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Accounting for Research and Productivity Growth Across Industries

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Abstract
What factors underlie industry differences in research intensity and productivity growth? We develop a multi-sector endogenous growth model allowing for industry specific parameters in the production functions for output and knowledge, and in consumer preferences. We find that long run industry differences in both productivity growth and R&D intensity mainly reflect differences in "technological opportunities", interpreted as the parameters of knowledge production. These include the capital intensity of R&D, knowledge spillovers, and diminishing returns to R&D. To investigate the quantitative importance of these factors, we calibrate the model using US industry data. We find that the observed variation in the capital intensity of research cannot account for industry differences in productivity growth rates, and that variation in intertemporal knowledge spillovers has counterfactual predictions for R&D intensity when it is an important factor behind differences in productivity growth rates. This suggests that diminishing returns to research activity is the dominant factor.

JEL Codes: D24, O3, O41
Keywords: Multisector growth, total factor productivity, R&D intensity, technological opportunity

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1 Introduction

Total factor productivity (TFP) growth rates differ widely across industries, and these differences appear linked to persistent cross-industry differences in R&D intensity. This link is sometimes interpreted as causation. However, a priori it is not clear why the *level* of industry R&D should affect industry productivity *growth*, a point that has been made by Jones (1995) for the aggregate economy. Rather, both R&D and productivity growth depend on the response of firms to deeper industry parameters.

We develop a general equilibrium model in which both research activity and productivity growth vary endogenously across industries, to identify the factors that account for differences in each. We show that the factors that influence TFP growth also have an impact on R&D intensity. However, we also show that the converse is not true: there exists a set of industry characteristics that affect the level of industry R&D, but not necessarily industry productivity growth rates.

We build the model according to criteria that we believe define a natural benchmark. First, industries differ in terms of factors commonly identified in the empirical literature as being potential determinants of research intensity: technological opportunity (factors that affect the efficiency of research), appropriability (the extent to which R&D benefits the innovator) and demand (which influences the returns to research). Second, these factors are implemented in the model using standard preference and technology parameters drawn from growth theory. The industry-specific factors we study are: diminishing returns to research, knowledge spillovers over time, knowledge spillovers across firms, capital intensity in the production of goods, capital intensity in the production of ideas, the elasticity of substitution across different varieties of goods within each industry, and the industry's market size.

Comparing across industries, we find that differences in TFP growth rates depend only on factors of technological opportunity. These include the extent to which research is subject to diminishing returns, the capital share of research spending, and total knowledge spillovers. By contrast, differences in R&D intensity also depend on appropriability, defined as the extent to which knowledge spillovers accrue from the firm's *own* stock of knowledge. Product demand is fundamental in providing incentives to perform research: nevertheless, we find that in an equilibrium where the distribution of firm productivity is stable within industries, *industry differences* in equilibrium TFP growth rates and R&D intensity do not depend on the parameters that underlie product demand.[1]

To narrow down which factors of technological opportunity might best account for cross-industry comparisons in a production-function based framework, we calibrate as many model parameters as possible using US industry data, and use simulations to investigate the impact of the remaining parameters. The industry parameters we calibrate are the capital intensity in the production of goods, capital intensity in the production of ideas, the elasticity of substitution across different varieties of goods within each industry, and the industry's market size. We use simulations to assess the impact of diminishing returns to research, knowledge spillovers over time, and knowledge spillovers across firms.[2]

We find that the capital intensity of research cannot account for the industry differences in productivity growth rates in the data. Thus, the model indicates that variation in diminishing returns to research or in the magnitude of spillovers must be responsible. Furthermore, we find that variation in appropriability has little impact on industry variation in

[1]For example we show that, while the price elasticity of demand affects the potential returns to innovation in partial equilibrium, this may not affect returns in general equilibrium when all firms are conducting research and trying to keep pace with each other. Indeed, the survey of Cohen and Levin (1989) finds at best weak links between demand factors and industry R&D intensity, consistent with a pervading sense among historians of technical change that the pace and direction of technical progress is primarily supply-driven, such as Rosenberg (1969) and Nelson and Winter (1977).

[2]In addition, we also used the NBER patent citation database as an indicator of knowledge flows to calibrate the spillover parameters, finding similar results.

R&D intensity even when we choose appropriability values so as to maximize their influence. As a result, the model indicates that variation in diminishing returns to research and in the magnitude of spillovers must jointly account for patterns of productivity growth and research activity.

Finally, we use simulations to compare different possible combinations of the two highlighted factors. We find that intertemporal knowledge spillovers have counterfactual predictions for R&D intensity when they are assumed to be an important factor behind variation in productivity growth rates, as they imply a non-monotonic relationship between these variables. By contrast, the model is able to account jointly for the observed variation in productivity growth rates and in R&D intensity when differences in diminishing returns are emphasized instead. This suggests that variation in diminishing returns to research activity is the dominating factor among those we consider.[3] We find that the correlation between industry R&D in the model and in the data can be as high as 72%, indicating that the mechanisms in the model are able to account simultaneously for industry variation in both productivity growth and research intensity.

In related work, Klenow (1996) studies the determinants of cross-industry differences in TFP growth and R&D intensity in a 2-sector version of the Romer (1990) model. Krusell (1998) develops a 2-sector framework to endogenize the decline in the price of capital relative to consumption goods documented by Greenwood, Hercowitz and Krusell (1997), and Vourvachaki (2006) also features a two-sector endogenous growth model: however, in these papers, there is only research in one sector, and the focus is not on the factors that determine industry TFP growth rates. In the partial equilibrium model of Nelson (1988), the extent to which knowledge spills from a firm to its competitors affects R&D intensity but not TFP growth rates, and our general equilibrium environment also yields this result. Klevorick et al (1995) and Nelson and Wolff (1997) provide evidence supporting this claim.

Section 2 provides an overview of the related literature. We do this to line up the factors we wish to embody later in our model. Section 3 describes the structure of the model and outlines the main results, and Section 4 studies its long run behavior. Section 5 uses a calibration of the model to determine the relative importance of different potential determinants of research and productivity differences. Section 6 discusses possible extensions.

2 Related Literature

Industry TFP growth rates appear linked to cross-industry variation in R&D intensity – see Terleckyj (1980) for an early survey. Many studies have attempted to identify the determinants of industry variation in R&D. While some studies assume that research *causes* productivity growth, others take our view that both are determined by deeper "fundamentals" of each industry. Consistent with our approach, Nelson and Wolff (1998) are able to identify factors that explain R&D intensity that do not account for TFP growth rates.

The literature has focused on three sets of factors that might drive industry research activity and TFP growth: product demand, technological opportunity, and appropriability.

[3] Thus, our results do not hinge on the particular set of industries used, but on the fact that R&D intensity and industry productivity growth are positively related in the data.

Technological opportunity encompasses factors that lead research to be more productive in some industries than others. Opportunity has been modeled in different ways – for example, in Klenow (1996) it is a constant Z_i in the knowledge production function for industry i. Nelson (1988) interprets opportunity in terms of knowledge spillovers from different sources. Measuring opportunity is difficult: however, using surveys of R&D managers, Levin et al (1985), Cohen et al (1987) and Klevorick et al (1995) try to identify different kinds of spillovers, relating them to R&D activity and to technical change.

Appropriability relates to the extent that an innovating firm (as opposed to its competitors) benefits from its own newly generated knowledge. Cohen et al (1987), Klevorick et al (1995) and Nelson and Wolff (1997) find evidence that appropriability is related to R&D intensity and, interestingly, Klevorick et al (1995) and Nelson and Wolff (1997) argue that the survey data are consistent with an influence of opportunity factors on both R&D intensity and technical change, whereas appropriability is only related to R&D intensity.[4]

Demand factors affect the returns to R&D. In Schmookler (1966), larger product markets encourage innovation by offering higher returns to innovators, whereas in Kamien and Schwartz (1970) the gains from reducing production costs may be larger when demand is more elastic. The survey of Cohen and Levin (1989) suggests that the evidence concerning demand factors is weak. For example, Levin et al (1985) find that they lose significance in cross-industry R&D regressions when indicators of opportunity and appropriability are included. Independently, several case-based and historical studies suggest that technical change is driven by scientific or engineering considerations rather than by demand conditions.[5]

The following stylized facts emerge from the empirical literature: (1) There is evidence that *opportunity* affects both statistics of interest; (2) *appropriability* is easier to relate to R&D intensity than to TFP growth rates; (3) the link between *demand factors* and research intensity (as well as rates of TFP growth) is not robust.

We wish to articulate opportunity, appropriability and demand factors within a general equilibrium growth model, based on primitives of preferences and technology drawn from the growth literature. Given the measurement difficulties inherent in studying the role of knowledge in technical progress, we use the structure of the model to guide us regarding the relationships that hold between R&D, TFP growth, and each of these factors. As a benchmark, we use a model of knowledge generation that is intentionally close to the production function approach common in both the theoretical and the empirical literature. Our model

[4]Cohen et al (1987) do find a positive link between appropriability and an indicator of innovation, also using survey data. What clouds these results is that the appropriability measure in all these papers may not distinguish sharply between appropriability and opportunity. The measure is based on the response to the question "in this line of business, how much time would a capable firm typically require to effectively duplicate and introduce a new or improved product developed by a competitor?" This may not distinguish between (a) the ease with which a competitor might access a firm's knowledge, and (b) the ease in general with which preexisting knowledge can be used to generate new knowledge. In particular, if appropriability itself is generally low, then the measure may reflect mostly differences in opportunity.

[5]Nelson and Winter (1977) coin the term "natural trajectories" to describe the phenomenon that "innovation has a certain inner logic of its own [...] – particularly in industries where technological advance is very rapid, advances seem to follow advances in a way that appears somewhat 'inevitable' and certainly not fine tuned to the changing demand and cost conditions." There is some evidence of an impact of market size on innovative activity at the firm or product level: however, these findings do not relate to *industry* differences. More discussion will follow in Section 4.

maps naturally into the frameworks of Jones (1995) and Krusell (1998). The functional forms we use are necessary for balanced growth.

3 Economic Environment

3.1 Knowledge Production

The economy consists of $z \geq 2$ industries. Consider a firm $h \in [0, 1]$ in industry i, with a level of productivity that depends upon the stock T_{iht} of technical knowledge at its disposal at date t. Knowledge accumulates over time according to the function

$$T_{ih,t+1} = F_{iht} + T_{iht} \tag{1}$$

where F_{iht}.[6]

New knowledge F_{iht} is generated using a knowledge production function, using the firm's research input and spillovers from other firms.[7] The knowledge production function is:

$$F_{iht} = Z_i T_{iht}^{\kappa_i} T_{it}^{\sigma_i} \left(Q_{iht}^{\eta_i} L_{iht}^{1-\eta_i} \right)^{\psi_i}. \tag{2}$$

where $\eta_i, \psi_i \in (0, 1]$, and Q_{iht} and L_{iht} are capital and labor used in the production of knowledge. The productivity index for industry i as a whole is $T_{it} \equiv \int_0^1 T_{iht} dh$, which firm h takes as given. Let $\gamma_{iht} \equiv T_{iht+1}/T_{iht}$ be the growth factor of T_{ih}.

Parameters Z_i, κ_i, σ_i, ψ_i and η_i represent technological opportunity, as they affect the productivity of research input. Parameter Z_i is an efficiency parameter for carrying out research in industry i.[8] It could be linked to the nature of research in the industry, or to the institutional environment. Parameter κ_i represents the effect of in-house knowledge on the production of new ideas, and is known in the growth literature as the intertemporal knowledge spillover. Parameter σ_i represents spillovers across firms within sector i. The total knowledge spillover $\rho_i \equiv \kappa_i + \sigma_i$ is the extent to which the production of new knowledge in sector i benefits from prior knowledge. Parameter ψ_i indicates decreasing returns to research inputs. One interpretation for $\psi_i < 1$ is that there is duplication in research, whereby some of the knowledge created by a firm in sector i might not be new. Parameter η_i captures the share of capital in R&D spending.

Conditional on total knowledge spillovers, industries may differ in the importance of in-house knowledge relative to knowledge spillovers from its competitors. We define *appropriability* A_i as the share of total spillovers accounted for by in-house knowledge: $A_i \equiv \kappa_i/\rho_i$.

[6] It is common to assume that ideas depreciate. There is a distinction between physical depreciation and economic depreciation, however. For ideas to physically depreciate would imply that some share of them is exogenously forgotten. Economic depreciation, on the other hand, implies that old knowledge becomes less valuable (obsolete) as newer knowledge accumulates, and rates of economic depreciation will be endogenous in our model. See Laitner and Stolyarov (2008) for a different approach based on new knowledge sometimes reducing the value of existing knowledge to zero.

[7] We focus for now on within-industry spillovers, and later discuss the impact of cross-industry spillovers.

[8] Nelson (1988) allows Z_i grows at an exogenous rate. Since the trademark of R&D-based growth models is that technical progress is endogenous, our model does not feature exogenously growing factors other than the population.

The last set of factors considered by the empirical literature relates to demand, which we present later when we close the model using standard household preferences.

3.2 Firm's problem

Each sector $i \leq z$ is monopolistically competitive. Firm h in sector i produces a differentiated variety $h \in [0,1]$ of good i. Output of variety h of good i is

$$Y_{iht} = T_{iht} K_{iht}^{\alpha_i} N_{iht}^{1-\alpha_i}, \ \alpha_i \in (0,1) \tag{3}$$

where Y_{iht} is output, K_{iht} is capital and N_{iht} is labor.

