What Do Internal Capital Markets Do?

Redistribution vs. Incentives

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Abstract

In this paper we explain the apparent "diversification discount" of conglomerates without assuming inefficient-cross subsidisation through internal capital markets. Instead we assume that an internal capital market efficiently redistributes scare resources across a conglomerate's divisions between successive production periods. The need for redistribution arises from the fact that resources may sometimes be produced by divisions which happen to be successful in an earlier production stage but which do not have the best investment opportunities in future production stages.

In contrast to the existing literature we consider explicitly the incentive problem between corporate headquarter and divisional managers using a standard Moral-Hazard framework. We show that although a complete incentive contract can be written bi-laterally between headquarter and divisional managers, the redistribution of resources across divisions creates additional agency costs in a conglomerate.

Moreover, assuming that no complete contract can govern the interim redistribution policy by the headquarter, we show how the agency problem with divisional managers constrains headquarters interim redistribution to be ex ante inefficient.

JEL-Classification codes: G31, G34, L23

1 Introduction

The main economic effect that our paper aims to capture is that of a classical capital budgeting process within a conglomerate. There a scarce resources and the conglomerate's board of directors, i.e. corporate headquarter, has to decide which of the conglomerate's divisions should be allocated those funds knowing that not all projects can be funded. We show that in a conglomerate the reallocation of scarce resources and the provision of incentives to produce those resources are intricately linked.

It has long been argued informally that the reallocation of scare resources is one of the most important tasks for corporate headquarter: "In many respects, this assignment of cash flows to high yield uses is the most fundamental attribute of the *M-form* enterprise." (Williamson (1975), p. 147f, our italics) or "The most critical choices *top management makes* are those that allocate resources among competing strategic investment opportunities." (Donaldson (1984), p.95, our italics). Lamont (1997) and Shin & Stulz (1998) provided evidence that conglomerates indeed do redistribute resources using an active internal capital market.

Although internal capital markets play an important role in conglomerates, there is a puzzle. At first sight it seems possible to argue that an internal capital market adds value due to its information advantage over external capital markets (see for example Alchian (1969), p. 349) The argument however is at odds with evidence that conglomerates destroy value (Lang & Stulz (1994), Berger & Ofek (1995), Comment & Jarrell (1995)). They typically trade at a discount compared to a portfolio of stand-alone firms which replicates the conglomerates operating divisions.

An explanation of the conglomerate discount in terms of internal capital markets is that corporate headquarter instead of distributing resources towards the most productive divisions, inefficiently cross-subsidizes them (Scharfstein (1998), Scharfstein & Stein (2000), Rajan et al. (2000)).[1] The reason why internal capital markets are inefficient, these papers argue, is that they are captured by managers and directors to fight power struggles over the control of the conglomerate's resources.

The empirical evidence supporting the cross-subsidisation, or "corporate socialism", hypothesis however has recently come under some criticism. Chevalier (2000) shows that one can replicate the evidence that is cited in support of "corporate socialism" by looking at firms that are going to merge in the future but which are not integrated yet. Without integration there cannot be cross-subsidisation to explain the evidence. Maksimovic & Phillips (2000) shows that there are no signs of inefficient cross-subsidisation when more micro-level data is used than what is usually done.

Our paper shows how the conglomerate discount can be explained without assuming inefficient cross-subsidisation through inefficient internal capital markets.

Suppose that we have a conglomerate with an efficient internal capital market, i.e. corporate headquarter, which owns the production assets of the conglomerate, allocates scarce resources to the most productive division. Moreover, it is possible that a division that has performed well in the past may no be the most productive division in the future. The internal capital market serves to channel scarce resources between production periods from previously successful divisions to divisions that will be successful in the future.

There is one more important task for corporate headquarter beyond channelling scarce resources across divisions during production. Corporate headquarter lacks the skill of running production assets and therefore has to employ managers to run the divisions. Since divisional managers dislike hard work, they have to be induced to work hard through appropriate incentive contracts. The other important task for corporate headquarter then is to hire divisional managers and design their incentive contracts.

[1] There is a large literature on the diversification discount not based on internal capital markets. For references, see Villalonga (2000).

Our main result is that there are costs of running an internal capital market although there is no inefficient cross-subsidisation as such.[2] Firstly, the possibility of reallocating resources within a conglomerate increases the cost of inducing divisional managers to perform well. Secondly, a reallocation of resources leads to an inefficient continuation of investment projects. And thirdly, a reallocation leads to an inefficient transfer of funds.

Our main assumption is that corporate headquarter can bi-laterally write complete incentive contracts with each divisional manager but there is no complete, multi-lateral contract that governs the corporate headquarter's decision of how to channel scarce resources across divisions. The inability of fixing ex-ante, i.e. before managers decide whether to work hard, which division looses its resources and which division gains new ones, is at the root of the second and third cost of running an internal capital market: inefficient continuation and transfers.

As far as we are aware there is no other paper that analyses the working of internal capital markets and explicitly considers managerial incentive contracts. Managerial pay is a variable in our model as opposed to a parameter like in the inefficient cross-subsidisation literature. There, managers mechanically receive an unspecified "private benefit" that is in fixed proportion to the funds they control.

Efficient internal capital markets were first analysed by Stein (1997) but his model neither considers managerial moral hazard nor are funds reallocated *between* production periods. Brusco & Panunzi (2000) have a model that is similar in spirit to ours since they consider the impact of a reallocation of funds on managerial incentives. In substance however, their model is quite different. Firstly, there is no explicit incentive contracting problem. They use the private benefits framework where managerial pay-off is exogenous. Secondly, headquarter does not reallocate funds since productive divisions happen to be poor in resources but because headquarter discovers new information about the productivity of divisions. Thirdly, in their model the manager of the more productive division has more incentives to work hard. In our model, he has less incentives to work hard. Fourthly, they do not consider the constraints on headquarter's decision to continue divisions and transfer funds at the interim stage. And finally, they use a model specification that makes the conglomerate's total value independent of managerial effort.

Section 2 introduces our model of a two-divisional firm that operates for two production periods. In order to highlight the role of an internal capital market in our set-up, section 3 takes a step backward and analyses the Stand-Alone benchmark case when there is no internal capital market. Section 4 then examines the case when there is an internal capital market. Section 5 shows that there is a conglomerate discount when the productivity difference between divisions is neither too large nor too small. Section 6 concludes.

2 The Model

Consider a firm that operates for two periods and that is composed of two divisions and a headquarter. The headquarter (the principal) owns all productive assets but it has no expertise in managing them. Hence, it employs two self-interested managers (the agents) to run the divisions and it controls them through incentive contracts. If divisional managers work hard in the first period, they positively affect their divisions performance at the end of the first period. At that interim stage, the headquarter can then decide what to do with a division's resources. The headquarter can either decide to continue a division for the second period, it can liquidate its assets or it can transfer its assets to another division. In other

[2] This means that if the difference in productivity across division is weak then corporate headquarter may find it optimal to take resources away from "weak" divisions and give them to "strong" divisions. It does not mean that corporate headquarter deliberately gives resources to weak divisions.

words, the headquarter can operate an internal capital market between the two production periods. Figure 1 illustrates the sequence of events.

In our set-up all parties are risk-neutral and the risk-free interest rate is normalised to zero.

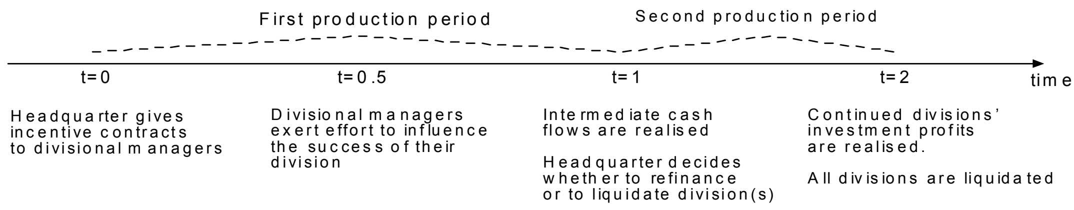

Figure 1: The timing of events

2.1 Production technology in a division

Production takes place for two periods. The higher the effort put in by the manager of division i, the better the interim performance of his division at the end of the first period. More precisely, it he exerts a high effort e^h then with probability p the division is worth 1 and with probability $(1-p)$ it is worth nothing. If he exerts a low effort e^l then with probability $q < p$ the division is worth 1 and with probability $(1-q)$ it is worth nothing. Exerting a high effort costs the manager c while exerting a low effort costs him nothing.

There will be production in a division for a second period only if 1 unit of resources is reinvested at the interim stage between the production periods. If a division continues to operate for a second period then it yields a final value of $\gamma_i\alpha$ for sure. Hence, the ex-ante expected value of a division that is continued and where the manager works hard is $p\gamma_i\alpha$. Note that the division has a positive Net Present Value (NPV) in the second period if $\gamma_i\alpha > 1$.

The divisions' production technologies are independent in the sense that division i's production technology does not depend on division j's production technology and a divisional manager's effort only affects the performance of his own division.

Note that α, as opposed to γ, is not indexed by i: α represents common productivity across all divisions while γ_i which represents the extra profitability of division i. Except for γ_i all divisions possess the same production technology.

2.2 The internal capital market

Suppose for a moment that the headquarter does not have the possibility to redistribute resources at the interim stage between the two production periods. In that case, each division must rely on its own resources in order to be refinanced and to continue production for a second period. If a division was successful in the first period and produced 1 unit of interim resources then this 1 unit can be used to refinance and continue the division to yield a final value $\gamma_i\alpha$. Alternatively, a successful division could be stopped and its interim resources be liquidated for their full value of 1. It is efficient to refinance a division if it has a positive NPV in the second production period, i.e. if $\gamma_i\alpha > 1$.

If in contrast a division was unsuccessful in the first period then there is no choice. In the absence of any interim resources of its own, a division must be stopped even though it may be very profitable in the second period.

Both divisions succeed	Only division 1 succeeds	Only division 2 succeeds	Both divisions fail
continue both continue 1, stop 2 stop 1, continue 2 stop both	continue 1, stop 2 stop both	stop 1, continue 2 stop both	stop both

Table 1: No internal capital market: possible strategies

Both divisions succeed	Only division 1 succeeds	Only division 1 succeeds	Both divisions fail
continue both continue 1, stop 2 stop 1, continue 2 stop both	continue 1, stop 2 stop both *stop 1, continue 2*	stop 1, continue 2 stop both *continue 1, stop 2*	stop both

Table 2: Internal capital market: possible strategies

Table 1 summarises the headquarter's possible actions between the production periods in the case of two divisions when it cannot redistribute resources. The four contingencies that the headquarter must consider are given in the title row. Each column shows which actions are possible. For example, if only division 1 has succeeded then, without an internal capital market, division 1 can be stopped or continued but division 2 must be stopped.

If the headquarter does have the possibility to redistribute resources at the interim stage, i.e. if there is an internal capital market, then it is possible to refinance and to continue a division that was unsuccessful in the first production period. For example, even though division 1 was not able to generate resources on its own it can still be refinanced if headquarter transfers resources from division 2 to division 1. This requires that division 2 was successful in the first production period and that division 2 is then stopped and liquidated.[3] We assume that there cannot be partial liquidation or partial continuation. A division's resources are just enough to refinance another division.[4]

We have deliberately introduced the notion of indivisibility since it captures the fact that in a capital budgeting process, corporate headquarter has to make decisions between various mutually exclusive alternatives. For example an R&D project can only be brought to the production stage if the production plant is built. If the plant is not built then all the budgeted money is available to other projects. If the plant is built then the money is not available.[5]

Table 2 summarises the headquarter's possible strategies when there is an internal capital market. The difference to case without an internal capital market is that we add a possible action in the cases when only one division succeeds. Headquarter can now stop the successful division and continue the unsuccessful one. If both divisions have a positive NPV in the second production period then it is efficient to transfer resources from division 2 to division 1 when the latter is more profitable, i.e. when $\gamma_1 > \gamma_2$.

At the beginning of the first production period the divisions are already endowed with resources and all the resources that are redistributed between the first and the second production period are internally generated. There is no access to external capital market in our

[3] For simplicity we assume that there is no difference in liquidating inside an internal capital market or outside it. Furthermore, in both cases it is possible to liquidate efficiently. Gertner et al. (1994) however sees efficient liquidation as one of the main advantages of an internal capital market as opposed to liquidation through an external investor, say a bank.

[4] If there were some "cash left on the table" at the interim stage we would complicate our model without gaining additional insights. Alternatively one could for example assume that all free-cash is paid out to shareholders.

[5] Different decisions by corporate headquarter will result in different incentive constraints for it at the interim stage between production periods. This insight is lost if we assume that capital budgeting is a perfectly divisible process.

set-up.[6,7]

2.3 Information and contracts

Headquarter has no expertise in managing production and therefore has to employ self-interested managers. To control their behaviour and to compensate them for their costly effort, it gives them incentive contracts at the beginning of the first production period. Since divisional managers' effort is not observable, their incentive contracts cannot be directly contingent it. In other words, there is a Moral-Hazard problem at the divisional level.

The incentive contract for divisional manager i specifies two payments. If his division is refinanced for a second production period he receives a share $\delta_i \in [0,1]$ of the final liquidation proceeds $\gamma_i \alpha$ at the end of the production process. A manager's division can also be liquidated at the end of the first production period no matter whether the division has succeeded or failed. Since we impose that all contracting parties are protected by *limited liability,* a divisional manager cannot receive anything if his division is liquidated after failing in the first period.[8] In that case there are no liquidation proceeds. If his division is liquidated after succeeding in the first period period he receives a share $W_i \in [0,1]$ of the 1 unit of interim liquidation proceeds.

