FOREIGN EXCHANGE INTERVENTION: HOW TO SIGNAL POLICY OBJECTIVES AND STABILISE THE ECONOMY

Paolo Vitale[*]
London School of Economics and CEPR[†]

Abstract

Within a simple model of monetary policy for an open economy, we study how foreign exchange intervention may be used as a costly signal of the policy makers' objectives. Our analysis indicates that: i) foreign exchange intervention typically stabilises the national economy, reducing the fluctuations of employment and output; ii) this result is sensitive to the institutional structure of decision-making, in that a larger stability gain is obtained when foreign exchange intervention and monetary policy are kept under the jurisdiction of different governmental agencies.

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[*]Correspondence should e addressed to Paolo Vitale, Department of Accounting and Finance, London School of Economics, Houghton Street, London WC2A 2AE, UK; tel.: +44-(0)20-7955-7230; e-mail: p.vitale@lse.ac.uk.

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1 Introduction

In a recent survey by Neely (2000) most central bankers agreed with the thesis that foreign exchange intervention has an impact on the market for foreign exchange. This thesis is reinforced by some recent empirical research, notably Dominguez and Frankel (1993a, 1993b), Fischer and Zurlinden (1999) and Payne and Vitale (2001), on the effectiveness of foreign exchange intervention, which suggests that sterilised purchases and sales of foreign currencies on the part of central banks influence exchange rates. However, there is no consensus among practitioners and researchers on what foreign exchange intervention can achieve.

In Neely's survey 47 percent of the respondents claimed that foreign exchange intervention is aimed at resisting short-term trends, 22 percent suggested that its main goal is to eliminate misalignements from fundamental values, while the rest indicated different, unspecified reasons for intervention. Likewise, in the academic arena several theses have been suggested: foreign exchange intervention may indicate future changes in the monetary policy (Mussa, 1981), new target levels for the exchange rates (Bhattacharya and Weller, 1997, Vitale, 1999), and may also be used to reduce market instability and smooth exchange rate movements (Dominguez and Frankel, 1993a).

We go beyond the analysis of the simple effects of foreign exchange intervention in the market for foreign exchange and look at its more general macroeconomic implications. We argue that: i) foreign exchange intervention may represent an independent instrument of policy making, as it can be an effective channel through which policy makers can *signal* their objectives, reduce agents' uncertainty on future policy decisions and hence stabilise the national economy; ii) the institutional arrangements governing the instruments of policy making influence such a stability gain, for this is more pronounced when foreign exchange intervention and monetary policy fall under the jurisdiction of two different governmental agencies.

To highlight the signalling role of foreign exchange intervention and present its macro-level effects, we propose a model which combines the classical macroeconomic approach to monetary policy with the more recent market microstructure approach to exchange rates. Indeed, a recent popular view among researchers, notably Frankel *et al.* (1996), Lyons (2001) and Evans and Lyons (2002), is that the traditional asset market approach fails to recognise the role that the structure of the foreign exchange market, with its trading protocols and organisation, plays in the determination of exchange rates.

In this respect our contribution offers a first formal analysis of the interplay between the microstructure of the foreign exchange market and its macro-level implications. We propose a market microstructure framework for the market for foreign exchange that presents two important features:

i) it allows identification of a clear *link* between the intervention operations of central banks, agents' expectations, exchange rates and employment; ii) it produces a series of testable implications which are consistent with a large body of empirical evidence concerning the statistical relations between intervention operations, exchange rates and monetary aggregates.

Through this market microstructure framework we are then able to analyse the macro-level effects of foreign exchange intervention. In particular, within a monetary policy model *à la* Barro and Gordon we see that the policy makers attempt to raise employment above its natural level with unexpected monetary shocks. If individual agents are rational and fully understand the motives of the policy makers, this attempt fails whilst an inflationary bias emerges. On the other hand, if there is incomplete information on their objectives, the policy makers might be able to stimulate the economy or reduce the inflationary bias if they can manage to manipulate agents' beliefs and mis-represent their objectives.

Manipulating agents' beliefs may be difficult. In fact, precise public announcements on the policy objectives will lack credibility, as the policy makers possess an incentive to lie, and cannot be used. Imprecise statements can instead be credibly employed, but since different announcements are *per se* all the same, in the sense that they are all equivalently *inexpensive*, multiple signals of different informative quality can be used at the same time. On the contrary, we see that this problem of the multiplicity of the equilibria does not emerge if a potentially *expensive* signalling mechanism is employed. We show that foreign exchange intervention i) permits the policy makers to obtain some degree of credibility, since the central bank stakes its own wealth in support of its signals, and ii) delivers a uniquely informative message.

While the policy makers will try to exploit this signalling channel to mis-represent their objectives and obtain a policy gain, in equilibrium their attempt to eliminate the inflationary bias and simultaneously stimulate the domestic economy is frustrated. And yet, the policy makers will benefit from foreign exchange intervention (even though not in the way they hoped for) as this helps to stabilise the economy by reducing the volatility of the employment level.

Our analysis also shows that in equilibrium the policy makers exploit their informational advantage and gain profits from their intervention operations. However, we see that there exists a trade off between these profits and the stability gain foreign exchange intervention brings about. This conclusion is reinforced when we compare different institutional arrangements concerning the division of power over monetary policy and foreign exchange intervention. We find that full coordination of these two instruments of policy making will result in larger profits at the expense of a more volatile employment level.

Therefore, a normative implication of our analysis is that foreign exchange intervention and monetary policy should be kept separate to reduce the instability of the economy. This is a quite novel and important conclusion, in that the institutional arrangements governing these instruments of policy making differ from country to country and have been completely overlooked in the existing monetary policy literature.

The paper is organised as follows. In section 2 we present a simple model of monetary policy for an open economy. We discuss the usual problem of time consistency and show that under incomplete information the policy makers may benefit from the manipulation of the agents' beliefs. In section 3 we present a market microstructure analytical framework for the analysis of the foreign exchange market. In the following section we characterise the equilibrium of the model discussing the stabilising effects of foreign exchange intervention on the employment level. In section 5 we extend our analysis investigating the effects of different institutional arrangements and considering some comparative analysis exercises. In section 6 we propose some concluding remarks. In the appendix we present brief proofs of the Propositions of the paper.

2 Monetary Policy in an Open Economy

We introduce a simplified model of monetary policy for an open economy as a basic set-up for the analysis of the relation between foreign exchange intervention monetary policy, and macroeconomic stability.[1]

2.1 A basic set-up

Let us consider two countries, a domestic economy and a foreign one, and the corresponding nominal exchange rate. We assume that the nominal exchange rate reacts to monetary injections in the home country according to the following formulation

$$s_t = m_t, \tag{1}$$

where s_t indicates the log of the nominal exchange rate between the home and foreign country, i.e. the units of domestic currency required to purchase one unit of the foreign one, and m_t denotes the log of the money supply in the home country in period t. Not only can a monetary injection create inflation and depreciate the national currency, but it might also stimulate the domestic economy increasing national employment and output. In particular, we suppose that

$$n_t = m_t - w_t, \tag{2}$$

[1] This model is a special case of the analytical framework proposed y Canzoneri and Henderson (1991).

where n_t denotes the log of the employment level and w_t denotes the log of the nominal wage in the domestic economy. In period t the nominal wage is negotiated before the monetary injection takes place. The wage setters will set it equal to the expected value of the money supply,

$$w_t = m_t^e, \tag{3}$$

in an attempt to stabilise the domestic employment level. Then, the domestic policy makers might attempt to push employment beyond its natural level, normalised to 0, via unexpected monetary shocks and inflation.

As common in the monetary policy literature, we suppose that the natural level of employment is suboptimal due to some distortion in the economy. Unexpected inflation may also be desirable *per se* as it guarantees a seignorage tax and reduces the cost of servicing government debt. In practice, we suppose that there is an incentive on the part of the government to surprise the wage setters and inflate the economy. However, the policy makers also pursue the complementary goal of stabilising the price level or, equivalently, the nominal value of the domestic currency.

In order to capture the tension between these two complementary goals we assume that in the home country the term of office of the policy makers is confined to one period and that they will then minimise the expected value of the following loss function

$$\mathcal{L}_p = (n_1 - \bar{n})^2 + (s_1 - \bar{s})^2, \tag{4}$$

where $\bar{n}$ and $\bar{s}$ represent optimal values for the employment level and the exchange rate.