Firms are competitive in the input markets. Taking input prices (w_t, R_t) and its demand function $p_{iht}(.)$ as given, firm h in sector i chooses both production inputs (K_{iht}, N_{iht}) and R&D inputs (Q_{iht}, L_{iht}) to maximize the discounted stream of real profits:

$$\sum_{t=0}^{\infty} \lambda_t \frac{\Pi_{iht}}{p_{ct}}; \quad \Pi_{iht} \equiv p_{iht} Y_{iht} - w_t \left(N_{iht} + L_{iht}\right) - R_t \left(K_{iht} + Q_{iht}\right), \tag{4}$$

where p_{ct} is the aggregate price-index for consumption goods, λ_t is the discount factor at time t, with $\lambda_0 = 1$, $\lambda_t = \prod_{s=1}^{t} \frac{1}{1+r_t}$ for $t \geq 1$, and r_t is the real interest rate. The transversality condition is $\lim_{t \to \infty} \chi_{iht} T_{iht+1} = 0$, where χ_{iht} is the shadow price of T_{iht+1}.[9] The complete derivation of the firm's maximization problem is given in Appendix B.1.

3.3 Equilibrium Productivity Growth

Given free mobility of inputs and competitive input markets, marginal rates of substitution are equal across activities within the firm (5), across firms within each industry (6), and also across sectors (7):[10]

$$\frac{1-\eta_i}{\eta_i} \frac{Q_{iht}}{L_{iht}} = \frac{1-\alpha_i}{\alpha_i} \frac{K_{iht}}{N_{iht}}, \tag{5}$$

$$\frac{Q_{iht}}{L_{iht}} = \frac{Q_{it}}{L_{it}}; \quad \frac{K_{iht}}{N_{iht}} = \frac{K_{it}}{N_{it}}, \tag{6}$$

$$\frac{1-\eta_i}{\eta_i} \frac{Q_{it}}{L_{it}} = \frac{1-\eta_j}{\eta_j} \frac{Q_{jt}}{L_{jt}} = \frac{1-\alpha_i}{\alpha_i} \frac{K_{it}}{N_{it}} = \frac{1-\alpha_j}{\alpha_j} \frac{K_{jt}}{N_{jt}}. \tag{7}$$

[9] Note that the transversality condition implies that the shadow price χ_{iht} is falling in any equilibrium where T_{iht} is growing. In the rest of the paper we will continue to refer the expression χ_{iht+1}/χ_{iht} as the growth factor of the shadow price.

[10] The linearity of quations $(5)-(7)$ stems from our use of Cobb-Douglas production functions. If we were to allowed for a general production function with constant elasticity of substitution, then marginal rates of substitution would be log-linear in capital-labor ratios with a coefficient equal to the elasticity of substitution. A linear relationship would still hold if the elasticity of substitution were identical across activities, firms and sectors. We choose to focus on the Cobb-Douglas productions both for the lack of measure for the sector-specific elasticity of substitution and for the possibility of deriving the balanced growth path later.

It follows that:

$$\frac{Q_{iht+1}/L_{iht+1}}{Q_{iht}/L_{iht}} = \frac{K_{iht+1}/N_{iht+1}}{K_{iht}/N_{iht}} = g_{kt} \quad \forall i, h. \tag{8}$$

Using (1) and (6), the productivity growth of firm h in sector i depends on

$$\gamma_{iht} - 1 = \frac{F_{iht}}{T_{iht}} = Z_i \left(\frac{T_{iht}}{T_{it}}\right)^{\kappa_i - 1} T_{it}^{\rho_i - 1} \left(\frac{Q_{it}}{L_{it}}\right)^{\eta_i \psi_i} L_{iht}^{\psi_i}. \tag{9}$$

As our interest is in industry comparisons, we focus on equilibria where the distribution of productivity within sectors is stable and *rank-preserving* i.e. in each industry $\gamma_{iht} = \gamma_{it}$ $\forall h$.[11] Then, (9) implies $L_{iht} = L_{it}$. To make meaningful comparisons across sectors, we also focus on equilibria with constant productivity growth, using (8) and (9):

Lemma 1 *In any rank-preserving equilibria, constant γ_i satisfies*

$$\gamma_i = \left[g_{kt}^{\eta_i} g_N \left(\frac{l_{it+1}}{l_{it}}\right)\right]^{\frac{\psi_i}{1-\rho_i}}, \quad \forall i, \tag{10}$$

where $l_{it} \equiv \frac{L_{it}}{N_t}$ is the fraction of labor allocated to research in sector i.

Three terms affect cross-industry comparisons of productivity growth: (i) the expression $\frac{\psi_i}{1-\rho_i}$, (ii) capital intensity of research activities η_i, and (iii) growth in the fraction of labor allocated to research $\left(\frac{l_{it+1}}{l_{it}}\right)$.

The expression $\frac{\psi_i}{1-\rho_i}$ is related to the historical work of Rosenberg (1969) and Nelson and Winter (1977) that underlines technological opportunity as a factor of productivity growth. Specifically, our model emphasizes the degree of decreasing returns to research input, the extent of intertemporal knowledge spillovers κ_i, and the magnitude of spillovers across firms σ_i. Interestingly, as far as spillovers are concerned, only total spillovers $\rho_i = \kappa_i + \sigma_i$ are important, whereas the *source* of spillovers is not.

We are not aware of a precedent to the second factor – *the capital intensity of research activity.* Technical improvements in the production of capital goods lead to capital deepening, and the extent to which this encourages research depends on η_i. Rosenberg (1969) and Nelson and Winter (1977) suggest that capital-intensive industries may enjoy inherently high TFP growth. However, equation (10) shows that what matters is not capital intensity per se, but the *capital-intensity of research activity.* The capital-intensity of production activity may affect the measurement of productivity, but not equilibrium rates of productivity growth.

Industry-specific demand factors and appropriability $A_i \equiv \kappa_i/\rho_i$ can only matter for cross-industry productivity growth comparisons if they alter the growth rate of labor allocated to research across sectors through $\left(\frac{l_{it+1}}{l_{it}}\right)$.

[11] This includes the case of symmetric equilibria ($T_{iht} = T_{it}$ $\forall h$).

3.4 Equilibrium research activity

Let χ_{iht} be the shadow price of knowledge T_{iht+1}, which is determined by the arbitrage condition for allocating inputs across activities. In the case of capital:

$$\chi_{iht} = -\left(\frac{\lambda_t}{p_{ct}}\right)\frac{\partial \Pi_{iht}/\partial Q_{iht}}{\partial F_{iht}/\partial Q_{iht}}. \tag{11}$$

The firm's dynamic optimization condition implies that

$$\chi_{iht} = \underbrace{\left[\frac{\lambda_{t+1}}{p_{ct+1}}\frac{\partial \Pi_{iht+1}}{\partial T_{iht+1}}\right]}_{(a)\ \text{production}} + \chi_{iht+1}\underbrace{\left[\frac{\partial F_{iht+1}}{\partial T_{iht+1}}\right.}_{(b)\ \text{research}}\underbrace{\left.+1\right]}_{(c)\ \text{future knowledge}}, \forall i \leq z. \tag{12}$$

Equation (12) reflects three benefits to the firm of producing more knowledge: (a) more efficient *production of goods and services*, (b) more efficient *production of knowledge*, and (c) a larger stock of future knowledge.

To determine the extent to which resources are directed towards research (as opposed to production), we define research intensity as the share of research spending in total costs:

$$RND_{iht} \equiv \frac{w_t L_{iht} + R_t Q_{iht}}{w_t\left(L_{iht} + N_{iht}\right) + R_t\left(Q_{iht} + K_{iht}\right)}.$$

Using (12) we have:

$$RND_{iht} = \left[1 + \frac{1}{\psi_i}\left(\frac{\frac{\chi_{iht}}{\chi_{iht+1}} - 1}{\gamma_i - 1} - \kappa_i\right)\right]^{-1}, \tag{13}$$

where by the definition of χ_{iht} in (11):

$$\frac{\chi_{iht}}{\chi_{iht+1}} = \frac{\lambda_t p_{iht}/p_{ct}}{\lambda_{t+1} p_{iht+1}/p_{ct+1}}\gamma_{iht}^{\kappa_i - 1}\gamma_{it}^{\sigma_i} g_{kt}^{\eta_i\psi_i - \alpha_i}\left(g_N \frac{l_{iht+1}}{l_{iht}}\right)^{\psi_i - 1}. \tag{14}$$

It follows from (10) that in any rank-preserving equilibria with constant γ_i,

$$\frac{\chi_{iht}}{\chi_{iht+1}} = \frac{\lambda_t p_{iht}/p_{ct}}{\lambda_{t+1} p_{iht+1}/p_{ct+1}} g_{kt}^{-\alpha_i}\left(g_N \frac{l_{it+1}}{l_{it}}\right)^{-1}. \tag{15}$$

Growth in the price of i relative to consumption $\frac{p_{iht}/p_{ct}}{p_{iht+1}/p_{ct+1}}$ requires knowledge of the demand function faced by each firm. Assume that price elasticities $\mu_i \equiv \frac{\partial p_{iht}/\partial Y_{iht}}{p_{iht}/Y_{iht}}$ are sector-specific constants (i.e. identical across firm within any sector i). Equating the value of marginal products of labor across firms, together with (6):

$$\frac{p_{iht+1}/p_{iht}}{p_{ih't+1}/p_{ih't+1}} = \frac{\gamma_{ih't}}{\gamma_{iht}}; \quad \forall i, h, h', \tag{16}$$

which implies that in any rank-preserving equilibria with constant γ_i :

$$\frac{p_{iht+1}/p_{iht}}{p_{jht+1}/p_{jht}} = \frac{p_{it+1}/p_{it}}{p_{jt+1}/p_{jt+1}} = \frac{\gamma_{jt}}{\gamma_{it}} g_{kt}^{\alpha_j - \alpha_i}, \tag{17}$$

where the last equality follows from using (7) and equating the value of marginal products of labor across sectors. Substituting into (15), we have:

Lemma 2 *If price elasticity μ_i is a sector-specific constant then, in any rank-preserving equilibria with constant productivity growth, research intensity for any sector i satisfies:*

$$RND_{it} = \left[1 + \frac{1}{\psi_i}\left(\frac{\frac{\chi_{it}}{\chi_{it+1}} - 1}{\gamma_i - 1} - \kappa_i\right)\right]^{-1}; \quad \forall i, \tag{18}$$

where

$$\frac{\chi_{it}/\chi_{it+1}}{\chi_{jt}/\chi_{jt+1}} = \left(\frac{\gamma_i}{\gamma_j}\right)\frac{l_{it}/l_{it+1}}{l_{jt}/l_{jt+1}}; \quad \forall i. \tag{19}$$

In addition to the factors that determine γ_i, there are three additional terms affecting cross-industry comparisons of research intensity: (i) the degree of diminishing returns to research input ψ_i, (ii) the effect of in-house knowledge on the production of new ideas κ_i, and (iii) growth in the fraction of labor allocated to research $\left(\frac{l_{it+1}}{l_{it}}\right)$.

Recall that $\kappa_i = A_i\rho_i$, implying that research intensity is affected by both opportunity and appropriability. Moreover, if price elasticities are sector-specific constants, industry-specific demand factors can only matter for cross-industry R&D intensity comparisons if they alter the growth rate of labor allocated to research across sectors.

3.5 Relating the model to the literature

We now compare our results so far with the empirical findings reviewed in Section 2.

Consistent with evidence, comparisons of industry TFP growth rates depend on factors of technological opportunity, whereas R&D intensity also depends upon appropriability. Low appropriability lowers R&D intensity without affecting productivity growth rates, so a prediction is that there should be a *negative* relationship between measures of *intra*-industry spillovers and R&D intensity, controlling for other variables. This is exactly what Nelson and Wolff (1997) find.

Klevorick et al (1995) identify two effects of appropriability on R&D intensity. First, in their terminology, there is an "incentive effect" whereby large, un-internalized spillovers reduce R&D activity, causing the negative relationship between appropriability A_i and R&D intensity in Lemma 2. Second, there is also an "efficiency" effect, whereby larger spillovers may *encourage* R&D at other firms. The efficiency effect is seen in that, conditional on κ_i, a larger value of σ_i raises ρ_i while leaving $A_i\rho_i$ constant, so that R&D intensity rises. However, in our model, the "efficiency" effect is related to the magnitude of spillovers, not to appropriability *per se* and, as suggested by Klevorick et al (1995), this effect disappears once opportunity is kept constant.

Demand parameters can only affect comparisons of TFP growth rates and research intensities through the growth in the fraction of labor allocated to research l_{it+1}/l_{it}, which is unlikely to be affected stationary demand parameters such as industry size and the price elasticity of demand. This is broadly consistent with the evidence in Section 2. We return to this point after presenting the demand side of the model.

3.6 Closing the model: Households

We now close the model by specifying the demand side of the economy.

There is a continuum of households, each of measure $N_t = g_N^t$, where g_N captures the constant population growth. In what follows, we use lower case letters to denote per-capita variables. Goods $i \in \{1, ..m-1\}$ are consumption goods while goods $j \in \{m, ...z\}$ are investment goods.

