Financing and Corporate Growth

under Repeated Moral Hazard

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Financing and Corporate Growth under Repeated Moral Hazard [1]

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Abstract

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This paper considers the impact of financial contracting on growth by exploring a model where entrepreneurs initially do R&D but subsequently need both outside investors to provide funds for capital investments and outside managers to operate the firm efficiently some time after assets are in place. The source of contracting inefficiency is that insiders can divert cash flows for their own benefit. We employ a repeated game framework which allows us to model *outside equity* as well as inside equity and debt. We call our framework the two-stage model of firm growth. A key finding is that outside equity promotes *ex post* efficiency (second stage growth) at the expense of *ex ante* efficiency (first stage growth), while debt works the opposite way. This is because equity promotes replacement of the entrepreneur, while debt promotes entrenchment. So debt has the disadvantage that it is less conducive to the implementation of second stage growth than equity, but the advantage that it provides the entrepreneur with more incentives to do R&D in the first place. Furthermore, equity is fragile, in the sense that moral hazard may be so high that investors will not finance the firm, regardless of the discount rate. In contrast, debt financing definitely can be raised for low discount rates. A prediction of the model is that in a cross-section of firms, we should observe a preponderance of highly levered, closely-held firms which have stagnated after an early growth phase.

Keywords: corporate growth, incomplete financial contracting, outside equity, debt, repeated moral hazard
JEL Classification Numbers: G30, O33, L14

... When, Caius, Rome is thine,
Thou art poor'st of all; then shortly art thou mine.

Coriolanus

1 Introduction

This paper considers the impact of finance on growth by exploring a model where entrepreneurs need both outside investors to provide funds and outside managers to operate the firm efficiently once assets are in place. In particular, we examine the implications of financial contracts for the growth of the firm both at initial stages when the product idea is developed and at later stages when the firm can take its operations to a higher, more profitable level. The possible financial contracts we consider are inside equity, outside equity, and debt. The source of contracting inefficiency is that insiders can divert cash flows for their own benefit. In addition, our framework is designed to capture the simple fact that the insider who can contribute most to the firm at one stage of its development may well become a source of under-performance later. We model these features by considering the problem of an entrepreneur who in an initial stage chooses whether to undertake R&D. Given a successful outcome of R&D, the entrepreneur attempts to implement the product idea by obtaining external financing, needed to make capital investments, and then by initially managing the firm. At some stage, however, the firm can be made more profitable by the appointment of a more able, outside manager. We call this framework the two-stage model of firm growth.

In this context, there are several distinct obstacles to achieving efficiency. First, positive NPV R&D projects which could obtain external financing may not be undertaken because the entrepreneur's returns are too low. Second, post R&D, positive NPV capital investments may not be done because sufficient external financing cannot be raised. Third, post capital investment, the firm's assets may be operated inefficiently. The combination of these three elements is at the heart of the interaction between financial contracting and the creation of growth opportunities. Showing how this interaction under outside equity financing is fundamentally different than under debt financing is the principal contribution of our paper. A key finding is that equity promotes *ex post* efficiency (second stage growth) at the expense of *ex ante* efficiency (first stage growth), while debt works the opposite way.

Under outside equity financing our main results turn on the different degrees of managerial moral hazard that can occur in equilibrium at the production stage. There are two main cases. In the first, the firm ends up being operated by the entrepreneur (*entrepreneurial equilibrium)* in which case second stage growth potential is not realized. In contrast, if second stage growth is implemented, the firm will be operated by an outside manager (*managerial equilibrium).* These production stage outcomes feed back to initial investment decisions. A necessary condition for R&D is that, given a successful outcome, an IPO can be done. However, even if an IPO would be successful, underinvestment in R&D can arise for any of three reasons:

- When managerial moral hazard of outsiders is high, entrepreneurial equilibria tend to occur, but the cash flow generated by the entrepreneur may be too low to cover the costs of R&D;

- When managerial moral hazard of outsiders is low, managerial equilibria tend to occur, but the perk consumption by the outside manager acts as a dead-weight cost so that returns to the entrepreneur may be insufficient to cover R&D costs;

- When outside managerial moral hazard is intermediate, managerial equilibria may occur even though *ex ante* the entrepreneur prefers an entrepreneurial equilibrium which would provide returns sufficient to cover the costs of R&D. This points out an important *time inconsistency problem with outside equity financing.*

Finally, we show that moral hazard may be so high that investors will not finance the firm regardless of the discount rate. This illustrates the fragility of outside equity.

The main contrasts between debt and equity in our framework arises from differences in control rights; whereas equity provides investors with unconditional control rights, debt bestows control rights to creditors only in case of default. We show that debt facilitates the entrenchment of the entrepreneur when that it is optimal from his perspective *ex ante*. As a result, debt eliminates the time inconsistency problem with outside equity and therefore enhances the entrepreneur's incentive to do R&D. The contingent control rights of debt also eliminates the fragility problem of outside equity.

The main downside with debt is that the entrenched entrepreneur will not realise the second stage growth potential of the firm. Furthermore, under managerial equilibria there is a double moral hazard problem. Specifically, both the manager and the entrepreneur are involved in the decision to service the debt. Each in his turn is tempted by the alternative of taking the available cash flows. The upshot is that for sufficiently high discount rates some projects that could be financed with equity cannot be financed using debt.

Our model is consistent with the empirical observation that debt finance dominates among smaller firms.[1] As we have emphasized, debt is well-suited to giving incentives for the creation of early growth opportunities, but may be an obstacle to later growth. Therefore, in a cross-section we would expect to see a preponderance of smaller, relatively highly levered firms which have stagnated after an initial growth spurt.[2] Our model is also consistent with the idea that equity will be chosen by technological firms with large amounts of intangible assets. Our interpretation of this is that initial insiders of such firms have inalienable human capital which depreciates relatively slowly. Therefore, they may enjoy a relatively long period during which they will be retained by outside shareholders. As a result, equity finance will give them adequate incentives to develop the product idea in the first place.

Our basic line of argument can be stated as follows. The model starts with a potential entrepreneur who must decide whether or not to undertake R&D, which will result in an idea for a product. Bringing this idea to fruition requires making some capital investment which, since the entrepreneur is cash constrained, must be financed

[1] *Finance for Small Firms – An Eighth Report*, Bank of England, March 2001.

[2] See the empirical study of Lang, Ofek, and Stulz [1996] who find a negative correlation between leverage and firm growth.

by raising external financing. Once financing has been raised and the investment has been made, the project will return a stream of cash flows forever unless it is liquidated. At the outset the firm must be managed by the entrepreneur who produces a cash flow, π_1, each period. After a time, however, it will be apparent how the machines are operated, so that outside shareholders can replace the entrepreneur with a new manager who produces a cash flow π_2 which may be greater than π_1, reflecting the fact that managing the firm may require a different mix of talents than developing the initial product idea. The insider who controls the firm has the opportunity to divert part of the cash flows for personal consumption. If the majority of shareholders are unhappy with the level of reported cash flows, they may fire old management and hire new ones to run the assets in place. Alternatively, shareholders can liquidate the firm and obtain its scrap value.

Under equity finance, once the firm is being operated under an outside manager, it will continue as long as the manager pays sufficiently large dividends. We show that depending upon the discount rate, r, there may be a range of possible dividend pay out rates, y_2, which will sustain the firm as a going concern. However, managers will never pay out everything ($y_2 < 1$) so that there is always some degree of managerial moral hazard. Consequently, there is some maximum discount rate beyond which positive NPV capital investments will not be undertaken.

Earlier, when the entrepreneur is still in charge but has become replaceable, the entrepreneur can attempt to save his job by paying a dividend which matches the dividend expected under an outside manager. He will choose to match dividends if and only if the outside managers' payout rate, y_2, is less than a critical value which is strictly less than the upper bound, π_1/π_2, for which it is feasible to do so. When outside managers pay out more than this critical value, the entrepreneur will cede managerial control. In this case, given that his days are numbered, the entrepreneur will consume maximum perquisites during the period that he is in control of the firm. Thus even though the second stage growth of the firm is socially efficient (since $\pi_1 < \pi_2$) the diversion of cash flow (π_1) to the entrepreneur when he is in charge aggravates the underinvestment problems caused by the partial loss of cashflows to the outside manager. As with other dynamic agency models, *ex post* efficiency may conflict directly with *ex ante* efficiency. Specifically, for some cases R&D will be undertaken if and only if subsequently the firm will be operated by the entrepreneur and therefore fail to realise its second stage growth potential.

In our framework debt acts quite differently than does equity. In particular, since creditors' control rights are conditional on default, there is no pressure for the entrepreneur to match the dividends of more able outside managers. All the entrepreneur needs to do to retain control is to service debt. As a result, debt can favor entrenchment by the entrepreneur and, consequently, can serve as a more potent incentive for doing the R&D needed for the firm to grow at the outset.

We focus on debt and equity because the inefficiencies that we have uncovered in our dynamic agency model of two-stage growth have not been identified previously. Furthermore, we think that these are of major importance in understanding the financing patterns and determinants of growth in many environments, particularly relatively unsophisticated ones. The insights of our analysis can be used to study more complicated and possibly superior contracts and may underly the motives for many real

world financial innovations.

The remainder of the paper is organized as follows. In Section 2, we discuss the relationship of our analysis with the literature. In Section 3, we present the model in the case of equity finance. In Sections 4 and 5, we analyze equity financing. In Section 6, we introduce debt financing and compare this with equity financing. In Section 7, we explore the two-stage firm growth framework under other contractual forms. We also consider how the model can be extended to take into account uncertain cash flows and unanticipated technological shocks. Section 8 is devoted to conclusions. An appendix contains proofs not supplied in the text.