The form of incentive contracts is therefore quite simple. The share δ_i represents a continuation reward, for example a wage or a bonus, and the share W_i represents a form of severance pay or "golden parachute" that the manager receives when his division is liquidated although it has been successful.[9]

Interim cash-flows are observable but not contractible. Since headquarter owns productive assets, it possesses the residual controls right over them. Therefore, it is the headquarter which uses the internally generate resources at the interim stage as it sees fit. Were interim cash-flows contractible then there is no need for corporate headquarter to operate an internal capital market since it could write a comprehensive contract with all divisions that specifies which division is refinanced under which circumstances.[10]

There is no asymmetric information between headquarter and divisional managers (and among divisional managers) beyond their individual effort levels. In particular, the second period productivity of each division, $\gamma_i \alpha$, is known to everybody. This means that headquarter will be able to operate an efficient internal capital market by channelling funds to more productive divisions.[11]

Finally, we assume that there is no conflict of interest between the owners of the firm and corporate headquarter. Headquarter maximises the total value if the firm net of incentive payments to managers. Managers however do not care about the value of the firm, they are

[6] This is for simplicity only. All we need is that resources are scarce. If we want to be more explicit, we could for example argue along the lines of Stein (1997) and say that access to external capital markets is limited due to asymmetric information.

[7] Models that consider the role of internal capital markets in relation to external finance are Scharfstein & Stein (2000) and Inderst & Müller (2000).

[8] That limited liability imposes a zero payment in one of the contingencies creates a standard moral-hazard problem although all parties are risk-neutral.

[9] Note that a divisional manager's incentive contract only depends on the performance of his own division. What we effectively assume is that a conglomerate does not use more complicated incentive contracts than a one-divisional firm. We come back to this crucial issue in section 6

[10] If interim cash-flows were contractible one could also write much more complicated incentive contracts. So far, the incentive contracts only require that liquidation and its proceeds are verifiable. For a discussion of these issues see section 6.

[11] In contrast, Stein (1997) assumes that divisional managers have superior information and that the task for the headquarter is to elicit that information. In Brusco & Panunzi (2000) headquarter receives a signal that informs him about the productivity of divisions. In Inderst & Laux (2000) the productivity of divisions itself depends on managerial effort.

only interested in the incentive payments they receive.[12]

3 No internal capital market

In this section we concentrate on the case when there is no internal capital market to redistribute resources. Initially we consider a one firm with one single division i in order to illustrate the moral-hazard problem between the headquarter and the division manager and to show how the incentive contract written by the headquarter interacts with its decision to continue or stop a division. We first present a First-Best situation where effort is contractible and then look at the Second-Best situation where effort is not contractible. We also introduce our Stand-Alone benchmark for the remainder of the analysis by asking: what is the value of a portfolio of two independent single firms with one division each.

The section also introduces some assumptions that make the analysis of the case with an internal capital market more tractable.

3.1 A single firm with one division: First Best

Suppose for a moment that the effort level of a divisional manager is contractible so that there is no moral hazard. The aim of this section is to illustrate the constraints on headquarter's behaviour at the interim stage between the two production periods.

If division i was successful and has generated 1 unit of resources in the first production period, then headquarter can either continue or liquidate the division. As the decision is not contractible, headquarter continues the division if and only if its continuation value $C_i(\delta_i) \equiv (1-\delta_i)\gamma_i\alpha$ exceeds its liquidation value $L_i(W_i) = 1 - W_i$ at the interim stage:

$$C_i(\delta_i) \geq L_i(W_i) \Leftrightarrow (1-\delta_i)\gamma_i\alpha \geq 1 - W_i \tag{1}$$

It is clear that by choosing the continuation reward δ_i and the golden parachute W_i headquarter not only compensates divisional managers for their effort, but also modifies its decision to continue or liquidate a successful division.

Let us now go through both possibilities, continuation and liquidation, bearing in mind that effort is, for the sake of this section only, observable. In other words, headquarter can just tell his divisional manager whether to work hard or not. Consider first the case when the headquarter wants to liquidate a successful division so that the relevant payment to his manager is the golden parachute W_i. If headquarter imposes a high effort then it just pays enough to so that his manager is indifferent between working for the firm and quitting, i.e. until his Participation Constraint binds: $pW_i = c$. Headquarter finds it optimal to liquidate a successful division at the end of the first production period iff

$$C_i(\delta_i) < L_i(\frac{c}{p}) \Leftrightarrow (1-\delta_i)\gamma_i\alpha \geq 1 - \frac{c}{p}$$

We see that by choosing a sufficiently large continuation reward, for example $\delta_i = 1$, headquarter can always commit itself ex-ante to liquidate a successful division at the interim stage.

Headquarter wants to impose a high effort when it is more profitable than a low effort, $p(1-\frac{c}{p}) \geq q \Longleftrightarrow p - q \geq c$, i.e. when the marginal benefit of high effort exceeds its marginal cost.[13]

[12] This is different from models in the private benefits tradition. There both headquarter and managers are interested in value maximising. Inefficiencies there typically arise from the fact that each party wants to maximise the value of his *own* realm.

[13] Should the headquarter still want to liquidate a successful division but impose a low effort on his manager then it need not pay him any golden parachute $W_i = 0$. Again, headquarter can ex-ante commit to liquidation with $\delta_i = 1$. The expected benefit to headquarter of these actions is q.

Consider now the case when headquarter wants to continue a successful division. Now the relevant payment to his divisional manager is δ_i. If headquarter imposes a high effort then it just needs to satisfy the manager's Participation Constraint, $p\gamma_i\alpha\delta_i = c$. Headquarter finds it optimal to continue a successful division iff

$$C_i(\frac{c}{p\gamma_i\alpha}) \geq L_i(W_i) \Leftrightarrow (1 - \frac{c}{p\gamma_i\alpha})\gamma_i\alpha \geq 1 - W_i$$

Headquarter can commit itself ex-ante to continue a successful division by paying a very large golden parachute, for example $W_i = 1$. It prefers to impose a high effort on its manager when the division's second period NPV exceeds the ratio of marginal cost to marginal benefit, $p\gamma_i\alpha(1 - \frac{c}{p\gamma_i\alpha}) \geq q\gamma_i\alpha \Longleftrightarrow \gamma_i\alpha \geq \frac{c}{p-q}$.[14]

The following proposition states that in the First-Best case in a one-divisional firm, headquarter continuation decision is efficient. Without divisional moral-hazard managers work hard when the marginal benefit of high effort exceeds its marginal cost.

Proposition 1 *When managerial effort is contractible then a single division is continued iff it has a positive NPV for the second period, $\gamma_i\alpha \geq 1$. Headquarter always imposes a high effort on the divisional manager when the marginal benefit of high effort exceeds its cost, $p - q \geq c$.*

Proof. In the appendix. ■

The intuition for proposition 1 is that since headquarter can perfectly commit itself ex-ante, it pays the manager the same *in expectation* in both the liquidation and continuation case. Hence, the ex-ante decision to continue or liquidate is identical to the efficient interim decision.

To have an interesting problem we will assume that the headquarter wants the manager to always exert a high effort in the First Best benchmark.

Assumption 1 *The marginal cost of high effort exceeds its marginal cost: $\frac{c}{p-q} < 1$.*

The First-Best continuation reward contract then is, $\delta_i^{FB} = \frac{c}{p\gamma_i\alpha}$.

3.2 A single firm with one division: Second Best

If effort is not contractible then there is moral-hazard at the divisional level. Instead of imposing the desired effort level, headquarter must now induce it through appropriate incentive payments. Again, we have to consider both the continuation and the liquidation case separately.

If headquarter wants to continue a successful division and wants the manager to exert a high effort then the continuation reward δ_i must be incentive compatible, i.e. with that reward the divisional manager must indeed prefer to work hard than to not work hard:

$$p\gamma_i\alpha\delta_i - c \geq q\gamma_i\alpha\delta_i \tag{2}$$

Headquarter pays the least possible amount $\delta_i = \frac{c}{(p-q)\gamma_i\alpha}$. The amount is higher than the corresponding payment in the First-Best case and therefore satisfies the Participation Constraint with slack. Since the manager can no longer be told what to do, the manager must expect to get some share of the production surplus.

Headquarter finds it optimal to continue a successful division iff

$$C_i(\frac{c}{(p-q)\gamma_i\alpha}) \geq L_i(W_i) \Leftrightarrow (1 - \frac{c}{(p-q)\gamma_i\alpha})\gamma_i\alpha \geq 1 - W_i$$

[14] The incentive payments for the low effort and continuation case are $\delta_i = 0$ and $W_i = 1$ so that the expected pay-off to headquarter is $q\gamma_i\alpha$.

so that headquarter can again commit itself ex-ante to continuation with a very large golden parachute, e.g. $W_i = 1$.[15]

If headquarter wants to liquidate a successful division and wants to induce the manager to work hard, then it must pay an incentive compatible golden parachute, $pW_i - c \geq qW_i$. Headquarter pays the least possible amount $W_i = \frac{c}{p-q}$ which again leaves some of the surplus to the manager. It is easily verified that again headquarter can perfectly commit ex-ante to liquidation by $\delta_i = 1$ for example.

The impact of effort not being observable is most clearly seen in the liquidation case where production ends after the first period.[16] In the liquidation case headquarter finds it profitable to induce a high effort if $p(1 - \frac{c}{p-q}) \geq q \iff \frac{p-q}{p} \geq \frac{c}{p-q}$. In contrast to the First Best case, having the marginal benefit of a high effort exceed its marginal cost is no longer sufficient to ensure a high effort. Instead, it must be that the ratio of marginal cost to marginal benefit must be smaller than the relative impact of a high effort.

As in the First-Best case, we can show that the headquarter's continuation decision is efficient. What changes when effort is not contractible is that it is more difficult to induce a divisional manager to work hard.

Proposition 2 *When managerial effort is not contractible then a single division is continued iff it has a positive NPV for the second period, $\gamma_i \alpha \geq 1$. The headquarter always induces a high effort level from the divisional manager when ratio of the marginal cost of high effort to its marginal benefit is smaller than relative impact of a high effort, $\frac{c}{p-q} \leq \frac{p-q}{p}$.*

Proof. As in proposition 1 ■

The second part of proposition 2 is a standard Moral-Hazard effect. The first part is surprising since one often encounters models where Moral Hazard leads to inefficient liquidation (e.g. Bolton & Scharfstein (1996)). The difference is that headquarter can, as in First-Best case, perfectly commit itself ex-ante and that headquarter pays the manager the same (in ex-ante expectation) in both liquidation and continuation. Hence, the continuation decision is not distorted.

To keep the subsequent analysis tractable we reduce the set of possible outcomes. First we assume that the headquarter always wants to induce his divisional manager to work hard. Thus, we strengthen assumption 1 to say:

Assumption 2 *The ratio of the marginal cost of high effort to its marginal benefit is smaller than relative impact of a high effort: $\frac{c}{p-q} \leq \frac{p-q}{p} < 1$.*

Second we assume that divisions always have a positive NPV for the second production period.

Assumption 3 *A division always has a positive Net Present Value for the second production period: $\gamma_i \alpha > 1$.*

The consequence is that we are left with just one outcome when there is no internal capital market. The manager always works hard in the first production period and his division is continued for a second period if and only if it is successful. The Second-Best continuation reward is $\delta_i^{SB} = \frac{c}{(p-q)\gamma_i \alpha}$. The manager's expected profit is $p\frac{c}{p-q}$ and headquarter's profit, i.e. firm value, is

$$V_i = p\gamma_i \alpha - p\frac{c}{p-q} \tag{3}$$

With these preliminaries we proceed to introduce our benchmark against which we measure the value of the conglomerate.

[15] A high effort in the continuation case is more profitable for headquarter if $p\gamma_i\alpha(1 - \frac{c}{(p-q)\gamma_i\alpha}) \geq q\gamma_i\alpha \iff \gamma_i\alpha\frac{p-q}{p} \geq \frac{c}{p-q}$.

[16] Remember that there is a moral-hazard problem only in the first production period.