The life-time utility of a household is

$$\sum_{t=0}^{\infty} (\beta g_N)^t \frac{c_t^{1-\theta} - 1}{1-\theta} \tag{20}$$

$$c_t = \prod_{i=1}^{m-1} \left(\frac{c_{it}}{\omega_i}\right)^{\omega_i}, \; c_{it} = \left(\int_0^1 c_{iht}^{\frac{\mu_i - 1}{\mu_i}} dh\right)^{\frac{\mu_i}{\mu_i - 1}} \qquad i \in \{1, .., m-1\} \tag{21}$$

where β is the discount factor, and $1/\theta$ is the intertemporal elasticity of substitution. We assume that $\beta g_N < 1, \theta > 0, \mu_i > 1, \omega_i > 0$ and $\sum_{i=1}^{m-1} \omega_i = 1$. Parameters μ_i and ω_i capture the industry-specific demand factors considered in the literature. μ_i is the elasticity of substitution across different varieties of good i which, in equilibrium, determines the price elasticity of demand, while ω_i determines the spending share of each good (market size).

Each household member is endowed with one unit of labor and k_t units of capital, and receives income by renting capital and labor to firms, and by earning profits from the firms. Her budget constraint is

$$\sum_{i=1}^{m-1} \int p_{iht} c_{iht} dh + \sum_{j=m}^{z} \int p_{jht} x_{jht} dh \leq w_t + R_t k_t + \pi_t \tag{22}$$

where x_{jht} is investment in variety h of capital good j, p_{iht} is the price of variety h of good i, w_t and R_t are rental prices of labor and capital, and $N_t \pi_t \equiv \sum_{i=1}^{z} \int_0^1 \Pi_{iht} dh$ equals total profits from firms. Her capital accumulation equation is

$$g_N k_{t+1} = x_t + (1 - \delta_k) k_t. \tag{23}$$

The composite investment good x_t is produced using all capital types j:

$$x_t = \prod_{j=m}^{z} \left(\frac{x_{jt}}{\omega_j}\right)^{\omega_j}, \; x_{jt} = \left[\int x_{jht}^{(\mu_j - 1)/\mu_j} dh\right]^{\mu_j / (\mu_j - 1)} \qquad j \in \{m, ..., z\}, \tag{24}$$

where $\mu_j > 1$, $\omega_j > 0$ and $\sum_{j=m}^{z} \omega_j = 1$.[12] Finally, the transversality condition for capital is $\lim_{t \to \infty} \zeta_t k_t = 0$, where ζ_t is the shadow price of capital. Define the price index for the consumption composite c_t and the investment composite x_t respectively as:

$$p_{ct} \equiv \frac{\sum_{i=1}^{m-1} \int_0^1 p_{iht} c_{iht} dh}{c_t}; \qquad p_{xt} \equiv \frac{\sum_{j=m}^{z} \int_0^1 p_{jht} x_{jht} dh}{x_t}. \tag{25}$$

[12] Cobb-Douglas aggregation across goods allows us to derive an aggregate balanced growth path. In a multi-sector model with exogenous technological progress, Ngai and Pissarides (2007) show that Cobb-Douglas aggregation across capital goods is necessary for deriving an aggregate balanced growth path.

4 Decentralized Equilibrium

The decentralized equilibrium is standard, where the firms' and consumers' problems are defined as in Section 3. In any period t, prices must clear all goods and input markets:

$$Y_{iht} = c_{iht} N_t; \quad i < m; \quad Y_{jht} = x_{jht} N_t, \quad j \geqslant m; \tag{26}$$

$$K_t = \sum_{i=1}^{z} \int_0^1 (K_{iht} + Q_{iht})\, dh, \; N_t = \sum_{i=1}^{z} \int_0^1 (N_{iht} + L_{iht})\, dh. \tag{27}$$

Our aim is to compare productivity dynamics across industries, and not across different varieties of any given good. Therefore, we focus on equilibria that treat varieties within each sector i symmetrically, and suppress the firm index h henceforth.[13]

Full derivation of the household's utility maximization is given in the Appendix. The implied Euler condition is:

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{p_{xt+1}/p_{ct+1}}{p_{xt}/p_{ct}} \left(1 - \delta_k + \frac{R_{t+1}}{p_{xt+1}}\right), \tag{28}$$

which implies the real discount factor:

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{1 + r_{t+1}} = \frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{p_{xt}/p_{ct}}{G_{t+1} p_{xt+1}/p_{ct+1}}, \tag{29}$$

where $G_{t+1} \equiv 1 - \delta_k + \frac{R_{t+1}}{p_{xt+1}}$ is the gross return on capital.

4.1 Balanced growth

We look for a balanced growth path equilibrium (BGP), along which aggregate variables are growing at constant rates although industry TFP growth rates may be different. Such a BGP requires a constant ratio of consumption to capital: $c/(qk)$, where q is the relative price of capital. Define Φ and γ_{xt} as:

$$\Phi = \frac{\sum_{j=m,z} \frac{\omega_j \psi_j}{1-\rho_j}}{1 - \sum_{j=m,z} \frac{\omega_j \psi_j}{1-\rho_j} \frac{\eta_j}{1-\alpha_x}}; \quad \gamma_{xt} \equiv \prod_{j=m,z} \gamma_{jt}^{\omega_j}; \quad \alpha_x \equiv \sum_{j=m,z} \omega_j \alpha_j. \tag{30}$$

Proposition 1 *Suppose there exists an equilibrium with constant $l_i, n_i > 0$ that satisfies the transversality conditions for T_i and k. If $\Phi > 0$, then there exists a unique balanced growth path. Along this path c/q and k grow by a constant factor $(\gamma_x)^{1/(1-\alpha_x)}$ where $\gamma_x = g_N^{\Phi}$, and γ_i is constant and satisfies* (10).

The proof observes that the return to investment G is constant if k grows by a factor $\gamma_{xt}^{1/(1-\alpha_x)}$, which by (30) is constant if γ_i is constant in all capital good sectors. The restriction for constant γ_i follows from Section 3.3, and γ_x is derived from (30).[14]

[13] In notes available upon request, we show asymmetric rank-preserving equilibria exist in which all the results of the paper hold.

[14] The Appendix reports sufficient conditions for the existence of a BGP with R&D activity in all sectors.

Proposition 1 contrasts with the behavior of the one-sector model of Jones (1995). In Jones (1995), Φ is replaced by $\frac{\psi_1}{1-\rho_1}$, so balanced growth path requires $\rho_1 < 1$ (where 1 indexes the only industry in the economy). There are two important differences compared to our requirement that $\Phi > 0$. First, suppose $\eta_j = 0$, i.e. capital is not used in the production of knowledge. Then $\Phi > 0$ is equivalent to $\sum_{j=m}^{z} \frac{\omega_j \psi_j}{1-\rho_j} > 0$, so the Jones (1995) restriction applies to the weighted average of $\frac{\psi_j}{1-\rho_j}$ across capital goods in the multi-sector model.[15] Second, the restriction $\sum_{j=m}^{z} \frac{\omega_j \psi_j}{1-\rho_j} > 0$ is not sufficient when capital is used in the production of knowledge ($\eta_j > 0$ for some $j \geq m$), as productivity improvements targeting capital goods become a factor of aggregate productivity growth by inducing capital deepening in R&D.

4.2 Comparing industries

In equilibrium, industries with the same level of technological opportunity (i.e. the same values of $\psi_i, \rho_i = \kappa_i + \sigma_i$ and η_i but different appropriability $A_i = \kappa_i/\rho_i$) display different R&D intensity, even if they have the same TFP growth rate. It follows from Lemmae (1) and (2) that:

Proposition 2 *Along the balanced growth path,*

(i) Cross-industry comparisons of productivity growth depend only on the technological opportunity factors ρ_i, ψ_i and η_i.

(ii) In addition to these factors, cross-industry comparisons of R&D intensity depend also on appropriability A_i.

Notice that differences in demand parameters affect neither comparisons of productivity growth rates nor of R&D intensity when l_i are constants.[16] General equilibrium mechanisms play a key role in this result.

In the model there are two industry demand parameters: ω_i, the weight of good i in the demand function, and μ_i, the elasticity of substitution across varieties of i. The spending share of each good depends on ω_i, and the elasticity of a firm's demand function depends on μ_i. Since ω_i affects the level of returns to production at all dates, but not their growth rate, it does not affect the decision of whether to use resources for investment in future production (via increases in knowledge) instead of current production.

The reason μ_i may matter in partial equilibrium is that elastic demand allows an innovator to increase market share without having to lower her output price to the same extent as the cost reduction. However, in equilibrium, all firms are performing research: R&D by the firm's *competitors* results in a commensurate fall in the relative price of *their* goods, so that this partial equilibrium benefit of research need not materialize in general equilibrium.

It is worth elaborating upon this last point. The literature on appropriability distinguishes between two channels whereby research by a firm might affect its competitors. The

[15] From (10), given $g_k, g_N \geqslant 1$, productivity growth in sector i is positive only if $\rho_i < 1$.

[16] Some empirical studies do find a demand-innovation link – for example, Newell et al (1999), Popp (2002) and Acemoglu and Linn (2004). These findings underline the importance of demand in providing incentives for R&D, but do not provide evidence relating to industry *differences* in productivity growth nor R&D intensity.

first is the "spillover effect" (captured by σ_i in our model) whereby innovations by one firm may be used by another. The second is the "business stealing" or "product rivalry" effect whereby innovations by a firm's competitors decreases its market share. In our model, the severity of this rivalry depends on μ_i. To see this, note that c_{ih} is proportional to $p_{ih}^{-\mu_i}$, so that the relative market share of two firms h and h' in the same industry is:

$$\frac{p_{ih}c_{ih}}{p_{ih'}c_{ih'}} = \left(\frac{p_{ih}}{p_{ih'}}\right)^{1-\mu_i} = \left(\frac{T_{ih}}{T_{ih'}}\right)^{\mu_i-1} \tag{31}$$

where $p_{ih}c_{ih}$ are the sales of firm h. Consider two firms that start period t with equal productivity. A given productivity improvement in one firm relative to the other will result in a larger increase in demand for higher values of $\mu_i > 1$.

Even though the rivalry effect is present in the model, this does not imply that μ_i affects equilibrium TFP growth rates, as these considerations influence R&D incentives at *all* firms in the industry. In a symmetric equilibrium, firms keep pace with each other technologically so that μ_i does not affect equilibrium research expenditure, as it does not affect equilibrium returns. The results hold in any rank-preserving equilibrium. Consistent with our results, Bloom et al (2007) estimate that the rivalry effect is quantitatively dominated by the "spillover effect" as a determinant of research activity.

The model suggests some caution in linking research intensity to demand factors empirically. The most common measure of research intensity is R&D spending divided by sales or, in terms of the model, $RND^{Sales} \equiv \frac{wL_i+RQ_i}{p_iY_i}$. Combined with the conditions for optimal input allocation, equation (12) becomes:

$$RND^{Sales} = \left(1 - \frac{1}{\mu_i}\right)\psi_i \left[\frac{\frac{\chi_{it}}{\chi_{it+1}} - 1}{\gamma_i - 1} - \kappa_i\right]^{-1} \tag{32}$$

This formula would appear to indicate an influence of demand parameters μ_i on research spending in the model, and indeed Cohen et al (1987) find some indicators of industry concentration to be related to the ratio of research spending to sales. However, in an environment with imperfect competition, the volume of sales contains a markup over cost, which is not an indicator of the quantity of resources devoted to research as opposed to other activities. The denominator in this measure of research activity contains demand side variables *by construction.* Future empirical work may turn out to substantiate an economic link between R&D and markups or other demand factors: however, the model suggests caution in employing sales-based measures of R&D activity in such work.

5 Quantitative findings

We now calibrate our model using US industry data to identify whether set of opportunity factors can account for observed industry differences in R&D and productivity growth. We match the model to United States data because of the rich sources of information available, because the US is arguably at the technological frontier in most industries, and because

GDP has grown at a stable rate for over a century, which is consistent with our focus on the balanced growth path of our model.[17]

We address the following questions:

1. The model predicts that productivity growth should be positively linked to the opportunity parameters ψ_i, ρ_i and to η_i. Which of these parameters do the data suggest to be the main factor?

2. The model suggests that R&D intensity should be linked to opportunity parameters, but also to appropriability A_j. Which of these parameters do the data suggest to be the main factor?

3. What values of these parameters best account for industry variation in productivity change and research intensity in the data?

To answer these questions, we proceed as follows. We first calibrate as many parameters as possible in the model using post-war US data. Then, we ask what combinations of the remaining parameters allow the model to best match industry data on productivity growth and R&D intensity. This allows us to assess whether variation in certain variables is sufficient to account for observed industry differences, and the circumstances under which the variables we do not observe directly are or are not able to account for the data.

We do not match measured TFP growth rates directly. For example, several of the long-term rates of TFP growth estimated by Jorgenson et al (2006) are negative, and we do not believe that productivity can decline in absolute terms in the long run when it is driven by knowledge accumulation. We take seriously the view of Greenwood et al (1997) among others that quality improvements are an important source of productivity change. Thus, we calibrate model TFP growth rates using quality-adjusted relative prices. Specifically, equation (17) implies a relationship between relative rates of price decline, capital shares, and TFP growth, and we use these to compute relative TFP growth rates.

