always larger in the country with the positive shock since the expected market size is bigger. The economy with the negative shock will have a smaller firm mass and higher price aggregate. Without free trade, aggregate prices are not equalised with asymmetric shocks. Given that consumers are risk-averse, this is welfare-reducing.

[17] Note that since both economies are in the same state, the cutoffs are unchanged from the autarky counterparts, which are φ_B^* for the bad state and φ_G^* for the good state.

A smaller k implies that firms are more heterogenous while a larger k implies that firms are more homogenous. A decrease in k would lead to an increase in $\frac{k+1-\sigma}{k(\sigma-1)}$, propagating the aggregate price fluctuations. In other words, if firms are more heterogeneous, the net effect (after accounting for firm entry and changes in aggregate productivity) is greater price fluctuation. This suggests that the diversification gains from trade are higher when firms are more heterogeneous.

5.2 Why Iceberg Trade Cost Matters: Comparison with Melitz Model

In the Melitz model with the absence of fixed export cost, the passage from autarky to free trade (by τ falling from infinity to 1) increases welfare through the CES price aggregates, with no further impact on firm level variables. The reason for this is that a fall in τ increases local competitive intensity through the price index but also increases export revenue, leaving the firm exactly indifferent.

However, the level τ is crucial here and affects the productivity cutoffs. The key here is to realise that Melitz presents a model which is a long run stable equilibrium of countries of symmetric sizes, "Firms correctly anticipate this stable aggregate environment when making all relevant decisions. The analysis then focuses on the long run effects of trade and the relative behaviour and performance of firms with different productivity levels." In that model, both consumption demand and investment into firms are constant. The presence of iceberg cost therefore does not have any further effect since it preserves the homotheticity amongst all firms.

In this paper, even though both economies have a long run average size of LH, each of them fluctuates around two states defined by the Markov process. The pool of aggregate savings in each economy changes according to the shocks, thereby changing the resource available for investment, and in the process altering the survivability conditions in different states.

5.2.1 Cross Border Capital Flow

Given that aggregate savings are not the same when the countries are faced with assymetric shocks, there will be cross border capital flow.

Through its effects on expected market potentials, τ changes the incentive for cross-border capital flows. A lower τ provides higher incentive for the high savings economy (good state) to invest into the low savings economy (bad state), until the productivity cutoffs are completely equalised with free trade. Conversely, a higher trade cost τ creates a divergence between the perceived market sizes when the economies are hit with asymmetric shocks and reduces the diversification effect. Trade liberalisation therefore dampens differences in productivity cutoffs between two economies when they are hit with asymmetric shocks, leading to lower price-output fluctuations.

The key point is this: free capital mobility, in the presence of positive trade costs, cannot equalise the productivity cutoffs between two economies when they are hit with asymmetric shocks. Therefore, free capital mobility alone cannot replicate free trade outcomes. Since productivity cutoffs are unequal with asymmetric shocks and positive trade costs, market shares are not allocated in the most efficient way between heterogeneous firms across the two economies. Only with free trade will there be optimal allocation of market shares between productively heterogeneous firms across countries - that is, an equally productive subset of producers in each country (above a common productivity cutoff of φ_M^* when there is asymmetric shocks) will stay in the market.

5.3 Extension to Output and Multiple Countries

In the setup of the model, the paper has modelled welfare changes to the consumer through the impact of trade on the CES price aggregates. However, there is an simple conceptual extension to output. If one considers the differentiated sector as an immediate sector supplying a final competitive sector as in the Ethier production function [see also Anthony J. Venables (1996)], the smaller price fluctuation shown here directly translates into smaller output fluctuation of the final sector. Free trade therefore reduces output fluctuation in this interpretation. So long as the consumer is risk-averse, the lower fluctuation of price-output will be a source of welfare gain.

Furthermore, if a large number of countries with uncorrelated γ shocks are engaged in free trade, all of them will converge to φ_M^*, completely stabilising aggregate price-output across all economies. Except to note that this result is obvious from the Central Limit Theorem, this will not be given any formal proof.

6 Conclusion

This paper has built on recent trade and firm heterogeneity literature, in particular the aggregation properties of Melitz (2003) in the presence of firm heterogeneity. Real Business Cycles type aggregate productivity shocks, with consumers who optimise inter-temporally, are introduced into a setting where firms have to invest in fixed assets before the realisation of the shocks. This model therefore makes the firm's problem more realistic compared to traditional trade models.