2 Relation to Literature

Our paper is related most directly to Myers [2000], who argues that to understand the fundamental differences between debt and equity in the absence of institutional arrangements that facilitate distribution of cash flows it is necessary to revert to the paradigm of non-contractible cash flows. Furthermore, in this setting, outside equity only makes sense in a dynamic context which creates a tradeoff for managers between short term gains from retaining current cash flows for themselves, on the one hand, and long term benefits from continued employment, on the other. This tension is present in the two main cases he explores (the "partnership model" and the "corporate model") and is the springboard for a variety of the suggestive observations he makes. We pursue this line of research by introducing the model of two-stage growth and by explicitly characterizing the implications of financial contracting for growth at both stages. Once second stage growth has been achieved our model operates very much like Myers' so that the intertemporal tradeoff faced by managers is at the heart of our analysis just as well. However, what is completely new here is the explicit study of the implications of the insider becoming an obstacle to growth once what was his specific expertise becomes available more generally.[3]

Our result that debt facilitates precommitment by the entrepreneur to pay out cash is related to the literature on the disciplinary role of debt. The traditional argument, advanced in an influential contribution by Jensen [1986], is essentially that when investors are uncoordinated, the hardness of debt is a useful disciplinary device since the failure to service debt automatically leads to bankruptcy, which is costly for managers because they may lose their jobs and possibly reputation. In contrast, the softness of equity allows managers great freedom in diverting free cash flows to uses that benefit them at the expense of investors. Our line of argument is quite different. In our model, equityholders are sufficiently coordinated to fire the incumbent and there are alternative outside managers. This puts pressure on the entrepreneur to perform since equityholders have unconditional control rights and can therefore fire the incumbent at any time if they think an outside manager will pay larger dividends. The problem is that if this pressure is too great, the entrepreneur will simply divert everything he can

[3]Our paper is also related to that of Fluck [1998] who analyses a repeated moral hazard model similar to Myers. She shows that when cash flows are non-contractible, outside equity is only feasible as an infinite (or uncertain) horizon claim. Consequently, we adopt the framework of an infinitely repeated game.

to himself while he is in charge of the company, knowing full well that this will lead to equityholders firing him. In contrast, under debt financing, there is no similar outside pressure since investors now do not have the right to replace the entrepreneur, unless he defaults. Thus, counterintuitively, in our model debt financing encourages pay outs by the entrepreneur because debt limits the disciplinary toolbox available to investors.

The underlying premise of Jensen's argument is that it is difficult to make financial contracts contingent on cash flows. This idea was earlier formalized by Townsend [1978] and Gale and Hellwig [1985] who arrive at the conclusion that debt will arise naturally in a setting where cash flows are costly to verify (see also Grossman and Hart [1982]). More recently, Hart and Moore [1998] and Bolton and Scharfstein [1990, 1996] use an incomplete contracts framework to study characteristics of optimal debt structures when cash flows are not verifiable at any cost. In a model where cash flows are verifiable but managerial effort is not, Aghion and Bolton [1992] show that debt can play a role in precommiting investors to punish low managerial effort (see also Berkovitch and Israel [1996] and Berkovitch, Israel and Spiegel [2000]). Dewatripont and Tirole [1994] have utilized this idea in a model with multiple investors where optimal capital structure involves a mixture of debt and equity. Outsiders are willing to hold equity in these models since cash flows are assumed to be contractible.

Our result that debt can be used to entrench the entrepreneur bears resemblance in appearance to results found by Stulz [1988] and Harris and Raviv [1988], but in these papers the channel through which entrenchment is facilitated is through a reduction in outsiders' voting shares arising from an increase in leverage. In our paper, entrenchment arises as a consequence of the precommitment function of debt. Shleifer and Vishny [1989] have suggested that entrenchment can also result from incumbents overinvesting in projects that are specific to the expertise of the incumbents. A paper more related to ours is Zwiebel [1996] who presents a finite-horizon model where commitment effects and entrenchment effects are both present. In particular, he considers a model where a manager chooses to take on debt in order to curb his natural predilection for empire building and, in this way, succeeds in entrenching himself. Zwiebel does not consider the original financing of the firm and therefore is not concerned with the existence of outside equity as such. Furthermore, the crucial ingredient that allows debt to serve as a commitment for Zwiebel is that managers pay out all available cash as dividends. This makes sense in his context because he assumes managers are only motivated by the private benefits of control and derive zero benefit from sharing in the firm's cash flows. As a consequence, his framework is quite distant from ours.

The theme of growth and finance is taken up in two other literatures, but in ways that are only distantly related to our paper. The literature on venture capital explores the consequences of finance arrangements between the venture capitalist and the entrepreneur. See Kaplan and Strömberg [2000] for references. This paper also documents the importance of control rights in financial contracting in practice. Our model cannot be considered a proper model of venture capital because we treat this whole process as a black-box whereby a single agent (the entrepreneur) chooses whether or not to undertake costly R&D which, if undertaken, will be successful for sure. What we explore, and what the venture capital literature does not, is the consequence of subsequent financing choices (the IPO phase) for the total value of a successful R&D outcome.

Finally, in the macroeconomics literature finance and growth is explored in a number of international comparisons which have documented a positive correlation between measures of financial development and growth (see, e.g., Goldsmith [1969]). Recently, Rajan and Zingales [1998] find evidence supporting the hypothesis that financial development stimulates growth. At the theoretical level, Greenwood and Jovanovic [1990] make the case for two-way causality. That is, financial development fosters growth by increasing the return on capital, and growth in turn provides the funds to invest in financial institutions. This literature does not explicitly address the source of imperfection in the financial sector. Our paper may be seen as providing a micro foundation for the interaction between finance and growth by providing a dynamic agency model in which different financial structures have an explicit impact on the pace of technological change both through the rate of creation of growth opportunities and in the pace of the adoption of new techniques. The model studied here might be a useful building block in the construction of an integrated theory of how financial structure evolves over time and interacts with the emergence of growth opportunities.

3 Description of the Model under Equity Financing

We use the following notation.

- K, the entrepreneur's cost of developing a product idea;
- I, the cost of investment to implement the product idea;
- γ, the fraction of common stock retained by the entrepreneur after the sale of shares to outsiders;
- π_1, the cash flow per period if the entrepreneur runs the firm (non-contractible);
- π_2, the cash flow per period if an outside manager runs the firm (non-contractible);
- y_{it}, the payout ratio at date t ; i.e., the fraction of cash flow that a manager of type i reports to investors ($i = 1$ for "entrepreneur" and $i = 2$ for "outside manager");
- L, the liquidation value of the firm. We assume that $L \leq I$. Hence, investing in the project only to collect the liquidation value at date 1 yields a negative NPV;
- r, the discount rate per period.

The model has three sets of players; the entrepreneur, outside managers, and investors. All parameter values are common knowledge, but cash flows are non-contractible. The strategic focus of the model is on the entrepreneur's and outside managers' choices of payout rates and how investors react to them. The timing of events and players' decision sets are as follows:

Time $t = 0$. The entrepreneur decides whether or not to do R&D. If he does so, he pays K and receives the product idea.

Time $t = 0^+$. Given the product idea, the entrepreneur attempts to realise it through a capital investment costing I. To finance this he sells $(1-\gamma)$ of the shares to outside investors which leaves him holding γ. Control is given to outside investors (the shares received by the entrepreneur can be viewed as non-voting shares with the same dividend rights as investors' shares). The firm begins operations under the entrepreneur who generates a cash flow π_1. The entrepreneur must then decide the initial payout ratio, $y_{10} \in [0,1]$. He pays out $y_{10}\pi_1$ to shareholders and consumes the rest.

Time $t \geq 1$. Once the firm is up and running, the Time 1 stage game, described here, is repeated until the firm is liquidated (which may be never). Investors have the first move, which is to choose

$$s_t \in \{retain, replace, liquidate\}.$$

That is, investors decide whether to *retain* the incumbent manager, *replace* the incumbent manager and continue with a new manager, or *liquidate* the firm. In case of liquidation, the game ends, shareholders receive the liquidation value L, and managers receive nothing unless they also are shareholders. If instead the firm is kept alive, the cash flow π_1 is produced if the entrepreneur has been retained as manager. If another manager has been appointed, the cash flow is $\pi_2 \geq \pi_1$. The manager who is in charge, decides the payout ratio, $y_{it} \in [0,1]$. Shareholders receive a total dividend of $y_{it}\pi_i$ and the current manager receives $(1-y_{it})\pi_i$ (plus dividends if he is also a shareholder).

For simplicity, we assume that managers (including the entrepreneur) do not draw any salary. This would emerge for example if the reservation wage of managers is 0 and competition among them forces their contractual wage to this level. Their compensation is therefore completely determined by the portion of the cash flows they do not report to investors (i.e. their perk consumption). Managers are assumed to having no money initially so that shareholders cannot require newly engaged managers to pay for the right to extract perks in the future. We also assume that the managerial labor pool is infinitely deep. Hence, once a manager has been fired, he is re-hired with probability zero. Outside managers are assumed to be distinct from outside investors. In particular, this means that outside managers do not own shares.

Payoffs

Players' payoffs are calculated by discounting cash flows at the rate r. If the firm is up and running, cash flows at time $t \geq 0^+$ are

$$\{c_{et}, c_{It}, c_{mt}\} = \begin{array}{ll} \{\gamma L, (1-\gamma)L, 0\} & \text{if } s_t = liquidate \\ \{(1-y_{1t}+\gamma y_{1t})\pi_1, (1-\gamma)y_{1t}\pi_1, 0\} & \text{if } t = 0^+ \text{ or } s_\tau = retain \ \forall \tau \leq t \\ \{\gamma y_{2t}\pi_2, (1-\gamma)y_{2t}\pi_2, (1-y_{2t})\pi_2\} & \text{otherwise,} \end{array} \tag{1}$$

where c_{et}, c_{It}, and c_{mt} are the cash flows to the entrepreneur, investors, and outside managers, respectively. Furthermore, $\{c_{et}, c_{It}, c_{mt}\} = \{0,0,0\}$ if the firm is not up and running, either because it has been liquidated, financing was not obtained, or R&D was not done. So, conditional on the entrepreneur doing R&D and investors providing financing, payoffs are as follows for the entrepreneur, investors, and outside managers,

respectively:

$$\begin{aligned} V_e &= -K + c_{e0^+} + \sum_{t=1}^{\infty} \frac{c_{et}}{(1+r)^t}; \\ V_I &= -I + c_{I0^+} + \sum_{t=1}^{\infty} \frac{c_{It}}{(1+r)^t}; \text{ and} \\ V_m &= \sum_{t=1}^{\infty} \frac{c_{mt}}{(1+r)^t}. \end{aligned}$$

Strategies and Equilibrium
We focus on subgame perfect Nash equilibria in pure strategies, which are deterministic functions mapping the history of the game into the decision sets described above. Throughout the paper, when we refer to a set of strategies as constituting "equilibrium," this means that the strategies are subgame perfect.

