Abstract
This paper shows that, with (partial) irreversibility, higher uncertainty reduces the impact effect of demand shocks on investment. Uncertainty increases real option values making firms more cautious when investing or disinvesting. This is confirmed both numerically for a model with a rich mix of adjustment costs, time-varying uncertainty, and aggregation over investment decisions and time, and also empirically for a panel of manufacturing firms. These cautionary effects of uncertainty are large - going from the lower quartile to the upper quartile of the uncertainty distribution typically halves the first year investment response to demand shocks. This implies the responsiveness of firms to any given policy stimulus may be much lower in periods of high uncertainty, such as after major shocks like OPEC I and 9/11.

Keywords: Investment, uncertainty, real options, panel data
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1. Introduction

Recent theoretical analyses of investment under uncertainty have highlighted the effects of irreversibility in generating 'real options' (e.g. Dixit and Pindyck (1994)). In these models uncertainty increases the separation between the marginal product of capital which justifies investment and the marginal product of capital which justifies disinvestment. This increases the range of inaction where investment is zero as the firm prefers to 'wait and see' rather than undertaking a costly action with uncertain consequences. In short, investment behaviour becomes more cautious.

Firm-level data is attractive for investigating this effect of uncertainty on the degree of caution since empirical measures of uncertainty can be constructed based on share price volatility (e.g. Leahy and Whited (1996)). One important difficulty for direct testing of real options models of investment under uncertainty using firm data, however, is the extreme rarity of observations with zero investment in annual consolidated accounts. If we believed that these firms make a single investment decision in each year this lack of zeros would reject the canonical real options model of a single investment decision with its region of inaction. However, given the extensive evidence of discrete and lumpy adjustments in more disaggregated plant-level data (e.g. Doms and Dunne (1998)), this lack of zeros at the firm level is suggestive of aggregation over types of capital, production units and time.

Previous research has shown that aggregation does not eliminate the impact of lumpy micro investment decisions for more aggregated investment dynamics.[1] This raises the question of whether the effects of uncertainty and irreversibility on short run investment dynamics can be detected in an econometric study of firm-level investment spending. To investigate this issue we develop a model of the firm's investment decisions that allows for two types of capital, a rich specification of adjustment costs, time-varying uncertainty, alternative functional forms for the

[1] See, for example, Bertola and Caballero (1994), Caballero and Engel (1999), Abel and Eberly (2001), and Doyle and Whited (2001). Thomas (2002) and Veracierto (2002) find that in general equilibrium models the impact of non-convex investment costs on the business cycle may be small. These papers are necessarily based on relatively simple models of firm investment - including a constant level of uncertainty - to enable complex general equilibrium modelling. Our focus here is on much richer (partial equilibrium) micro models that include fluctuations in the level of uncertainty. These are appropriate for estimation on firm-level data.

revenue function and extensive aggregation over time and over production units. We solve this theoretical model numerically and simulate firm-level panel data. We use this simulated data in two ways. First we analyse it directly to confirm two properties of firm-level investment dynamics in this framework. One property is the effect of higher uncertainty on the degree of caution in investment decisions as noted above. We show that, with (partial) irreversibility, the impact effect on investment of a given firm-level demand shock tends to be weaker for firms that are subject to a higher level of uncertainty. We also show that the response of investment to demand shocks tends to be convex, as larger shocks induce firms to invest in more types of capital and at more production units (the extensive margin). This in turn induces more adjustment at the intensive margin, with these aggregation effects being reinforced by supermodularity in the production technology.

We also use our simulated data to show that both of these effects can be detected using a relatively simple dynamic econometric specification to approximate the complex firm-level investment dynamics implied by this framework. Our starting point is an error correction model (ECM) of investment that has been widely used in firm-level studies. We add two types of terms. First, an interaction between real sales growth and measured uncertainty tests for the more cautious response of investment to demand shocks at higher levels of uncertainty. Second, a non-linear sales growth term to test for convexity in the response of investment to demand shocks. Generalised Method of Moments (GMM) estimation on the simulated panel data indicates that we can reject the null hypothesis of a common, linear response of investment to demand shocks, provided the dynamic specification used is sufficiently rich for standard tests of overidentifying restrictions not to indicate severe misspecification of the econometric model.

We then apply the same econometric approach to study the investment behaviour of a sample of 672 publicly traded UK manufacturing companies over the period 1972 to 1991. We find evidence both of more cautious investment behaviour for firms subject to greater uncertainty, and of a convex response of investment to real sales growth. While there may be other explanations for these patterns in company investment dynamics, we conclude that the investment behaviour of large firms is consistent with a partial irreversibility model in which uncertainty dampens the short run adjustment of investment to demand shocks.