To our knowledge, comparable quality adjusted prices are available only for durable goods. Hence, we assume that $m = 2$, so that there is only one sector producing non-durables. We set $z = 15$, so that there are 14 capital-producing industries. This partition was the finest that allowed us to match the relative price data with the patent data we employ later to measure knowledge spillovers.

It is worth pointing out that our quantitative conclusions turn out not to depend on the use of these particular industries. The main sources of discipline on our quantitative exercise turn out to be (a) the extent of variation in productivity growth rates, and (b) the fact that productivity growth and R&D intensity are positively linked across industries. Ilyina and Samaniego (2009) find support for this positive relationship in post-war US data for a comprehensive sample of industries.

[17] The model ranking of TFP and R&D intensity is stable in a rank-preserving equilibrium. To make industry comparisons of TFP growth rates and research intensity requires those features to be stable over time in the data. We computed TFP growth rates for durable goods over non-overlapping 10-year periods, using the procedure below. We found that the correlations between cross sections were always 80% or higher. Ilyina and Samaniego (2008) find that the decade-to-decade correlation of R&D intensity across US manufacturing industries is over 90%.

5.1 Calibration

We calibrate the model economy as follows. We compute industry spending shares ω_i from the Bureau of Economic Analysis' capital flow tables, 1947-2007. These shares for each of our 14 capital goods sectors appear to be fairly stable during the post-war era, consistent with our Cobb-Douglas capital good aggregator (24). We draw values of α_i from the BEA industry GDP tables. The implied value of $\alpha_x \equiv \sum_{j=2}^{15} \omega_j \alpha_j$ equals 0.3. Using (25), the growth of the relative price of capital, p_x/p_c, is $g_q = \prod_{j=2}^{15} \left(\frac{p_{jt+1}/p_{ct+1}}{p_{jt}/p_{ct}} \right)^{\omega_j}$. We compute g_q using $\frac{p_{jt+1}/p_{ct+1}}{p_{jt}/p_{ct}}$ (the growth of the quality-adjusted capital price relative to consumption) from Cummins and Violante (2002) for our 14 capital goods.

Ours is a multi-industry value-added growth model. Ngai and Samaniego (2009) argue that relative prices in the data do not correspond to relative productivity indices in a value added model. However, assuming that the share and composition of intermediate goods in gross output are each similar across sectors, they show that there is a simple transformation between relative prices in a value added model and relative prices in the data. If the relative price of a good in a value added model is $\frac{p_{it}}{p_{ct}}$, and the relative price of good i in the data is $\frac{\tilde{p}_{it}}{\tilde{p}_{ct}}$ (measured at the level of the good, i.e. gross output) then $\frac{p_{it}}{p_{ct}} = \left(\frac{\tilde{p}_{it}}{\tilde{p}_{ct}} \right)^{\frac{1}{1-\alpha_m}}$ where α_m is the share of intermediate goods in gross output. We set $\alpha_m = 0.45$, and find that $g_q = 1.052^{-1}$. In the working version of the paper, however, we show that results are strikingly similar when we use unadjusted prices.

Let g_y equal the growth factor of real output measured in units of consumption. US National Income and Product Accounts indicate that $g_y = 1.022$ in consumption units. In the model, g_y also represents the growth of real consumption, so we can compute the growth rate of capital in quality-adjusted units $g_k = g_y/g_q$. The model implies that $g_k = \gamma_x^{1/(1-\alpha_x)}$, which implies that $\gamma_x = 1.052$. The growth rate of the population is reported by the US Census Bureau. Table 1 summarizes the values of these variables.

The final two variables required for calibration are γ_i and η_i. As discussed earlier, we compute TFP growth to match the decline in the quality-adjusted relative prices for our 14 capital goods industries. This mapping is slightly complicated in our model compared to Greenwood et al (1997) because we allow input shares to differ across industries. Using (17), the definition of p_x and the calibrated value of γ_x, we compute γ_i as follows:

$$\frac{p_{xt+1}/p_{xt}}{p_{it+1}/p_{it+1}} = \frac{\gamma_i}{\gamma_x} g_k^{\alpha_i - \alpha_x} \Longrightarrow \gamma_i = \gamma_x g_k^{\alpha_x - \alpha_i} g_q \left(\frac{p_{it+1}/p_{it}}{p_{ct+1}/p_{ct+1}} \right)^{-1}, \tag{33}$$

where values of $\frac{p_{it+1}/p_{it}}{p_{ct+1}/p_{ct+1}}$ are drawn from Cummins and Violante (2002), and adjusted as discussed above.

We measure η_i as the capital share of research expenditures using data from the National Science Foundation Industrial Research and Development Survey.[18] The values of γ_i and η_i are reported in Table 2.

[18] See the Appendix for further notes on the data used for ω_i, α_i and η_i.

5.2 Opportunity and TFP growth rates

The model implies that variation in industry productivity growth rates depends on technological opportunity. We now use the data to learn about which of these factors appear quantitatively important. Using (10) and the restrictions imposed by balanced growth, industry productivity growth follows:

$$\gamma_i = (g_k^{\eta_i} g_N)^{\frac{\psi_i}{1-\rho_i}}; \quad \forall i. \tag{34}$$

Our first step is to ask whether variation in η_i can account for industry variation in γ_i. To this end, we use (34) to compute $\frac{\psi_i}{1-\rho_i}$ as a residual. Results are reported in Table 2. The correlation between γ_i and $\frac{\psi_i}{1-\rho_i}$ is 0.985. There are two reasons why the contribution of η_i to industry growth differences is low. First, as seen in Figure 1, the correlation between η_i and γ_i is not statistically significant (although it is positive, as implied by the model). Second, most importantly, variation in η_i is not of sufficient *magnitude* to generate large differences in γ_i on its own. To see this, we re-compute γ_i from (34) under the assumption that $\frac{\psi_i}{1-\rho_i}$ was equal in all industries. When we set $\frac{\psi_i}{1-\rho_i}$ to equal the weighted average across industries, we found that productivity growth rates ranged from 2.9% to 4.7%, which accounts for only about a tenth of the variation in Table 2. Thus, industry differences in productivity growth reflect significant variation in technological opportunities, as captured by ψ_i and ρ_i.

Determining whether ρ_i or ψ_i is responsible for differences in γ_i requires measures of at least one of these two parameters. Distinguishing between ρ_i and ψ_i is also needed later to compute R&D intensities. In what follows, we follow two approaches:

1. First, we discuss a possible measure of ρ_i, based on patent data. We then think of ψ_i as a residual, computing it from the values of $\frac{\psi_i}{1-\rho_i}$ in Table 2 for given values of ρ_i. We will find that variation in ρ_i is unable to jointly account for industry differences in productivity growth rates and R&D intensity, whereas variation in ψ_i is able to do so.

2. Second, we assume that ρ_i is perfectly correlated with γ_i, to give variation in ρ_i its "best shot." Thus, there will be parameterizations under which variation in ρ_i accounts for all industry differences in γ_i, and parameterizations under which ψ_i accounts for all differences in γ_i. Even so, in this case we still find that variation in ρ_i is unlikely to matter much.

Parameter ρ_i is linked to the magnitude of knowledge spillovers received by each industry. Following Jaffe et al (2000), we measure knowledge spillovers using the NBER patent citation database described in Hall et al (2001). For each patent granted over the period 1975-1999, the database mentions every patent that it cites – its bibliography. The database also includes patent categories for patents granted 1963-1999, at the 2-digit SIC level and also more finely. We use this information to assign patents to industries.

At the United States Patent and Trademark Office, one role of the patent examiner is to determine that the applicant has cited all relevant "prior art," and the presumption is that this mechanism ensures that patent citations accurately report the intellectual precursors of the patent under review. The examiner's name is reported on the patent, so the examiner is responsible for any mis-attributions. This suggests that patent citations accurately reflect knowledge spillovers from sources of patented knowledge. Since the bibliography does not include knowledge that is not patented, the presumption is also that the ranking of extent to which different sectors build on unpatented knowledge is not too different from ranking constructed using patented knowledge.

Parameter ρ_i represents the extent to which new knowledge in industry i "stands on the shoulders" of prior knowledge. Hence, to get a sense of relative magnitudes of ρ_i between industries we examine the rate at which patents in a given industry cite other patents – the rate of "backwards citations". We call this CIT_i. We assume that relative CIT_i is an indicator of relative ρ_i, and use CIT_i to examine a variety of possible parameterizations of the model, by changing the range over which ρ_i varies while assuming that ρ_i and CIT_i are correlated.

Aggregate estimates of the decreasing returns to research investment (analogous to ψ_i) vary between 0.1 and 0.6 (see Kortum (1993) and Samaniego (2007) for surveys): however, to our knowledge industry level estimates do not exist. Hence, we will think of ψ_i as a residual, computing it from $\frac{\psi_i}{1-\rho_i}$ for given values of ρ_i. We remind the reader that we follow two approaches to selecting ρ_i: we use patent citation data, and we also give variation in ρ_i its "best shot" by assuming it is perfectly correlated with productivity growth rates.

Figure 1 reports the correlation between CIT_i and γ_i. We do indeed find a correlation between backwards citations and productivity growth.[19] This suggests that ρ_i may be important for industry variation in productivity growth rates – although, just because ρ_i is correlated with γ_i does not mean that it is a *quantitatively* important factor behind variation in γ_i, something we will check below.

5.3 Decomposing opportunity

To make further progress in decomposing the sources of opportunity that account for industry growth, we turn to the model predictions for research intensity.

Assuming that CIT_i is an indicator of relative values of ρ_i, we examine a broad set of possible mappings between CIT_i and ρ_i. Given the series for $\frac{\psi_i}{1-\rho_i}$ in Table 2, a choice of ρ_i implies values of ψ_i for each industry. Specifically, given a lower bound $\underline{\rho}$ and an upper bound $\bar{\rho}$, we assume that ρ_i is perfectly correlated with CIT_i between these two parameters. We explore all values of $\underline{\rho}$ and $\bar{\rho}$ in the range $[-1, 1)$, provided $\underline{\rho} \leq \bar{\rho}$. We set $\bar{\rho} < 1$ because this is required for positive industry productivity growth rates. The bound $\underline{\rho} \geq -1$ is arbitrary but, as we shall see, results for lower values are straightforward to infer. Note that, given a value of γ_i, parameters ρ_i and ψ_i are negatively related.

The correlation between ρ_i and γ_i equals the correlation between CIT_i and γ_i by construction, which is 60% (the P-value is 2%). However, the correlation between ψ_i and γ_i is also high for most parameterizations – see Figure 2. Thus, correlations between parameters and productivity are not enough to indicate whether variation in γ_i is mainly due to ρ_i or ψ_i. It is interesting to note that the highest correlations between ψ_i and γ_i occur when the upper

[19] For robustness, we also checked the same link with productivity growth rates measured by Jorgenson et al (2006). Results were similar but statistical significance hinged on an outlier (Computers and Office Equipment).

and lower bounds on ρ_i are close together (near the 45 degree line): in this case, although ρ_i and γ_i are significantly correlated, most of the variation in productivity is in fact due to differences in ψ_i. On the 45 degree line itself, $\underline{\rho} = \bar{\rho}$, so that variation in ρ_i accounts for none of industry variation in productivity growth.

5.4 Opportunity, appropriability and research

To further narrow down the parameters that best account for the data, we use the model to compute predicted research intensity at the industry level, and examine for what parameter values the model generates research intensity values that most resemble those in the data. We look at two different criteria:

1. Is research intensity in the model correlated with research intensity in the data?

2. Is the *magnitude* of research intensity in the model close to that in the data?

Following the literature we measure R&D intensity as the *median ratio* of R&D expenditures to sales among firms in Compustat over the period 1950-2000.[20] The maintained assumption is that the *median firm* in Compustat is subject to weak if any financial constraints, so that its R&D behavior should reflect the "pure" technologically determined level of R&D intensity for the industry. See Rajan and Zingales (1998) and Ilyina and Samaniego (2008) on the use of median firms to detect technological characteristics. We discard the top and bottom 1% of observations in the sample, to reduce the influence of outliers and of possible measurement error.[21]

The model R&D spending to sales ratio $\left(RND^{Sales}\right)$ is determined by equation (32), which requires an expression for the growth rate in the shadow price of knowledge $\frac{\chi_{it+1}}{\chi_{it}}$ along a balanced growth path. Using (15), (17) and (29) this expression is:

$$\frac{\chi_{it+1}}{\chi_{it}} = \frac{\gamma_x}{G\gamma_i} g_k^{\alpha_x} g_N = \frac{g_k g_N}{G\gamma_i}. \tag{35}$$

Thus, computing RND^{Sales} in (32) requires values for industry-specific parameters ρ_i, ψ_j, A_i and μ_i, as well as G, which is common across industries. Again, we examine a variety of values of ρ_i and ψ_i by assuming a linear mapping between ρ_i and CIT_i as described above, and alternatively by assuming ρ_i is correlated with γ_i.[22]

As for appropriability A_i, we follow two approaches.

1. We use patent data to get a sense of the likely values of A_i.

[20] Ideally we would like to use an expenditure-based (RND_{it}) rather than a sales-based measure $\left(RND^{Sales}\right)$ to compute R&D intensity, however, we could not use Compustat to construct the shares of R&D as labor expenditures were very sparsely reported.