4 Subgame: Outside Manager Runs Firm

The analysis of the model proceeds recursively. Thus we start with the stage where the firm is up and running and the entrepreneur has been replaced with an outside manager. We focus on going-concern equilibria in which the outside manager plays a time invariate strategy so that $y_{2t} = y_2, \forall t$. This is a natural case to consider since the model at this point is stationary and since outside managers are fundamentally identical.[4]

We start the analysis by taking y_2 as given and asking what is the best response for shareholders. If shareholders never fire the current outside manager, they receive dividends of $y_2\pi_2$ every period forever. Moreover, shareholders cannot improve upon this by replacing the current manager, since any other outside manager would pay identical dividends. Therefore, at any date, the shareholders' will refrain from liquidation only if

$$y_2\pi_2 \frac{1+r}{r} \geq L.$$

This can be rearranged as

$$\frac{rL}{(1+r)\pi_2} \leq y_2, \tag{2}$$

which shows that the investors' incentive compatibility constraint imposes a lower bound on y_2.

To get the sitting management to pay out $y_2\pi_2$ at every date, as has been assumed in (2), it is necessary for him to be punished in some way if he pays less. Since any other outside manager is in principle as good as the current manager, it is credible for shareholders to replace the current manager whenever he pays out less than $y_2\pi_2$. Moreover, this also minimizes the incentive for the current manager to pay out less. In

[4] At certain points below we also discuss equilibria involving time-varying strategies.

short, a best reply for shareholders is to choose

$$s_{t+1} = \begin{array}{ll} \text{retain} & \text{if } y_{2t} \geq y_2 \text{ and } y_2 \geq rL/(1+r)\pi_2 \\ \text{replace} & \text{if } y_{2t} < y_2 \text{ and } y_2 \geq rL/(1+r)\pi_2 \\ \text{liquidate} & \text{if } y_2 < rL/(1+r)\pi_2. \end{array} \tag{3}$$

Given that investors use (3), what is the best response of the current manager? This depends on y_2. If investors' incentive compatibility constraint (2) is not satisfied, the best thing for the manager to do is to consume the entire current cash flow himself, i.e., set $y_{2t} = 0$, since shareholders will liquidate no matter what he does. More interestingly when the investors' incentive compatibility constraint is satisfied, the current manager knows that he will be retained as long as he pays a dividend of $y_2\pi_2$. If so, he will receive a constant consumption stream of $(1 - y_2)\pi_2$ from the current period on into the future. His best alternative is to consume all the current cash flow and be fired. Thus the manager pays out $y_2\pi_2$ only if

$$(1 - y_2)\pi_2 \frac{1+r}{r} \geq \pi_2.$$

This can be written as

$$y_2 \leq \frac{1}{1+r}, \tag{4}$$

which establishes that the manager's incentive compatibility constraint imposes an upper bound on y_2.

The LHS of the investors' incentive compatibility constraint (2) is increasing in r, whereas the RHS of the manager's incentive compatibility constraint (4) is decreasing in r. This contrast reflects the conflicting objectives of investors and the manager. For small discount rates, there are multiple payout ratios that simultaneously satisfy both incentive compatibility constraints. However, no such payout ratios exists for sufficiently high r. By equating the two expressions (2) and (4), we see that there is an incentive compatible payout ratio if and only if r does not exceed

$$r^* \equiv \frac{\pi_2}{L}. \tag{5}$$

This establishes the main part of the following lemma.[5]

Lemma 1 *Suppose an outside manager has been appointed. There is a stationary equilibrium in which the firm is maintained as a going concern if and only if the discount rate is less than r^*. The range of discount rates for which the firm can be kept alive in a non-stationary subgame perfect equilibrium is also $[0, r^*]$. Regardless of the discount rate, it is also equilibrium for the manager to pay out nothing and for investors to liquidate.*

(The proof is completed in the appendix). Combined with the preceeding analysis, Lemma 1 makes three main points: First, provided the discount rate is sufficiently small, there are multiple stationary going concern equilibria, each corresponding to a

[5]Fluck's [1998] Proposition 2 is similar to our Lemma 1 .

different value of y_2. However, nothing in the structure of the model allows us to say with great confidence which of these equilibria will emerge. Recalling that y_2 is the rate of payout that shareholders would expect from a new manager, *we can interpret y_2 as an index of managerial moral hazard prevailing in the firm's operating environment.* The case where y_2 satisfies the shareholders' incentive compatibility condition (2) with equality corresponds to the case of "maximum managerial moral hazard" consistent with the continued operation of the firm under an outside manager. When y_2 satisfies the manager's incentive compatibility condition (4) with equality, we have the case of "minimum managerial moral hazard" consistent with the continued operation of the firm under an outside manager. The multiplicity of equilibrium payout rates will lie behind most of the important problems with equity discussed later.

Second, liquidation equilibria always exist regardless of the discount rate. The extent to which this feeds back to the earlier stages of the model, including the financing stage will be explored in subsequent sections.

Third, when $r \in (r^*, \pi_2/(L - \pi_2))$ an inefficient liquidation would definitely occur. The inefficiency can be seen from the size of the threshold discount rate, r^*, being lower than the yield that would accrue from maintaining the firm as a going concern, $\pi_2/(L - \pi_2)$. The intuition relates to the managerial moral hazard problem arising from the twin assumptions of non-contractibility and managerial self-interest. These imply that at any time, at the very least, the manager's overall payoff must be equal to the most he can take out of the firm in the current period, i.e., π_2. Therefore, at most, outside investors receive an overall payoff which is equivalent to no dividend in the current period and π_2 forever thereafter. Since the opportunity cost of keeping the firm alive is L, the maximum yield earned by outside investors is then π_2/L.

5 Equilibria under Equity Financing: R&D, Financing, and Managerial Replacement

This section completes the analysis of equity financing by considering in turn, the decision to replace the entrepreneur, the earlier IPO process, and finally the initial R&D decision.

Once financing has been raised and the capital investment has been made, there are potentially two types of going concern equilibria: managerial (outside manager in charge) and entrepreneurial (entrepreneur in charge). The social first best would be to replace the entrepreneur by an outside manager as soon as possible. This is not necessarily what will happen, nor is it necessarily desirable from the entrepreneur's perspective *ex ante*. Which type of going concern equilibrium obtains, if any, depends on the level of managerial moral hazard and the discount rate. Given stationarity in the managerial subgame, if the entrepreneur prefers to cede management at date t, then he also prefers to do so at date 1. In this case, the entrepreneur optimally consumes the entire cash flow produced at date 0^+ and is replaced by outside investors at date 1.

5.1 The Replacement Decision

If the firm were not viable as a going concern under outside management, the entrepreneur does not need to worry about being replaced. However, he does need to worry about investors deciding to liquidate the firm. In this case, our focus is on equilibria where the entrepreneur's strategy is to pay out a dividend of $y_1\pi_1$ every period. Using a similar argument as in Section 4, we can establish the following:

Lemma 2 *Suppose the firm would be liquidated under outside management. There is an equilibrium in which the firm is maintained as a going concern under the entrepreneur with a time invariant payout rate if and only if the discount rate is less than $R(\gamma)$, where $R(\gamma)$ is increasing in the entrepreneur's shareholding, γ, and $R(0) = \pi_1/L$ and $R(1) = \pi_1/(L-\pi_1)$. It is also impossible to keep the firm alive for a larger discount rate than $R(\gamma)$ when the equilibrium dividend payment is time dependent. Regardless of the discount rate, it is also equilibrium for the entrepreneur to pay out nothing and for investors to liquidate.*

There is a slight difference between the current situation and that in Section 4. In particular, the threat that currently hangs over the entrepreneur and drives him to pay out $y_1\pi_1$ in dividends every period is that the firm will be liquidated if he does not do so. In contrast, in Section 4, the threat faced by the outside manager who had replaced the entrepreneur was that he, in turn, would be replaced by yet another manager. This is fleshed out in the proof of Lemma 2. In other words, *the lemma shows that the threat of liquidation is just as effective as the threat of replacement by an equally productive manager in supporting outside equity in a non-contractible cash flow model.*

Another important message of Lemma 2 is that liquidation equilibria always exist at the production stage. In this case, by our assumption that $L \leq I$, it would be impossible to raise equity financing. This illustrates the fragility of equity financing. *Even when the discount rate is small, one cannot be sure that equity can be raised.*

Next, we consider the more interesting case that the firm is viable as a going concern under outside management, because y_2 satisfies the incentive compatibility constraints discussed above. In this case, the threat facing the entrepreneur is replacement by a more able outside manager. Clearly, to avoid being replaced he must at the very least match the dividends that investors expect that an outside manager would pay. Since there is no reason for the entrepreneur to pay more than needed, this means that to avoid being replaced, the entrepreneur must follow a strategy of paying a dividend of [6]

$$y_1\pi_1 = y_2\pi_2 \tag{6}$$

every period. When y_2 is very high, it will be impossible for the entrepreneur to match dividends, and so the entrepreneur will be replaced for sure at date 1. In this case, the optimal action for the entrepreneur is to consume all cash flows before being replaced.

[6] While one can construct equilibria where the entrepreneur must pay more in dividends than an outsider manager would, we do not believe these equilibria are very reasonable. We therefore focus on entrepreneurial equilibria with the matching condition (6).

More generally, when y_2 is sufficiently small that the entrepreneur is able to match, the entrepreneur prefers to do so if and only if

$$(1-y_1)\pi_1\frac{1+r}{r}+\gamma y_1\pi_1\frac{1+r}{r}\geq\pi_1+\gamma y_2\frac{\pi_2}{r}, \tag{7}$$

where γ is the entrepreneur's shareholding. On the left hand side of this relation is the present value of cash flows to the entrepreneur from matching; where we have used the fact that the stationarity of an outside manager's strategy implies that if the entrepreneur prefers to match once, then he will do so forever. On the right hand side is the current cash flow plus the present value of the future equilibrium dividends the entrepreneur will collect under an outside manager; here we have used the fact that if the manager will be replaced next period, it is optimal for him to consume the entire current cash flows, π_1. By substituting in $y_1\pi_1=y_2\pi_2$, we can rewrite (7) as

$$y_2\leq\frac{\pi_1}{\pi_2[1+(1-\gamma)r]}, \tag{8}$$

which shows that the entrepreneur will match provided y_2 is sufficiently low, which is intuitive.