Finally, simple simulations using our estimated econometric model suggest that observed fluctuations in uncertainty can play an economically important role in shaping firm-level investment decisions. For example, we find that a one standard deviation increase in our measure of uncertainty, as occurred after 9/11 and the first OPEC oil crisis, can halve the impact effect of demand shocks on company investment. While we do not model the behaviour of labour demand, the existence of similar labour hiring and firing costs would imply that higher uncertainty would also make employment responses to demand shocks more cautious. This suggests that firms will generally be less responsive to monetary and fiscal stimulus in periods of high uncertainty, which is important for policy-makers trying to respond to major shocks during periods of high uncertainty.[2] Several papers have also reported evidence of an increase in firm-specific uncertainty in the US and other OECD countries in recent years,[3] which our analysis indicates could have significant effects on investment dynamics.

The plan of the paper is as follows. Section 2 considers two implications of uncertainty and irreversibility for investment behaviour, and confirms these numerically using simulated data. Section 3 develops our econometric investment equation and shows, using the simulated data, that tests based on this model can detect these effects on investment dynamics. Section 4 takes this econometric model to real company investment data to test for the presence of these effects, while section 5 examines their magnitude. Section 6 offers some concluding remarks.

2. Simulating investment dynamics under uncertainty

The typical model in the literature considers investment in a single partially irreversible capital good, with a Cobb-Douglas revenue function and demand conditions which follow a Brownian motion process with constant variance. Investment only

[2] See Bloom (2006) on the evidence for steep rises in uncertainty after major macro shocks.

[3] Campbell et al. (2001) study US firms in the period 1962-1997 and find an increase in the firm-level (but not market-level) volatility of annualised daily stock returns in the 1980s and 1990s compared to the 1960s and 1970s. See also Philippon (2003) for evidence of increased sales growth volatility for US firms, and Thesmar and Thoenig (2003) for similar evidence on French firms.

occurs when the firm's marginal revenue product of capital hits an upper threshold, given by the traditional user cost of capital plus an option value for investment. Similarly disinvestment only occurs when the marginal revenue product hits a lower threshold, given by the user cost for selling capital less an option value for disinvestment. The firm chooses to wait and do nothing if its marginal revenue product of capital lies between these two thresholds.

As the marginal revenue product of capital evolves stochastically over time this approach predicts that the firm will undertake sporadic bursts of investment or disinvestment, consistent with the typical evidence from plant-level data (see, for example, Doms and Dunne (1998) or Nilson and Schiantarelli (2003)). Abel and Eberly (1996) show by comparative statics that the option values are increasing in the (time invariant) level of uncertainty. This suggests that firms which face a higher level of uncertainty are less likely to respond to a given demand shock.

2.1. Aggregation and firm-level investment

Annual investment data for publicly traded UK and US firms, however, do not display the discrete switches from zero to non-zero investment regimes indicated by this basic model. In particular observations with zero investment spending are almost completely absent from their company accounts. Table 1 reports evidence from our sample of 672 UK manufacturing companies, and from a sample of UK manufacturing establishments that contain one or more plants at the same location. There are two distinct patterns of aggregation that can be observed: first aggregation across types of capital (structures, equipment and vehicles); and second aggregation across plants within the establishment or the firm. In both cases we observe a higher proportion of observations with zero investment when we consider more disaggregated data. There is also likely to be a third type of aggregation - temporal aggregation - as the frequency of shocks and investment decisions is likely to be much higher than that of the (annual) data.

[Table 1 about here]

In view of this we explicitly consider a framework in which firms invest in multiple types of capital goods, across multiple production units, and there is aggregation over time. These production units experience idiosyncratic unit-level productivity shocks as well as a common firm-level demand shock. In this more general framework, but in a model with a constant level of uncertainty and partial irreversibilities only, Eberly and Van Mieghem (1997) have shown that the optimal investment decisions for each unit will follow a multi-dimensional threshold policy. Extending this to allow for time-varying uncertainty and temporal aggregation provides two implications which are the focus of our simulation and empirical investigation.

The first implication is that the response of company investment to demand shocks should be lower at higher levels of uncertainty due to the "cautionary" effect of uncertainty. For each production unit or type of capital the option to wait and do nothing is more valuable for firms that face a higher level of demand uncertainty. Following a given positive demand shock investment by such firms is expected to be lower, as both less units (or types of capital) will invest (the extensive margin) and each unit (type) that does invest will invest less (the intensive margin), with any supermodularity in the production technology reinforcing these effects.[4] Similarly the impact of a given negative demand shock on firm-level disinvestment is also expected to be smaller for firms that face a higher level of uncertainty.