The particular choice of the policy makers' loss function is inconsequential for the results of our analysis. We could use alternative specifications, such as those put forward by Barro and Gordon (1983), Canzoneri (1985), Cukierman and Meltzer (1986), where the nominal exchange rate is replaced by the inflation rate or the price level. The conclusions of our analysis would not change.[2]

We can easily justify the particular choice we have made. Many small or developing countries, notably Argentina, Mexico and other Latin American countries, use or have used the exchange rate as a nominal *anchor* to stabilise their domestic economies. The recent experience of the European Monetary System also suggests that even large industrialised economies have used this nominal anchor to fight inflation. Japan and the United States have also tried to target the value of the dollar in the 1980s in order to stabilise their terms of trade.[3] Finally, United Kingdom may decide

[2] It is not difficult to show that simple linear mappings exist etween the nominal varia les in Canzoneri and Henderson's analytical framework. This means that we could easily reformulate our loss function in terms of the price level.

[3] Agenor and Montiel (1996) offer detailed accounts of the sta ilisation programmes attempted in Latin America

to join EMU in the near future. In this case the British government would probably target the nominal exchange rate of the pound before a conversion rate were negotiated.

2.2 Incomplete information and the manipulation of agents' beliefs

When the loss function of the policy makers, $\mathcal{L}_p$, is common knowledge the values for the money supply and the nominal wage in a *discretionary* equilibrium are

$$m_1 \;=\; w_1 \;=\; \bar{s} \,+\, \bar{n}, \tag{5}$$

while the equilibrium value of $\mathcal{L}_p$ is

$$\mathcal{L}_p \;=\; \mathbf{l}_c \;\equiv\; 2\bar{n}^2. \tag{6}$$

As usual, under discretion the condition of time consistency induces an *inflationary bias*, $\bar{n}$, whilst the policy makers are totally unsuccessful in their attempt to raise the employment level. In this case the policy makers would be better off if they could commit to a non-inflationary policy where the monetary supply were fixed at $m_1 = \bar{s}$. They would still be unable to stimulate the economy, as n_1 would remain at its natural level zero, but at least the exchange rate would converge to the target level, $\bar{s}$.

In this context foreign exchange intervention cannot play any role, *unless* we consider some form of asymmetric information. Indeed, we can claim that in general the policy makers possess various forms of private information. This often stems from incomplete information on their preferences, as economic agents might not know what levels of employment and inflation the policy makers prefer or there can be uncertainty on their *type*. For instance, in Barro (1986) and Cukierman and Meltzer (1986) agents do not know how strongly the policy makers are committed to fighting inflation.

Here we follow Stein (1989) in assuming that there is incomplete information on the target level for the exchange rate, $\bar{s}$. In particular, we assume that the wage setters do not know the exact value of the target level. They just know that $\bar{s}$ is drawn from a normal distribution with mean $\bar{s}^e$ and variance at the beginning of the term of office. No uncertainty exists on the target value for the employment level, $\bar{n}$, while the distribution of $\bar{s}$ is common knowledge and so is the form of the loss function.

We can justify this particular formulation of incomplete information on the basis of past experience. In fact, in the 1980s reference values were set for the exchange rates of the main currencies

in the 1970s and 1980s, while Giavazzi and Pagano (1988) discuss the anchoring of France and Italy to the German monetary policy in the 1980s through the European ERM. Funa ashi (1988) discusses the attempts of the G-5 to target the exchange rates of the dollar, the deutsche mark and the yen in the 1980s.

by the G-5, but were neither officially endorsed nor publicly announced.[4] We could also argue that the British government would desire to target the pound/euro nominal exchange rate before joining EMU, without having the possibility of announcing its future plans.

Under this new assumption we find that the wage setters are not capable of predicting the exact value of the monetary injection. In a discretionary equilibrium the nominal wage and the money supply are now as follows

$$w_1 = \bar{s}^e + \bar{n}, \tag{7}$$

$$m_1 = \bar{s} + \bar{n} - \frac{1}{2}(\bar{s} - \bar{s}^e), \tag{8}$$

where $\bar{s}^e$ denotes the expectation of the target level conditional on the information of the wage setters in period 1. The prediction error of the wage setters, $\bar{s} - \bar{s}^e$, affects the equilibrium value of the policy mak

Only in the special case in which $\bar{n}$ is equal to zero can the policy makers credibly announce the true value of the target level for the spot rate, $\bar{s}$. They would not have an incentive to lie, as the two goals of stimulating the economy and stabilising the nominal exchange rate would not be in conflict. For $\bar{n} \neq 0$, in contrast, the policy makers possess an incentive to mis-represent their preferences and a precise statement on their part could not be believed by the wage setters.

While, as suggested by Stein (1989), some imprecise statements could still be used to convey credible signals to the market, we focus on an alternative mechanism of communication based on foreign exchange intervention. Indeed, operations in the foreign exchange market by a central bank may be used to "buy credibility" and convey signals to the market, in that the central bank stakes its own capital in support of the signal. We are now going to see how this is possible: first we will see how foreign exchange intervention can be modelled and then we will study its macroeconomic consequences.

Before introducing foreign exchange intervention we conclude this section by deriving the unconditional variance of the employment level and the unconditional expectation of the policy-makers' loss in the equilibrium *without* intervention. These values will then be used as yardsticks to study the effects of foreign exchange intervention. Simple calculations give the following values

$$\mathrm{Var}_{\emptyset}\left[n_1\right] = \frac{1}{4} , \tag{9}$$

$$E_{\emptyset}\left[\mathcal{L}_p\right] = 2\bar{n}^2 + \frac{1}{2} , \tag{10}$$

where henceforth the subscript $\emptyset$ refers to the equilibrium without foreign exchange intervention.

3 Foreign Exchange Intervention

According to Mussa (1981) operations in the foreign exchange market by a central bank may be used to signal future changes in the monetary policy. Purchases (sales) of foreign exchange should signal a forthcoming monetary expansion (contraction) more effectively than a simple announcement, because the central bank stakes its own capital in support of the future policy. In fact, when a sale of foreign assets is followed by a monetary expansion that forces a devaluation of the domestic currency, the central bank incurs a net loss.

In our context we argue that the policy makers might attempt to manipulate the wage setters' expectations of the target level, $\bar{s}$, and of the money supply, m_1, before the nominal wage, w_1, is set, by buying or selling the foreign currency in the foreign exchange market. In order to show how this "signalling mechanism" can be exploited we need to introduce an analytical framework for foreign exchange intervention. To do so let us see how foreign exchange intervention operates.

3.1 The practice of foreign exchange intervention

The practice of foreign exchange intervention depends on the complex and evolving structure of the market for foreign exchange. This is a fragmented dealership market, where bilateral direct inter-dealer transactions, conducted on the phone or via electronic communication systems, coexist with brokered ones, mediated by brokerage firms or via electronic brokerage services. This implies that when undertaking an intervention operation a central bank can either trade via a broker or place individual orders directly with one or more dealers.

In the past the Federal Reserve would have preferred the latter route when maximum visibility was desired, while the former would have been favoured whenever secrecy was required. These days, however, electronic limit order books have largely replaced brokerage firms and hence the Federal Reserve splits its activity between EBS, the main electronic brokerage service in the United States, and direct trading with many different dealers. Other central banks use similar practices.

Despite the massive size of the foreign exchange market, its fragmented structure allows a central bank to operate with individual dealers and influence their quotes with small transactions, as the activity of single dealers remain within limited size. Furthermore, operating through different routes a central bank is able to provoke a wave of inter-dealer transactions that quickly spreads news of intervention in the market, as Chaboud and LeBaron (2001) find for the intervention activity of the Federal Reserve. The diffusion of news of foreign exchange intervention is so rapid that newswire services report intervention activity in the space of few minutes.

Other important features of foreign exchange intervention are that: i) no bank official makes a public statement announcing the *exact* size of any intervention operation, and ii) despite the recent consolidation brought about by electronic brokerage services, such as EBS and Reuters 2000-2, the market for foreign exchange remains fragmented and opaque. This means that, since most transactions are the result of private bilateral meetings between sellers and buyers, single dealers cannot observe *all* market orders and prices cannot immediately incorporate *all* private information contained in individual trades, implying that news of intervention will diffuse in the market with some noise, in the sense that the *average* dealer will estimate its exact amount imprecisely.