But perhaps the most important lesson from (8) is that the entrepreneur may decide not to match in some cases when the dividend paid by an outside investor is less than the cash flow that the entrepreneur would generate himself, i.e., in some cases when $y_2\pi_2<\pi_1$. This is the source of the *ex post* versus *ex ante* tension that exists under equity financing. *Ex ante*, the entrepreneur would prefer the outside manager to run the firm if and only if the outside manager would pay larger dividends than what the entrepreneur could generate himself, since this maximizes the size of the pie available to the entrepreneur and investors and since investors' equilibrium slice of the pie is a constant, I. But *ex post*, after investors have put up the cash required for capital investments and received a fixed number of shares in return, the entrepreneur has an incentive to exploit the non-contractibility of cash flows by consuming all the cash flows that he generates. This will result in his replacement and therefore he loses the ability to divert cash flows to himself in the future, but the time value of money may make it attractive nevertheless. This is most obvious when the outside manager's payout ratio is so high that $y_2\pi_2=\pi_1$. If the entrepreneur matches this, his perquisite consumption is squeezed to zero. Therefore, in this case, the entrepreneur prefers to pay out no dividends initially, diverting all of π_1 to himself. Getting fired as a result of this constitutes no loss to the entrepreneur, since he still collects the same dividends as he would if he ran the firm and matched the outside manager's dividends. Below, we show how this tension between *ex ante* and *ex post* efficiency is resolved in equilibrium.

5.2 Equilibrium Shareholdings

As seen in (8), whether or not the entrepreneur will match the anticipated dividends of an outside manager depends upon the shareholdings of the entrepreneur. The lower are his shareholdings, the less inclined will he be to match dividends. So whether we have an entrepreneurial or managerial equilibrium depends upon the shareholdings of the entrepreneur, which are determined at date 0^+. But it is also true that this shareholding depends upon whether investors anticipate an entrepreneurial or managerial

equilibrium. For if the entrepreneur matches dividends, shareholders receive dividends of $y_2\pi_2$ every period forever, starting at date 0^+. In contrast, if the entrepreneur prefers not to match dividends, shareholders only start to receive their $y_2\pi_2$ dividend when the outside manager comes along at date 1. Since, in equilibrium, outside shareholders must receive shares worth I, we see that under an entrepreneurial equilibrium, the entrepreneur's shareholding will be

$$\gamma_e = 1 - \frac{rI}{y_2\pi_2(1+r)}, \tag{9}$$

and under a managerial equilibrium, the entrepreneur's shareholding will be

$$\gamma_m = 1 - \frac{rI}{y_2\pi_2}. \tag{10}$$

Notice that γ_m is higher than γ_e, which is intuitive since in a managerial equilibrium shareholders start to receive dividends later than in an entrepreneurial equilibrium. With these expressions in hand, we can go to examine the conditions under which the entrepreneur will be successful in raising equity financing and under which there will be an entrepreneurial or managerial equilibrium.

5.3 Financing and Entrepreneurial versus Managerial Equilibria

We continue to focus on the case that the firm is viable as a going concern under outside management.[7] Since negative shareholdings are not possible, (10) shows that the entrepreneur can successfully raise financing under a managerial equilibrium provided that

$$y_2 \geq \frac{rI}{\pi_2}. \tag{11}$$

Recall now that keeping the firm alive under an outside manager is only feasible if the investors' and the outside manager's incentive compatibility constraints are satisfied. The former, (2), puts a lower bound on y_2 and the latter, (4), puts an upper bound on y_2. Condition (11) shows that raising financing under a managerial equilibrium raises the lower bound on y_2; i.e., it is "more difficult" to raise financing than maintaining the firm as a going concern once it is up and running. Intuitively, the reason is the delay in dividend payments under a managerial equilibrium that we discussed above. Furthermore, combining (11) and the upper bound (4), we see that the largest discount rate for which financing can be provided at date 0 under a managerial equilibrium, r_m^*, satisfies

$$r_m^*(1 + r_m^*) \equiv \frac{\pi_2}{I}. \tag{12}$$

Note that this implies that $r_m^* < r^*$, which means that it may be impossible to raise equity financing under a managerial equilibrium even though the firm can be maintained as a going concern under an outside manager. The (implicit) formula for the

[7] The case when outside management is not a viable alternative to entrepreneurial management is dealt with in Lemma 2 and Theorem 1.

threshold discount rate, r_m^*, is quite intuitive. Under a managerial equilibrium there are two sources of moral hazard. First, the entrepreneur consumes the entire date 0^+ cash flow; and second, the outside manager must receive compensation for not consuming the entire date 1 cash flow. Hence, from investors perspective, this is the same as saying that the first two cash flows are lost, and equation (12) is an immediate implication since investors need to break even. This also explains in an intuitive way why $r_m^* < r^*$.

Turning now to the case that the entrepreneur is able to and prefers to match dividends, (9) shows that the entrepreneur can successfully raise financing under an entrepreneurial equilibrium provided that

$$y_2 \geq \frac{rI}{(1+r)\pi_2}. \tag{13}$$

This puts a lower bound on y_2. When this relation is satisfied as an equality, $\gamma_e = 0$. In this case, the condition under which the entrepreneur prefers to match, (8), reduces to $y_2 \leq \pi_1/\pi_2(1-r)$, which puts an upper bound on y_2. Equating this upper bound with the lower bound in (13), we see that financing can be provided under an entrepreneurial equilibrium provided that the discount rate is less than

$$r_e^* \equiv \pi_1/I. \tag{14}$$

Notice that this is less than or equal to the threshold for keeping the firm alive under the entrepreneur when it cannot be kept alive by an outside manager (see Lemma 2).

This shows that the range of discount rates for which an entrepreneurial equilibrium can be supported is generally different from the range of discount rates for which a managerial equilibrium can be supported. More importantly, this also shows that there is some irreducible form of moral hazard in our model, since both r_m^* and r_e^* are less than the project's internal rate of return as of date 0^+. Hence, not all positive NPV investment projects can get financing.

We now have mapped out the critical elements of the model needed to pinpoint the conditions under which financing can be raised and an entrepreneurial or managerial equilibrium may obtain. This is put together and synthesised in the following theorem.

Theorem 1
(1) Suppose the firm would be sustained as a going concern if an outside manager were appointed.[8] *It is possible for the firm to raise financing under a managerial equilibrium if and only if (i)* $r \leq r_m^*$ *and (ii)*

$$y_2 \geq \frac{\pi_1}{\pi_2} - \frac{r^2 I}{\pi_2}. \tag{15}$$

It is possible for the firm to raise financing under an entrepreneurial equilibrium if and only if (iii) $r \leq r_e^*$ *and (iv)*

$$y_2 \leq \frac{\pi_1}{\pi_2} - \frac{r^2 I}{(1+r)\pi_2}. \tag{16}$$

[8] This means that (2) and (4) hold.

(2) Suppose the firm would be liquidated if an outside manager were appointed. It is possible for the entrepreneur to raise financing if and only if $r \leq r_e^*$.
(3) For every discount rate, there is an equilibrium in which equity financing cannot be raised.

Part (3) of the theorem illustrates the fragility problem with equity financing: even for very low discount rates, it may be equilibrium for investors not to advance equity capital, because subsequent moral hazard will be too large. In the case that financing can be raised, Parts (1) and (2) show that entrepreneurial equilibria tend to obtain for low y_2's and small r's, whereas managerial equilibria obtain for high y_2's and large r's. This and other aspects of the theorem are illustrated in Figure 1.[9]

The shaded parts in Figure 1 represent the combinations of y_2's and r's for which equity financing can be raised. The figure illustrates the point that not all positive NPV projects can get financing, since the threshold discount rate, r_m^*, is less than the yield of the project, $\pi_2/(I - \pi_1)$, in the first best state.

A more subtle impediment to raising equity originates from the outside pressure on the entrepreneur. Since equity investors have unconditional control rights, for the entrepreneur to be retained as manager of the firm, he needs to match the dividends that investors would expect to receive from an outside manager. As shown in Figure 1, if the entrepreneur attempts to match dividends when y_2 is intermediate ("close to" but less than π_1/π_2), the present value of his future perk consumption becomes so small that the entrepreneur prefers to pay out nil at date 0^+ and be replaced at date 1. This is an impediment to raising financing if reported cash flows by outsiders alone are not sufficiently large to cover the cost of investment. Figure 1 illustrates that this problem becomes particularly acute at the financing stage when the discount rate is "large," as shown by the non-shaded triangular region which is bounded below by line e, above by c and to the right by r_e^*. In this region, if the entrepreneur could commit to matching dividends, financing would actually be available (since $r \leq r_e^*$). The problem here is that, once financing is obtained, the entrepreneur prefers to divert all initial cash flows to himself. Anticipating this, investors are not willing to advance financing in the first place.

The intermediate moral hazard scenario shows that there can be a conflict between efficiency at the production and financing stages under equity financing. It also illustrates that *ex ante* welfare can be reduced by the existence of managers with su-

[9]Figure 1 is drawn under the condition that π_2 is so much larger than π_1 that $r_m^* > r_e^*$. Note that $r_m^* > r_e^*$ if and only if $(1 + \pi_1/I)\pi_1/I < \pi_2/I$, that is, if and only if $\pi_2 > \pi_1(1 + \pi_1/I) = \pi_1(1 + r_e^*)$. Figures for $r_e^* \geq r_m^*$ would look very similar. The comments in the text are not limited to the case depicted. As noted in the legend for the figure, lines a and b represent the incentive compatibility constraints of an outside manager to pay out $y_2\pi_2$ per period and of investors not to liquidate, respectively. Hence the "triangle" formed by lines a and b represents the set of y_2's and r's for which the firm can be maintained as a going concern by an outside manager. Lines a and c form a smaller "triangle" representing the set of y_2's and r's for which financing can be raised if the firm will subsequently be run by an outside manager. Line c lies above line b because in a managerial equilibrium, the entrepreneur consumes the entire date 0^+ cash flow, whereas the outside manager will be paying out $y_2\pi_2$ from period 1 onwards. Additionally, if y_2 is above line b, financing can be raised in an entrepreneurial equilibrium. If y_2 is below line e, the entrepreneur prefers not to relinquish control if investors think that he will not. If y_2 is above line f, the entrepreneur prefers to relinquish control if investors think that he will do so.

perior abilities, since if outside managers had the same ability as the entrepreneur, the entrepreneur would never relinquish control in equilibrium. Of course, in an entrepreneurial equilibrium, there is social inefficiency at the production stage since second stage growth is not implemented.

The crucial property which drives these results is that the incumbent manager who sees that his days are numbered will act so as to maximize his short-term benefits. In the current model this is limited to consuming all the cash flows during one period. More generally there may be a substantial delay between the decision to fire the incumbent and the arrival of his replacement. During this period there may be a substantial loss of value for outside investors. The longer is this period, or the longer is the initial period when the entrepreneur is the only one who can run the firm, the larger is the problematic region of intermediate moral hazard discussed above.