Second, the investment response will be convex in response to positive demand shocks and concave in response to negative demand shocks. When the firm experiences a positive demand shock it may invest in a greater number of production units or types of capital (the extensive margin) and it may invest more in each unit or type of capital (the intensive margin). Larger demand shocks will affect both margins, and any supermodularity in the production technology would make these two effects reinforcing. Thus, the more types of capital the firm is induced to invest in, the more it wants to invest in those types of capital which are already adjusting, generating a convex response. The same reasoning also suggests that the response of firm-level disinvestment to negative demand shocks will be concave.

[4] Supermodularity is a general concept for complementarity. A function $f:\mathbf{R}^n \rightarrow \mathbf{R}$ is defined as supermodular if $\forall\ x, x' \in \mathbf{R}^n,\ f(x) + f(x') \leq f(\min(x, x')) + f(\max(x, x'))$. If f is twice differentiable this implies $\frac{\partial^2 f(x_1, x_2, ..x_n)}{\partial x_i \partial x_j} \geq 0\ \forall\ i \neq j$. The Cobb-Douglas and CES production functions are both supermodular.

As these investment models do not have closed form solutions we cannot prove these properties analytically. In the next section we confirm them using numerical simulations.

2.2. The simulation model

We start by parameterising one model from the general class of supermodular homogeneous models that we are considering. Firms are assumed to operate a large collection of individual production units, with the number chosen to ensure that full aggregation has occurred. In the simulation this is set at 250 units per firm, chosen by increasing the number of units until the results were no longer sensitive to this number.[5]

Each unit faces an iso-elastic demand curve for its output, which is produced using labour and two types of capital. Demand conditions evolve as a geometric random walk with time-varying uncertainty, and have a unit-specific idiosyncratic component and a common firm-level component. Demand shocks, uncertainty shocks and optimisation occur in monthly discrete time. Labour is costless to adjust while both types of capital are costly to adjust.

2.2.1. The production unit model

In the basic model each production unit has a reduced form supermodular revenue function $R(X, K_1, K_2)$

$$R(X, K_1, K_2) = X^{\gamma} K_1^{\alpha} K_2^{\beta} \tag{2.1}$$

based on an underlying Cobb-Douglas production function after labour, a flexible factor of production, has been optimised out. Demand and productivity conditions have been combined into one index, X, henceforth called demand conditions. For computational tractability we normalize this demand conditions parameter through the substitution, $P^{\frac{1-\alpha-\beta}{\gamma}} = X$, so that the revenue function is homogeneous of degree

[5] In the UK Census of Production microdata the average size of a manufacturing production unit is about 20 employees. The mean size of firms in our sample is 4,440 employees, suggesting a mean of around 220 units per firm. Tests on specifications with different degrees of cross-sectional aggregation (5, 10 and 50 units per firm) and temporal aggregation (2, 4 and 6 periods per year) confirm the robustness of our results to these assumptions.

one in (P, K_1, K_2), where

$$R(X, K_1, K_2) = \widetilde{R}(P, K_1, K_2) \tag{2.2}$$
$$= P^{1-\alpha-\beta} K_1^{\alpha} K_2^{\beta}. \tag{2.3}$$

In the simulation we set $\alpha = 0.4$ and $\beta = 0.4$, corresponding to a 25% mark-up and constant returns to scale in the physical production function, with equal coefficients on each type of capital.

Demand conditions are a composite of a unit-level (P^U) and a firm-level (P^F) component, $P = P^U \times P^F$. The unit-level demand (or productivity) conditions evolve over time as an augmented geometric random walk with stochastic volatility:

$$P_t^U = P_{t-1}^U (1 + \mu(\sigma_t) + \sigma_t V_t^U) \qquad V_t^U \sim N(0,1) \tag{2.4}$$
$$\sigma_t = \sigma_{t-1} + \rho_\sigma(\sigma^* - \sigma_{t-1}) + \sigma_\sigma W_t \qquad W_t \sim N(0,1). \tag{2.5}$$

Here $\mu(\sigma_t)$ is the mean drift in unit-level demand conditions, σ_t^2 is the variance of unit-level demand conditions, σ^* is the long run mean of σ_t, ρ_σ is the rate of convergence to this mean, and σ_σ^2 is the variance of the shocks to this variance process. The terms V_t^U and W_t are the i.i.d. shocks to unit-level demand and variance conditions respectively.