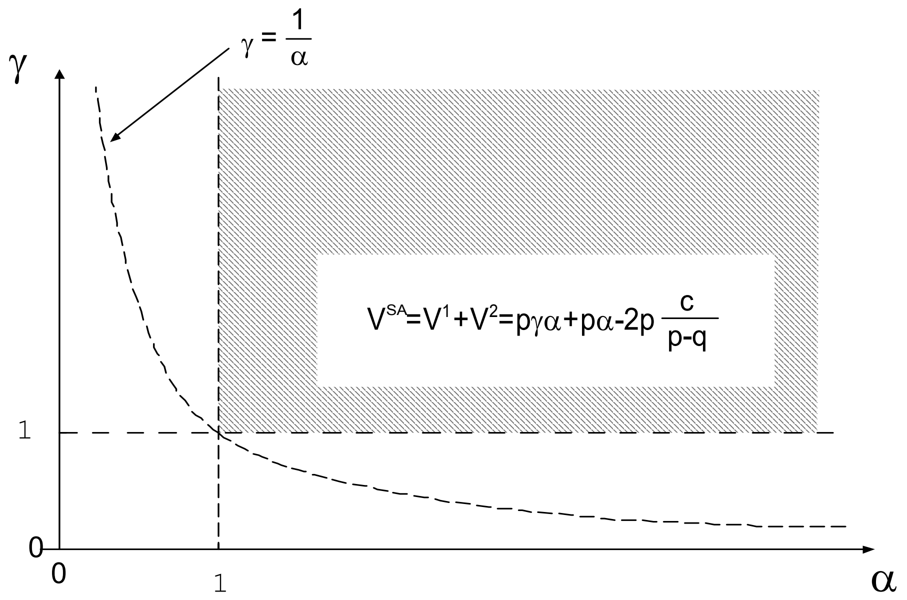

Figure 2: The Stand-Alone benchmark

3.3 The Stand-Alone benchmark: two firms with one division each

In order to establish whether the ability to redistribute resources across divisions of a conglomerate creates or destroys value, we need a benchmark. As in the empirical literature, the benchmark considers the value of portfolio of focused firms.[17] The previous section showed that under assumptions 1, 2 and 3 a single firm with one division is worth $V_i = p\gamma_i\alpha - p\frac{c}{p-q})$. Since we will only consider a conglomerate with two divisions $i = 1, 2$, we simplify γ_2 to 1 and $\gamma_1 = \gamma$. Our divisions are identical so that we can assume without loss of generality that $\gamma \geq 1$. The parameter γ then describes the extra productivity of division 1 over division 2 in the second production period.

The value of a portfolio of two independent firms with one division each is

$$V^{SA} = V_1 + V_2 = p\gamma\alpha + p\alpha - 2p\frac{c}{p-q} \tag{4}$$

The first term in (4) describes the expected benefit from production in firm/division 1 over two periods, the second describes the expected benefit from firm/division 2 and the third term describes the expected cost of inducing two divisional managers to exert a high effort.

Figure 2 illustrates the Stand-Alone benchmark in term of the overall productivity (α) and the extra productivity of firm/division 1 (γ). In the shaded area γ and α are such that both divisions have a positive NPV in the second production period, $\gamma\alpha \geq 1$ and $\alpha \geq 1$, and division 1 is weakly more profitable, $\gamma \geq 1$. The shaded area depicts the set of admissiable parameter values.

4 The conglomerate with an internal capital market

In this section we develop the model when there is an internal capital market, i.e. when headquarter can redistribute resources across its two divisions between the two production

[17] The difference is that in a theoretical model we do not have to worry about a possible selection bias.

periods. Adding an internal capital market has two consequences. First, we add strategies for the headquarter at the interim stage (this was illustrated in table 2). This means that if the headquarter wants to continue only a successful division, i.e. replicate the Stand-Alone outcome, then it must not only prefer continuation to liquidation but it must also prefer continuation to transferring resources to another division. Hence there will be further constraints on the incentive contracts headquarter can write.

Second, divisional managers will anticipate in the first period that there may be a redistribution of resources across divisions at the interim stage. A manager will have different incentives to exert high effort if he knows that he will be refunded only if he had success or if he knows that he will be refunded no matter what or if he knows that he will never be refunded.

4.1 Autarkic divisions

We now explore the first consequence of adding an internal capital market, i.e. what are the constraints headquarter's behaviour at the interim stage? In order to avoid the second consequence, i.e. the incentive effect on managers we ask: when can a conglomerate replicate the Stand-Alone outcome and attain the benchmark value V^{SA}? In replicating the Stand-Alone outcome, each division will be autarkic and managers act independently so that their incentive problem is identical to the Stand-Alone benchmark.

Given that the productivity of division 1 is common knowledge, headquarter may a prior have an incentive to redistribute funds to division 1 which is more productive. By assuming for a moment that headquarter keeps divisions autarkic, we explore a pure negative effect due to the *possibility* of operating an internal capital market in this section.

If the conglomerate would never do worse with autarkic divisions there would be no point in continuing our analysis of a conglomerate discount.

4.1.1 Implementation constraints

There are four contingencies that the headquarter can encounter at the interim stage. Either both division succeeded, or just division 1 succeeded, or just division 2 succeeded or neither division succeeded. The four cases correspond to the columns in table 2. In each case the headquarter has various choices shown in the rows of the table. In order to implement a certain redistribution policy, in this case the policy of no redistribution, the corresponding choice must be the preferred one. So when both divisions succeeded then headquarter must prefer to continue both divisions to the other three possible choices. Formally, it must be that

$$C_1 + C_2 \geq C_1 + L_2 \tag{5}$$

$$C_1 + C_2 \geq C_2 + L_1 \tag{6}$$

$$C_1 + C_2 \geq L_1 + L_2 \tag{7}$$

The first inequality says that headquarter must prefer to continue both divisions to just continue division 1 and stop division 2. Similarly, the third inequality says that headquarter must prefer to continue both divisions to stopping both.

When only division 1 succeeds then the headquarter must prefer to continue only division 1 so that

$$C_1 \geq L_1 \tag{8}$$

$$C_1 \geq C_2 - W_1 \tag{9}$$

The second inequality says that profit from continuing division 1 must be weakly greater than the profit from continuing division 2 after compensating manager 1 whose division is stopped.[18]

When only division 2 succeeds then the headquarter must prefer to continue only division 2 so that

$$C_2 \geq L_2 \tag{10}$$
$$C_2 \geq C_1 - W_2 \tag{11}$$

In fact, we only need to consider the last four inequalities since together they imply the first three inequalities. What counts are those contingencies in which there is a potential for redistribution, i.e. those in which only one division succeeds. Conditions (8) and (10) are the continuation constraints that we also encountered in the previous section where there was no internal capital market. What is new is the presence of the transfer constraints (9) and (11). If headquarter wants to implement a policy of no redistribution at the interim stage, then the two incentive contracts for the divisional managers, (δ_1, W_1) and (δ_2, W_2), must satisfy those four constraints.

Even though divisions are autarkic, the mere possibility of transfers links the divisions via the transfer constraints to be not too dissimilar. Since the net continuation profit C_i is a function of the incentive payment the manager receives when his division is continued, δ_i, there is now an important externality between the redistribution policy and incentive contracting.

4.1.2 Managerial effort

Given that the headquarter does not redistribute resources at the interim stage, how does a divisional manager behave in the first production period? In the absence of any redistribution each division is autarkic so that each manager behaves exactly as in the case without any internal capital market. For example, the manager of division 1 exerts a high effort only if it is profitable for him to do so, i.e. when

$$p\gamma\alpha\delta_1 - c \geq q\gamma\alpha\delta_1 \tag{12}$$

If the manager works hard then he incurs the cost of effort c and with probability p the division succeeds so that he is paid a fraction δ_i of the continuation pay-off $\gamma\alpha$. If he does not work hard then he does not incur the cost of effort but only gets the continuation payment with probability q.[19] Note that the performance of division 2 does not affect his incentives, (12) is identical to (2) which described the incentive constraint for a firm with one single division. The following proposition summarises the behaviour of divisional managers when the internal capital market is inactive.

Proposition 3 *In a conglomerate with autarkic divisions the managerial incentive problem is as in the Stand-Alone benchmark. It costs the same to induce a high effort from divisional managers and they behave independently from each other.*

Figure 3 illustrates the managerial incentive problem under autarky. It shows the effort of both managers (e_1, e_2) as a function of the payment δ_i each manager receives if his division

[18] There are many more possible choices that we do not present explicitly. For example, the headquarter could also stop both divisions and use the resources from division 1 to pay the manager of division 2 W_2. Such awkward possibilities are not optimal given our assumption of the positive Net Present Value of each division. The headquarter always wants to continue successful divisions.

[19] Given assumptions 2 and 3 we can focus on the case where the headquarter wants the manager to work hard and where the headquarter wants to continue divisions iff they were successful.

is continued. The payment threshold beyond which they exert a high effort is the same as in the Second-Best case: $\delta_1 = \frac{c}{(p-q)\gamma\alpha}, \delta_2 = \frac{c}{(p-q)\alpha}$. Moreover, manager i's payment threshold is independent of whether manager j exerts a high effort or a low effort.

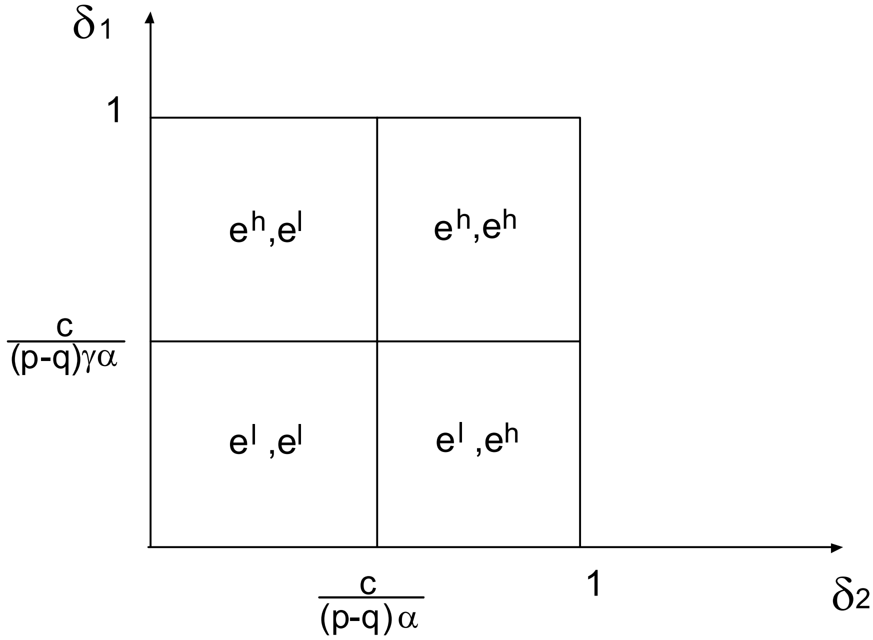

Figure 3: Managerial effort as a function of continuation payments under autarky

4.1.3 When can a conglomerate replicate the Stand-Alone benchmark?

We are now ready to answer our initial question: when can a conglomerate replicate the Stand-Alone benchmark, i.e. the outcome where there is no redistribution and attain the benchmark value V^{SA}? In order to replicate the benchmark, the headquarter must solve the following optimisation program:

$$\max_{\delta_1,\delta_2,W_1,W_2} p^2(C_1(\delta_1) + C_2(\delta_2)) + p(1-p)C_1(\delta_1) + (1-p)pC_2(\delta_2) \tag{13}$$

subject to:

$$\begin{aligned} C_1(\delta_1) &\geq L_1(W_1) \\ C_2(\delta_2) &\geq L_2(W_2) \end{aligned} \qquad \text{(Continuation Constraints)}$$

$$\begin{aligned} C_1(\delta_1) &\geq C_2(\delta_2) - W_1 \\ C_2(\delta_2) &\geq C_1(\delta_1) - W_2 \end{aligned} \qquad \text{(Transfer Constraint)}$$

$$\begin{aligned} \delta_1 &\geq \tfrac{c}{(p-q)\gamma\alpha} \\ \delta_2 &\geq \tfrac{c}{(p-q)\alpha} \end{aligned} \qquad \text{(Incentive Constraints)}$$

The headquarter's objective function consists of four terms that reflect i) the four contingencies at the interim stage, ii) the transfer decision taken at that stage and iii) the managerial effort level in the first production period. For example the second term $p(1-p)C_1(\delta_1)$ is the expected profit in the case that only division 1 succeeds, that division 1 is continued and that both managers exert a high effort level.

The first four constraints reflect the headquarter's transfer policy, here no-redistribution, at the interim stage. The last two constraints are the incentive constraints for the divisional

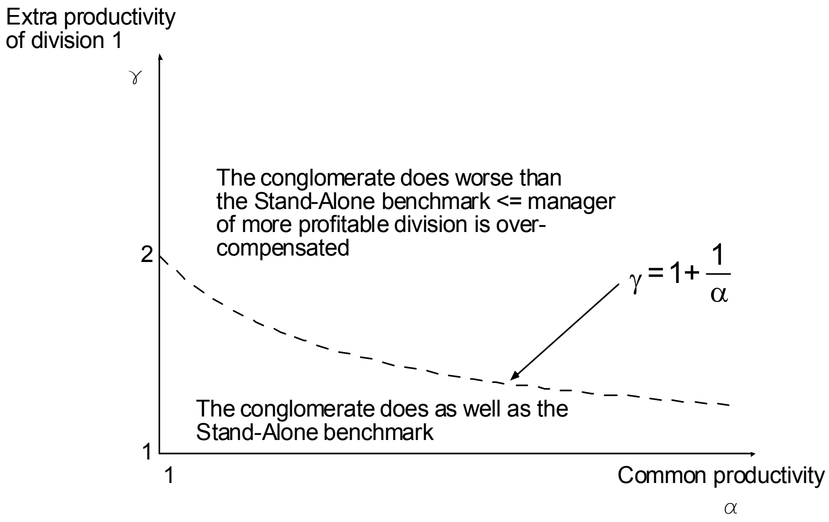

Figure 4: A conglomerate with autarkic divisions can never do better than the Stand-Alone benchmark

managers. To attain the benchmark value V^{SA} the headquarter must induce a high effort level from both managers with the efficient, i.e. Second Best, continuation payment $\delta_1 = \frac{c}{(p-q)\gamma\alpha}$ and $\delta_2 = \frac{c}{(p-q)\alpha}$. Given that headquarter optimally maximises the "golden parachute" for both managers, $W_1 = W_2 = 1$, the transfer constraint then gives the following result.