5.4 The R&D Decision

We close the analysis of the model under equity financing by addressing the question as to whether, at time 0, the budding entrepreneur will do R&D. The first best is for him to do so whenever NPV is positive, i.e.,

$$\pi_1 + \pi_2/r \geq I + K. \tag{17}$$

The first criterion that must be satisfied in order for the entrepreneur to be willing to do R&D is that financing can be raised. Conditional upon that, if the entrepreneur anticipates that a managerial equilibrium obtains, the entrepreneur is willing to do R&D if and only if

$$\pi_1 + \frac{y_2\pi_2}{r} \geq I + K, \tag{18}$$

since, in equilibrium, investors will have shares worth exactly I at date 0^+ [see (10)].

Similarly, under an entrepreneurial equilibrium, the entrepreneur is willing to do R&D if and only if financing can be raised and

$$\pi_1 + \frac{\pi_1}{r} \geq I + K. \tag{19}$$

The first thing to note from these three equations is that first best is not achieved at the R&D stage since $y_2 \leq 1/(1+r)$ [by (4)]. Hence, some R&D opportunities with "low" NPV's will be foregone by the entrepreneur, since the returns to the entrepreneur are too low to cover his cost of R&D. Under outside equity financing in our model, the non-contractibility of cash flows means that there is an irreducible amount of moral hazard which ultimately leads to underinvestment in R&D.

The three equations also illustrate that equilibrium underinvestment in R&D relates to the degree of moral hazard associated with an outside manager. First, when moral hazard is "high" (y_2 is "low"), the entrepreneur anticipates that he will not be replaced by an outside manager once the firm is up and running, however, his share of the cash flows may not be sufficient to cover the cost of R&D. This can be seen immediately from (19).

Second, when moral hazard is "low" (y_2 is "high"), the entrepreneur anticipates that he eventually will be replaced by an outside manager, however, his initial perquisite

consumption plus his share of future dividends may not cover his R&D costs. This can be seen from (18), since incentive compatibility requires $y_2 < 1$. Intuitively, what happens here is that hiring an outside manager creates a deadweight cost equal to the present value of the outside manager's on-the-job perk consumption. This can be seen by observing that (18) can be rewritten as

$$\pi_1 + \frac{\pi_2}{r} \geq I + K + \frac{(1-y_2)\pi_2}{r}.$$

The final term is the value of the outside manager's perk flow. This contrasts with an entrepreneurial equilibrium where all cash flows generated by the project go either to the entrepreneur or to investors, who are compensated for a large perk consumption by the entrepreneur through a high shareholding. In contrast, the entrepreneur has no recourse to compensation from the outside manager's perk consumption. Put another way, if the entrepreneur will be replaced by the efficient outside manager starting in $t = 1$, the entrepreneur will not internalize the consumption of future perquisites in his decision making. As a result, underinvestment in R&D may occur.

Third, when moral hazard is "intermediate" (y_2 is intermediate), there is a tension between *ex post* and *ex ante* efficiency. In this case, when $y_2\pi_2$ is "close to" but less than π_1, managerial equilibria tend to occur, as illustrated in Figure 1. What happens here is that once financing has been sunk and investors' shareholding has been fixed, the entrepreneur maximizes his wealth by relinquishing control, since in doing so, he gets to divert all the initial cash flows to himself. This *ex post* maximization by the entrepreneur is also socially *ex post efficient*, since the outside manager generates larger cash flows. However, a simple comparison of (18) and (19) shows that from an *ex ante* perspective, this can be inefficient since it reduces the incentive to do R&D.

As was emphasized in the introduction, one of our main objectives in this paper has been to understand how problems in financial contracting can impact on growth. The results of this section show the complex nature of these relations. On the one hand managerial moral hazard may reduce the incentive for shareholders to grow the firm by appointing more able outside management. On the other hand contracting arrangements which make it easy to improve the firm by changing management may discourage the creation of growth opportunities in the first place. Stated otherwise, entrenchment has both its negative side and its positive side. Agents may be willing to take the steps necessary to develop new lines of business only if the financial structure assures that they will retain effective control of the firm (and have privileged access to its cash flows) for a sufficiently long time once the project is underway.

6 Debt Financing

In this section we study debt which is the main alternative to outside equity financing. We will not obtain a full-blown theory of capital structure. However, it should be noted that if the capital investment is financed by debt and subsequently the firm operates under entrepreneurial management, the firm's capital structure consists of debt and inside equity. Alternatively, if the firm is financed by debt and has an outside manager, its capital structure consists of debt and outside equity (either held by the

original entrepreneur or by other investors if the entrepreneur cashes out). We will ask whether debt financing will be able to eliminate any of the possible inefficiencies we found with outside equity. We will find that the answer is yes, possibly, but that other inefficiencies will emerge.

In models of debt, it is widely recognized that often important results turn on whether or not debt contracts are renegotiable and whether or not there are bankruptcy costs. In particular, Bolton and Scharfstein [1996] and Franks and Nyborg [1996] have demonstrated that a complex debt structure can make debt hard to renegotiate (see also Berglöf and von Thadden [1994]). In practice, complexity can result from having multiple creditors, multiple layers of seniority, and multiple and fragmented collateral.[10] In this paper, we will focus on the case that the firm has a debt structure that is so complex as to be non-renegotiable. As a consequence, failure to service debt results in the liquidation of the firm. Claimants share in the liquidation value according to strict absolute priority. We choose this model of debt not because it is necessarily the most commonly observed form of debt, but because it will bring out clearly some important differences relative to equity financing. We will comment below on the extent to which results may change under alternative specifications of debt contracts, which may or may not be more relevant to a particular institutional context.

In terms of the timing of events, the model under debt financing parallels the equity financing model. However, since the debt is complex, failure to service the debt in full leads to liquidation. From time $t = 1$ onwards, the stage game proceeds as follows: (i) The entrepreneur decides whether to appoint a new manager.[11] (ii) Cash flows are produced. If an outside manager has been appointed, the manager reports a cash flow $y_{2t}\pi_2$. (iii) The entrepreneur decides whether to service the debt or default.[12] If he defaults, the firm will be liquidated. Since there is no benefit from defaulting partially, the entrepreneur's choice is between paying zero or the contractual debt service in full.

Thus under debt financing and outside management, the firm is doubly exposed to moral hazard. First, there is the risk that managers will consume perquisites and under-report true cash flows. Second, there is the risk that the shareholder/entrepreneur may prefer to consume all the reported cash flows rather than service the debt.

Since in our model, the project has an infinite life, the range of discount rates for which debt can be raised is maximized by considering perpetual debt. Furthermore, since the entrepreneur generates lower cash flows than outside managers, it is natural to consider debt contracts which may include an initial "grace period", where contractual debt service is reduced. Therefore, we consider debt contracts where the firm promises to pay a coupon of d_0 in the initial period and a coupon of $d_1 \geq d_0$ in all subsequent periods. We are interested in equilibria in which the firm is kept alive forever, i.e. in which there is no default. In such equilibria,

$$I = d_0 + \frac{d_1}{r}.$$

[10]For example, in Germany the fact that the vast majority of all firms in financial distress in the past have ended up closing down has been attributed to a high fragmentation of collateral among multiple creditors (see Franks, Nyborg, and Torous [1996]).

[11]It is never beneficial for the entrepreneur to decide to liquidate the firm.

[12]Since the entrepreneur is the sole shareholder, he controls the board of directors and therefore the debt service decision.

So in the special case that $d_0 = d_1 = d$, the coupon every period is

$$d = \frac{Ir}{1+r}. \tag{20}$$

If $d \leq \pi_1$, a grace period is not necessary, nor would it have any benefit. We have:

Lemma 3 *It is feasible for the entrepreneur to service debt where there is no grace period if and only if the discount rate is less than or equal to the yield of the project under the entrepreneur. That is, $\pi_1 > Ir/(1+r)$ iff $r \leq \phi$, where $\phi \equiv \pi_1/(I-\pi_1)$.* [13]

If $r > \phi$, it is not feasible for the entrepreneur to service the debt at date 0^+ unless d_0 is reduced. In such cases, we assume that $d_0 = \pi_1$ since this maximizes return to creditors and therefore reduces any underinvestment problem arising from the feasibility constraint.[14] In this case,

$$d_1 = (I - \pi_1)r, \tag{21}$$

which is indeed greater than d_0 whenever $r > \phi$. We will see below when, in equilibrium, a grace period will be used. As before, we focus on stationary subgame perfect equilibria where outside managers use a constant payout ratio. Also, as before, we will see that there are numerous equilibria, depending on y_2.

6.1 The Replacement Decision

Observe first that a necessary condition for a managerial equilibrium is that an outside manager would place a greater cash flow in the entrepreneur's hand than what he could generate himself, that is,

$$y_2\pi_2 > \pi_1. \tag{22}$$

This is in sharp contrast to the result under equity financing, where a managerial equilibrium could obtain even if $y_2\pi_2 < \pi_1$. This reflects the combined effects of differences in control and cash flows rights between debt and equity. If the entrepreneur uses equity financing, he loses the right to decide over the use of corporate resources, including his own replacement decision. If it becomes too onerous for him to match the dividends that would be paid by an outside manager, the entrepreneur will simply pay no dividends and instead consume all current cash flows before getting replaced by an outside manager next period. In contrast, under debt financing, the entrepreneur makes the decision whether to replace himself and whether to service debt (which here is less critical). Given y_2, at any time $t \geq 1$, the replacement decision is basically a decision between a pre debt service cash flow of $y_2\pi_2$ versus π_1. So, in equilibrium, the entrepreneur replaces himself if and only if (22) holds, since the contractual debt service is independent of reported cash flows.

[13] **Proof:** In (20), set $d = \pi_1$ and rearrange.

[14] The qualitative results are not affected by the choice of $d_0 \leq \pi_1$.