The firm-level demand process is also an augmented geometric random walk with stochastic volatility, which for tractability we assume has the same mean and variance:

$$P_t^F = P_{t-1}^F (1 + \mu(\sigma_t) + \sigma_t V_t^F) \qquad V_t^F \sim N(0,1). \tag{2.6}$$

Hence, the overall demand process $\log P$ has drift $2\mu(\sigma_t)$ and variance $2\sigma_t^2$. While this demand structure may seem complex, it is formulated to ensure that units within the same firm have linked investment behaviour due to the common firm-level demand shocks and level of uncertainty, but also display some independent behaviour due to idiosyncratic shocks. The baseline value of $2\mu(\sigma_t)$ is set to 4% (average real sales growth), invariant to the level of uncertainty, although we also report below some experiments that allow for more general drifts.

The two types of capital are costly to adjust. We start by modelling only partial irreversibility adjustment costs whereby the resale price of a unit of capital is less than the purchase price. Capital type 1 is assumed more costly to adjust (for example, specialised equipment), while capital type 2 is less costly to adjust (for example, vehicles). For the simulation we set the resale loss for capital of type 1 to 50% and the resale loss for capital of type 2 to 20%.[6]

These adjustment costs are defined by the firm's adjustment cost function, $C(P, K_1, K_2, I_1, I_2)$. We assume, for numerical tractability, that newly invested capital enters production immediately, that both types of capital depreciate at an annualized rate of 10%, and that the firm has an annualized discount rate of 10%.

2.2.2. Solving the production unit model

The complexity of the model necessitates numerical simulation, but analytical results can be used to show that the problem has a unique-valued continuous solution,[7] and an (almost everywhere) unique policy function. This means our numerical results will be convergent with the unique analytical solution.

In principle we have a model with too many state variables to be solved using numerical methods given current computing power. The unit's optimization problem, however, can be simplified by noting that the revenue function, adjustment cost function, depreciation schedules and expectations operators are all jointly homogeneous of degree one in (P, K_1, K_2). This allows us to normalize by one state variable - capital type 1 - simplifying the model and dramatically increasing the speed of the numerical solution routine. This effectively gives us one state "for free", in that we estimate on two major state spaces ($\frac{P}{K_1}$ and $\frac{K_2}{K_1}$) but for three underlying state variables.

[6]Our choice of adjustment cost parameters is based on the literature where available, in particular Cooper and Haltiwanger (2006). The qualitative results from our analysis of the simulated data are not sensitive to moderate changes to the adjustment cost parameter values, although as discussed in section 3.2, they are sensitive to the type of adjustment costs considered.

[7]An application of Stokey and Lucas (1989) for the continuous, concave and almost surely bounded normalized returns and cost function for models with partial irreversibilities (this section) and quadratic adjustment costs; and Caballero and Leahy (1996) for the extension to models with fixed costs in section (3.2.2).

The optimization problem (before normalization) can be stated as:

$$\begin{aligned} V(P_t, K_{1t}, K_{2t}, \sigma_t) &= \max_{I_{1t}, I_{2t}} \widetilde{R}(P_t, K_{1t,} + I_{1t}, K_{2t} + I_{2t}) - C(P_t, K_{1t}, K_{2t}, I_{1t}, I_{2t}) \\ &+ \frac{1}{1+r} E[V(P_{t+1}, (K_{1t} + I_{1t})(1-\delta), (K_{2t} + I_{2t})(1-\delta), \sigma_{t+1})] \end{aligned}$$

where r is the discount rate, δ is the depreciation rate, $E[.]$ is the expectations operator, I_{jt} is investment in type j ($j = 1, 2$) capital at time t and K_{jt} is the stock of type j capital. Using the homogeneity in (P, K_1, K_2) this can be re-written as:

$$\begin{aligned} K_{1t} V(P_t^*, 1, K_{2t}^*, \sigma_t) &= \max_{I_{1t}^*, I_{2t}^*} K_{1t} \widetilde{R}(P_t^*, 1 + I_{1t}^*, K_{2t}^*(1 + I_{2t}^*)) - K_{1t} C(P_t^*, 1, K_{2t}^*, I_{1t}^*, I_{2t}^* K_{2t}^*) \\ &+ \frac{1}{1+r} K_{1t+1} E[V(P_{t+1}^*, 1, K_{2t+1}^*, \sigma_{t+1})] \end{aligned}$$

where starred variables are $K_2^* = \frac{K_2}{K_1}$, $P^* = \frac{P}{K_1}$, $I_1^* = \frac{I_1}{K_1}$ and $I_2^* = \frac{I_2}{K_2}$. Upon normalization by K_{1t} this simplifies to:

$$\begin{aligned} V(P_t^*, 1, K_{2t}^*, \sigma_t) &= \max_{I_{1t}^*, I_{2t}^*} \widetilde{R}(P_t^*, 1 + I_{1t}^*, K_{2t}^*(1 + I_{2t}^*)) - C(P_t^*, 1, K_{2t}^*, I_{1t}^*, I_{2t}^* K_{2t}^*) \\ &+ \frac{(1 + I_{1t}^*)(1-\delta)}{1+r} E[V(P_{t+1}^*, 1, K_{2t+1}^*, \sigma_{t+1})] \end{aligned}$$