[21] The medians were in fact quite close to R&D/sales numbers reported by the NSF, so we view them as accurate (NSF values were not available for all industries, which is why we did not use them directly).

[22] Notice that $1 - \frac{\chi_{it+1}}{\chi_{it}}$ is the rate of economic depreciation of knowledge in our model. We find that computed values of this expression are almost perfectly correlated with γ_i, and vary between 5% and 23% across industries. Samaniego (2007) surveys values for the rate of economic depreciation of knowledge in a 1-sector economy that range between 10% and 25%.

2. We select $A_i \in [0,1]$ so as to maximize the influence of appropriability on R&D intensity.

Appropriability is related to whether spillovers across firms are a significant source of knowledge. As before, we use patent data to get a sense of the magnitude of these spillovers. The NBER patent citation database reports the assignee of each patent awarded since 1969. Consequently, we can establish what proportion of own-industry citations are in fact self-citations. We define appropriability A_i as this ratio. The required assumption is that A_i does not differ significantly for a given industry depending on whether or not knowledge is patented. If unpatented knowledge flows across firms more easily than patented knowledge, then the measure of spillovers implied by the patent data is an upper bound on A_i. On the other hand, if ideas that flow most easily across firms are the ones patented, then our numbers represent a lower bound on A_i. As we shall see, appropriability differences between patented and unpatented knowledge must be quite drastic to affect our results (in fact, when we assumed that A_i varied between 0 and 1 and that it was perfectly correlated with R&D intensity, our results were almost identical). Table 2 finds that appropriability A_i is generally quite low – 18.5% on average. In addition, it appears to vary little across industries, ranging in the interval $[0.12, 0.34]$. Thus, R&D intensity in equation (18) will be mainly determined by differences in ρ_i and ψ_i.

We calibrate μ_i using industry markups from Oliveira, Scarpetta and Pilat (1996). These authors report markups over average cost. In the model, μ_i is linked to the markup over *production* cost – which could be significantly larger than the markup over total cost in very research-intensive industries. In the Appendix we discuss the mapping between the reported markups and those required to calibrate μ_i.

Finally we set values for G. Using the Euler condition (29), the gross return to capital, $G = (1+r)/g_q$. We match the real rate of return on capital to be 7% as in Greenwood et al (1997). Hence the gross return in terms of capital goods is $G = 1.07/g_q$.

We find that the strongest correlations between R&D in the model and in the data (about 0.72) are generated by parameterizations under which $\underline{\rho}$ and $\bar{\rho}$ are close. See Figure 3. However, the *magnitude* of R&D intensity in the model is much larger than in the data for most parameterizations. Magnitudes are comparable only when $\underline{\rho}$ and $\bar{\rho}$ are both high. See Figure 4.

Along the locus of parameterizations such that overall R&D intensity in the model economy matches that in the data, the parameters that generate model industry R&D intensity numbers that correlate most strongly with those in the data satisfy $\underline{\rho} = \bar{\rho} = 0.94$, so that differences in ρ_i account for little of the variation in γ_i. Thus, industry *variation* in R&D intensity indicates that $\underline{\rho}$ and $\bar{\rho}$ are close, whereas the *magnitude* of R&D intensity indicates that the values of $\underline{\rho}$ and $\bar{\rho}$ are high. As a result, for the parameterization preferred by the data, variation in ψ_i is primarily responsible for industry differences in both R&D intensity and productivity growth.

Note that this is not because we have an imperfect measure of ρ_i. We repeated this exercise assuming that ρ_i was perfectly correlated with γ_i, also obtaining the result that the data prefer a parameterization under which variation in ψ_i is the paramount factor. Indeed, the results for the figures are almost identical.

There are two reasons for this result. First, (32) and (35) together imply that:

$$RND^{Sales} = \frac{(1 - 1/\mu_i)\,\psi_i}{1 - A_i\rho_i + \left(\frac{G}{g_k g_N} - 1\right)/(\gamma_i - 1)}, \tag{36}$$

Thus, given the calibrated value of γ_i, research intensity depends on ρ_i only through A_i. Our measures of A_i are not correlated with RND^{Sales} and are in fact negatively correlated with CIT_i (which we use to rank ρ_i across industries), so that industry variation in ρ_i does not necessarily translate into industry differences in research intensity.

The main reason is much simpler, however, and in no way hinges on the use of patent data nor on the fact that we are looking at a particular set of industries. The link between ψ_i and RND^{Sales} is monotonic, whereas that between ρ_i and RND^{Sales} is non-monotonic – in fact, it is an inverted U-shaped. To see this, we set μ_i, A_i, α_i and η_i to be equal across industries, so that $\frac{\psi_i}{1-\rho_i}$ is be perfectly correlated with $\log \gamma_i$. Moreover, we assumed that γ_i and ρ_i were perfectly correlated. Eliminating variation in A_i slightly increased correlations between R&D in the model and the data. However, once more, the correlation between R&D intensity in the model and the data was highest when $\underline{\rho} \approx \bar{\rho}$, and the data preferred a specification with $\underline{\rho} = \bar{\rho} \approx 0.9$. What happens is that, when values of ρ_i differ and the value of ρ_i is very high for the highest-growth industries, the value of ψ_i in those industries is driven towards zero. Since ψ_i enters the R&D expression multiplicatively, this drives research intensity to zero in those industries. As a result, when ρ_i is very high in some industries, there is no longer a monotonic relationship between γ_i and R&D intensity in the model. Since the data indicate that γ_i and R&D intensity are correlated, the presence of variation in ρ_i leads patterns of R&D intensity in the model to differ from those in the data. Interestingly, the parameters that give a better correlation between model research intensity and productivity change are those that also yield the strongest link between R&D intensity in the model and the data. For the preferred parameterization, the correlation between R&D intensity in the model and in the data is fully 0.72.

We began the paper noting that several authors have found an empirical link between industry R&D intensity and TFP growth rates – including Terleckjy (1980) and more recently Ilyina and Samaniego (2009). For the preferred parameterization, the correlation between the research-to-sales ratio in the data and the values of γ_i computed using the model is fully 84%. We also find that the correlation between model R&D intensity and γ_i is very high for most parameterizations – see Figure 5.

In the preferred parameterization, since values of ρ_i appear close to one, we expect values of ψ_i to be relatively small. Table 3 reports values of ψ_i that correspond to this parameterization. Are they consistent with the data? As mentioned, *aggregate* measures of the counterpart of ψ_i vary between 0.1 and 0.6, whereas the ω_i-weighted average value in the model economy is 0.12, which lies in this range (when ψ_i is weighted by R&D intensity, the average value is 0.26). We conclude that the data and the structure of the model together indicate that most of the dispersion in both productivity growth and research intensity is driven by industry differences in ψ_i – the extent to which there are decreasing returns to R&D – and that values of ψ_i are likely to be small in most industries, consistent with independent data on decreasing returns to research at the aggregate level.

To further assess the robustness of these conclusions, we maximized the ability of A_i to account for the R&D data, assuming that A_i is distributed between zero and one and that it is perfectly correlated with RND^{Sales}. Our results were essentially the same. Simply put, variation in A_i is small compared to variation in γ_i, so that the denominator is not significantly affected by appropriability.

6 Discussion and extensions

We have abstracted from cross-industry spillovers to keep the mechanism transparent, but it would be interesting to include them in the model. There are two reasons why allowing them is unlikely to change our results. First, the model does not suggest that knowledge spillovers are the driving force behind industry differences in productivity: ψ_i takes center stage. Second, cross-industry spillovers appear small compared to within-industry spillovers.

To see the second point, we use the patent citation database to estimate the importance of cross-industry citations. This is analogous to classifying all Economics papers by field, and looking at the rates at which papers in any given field cite papers in any other given field. As discussed in Hall et al (2001), industries seem to vary in their propensity to patent. We handle this by normalizing cross-citations by the total number of patents in the citing industry. Thus, the citation matrix we construct reflects the average rate at which patents in industry i cite patents in any industry j.

Table 4 reports the patent citation matrix. Each row corresponds to the average number of citations made by a given industry. Numbers on the diagonal represent within-industry citations. CIT_j is the sum of each row, the average number of citations per patent in each industry. For all industries, citations are dominated by within-industry citations, suggesting that cross-industry spillovers are relatively small.

We do not distinguish between product and process innovation, for several reasons. First, much (although by no means all) of the related empirical literature neglects the distinction. Second, it is rare that a "truly new" product is introduced. Rather, thinking of industries as being defined at the 2- or 3-digit SIC level, both product and process innovations may result in improved (or cheaper) consumer (or capital) services of a given type. Thus, our modeling approach is consistent with our use of quality-adjusted price data. Third, although one-sector growth models that distinguish between product and process innovation sometimes have different properties (such as Young (1998)), Jones (1999) argues that these properties require a "knife-edge" condition on the parameter linking the rate of product innovation to the scale of the economy. Still, it would be interesting to perform our analysis in a model that allows for product innovation.

There are three ways for a firm to acquire knowledge for use in production. First, firms may produce knowledge by investing in R&D, as in our model. Second, knowledge that spills between firms may be used as an input into R&D. This activity is free in the sense that, for example, if one patent cites another, there is no requirement that any payments be made between patent holders. While our model allows for such spillovers, the knowledge production function (1) implies that a firm can only receive spillovers from other firms if it is also carrying out research, as argued by Cohen and Levinthal (1990). Third, firms may employ the knowledge produced by other firms *in production,* by means of a license payment

– as in Klenow (1996). However, Arora et al (2002) find that revenues from licensing equal about 4% of R&D expenditure, suggesting that licensing is not a major incentive behind R&D activity in general. We abstract from this third form of knowledge transfers, as the other two appear to be more quantitatively important. Still, an extension of the model could be useful for studying patterns of licensing activity.

7 Concluding remarks

We develop a multi-sector, general equilibrium model of endogenous growth, incorporating a number of factors identified in the literature as potential determinants of the costs and benefits of research. We find that the main determinant of productivity growth differences across sectors are the technological opportunity parameters, especially the extent of decreasing returns to research activity. Although this parameter has not been identified as a potentially important source of cross-industry differences in the related literature, it turns out to play a pivotal role in a growth model that is consistent with stable growth over the long run. Theoretically, we find that two more factors of opportunity may be important – the extent to which new knowledge "stands on the shoulders" of prior knowledge, and the capital share of research activity – although quantitatively they do not appear to play an important role.

The fraction of total spillovers that accrues from the firm's own stock of knowledge affects research intensity but not TFP growth, whereas differences in demand factors affect neither, consistent with the lack of robustness in the empirical literature on the role of demand, and in line with a sense in the technology literature that technical change is primarily supply-driven. Nelson and Winter (1977) argue that innovations follow "natural trajectories" that have a technological or scientific rationale rather than being driven by movements in demand and, similarly, Rosenberg (1969) writes of innovation following a "compulsive sequence." In our model, equilibrium differences in long run productivity growth rates depend on opportunity parameters, so that long-run TFP growth rates are determined by technological factors: "natural trajectories" are an *equilibrium outcome*.

We see several directions for future work. It would be interesting to provide microfoundations for different factors of opportunity and appropriability. For example, could the magnitude of knowledge spillovers or the extent to which they accrue to different agents depend on the institutions that govern research, or even on organizational structure? Also, we have not used our model to explore policy implications. However, one of the key implications of our results is that a "one-size fits all" R&D subsidy may not be an optimal policy when technological opportunities vary significantly across industries. We leave this topic for future work.

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A Data

A.1 Patent data

The NBER patent database, described in detail in Hall et al (2001), classifies patents according to their industry of origin and type of innovation. This involves tracking the industry of origin of each patent, and of the patents that each patent cites, for 16,522,438 citation entries. While data on patents begin in 1963, citations are only available for patents granted since

1975. For most of the paper we place durables into 14 categories we could identify in the citation data. The industry classification in Hall et al (2001) mostly coincides with that in Table 2. The exceptions were Aircraft, Ships and Boats, Autos and Trucks, and Structures, which we put together from their finer classification, including only rubrics that we could definitively associate with the industry in question. Autos and Trucks combines classes 180, 280, 293, 278, 296, 298, 305 and 301. Structures combines classes 14 and 52 (Bridges and Static Structures). Aircraft equals class 244 (Aeronautics), and Ships and Boats is class 114 (Ships). The full list of categories may be found at http://www.nber.org/patents/list_of_classes.txt

Patents from other categories were counted as "Other" (i.e. non-durables). There is also an issue with the 15% of patent citations where the industry of origin of the cited patent was not available (i.e. the cited patent was older than 1963). When the industry of a citation was not known, we assumed that the industry distribution of citations was the same as for citations with a reported industry (which make up 85% of the database). Excluding these patents, or counting them as "Other", did not affect results. Assuming no spillovers between capital and "Other" also had little impact on the matrix for capital.

A.2 Capital shares

The NSF does not report capital expenditures related to R&D, rather they report a value of depreciation costs. Using a perpetual inventory method and the physical depreciation rates in the model, we derive the capital stocks implied by the depreciation costs and use them to impute the values of η_i reported in Table 2. This requires a value of the depreciation rate for capital δ: we use a value of 0.056, which we calibrate as in Greenwood et al (1997). We use the 2003 edition of the Industrial Research and Development Survey.