Reports of central bank intervention commonly appear in the press, but they are usually imprecise: sometimes central banks intervene and no report appears in the press or reports appear and intervention operations have not occurred;[5] likewise, quantities are rarely indicated and when

[5]Klein (1993) finds that in the 1980s the likelihood of intervention eing reported when it actually occurred was 72 percent, while the likelihood of intervention actually occurring when it was reported was 88 percent. Hung (1997) finds that in the same period 40 percent of intervention operations conducted y the Federal Reserve was not reported y either newspapers or newswire services.

they are the reported figures are way out from the actual ones and generally smaller. Moreover, as Chaboud and LeBaron (2001) suggest, these reports *neither* increase trading volume in the foreign exchange market *nor* influence exchange rates, while actual intervention operations *do*.

3.2 A market micro-structure framework for foreign exchange intervention

To capture these features of foreign exchange intervention in a formal model we suppose that the term of office is divided in two stages: 0 and 1. At the beginning of stage 0 $\bar{s}$ is realised. Again we assume that the unconditional distribution of $\bar{s}$ is normal with mean $\bar{s}^e$ and variance and that this information is common knowledge. At the beginning of stage 1 the nominal wage is set. After observing the value of w_1, the *domestic authorities* fix the money supply, m_1.[6]

We assume that: i) the domestic authorities may intervene in the market for foreign exchange both in stage 0, after the target level is realised, and in stage 1, after the nominal wage and the money injection have been selected, trading an undisclosed quantity of the foreign currency, I_τ ($\tau = 0, 1$); ii) in stage τ ($\tau = 0, 1$) a representative dealer will determine the equilibrium exchange rate, s_τ ($\tau = 0, 1$), right after the intervention operation of the domestic authorities.

Given the fragmented structure of the foreign exchange market and its *lack* of transparency, we suppose that this representative dealer cannot observe the actual value of I_τ, but just a noisy signal, $x_\tau = I_\tau + u_\tau$, where u_τ is normally distributed with mean 0 and variance σ_u^2. The noisy term, u_τ, is a proxy for the opaqueness of foreign exchange market and its variance, σ_u^2, can be considered a measure of the lack of transparency of this market.[7]

To establish the effect of the intervention operations we assume that the foreign exchange market is competitive, so that the representative dealer will set the exchange rate equal to the expected fundamental value of the foreign currency, f, given the information he obtains from the signal, x_τ,[8]

$$s_\tau \; = \; E[f \,|\, x_\tau], \quad \tau = 0, 1.$$

For simplicity we assume that the dealer and the wage setters share the same prior on the value of the target level, $\bar{s}$. In addition, the dealer can observe the nominal wage and the money supply before fixing the spot rate in stage 1, s_1. On the other hand, both the wage setters and the domestic authorities can observe the exchange rate fixed by the dealer in stage 0, s_0, before selecting the nominal wage and the money supply in stage 1. In this way, at the end of stage 0 the dealer and the wage setters will *share* the same expectations of the target level, $\bar{s}$. Finally, to *isolate*

[6] Later on we discuss the details of the governance of monetary policy and foreign exchange intervention.

[7] See Vitale (1999) for a more extensive discussion of this analytical framework.

[8] We suppose that the two stages are close in time, so that we can disregard any discounting.

the signalling role of foreign exchange intervention, we assume that all purchases and sales of the foreign currency by the domestic authorities are automatically *sterilised* through compensatory open market operations, as it is generally the case in the practice of intervention.[9]

Under these assumptions the fundamental value of the foreign currency corresponds to the equilibrium exchange rate after the nominal wage and the money supply have been fixed. From equation (1) we know that this is equal to the money supply,

$$f = m_1.$$

The money supply, m_1, depends on the nominal wage set by the wage setters in stage 1. This in turn is a function of their expectations on the target level, $\bar{s}$. By trading in stage 0 the domestic authorities will be able to influence the expected value of $\bar{s}$ and hence the same fundamental value, f. This is an important conclusion, because it is commonly argued that, in the absence of portfolio balance effects, sterilised foreign exchange intervention cannot alter the fundamental value of the exchange rate.[10]

Since in stage 1 all uncertainty on the fundamental components of the exchange rate is resolved, foreign exchange intervention will not be useful and the policy makers will concentrate their intervention activity in stage 0. In Figure 1 we present a time line summary of the structure of the model.

To determine what the optimal intervention strategy is in stage 0 we need to analyse its ultimate goals. In this sense we need recognising that foreign exchange intervention can be expensive. Therefore, i) authorities which are responsible for foreign exchange intervention will be accountable for the possible losses intervention brings about and take them into account when selecting operations in the market for foreign exchange; ii) if a single authority controls both the money supply and foreign exchange intervention the potential cost of its intervention operations also affects the monetary policy.

To capture both the macroeconomic objectives of the policy makers and the cost of intervention we assume that a single *central authority* possesses full control on both monetary policy and foreign exchange intervention and acts on behalf of the policy makers. In this case, which we will refer as *of coordination* ($\mathcal{C}_o$), the intervention operation, I_0, and the monetary injection, m_1, are selected by the central authority in order to minimise the expected value of a modified loss function, $\mathcal{L}_a$.

[9]Henceforth for foreign exchange intervention we will mean sterilised intervention operations in the market for foreign exchange.

[10]Interestingly this fundamental value might e *indeterminate* if either no signalling equili rium exists or multiple signalling equili ria prevail in stage 0.

This modified loss function is defined as follows

$$\mathcal{L}_a = (n_1 - \bar{n})^2 + (s_1 - \bar{s})^2 + 2Q\,\mathcal{C}, \tag{11}$$

where Q is a positive constant measuring the weight attached to the cost of intervention, $\mathcal{C}$, which is equal to $(s_0 - s_1)I_0$.[11] Again, in the definition of this modified loss function we assume that its form and the value of Q are common knowledge.[12]

4 Equilibrium under Coordination

We now investigate the consequences of foreign exchange intervention: first, we analyse the sequential equilibrium of the model by characterising the unique signalling equilibrium that exists in the market for foreign exchange in stage 0; second, we discuss the macroeconomic implications of this equilibrium.

4.1 Signalling equilibrium under coordination

To determine the sequential equilibrium of this model we need to consider that:

1. Given the amount of foreign exchange intervention in stage 0, I_0, changes in the money supply in stage 1 affect the cost of intervention, $\mathcal{C}$, and the loss function of the central authority, $\mathcal{L}_a$. This means that the intervention operation undertaken in stage 0, I_0, conditions the choice of the money supply in stage 1, m_1, and hence has a *feed-back* effect on the monetary policy.
2. In stage 0 the dealer and the wage setters observe the signal x_0. Since this signal depends on the activity of intervention of the central authority in the market for foreign exchange, it conveys some information on the future monetary injection.

In stage 1 we solve for the discretionary equilibrium and find that the equilibrium values for the nominal wage, w_1, and the money supply, m_1, differ from those derived in section 2 for the

[11] Since we use natural logs, the difference $s_1 - s_0$ measures the log of the gross rate of return on the foreign currency etween stage 0 and 1, $\ln(1 + R)$. For R small we can use the approximation $\ln(1 + R) \approx R$, so that $\mathcal{C}$ is an approximation of the cost of intervention.

[12] The particular value taken y Q might well e very small, in that when intervening the central authority is not principally motivated y speculative purposes and may have access to large foreign exchange reserves. However, it could not sustain the political urden of infinite losses, a condition which is captured in the model y a value of Q strictly greater than zero.

equilibrium *without* intervention, as now equations (7) and (8) are replaced by the following

$$w_1 = \bar{s}_0^e + \bar{n} + Q\,I_0^e, \tag{12}$$

$$m_1 = \bar{s} + \bar{n} + Q\,I_0 - \frac{1}{2}(\bar{s} - \bar{s}_0^e) - \frac{Q}{2}(I_0 - I_0^e), \tag{13}$$

where $\bar{s}_0^e$ and I_0^e indicate respectively the expected value of the target level, $\bar{s}$, and the intervention operation, I_0, *given* the information the dealer and the wage setters possess at the end of stage 0.