6.2 Debt Service and Financing

Consider first the case that $y_2\pi_2 \leq \pi_1$ so that the entrepreneur would not replace himself with an outside manager. We are interested in conditions for debt service. In this case, there is no need for a grace period, so the coupon is given by (20). Hence, the entrepreneur will service this debt if and only if

$$\pi_1 - \frac{Ir}{1+r} \quad \frac{1+r}{r} \geq \pi_1, \tag{23}$$

since the entrepreneur's best alternative at any given time is to consume the entire cash flow and see the firm liquidated. Note that (23) is equivalent to saying that $r \leq r_e^*$ [see (14)]. An important observation here is therefore that *if $r \leq r_e^*$ then debt financing definitely can be raised.* This contrasts with our result under equity financing which says that *it is possible* to raise equity financing if and only if r_e^*. This contrast has its roots in the limited control rights of debt versus equity. Under equity financing, there are numerous equilibria in the subgame after financing has been raised, depending upon the payout rates used by the entrepreneur (and outside managers). Liquidation equilibria always exist because the entrepreneur cannot be sure that outside investors will not replace him or liquidate the firm, which they can do *because as equityholders they have unconditional control rights.* In contrast, under debt financing, the entrepreneur knows that as long as he services the debt, he will remain in charge, *since creditors only have conditional control rights*; they are activated only in the event of a default. As a result, when $r \leq r_e^*$, all players know that the entrepreneur will service the debt and so financing can be raised for sure.

Consider next the case that $y_2\pi_2 \geq \pi_1$ so that the entrepreneur would replace himself with an outside manager. Note first that not all "large" y_2's would be consistent with equilibrium. First, the manager must prefer reporting $y_2\pi_2$ to reporting nothing and thereby precipitating the liquidation of the firm, i.e.,

$$y_2 \leq \frac{1}{1+r}. \tag{24}$$

This is just as in the case of equity financing. Second, given that the outside manager reports $y_2\pi_2 > d_1$, the shareholder/entrepreneur must do better by servicing the debt than defaulting and seeing the firm terminated, i.e.,

$$(y_2\pi_2 - d_1)\frac{1+r}{r} \geq y_2\pi_2. \tag{25}$$

The manager recognizes that if he reports a cash flow that is too low, the shareholder/entrepreneur will prefer to default on the debt contract, leading to the liquidation of the firm and the loss of perks to the manager. Hence, a going concern equilibrium under an outside manager is possible only if

$$\frac{d_1(1+r)}{\pi_2} \leq y_2 \leq \frac{1}{1+r}. \tag{26}$$

Of course, to raise financing in the first place, it must also be the case that before replacing himself (at date 1), the entrepreneur must prefer servicing debt at date 0^+.

Hence, we must also have

$$\pi_1 - d_0 + \frac{y_2\pi_2 - d_1}{r} \geq \pi_1. \tag{27}$$

Since $d_0 \leq d_1$, we see that (27) is satisfied whenever (25) and (22) are satisfied, implying that the entrepreneur will wish to service debt initially if he will also do so once an outside manager has been appointed. Therefore, if no grace period is needed, (25) tells us that there is y_2 such that it is incentive compatible to service debt if and only if $r \leq r_m^*$ [see (12)], which parallels the result under equity financing.

The following theorem summarizes the analysis so far and completes the analysis in the case that a grace period is needed.

Theorem 2
(1) Suppose $r_m^* \leq \phi$. It is possible to raise complex debt financing in a managerial equilibrium if and only if $r \leq r_m^*$. In a managerial equilibrium, there is no grace period and y_2 satisfies (22) and (26), with $d_1 = d$ as given by (20).
(2) Suppose $r_m^* > \phi$. There is $\phi^* \in (\phi, r_m^*)$ such that it is possible to raise complex debt financing in a managerial equilibrium if and only if $r \leq \phi^*$. In a managerial equilibrium, there is a grace period iff $r > \phi$ and y_2 satisfies (22) and (26), with d_1 given by (20) if $r \leq \phi$ and (21) if $r \in (\phi, \phi^*]$.
(3) The entrepreneur will raise complex debt financing under an entrepreneurial equilibrium if and only if $r \leq r_e^*$ and the conditions for a managerial equilibrium in (1) or (2) are not satisfied.

Theorem 2 when $r_m^* > \phi$ is summarized in Figure 2. The figure shows that managerial equilibria tend to occur for large payout rates and entrepreneurial equilibria tend to occur for small payout rates. [15] This parallels what we saw in Section 5 under equity financing (Figure 1).

6.3 The Contrast Between Debt and Equity

The equity analysis showed that, beyond the intrinsic moral hazard problem that exists in our model, equity financing has two main problems.

Time inconsistency: Comparing Figures 1 and 2 (and Theorems 1 and 2) we see that under debt financing, we never get a managerial equilibrium when $y_2\pi_2 < \pi_1$. In contrast, this can happen under equity financing. Hence there is no time inconsistency problem under debt financing.

This difference between debt and equity emerge in our model primarily because of differences in control rights. Under equity financing, investors have unconditional control rights and the cash flows they receive are highly sensitive to reported cash flows. Hence, if the entrepreneur tries to stay on as manager he will find his perk consumption squeezed by the threat of being replaced by an outside manager. In the face of this, the entrepreneur often will prefer to cede control, consume as much as he can before he is replaced and benefit from the future dividends given that he is a shareholder. In contrast, under debt financing, since creditors have conditional control rights, the

[15] This is also true when $\phi \geq r_m^*$.

entrepreneur need only meet contractual debt service to assure that he continues to manage the firm indefinitely into the future. This is less onerous than matching dividends.[16] Furthermore, under complex debt, the consequences of consuming maximum perks are harsher. The firm is liquidated and the entrepreneur loses all further benefits. The upshot is that debt promotes entrenchment. The entrepreneur's incentive to do R&D is enhanced and therefore *ex ante* efficiency is improved.

Fragility: As shown in Lemma 2, for any discount rate, once equity financing has been raised and the capital investment has been sunk, there is an equilibrium in the ensuing subgame in which the firm is liquidated immediately. In this case, equity financing will of course not be advanced in the first place and, anticipating this, the entrepreneur will not do R&D. In contrast, complex debt financing can always be raised when $r \leq r_e^*$.

This contrast also arises because of the different control rights bestowed upon investors by debt and equity. Investors advance equity financing if they believe that dividends, either under the entrepreneur or an outside manager, will be large enough to cover the cost of investment. Conversely, the entrepreneur or an alternative manager, will pay out these dividends if he believes that investors will not replace him or liquidate the firm. The repeated interaction between the players creates a self-enforcing mechanism that fosters the virtuous cycle of beliefs. However, this is a fragile mechanism, and a vicious cycle can emerge as well. Since equity investors have unconditional control rights, they can replace the incumbent at any time, and they will do so if they believe that he will not pay out sufficiently large dividends in the future. Conversely, if the incumbent thinks that he will be replaced, he will not pay out any dividends to equity investors. By giving investors control rights only in the case of default, debt breaks this vicious circle of mistrust. The upshot is that debt financing can be raised whenever it is in the entrepreneur's unilateral interest to service it, i.e., whenever $r \leq r_e^*$.

However, debt does not dominate equity because it has drawbacks as well, as seen by a comparison between Figures 1 and 2. Specifically, when $r_m^* > \phi$, the maximum discount rate for which debt financing can be raised may be less than the maximum discount rate for which equity financing can be raised. Intuitively, this happens because when $r_m^* > \phi$ a grace period is needed and this involves increasing subsequent coupons to compensate for lowering the initial coupon. Hence, the double moral hazard problem of debt becomes more severe. Hence, equity is (weakly) preferred when discount rates are relatively high while debt is (weakly) preferred when discount rates are low.

In comparing the growth implications of debt versus equity, we see that debt promotes the creation of growth opportunities, possibly at the expense of efficiency once the firm is up and running. The reason is that debt allows the entrepreneur to entrench himself and therefore encourages him to do R&D in the first place. In contrast, equity promotes the implementation of improvements, possibly at the expense of the creation of growth opportunities. In sum, debt favours first stage growth; equity favour second stage growth.

As discussed in Section 2, our model delivers a result which bears superficial resemblance to Jensen's [1986] free cash flow hypothesis. In particular, debt tends to have an

[16] Fluck [1999] argues that an advantage of the unconditional control rights of equity is that it allows investors to punish managers for things like not maintaining plant and machinery to an appropriate standard.

advantage over outside equity when outside managerial moral hazard is relatively large ($y_2 < \pi_1/\pi_2$). However, we reach this conclusion in a dynamic analysis that allows us to model outside equity and which highlights the importance of control rights in financial contracting.[17]

To what extent are our results driven by the assumption that debt structure is complex? To address this, suppose instead that debt structure is "simple", in the sense that creditors are sufficiently coordinated to negotiate with equityholders and to take over the firm upon default and run it as a going concern. More precisely, if the firm defaults creditors can either (i) do nothing, (ii) liquidate, or (iii) take over the firm and keep it going under (possibly) new management. The general conclusion here is that simple and complex debt are approximately the same. They differ only if the entrepreneur has a chance of being retained as manager upon default. When this is not the case, the entrepreneur's and outside managers' debt servicing incentive compatibility constraints would be the same whether we are dealing with complex or simple debt, since the defaulting manager would not be retained by creditors. Thus when there are alternative outside managers as in our model, the important difference between debt and equity lies in the conditionality of control rights under debt – not in the hardness of the claim.

As we have already noted, our model does not state how a particular equity payout policy will be determined. It may be that the prevailing degree of moral hazard in the economy reflects cultural factors or, as has been stressed by La Porta et al [1998], legal factors. If so, our model tell us something about the kind of contracting that will arise. When managerial moral hazard is great, we would expect debt contracts to be relatively common and outside equity finance relatively uncommon. In contrast, when managerial moral hazard problems are less severe, then outside equity may be relatively more widespread. Some readers may see here the makings of an explanation for why equity markets have recently emerged in some developing economies where previously they did not exist. In addition to the explanation that liberalization may have removed administrative obstacles, our analysis here suggests an alternative explanation. Specifically, the hypothesis would be that institutional changes have provided shareholders greater means of disciplining managers and that financing shifted from debt to equity as a result. Exploring this idea would take us very far from our analytical framework, so we do not pursue it here.

7 Extensions

7.1 Alternative Financial Contracts

We feel that our study of debt and equity contracts has been useful in demonstrating inefficiencies that arise in dynamic agency models and their implications for firm growth. This is not meant to say that alternative financial contracts cannot alleviate these inefficiencies in some contexts. Indeed, given our understanding of the structure of our model it is an interesting exercise to see what some more complex contract can

[17] Of course, the importance of conditional control rights has been emphasized in the static, incomplete contracts literature, for example, by Aghion and Bolton [1992].

achieve. For example, a problem with outside equity that has not been resolved by debt is the fact that minimum moral hazard [$y_2 = 1/(1+r)$] cannot be enforced. As seen in Figure 2, there is a wide range of equilibrium payout rates also under debt financing. Here, we ask the question whether there is some contractual modification that can simultaneously eliminate the time inconsistency and fragility problems of outside equity, guarantee minimum moral hazard, *and* eliminate the double moral hazard problem of debt. The answer is "yes"; this can be achieved by doing a leveraged buyout (LBO) at date 1. By selling his equity stake to an outside manager and taking debt in return, the entrepreneur creates a single layer of agency. Furthermore, by setting the debt at a sufficiently high level, the entrepreneur can assure minimum managerial moral hazard under an outside manager.