which is a function of only the state variables $(\frac{P}{K_1}, \frac{K_2}{K_1}, \sigma)$. We let uncertainty, σ_t, take five equally-spaced values from 0.05 to 0.5, with a symmetric monthly transition matrix that is approximately calibrated against (the variance and autocorrelation of) our stock-returns measure of uncertainty for UK listed firms, described in section 4.1 below. The simulation is run on a state space of $(\frac{P}{K_1}, \frac{K_2}{K_1}, \sigma)$ of (100,100,5).[8]

2.2.3. Aggregation to firm-level data

Simulated data is generated by taking the numerical solutions for the optimal investment functions and feeding in demand and uncertainty shocks at a monthly frequency. The simulation is run for 60 months to generate an initial ergodic distribution. Annual firm-level investment data is then generated by aggregating across the two types of capital, across the 250 units and across 12 months within each year. Capital stocks and the level of the demand conditions are summed across all units at the end of each year, while uncertainty is measured as the average yearly value.

[8] We also need the optimal control space of (I_1^*, I_2^*) of dimension (100,100), so that the full returns function in the Bellman equation has dimensionality (100,100,100,100,5). The program and a manual explaining the underlying techniques are available at http://cep.lse.ac.uk/matlabcode or from nbloom@stanford.edu.

2.3. Investigating the theoretical implications

Using the model and solution method outlined above we generate simulated investment and demand data for a panel of 50,000 firms and 25 years. We confirm the two implications for short run investment dynamics highlighted in section 2 by considering the relationship between firm-level annual investment rates and demand growth in this simulated data. As the drift in the demand process is common to all firms, and the idiosyncratic shocks are averaged across 250 production units, there is a simple correspondence between demand growth and the firm-level demand shock in this simulation.

Figure 1 presents Lowess smoothed non-parametric plots[9] of investment against demand growth for observations around the 10$^{\text{th}}$, 25$^{\text{th}}$, 50$^{\text{th}}$, 75$^{\text{th}}$ and 90$^{\text{th}}$ percentiles of the distribution of uncertainty (σ_t).[10] Investment rates are measured as annual investment divided by the capital stock at the beginning of the year, and annual demand growth is measured as the percentage change comparing the beginning and the end of the year. The first implication - that the short run response of investment to demand shocks will be lower at higher uncertainty - indicates that the slope of these response functions is lower at higher levels of uncertainty. It is evident that these non-parametric regression estimates do indeed become flatter as the level of uncertainty rises, consistent with the first implication. In quantitative terms, comparing investment responses to -10% and +25% demand growth, the gradient of the investment response to demand growth approximately doubles when moving from the third quartile to the first quartile of the distribution of uncertainty, and approximately triples when moving from the 90th percentile to the 10$^{\text{th}}$ percentile. Hence, differences in the level of uncertainty generate substantial variation in the short run response of investment to demand shocks, and this is clearly seen in our

[9] Lowess smoothing estimates a linear regression at each data point, using Cleveland's (1979) tricube weighting over a moving window of 5% of the data, to generate a non-parametrically smoothed data series. Lowess is similar to Kernel smoothing but uses information on both the mean and the slope of the data, and so is more efficient in estimating functions with continuous first derivatives, which our aggregated data has asymptotically (in the number of production units).

[10] As this variance parameter has a five point process in the underlying monthly model, we obtain considerable clustering of observations around these values, even in the average annual firm data. Although we use 1.25 million generated observations, there are no observations in the sample at the 10th and 25th percentiles of uncertainty with annual demand growth above 27% and 64% respectively, so the lines are not estimated beyond these points.

simulated firm-level data despite extensive aggregation across two types of capital, 250 production units and 12 monthly decision periods.

The second implication - that the short run response of investment to demand shocks is non-linear - indicates that these response functions are convex for positive investment and concave for negative investment. Focusing first on positive investment, it is evident that all five curves are indeed convex, with a proportionally larger response to larger positive shocks. Looking at negative investment the picture is unclear because, even for large negative demand growth of -25%, most firms are still undertaking positive gross investment. This reflects the combination of longer run dynamics with pent-up investment demand, 4% demand drift and 10% depreciation, which even in the presence of relatively low degrees of irreversibility generates very few firm-level disinvestment observations (2% in our simulated data sample and 3% in the real UK data). Thus, we cannot identify the concavity in the disinvestment responses in either the simulation or in actual UK data, and therefore we concentrate on the convex response for positive investment in the remainder of the paper.