Proposition 4 *A conglomerate that operates a no-transfer policy can only do as well a corresponding portfolio of Stand-Alone firms, i.e. attain the benchmark value V^{SA}, iff the heterogeneity across divisions is not too strong $\gamma \leq 1 + \frac{1}{\alpha}$.*

Figure 4 illustrates the result. Note that for a conglomerate with autarkic divisions it become more difficult to do as well as the Stand-Alone benchmark when the overall productivity increases.

The proposition illustrates the negative side of having an internal capital market. There exists a tension between the incentives for divisional managers and the incentives for the headquarter to redistribute funds. Should the headquarter want to keep the division separate and commit to a no-transfer policy then he must obtain a similar net continuation profit from both divisions. Otherwise, there is an incentive for redistribution should a less profitable division fail. But since net continuation profits depend on the incentive payment to managers, there is now a strong restriction on the incentive payments the headquarter can make. In order to attain the benchmark value of 2 stand-alone divisions, each manager must be induced to work hard. If now for example division 1 is a lot more profitable, i.e. $\gamma > 1 + \frac{1}{\alpha}$, then manager 1 must be paid more than the efficient, Second Best, continuation payment $\delta_1^{SB} = \frac{c}{(p-q)\gamma\alpha}$ in order to satisfy the transfer constraints. But since more is paid to manager 1 than is necessary to induce a high effort, not because the headquarter induces the manager of the more profitable division to work harder but in order to make the no-transfer policy credible, there is a loss in firm value.

4.1.4 Full characterisation with autarkic divisions

Note that proposition 4 only tells us when the conglomerate, by choosing not use the internal capital market, can do as well as the portfolio of two firms with one division each. The proposition is not a full characterisation of the optimisation program nor does the optimisation program fully describe the no-transfer case.

As we have seen in proposition 4, when the headquarter wants to implement the no-transfer policy, it must be optimal to refinance the less productive division when this division is the sole division that succeeds. In other words, the net continuation value of division 2 should be greater than the net continuation value of division 1 net of the payment to manager 2 if his division is successful but not refinanced. The relevant transfer constraint of the problem is the second one: $C_2 \geq C_1 - W_2$. To satisfy this constraint, the headquarter has three possibilities. First, he could increase the payment to manager 2 when his successful division is liquidated. But the headquarter cannot pay more than what he has. W_2 cannot exceed the liquidation value of division 2[20]. Second, the headquarter could decrease the net continuation profit of the most profitable division by paying the manager more than what he needs to do a high effort. Increasing δ_1 above δ_1^{SB} decreases the profit in division 1 and hence makes continuation of this division less attractive. Last, the headquarter could increase the net continuation value of the less productive division by decreasing the payment δ_2 to the manager. But a decrease in δ_2 implies that the incentive constraint of the second manager will no longer be satisfied and the manager of the less productive division will not work hard. And hence, if the headquarter decides to decrease δ_2, it will be set at its lowest possible value: $\delta_2 = 0$.

When the conglomerate does not have the benchmark value V^{SA}, if the headquarter wants to implement the no-transfer policy at the interim stage, he should either over-compensate the first manager or rely on a low effort from the second manager (or both). In appendix A, we compare the benefits, in term of conglomerate value, of these strategies and the results are summarized in a proposition:

Proposition 5 *Under autarky, the headquarter induces both managers to work hard if either the overall productivity is sufficiently high, $\alpha \geq 2(\frac{c}{p-q})(\frac{p}{p-q})$, or if the extra productivity of division 1 is sufficiently low, $\gamma \leq (1 + \frac{p-q}{p}) + (1 - \frac{c}{p-q})\frac{1}{\alpha}$. If not, then only the manager of the more profitable division works hard.*[21]

Figure 5 illustrates the proposition.

What is the intuition for that result, which is surprising given that managers are autarkic (remember that in the Stand-Alone benchmark, both managers always work hard)?

We need to explain why the headquarter does better with a low effort from manager 2 when there is little overall profitability but a lot of heterogeneity. In that case there is big difference in the *gross* continuation value between division 1 and division 2. If both managers were to work hard then only way to achieve equal net values is to give away the extra profitability of division 1 to its manager.[22] Given that division 1 is quite a bit more profitable than division 2, giving away the extra profitability of division 1 is very costly. A cheaper way is to have the manager of division 2 work little. True, this means that division 2 will be less often continued but that division is not very profitable, relative to division 1, anyway. What matters more for headquarter is that since manager 2 is no longer given a share of his division, the net value of division 2 increases which in turn eases the transfer constraint and allows headquarter to profit more from the strong division 1.

[20]

[21] The corresponding incentive contracts can be found in the derivation of the solution in appendix A.

[22] The headquarter cannot raise the net value of division 2 by reducing δ_2 since its manager must be induced to work hard.

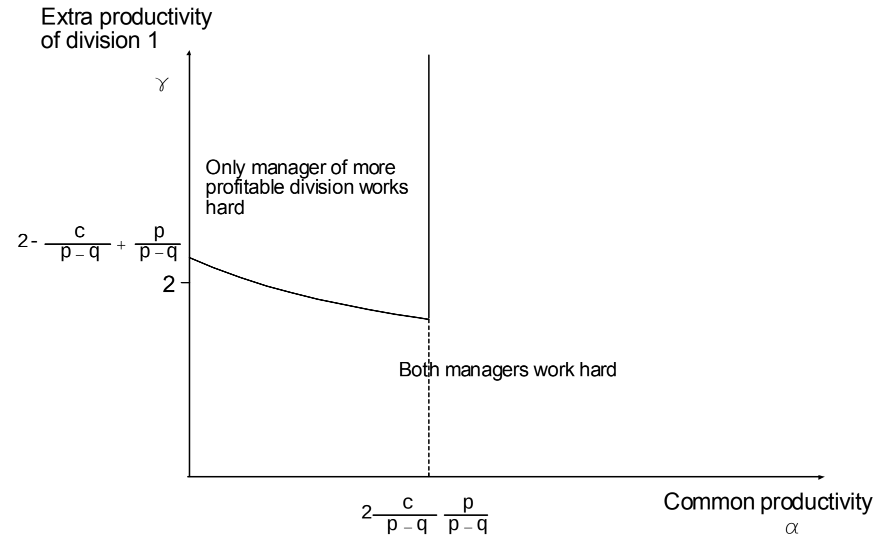

Figure 5: Divisional manager's effort as a function of overall profitability and heterogeneity where there is a no-transfer policy

4.2 Redistribution

Proposition 4 shows that the conglomerate with an inactive internal capital market always does worse than the corresponding portfolio of Stand-Alone firms if the productivity difference across divisions is large. In other words, the previous section showed a pure negative aspect of an internal capital market.

division 1 than to just stop division 2.[25] The second inequality says that it must be more profitable to stop division 2 and continue division 1 than to continue division 2. Note that if the two continuation constraints (10) and (14) as well as the transfer constraint (15) are satisfied then the remaining 4 constraints hold automatically.

However, limited liability needs to be respected. Headquarter must set the payment when stopping division 2 to zero, $W_2 = 0$, since there is only 1 unit of resources available when only division 2 succeeded. And that unit is used to refinance the failed division 1. Hence there is nothing left to compensate the manager of division 2, a division that is stopped although it has succeeded.[26]

The continuation and transfer constraints then simply are

$$C_1 \geq C_2 \geq 1$$

In a conglomerate that reallocated resources to the more productive division the net continuation value of the more profitable division must be bigger than the net continuation value of the less profitable division. In addition, the net continuation value of both divisions should be greater than their liquidation value. This means that, there will be inefficient continuation (all the NPV projects will not be continued) if the manager of the less profitable division is induced to work hard.

4.2.2 Managerial effort

Whereas under a no-transfer policy both managers behave independently from each other there is now an externality between them. Whereas under a no-transfer policy *any* manager that works hard gets the continuation payment with probability $p^2 + p(1-p) = p$, this is no longer true when the headquarter always refinances the more profitable division.

Incentives for the manager of the more profitable division The manager of division 1 is always refinanced so that also gets a share of the surplus when his division fails but the other division succeeds. The probability of him receiving the continuation payment δ_1 therefore depends not only on his own effort but also on the effort of the other manager. If manager 2 works hard then manager 1 receives his continuation payment δ_1 with probability $p + (1-p)p$ if he himself works hard and with probability $q + (1-q)p$ if he does not work hard. A high effort by manager 1, e_1^h, therefore is a best reply to a high effort by manager 2, e_2^h, if:

$$\delta_1 \geq \frac{c}{(1-p)(p-q)\alpha\gamma} \tag{16}$$

If manager 2 does not work hard then manager 1 receives his continuation payment with probability $p + (1-p)q$ if he works hard and with probability $q + (1-q)q$ if he himself works little. Thus, e_1^h is a best reply to e_2^l if:

$$\delta_1 \geq \frac{c}{(1-q)(p-q)\alpha\gamma} \tag{17}$$

There two effects to note. First, it is now harder to induce the manager of the more profitable division to work hard (compare (16) and (17) with (12)). His incentive to work hard is weakened since he knows that even if his division fails more often due to his lower effort, his division is still continued due to the transfer of funds from the other division.

[25] We do not distinguish between a W_i that is paid because i's resources are used elsewhere in the conglomerate or not.

[26] This is for simplicity only. What is important is that in a capital budgeting process choices must be made. Besides, headquarter would want to reduce W_2 as much as possible anyway.

Second, it is more difficult to induce manager 1 to work hard when manager 2 works hard too. This mirrors the first effect. If manager 2 works hard then manager 1 is more often refinanced when his division fails. Due to manager 2's hard work, division 2 succeeds more often which means that the funds for redistribution are available more often.

Incentives for the manager of the less profitable division The manager of division 2 is only refinanced when both divisions succeed so that the probability of him receiving the continuation share δ_2 also depends not only on his own effort but also on the effort of manager 1. If manager 1 works hard then manager 2 receives δ_2 with probability p^2 when he also works hard and with probability pq if he does not work hard. So e_2^h is a best reply to e_1^h if:

$$\delta_2 \geq \frac{c}{p(p-q)\alpha} \tag{18}$$

If manager 1 does not work hard then manager 2 receives δ_2 with probability pq if he works hard and with probability q^2 if he also does not work hard. So e_2^h is a best reply to e_1^l if:

$$\delta_2 \geq \frac{c}{q(p-q)\alpha} \tag{19}$$

Again there are two effects at play. As in the case of the manager of division 1 it is also more difficult to induce the manager of division 2. But the reason is a different one. Manager 2 simply gets paid the continuation payment less often. The second effect confirms the difference since when the other manager works hard, it is now *easier* to induce manager 2 to work hard too.

We summarise the impact of redistribution on the Moral-Hazard problem in the following proposition.

Proposition 6 *In a conglomerate that reallocates resources towards the more productive division it is more difficult to induce both divisional managers. Moreover, the impact of redistribution is asymmetric. On the one hand, it is more difficult to induce the manager of the more profitable division to work hard when the manager of the less profitable division works hard. On the other hand, it is easier to induce the manager of the less profitable division to work hard when the manager of the more profitable division works hard.*

Figure 6 illustrates that i) the Moral-Hazard problem now is more severe than under a no-transfer policy and ii) that the Moral-Hazard problem is no longer symmetric across divisions.

The best pair of incentive payments, δ_1 and δ_2, that induce both managers to work hard under a reallocation of funds are

$$\begin{aligned} \delta_1^{TB} &= \frac{c}{(p-q)(1-p)\alpha\gamma} \\ \delta_2^{TB} &= \frac{c}{(p-q)p\alpha} \end{aligned}$$

We call them the Third-Best incentive contracts.

4.2.3 Redistributing towards the more profitable division and having both managers work hard

We now describe the case when headquarter wants to implement a transfer policy by which the more profitable division is always refinanced and when headquarter induces both managers

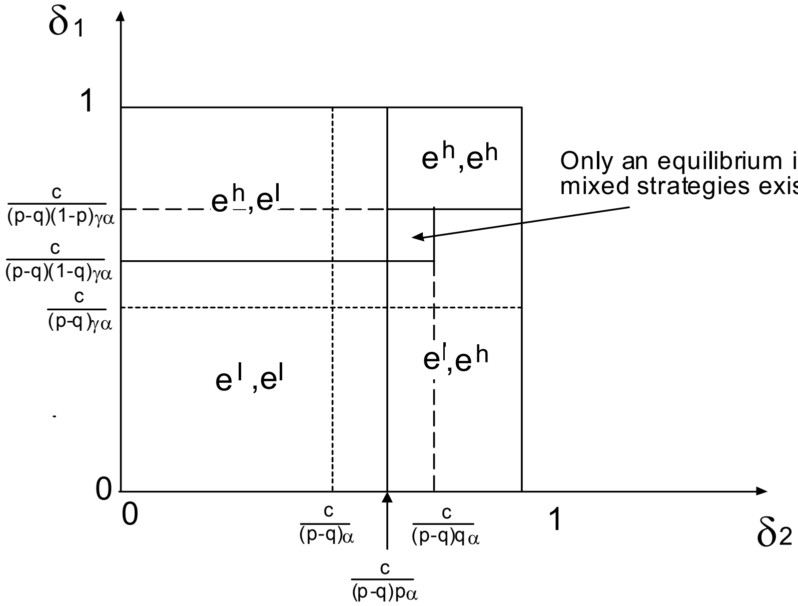

Figure 6: Managerial efforts as a function of continuation payments when division 1 is always refinanced

to work hard. We compare this solution with the stand alone benchmark and we introduce latter the full solution to the conglomerate problem.