Equation (13) indicates that when the monetary policy and foreign exchange intervention are coordinated the latter has a feed-back effect on the former through *two* different channels. First, we have an *expectations channel*, in that the signal observed in the foreign exchange market in stage 0 conditions the wage setters' expectations of the target level, $\bar{s}_0^e$, and hence the nominal wage, w_1, as indicated by the first term of equation (12). The central authority is then induced to revise the optimal value of the money supply, as shown by the fourth component of equation (13).

Second, we have a *cost-of-intervention channel*, in that the value of the money supply fixed in stage 1, m_1, determines the exchange rate in stage 1, s_1, and hence the cost of intervention, $\mathcal{C}$. Such cost alters the choice of the money supply, m_1, as indicated by the two extra terms in equation (13), QI_0 and $Q(I_0 - I_0^e)/2$, function respectively of the actual value of intervention and its unexpected component. The nominal wage, w_1, is also affected, as shown by the last term in equation (12).

By backward induction we can determine the optimal intervention strategy of the central authority and pricing policy of the dealer in stage 0 and hence complete the analysis of the sequential equilibrium of the model. In stage 0 the dealer and the central authority play a signalling game. The former seeks to extract all possible information on the target level, $\bar{s}$, contained in the signal he observes, x_0, in an attempt to correctly price the foreign currency. The latter intervenes in the foreign exchange market in an attempt to manipulate the expectations of the dealer and the wage setters. Indeed, the loss function of the central authority can be decomposed as follows

$$\mathcal{L}_a = \mathbf{l}_a \equiv \mathbf{l}_c - \mathbf{g}_a,$$

where the *gain* from manipulation is now given by the following expression

$$\mathbf{g}_a \equiv \left\{ 2\,\Delta\,\bar{n} + 2Q\,(\Delta - \bar{n})\,I_0 - \frac{1}{2}\Delta^2 - Q^2\,I_0^2 \right\},$$

$$\text{with} \qquad \Delta \equiv (\bar{s} - \bar{s}_0^e) + Q\,(I_0 - I_0^e).$$

The term Δ can be considered a measure of the ignorance of the wage setters, as it is a function of their prediction errors with respect to the target level and the intervention operation. Since $\mathbf{g}_a$ can be positive for some values of Δ, this decomposition shows that the central authority attempts to exploit the wage setters' ignorance and manipulate their beliefs in order to reduce the value of its loss. In the appendix we prove the following Proposition, which characterises the unique linear signalling equilibrium that exists in the market for foreign exchange.

Proposition 1 *Under cooperation in stage 0 there exists a unique linear signalling equilibrium, in which the central authority conditions the dealer's beliefs with an intervention operation in the market for foreign exchange, I_0, while the dealer rationally updates his prior beliefs on the target level, $\bar{s}$, and the intervention operation, I_0, using all the information contained in the signal he observes, x_0. The intervention operation of the central authority and the dealer's expected values for the target level and the intervention operation are*

$$\begin{aligned} I_0 &= \beta_{\mathcal{C}_o} (\bar{s} - \bar{s}^e) - \theta_{\mathcal{C}_o} \bar{n}, \\ \bar{s}_0^e &= \bar{s}^e + \lambda_{\mathcal{C}_o} (x_0 + \theta_{\mathcal{C}_o} \bar{n}), \\ I_0^e &= \beta_{\mathcal{C}_o} \lambda_{\mathcal{C}_o} (x_0 + \theta \bar{n}) - \theta_{\mathcal{C}_o} \bar{n}, \end{aligned}$$

where $\beta_{\mathcal{C}_o}$ is the unique positive root of the following equation

$$\sigma_u^2 (\sigma_u^2 Q + \quad \beta_{\mathcal{C}_o}) = 2Q \quad ^2 \beta_{\mathcal{C}_o}^4, \tag{14}$$

and $\lambda_{\mathcal{C}_o}$ and $\theta_{\mathcal{C}_o}$ are positive coefficients given in the appendix.

4.2 Foreign exchange intervention and signalling

Proposition 1 proposes some interesting conclusions.

• Through foreign exchange intervention the central authority can influence the expectations of the dealer and the wage setters and hence the exchange rate, s_0 and s_1, the nominal wage, w_1, and the employment level, n_1. The exact size of its intervention operation, I_0, need not be publicly announced and any public statement on its value would be equivalent to one regarding the target level, as in equilibrium I_0 is a simple function of $\bar{s}$. This means that, as already established for the objectives of the policy makers, we can rule out precise statements on the intervention operation of the central authority as they would not be equally credible.[13]

• While Stein's work suggests that imprecise statements on the objectives of the policy makers could be useful and credible, signalling equilibria based on *cheap talk* are plagued by the problem of their multiplicity. This problem does not emerge when foreign exchange intervention is employed. This is

[13] In the fall of 1985 the G-5 heavily intervened in the foreign exchange markets to signal the reference levels set at the Plaza meeting for the exchange rates etween the dollar, the deutsche mark and the yen, without precise announcements on these levels or on the intervention activity.

because, differently from public statements, foreign exchange intervention represents a potentially *costly* instrument of communication. In effect, one should observe that only if Q is strictly larger than 0, i.e. only if the central authority cares about the cost of this instrument, it "means what it says" and we can have a proper equilibrium.

• It is not always the case that the central authority "tells the truth". In fact, its intervention operation contains two parts: a "true" signal component, $\beta_{\mathcal{C}_0}(\bar{s} - \bar{s}^e)$, that indicates the prediction error of the dealer and the wage setters, and a "false" signal term, $-\theta_{\mathcal{C}_o}\bar{n}$, through which the central authority tries to manipulate the expectations of the wage setters and stimulate the economy. Then, there can be instances in which the central authority buys (sells) the foreign currency when it is overvalued (undervalued), i.e. when $\bar{s} < \bar{s}^e$ ($\bar{s} > \bar{s}^e$). This means that at times foreign exchange intervention and the monetary policy might be *inconsistent*. In this respect, it is at least reassuring that, as documented by Dominguez (1992) and Kaminsky and Lewis (1996), on several occasions in the 1980s intervention operations in the United States were not coherent with subsequent open market operations.

• The dealer and the wage setters rationally update their beliefs using the information contained in the signal x_0. However, since they cannot directly observe the central authority's intervention operation, I_0, on average they underestimate its size, as is usually the case when intervention events are reported in the press.[14] Moreover, foreign exchange intervention and the exchange rate, s_0, are not always consistent, as the latter may move in the opposite direction to that of the intervention operation. This happens when the central authority purchases (sells) the foreign currency, $I_0 > 0$ ($I_0 < 0$), while the noisy component of the signal, u_0, is negative (positive) and larger in size, i.e. $u_0 < 0$ ($u_0 > 0$) with $|u_0| > |I_0|$. Once again, this feature of our analysis is coherent with those many events in the 1980s and 1990s in which after an intervention operation the exchange rate did not move in the direction of the operation.

• Foreign exchange intervention dramatically *reduces* the uncertainty of the dealer and the wage setters on the target level, $\bar{s}$. In fact, let ${}_{0|\mathcal{C}_o}$ denote the variance of the target level conditional on the information contained in the signal x_0. Applying the projection theorem for normal distributions, we find that this conditional variance is

$$ {}_{0|\mathcal{C}_o} \equiv \mathrm{Var}\,[\bar{s} \,|\, x_0] = (1 - \lambda_{\mathcal{C}_o}\beta_{\mathcal{C}_o}) \quad , $$

where $(1 - \lambda_{\mathcal{C}_o}\beta_{\mathcal{C}_o}) \in (0, \sqrt{2}/(\sqrt{2}+1))$ and is the unconditional variance of the target level, $\bar{s}$, i.e. the level of uncertainty of the wage setters on the value of $\bar{s}$ in the absence of foreign exchange intervention. This indicates a reduction of a least 40% in the uncertainty of the wage setters, a result which has important stabilising consequences for the economy.

[14] We can prove that given the information possessed y the central authority the a solute expected value of I_0^e is smaller than the actual size of intervention, i.e. $|E[I_0^e \mid I_0]| < |I_0|$.