3. Evaluating our empirical specification

The next step is to investigate the empirical importance of these properties of short run investment dynamics in actual firm-level data, which requires an appropriate econometric specification. If we observed the true underlying demand shocks and demand variance this would be relatively straightforward as we could, for example, use the same the non-parametric approach used in the previous section to analyse short run investment responses to exogenous demand shocks. However, in real firm-level datasets we only observe proxies for demand growth such as sales growth and proxies for uncertainty such as share price volatility. Among other issues, this requires us to deal with the problem that outcomes like sales and share prices are jointly determined with the firm's investment decisions. To do this we consider GMM estimates of dynamic econometric investment equations.

Our starting point is a reduced form error correction model that provides a flexible distinction between short run influences on investment rates and longer term influences on capital stocks. This has been widely used in recent empirical studies of company investment behaviour.[11] Bloom (2000) shows that the actual capital stock series chosen by a firm under partial irreversibility has a long run growth rate equal to that of the hypothetical capital stock series that the same firm would choose under costless reversibility, essentially because the gap between these two series is bounded. This implies that the logarithms of the two series should be cointegrated, and thus provides one motivation for considering an error correction model of capital stock adjustment.[12]

This cointegration result indicates that

$$\log K_{it} = \log K_{it}^{*} + e_{it} \tag{3.1}$$

where K_{it} is the actual capital stock for firm i in period t, K_{it}^{*} is the capital stock this firm would have chosen in the absence of adjustment costs, and e_{it} is a stationary error term. We specify this hypothetical frictionless level of the capital stock as

$$\log K_{it}^{*} = \log Y_{it} + A_{i}^{*} + B_{t}^{*} \tag{3.2}$$

where Y_{it} is the (real) sales of firm i in period t, and A_{i}^{*} and B_{t}^{*} are unobserved firm-specific and time-specific effects reflecting possible variation across firms in the components of and response to the user cost of capital (Chetty, 2006). This formulation is consistent, for example, with the frictionless demand for capital for a firm with a constant returns to scale CES production function and iso-elastic demand, and implies that the logs of the actual capital stock and real sales are cointegrated, provided the user cost of capital is stationary.[13] Note that this does not impose that the actual capital stock and its hypothetical frictionless level are equal on average, since the error term e_{it} need not be mean zero. However, the partial irreversibility framework indicates that e_{it} will be serially correlated in a

[11] See, for example, Hall, Mairesse and Mulkay (1999) and Bond, Harhoff and Van Reenen (2003).

[12] The representation theorem of Engle and Granger (1987) shows that the dynamic relationship between two I(1) series that are cointegrated can be formulated as an error correction relationship.

[13] Both this specification and the results in Bloom (2000) are based on a single production unit with one type of capital. To check that this provides an accurate approximation for our aggregated firm-level data, we confirmed that log capital was cointegrated with log sales in our simulated data, with a coefficient of 1.008 on log sales. In sections 3.2 and 4.4 we also consider relaxing the restriction in (3.2) that this coefficient is unity.

highly complex way. Any parsimonious specification of these dynamics should be viewed as an approximation, the quality of which we will investigate using simulated investment data in the next section.

A basic error correction representation of the dynamic relationship between $\log K_{it}$ and $\log K_{it}^{*}$, using equation (3.2), would have the form

$$\Delta \log K_{it} = \beta \Delta \log Y_{it} + \theta(\log Y_{i,t-s} - \log K_{i,t-s}) + A_i + B_t + v_{it} \qquad (3.3)$$

where A_i and B_t are again unobserved firm-specific and time-specific effects and v_{it} is, at least approximately, a serially uncorrelated error term. A key property is that the coefficient θ on the error correction term should be positive, so that firms with a capital stock level below their target will eventually adjust upwards, and vice versa.

We use the approximation $\Delta \log K_{it} \approx (I_{it}/K_{i,t-1}) - \delta_i$, where I_{it} is gross investment and δ_i is the (possibly firm-specific) depreciation rate. To test for the effect of uncertainty on the impact effect of demand shocks (the first implication), we add an interaction term between a measure of uncertainty (SD_{it}) and current sales growth ($\Delta \log Y_{it}$). A negative coefficient on this interaction term would indicate that the short run response of investment to demand shocks is indeed lower at higher levels of uncertainty. To allow for other possible effects of uncertainty on the level of the capital stock in either the short run or the long run, we also consider further terms in both the change (ΔSD_{it}) and the level (SD_{it}) of our measure of uncertainty. To test for non-linearity in the short run response of investment to demand shocks (the second implication), we add a higher order term in current sales growth $(\Delta \log Y_{it})^2$. A positive coefficient on this squared term would be consistent with this implication, indicating a convex relationship between investment and demand shocks, recalling that our samples are dominated by observations on firms with positive gross investment.