If headquarter wants to induce both managers to work hard, it must solve the following optimisation programme:

$$\max_{\delta_1,\delta_2} p^2(C_1(\delta_1)+C_2(\delta_2))+p(1-p)C_1(\delta_1)+(1-p)pC_1(\delta_1)$$

subject to:

$$\begin{array}{cl} \begin{array}{c} C_1(\delta_1)\geq 1 \\ C_2(\delta_2)\geq 1 \end{array} & \text{(Continuation Constraints)} \\ C_1(\delta_1)\geq C_2(\delta_2) & \text{(Transfer Constraint)} \\ \begin{array}{c} \delta_1\geq\frac{c}{(p-q)(1-p)\gamma\alpha} \\ \delta_2\geq\frac{c}{(p-q)p\alpha} \end{array} & \text{(Incentive Constraints)} \end{array}$$

Note how this programmes compare to the optimisation programme when headquarter wants autarkic divisions. The third term in the objective function changes to reflect that when division 1 fails but division 2 succeeds then it is *division 1* that is continued. The continuation constraints no longer include the "golden parachutes" W_1 and W_2. There is only one transfer constraint but again, the "golden parachutes" are missing. Finally, the incentive constraints show that it is more difficult to induce managers to work hard.

The following proposition shows when a solution to the optimisation programme exists.

Proposition 7 *The headquarter can commit to always transfer resources towards the more productive division and induce both managers to work hard if the overall productivity is high enough,* $\alpha\geq 1+\frac{c}{p-q}\frac{1}{1-p}$. *If* $1+\frac{c}{p-q}\frac{1}{p}\geq\alpha>1+\frac{c}{p-q}\frac{1}{1-p}$ *then an additional condition is that the extra productivity of division 1 is high enough* $\gamma\geq\frac{1}{\alpha}(1+\frac{c}{p-q}\frac{1}{1-p})$. *If the extra productivity of division 1 is high,* $\gamma>1+\frac{c}{p-q}\frac{2p-1}{p(1-p)}$, *then the manager of division 2 is overcompensated.*

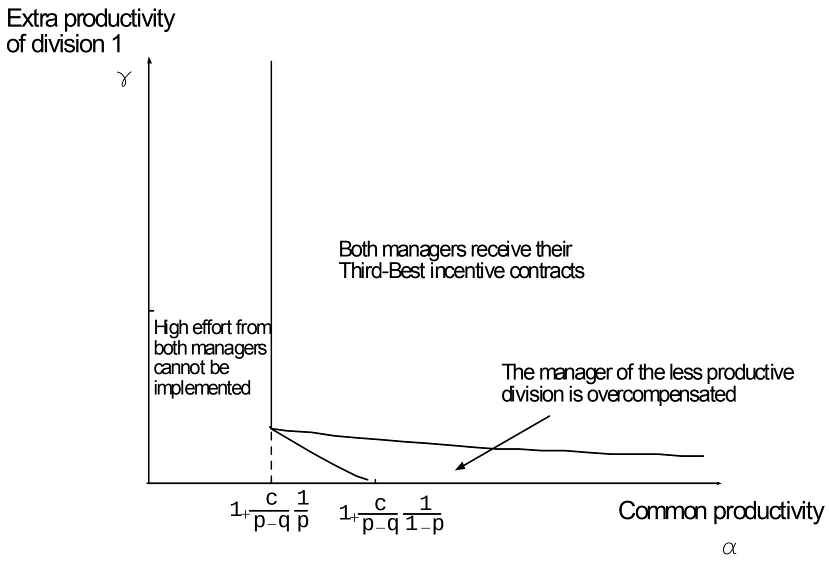

Figure 7: When can corporate headquarter reallocate resources to the more productive division with both managers working hard?

Proof. In the appendix ■

The various constraints are illustrated in figure 7. The first thing to note is that there is inefficient continuation, i.e. the Net Present Value rule does not hold. If the overall productivity is low then it is not possible to implement the transfer policy with both managers working hard. The inefficiency comes from the fact that since the "golden parachute" is paid to manager 2 even if headquarter continues division 1, the "golden parachute" can no longer serve to commit to continuation. Next, we see that the manager of the less profitable division must be overcompensated if the extra profitability of division 1 is low. In that case, the transfer constraint is binding. In order to reduce the net continuation value of division 2, its manager receives a larger incentive payment than what is needed to induce him to work hard. Note how this overcompensation feeds into the continuation constraint. Since manager 2 must receive a large payment due to transfer reasons, this makes it even more difficult to meet the continuation constraint.

4.2.4 What are the costs and benefits of an active internal capital market?

We can now discuss the costs and benefits of running an efficient internal capital. The Stand-Alone benchmark value V^{SA} was given in equation (4). The value of a conglomerate that always refinances the more profitable division and where both divisional managers work hard is at most:

$$V_{hh}^1 = (2p-p^2)\gamma\alpha + p^2\alpha - p\frac{c}{p-q} - (2p-p^2)\frac{c}{p-q}\frac{1}{1-p}$$

The first term is the expected benefit from continuing division 1, the second term is the expected benefit from continuing division 2, the third term is the expected cost of inducing manager 2 to work hard and the last term is the expected cost of inducing manager 1 to work hard. When is $V_{hh}^1 > V^{SA}$? The inequality can be written explicitly as

$$(2p-p^2)\gamma\alpha + p^2\alpha - p\frac{c}{p-q} - (2p-p^2)\frac{c}{p-q}\frac{1}{1-p} > p\gamma\alpha + pa - 2p\frac{c}{p-q}$$

or as

$$(1-p)p(\gamma-1)\alpha > \frac{p}{1-p}\frac{c}{p-q} \tag{20}$$

The term on the left-hand is the net benefit of reallocating resources to the more productive division. With probability $(1-p)p$ we have the situation that there are no resources in the more productive division and there are resources in the less productive division. If the resources are left where they are then gross profits are α. If they are reallocated towards the more productive division then gross profits are $\gamma\alpha$. Hence the net benefit of reallocation is $(1-p)p(\gamma-1)\alpha$.

The term on the right-hand side is the net cost of reallocation. In the Stand–Alone case, the manager of the more profitable firm obtains his share $\frac{c}{p-q}$ with probability p. Now this manager is paid more often (he receives his share with probability $p+p(1-p)$). In addition, he must be paid a larger share $\frac{c}{p-q}\frac{1}{1-p}$ since, knowing that he receives his share also when has failed provided the other manager succeeds, reallocation weakens his incentives.

Rewriting the cost-benefit inequality in terms of the extra productivity of division 1, γ, we can state the next result.

Proposition 8 *By redistributing towards the more productive division and inducing both managers to work hard, the conglomerate can do better than the Stand-Alone benchmark if the extra productivity of that division is sufficiently high* $\gamma > 1+\frac{1}{\alpha}\frac{c}{p-q}\frac{1}{(1-p)^2}$. *But it can only do better when both managers get their third best incentive contract.*

Proof. We must show that when $\gamma \leq 1+\frac{1}{\alpha}\frac{c}{p-q}\frac{2p-1}{p(1-p)}$ (the manager of the second division receives more than δ_2^{TB}), γ is smaller than $1+\frac{1}{\alpha}\frac{c}{p-q}\frac{1}{(1-p)^2}$. Which is true if: $\frac{1}{(1-p)^2} > \frac{2p-1}{p(1-p)} \Leftarrow p^2-p+\frac{1}{2} > 0$. Which is true for all p. ■

Figure 8 illustrates the costs and benefits of running an active internal capital market. We saw in the previous section that it is not always possible to implement the case with both managers working hard. That restriction is a cost since it means that some manager will not work hard which, a priori, reduces the expected value of the conglomerate. More precisely, there is inefficient continuation when overall productivity is low and there is inefficient transfer when the extra productivity of division 1 is low. As the extra productivity increases we still have the pure agency cost due to the reallocation of funds. It is only when the extra productivity is very high that the conglomerate does better than the Stand-Alone due to the possibility of channelling funds to the most productive division.

4.2.5 Redistribution toward the more productive division: complete solution

The solution with transfers and two high efforts has three additional problems compared to the stand alone benchmark. First, there is over-compensation of the manager of the more productive division. It is more difficult to motivate this manager to exert a high effort, and hence, his expected payment is higher than in the Stand Alone case. Second, the NPV rule does not hold. The continuation decision is inefficient when both managers work hard. For low values of the overall productivity α, even though both divisions have a positive NPV, headquarter prefers to liquidate them and collect the liquidation value of the divisions. Third, the manager of the less productive division may be over-compensated, if the differential in productivity γ is low.

For these reasons, nothing guarantees that headquarter wants both managers to work hard when it transfers resources across divisions. Low effort from some divisional manager may indeed alleviate some of the problems mentioned above.

If the manager of the less productive division does not work hard, headquarter can set $\delta_2 = 0$. The continuation value of the second division then is α, and the division is continued

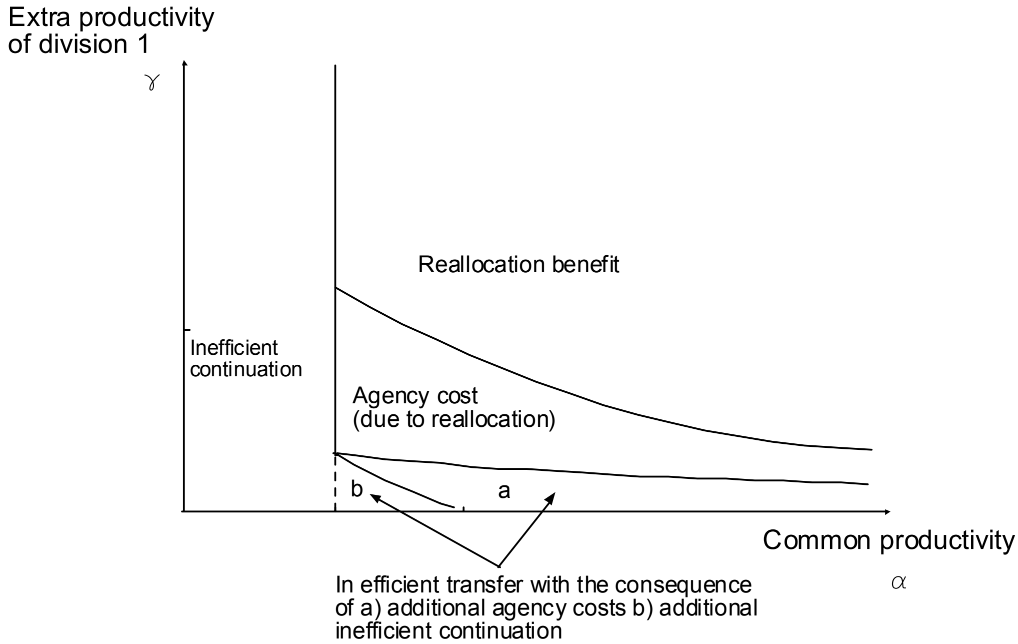

Figure 8: Costs and benefits of running an active internal capital market

when $C_2 = \alpha \geq 1$, which is the NPV rule. Since $C_1 \geq C_2$ the first division is also always continued. A low effort in the less productive division therefore solves the inefficient continuation problem. But it does not solve the over-compensation of manager 1. It is only when manager 1 does not work hard that the problem vanishes.

A low effort by the manager of the less productive division restores the NPV rule. And a low effort by the manager of the more productive division suppresses the problem of expected over-compensation. In appendix B, we describe the optimal contract for all the possible effort combinations and compare them. The optimal effort combination is represented in figure 9.

From the picture and the discussion in the appendix, we can draw the following conclusions. When the conglomerate transfers its resources to the most productive division, two high efforts is not always the optimal effort combination. The corporate headquarter has two reasons to lower the amount of effort. For low values of α, there will be less effort to preserve the NPV rule. For low values of γ, when the benefits of redistribution are not so high, the amount of effort is lowered to suppress the problem of higher payment to manager 1.

5 Conglomerate vs. Stand-Alone: is there a discount?

Now that we have described the complete solution to the conglomerate problem when the headquarter does and does not transfer funds across divisions we can compare the conglomerate's value with the value of the portfolio of single-divisional firms.

Proposition 9 *There exists a function $\gamma^*(\alpha)$, such that:*
(i) for all (γ, α) such that $\gamma > \gamma^*(\alpha)$, the conglomerate has a strictly higher value than V^{SA}, if the headquarter transfers resources to the most productive division.
(ii) for all (γ, α) such that $\gamma^*(\alpha) > \gamma > 1 + \frac{1}{\alpha}$, the conglomerate has a strictly lower value than V^{SA}, whatever the redistributive policy.
(iii) for all (γ, α) such that $\gamma < 1 + \frac{1}{\alpha}$, the conglomerate has the same value than V^{SA}, if the headquarter does not transfer resources.
(iv) the function $\gamma^*(\alpha)$ is decreasing in α.