Values of α_i come from the Bureau of Economic Analysis Industry GDP tables. Not all industries were specifically listed as the industry classification of the BEA is coarser than ours. Thus, for example, the BEA entry for "Machinery" included both our "Machinery" and our "Mining and oilfield Machinery". In this case we used the same value for both sub industries. We used tables for 1987-1997 as earlier years were even more aggregated. We followed the same procedure for η_i.

A.3 Research intensity

Research intensity numbers from Compustat include labor and materials costs but not capital. In the model we have removed intermediates, and we also include capital. To make the numbers comparable, first, we remove materials using the materials share of R&D in NSF data (which is small and averages around a fifth of labor spending). Then, to impute capital expenditures related to research, we use the values of η_i reported earlier. Finally, formal R&D spending does not necessarily reflect all the costs of conducting R&D. For example, the Bureau of Labor Statistics Occupational Employment Statistics 2007 report that, for firms in NAICS 541700 (Scientific Research and Development Services) scientists and engineers make up about 40 percent of the wage bill. Assuming that the activities of pure research firms are broadly similar to those at research units within firms that do not outsource their R&D, this suggests multiplying the Compustat R&D numbers by a factor of 2.5. The effect of the above adjustments was to increase the values of RND^{Sales} somewhat above the raw numbers in Compustat, but the results that follow were qualitatively unchanged by using the "raw" numbers from Compustat instead.

A.4 Markups

Markups are from Oliveira, Scarpetta and Pilat (1996). Where industry values were not available for the US, we took them from Canada (or in the case of Aircraft from Italy[23]). These are markups over average cost. In the model calibrating μ_i requires a measure of the markup over *production* cost – which could be significantly larger than the markup over total cost in very research-intensive industries. Let M be the markup in the model, so $M = \frac{1}{\mu}$. N is the measured markup, which is the markup over average cost. Suppose P is sales, R is research cost and C is production cost. Then, the measured markup $N = \frac{P-R-C}{R+C}$. Let r equal R&D intensity as measured in the data (relative to sales), so that $R = rP$. Then, it can be shown that $M = \frac{r(N+1)+N}{1+r(N+1)}$, so the measured markups can be derived from those reported in the data using R&D intensity numbers.

B Derivations and Proofs

B.1 Firm's maximization

Taking the demand function $p_{iht}(.)$ and input prices $\{w_t, R_t\}$ as given, the firm chooses $\{N_{iht}, K_{iht}, Q_{iht}, L_{iht}\}_{t=0,..}$ to maximize (4) subject to (1)-(3). Optimal conditions imply:

$$w_t = p_{iht}\frac{\partial Y_{iht}}{\partial N_{iht}}\left(1+\frac{Y_{iht}}{p_{iht}}\frac{\partial p_{iht}}{\partial Y_{iht}}\right); \quad R_t = p_{iht}\frac{\partial Y_{iht}}{\partial K_{iht}}\left(1+\frac{Y_{iht}}{p_{iht}}\frac{\partial p_{iht}}{\partial Y_{iht}}\right). \tag{37}$$

Using (3), relative prices are,

$$\frac{p_{iht}}{p_{jht}} = \frac{\left(1+\frac{Y_{jht}}{p_{jht}}\frac{\partial p_{jht}}{\partial Y_{jht}}\right)(1-\alpha_j)\, T_{jht}k_{jht}^{\alpha_j}}{\left(1+\frac{Y_{iht}}{p_{iht}}\frac{\partial p_{iht}}{\partial Y_{iht}}\right)(1-\alpha_i)\, T_{iht}k_{iht}^{\alpha_i}}. \tag{38}$$

All firms take $\{w_t, R_t\}$ as given, so marginal rate of substitution between capital and labor are equal across activities, firms and sectors: $\frac{\partial Y_{iht}/\partial N_{iht}}{\partial Y_{iht}/\partial K_{iht}} = \frac{w_t}{R_t} = \frac{\partial F_{iht}/\partial L_{iht}}{\partial F_{iht}/\partial Q_{iht}}$. Using (37), the capital-labor ratios in equations (5)-(7) follows from (2)&(3) . Let $k_{iht} \equiv \frac{K_{iht}}{N_{iht}}$, (5)-(7) imply:

$$k_{iht} = k_{it}, \quad k_{jt} = \frac{\alpha_j}{1-\alpha_j}\frac{1-\alpha_i}{\alpha_i}k_{it}; \quad \frac{Q_{iht}}{L_{iht}} = \frac{\eta_i}{1-\eta_i}\frac{1-\alpha_i}{\alpha_i}k_{it}. \tag{39}$$

So (17) follows from focusing on a rank-preserving equilibrium where $\gamma_{iht} = \gamma_{it}$ and assuming price elasticities of demand are constants and sector-specific.

R&D Intensity: The firm's optimal allocation of capital across activities implies (11) and its optimal condition for T_{iht+1} implies (12). Using (11),

[23] Aircraft values were also available for France but French markups appeared systematically larger.

$\frac{1}{\chi_{iht+1}}\frac{\lambda_{t+1}}{p_{ct+1}}\frac{\partial \Pi_{iht+1}}{\partial T_{iht+1}} = -\frac{\partial \Pi_{iht+1}/\partial T_{iht+1}}{\partial \Pi_{iht+1}/\partial Q_{iht+1}}\frac{\partial F_{iht+1}}{\partial Q_{iht+1}}$. Together with (2)-(4),

$$\begin{aligned}
\frac{\frac{\lambda_{t+1}}{p_{ct+1}}\frac{\partial \Pi_{iht+1}}{\partial T_{iht+1}}}{\chi_{iht+1}} &= \frac{\left(1+\frac{Y_{iht}}{p_{iht}}\frac{\partial p_{iht}}{\partial Y_{iht}}\right)p_{iht+1}\frac{Y_{iht+1}}{T_{iht+1}}}{\left(1+\frac{Y_{iht}}{p_{iht}}\frac{\partial p_{iht}}{\partial Y_{iht}}\right)\frac{\alpha_i p_{iht+1}Y_{iht+1}}{K_{iht+1}}}\frac{\eta_i \psi_i F_{iht+1}}{Q_{iht+1}} \\
&= \frac{K_{iht+1}}{T_{iht+1}\alpha_i}\frac{\eta_i \psi_i F_{iht+1}}{Q_{iht+1}} = \frac{1-\eta_i}{1-\alpha_i}\frac{N_{iht+1}}{L_{iht+1}}\psi_i\left[\gamma_{iht+1}-1\right].
\end{aligned}$$

where the last equality follows from (1) & (39). Rearrange (12) as:

$$\begin{aligned}
\frac{\chi_{iht}}{\chi_{iht+1}} &= \frac{\frac{\lambda_{t+1}}{p_{ct+1}}\frac{\partial \Pi_{iht+1}}{\partial T_{iht+1}}}{\chi_{iht+1}} + \frac{\partial F_{iht+1}}{\partial T_{iht+1}} + 1 \\
&= \frac{1-\eta_i}{1-\alpha_i}\frac{N_{iht+1}}{L_{iht+1}}\left[\psi_i\gamma_{iht+1}-1\right] + \kappa_i\left[\gamma_{it+1}-1\right] + 1,
\end{aligned}$$

where the equality follows from (1), finally,

$$\frac{n_{iht+1}}{l_{iht+1}} = \left(\frac{1-\alpha_i}{1-\eta_i}\right)\frac{1}{\psi_i}\left[\frac{\frac{\chi_{iht}}{\chi_{iht+1}}-1}{\gamma_{iht+1}-1}-\kappa_i\right], \tag{40}$$

where $n_{iht} \equiv N_{iht}/N_t$ and $l_{iht} \equiv L_{iht}/N_t$. Using (37),
$w_t L_{iht} + R_t Q_{iht} = \left(p_{iht} + Y_{iht}\frac{\partial p_{iht}}{\partial Y_{iht}}\right) T_{iht} k_{iht}^{\alpha_i}\left[(1-\alpha_i)L_{iht} + \alpha_i Q_{iht}\right]$.

$$\begin{aligned}
\text{Use (39)} \quad &: \quad w_t L_{iht} + R_t Q_{iht} = \left(1+\frac{Y_{iht}}{p_{iht}}\frac{\partial p_{iht}}{\partial Y_{iht}}\right)\frac{1-\alpha_i}{1-\eta_i}p_{iht}Y_{iht}\frac{L_{iht}}{N_{iht}}. \qquad (41)\\
\text{Similarly} \quad &: \quad w_t N_{iht} + R_t K_{iht} = \left(1+\frac{Y_{iht}}{p_{iht}}\frac{\partial p_{iht}}{\partial Y_{iht}}\right)p_{iht}Y_{iht}. \\
\text{So} \quad &: \quad \frac{w_t N_{iht} + R_t K_{iht}}{w_t L_{iht} + R_t Q_{iht}} = \left(\frac{1-\eta_i}{1-\alpha_i}\right)\frac{N_{iht}}{L_{iht}} = \frac{1}{\psi_i}\left(\frac{\frac{\chi_{iht}}{\chi_{iht+1}}-1}{\gamma_{iht+1}-1}-\kappa_i\right),
\end{aligned}$$

where the last equality follows from (40). Substituting into the definition of RND_{iht} to obtain (13). To derive χ_{iht}/χ_{iht+1}, use (2) & (39): $\frac{\partial F_{iht+1}/\partial Q_{iht+1}}{\partial F_{iht}/\partial Q_{iht}} = \gamma_{iht}^{\kappa_i}\gamma_i^{\sigma_i}g_{kt}{}^{\eta_i\psi_i-1}\left(\frac{l_{iht+1}}{l_{iht}}\right)^{\psi_i-1}$.

$$\text{Use (11)\&(37)}: \frac{\chi_{iht}}{\chi_{iht+1}} = \left(\frac{\lambda_t p_{iht}/p_{ct}}{\lambda_{t+1}p_{iht+1}/p_{ct+1}}\right)\gamma_{iht}^{\kappa_i-1}\gamma_{it}^{\sigma_i}g_{kt}{}^{\eta_i\psi_i-\alpha_i}\left(g_N\frac{l_{iht+1}}{l_{iht}}\right)^{\psi_i-1}. \tag{42}$$

B.2 Household maximization

We first determine the consumer's optimal spending across goods taking as given the total per capita spending on consumption s_c and investment s_x. Omitting time subscripts:

$$\begin{aligned}
\max_{\{c_{ih}\}} c \qquad s.t. \qquad s_c &= \sum\nolimits_{i=1}^{m-1}\int_0^1 p_{ih}c_{ih}dh, \qquad \text{and} \\
\max_{\{x_{jh}\}} x \qquad s.t. \qquad s_x &= \sum\nolimits_{j=m}^{z}\int p_{jh}x_{jh}dh
\end{aligned}$$

where c and x are defined (21) and (24). The optimal spending across varieties of consumption goods: $(c_{ih}/c_{ih'})^{-1/\mu_i} = p_{ih}/p_{ih'} \Longrightarrow c_{ih'} = c_{ih}(p_{ih}/p_{ih'})^{\mu_i}$, so $c_i = \left(\int_0^1 c_{ih'}^{\frac{\mu_i-1}{\mu_i}} dh'\right)^{\frac{\mu_i}{\mu_i-1}} = c_{ih}\left[\int (p_{ih}/p_{ih'})^{\mu_i-1} dh'\right]^{\frac{\mu_i}{\mu_i-1}}$. Define $p_i \equiv \left[\int p_{ih}c_{ih}dh\right]/c_i = \left[\int p_{ih}^{1-\mu_i}dh\right]^{1/(1-\mu_i)}$ to rewrite $c_i = c_{ih}(p_{ih}/p_i)^{\mu_i}$. Thus across good i, $p_i c_i/(p_j c_j) = \omega_i/\omega_j \Longrightarrow p_i c_i = \omega_i s_c$, together with (21),

$$c_{ih} = s_c (p_i/p_{ih})^{\mu_i}\omega_i/p_i; \quad p_c \equiv s_c/c = \prod_{i=1}^{m-1} p_i^{\omega_i}. \tag{43}$$

The result follows analogously for investment,

$$x_{jh} = s_x (p_j/p_{jh})^{\mu_j}(\omega_j/p_j) \text{ and } x_j = s_x(\omega_j/p_j), \tag{44}$$

$$p_j \equiv \frac{\int p_{jh}c_{jh}dh}{x_j} = \left[\int p_{jh}^{1-\mu_j}dh\right]^{1/(1-\mu_j)}; \quad p_x \equiv s_x/x = \prod_{j=m}^{z} p_j^{\omega_j} \tag{45}$$

Finally, the dynamic problem is to maximize $\sum_{t=0}^{\infty}(\beta g_N)^t u(c_t)$ by choosing $\{c_t, x_t\}_{t=0,..}$ s.t.

$$p_{ct}c_t + p_{xt}x_t = w_t + R_t k_t + \pi_t \quad and \quad g_N k_{t+1} = x_t + (1-\delta_k)k_t.$$

The solution implies (28) and (29).