These additional terms then give us an empirical specification of the form

$$\begin{aligned} \frac{I_{it}}{K_{i,t-1}} &= \beta_1 \Delta \log Y_{it} + \beta_2 (\Delta \log Y_{it})^2 + \beta_3 (SD_{it} * \Delta \log Y_{it}) \\ &\quad + \theta(\log Y_{i,t-1} - \log K_{i,t-1}) + \gamma_1 SD_{it} + \gamma_2 \Delta SD_{it} + A_i + \delta_i + B_t + v_{it}. \end{aligned} \qquad (3.4)$$

3.1. Testing our empirical specification on simulated data

To investigate whether this econometric approach can detect the properties of short run investment dynamics highlighted in section 2, we use our simulation model to generate data for a panel of 1,000 firms and 15 years. This allows us to consider whether this relatively simple dynamic econometric specification provides an adequate approximation to the complex investment dynamics suggested by models with partial irreversibility, and to compare specifications that use sales and a stock-returns measure of uncertainty with specifications that use the true underlying demand and uncertainty variables. Sales (Y_{it}) are generated from the revenue function and aggregated across production units and months. Monthly stock returns are generated by aggregating the value function across units and adding in monthly net cash flows (revenue less investment costs). The within-year standard deviation of these monthly returns (SD_{it}) provides our firm-level measure of uncertainty, which mimics the kind of measure used in our empirical analysis in section 4. Table 2 reports the sample correlation matrix for key variables in our simulated dataset. This demonstrates that the standard deviation of monthly stock returns is positively correlated with the underlying standard deviation of demand shocks (σ_{it}), supporting the use of this as an empirical measure of uncertainty. In what follows we use lower cases to denote natural logarithms, so for example, $y_{it} = \log Y_{it}$.

[Tables 2 and 3 about here]

In Table 3 we present the results of estimating the augmented error correction model of investment using this simulated firm-level panel. In column (1) we first report OLS estimates using as explanatory variables the annual measures of the 'true' demand (P) and uncertainty (σ) variables that were used to generate this simulated investment data. Our tests detect significant heterogeneity in the impact effect of demand shocks on firm-level investment, depending on the level of uncertainty, and significant convexity in the response of investment to demand shocks. We also find evidence of 'error correcting' behaviour, with the actual capital stock adjusting in the long run towards a target that is cointegrated with its frictionless level. We find no evidence here that a permanent increase in the level of uncertainty would affect the level of the capital stock in the long run, but there is an indication that increases in uncertainty reduce investment in the short run in ways that are not fully captured by our multiplicative interaction term.

Column (2) of Table 3 uses instead the empirical counterparts to the demand and uncertainty variables, based on annual levels of simulated sales (Y_{it}) and the within-year standard deviation of simulated monthly stock returns (SD_{it}). As these variables are jointly determined with investment decisions we treat them as endogenous and report GMM estimates. To mimic our empirical analysis of real company data more closely, we also allow for the possibility of unobserved firm-specific effects here, and estimate this specification in first-differences. The instruments used are the second and third lags of our simulated investment, capital, sales and uncertainty measures, following Arellano and Bond (1991). A Sargan-Hansen test of overidentifying restrictions does not reject this specification, and there is no significant evidence of second-order serial correlation in the first-differenced residuals. While the parameter estimates are less precise in this case, we again detect significant evidence that uncertainty influences the short run response of investment to demand shocks, and that this response is convex. It should be noted, however, that this was not always the case if we imposed simpler dynamic specifications that were rejected by the test of overidentifying restrictions (for example, if we omit the error correction term). This illustrates the potential importance of controlling for longer run investment dynamics when testing the properties of the short run responses to demand shocks. For other calibrations of the simulation model we found that alternative dynamic specifications or instrument sets may be required. The negative coefficient on the interaction term and the positive coefficient on the squared term, however, were found consistently across specifications that were not rejected by the test of overidentifying restrictions.

Considering the magnitude of this effect of uncertainty, we find that the predicted impact effect of sales growth on investment rates increases by 79% when moving from the third quartile to the first quartile in the distribution of measured uncertainty, and by 168% when moving from the 90th percentile to the 10th percentile. These differences are quantitatively similar to those that we estimated directly for the underlying model in section 2.3.

This suggests that our econometric tests have power to detect these properties of short run investment dynamics, at least using this simulated dataset. Interestingly we also find that the longer run capital stock adjustment process is approximated quite well by our error correction specification, and that our GMM estimates using measured sales and uncertainty variables even provide quantitative estimates of the effect of uncertainty on short run responses to demand shocks that are in the right ballpark.