4.3 Policy gain and macroeconomic stability

We have seen that the intervention operation I_0 contains both a "true" signal component, $\beta_{\mathcal{C}_o}(\bar{s}-\bar{s}^e)$, and a "false" signal one, $-\theta_{\mathcal{C}_o}\bar{n}$, through which the central authority tries to manipulate the expectations of the wage setters in an attempt to stimulate the economy. However, this attempt is frustrated: the dealer and the wage setters are capable of filtering out the "false" signal component, $-\theta_{\mathcal{C}_o}\bar{n}$, and hence extract the relevant information from the signal, x_0, to properly estimate the target level, $\bar{s}$. Consequently, the unconditional expectations of the exchange rate and the employment level correspond to the equilibrium values that prevail when the central authority does *not* intervene in the foreign exchange market. Formally, one can prove that $E_{\emptyset}[s_1] = E_{\mathcal{C}_o}[s_1]$ and $E_{\emptyset}[n_1] = E_{\mathcal{C}_o}[n_1]$.

This shows that there is *no* systematic policy gain from foreign exchange intervention, because the central authority is able neither to *fool* the wage setters and stimulate the economy, nor to *reduce* the misalignement of the exchange rate. One might then be drawn to the conclusion that foreign exchange intervention is useless, as it fails to achieve the goals it is aimed for.

However, foreign exchange intervention can bring about an undesired but positive stabilising effect on the economy through its second moment effects on the employment level, n_1, in that for small values of Q the unconditional variance of the employment level is *smaller* than the equivalent value for the equilibrium without intervention, $\text{Var}_{\mathcal{C}_o}[n_1] < \text{Var}_{\emptyset}[n_1]$. This is because, by reducing the uncertainty of the wage setters, the central authority drastically *reduces* the probability of extreme events, in which i) the wage setters greatly overestimate or underestimate the objectives of the policy makers and ii) the employment level dramatically swings from its natural level.

Notice that if the policy makers could commit to reveal the true target level, $\bar{s}$, the fluctuations in the employment level induced by the uncertainty on their objectives could be completely eliminated and there would be *no* role for foreign exchange intervention. In other words, the policy makers had better commit to truly reveal their policy objectives than trying to manipulate agents' beliefs through foreign exchange intervention.

Even though desirable, true revelation of these policy objectives is often not implementable, as the policy makers possess an incentive to lie. On the other hand, the sequential equilibrium with foreign exchange intervention that we have characterised is fully time consistent. Therefore, we can argue that when a commitment technology does not exist intervention operations can supplement the lack of credibility of simple announcements and represent a useful policy instrument.

In some instances the central authority may incur in intervention losses with its intervention activity, but on average it gains profits exploiting its superior information. These profits do not contradict the claim that since it is potentially expensive foreign exchange intervention i) buys credibility and ii) guarantees a unique linear signalling equilibrium. Given that two operations of different size will imply two different (even if possibly negative) intervention costs, the central authority will be induced to discriminate and deliver a unique informative signal.

The central authority gains speculative profits because, leaving aside the "false" signal component, $-\theta_{\mathcal{C}_o}\bar{n}$, it buys (sell) the foreign currency to signal a greater (smaller) than expected target level for the exchange rate, $\bar{s} > \bar{s}^e$ ($\bar{s} < \bar{s}^e$). That is, it buys (sell) when the foreign currency is undervalued (overvalued). Then, since $\bar{s}$ and $\bar{n}$ are uncorrelated, on average the central authority makes some profits, as formally stated in the following Proposition.

Proposition 2 *Under coordination the unconditional expected profits the central authority gains from trading in the foreign exchange market, $E_{\mathcal{C}_o}[\pi]$, are positive and increasing in the lack of transparency, measured by σ_u^2,*

$$E_{\mathcal{C}_o}[\pi] \;=\; \frac{1}{2}(1 + Q\,\beta_{\mathcal{C}_o})\,\lambda_{\mathcal{C}_o}\,\sigma_u^2.$$

This result is coherent with: i) Friedman's prescription that foreign exchange intervention should be equivalent to stabilising speculation (and hence profitable) and ii) some recent empirical results (Sweeney 1997, 2000), which show that in the medium run the risk-adjusted profits central banks obtain when intervening in the foreign exchange market are positive.

4.4 Foreign exchange intervention and institutions

The *stability gain* from foreign exchange intervention is not a general result under coordination, since we can conceive perverse scenarios in which intervention operations actually *increase* the volatility of the employment level. As an example assume that Q takes the relatively large value $(\;\;/\sigma_u^2)^{1/2}$. We can show that in this case the unconditional variance $\text{Var}_{\mathcal{C}_o}[n_1]$ is equal to $\;\;/2$, i.e. is *twice* the volatility of the employment level in the equilibrium without intervention ($\text{Var}_{\mathcal{C}_o}[n_1] = 2\text{Var}_{\emptyset}[n_1]$).

This shows that while foreign exchange intervention might potentially be useful in stabilising the economy it can also achieve the opposite perverse effect. Some would then argue that foreign exchange intervention possesses a controversial role, since its effects crucially depend on the particular weight assigned to contrasting aims, and does not represent a useful instrument of policy making.

This criticism hinges on the particular institutional arrangements we have investigated. While it seems natural to study the case of coordination, in which a single governmental body possess full jurisdiction over both monetary policy and foreign exchange intervention, there is no reason why we should not consider other institutional specifications. In this respect we notice a variety of alternative solutions adopted by the main industrialised countries.

• In Japan the Minister of Finance is *sole* responsible for intervention in the market for foreign exchange, while the Bank of Japan acts *only* as an agent, carrying out the intervention operations selected by the Minister of Finance through the Special Accounts of its Foreign Exchange Fund.

• In the United States intervention operations are the result of the initiative of the Treasury, in consultation with the Federal Reserve, and are carried out by the Federal Reserve in New York. This splits the cost of intervention with the Treasury drawing funds in equal shares from its SOMA (System Open Market Account) account and the Treasury's ESF (Exchange Stabilization Fund) account.[15]

• In Germany, before the launch of the euro, the Minister of Finance would select the appropriate exchange rate regime. Within that regime the Bundesbank had full jurisdiction over foreign exchange intervention and would use its own funds to finance it.[16] With the launch of the euro these features have changed: in Euroland the Council of the Eco-Fin Ministers called for some action before the European Central Bank intervened to sustain the value of the euro in September 2000. However, afterwards the European Central Bank claimed, through its President, to have intervened of its own choice.

Given the different practices and the monetary independence that central banks possess in these countries, we can say that there exist important differences with respect to the control of monetary policy and foreign exchange intervention. While the German experience is well represented by the coordination case we have investigated so far, this is not true for the American and especially the Japanese one. In order to span the rich variety of the actual institutional arrangements adopted by these countries and underline the possible advantages that different institutional arrangements may bring about, we now consider an alternative case. In this case, that we will refer as *of separation* ($\mathcal{S}_e$), the policy makers delegate monetary policy and foreign exchange intervention to *two* different governmental agencies.

We assume that in the separation case the *monetary authority* does not care about the cost

[15]Furthermore, while all the 7 mem ers of the Board of Governors of the Federal Reserve System plus 5 of the Presidents of the 12 local Reserve Banks participate to the FOMC meetings, in which decisions over monetary policy are taken, only the Chairman of the Board of Governors and the Treasury Secretary meet to agree upon foreign exchange intervention.

[16]Even in the United Kingdom foreign exchange intervention and monetary policy are under the control of the Bank of England. Notice, however, that different groups of officials actually participate to the meetings in which decisions over foreign exchange intervention and monetary policy are taken.

of intervention but pursues the same goals of the policy makers. This means that it chooses the money supply, m_1, in order to minimise the expected value of the original loss function, $\mathcal{L}_p$, given in (6). The governmental agency controlling foreign exchange intervention, conventionally denoted as the *foreign exchange (forex) authority*, instead chooses its intervention operation, I_0, in order to minimise the expected value of a modified loss function, $\mathcal{L}_f$, that captures both the macroeconomic objectives of the policy makers and the cost of intervention. This is once again given in (11).

The delegation of monetary policy and foreign exchange intervention to two separate governmental agencies poses some new issues we do not discuss in the present context. As we assume that the two authorities agree on their macroeconomic objectives, i.e. on the target values for the exchange rate and the employment level, $\bar{s}$ and $\bar{n}$, and their relative importance, we rule out all possible inconsistencies and disagreements that emerge when differences exist on the ultimate macroeconomic goals of policy making. We also rule out possible conflicts that emerge between the policy makers and the two agencies when, in the face of their independence, the latter pursue their own goals.