Without firm heterogeneity, a negative shock would result in all firms making losses, thereby rendering any between-firms analysis meaningless. As it is now possible to solve for market and firm level outcomes in the presence of firm heterogeneity, it has become meaningful to analyse the reallocative impact of such shocks. Different firms will be affected differently while still allowing for market aggregates to be solved analytically.

When trade is not totally free, the productivity cutoffs cannot be equalised, and some producers are shielded by trade barriers and will continue to have positive market shares. With free trade, productivity cutoffs are always equalised in both economies in a full diversification outcome - which means that an equally productive subset of producers remain in the market. Nevertheless, the diversification equilibrium also implies that aggregate firm-level productivity does not always increase with trade. Weak companies that would have quit in a negative demand shock in autarky can continue to operate given diversification possibilities.

Despite this, the model offers a comforting result for trade economists by identifying another source of trade gains. The key to unlocking the insight from the model lies in understanding that opening to trade results in smaller fluctuation in the aggregate price levels and may therefore raise the welfare of risk-averse consumers.

7 Appendix

7.1 Distribution of Aggregate Revenues and Profits

This subsection highlights the distribution of aggregate revenues and profits across firms. It will show that aggregate revenues are independent of the number of existing firms. The stream of dividend to the consumers, which depends on aggregate revenues only, is therefore also unaffected by the number of firms. This shows that the good and bad state resource constraints in equations (8) and (9) are also independent of firm level considerations.

The consumer is forward looking. Once the current state is realised, his adjustment to E_G or E_B is instant, pinning down the current period's market size or aggregate revenue for the industry (R_G or R_B). However, the revenue per firm depends on the number who invested f in the previous period, which depends on the last period's realised state. There could either be M_G or M_B firms investing f previously, who will share revenue this period

$$R_G = M_B \bar{R}_{G,B} = M_G \bar{R}_{G,G}$$

where $\bar{R}_{G,B}$ denotes the average per firm revenue conditioned on a previously bad state (analogous for $\bar{R}_{G,G}$). Since $M_G > M_B$, the per firm revenue is higher when there are fewer competitors $\bar{R}_{G,B} > \bar{R}_{G,G}$. Similarly for the bad state

$$R_B = M_B \bar{R}_{B,B} = M_G \bar{R}_{B,G}$$

where $\bar{R}_{B,B} > \bar{R}_{B,G}$. Therefore, conditioning out the current state, average revenue is always higher when the previous state is bad. Since the ratio of average productivity is directly related to the ratio of average productivity (to the power of $\sigma - 1$), this shows that average productivity of firms is higher following a bad state.

$$\frac{\bar{R}_{G,B}}{\bar{R}_{G,G}} > 1 \Rightarrow \left(\frac{\tilde{\varphi}_B}{\tilde{\varphi}_G}\right)^{\sigma-1} > 1 \Leftarrow \frac{\bar{R}_{B,B}}{\bar{R}_{B,G}} > 1 \tag{A1}$$

This result shows that $\varphi_B^* > \varphi_G^*$. Conditioned on the current state, average profit is therefore also higher if the previous state is bad.

Per firm profit is higher following a bad state To develop this idea more formally, one can show that

$$R_{G,B} = \int_{\varphi_B^*}^{\infty} \frac{p(\varphi)^{1-\sigma} E_G}{P_B^{1-\sigma}} M_B g(\varphi) d\varphi = \int_{\varphi_G^*}^{\infty} \frac{p(\varphi)^{1-\sigma} E_G}{P_G^{1-\sigma}} M_G g(\varphi) d\varphi = R_{G,G}$$

It does not matter what the previous state is, aggregate revenue R_G depends on only the current state. Furthermore, operating profit will also be $\frac{R_G}{\sigma}$ if today is a good state. Similarly

$$R_{B,B} = \int_{\varphi_B^*}^{\infty} \frac{p(\varphi)^{1-\sigma} E_B}{P_B^{1-\sigma}} M_B g(\varphi) d\varphi = \int_{\varphi_G^*}^{\infty} \frac{p(\varphi)^{1-\sigma} E_B}{P_G^{1-\sigma}} M_G g(\varphi) d\varphi = R_{B,G}$$