B.3 Market Equilibrium

The goods market clearing condition (26) together with the demand for goods ih in (43) and (44) imply $\frac{Y_{iht}}{p_{iht}}\frac{\partial p_{iht}}{\partial Y_{iht}} = \frac{1}{\mu_i}$ together with (39), equations (37) and (38) become:

$$R_t = \alpha_i p_{iht}T_{iht}k_{it}^{\alpha_i-1}(1-1/\mu_i); \quad w_t = (1-\alpha_i)p_{iht}T_{iht}k_{it}^{\alpha_i}(1-1/\mu_i), \tag{46}$$

$$\frac{p_{iht}}{p_{jht}} = \frac{(1-1/\mu_j)(1-\alpha_j)T_{jht}k_{jt}^{\alpha_j}}{(1-1/\mu_i)(1-\alpha_i)T_{iht}k_{it}^{\alpha_i}}. \tag{47}$$

The capital market clearing condition (27) and (39) imply

$$k_{jht} = \left(\frac{\alpha_j}{1-\alpha_j}\right)\frac{k_t}{\Psi_t}; \quad \Psi_t = \sum_{i,h}\left(\frac{\alpha_i}{1-\alpha_i}n_{iht} + \frac{\eta_i}{1-\eta_i}l_{iht}\right). \tag{48}$$

In any rank-preserving equilibrium, we know (8) and (17), using (25),

$$\frac{p_{xt+1}/p_{xt}}{p_{it+1}/p_{it+1}} = \prod_{j=m}^{z}\left(\frac{p_{jt+1}/p_{jt}}{p_{it+1}/p_{it+1}}\right)^{\omega_j} = \frac{\gamma_{it}}{\gamma_{xt}}g_{kt}^{\alpha_i-\alpha_x}, \tag{49}$$

$$\frac{p_{ct+1}/p_{ct}}{p_{it+1}/p_{it+1}} = \prod_{j=1}^{m-1}\left(\frac{p_{jt+1}/p_{jt}}{p_{it+1}/p_{it+1}}\right)^{\omega_j} = \frac{\gamma_{it}}{\gamma_{ct}}g_{kt}^{\alpha_i-\alpha_c}$$

$$\gamma_{xt} \equiv \prod_{j=m}^{z}\gamma_{jt}^{\omega_j}; \quad \gamma_{ct} \equiv \prod_{i=1}^{m-1}\gamma_{it}^{\omega_i}; \quad \alpha_c \equiv \sum_{i=1}^{m-1}\alpha_i\omega_i; \quad \alpha_x \equiv \sum_{j=m}^{z}\alpha_j\omega_j \tag{50}$$

$$\text{So: } \frac{q_{t+1}}{q_t} = \frac{\gamma_{ct}}{\gamma_{xt}}g_{kt}^{\alpha_c-\alpha_x}. \tag{51}$$

In any rank-preserving equilibrium, substituting (29) into (42) implies $\frac{\chi_{iht}}{\chi_{iht+1}} = \frac{\chi_{it}}{\chi_{it+1}} = \left(\frac{G_t p_{iht}/p_{xt}}{p_{iht+1}/p_{xt+1}}\right) \gamma_{it}^{\rho_i - 1} g_{kt}^{\eta_i \psi_i - \alpha_i} \left(g_N \frac{l_{it+1}}{l_{it}}\right)^{\psi_i - 1}$. Use (49):

$$\frac{\chi_{it}}{\chi_{it+1}} = \frac{G_t}{\gamma_{xt}} \gamma_{it}^{\rho_i} g_{kt}^{\eta_i \psi_i - \alpha_x} \left(g_N \frac{l_{it+1}}{l_{it}}\right)^{\psi_i - 1}. \tag{52}$$

Symmetric Equilibrium

We now focus on the symmetric equilibrium across firms and omit index h. Using (47)&(25), $\frac{p_i}{p_x} = \prod_{j=m}^{z} \left(\frac{p_i}{p_j}\right)^{\omega_j} = \prod_{j=m}^{z} \left(\frac{(1-1/\mu_j)(1-\alpha_j) T_{jt} k_{jt}^{\alpha_j}}{(1-1/\mu_i)(1-\alpha_i) T_{it} k_{it}^{\alpha_i}}\right)^{\omega_j}$.

$$\text{Use (39)} \quad : \quad \frac{p_{it}}{p_{xt}} = \frac{(1-1/\mu_x) T_{xt} k_{it}^{\alpha_x - \alpha_i} \prod_{j=m}^{z} \left[\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}\right]^{\omega_j}}{(1-1/\mu_i) T_{it} \alpha_i^{\alpha_x} (1-\alpha_i)^{1-\alpha_x}}. \tag{53}$$

$$(1-1/\mu_x) \equiv \prod_{j=m}^{z} (1-1/\mu_j)^{\omega_j}, \quad T_{xt} \equiv \prod_{j=m}^{z} T_{jt}^{\omega_j}.$$

$$\text{Use (46)\&(48)} : \frac{R_t}{p_{xt}} = T_{xt} (1-1/\mu_x) k_t^{\alpha_x - 1} {\Psi_t}^{1-\alpha_x} \prod_{j=m}^{z} \left[\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}\right]^{\omega_j}. \tag{54}$$

Market clearing for consumption goods: $p_{it} T_{it} k_{it}^{\alpha_i} n_{it} = \omega_i p_{ct} c_t \Longrightarrow n_{it} = \frac{\omega_i p_{ct} c_t}{p_{it} T_{it} k_{it}^{\alpha_i}}$.

$$\text{Use (48)\&(53)} : n_i = \frac{c_t/q_t}{T_{xt} k_t^{\alpha_x}} \left(\frac{1-1/\mu_i}{1-1/\mu_x}\right) \frac{\omega_i (1-\alpha_i)}{\Psi_t \prod_{j=m}^{z} \left[\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}\right]^{\omega_j}}; \quad i = 1, .m-1 \tag{55}$$

Similarly, use market clearing for investment goods, (53) and (48),

$$n_{jt} = \frac{x_t}{T_{xt} k_t^{\alpha_x}} \left(\frac{1-1/\mu_j}{1-1/\mu_x}\right) \frac{\omega_j (1-\alpha_j)}{\Psi_t \prod_{j=m}^{z} \left[\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}\right]^{\omega_j}}; \quad j = m, ..z. \tag{56}$$

Balanced Growth Path

Proof of Proposition 1. We look for a balanced growth path (BGP) such that x, k and c are growing at constant rates. Household's (22) and (23) require x/k and $c/(qk)$ to be constants. Define $k_{et} = k_t T_{xt}^{-1/(1-\alpha_x)}$. Let $g_x \equiv x_{t+1}/x_t$ for all variables x. Note when n_i and l_i are constants, (48) implies Ψ is constant, so (48) implies k_{it}/k_t is constant. Using (28),

$$\frac{1}{\beta} \left(\frac{c_{t+1}}{c_t}\right)^{\theta} = \frac{u'(c_t)}{\beta u'(c_{t+1})} = g_{qt} G_{t+1}, \tag{57}$$

so g_c is constant if g_q and G are constants. Using (54), G is constant if and only if k_e is constant, i.e. $g_{kt} = \gamma_{xt}^{1/(1-\alpha_x)}$. It follows from (55) and (56) that both n_i are constants $\forall i$.

Substituting (3) into (21) and (24): $x_t = \prod_{j=m}^{z} \left(\frac{T_{jt} k_{jt}^{\alpha_j} n_{jt}}{\omega_j} \right)^{\omega_j}$ and $c_t = \prod_{i=1}^{m-1} \left(\frac{T_{it} k_{it}^{\alpha_i} n_{jt}}{\omega_j} \right)^{\omega_j}$. Using (48) and (50), $g_x = \gamma_x g_k^{\alpha_x}$, together with (51), $g_c = \gamma_c g_k^{\alpha_c} = g_q \gamma_x g_x^{\alpha_x}$. Given $g_k = \gamma_x^{1/(1-\alpha_x)}$, both x/k and $c/(qk)$ are constants when g_k and g_q are constants. But g_k is constant if γ_j are constants $\forall j = m, .z$. Lemma (1) implies (10) holds for $\forall j = m, .z$, using (50), $\gamma_x = \prod_{i=m}^{z} \gamma_i^{\omega_i} = \prod_{i=m}^{z} (g_N g_k^{\eta_i})^{\frac{\omega_i \psi_i}{1-\rho_i}} = \prod_{i=m}^{z} \left(g_N \left(\gamma_x^{\eta_i/(1-\alpha_x)} \right) \right)^{\frac{\omega_i \psi_i}{1-\rho_i}}$, which implies (30) and γ_x in Proposition 1. It follows from (51) that g_q is constant if and only if γ_c is constant, i.e. γ_i are constants $\forall i = 1, ..m-1$. Given n_i, g_k and γ_i are constants, (52) and (40) imply χ_{it+1}/χ_{it} and n_{it}/l_{it} are constants $\forall i$, so l_i are constants $\forall i$. ■

Corollary 1 *Let $y = \sum \frac{p_{it}}{p_{ct}} y_{it}$. Along BGP, c/y, real interest rate and R&D spending to GDP ratio are constants. Moreover,*

$$g_q = \gamma_c \gamma_x^{\frac{\alpha_c - 1}{1-\alpha_x}}; \quad g_c = \gamma_c \gamma_x^{\frac{\alpha_c}{1-\alpha_x}}; \quad \gamma_c = g_N^{\Upsilon}; \quad \Upsilon \equiv \sum_{i=1}^{m-1} \left(\frac{\eta_i \Phi}{1-\alpha_x} + 1 \right) \frac{\omega_i \psi_i}{1-\rho_i}. \qquad (58)$$

Proof. Given G and g_q are constants, it follows from (29) that real interest rate r is constant. Using (53) and (48), GDP per head:

$$\begin{aligned} \sum_{i=1}^{z} \frac{p_{it} Y_{it}}{N_t} &= p_{xt} \sum_{i=1}^{z} \frac{(1-1/\mu_x) T_{xt} k_{it}^{\alpha_x} n_i \prod_{j=m}^{z} \left[\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j} \right]^{\omega_j}}{(1-1/\mu_i) \alpha_i^{\alpha_x} (1-\alpha_i)^{1-\alpha_x}} \\ &= p_{xt} k_t T_{xt} k_t^{\alpha_x - 1} \sum_{i=1}^{z} \frac{n_i (1-1/\mu_x) \prod_{j=m}^{z} \left[\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j} \right]^{\omega_j}}{(1-1/\mu_i) \Psi (1-\alpha_i)}, \end{aligned} \qquad (59)$$

so y/c is constant given $T_{xt} k_t^{\alpha_x - 1}$ and $c/(qk)$ are constants. Using (41) and (39),

$$\sum_{i=1}^{z} (L_i w_t + Q_i R) = N_t R_t \frac{k_t}{\Psi} \sum_{i=1}^{z} \frac{l_i}{1-\eta_i},$$

Given constant R_t/p_{xt} and (59), the R&D spending to GDP ratio is constant. For (58), γ_c follows from substituting (10) and $g_k = g_N{}^{\Phi/(1-\alpha_x)}$ into (50). Given $g_k = \gamma_x^{1/(1-\alpha_x)}$, g_c and g_q follow from (51) and constant $c/(qk)$. ■

Proposition 3 *Along the BGP, the non-negativity constraints on l_i and n_i do not bind and the transversality conditions for T_i and k are satisfied if $g_N^{\left(1+\frac{\eta_i \Phi}{1-\alpha_x}\right)\frac{\psi_i}{1-\rho_i}} \geq 1$, $\kappa_i < 1$, $\forall i$ and $\beta < \left\{ g_N^{-1}, \bar{\beta} \right\}$, where $\bar{\beta} \equiv (1/g_N)^{1+(1-\theta)\left(\frac{\alpha_c \Phi}{1-\alpha_x} + \Upsilon\right)}$ and Υ is defined in (58).*

Proof of Proposition 3. First note that $\beta g_N < 1$ is required for the household maximization to be well-defined. The transversality conditions (TVC) are: $\lim_{t\to\infty} \zeta_t k_{t+1} = \lim_{t\to\infty} \chi_{it} T_{it+1} = 0$, $\forall i$. χ_{it} and ζ_t are the corresponding shadow values. Using (35),

$$\frac{\chi_i T_{it}}{\chi_{it-1} T_{it-1}} = \frac{\gamma_x}{G} g_k^{\alpha_x} g_N = \gamma_x g_k^{\alpha_x} g_N \beta g_q g_c^{-\theta} = \beta g_N g_c^{1-\theta}, \qquad (60)$$

where it uses (57), $g_k = \gamma_x^{1/(1-\alpha_x)}$ and constant $c/(qk)$. Using (58), $\lim_{t\to\infty} \chi_{it} T_{it+1} = \chi_{i0} T_{i0} \lim_{t\to\infty} \left(\beta g_N^{1+(1-\theta)\left(\frac{\alpha_c \Phi}{1-\alpha_x}+\Upsilon\right)} \right)^t$. So TVC for T_i holds if $\beta < \bar{\beta}$. The shadow price for k is the discounted marginal utility, $\zeta_t = (\beta g_N)^t \left(\frac{p_{xt}}{p_{ct}}\right) u'(c_t) = (\beta g_N)^t q_t c_t^{-\theta}$, constant $\frac{c}{qk}$ implies $\frac{\zeta_t k_t}{\zeta_{t-1} k_{t-1}} = \beta g_N g_c^{1-\theta}$, so TVC for k holds when TVC for T_i in (60) holds. Finally, from (1), $l_i > 0 \Leftrightarrow \gamma_i > 1$, so (10) implies $g_N^{\left(1+\frac{\eta_i \Phi}{1-\alpha_x}\right)\frac{\psi_i}{1-\rho_i}} \geq 1$. From (60), $\frac{\chi_{it}/\chi_{it+1}}{\gamma_i} = \frac{\chi_{it} T_{it}}{\chi_{it+1} T_{it+1}} = \left(\beta g_N g_c^{1-\theta}\right)^{-1} > 1$, for $\beta < \bar{\beta}$. So from (40), a sufficient condition for $n_i/l_i > 0$ (so $n_i > 0$) is $\kappa_i < 1$. ■

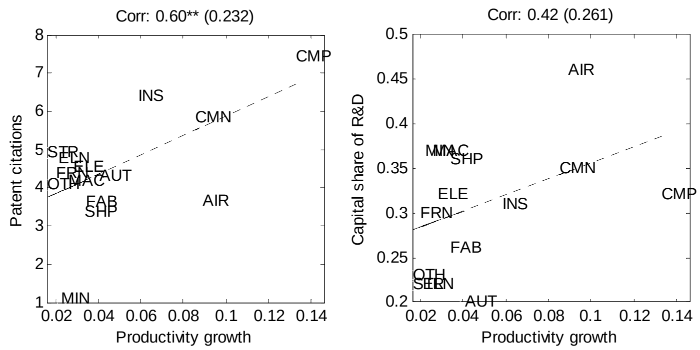

Figure 1 – Productivity growth, average patent citations and capital shares in research, by industry. Citations are the average backwards citations, the number of patents cited by a patent in the given industry. Productivity is measured using rate of decline of quality-adjusted prices relative to the consumption and services deflator. Sources – NBER patent citation database, Cummins and Violante (2002), Bureau of Economic Analysis, National Science Foundation.