5 Equilibrium under Separation

We now study the consequences of foreign exchange intervention under separation. As before, we study the sequential equilibrium of the model and investigate the macroeconomic implications of foreign exchange intervention, underlying the differences that exist with the coordination case.

5.1 Signalling equilibrium under separation

Proceeding as for the coordination case we solve for the discretionary equilibrium in stage 1 and find new equilibrium values for the nominal wage, w_1, and the money supply, m_1,

$$w_1 = \bar{s}_0^e + \bar{n}, \tag{15}$$

$$m_1 = \bar{s} + \bar{n} - \frac{1}{2}(\bar{s} - \bar{s}_0^e). \tag{16}$$

These expressions differ from those derived in section 4 for the equilibrium under coordination. Since the monetary authority does not care about the cost of intervention, foreign exchange intervention now possesses a feed-back effect on monetary policy *only* through the expectations channel. This is also reflected by the signalling equilibrium in the foreign exchange market in stage 0, as indicated in the next Proposition.

Proposition 3 *Under separation in stage 0 there exists a unique linear signalling equilibrium, in which the forex authority conditions the dealer's beliefs with an intervention operation in the market for foreign exchange, I_0, while the dealer rationally updates his prior belief on the target level, $\bar{s}$, using all the information contained in the signal he observes, x_0. The intervention operation of the forex authority and the expected target level of the dealer are*

$$\begin{aligned} I_0 &= \beta_{\mathcal{S}_e} (\bar{s} - \bar{s}^e) - \theta_{\mathcal{S}_e} \bar{n}, \\ \bar{s}_0^e &= \bar{s}^e + \lambda_{\mathcal{S}_e} (x_0 + \theta_{\mathcal{S}_e} \bar{n}), \end{aligned}$$

where $\beta_{\mathcal{S}_e}$ is the unique positive root of the following equation

$$\sigma_u^2 (\sigma_u^2 Q + \quad \beta_{\mathcal{S}_e}) = Q \quad {}^2 \beta_{\mathcal{S}_e}^4, \tag{17}$$

and $\lambda_{\mathcal{S}_e}$ and $\theta_{\mathcal{S}_e}$ are positive coefficients given in the appendix.

Proposition 3 confirms the first claim of our analysis: since an expensive instrument of communication is employed a unique linear signalling equilibrium exists. We argue that this is a robust result, irrespective of either the specific arrangements concerning the control of foreign exchange intervention or the details of the monetary policy model. Likewise, whilst we can again prove that foreign exchange intervention does not produce a systematic policy gain, in the form of an increase in the employment level or a reduction in the inflationary bias, we still observe a significant reduction in the uncertainty of the wage setters on the objectives of the policy makers.

In particular, the conditional variance of the target level, $\bar{s}$, given the information the wage setters possess at the end of stage 0, $_{0|\mathcal{S}_e}$, is always smaller than $/2$, with a reduction of *more* than 50% from the unconditional value, . This result presents important implications for the macroeconomic consequences of foreign exchange intervention.

5.2 Institutions, stability gain and intervention profits

By conveying a signal on the objectives of the policy makers, foreign exchange intervention reduces the volatility of the employment level, n_1, stabilising the economy and reducing the expected loss of the policy makers. Differently from the case of cooperation this is now a *general* result, which holds for all possible parametric configurations. This is stated in the following Proposition.

Proposition 4 *Under separation when the forex authority intervenes in the market for foreign exchange: 1) the unconditional variance of the employment level* more than halves *and 2) the unconditional expected loss of the policy makers* fall, *in that*

$$\text{Var}_{\mathcal{S}_e}[n_1] < \frac{1}{2}\text{Var}_{\emptyset}[n_1], \qquad E_{\mathcal{S}_e}[\mathcal{L}_p] < E_{\emptyset}[\mathcal{L}_p].$$

This result suggests that in terms of the macroeconomic objectives the separation of the control of foreign exchange intervention and monetary policy represents a *superior* institutional arrangement. This is confirmed by the following Proposition that compares the different implications of foreign exchange intervention under coordination and separation.

Proposition 5 *With respect to the equilibrium under cooperation separation of foreign exchange intervention and monetary policy will result in: i)* more *information on the target level, $\bar{s}$, revealed through foreign exchange intervention,* $_{0|\mathcal{S}_e} < {}_{0|\mathcal{C}_o}$*; ii) a* less *volatile economy, as the unconditional variance of the employment level, n_1, is* smaller *under the former institutional arrangement,* $\text{Var}_{\mathcal{S}_e}[n_1] < \text{Var}_{\mathcal{C}_o}[n_1]$*; iii) and* smaller *unconditional expected profits from intervention,* $E_{\mathcal{S}_e}[\pi] < E_{\mathcal{C}_o}[\pi]$.

To explain the results reported in Proposition 5 we need to consider two fundamental differences between the coordination and separation equilibria.

• As a straightforward comparison of equations (14) and (17) clearly indicates the value of β, the intensity of trading against the prediction error $\bar{s} - \bar{s}_0^e$, is larger in the case of separation than in that of coordination, i.e. $\beta_{\mathcal{S}_e} > \beta_{\mathcal{C}_o}$. This means that a *more* informative intervention operation is selected under separation, with a larger reduction in the uncertainty of the wage setters and a less volatile employment level.

• A second important distinction emerges with respect to the monetary rule. Comparing equations (13) and (16) we see that under coordination two extra terms, QI_0 and $Q(I_0 - I_0^e)/2$, affect the money supply. As we underlined in Section 4, when the two instruments of policy making are coordinated the central authority takes into account the impact on the cost of intervention of the monetary injection, so that these two quantities reflect the feed-back effect of foreign exchange intervention on the monetary policy via what we call the cost-of-intervention channel. Consequently, the unexpected quantity $Q(I_0 - I_0^e)/2$ appears in the equilibrium expression of the monetary shock, $m_1 - m_1^e$, and the employment level, n_1 (as $n_1 = m_1 - m_1^e$), representing an extra source of variability.

This source of variability turns out to be very significant when either the lack of transparency in the foreign exchange market, σ_u^2, or the weight the central authority attaches to the cost of intervention, Q, is large and may lead to the perverse destabilising effect of foreign exchange intervention outlined in section 4. In particular, a large value of σ_u^2 forces the central authority to employ a more aggressive intervention strategy in its attempt to manipulate the wage setters' beliefs. Then, given the larger noisy component of the signal x_0, the prediction error of the intervention operation, $I_0 - I_0^e$, increases, inducing a more volatile monetary shock, $m_1 - m_1^e$, and employment level, n_1.

Likewise, when Q is large the central authority augments the volatility of the monetary shock and the employment level to gain larger profits. Because its profits, π, are a *convex* function of the monetary shock, $m_1 - m_1^e$, via Jensen's inequality we know that a more volatile monetary shock will deliver larger expected profits. This trade off between the stability gain that a more informative signal may induce and the profits that a more aggressive monetary injection brings about is stressed by the last part of Proposition 5.[17] In effect, the bottom line of this Proposition is exactly this: when foreign exchange intervention and monetary policy are coordinated more weight is attached to the cost of intervention, so not surprisingly in this case larger expected profits are obtained at the expense of a less stable economy.

In synthesis, our analysis clearly indicates that the division of power over monetary policy and foreign exchange intervention is crucial and should be carefully considered. This is an important conclusion, in that the specific institutional arrangements that govern these two policy instruments differ among countries and have so far been overlooked. For this very reason we now turn to a brief comparative analysis.

5.3 Comparative analysis

We now discuss some interesting comparative statics results, investigating the characteristics and properties of the equilibria as we move Q, the weight attached to the cost of intervention, and σ_u^2, the measure of the lack of transparency of the market for foreign exchange. With respect to changes in the value of Q we can establish the following.

Proposition 6 *Both under coordination and separation the conditional variance of the target level* ($_{0|\mathcal{C}_o}$ *and* $_{0|\mathcal{S}_e}$), *the unconditional variance of the employment level* ($\mathrm{Var}_{\mathcal{C}_o}[n_1]$ *and* $\mathrm{Var}_{\mathcal{S}_e}[n_1]$), *the unconditional profits of fovfi0TT21s*

$(1/4) \quad + 2\bar{n}^2$.