The LBO would work as follows: The firm initially issues debt at date 0^+ which raises I. Denote the contractual debt service of this debt by d_0^s and d_1^s. Then $d_1^s = (I - d_0^s)r$. At date 1, the entrepreneur hires an outside manager and simultaneously exchanges his equity for (junior) debt and gives all the equity to the new manager. Let d_1^j denote contractual debt service on the entrepreneur's (junior) debt. Minimum moral hazard can be forced by setting: $d_1^j = \frac{\pi_2}{1+r} - d_1^s$. Thus, the total debt outstanding as of date 1 will be π_2/r. This will maximize cash flows to the entrepreneur, subject to incentive compatibility, and therefore maximizes his incentives to do R&D.

This benefit from an LBO in our basic framework should not be viewed as a statement about the general optimality of LBO's. If the model were enhanced to be more realistic, we would see that there are countervailing costs associated with that financial structure. The important point is that the fundamental inefficiencies we have identified will continue to be present in these more realistic settings.

7.2 Uncertainty

Probably the most obvious limitation to our analysis is that we have worked in a perfect foresight setting. For example, it is clear that stochastic cash flows could add an important disadvantage to complex debt because unexpectedly low cash flows may lead to the liquidation of the firm. This might create a role for simple debt structures, since the conditionality of control rights would be retained while introducing the flexibility to handle temporary downturns in business conditions.

7.3 Shocks

Short of a full analysis of uncertainty it is interesting to step back slightly from the perfect foresight setting we have adopted in our analysis and ask what is the effect of a shock to the parameters of the model. In particular, imagine that the firm is operating under entrepreneurial management, the investors have just decided to retain the entrepreneur one more period, but that this period's cash flow has not yet been realised. Suppose at this time there is a discovery of a technological improvement which could only be implemented by outside managers. That is, outsiders' productivity is increased to $\pi_2^* > \pi_2$. If under this new condition it is clear that shareholders will replace the entrepreneur, his reaction will be to consume all the current period's cash flow. It is interesting to note that this may *reduce* the value of shares immediately

upon the announcement of the technical improvement. For the value of shares just prior to the announcement is

$$\frac{y_2\pi_2(1+r)}{r}.$$

Just after the announcement it is

$$\frac{y_2\pi_2^*}{r}.$$

Next period its value will be

$$\frac{y_2\pi_2^*(1+r)}{r}.$$

If $\pi_2(1+r) > \pi_2^*$ share prices fall initially upon the announcement and then subsequently rise to a new higher level when finally the improvement is put into place.[18]

8 Conclusion

Despite a number of important contributions to the theory of dynamic capital structure, we are still looking for a coherent vision of how financial structure evolves over the life of the firm in response to growth opportunities and to changing managerial needs. Our contribution has been to study the two-way causality between the creation of growth opportunities and financial contracting. Through our model we have told a story which we believe repeats itself continually in many firms in many economies. The main features of this story are as follows. At some point insiders in the firm perceive the possibility of developing a growth opportunity. If they pursue this and have a new product idea in hand, they would need to change the firm by calling upon some outsiders to provide additional finance for capital investments and other outsiders to provide managerial expertise. Once the new direction would be taken, the initial insiders may find themselves increasingly alienated from the firm they have created. At some point it may become apparent that they lack the vision or the clout to shape the way the new firm should develop. They are as Coriolanus who, after leading the Volscian forces in imposing the new order on Rome, is effaced in a maneuver which exceeds his understanding.

Financial contracts are an important part of the story notably because they can determine who will retain effective control of the firm along the path that is pursued. A contract that allows initial insiders to entrench themselves cuts two ways with respect to the growth of the firm. On the one hand it encourages growth by allowing the initiators of a new product idea to reap for a longer time the benefits accruing to insiders. On the other hand, the entrenchment of an earlier generation of insiders may impede the introduction of improvements that are needed to allow a product to attain its full commercial potential.

Our model has been kept simple. The initial insiders are represented as a single entrepreneur who has the opportunity of creating a product idea for sure if he

[18]This feature of our model corresponds to the argument of Hobijn and Jovanovic [1999] in their analysis of the evolution of the stock market since the 1970's. Their thesis is that the prospect of new information technology many incumbent firms who were ill-placed to benefit from new technologies adopted defensive strategies which reduced values of traded shares. Only later as these firms were restructured or new firms entered the stock market did the stock market take off.

undertakes a costly R&D project. Later, given the project is up and running, the product can be "improved" by replacing the entrepreneur by a more skilled outside manager. Moreover, we have worked in a framework where cash flows are completely non-contractible. We have shown that under equity financing there is substantial outside pressure on the entrepreneur, since he has to match the expected dividends of the more skilled outside managers in order to retain his job. This pressure leads to a time inconsistency problem and reduces the entrepreneur's incentive to R&D. Debt resolves the time inconsistency problem and enhances the incentive to do R&D because debt gives investors control rights only in the event of default, thus reducing the outside pressure on the entrepreneur.

Another problem with the unconditional control rights of equity is that it may foster a vicious circle where managers pay no dividends and investors liquidate. Debt financing breaks this vicious circle because managers need not fear liquidation, provided that they service debt. Balanced against these advantages, the main drawback of debt is that it tends to stifle second stage growth. Debt may also introduce a double moral hazard problem, which reduces the range of discount rates for which it is possible to obtain debt financing relative to the range for which it is possible to obtain equity financing.

It would be interesting to enrich the analysis in any of a variety of ways. We have already mentioned that it would be desirable to introduce uncertainty about cash flows or other aspects of the project. Not only would this be important in better assessing the relative benefits of outside equity, debt or other financial contracts; but it would establish a link to contingent claims valuation (see Anderson and Sundaresan [1996]). Another interesting line of research would be to model the creation of growth opportu-

A Appendix

Proof of Lemma 1
All that remains to show is subgame perfection, the claim regarding non-stationary strategies, and the existence of liquidation equilibria.
Subgame Perfection
First, since investors suffer no loss in payoff from replacing incumbent management, they have a credible threat to fire the incumbent if he should choose a payout ratio below y_2. The incumbent therefore cannot benefit from paying a lower dividend than $y_2\pi_2$ (when (4) holds). Second, since there is an infinitely deep pool of alternative managers, the principle of induction establishes that the alternative manager also cannot do better than by playing the same strategy as the incumbent; the alternative manager is only placed in charge if the incumbent deviates, moreover, once this happens, the alternative simply becomes the new incumbent and is indistinguishable from the previous manager. Third, since outside managers use the same strategy, investors cannot gain by replacing the incumbent when he adheres to his strategy by paying out $y_2\pi_2$. This establishes subgame perfection.
Non-Stationary Strategies
We want to show that if there is a going concern equilibrium where outside managers use a strategy with time variant payout ratios $\{y_{2t}\}$, then there is also an equilibrium with a time invariant payout ratio, $\bar{y}$, which leaves all players equally well off. In the non-stationary going concern equilibrium, at date τ, the overall payoff to the incumbent manager is

$$\sum_{t=\tau}^{\infty} \frac{(1-y_{2t})\pi_2}{(1+r)^{t-\tau}} \quad ,$$

the overall payoff to outside shareholders is

$$\sum_{t=\tau}^{\infty} \frac{y_{2t}(1-\gamma)\pi_2}{(1+r)^{t-\tau}},$$

and the overall payoff to the alternative manager is zero. These payoffs are identical to the payoffs from a time-invariant payout ratio of

$$\bar{y} = \sum_{t=\tau}^{\infty} \frac{y_{2t}}{(1+r)^{t-\tau}} \quad .$$

Therefore, if the incentive compatibility constraints are satisfied under the time-invariant payout ratios at date τ, they are also satisfied under the constant payout ratio, $\bar{y}$ at date τ and at every other date. This is what we wanted to show, since an equilibrium where some outside managers use different strategies and the incumbent will be replaced at some point makes the incumbent's incentive compatibility constraint more stringent, leading to a threshold discount rate which is less than r^*.
Liquidation Equilibria Finally note that if investors always liquidate then a best response for the manager is to pay out nothing, and *vice versa*. It is straightforward that these strategies are subgame perfect. □

Proof of Lemma 2
Suppose the entrepreneur uses a strategy under which

$$y_{1t} = \begin{array}{ll} y_1 & \text{if } t = 0^+ \text{ or } [\ y_{1\tau} \geq y_1\ \forall \tau < t] \\ 0 & \text{if } y_{1\tau} = y_1 \text{ for some } \tau < t. \end{array} \tag{28}$$

Since the firm is not viable as a going concern under an outside manager, the best reply of investors is to choose

$$s_{t+1} = \begin{array}{ll} \text{retain} & \text{if } y_{1t} \geq y_1 \text{ and } y_1 \geq rL/(1+r)\pi_1 \text{ and } [\ y_{1\tau} \geq y_1\ \forall \tau < t] \\ \text{liquidate} & \text{if } y_1 < rL/(1+r)\pi_1 \text{ or } [y_{1\tau} = y_1 \text{ for some } \tau < t.] \end{array} \tag{29}$$

Given (29), at any time the entrepreneur prefers paying out y_1 forever as compared with paying out nothing and bringing about the liquidation of the firm provided that

$$(1-y_1)\pi\frac{1+r}{r} + \gamma y_1 \pi \frac{1+r}{r} \geq \pi + \gamma L,$$

which simplifies to

$$y_1 \leq \frac{1 - \frac{\gamma L r}{(1+r)\pi}}{(1-\gamma)(1+r)}. \tag{30}$$

In other words, (28) and (29) are best replies to each other if and only if (30) holds. Observe now that RHS(30) is decreasing in r while $rL/(1+r)\pi_1$ is increasing in r, and there is y_1 which simultaneously satisfies (30) and $y_1 \geq rL/(1+r)\pi_1$ if and only if $r \leq R(\gamma)$, where $R(\gamma)$ is implicitly defined by

$$\pi_1/L = R(\gamma)(1-\gamma) + \gamma R(\gamma)/(1+R(\gamma)). \tag{31}$$

From this expression, we see that $R(0) = \pi/L$, $R(1) = \pi/(L-\pi)$, and

$$R'(\gamma) = \frac{R(\gamma)^2(1+R(\gamma))}{\gamma(1-\gamma)} > 0.$$

We have shown that there is y_1 such that (28) and (29) are best replies to each other and the firm is not liquidated when these strategies are employed if and only if $r \leq R(\gamma)$. Subgame perfection follows by inspection of these strategies, and the claim regarding non-stationary strategies and the existence of liquidation equilibria follow along the same lines as in the proof of Lemma 1. □