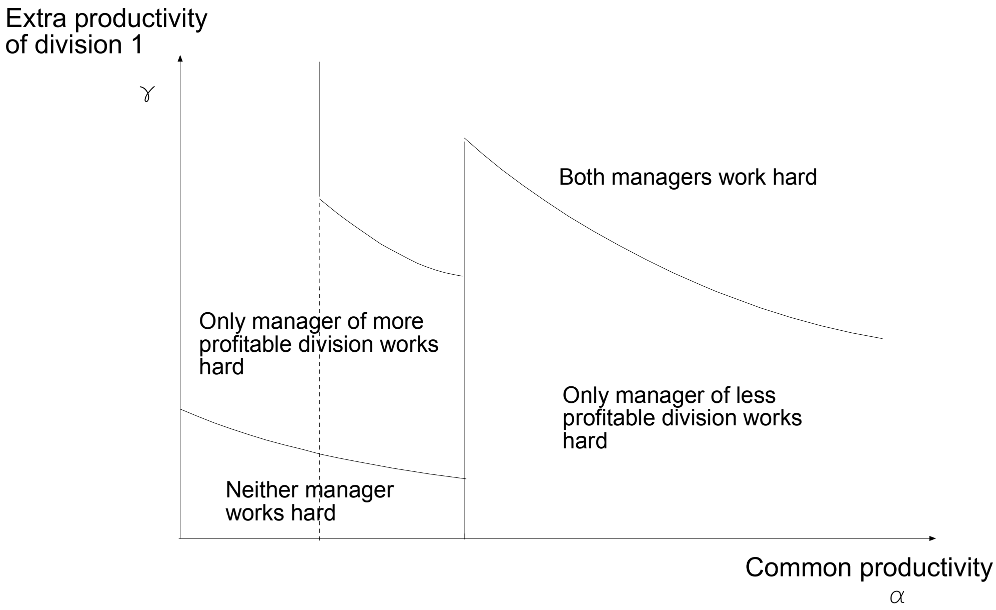

Figure 9: Optimal managerial effort levels when there is redistribution

Proof. In the appendix. ■

Proposition 9 is illustrated in figure 10.

The proposition is the central result of the paper and deserves some comments. As we already explained, an active redistributive policy inside the conglomerate has benefits as well as costs. The benefits are obvious, as the conglomerate headquarter could select the most efficient project when resources are scare. The costs have two sources: the interdepandance between the effort decisions of the managers when the headquarter decides to always refinance one division (this was explained in proposition 6) and the necessity of making the transfer policy ad-interim efficient. The costs of an active internal capital market take the form of either a lower effort by managers or a higher payment to managers or both. To create more value than comparable single division firms, the benefits of redistribution should exceed the costs.

The benefits of redistrbution increase both with γ and α, which explains the decreasing shape of the $\gamma^*(\alpha)$ function.

What proposition **??** show is that if γ is large enough, i.e. larger than $\gamma^*(\alpha)$, the benefits of an active internal capital market exceed the costs and the conglomerate creates more value than single-divisional firms. Consequently, when the benefits are not large enough, the conglomerate has a lower value than single-divisional firms, unless, the conglomerate is able to replicate the Stand-Alone solution which is the case for low values of γ.

We cannot conclude that conglomerates systematically destroy value. Depending on the conglomerate's heterogeneity among divisional performances, it could create either more or less value than comparable single division firms. Heterogeneity in the performance of conglomerates is consistent with the empirical evidence. For example although Berger & Ofek (1995) find that conglomerates *on average* have a lower value than Stand-Alone firm, they also note that some conglomerates perform better while others do worse.

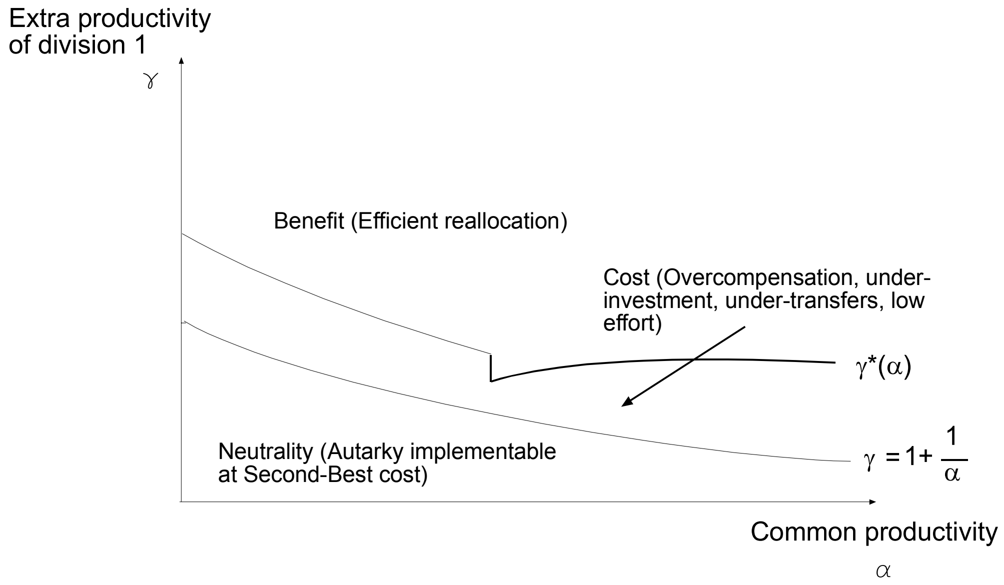

Figure 10: When does a conglomerate create or destroy value?

6 Conclusion

The aim of our paper was to explain the conglomerate discount with a model that a) does not assume that managerial pay-off is exogenous and b) does not assume inefficient cross-subsidisation. It is especially the second assumption that has recently be criticised on empirical grounds.

Our starting point was that an internal capital markets allows headquarter to make efficient use of limited resources. The main problem that headquarter faces is that those funds may be found in the "wrong" place. By this we mean that due to exogenous shocks, less productive divisions may turn out to be rich in resources while more productive divisions may turn out to be poor in resources. Managers' anticipation that headquarter has an incentive to reallocate funds creates new agency costs in a conglomerate. Moreover, headquarter must indeed find it optimal between production periods to reallocate funds, i.e. there are interim continuation and transfer constraints. On the whole there is a subtle interaction between the agency cost and the interim constraints so that neither all positive NPV divisions may be refinanced nor may more productive divisions always receive reallocated funds.

Our main conclusion is that a conglomerate discount exists and, what is noteworthy, that it exists when the productivity difference between divisions is neither too large nor too small. But note that we do not have a theory of diversification or corporate focus since i) our divisions are independent and ii) they are technologically identical but for their productivity.

Although we examine explicitly the role of managerial incentive contracts our analysis is still limited in that respect. We do not attempt to derive the optimal incentive scheme that would minimise the costs that we have identified. For example, an optimal contract would not only pay the manager a fraction of the continuation profits of his own division but it would also specify a payment out of the continuation profits from other divisions. It is also conceivable for divisional managers to hold option contracts on the value of the total firm. These contracts would pay them something even if their own division has been liquidated. Furthermore, we saw that the agency cost of reallocation is due to the fact that the manager of the more productive division is paid more and that he is paid more often. An optimal contract would then try to take something away from him, for example if his division is continued with its own resources. In other words, we would have a δ_i^i if division i is continued with its own resources and a δ_i^j if it is continued with a transfer from another successful division j. Such an approach would amount to an examination of transfer pricing in a conglomerate. This is indeed a fruitful area for future research.

What we have shown is that we do not need to resort to "private benefits" (or stealing) models with their ad-hoc assumptions about the nature of the conflict between headquarter and divisional managers if we want to explain the apparent conglomerate discount. Our paper therefore also touches on two more fundamental questions about which the existing theoretical literature remains largely silent. The first question is: in what respect are internal capital market really different than external capital markets. We showed that headquarter that operates an internal capital market can do both, redistribute funds and take into account incentive distortions. The second question is: why do firms merge and break-up? Models like Rajan et al. (2000) only explain the costs of running a conglomerate. According to their analysis, all conglomerates should be dismantled. Our paper shows that that we can have conglomerates doing better or doing worse or doing as well as a Stand-Alone benchmark. It all depends on the relationship of the relative to the absolute productivity of divisions.

A Full characterisation of the no-transfer case

A.1 Both managers exert high effort

We must solve the following optimisation programme

$$\max_{\delta_1,\delta_2,W_1,W_2} p^2(C_1(\delta_1)+C_2(\delta_2))+p(1-p)C_1(\delta_1)+(1-p)pC_2(\delta_2)$$

subject to:

$$\begin{array}{rll} \delta_1 \geq \frac{c}{(p-q)\gamma\alpha} & (\lambda_1) & \\ \delta_2 \geq \frac{c}{(p-q)\alpha} & (\lambda_2) & \text{(Incentive Constraints)} \\ \delta_2 \geq 1-\frac{1}{\alpha}-\gamma+\gamma\delta_1 & (\lambda_3) & \\ \delta_2 \leq 1+\frac{1}{\alpha}-\gamma+\gamma\delta_1 & (\lambda_4) & \text{(Transfer Constraint)} \end{array}$$

where we indicate the Lagrange multipliers in brackets. Remember that it is optimal to set $W_1 = W_2 = 1$. The first-order conditions wrt to δ_1 and δ_2 give

$$\begin{aligned} \lambda_1-\gamma(\lambda_3-\lambda_4+p\alpha) &= 0 \\ \lambda_2+\lambda_3-\lambda_4-p\alpha &= 0 \end{aligned}$$

We must distinguish three cases: either transfer constraint binds and the case when they both are slack. When the first transfer constraint binds, $\lambda_3 > 0$ and $\lambda_4 = 0$. This implies that $\lambda_1 > 0$ so that $\delta_1 = \frac{c}{(p-q)\gamma\alpha}$ and $\delta_2 = 1-\frac{1}{\alpha}-\gamma+\frac{c}{(p-q)\alpha}$. Since $\delta_2 \geq \frac{c}{(p-q)\alpha}$ it must be than $\gamma < 1-\frac{1}{\alpha}$ which is never the case. So the first transfer constraint is always slack, $\lambda_3 = 0$.

$$\begin{aligned} \lambda_1-\gamma(-\lambda_4+p\alpha) &= 0 \\ \lambda_2-\lambda_4-p\alpha &= 0 \end{aligned}$$

So $\lambda_2 > 0$ and hence manager 2 always receives his second-best contract: $\delta_2 = \frac{c}{(p-q)\alpha}$.

Manager 1's contract depends on whether the second transfer constraint binds or not. If not he gets his second-best contract too: $\delta_1 = \frac{c}{(p-q)\gamma\alpha}$. This is possible as long as $\gamma \leq 1+\frac{1}{\alpha}$. If the constraint binds, the δ_1 is given by the binding transfer constraint: $\delta_1 = \frac{c}{(p-q)\gamma\alpha}+1-\frac{1}{\gamma}-\frac{1}{\gamma\alpha}$.

Lemma 1 *Under autarky, corporate headquarter can always induce both managers to work hard. The manager of the less profitable division always receives his second best contract,* $\delta_2 = \frac{c}{(p-q)\alpha}$. *If the relative profitability is low,* $\gamma \leq 1 + \frac{1}{\alpha}$, *then corporate headquarter can pay manager 1 his second best contracts too,* $\delta_1 = \frac{c}{(p-q)\gamma\alpha}$. *If not then manager 1 must receive a lager share and gets a third best contract,* $\delta_1 = \frac{c}{(p-q)\gamma\alpha} + 1 - \frac{1}{\gamma} - \frac{1}{\gamma\alpha}$.

A.2 Only manager 2 exerts high effort

We have the following optimisation programme

$$\max_{\delta_1,\delta_2,W_1,W_2} qp(C_1(\delta_1) + C_2(\delta_2)) + q(1-p)C_1(\delta_1) + (1-q)pC_2(\delta_2)$$

subject to:

$$\begin{array}{rll} \delta_1 < \frac{c}{(p-q)\gamma\alpha} & (\lambda_1) & \multirow{2}{*}{(Incentive Constraints)} \\ \delta_2 \geq \frac{c}{(p-q)\alpha} & (\lambda_2) & \\ \delta_2 \geq 1 - \frac{1}{\alpha} - \gamma + \gamma\delta_1 & (\lambda_3) & \text{(Transfer Constraint)} \\ \delta_2 \leq 1 + \frac{1}{\alpha} - \gamma + \gamma\delta_1 & (\lambda_4) & \end{array}$$

We do not have to solve the entire optimisation programme since this case is always dominated by the case when both managers work hard. To see this note that if $\gamma > 1 + \frac{1}{\alpha}$ then it is impossible to satisfy the incentive constraints and the second transfer constraint. Thus a solution only exists when $\gamma \leq 1 + \frac{1}{\alpha}$ but then the corporate headquarter can induce both managers to work hard with the second-best contracts, which means that the conglomerate does as well as the Stand-Alone benchmark. Under autarky, the conglomerate can never do better, and certainly not with the manager of the more profitable division not working hard.

Lemma 2 *Under autarky, corporate headquarter never finds it optimal to induce only the managers of the less profitable division to work hard.*

A.3 Only manager 1 exerts high effort

The optimisation programme now becomes

$$\max_{\delta_1,\delta_2,W_1,W_2} pq(C_1(\delta_1) + C_2(\delta_2)) + p(1-q)C_1(\delta_1) + (1-p)qC_2(\delta_2)$$

subject to:

$$\begin{array}{rll} \delta_1 \geq \frac{c}{(p-q)\gamma\alpha} & (\lambda_1) & \text{(Incentive Constraints)} \\ \delta_2 < \frac{c}{(p-q)\alpha} & (\lambda_2) & \\ \delta_2 \geq 1 - \frac{1}{\alpha} - \gamma + \gamma\delta_1 & (\lambda_3) & \text{(Transfer Constraint)} \\ \delta_2 \leq 1 + \frac{1}{\alpha} - \gamma + \gamma\delta_1 & (\lambda_4) & \end{array}$$

As in the case with both manager working hard, we can show that first transfer constraint is slack. Suppose it is not then $\delta_1 = \frac{c}{(p-q)\gamma\alpha}$ so that $\delta_2 = 1 - \frac{1}{\alpha} - \gamma + \frac{c}{(p-q)\alpha}$. The problem is that this δ_2 is negative since $1 - \frac{1}{\alpha} - \gamma + \frac{c}{(p-q)\alpha} < 0 \Leftrightarrow \gamma > 1 - \frac{1}{\alpha}(1 - \frac{c}{p-q})$ $(1 - \frac{c}{p-q} > 0$ due to assumption X). Given that the first transfer contraint is slack, $\delta_2 = 0$.