2. *For $Q \downarrow 0$: $_{0|\Xi}$, $\mathrm{Var}_\Xi[n_1]$ and $E_\Xi[\pi]$ all converge to 0, while $E_\Xi[\mathcal{L}_a]$ converges $2\bar{n}^2$, when i) $\Xi = \mathcal{C}_o$ and ii) $\Xi = \mathcal{S}_e$.*

A graphical representation of this Proposition may help to interpret its results. In Figure 2 various characteristics of the equilibrium under coordination and separation are presented against different values of Q.[18] We immediately see that Proposition 6 underlines the existing trade off between the profitability of foreign exchange intervention and the stability gain it brings about. Under both institutional arrangements, for a larger weight attached to the cost of intervention, larger expected profits are obtained by reducing the informational content of the intervention operation at the expense of a more volatile employment level.

While these monotonicity results are quite intuitive, the limiting behaviour of the characteristics of the equilibrium is more surprising. In particular, when nearly no weight is attached to the cost of intervention, $Q \downarrow 0$, the residual uncertainty of the dealer and the wage setters almost vanishes, as $_{0|\Xi} \downarrow 0$ for $\Xi = \mathcal{C}_o$ and $\Xi = \mathcal{S}_e$, so that in the limit we approach a set of equilibria which are nearly *equivalent*, even if not *equal*, to that prevailing when the policy makers commit to the true revelation of the target level, $\bar{s}$, and no foreign exchange intervention is undertaken.

This shows a discontinuity, as for Q *exactly* null we would not have a signalling equilibrium in the market for foreign exchange. Yet, values of Q very close to zero are unrealistic, as a very small weight attached to the cost of intervention would imply a willingness on the part of the domestic authorities to consume all foreign reserves, a hypothesis that many would find hard to accept.

Proposition 6 also restates a fundamental difference between the two institutional arrangements we have already discussed. In the equilibrium under separation there are *always* benefits from foreign exchange intervention, both in terms of a larger stability gain and a smaller loss for the policy makers. This is no longer the case under coordination, as we see that for some values of Q both the unconditional variance of the employment level and the expected loss of the central authority are *larger* when foreign exchange intervention is undertaken than when it is not, i.e. $\mathrm{Var}_{\mathcal{C}_o}[n_1] > \mathrm{Var}_\emptyset[n_1]$ and $E_{\mathcal{C}_o}[\mathcal{L}_a] > E_\emptyset[\mathcal{L}_a]$. Therefore, under these circumstances this instrument of policy making might even be counterproductive.[19]

[18] In the construction of the panels presented in Figures 2 and 3 the actual values of the coefficients that characterise the equili ria, specifically β_Ξ and λ_Ξ for $\Xi = \mathcal{C}_o$ and $\Xi = \mathcal{S}_e$, are o tained numerically.

[19] To e precise, under separation when the forex authority just intervenes to profit from its superior information, i.e. for $Q \uparrow \infty$, the policy makers do not enefit at all from foreign exchange intervention, i.e. in the limit $E_{\mathcal{S}_e}[\mathcal{L}_p] \uparrow E_\emptyset[\mathcal{L}_p]$. Foreign exchange intervention reduces the volatility of the employment level, ut also increases that of the exchange rate. While in general the former effect dominates the latter, when $Q \uparrow \infty$ the two exactly offset each other.

Proposition 7 completes our comparative statics analysis with respect to changes in the value of σ_u^2. Once more Figure 3 may help to interpret its results more effectively.

Proposition 7 *Both under coordination and separation, the conditional variance of the target level* ($_{0|\mathcal{C}_o}$ *and* $_{0|\mathcal{S}_e}$)*, the unconditional variance of the employment level* ($\mathrm{Var}_{\mathcal{C}_o}[n_1]$ *and* $\mathrm{Var}_{\mathcal{S}_e}[n_1]$) *and the unconditional profits of foreign exchange intervention* ($E_{\mathcal{C}_o}[\pi]$ *and* $E_{\mathcal{S}_e}[\pi]$) *are increasing in* σ_u^2.

Both under coordination and separation, the conditional variance of the target level, the unconditional variance of the employment level, alongside with the unconditional expected profits of intervention and the unconditional expected loss converge to limit values when either $\sigma_u^2 \uparrow \infty$ *or* $\sigma_u^2 \downarrow 0$. *In particular:*

1. *For* $\sigma_u^2 \uparrow \infty$: *i)* $_{0|\mathcal{C}_o} \uparrow (\sqrt{2}/(\sqrt{2}+1))$, *while* $\mathrm{Var}_{\mathcal{C}_o}[n_1]$ *and* $E_{\mathcal{C}_o}[\pi]$ *all converge to infinity; ii)* $_{0|\mathcal{S}_e} \uparrow (1/2)$, $\mathrm{Var}_{\mathcal{S}_e}[n_1] \uparrow (1/8)$, $E_{\mathcal{S}_e}[\pi] \uparrow (1/4)(\sigma_u^2 \quad)^{1/2}$ *and* $E_{\mathcal{S}_e}[\mathcal{L}_p] \uparrow (1/4) \quad + 2\bar{n}^2$.
2. *For* $\sigma_u^2 \downarrow 0$: $_{0|\Xi}$, $\mathrm{Var}_{\Xi}[n_1]$ *and* $E_{\Xi}[\pi]$ *all converge to 0, when i)* $\Xi = \mathcal{C}_o$ *and ii)* $\Xi = \mathcal{S}_e$. *Moreover,* $E_{\mathcal{S}_e}[\mathcal{L}_p] \downarrow 2\bar{n}^2$, *while* $E_{\mathcal{C}_o}[\mathcal{L}_a] \downarrow (2+Q)\bar{n}^2$.

Proposition 7 advocates the use of foreign exchange intervention during periods of higher transparency of the foreign exchange market, in that a smaller value of σ_u^2 presents two important effects: i) it augments the dealer's ability to extract information from the signal x_0, while reducing the profitability of foreign exchange intervention and the volatility of the employment level and ii) it brings about a smaller expected loss for the policy makers.

Finally, notice that Proposition 7 also reinforces our original claim that the lack of transparency is fundamental for the effectiveness of foreign exchange intervention. For $\sigma_u^2 \downarrow 0$ we approach a set of equilibria which are nearly *equivalent* to that in which the policy makers commit to truly reveal the target level, $\bar{s}$, suggesting that in a perfectly transparent market we would not have a signalling equilibrium with foreign exchange intervention.

6 Concluding Remarks

Recently growing interest has emerged for i) the microstructure of the market for foreign exchange and ii) the impact of foreign exchange intervention on market sentiment and exchange rates. Within this strand of literature we have investigated the signalling role of foreign exchange intervention and its macroeconomic implications, employing an analytical framework which combines a simple monetary policy model *à la* Barro-Gordon with a market microstructure framework for the foreign exchange market.

Through this framework we have seen that foreign exchange intervention represents a genuine costly signalling device which can be employed to "buy" credibility. However, such a credibility cannot be "spent" to acquire a systematic policy gain, as foreign exchange intervention cannot stimulate the economy or tame inflation. Foreign exchange intervention is instead beneficial through a *second moment* effect, in that, when conducted by an independent authority, it reduces the volatility of the employment level and hence helps to stabilise the economy. This is an orthogonal conclusion to the contribution of Rogoff (1985), who suggests that a conservative independent central bank is capable of reducing the inflationary bias due to the lack of credibility of the policy makers, but at the cost of a more unstable economy.

We have also seen that the stability gain is accompanied by positive profits from the intervention activity, but that a trade off between these two effects exists in that when a larger weight is attached to the cost of intervention larger profits are obtained at the expense of a more volatile employment level. This trade off implies that the division of responsibility over monetary policy and foreign exchange intervention influences the effectiveness of these policy instruments. In other words, an important normative conclusion of our analysis is that the specific institutional arrangements concerning the control of foreign exchange intervention and monetary policy are not inconsequential and should be carefully considered.