Proof of Theorem 1
Part (1): Most of the theorem is already established in the text. To complete the proof, suppose that y_2 is such that the firm is viable as a going concern under an outside manager. Substituting in the expression for γ_e into (8), we see that there is an entrepreneurial equilibrium if $r \leq r_e^*$ and

$$y_2 \leq \frac{\pi_1}{\pi_2} - \frac{r^2 I}{(1+r)\pi_2}.$$

Next, substituting in the expression for γ_m into (8), we see that there is a managerial equilibrium if $r \leq r_m^*$ and

$$y_2 \geq \frac{\pi_1}{\pi_2} - \frac{r^2 I}{\pi_2}.$$

Part (1) of the theorem now follows by the discussion in the text.
Part (2): Denote the largest discount rate for which financing can be obtained by $r_{\max}$. To find $r_{\max}$, Lemma 2 shows that we need only consider strategies where the equilibrium payout rate is time invariant. Denote it by y_1. When the firm is kept alive forever by the entrepreneur under this payout ratio, the shareholding of the entrepreneur is

$$\gamma = 1 - \frac{rI}{y_1\pi_1(1+r)}. \tag{32}$$

In the remainder of this proof, γ will be taken to be defined by this equation. Notice that γ is decreasing in r and increasing in y_1. Hence $r_{\max}$ is the largest r for which there is y_1 such that $\gamma(y_1, r) \geq 0$ and the firm can be maintained as a going concern under the entrepreneur after financing has been raised given r and y_1. We want to show that $r_{\max} = \pi_1/I$. Recall from Lemma 2 that the threshold discount rate for which the firm can be kept alive after financing has been raised, $R(\gamma)$, is increasing in γ. Hence, $r_{\max}$ can be found by setting $\gamma(r_{\max}, y_1) = 0$ when y_1 is at its largest incentive compatible level, provided that $r_{\max}$ calculated this way is less than $R(0) = \pi_1/L$ (see Lemma 2). Now, from (30), when $\gamma = 0$, the largest incentive compatible y_1 is $1/(1+r)$. Using this in (32) confirms that $r_{\max} = \pi_1/I$, since $L \leq I$.
Part (3): This follows immediately from the last statement in Lemma 2. □

Proof of Theorem 2

Part 1: This follows immediately by the discussion in the text and Lemma 3.
Part 2: The case that $r \leq \phi$ follows from the discussion in the text. So suppose $r > \phi$. In this case, Lemma 3 tells us that a grace period is necessary. Thus, set $d_0 = \pi_1$ and $d_1 = (I - \pi_1)r$. (26) then implies that there is y_2 for which it is incentive compatible to service this debt when an outside manager is in place if and only if $r \leq \phi^*$, where ϕ^* is implicitly defined by

$$\phi^*(1+\phi^*)^2 = \frac{\pi_2}{I - \pi_1}.$$

By the observation immediately after equation (27), the entrepreneur's incentive compatibility constraint for servicing debt prior to the appointment of the outside manager will also be satisfied since $d_0 < d_1$.

To conclude the proof of *Part 2* we need to verify that $\phi < \phi^* < r_m^*$. By definition, r_m^* is such that $q(r_m^*) = g(r_m^*)$, where $q(r) \equiv 1/(1+r)$ and $g(r) \equiv Ir/\pi_2$. By definition, ϕ^* is such that $q(\phi^*) = h(\phi^*)$, where $h(r) \equiv (I - \pi_1)r(1+r)/\pi_2$.

First, $g(r)$ is strictly increasing and linear and $h(r)$ is strictly increasing and strictly convex. Furthermore, $g(0) = h(0) = 0$ and $g(\phi) = h(\phi)$. This implies that for all $r > \phi$, we have $h(r) > g(r)$.

Second, $q(r)$ is strictly decreasing and $q(\phi) > g(\phi) = h(\phi)$ (since $\phi < r_m^*$). It follows that the point of intersection between $h(r)$ and $q(r)$, at $r = \phi^*$, is larger than ϕ. It also follows that ϕ^* is smaller than the r at which $g(r)$ intersects $q(r)$. In other words, $\phi^* < r_m^*$.
Part 3: Follows directly from the discussion in the text. □

Legend for Figure 1

a Outside manager's incentive compatibility constraint

$$y_2 \leq \frac{1}{1+r}$$

b Investors' incentive compatibility constraint (do not liquidate outside manager)

$$y_2 \geq \frac{Ir}{(1+r)\pi_2}$$

c Financing provided in a managerial equilibrium

$$y_2 \geq \frac{Ir}{\pi_2}$$

d Feasibility of entrepreneur matching dividends of outside manager

$$y_2 \leq \frac{\pi_1}{\pi_2}$$

e Entrepreneur prefers to match dividends if $\gamma = \gamma_e$

$$y_2 \leq \frac{\pi_1}{\pi_2} - \frac{Ir^2}{(1+r)\pi_2}$$

f Entrepreneur prefers not to match dividends if $\gamma = \gamma_m$

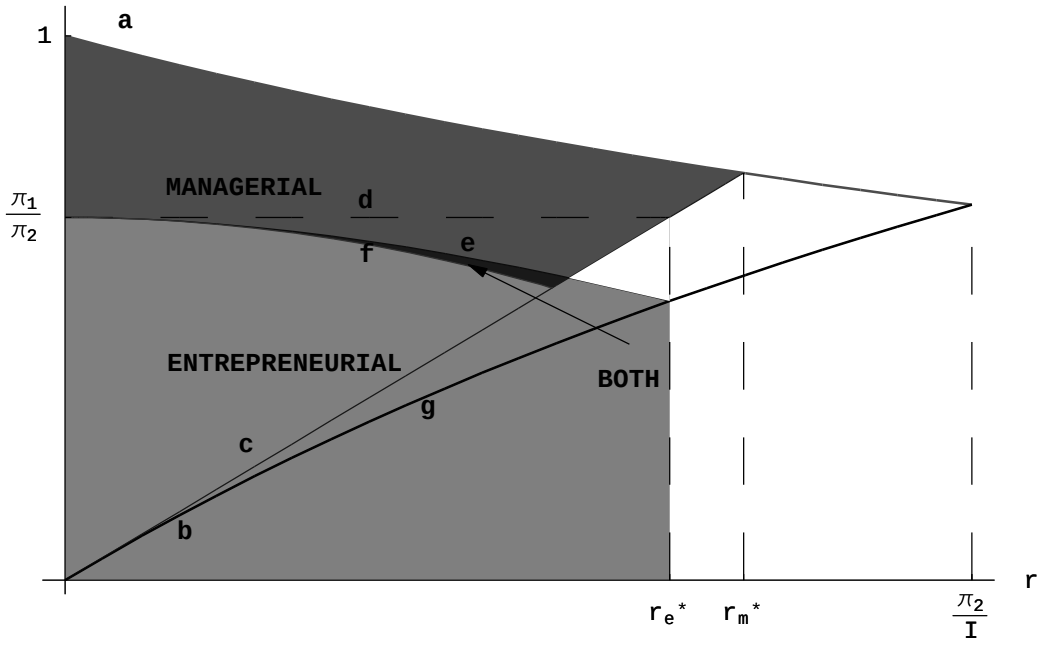
$$y_2 \geq \frac{\pi_1}{\pi_2} - \frac{Ir^2}{\pi_2}$$

g Financing provided in entrepreneurial equilibrium (assuming firm can be kept alive by outside manager)

$$y_2 \leq \frac{Ir}{(1+r)\pi_2}$$

This uses $y_1\pi_1 = y_2\pi_2$ and is functionally equivalent to constraint **b**.

Figure 1: Equilibria under Equity Financing



Legend for Figure 2

a Outside manager's incentive compatibility constraint

$$y_2 \leq \frac{1}{1+r}$$

b Financing in a managerial equilibrium without a grace period

$$y_2 \geq \frac{Ir}{\pi_2}$$

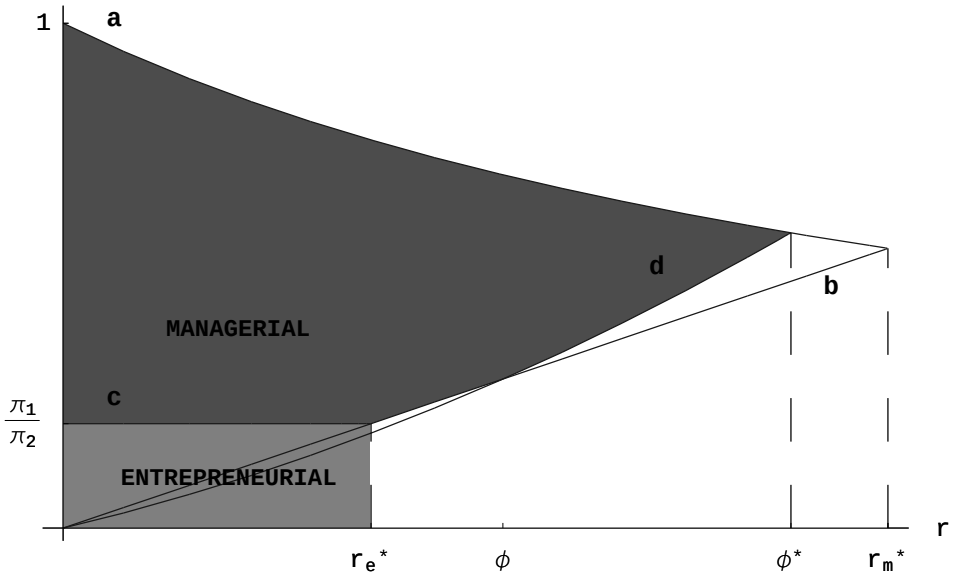
c Feasibility of entrepreneur matching dividends of outside manager

$$y_2 \leq \frac{\pi_1}{\pi_2}$$

d Financing provided in a managerial equilibrium with a grace period (and $d_0 = \pi_1$)

$$y_2 \geq \frac{(I-\pi_1)r(1+r)}{\pi_2}$$

Figure 2: Equilibria under Debt Financing



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