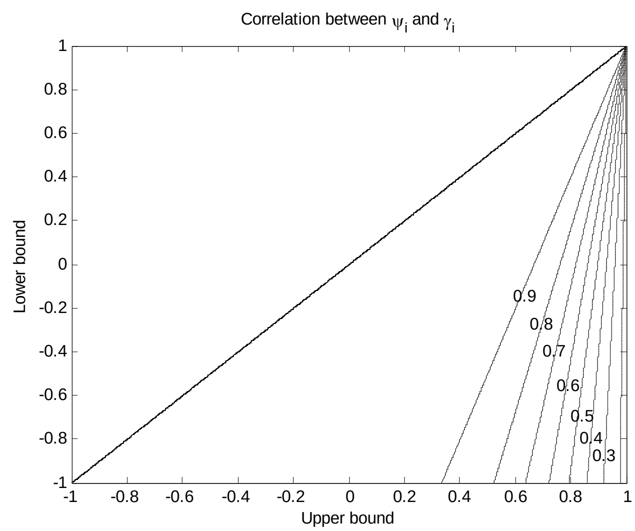

Figure 2 – Cross-industry correlation between ψ_i and productivity growth γ_i in the model, for different parameterizations. The correlation between patent citations CIT_i and ρ_i is assumed to be one, and ρ_i is distributed between the upper and lower bounds depicted in the graph. The upper left half of the graph is blank because the upper bound is necessarily higher than the lower bound. The same holds in Figures $2 - 5$.

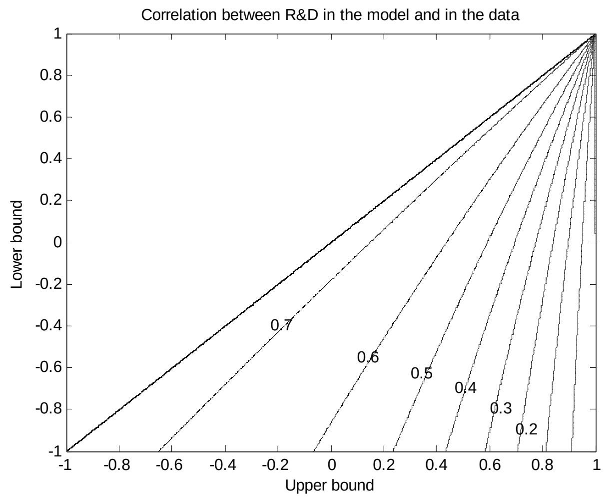

Figure 3 – Cross-industry correlation between R&D intensity in the model and the data, for different parameterizations. The correlation between patent citations CIT_i and ρ_i is assumed to be one, and ρ_i is distributed between the upper and lower bounds depicted in the graph.

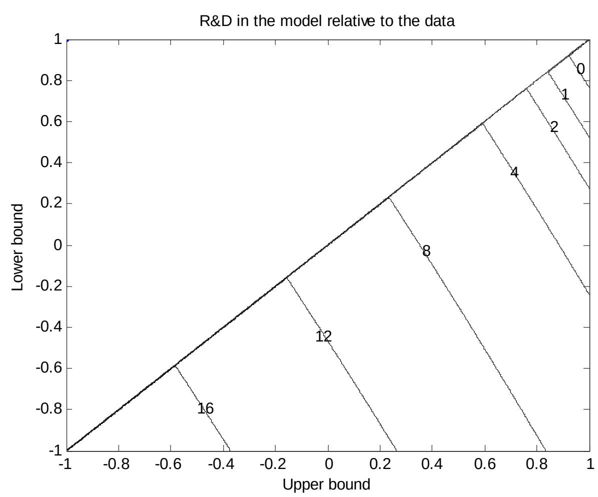

Figure 4 – Total R&D intensity in the capital sector of the model economy divided by the same statistic in the data, for different parameterizations. A value of one indicates that R&D intensity in the model is double that in the data. The correlation between patent citations CIT_i and ρ_i is assumed to be one, and ρ_i is distributed between the upper and lower bounds depicted in the graph.

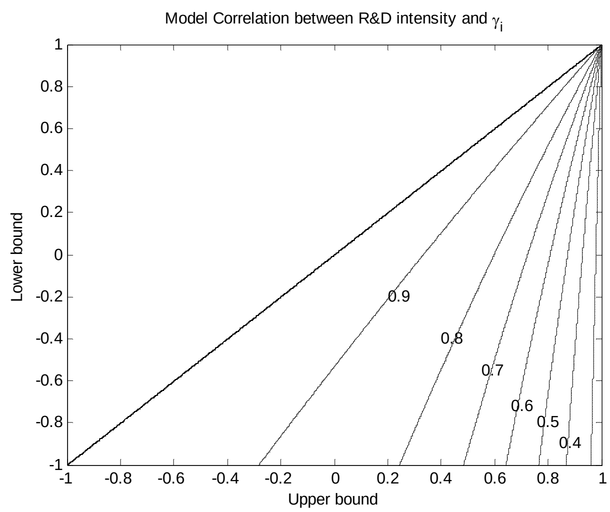

Figure 5 – Cross-industry correlation between R&D intensity and γ_i in the model, for different parameterizations. The correlation between patent citations CIT_i and ρ_i is assumed to be one, and ρ_i is distributed between the upper and lower bounds depicted in the graph.

Variable	α_x	g_y	g_q	γ_x	g_N
Value	0.3	1.022	1.0517^{-1}	1.0518	1.012

Table 1 – Calibrated aggregate values (see text).
Values for ω_i and α_i are reported in the Appendix.

Capital good sector	$\log \gamma_i$	η_i	$\frac{\psi_i}{1-\rho_i}$	CIT_i
Computers and office equipment	0.228	0.32	6.53	7.43
Communication equipment	0.145	0.35	3.90	5.85
Aircraft	0.148	0.46	3.28	3.64
Instruments and photocopiers	0.095	0.31	2.78	6.39
Fabricated metal products	0.049	0.26	1.62	3.62
Autos and trucks	0.060	0.20	2.28	4.32
Elec. trans. dist. and ind. app.	0.040	0.32	1.16	4.54
Other durables	0.018	0.23	0.64	4.07
Ships and boats	0.047	0.36	1.24	3.36
Electrical equipment, n.e.c.	0.028	0.22	1.02	4.75
Machinery	0.034	0.37	0.88	4.19
Mining and oilfield machinery	0.028	0.37	0.72	1.09
Furniture and fixtures	0.023	0.30	0.70	4.37
Structures	0.018	0.22	0.64	4.92

Table 2 – TFP growth rates and industry parameters. γ_i is based on the quality-adjusted relative price of capital from Cummins and Violante (2002). The capital share of R&D spending is η_i. Values of $\psi_i/(1-\rho_i)$ are computed using equation (34). CIT_i is the average number of backwards citations by patents in industry i.

Capital good sector	Model	Data	ψ_i
Computers and office equipment	0.220	0.250	0.50
Communication equipment	0.131	0.205	0.30
Aircraft	0.197	0.071	0.26
Instruments and photocopiers	0.139	0.158	0.21
Fabricated metal products	0.073	0.026	0.13
Autos and trucks	0.117	0.014	0.18
Elec. trans. dist. and ind. app.	0.043	0.043	0.09
Other durables	0.016	0.024	0.05
Ships and boats	0.048	0.022	0.10
Electrical equipment, n.e.c.	0.032	0.042	0.08
Machinery	0.028	0.096	0.07
Mining and oilfield machinery	0.023	0.048	0.06
Furniture and fixtures	0.022	0.006	0.05
Structures	0.014	0.000	0.05
TOTAL	0.057	0.057	–

Table 3 – R&D intensity in the model and in the data. Values of ρ_i and ψ_i are those that maximize the correlation between R&D in the model and the data – specifically, $\bar{\rho}$ and $\underline{\rho}$ equal 0.94. The column "Data" represents R&D spending at the median firm in Compustat, adjusted as described in the text. The third column represents the values of ψ_i used to compute R&D intensity in the model

		Spillover source															Total
Code	Spillover recipient	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	CIT
1	Comp. & Office	5.30	0.55	0.01	0.26	0.03	0.05	0.32	0.13	0.00	0.35	0.14	0.00	0.01	0.00	0.29	7.43
2	Commun.	0.49	4.17	0.01	0.27	0.01	0.02	0.36	0.05	0.01	0.23	0.04	0.00	0.01	0.00	0.19	5.85
3	Aircraft	0.10	0.11	2.11	0.07	0.01	0.04	0.19	0.22	0.04	0.06	0.12	0.00	0.06	0.03	0.48	3.64
4	Instr. & Photocop	0.15	0.16	0.00	4.69	0.03	0.00	0.21	0.07	0.00	0.22	0.13	0.00	0.03	0.00	0.69	6.39
5	Fab. Met. Prod	0.05	0.02	0.00	0.07	2.06	0.01	0.29	0.15	0.00	0.07	0.25	0.00	0.02	0.02	0.64	3.62
6	Autos and Trucks	0.06	0.03	0.01	0.02	0.01	2.99	0.11	0.18	0.03	0.02	0.28	0.00	0.13	0.01	0.44	4.32
7	Electrical transm.	0.14	0.11	0.01	0.10	0.09	0.01	3.46	0.05	0.00	0.12	0.09	0.00	0.01	0.00	0.35	4.54
8	Other Durables	0.10	0.04	0.01	0.09	0.07	0.04	0.09	2.59	0.01	0.04	0.31	0.00	0.04	0.02	0.62	4.07
9	Ships and boats	0.01	0.04	0.03	0.02	0.01	0.06	0.08	0.11	2.32	0.01	0.16	0.00	0.04	0.02	0.46	3.36
10	Electrical eq. n.e.c.	0.30	0.23	0.00	0.36	0.07	0.01	0.38	0.05	0.00	2.86	0.07	0.00	0.01	0.00	0.40	4.75
11	Machinery	0.06	0.02	0.00	0.08	0.07	0.04	0.10	0.18	0.01	0.03	2.86	0.00	0.02	0.01	0.72	4.19
12	Mining and oilfield	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.04	0.00	0.00	0.06	0.65	0.00	0.00	0.29	1.09
13	Furniture and fixt	0.01	0.02	0.01	0.08	0.02	0.07	0.06	0.14	0.01	0.03	0.11	0.00	3.00	0.05	0.77	4.37
14	Structures	0.00	0.01	0.01	0.02	0.05	0.01	0.09	0.23	0.01	0.01	0.12	0.00	0.12	3.16	1.08	4.92
15	Other	0.05	0.03	0.00	0.17	0.08	0.02	0.14	0.14	0.01	0.06	0.26	0.01	0.06	0.03	6.66	7.72

Table 4 – Patent citation matrix derived from the NBER patent citation database. We focus on 14 durable goods sectors to match between our patent citation data and the data we use to calibrate the model. "Other" indicates all industries other than these 14.

Capital good sector	ω_i	α_i	$1/\mu_i$	A_i
Computers and office equipment	0.049	0.24	0.55	0.16
Communication equipment	0.057	0.40	0.52	0.16
Aircraft	0.016	0.18	0.18	0.19
Instruments and photocopiers	0.042	0.35	0.20	0.17
Fabricated metal products	0.020	0.30	0.12	0.21
Autos and trucks	0.116	0.20	0.07	0.19
Elec. trans. dist. and ind. app.	0.028	0.40	0.18	0.16
Other durables	0.077	0.35	0.10	0.19
Ships and boats	0.007	0.18	0.21	0.16
Electrical equipment, n.e.c.	0.003	0.40	0.18	0.22
Machinery	0.203	0.26	0.24	0.18
Mining and oilfield machinery	0.009	0.26	0.20	0.34
Furniture and fixtures	0.028	0.26	0.06	0.13
Structures	0.346	0.32	0.17	0.12

Table A – Parameters used in calibrating industry R&D intensity in the model economy.

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