If the second transfer constraint is slack too, $\delta_1 = \frac{c}{(p-q)\gamma\alpha}$. This case is possible when $\gamma < 1 + \frac{1}{\alpha}(1 + \frac{c}{p-q})$. If $\gamma \geq 1 + \frac{1}{\alpha}(1 + \frac{c}{p-q})$ then the second transfer contraint binds and $\delta_1 = 1 - \frac{1}{\gamma} - \frac{1}{\gamma\alpha}$.

Lemma 3 *Under autarky, corporate headquarter can always induce only the manager of the more profitable division to work hard. The manager of the less profitable division always receives his second best contract, $\delta_2 = 0$. If the relative profitability is low, $\gamma \leq 1 + \frac{1}{\alpha}(1 + \frac{c}{p-q})$, then corporate headquarter can pay manager 1 his second best contracts too, $\delta_1 = \frac{c}{(p-q)\gamma\alpha}$. If not then manager 1 must receive a lager share and gets a third best contract, $\delta_1 = 1 - \frac{1}{\gamma} - \frac{1}{\gamma\alpha}$.*

A.4 Neither manager exerts high effort

We must solve

$$\max_{\delta_1,\delta_2,W_1,W_2} q^2(C_1(\delta_1) + C_2(\delta_2)) + q(1-q)C_1(\delta_1) + (1-q)qC_2(\delta_2)$$

subject to:

$$\begin{array}{rll} \delta_1 < \frac{c}{(p-q)\gamma\alpha} & (\lambda_1) & \\ \delta_2 < \frac{c}{(p-q)\alpha} & (\lambda_2) & \text{(Incentive Constraints)} \\ \delta_2 \geq 1 - \frac{1}{\alpha} - \gamma + \gamma\delta_1 & (\lambda_3) & \\ \delta_2 \leq 1 + \frac{1}{\alpha} - \gamma + \gamma\delta_1 & (\lambda_4) & \text{(Transfer Constraint)} \end{array}$$

Again, we do not have to solve the entire optimisation programme since this case is always dominated by the case when only manager 1 works hard. To see this note that if $\gamma > 1 + \frac{1}{\alpha}(1 + \frac{c}{p-q})$ then it is impossible to satisfy the incentive constraints and the second transfer constraint. Thus a solution only exists when $\gamma \leq 1 + \frac{1}{\alpha}(1 + \frac{c}{p-q})$ but then the corporate headquarter can induce manager 1 to work hard with the second-best contract. Since we assumed that high managerial effort is desirable in a second-best situation, the conglomerate can never do better with a low effort from manager 1.

Lemma 4 *Under autarky, corporate headquarter never finds it optimal to induce both managers to not work hard.*

A.5 Comparing profits

We have established a) that the manager of the more profitable division always works hard and b) that the manager of the less profitable division always receives his second best contract. The question then is: when does corporate headquarter find it profitable to induce both managers to work hard subject to the constraint that it must sometimes offer third-best incentive contracts.

When $\gamma \leq 1 + \frac{1}{\alpha}$ then it is possible to induce both managers to work hard with second-best contracts and headquarter cannot do better than that. But when $\gamma > 1 + \frac{1}{\alpha}$ then it must offer a third-best contract to the manager of the more profitable division. The question then is: is it cheaper to have the manager of the less profitable division not work hard?

When $1 + \frac{1}{\alpha} < \gamma \leq 1 + \frac{1}{\alpha}(1 + \frac{c}{p-q})$ then we compare the value of the conglomerate when manager 1 works hard due to a third-best contract with the value when manager 1 works little due to second-best contract. The former exceeds the latter when

$$\begin{aligned} p(2\alpha + 1) - 2p\frac{c}{p-q} &\geq p\gamma\alpha + q\alpha - p\frac{c}{p-q} \\ \gamma &\leq (1 + \frac{p-q}{p}) + (1 - \frac{c}{p-q})\frac{1}{\alpha} \end{aligned}$$

When $1 + \frac{1}{\alpha}(1 + \frac{c}{p-q}) < \gamma$ then we compare the value of the conglomerate when manager 1 works hard due to a third-best contract with the value when manager 1 works little due to third-best contract. The former exceeds the latter when

$$\begin{aligned} p(2\alpha+1) - 2p\frac{c}{p-q} &\geq p(1+\alpha) + q\alpha \\ \alpha &\geq 2\frac{c}{p-q}\frac{p}{p-q} \end{aligned}$$

B Full characterisation of the transfer case

C Complete solution to the conglomerate with transfers

The aim of this appendix is to describe the complete solution to the conglomerate problem when the corporate headquarter always refinance the most productive division.

Like in the case of two high efforts, the following continuation and transfer constraint should be satisfied in order to support the conglomerate refinancing policy:

$$\begin{aligned} C_1 &\geq C_2 \\ C_2 &\geq 1 \end{aligned}$$

In addition to these constraints, to each efforts combination corresponds a pair of incentive constraints. These constraints (one for each manager) are constructed from picture xx:

	(e_1^l, e_2^h)	(e_1^h, e_2^l)	(e_1^l, e_2^l)
IC_1	$\delta_1 \leq \frac{c}{(1-p)(p-q)\alpha\gamma}$	$\delta_1 \geq \frac{c}{(1-q)(p-q)\alpha\gamma}$	$\delta_1 \leq \frac{c}{(p-q)(1-q)\alpha\gamma}$
IC_2	$\delta_2 \geq \frac{c}{q(p-q)\alpha}$	$\delta_2 \leq \frac{c}{p(p-q)\alpha}$	$\delta_2 \leq \frac{c}{q(p-q)\alpha}$

C.1 Only manager 1 exerts a high effort

We have the following optimization program:

$$\max_{\delta_1,\delta_2}(p+(1-p)q)C_1 + qpC_2$$

subject to: IC_1, IC_2 and the continuation and transfer constraint.

Lemma 5 *It is possible to refinance the most productive division and having only the manager of the most productive division working hard if* $\gamma \geq 1 + \frac{c}{(p-q)\alpha}\frac{p+q-1}{(1-q)p}$. *The manager of the most productive division receives* $\delta_1 = \frac{c}{(1-q)(p-q)\alpha\gamma}$. *The manager of the less productive division receives* $\delta_2 = 0$ *if the relative productivity is high,* $\gamma \geq 1 + \frac{c}{(1-q)(p-q)\alpha}$. *Otherwise, there is inefficient refinancing and the manager of division 2 receives* $\delta_2 = 1-\gamma+\frac{c}{(1-q)(p-q)\alpha}$.

In the first case, the conglomerate worths: $V_1^{HL} = (p+q-pq)\alpha\gamma + pq\alpha - \frac{(p+q-pq)c}{(1-q)(p-q)}$ *and* $V_2^{HL} = (p+q)\alpha\gamma - \frac{(p+q)c}{(1-p)(p-q)}$ *in the second.*

Proof. From the constraints, it must be that δ_1 is set at its lowest possible value given by IC_1. If γ is greater than $1 + \frac{c}{(1-q)(p-q)\alpha}$, δ_2 can be set at its lowest possible value ($=0$). Otherwise, δ_2 is set to satisfy the transfer constraint: $\delta_2 = 1-\gamma+\frac{c}{(1-q)(p-q)\alpha}$ and there is inefficient refinancing. But at this value, the constraint IC_2 is satisfied only if $\gamma \geq 1 + \frac{c}{(p-q)\alpha}\frac{p+q-1}{(1-q)p}$ and the continuation constraint is satisfied if: $\gamma \geq \frac{1}{\alpha} + \frac{c}{(1-q)(p-q)\alpha}$ (which is a weaker condition). So IC_2 determines the validity set of the solution. ■

The solution presents characteristics that are similar to the case where both managers work hard. The manager of the most productive division gets in expectation more than what he gets in a stand alone firm. This creates two problems, there could be inefficient refinancing (when the transfer constraint binds) and continuation is not possible for all positive NPV projects.

C.2 Only manager 2 exerts a high effort

The optimization program is similar to the previous one except the incentive constraints. The solution is:

Lemma 6 *It is possible to refinance the most productive division and having only the manager of the less productive division working hard if* $\alpha \geq 1+\frac{c}{q(p-q)}$. *The managers receive shares* $(\delta_1, \delta_2) = (0, \frac{c}{(p-q)q\alpha})$ *and the conglomerate has value:* $V^{LH} = (q+p-pq)\alpha\gamma + qp\alpha - \frac{pc}{(p-q)}$.

There is no inefficient refinancing in this solution and the expect costs of effort is the same as in the stand alone case but this solution is not implementable if α is not large enough (because of the continuation problem).

C.3 Both managers do a low effort

$$\max_{\delta_1,\delta_2}(q+(1-q)q)C_1 + q^2 C_2$$

subject to: IC_1, IC_2 and the continuation and transfer constraint. The solution is:

Lemma 7 *It is always possible to refinance the most productive division and having both managers not working hard. The managers receive shares* $(\delta_1, \delta_2) = (0, 0)$ *and the conglomerate has value:* $V^{LL} = (2q - q^2)\alpha\gamma + q^2\alpha$.

C.4 Comparing profits

In this section, we derive the optimal effort combination that would be selected by the corporate owner when he implements the following transfer policy : always refinance the most productive division.[27]

To make the comparisons as clear as possible, we will make the following (technical) assumptions:

Assumption 4 $\frac{(p-q)}{p} = \frac{c}{(p-q)} = \psi$

Assumption 5 $q = \frac{1}{2}$

With these assumptions, we reduce the number of parameter to three: p (or c), α and γ.

We divide the parameter space into three different regions: in the first, when $\alpha \in [1, 1+\frac{\psi}{p}]$, the solutions with a high effort in the second division do not exist. In the second, when $\alpha \in [1+\frac{\psi}{p}, 1+2\psi]$, the solution with only the manager of the less productive division working hard does not exist. And finally for $\alpha \geq 1+2\psi$, all the solutions exist.

Let's start with this latest case and establish a first result:

Lemma 8 *When it exists, the solution with a high in division 2 only dominates both the solution with a high effort in division 1 only and the solution with two low efforts.*

Proof. The solution with a high effort in division 1 is dominated because the expected costs of a high effort in division 1 is greater than in division 2: $(\frac{3}{2}p+1)\psi > p\psi$

With assumption 2, we can establish that $V^{LL} < V^{LH}$ for all $\alpha \geq 1$. ■

By lemma 8 for $\alpha > 1+2\psi$, we are left with two possible solutions: either two high efforts or a high effort in division 2 only.

[27] We will see later when this transfer policy is the optimal one.

Consider first, the case where there is efficient refinancing ($\gamma \geq 1 + \frac{(2p-1)\psi}{(1-p)p\alpha}$). In this case, both managers working hard is optimal if : $V_1^{HH} \geq V^{LH} \Leftrightarrow \gamma \geq \frac{1}{\alpha}\frac{(2-p)}{(1-p)^2} - \frac{p}{1-p}$.

For α greater than $\tilde{\alpha} = \frac{p}{1-p} + \frac{2}{p} - \frac{1}{2p^2}$, with $\tilde{\alpha} > 1 + 2\psi$, when the solution with both managers working hard and efficient refinancing exists, it dominates the solution where only the manager of the lowest productive division works hard[28]. While for $\alpha \leq \tilde{\alpha}$, both managers working hard dominates only if: $\gamma \geq \frac{1}{\alpha}\frac{(2-p)}{(1-p)^2} - \frac{p}{1-p}$.

Now consider the case where there is inefficient refinancing when both managers work hard. In this case, the conglomerate worths $V_2^{HH} = 2p\alpha\gamma - 2\frac{p}{(1-p)}\psi$. This value is greater than V^{LH} if: $\gamma \geq \frac{p}{3p-1} + \frac{(2p-1)(1+p)}{\alpha(1-p)(3p-1)}$.

When $\alpha \leq \tilde{\alpha}$, when it exists the solution with inefficient refinancing is always dominated, while for $\alpha \geq \tilde{\alpha}$, two high efforts and inefficient refiancing dominates if: $\gamma \geq \frac{p}{3p-1} + \frac{(2p-1)(1+p)}{\alpha(1-p)(3p-1)}$.

To sum up, both managers working hard is the efficient solution for $\alpha \in [1 + 2\psi, \tilde{]}$ if: $\gamma \geq \frac{1}{\alpha}\frac{(2-p)}{(1-p)^2} - \frac{p}{1-p}$. While for $\alpha \geq \tilde{\alpha}$, both managers working hard is efficient if: $\gamma \geq \frac{p}{3p-1} + \frac{(2p-1)(1+p)}{\alpha(1-p)(3p-1)}$.

Now consider the values of α smaller than $1 + 2\psi$. For these values of α, we can establish that when there is inefficient refinancing, both managers not working hard is the optimal solution. In technical terms, it means that (i) when $\gamma \leq 1 + \frac{2\psi}{\alpha}$, V^{LL} is greater than V_2^{HL} and (ii) when $\gamma \leq 1 + \frac{(2p-1)\psi}{(1-p)p\alpha}$, V^{LL} is greater than V_2^{HH}.