This establishes the following inequalities

$$\bar{R}_{G,B} = \frac{R_{G,B}}{M_B} > \frac{R_{G,G}}{M_G} = \bar{R}_{G,G} \qquad \bar{R}_{B,B} = \frac{R_{B,B}}{M_B} > \frac{R_{B,G}}{M_G} = \bar{R}_{B,G} \quad \text{(A2)}$$

Since the consumers optimise instantly, R_G and R_B are pinned down immediately. However, the number of firms selling in this period has the lag effect of investing f the previous period. Aggregate revenue is therefore shared among the mass of firms determined in the previous period, and the market shares allocated as such. However, aggregate revenues are unaffected by the number of firms since $R_{G,B} = R_{G,G}$ (good state) and $R_{B,B} = R_{B,G}$ (bad state). The stream of dividend for consumers in each state is therefore also unaffected by the number of firms.

7.2 Equilibrium Characterisation with Fixed Export Cost

The firm heterogeneity literature is motivated by the empirical evidence that only a small and productive subset of firms engage in exporting activities. The presence of iceberg trade cost alone does not create this export partitioning, due to the CES preferences. In order to achieve export partitioning, a fixed export cost f_X has to be introduced. This paper assumes that f_X has exactly the same conditions attached to f - it is funded through aggregate savings and has to be invested one period before export can take place.

For exporters to be a small and more productive subset of all firms, there must exist firms with productivity below φ that find it profitable to operate domestically (with domestic revenue r_D) but not export (thereby foregoing revenue r_X). The two inequalities therefore become

$$\frac{r_D(\varphi)}{\sigma} - f > 0 \qquad\qquad \frac{r_D(\varphi)}{\sigma} + \frac{r_X(\varphi)}{\sigma} - f - f_X < 0$$

where $r_X(\varphi) = \tau^{1-\sigma} r_D(\varphi)$ due to the CES preference. Together, the partitioning condition implies that

$$f < \tau^{1-\sigma} f_X$$

which says that the combination of iceberg cost and fixed export cost must be high enough to deter some firms from exporting. Define φ_X^* as the marginal firm that just breaks even through exporting. The probability that a firm is strong enough to export is the conditional probability of a firm having a distribution above φ_X^*. This conditional probability $\tilde{p}_X$ is given as

$$\tilde{p}_{HX} = \frac{1 - G(\varphi_{HX}^*)}{1 - G(\varphi_H^*)}$$

Note that $r_D(\varphi_H^*) = \sigma \iota_H f$ and $r_X(\varphi_{HX}^*) = \phi r_D(\varphi_{HX}^*) = \sigma \iota_H f_X$. Taking ratios of the two gives the following relationship

$$\phi \left(\frac{\varphi_{HX}^*}{\varphi_H^*} \right)^{\sigma-1} = \frac{f_X}{f} \qquad \text{or} \qquad \varphi_{HX}^* = \frac{1}{\tau} \left(\frac{f_X}{f} \right)^{\frac{1}{\sigma-1}} \varphi_H^*$$

which says that the export cutoff φ_{HX}^* is a function of domestic cutoff φ_H^* only. This allows the conditional export probability $\tilde{p}_X$ to be determined as a function of parameters only.

Suppose Home and Foreign are in a good state. The aggregate resource constraint from equation (26) becomes modified as

$$[1 - G(\varphi_G^*)]M = \frac{H(1+\gamma) + E_G\left(\frac{\lambda-\sigma}{\sigma}\right)}{f + \tilde{p}_{HX} f_X} \tag{A3}$$

In other words, global aggregate savings have to be used to fund fixed cost f as well as the the f_X requirements of exporters. By inspecting equation (A3) and comparing it with equation (26), it is clear that the effect of fixed export cost will shift φ_G^* rightwards (higher). The effect of f_X creates another source of resource competition. As exporters demand f_X, there will be fewer firms in equilibrium and productivity cutoffs will have to increase. Similar analytical reasoning can be applied to when both economies are in a bad state or when they have asymmetric shocks. The effect of fixed export cost will always push productivity cutoffs higher.

Assuming that export partitioning holds, the conditional probability of exporting $\tilde{p}_{HX}$ is strictly less than 1. The effects of trade liberalisation (as characterised by a fall in τ) can be seen from the above equation. As τ falls, the conditional probability of exporting increases. This increases the denominator of equation (A3), leading to an increase in the productivity cutoffs φ_G^* through the competition of resource.

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