In columns (3) and (4) of Table 3 we confirm that these properties of short run investment dynamics are also found using two alternative specifications of our simulation model, which approximate Hartman (1972) and Abel (1983) type effects of uncertainty on the expected marginal revenue product of capital (MRPC).[14] In column (3) we set the drift in the demand process $2\mu(\sigma_t) = 0.04 + \frac{\sigma_t^2}{2}$, so that the expected MRPC is increasing in uncertainty. As expected, this generates a positive long run effect of the level of uncertainty on the level of the capital stock. Nevertheless we can still detect the negative effect of uncertainty on the short run response of investment to demand shocks, and the convex shape of these short run responses. In column (4) we set the drift $2\mu(\sigma_t) = 0.04 - \frac{\sigma_t^2}{2}$, so that the expected MRPC is decreasing in uncertainty. This generates a negative long run effect of uncertainty on the level of the capital stock, but has little impact on either the interaction term between demand growth and uncertainty or on our higher order demand growth term. This suggests, first, that our econometric tests of the properties of short run

[14] In a competitive model with shocks to output prices and a flexible factor (such as labour) the marginal revenue product of capital (MRPC) is convex in demand conditions, so uncertainty has a positive impact on the expected MRPC. For example, with a revenue function $R = ZK^aL^b$ (where Z is a demand process, K is capital and L is labour), after optimizing out labour net revenue equals $CZ^{\frac{1}{1-b}}K^{\frac{a}{1-b}}$ (where C is a constant) and the MRPC equals $\frac{aC}{1-b}Z^{\frac{1}{1-b}}K^{\frac{a+b-1}{1-b}}$. If Z is a geometric Brownian process with drift μ and variance σ then $E[dZ^{\frac{1}{1-b}}/Z^{\frac{1}{1-b}}] = (\mu + \frac{b}{1-b}\frac{\sigma^2}{2})\frac{dt}{1-b}$, so the expected growth of MRPC equals $(\mu + \frac{b}{1-b}\frac{\sigma^2}{2})\frac{1}{1-b}$, which is increasing in uncertainty. However, as Caballero (1991) notes, the sign of this effect is sensitive to assumptions such as the degree of imperfect competition, and whether the underlying shocks are to prices or quantities. Under alternative assumptions the marginal revenue product of capital can become concave in demand conditions, with a negative impact of uncertainty. To *qualitatively* simulate these positive and negative Hartman-Abel effects in our linear homogeneous specification, we adjust our demand drift term by $\pm\frac{\sigma^2}{2}$, noting that the *quantitative* effects would also depend on the exact convexity/concavity of the underlying MRPC in demand conditions.

investment dynamics appear to be robust (at least to these modifications), and secondly, that the longer run effects of uncertainty are theoretically ambiguous and need to be determined empirically. This echoes the discussions both in Leahy and Whited (1996), who outline a range of potentially positive and negative effects of uncertainty, and in Abel and Eberly (1999), who note the ambiguous long run effects of uncertainty on capital stock levels in a partial irreversibility framework.

3.2. Simulation robustness tests

To assess the generality of our predictions on the uncertainty-demand growth interaction term and on the demand growth squared term, we now investigate whether these effects are found for an alternative revenue function, and for alternative types of adjustment costs.

[Table 4 about here]

3.2.1. A CES specification

The simulation model and assumptions require only a supermodular homogeneous unit revenue function, so we can replace the Cobb-Douglas revenue function (2.1) with a function of a CES aggregator over the two types of capital

$$R(X, K_1, K_2) = X^{\alpha}(K_1^{\beta} + K_2^{\beta})^{\frac{\gamma}{\beta}} \tag{3.5}$$

The associated linear homogeneous revenue function is then defined by $\widetilde{R}(P, K_1, K_2) = P^{1-\gamma}(K_1^{\beta} + K_2^{\beta})^{\frac{\gamma}{\beta}}$, where $P = X^{\frac{\alpha}{1-\gamma}}$. We set $\beta = 0.5$ and $\gamma = 0.8$.

Column (1) of Table 4 presents OLS results for simulated firm-level data with this alternative CES specification, using the true demand and uncertainty variables. We again find that the short run response of investment to demand shocks is convex, and that higher uncertainty reduces this impact effect of demand shocks on investment. First-differenced GMM estimates, using sales as a measure of demand and stock-return volatility as a measure of uncertainty, also yielded a significant positive coefficient on the sales growth squared term and a significant negative coefficient on the uncertainty interaction term.[15] This suggests that our empirical tests can detect

[15]Coefficients (standard deviations) of 0.627 (0.132) on the sales growth term and -1.452 (0.467) on the uncertainty interaction term.