Proof. (i) $V^{LL} \geq V_2^{HL} \Leftrightarrow \gamma \leq \frac{1}{4p-1} + \frac{\psi}{\alpha}\frac{2(2p+1)}{(1-p)(4p-1)}$. The intersection of this curve and $1 + \frac{2\psi}{\alpha}$ is $\alpha = \frac{2\psi}{(1-p)(4p-2)}(2 - 3p + 4p^2)$. Given our assumptions, this value is greater than $1 + 2\psi$ meaning that (on the considered space), when V_2^{HL} exists, it is smaller than V^{LL}.

(ii) $V^{LL} \geq V_2^{HH} \Leftrightarrow \gamma \leq \frac{8\psi}{\alpha}\frac{p}{(1-p)(8p-3)} + \frac{1}{8p-3}$. Similarly, we can show that the intersection of this curve with $\gamma \leq 1 + \frac{(2p-1)\psi}{(1-p)p\alpha}$ is greater than $1 + 2\psi$ meaning that for all $\alpha \leq 1 + 2\psi$, $V^{LL} > V_2^{HH}$. ∎

For $\alpha \in [1, 1 + \frac{\psi}{p}]$, we have to solution to compare: both managers not working hard and only the manager 1 working hard (with efficient refinancing). Both managers not working hard is optimal if: $V^{LL} \geq V_1^{HL} \Leftrightarrow \gamma \leq \frac{2(p+1)}{p\alpha} - 1$. Otherwise, only the manager of the first division working hard is optimal.

In the last region, for $\alpha \in [1 + \frac{\psi}{p}, 1 + 2\psi]$, there are three possible solutions. Pairwise comparisons between the conglomerate value $(V_1^{HH}, V^{LL}, V_1^{HL})$ gives the following results:

$$\begin{aligned}
V_1^{HH} \geq V^{LL} &\Leftrightarrow \gamma \geq \frac{1}{\alpha}\frac{2}{(1-p)} - \frac{2p+1}{3-2p} \\
V_1^{HH} \geq V_1^{HL} &\Leftrightarrow \gamma \geq \frac{1}{(1-p)^2\alpha} - \frac{1}{p\alpha} - \frac{p}{1-p} \\
V_1^{HL} \geq V^{LL} &\Leftrightarrow \gamma \geq \frac{2(p+1)}{p\alpha} - 1
\end{aligned}$$

All these three curves intersect at a same point $\hat{\alpha} = 1 + \frac{1}{1-p} + \frac{3}{p} - \frac{2}{2p-1}$.

When α is smaller than $\hat{\alpha}$, the optimal combination is: both managers working hard if $\gamma \geq \frac{1}{\alpha}\frac{2}{(1-p)} - \frac{2p+1}{3-2p}$ and both managers not working hard otherwise.

While when α is greater than $\hat{\alpha}$, the optimal combination is both manager working hard for $\gamma \geq \frac{1}{(1-p)^2\alpha} - \frac{1}{p\alpha} - \frac{p}{1-p}$, the sole manager one working hard for $\gamma \in [\frac{2(p+1)}{p\alpha} - 1, \frac{1}{(1-p)^2\alpha} - \frac{1}{p\alpha} - \frac{p}{1-p}]$ and both managers not working hard for the remaining values of γ.

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[28] In technical terms, it means that for $\alpha \geq \tilde{\alpha}$, if $\gamma \geq 1 + \frac{(2p-1)\psi}{(1-p)p\alpha}$, then $V_1^{HH} \geq V^{LH}$.

For the considered parameter space ($\alpha \in [1+\frac{\psi}{p}, 1+2\psi]$), it is important to know the position of $\hat{\alpha}$ compared to these corner values. The position of $\hat{\alpha}$ is determined by p: if p is smaller than 0.64, then $\hat{\alpha} < 1+\frac{\psi}{p}$, while if $p > 0.642 \Rightarrow \hat{\alpha} > 1+2\psi$.

Last, we sill suppose that $p > 0.642$ and sum up the results into a proposition.

Proposition 10 *When the corporate headquarter always refinance the most productive division, the optimal managerial effort is:*
- Both managers not working hard if $\gamma \leq \frac{2(p+1)}{p\alpha} - 1$*, and* $\alpha \in [1, 1+2\psi]$.
- Only the manager of the most productive division works hard if $\gamma \geq \frac{2(p+1)}{p\alpha} - 1$*, and* $\alpha \in [1, 1+\frac{\psi}{p}]$ *or if* $\gamma \in [\frac{2(p+1)}{p\alpha} - 1, \frac{1}{(1-p)^2\alpha} - \frac{1}{p\alpha} - \frac{p}{1-p}]$ *and* $\alpha \in [1+\frac{\psi}{p}, 1+2\psi]$.
- Only the manager of the less productive division works hard if $\gamma \leq \frac{1}{\alpha}\frac{(2-p)}{(1-p)^2} - \frac{p}{1-p}$ *and* $\alpha \in [1+2\psi, \tilde{\alpha}]$ *or if* $\gamma \leq \frac{p}{3p-1} + \frac{(2p-1)(1+p)}{\alpha(1-p)(3p-1)}$ *and* $\alpha \geq \tilde{\alpha}$.
- Both manager working hard in all the other cases.

D Proofs

D.1 Proof of proposition 1

(i) Suppose that $p - q \geq c$. If the headquarter wants to liquidate the division then it chooses the contract $(1, \frac{c}{p})$ and always imposes a high effort. This yields a profit of $p - c$ for all values of $\gamma_i\alpha$. If the headquarter wants to continue the division then it chooses the contract $(\frac{c}{p\gamma_i\alpha}, 1)$ and imposes the high effort iff $\gamma_i\alpha \geq \frac{c}{p-q}$. Hence, the headquarter's profit in the continuation case is $p\gamma_i\alpha - c$ if $\gamma_i\alpha \geq \frac{c}{p-q}$ and $q\gamma_i\alpha$ if $\gamma_i\alpha < \frac{c}{p-q} \leq 1$.

Now if $\gamma_i\alpha \geq 1$ the headquarter imposes high effort in continuation and the NPV rule clearly holds: $p\gamma_i\alpha - c \geq p - c \Leftrightarrow \gamma_i\alpha \geq 1$. If $\gamma_i\alpha < 1$ then the headquarter prefers the low effort in continuation when $\gamma_i\alpha < \frac{c}{p-q}$ and the high effort otherwise (in which case the NPV rule clearly holds). The highest possible profit from the continuation profit with a low effort is $q\frac{c}{p-q}$ which is less than the profit of stopping given our initial supposition: $q\frac{c}{p-q} < p - c \Leftrightarrow p - q \geq c$.

(ii) Suppose that $p - q < c$. If the headquarter wants to liquidate the division then it always imposes a low effort. This yields a profit of q for all values of $\gamma_i\alpha$. The liquidation contract $(1, \frac{c}{p})$ is less profitable that the continuation contract $(\frac{c}{p\gamma_i\alpha}, 1)$ with the headquarter imposing a low effort there too iff $q\gamma_i\alpha \geq q \Leftrightarrow \gamma_i\alpha \geq 1$. Since a high effort dominates a low effort in the continuation case only if $\gamma_i\alpha \geq \frac{c}{p-q} > 1$ this establishes the NPV rule.

D.2 Proof of proposition 7

We must solve

$$\max_{\delta_1,\delta_2,W_1,W_2} p^2(C_1(\delta_1) + C_2(\delta_2)) + p(1-p)C_1(\delta_1) + (1-p)pC_1(\delta_1)$$

subject to:

$$\begin{aligned}
\delta_2 &\leq 1 - \tfrac{1}{\alpha} && \text{(Continuation Constraint)} \\
\delta_2 &\geq 1 - \gamma + \gamma\delta_1 && \text{(Transfer Constraint)} \\
\delta_1 &\geq \tfrac{c}{(p-q)(1-p)\gamma\alpha} && \text{(Incentive Constraints)} \\
\delta_2 &\geq \tfrac{c}{(p-q)p\alpha} &&
\end{aligned}$$

It is immediate that $\delta_1 = \frac{c}{(p-q)(1-p)\gamma\alpha}$ so that we have 2 possibilities for δ_2 depending on which constraint binds first, the incentive constraint or the transfer constraint.

i) Suppose that the incentive constraint binds first. Then $\delta_2 = \frac{c}{(p-q)p\alpha}$ and the continuation constraint becomes $\alpha \geq 1+\frac{c}{p-q}\frac{1}{p}$ and the transfer constraint becomes $\gamma \geq 1+\frac{1}{\alpha}\frac{c}{p-q}\frac{2p-1}{p(1-p)}$.

ii) Suppose then that the transfer constraint binds first. Then $\delta_2 = \frac{c}{(p-q)p\alpha} + 1 - \gamma$ and the continuation constraint becomes $\gamma \geq \frac{1}{\alpha}(1+\frac{c}{p-q}\frac{1}{1-p})$ and the incentive constraint becomes $\gamma \leq 1 + \frac{1}{\alpha}\frac{c}{p-q}\frac{2p-1}{p(1-p)}$.

So we see that both cases are mutually exclusive. Which one occurs depends on $\gamma \lesseqgtr 1 + \frac{1}{\alpha}\frac{c}{p-q}\frac{2p-1}{p(1-p)}$.

D.3 Proof of proposition 10

First, we show that for any α there is γ so that i) it is not possible to implement the autarkic solution that does as well as the Stand-Alone benchmark and that ii) by redistributing towards the more productive division and having both managers work hard the conglomerate does worse than the Stand-Alone benchmark.

For i) it must be that

$$\gamma > 1 + \frac{1}{\alpha}$$

For ii) it must be that

$$\gamma < 1 + \frac{1}{\alpha}\frac{c}{p-q}\frac{1}{(1-p)^2}$$

Then we must only show that there exists an α so that headquarter prefers both managers working hard to just the manager of the less profitable division to work hard, i.e.

$$(2p-p^2)\gamma a + p^2\alpha - \frac{c}{p-q}\frac{3p-2p^2}{1-p} > (p+q-pq)\gamma\alpha + pq\alpha - \frac{c}{p-q}p$$

which can be rewritten as

$$\gamma > \frac{p}{p-1} + \frac{1}{\alpha}\frac{c}{p-q}\frac{p(2-p)}{(1-p)^2}$$

If α is sufficiently large, the inequality is satisfied for any $\gamma > 1$.

References

Alchian, A. A. (1969), Corporate management and property rights, *in* H. G. Manne, ed., 'Economic Policy and the Regulation of Corporate Securities', American Enterprise Institute, pp. 337–360.

Berger, P. & Ofek, E. (1995), 'Diversifications's effect on firm value', *Journal of Financial Economics* **37**, 39–65.

Bolton, P. & Scharfstein, D. S. (1996), 'Optimal debt structure and the number of creditors', *Journal of Political Economy* pp. 1–25.

Brusco, S. & Panunzi, F. (2000), Reallocation of corporate resources and managerial incentive in internal capital markets. Financial Markets Group (LSE) DP 353.

Chevalier, J. A. (2000), What do we know about cross-subsidization? evidence from the investment policies of merging firms. Mimeo, GSB Chicago.

Comment, R. & Jarrell, G. A. (1995), 'Corporate focus and stock returns', *Journal of Financial Economics* **37**, 67–87.

Donaldson, G. (1984), *Managing Corporate Wealth*, Praeger Publishers, New York.

Gertner, R. H., S., S. D. & Stein, J. C. (1994), 'Internal versus external capital markets', *Quarterly Journal of Economics* **109**, 1211–1230.

Inderst, R. & Laux, C. (2000), Incentives in internal capital markets: Capital constraints, competition, and investment opportunities. Universität Mannheim.

Inderst, R. & Müller, H. M. (2000), Project bundling, liquidity spillovers, and capital market discipline. Universität Mannheim.

Lamont, O. (1997), 'Cash flow and investment: Evidence from internal capital markets', *Journal of Finance* **52**, 83–109.

Lang, L. H. & Stulz, R. (1994), 'Tobin's q, corporate diversification and firm performance', *Journal of Political Economy* **102**, 1248–1280.

Maksimovic, V. & Phillips, G. (2000), Do conglomerate firms allocate resources inefficiently? Mimeo, Rober H. Smith School of Business, University of Maryland.

Rajan, R., Servaes, H. & Zingales, L. (2000), 'The cost of diversity: The diversification discount and inefficient investment', *Journal of Finance* **55**, 35–84.

Scharfstein, D. S. (1998), The dark side of internal capital markets II: Evidence from diversified conglomerates. NBER WP 6352.

Scharfstein, D. S. & Stein, J. C. (2000), 'The dark side of internal capital markets: Divisional rent-seeking and inefficient investment', *Journal of Finance* **55(6)**.

Shin, H.-H. & Stulz, R. (1998), 'Are internal capital markets efficient?', *Quarterly Journal of Economics* **52**, 531–553.

Stein, J. C. (1997), 'Internal capital markets and the competition for corporate resources', *Journal of Finance* **52**, 111–133.

Villalonga, B. (2000), Does diversification cause the "diversification discount"? UCLA (available under http://personal.anderson.ucla.edu/belen.villalonga).

Williamson, O. E. (1975), *Markets and Hierarchies: Analysis and Antitrust Implications*, Collier Macmillan Plublishers, Inc., New York.