Our analysis advocates a clear separation between monetary policy and foreign exchange intervention. Yet, the delegation of these policy instruments to two different governmental agencies may lead to conflicts and inconsistencies. Some would also argue that separating monetary policy and foreign exchange intervention could undermine the independence of the monetary authorities and reduce their discipline. These are clearly important issues we have not investigated. They call for further analysis that we leave to future research.

7 Appendix

Proof of Proposition 1.
Assume that $I_0 = \beta_{\mathcal{C}_o}(\bar{s} - \bar{s}^e) - \theta_{\mathcal{C}_o}\bar{n}$, where $\beta_{\mathcal{C}_o}$ and $\theta_{\mathcal{C}_o}$ are non-negative constants. Given the assumption of normality for $\bar{s}$ and u_0, we can apply the projection theorem for normal distributions and find that

$$\begin{aligned} \bar{s}_0^e &\equiv E[\bar{s} \mid x_0] = \bar{s}^e + \lambda_{\mathcal{C}_o}(x_0 + \theta_{\mathcal{C}_o}\bar{n}), \\ I_0^e &\equiv E[I \mid x_0] = -\theta_{\mathcal{C}_o}\bar{n} + \lambda_{\mathcal{C}_o}(x_0 + \theta_{\mathcal{C}_o}\bar{n}), \\ &\text{where } \lambda_{\mathcal{C}_o} = \frac{\beta_{\mathcal{C}_o}\Sigma}{\beta_{\mathcal{C}_o}^2\Sigma + \sigma_u^2}. \end{aligned}$$

Suppose, then, that the dealer updates his expectations of the target level for the spot rate and the intervention operation according to the following expressions: $\bar{s}_0^e = \bar{s}^e + \lambda_{\mathcal{C}_o}(x_0 + h)$ and $I_0^e = -h + \lambda_{\mathcal{C}_o}(x_0 + h)$, where h is some constant. Inserting these expressions in equations (12) and (13), considering that $s_0 = w_1$ and $s_1 = m_1$, we find relations between the intervention operation, the spot rates in stages 0 and 1, and the employment level in stage 1. Plugging the new expressions for s_0, s_1, n_1 in the modified loss function (11), taking its expectation with respect to the central authority's information and minimising with respect to I_0, we obtain, after some long but simple algebra, that the optimal intervention operation is

$$\begin{aligned} I_0 &= \beta_{\mathcal{C}_o}(\bar{s} - \bar{s}^e) - \theta_{\mathcal{C}_o}\bar{n} - \beta_{\mathcal{C}_o}a_{\mathcal{C}_o}h, \\ \text{where} \quad \beta_{\mathcal{C}_o} &= \frac{a_{\mathcal{C}_o} + 2Q}{a_{\mathcal{C}_o}(a_{\mathcal{C}_o} + 4Q) + 2Q^2}, \quad \theta_{0|\mathcal{C}_o} = 2\beta_{\mathcal{C}_o}, \\ a_{\mathcal{C}_o} &= \lambda_{\mathcal{C}_o} + Q(\mu_{\mathcal{C}_o} - 1), \quad \text{and} \quad \mu_{\mathcal{C}_o} = \beta_{\mathcal{C}_o}\lambda_{\mathcal{C}_o}. \end{aligned}$$

Notice that to have a Nash equilibrium the intervention operation of the central authority and the expectations of the dealer must be consistent. This is the case if two conditions are met. First, the constants h, $\theta_{0|\mathcal{C}_o}$ and $\beta_{\mathcal{C}_o}$ are such that $h = \theta_{0|\mathcal{C}_e}\bar{n} + \beta_{\mathcal{C}_o}a_{\mathcal{C}_o}h$. This implies that $h = \theta_{\mathcal{C}_o}\bar{n}$ with $\theta_{\mathcal{C}_o} = (a_{\mathcal{C}_o} + 2Q)/(Q(a_{\mathcal{C}_o} + Q))$. Second, $\beta_{\mathcal{C}_o}$ and $\lambda_{\mathcal{C}_o}$ solve simultaneously the system of equations

$$\beta_{\mathcal{C}_o} = \frac{a_{\mathcal{C}_o} + 2Q}{a_{\mathcal{C}_o}(a_{\mathcal{C}_o} + 4Q) + 2Q^2}, \qquad \lambda_{\mathcal{C}_o} = \frac{\beta_{\mathcal{C}_o}\Sigma}{\beta_{\mathcal{C}_o}^2\Sigma + \sigma_u^2}. \tag{18}$$

Substituting the expression for $\lambda_{\mathcal{C}_o}$ into that for $\beta_{\mathcal{C}_o}$ after some massage we obtain the following equation

$$\sigma_u^2(1 + Q\beta_{\mathcal{C}_o})(\sigma_u^2 Q + \Sigma\beta_{\mathcal{C}_o}) = 2Q(1 + Q\beta_{\mathcal{C}_o})\Sigma^2\beta_{\mathcal{C}_o}^4.$$

Notice that the obvious solution $\beta_{\mathcal{C}_o} = -1/Q$ can be proved to violate the second order condition of the central authority's optimisation problem, $a_{\mathcal{C}_o}(a_{\mathcal{C}_o} + 4Q) + 2Q^2 > 0$. Simplifying we obtain equation (14). This presents two solutions, one positive and one negative. Only the positive one satisfies the second order condition. To complete the prove notice that we have characterised the unique *linear* equilibrium.

Proof of Proposition 2.
Consider first the change in the spot rate between stage 0 and 1. This is given by the following expression: $s_1 - s_0 = m_1 - w_1 = n_1$. From equations (12) and (13) we find that $n_1 = [(\bar{s} - \bar{s}_0^e) + Q(I_0 - I_0^e)]/2$. Thus, inserting the expression for s_0^e and I_0^e given in Proposition 1 we find that

$$n_1 = \frac{1}{2}(1 + Q\beta_{\mathcal{C}_o})\left((1 - \lambda_{\mathcal{C}_o}\beta_{\mathcal{C}_o})(\bar{s} - \bar{s}^e) - \lambda_{\mathcal{C}_o}u_0\right). \tag{19}$$

Given that $I_0 = \beta_{\mathcal{C}_o}(\bar{s}-\bar{s}^e)-\theta_{\mathcal{C}_o}\bar{n}$, we find that $\pi = \pi_1+\pi_2$, where $\pi_1 = (1+Q\beta_{\mathcal{C}_o})(1-\lambda_{\mathcal{C}_o}\beta_{\mathcal{C}_o})\beta_{\mathcal{C}_o}(\bar{s}-\bar{s}^e)^2/2$ and π_2 is a linear function of the cross products of $(\bar{s}-\bar{s}^e)$, $\bar{n}$ and u_0. Given the assumption of independence, the unconditional expected value of π is

$$E_{\mathcal{C}_o}[\pi] \;=\; \frac{1}{2}(1+Q\beta_{\mathcal{C}_o})\beta_{\mathcal{C}_o}(1-\lambda_{\mathcal{C}_o}\beta_{\mathcal{C}_o})\,\Sigma \;=\; \frac{1}{2}(1+Q\beta_{\mathcal{C}_o})\,\lambda_{\mathcal{C}_o}\,\sigma_u^2, \tag{20}$$

where the last equality is obtained considering the definition of $\lambda_{\mathcal{C}_o}$ in (18).

Proof of Proposition 3.
The proof is analogous to that proposed for Proposition 1. Assume first that $I_0 = \beta_{\mathcal{S}_e}(\bar{s}-\bar{s}^e)-\theta_{\mathcal{S}_e}\bar{n}$, where $\beta_{\mathcal{S}_e}$ and $\theta_{\mathcal{S}_e}$ are non-negative constants. Applying the projection theorem for normal distributions we find that

$$\begin{aligned}\bar{s}^e_0 \;&\equiv\; E[\bar{s}\mid x_0] \;=\; \bar{s}^e + \lambda_{\mathcal{S}_e}(x_0 + \theta_{\mathcal{S}_e}\bar{n}),\\ &\text{where}\quad \lambda_{\mathcal{S}_e} \;=\; \frac{\beta_{\mathcal{S}_e}\Sigma}{\beta^2_{\mathcal{S}_e}\Sigma + \sigma_u^2}.\end{aligned}$$

Suppose, then, that $\bar{s}^e_0 = \bar{s}^e + \lambda_{\mathcal{S}_e}(x_0 + h)$, where h is some constant. Inserting this expression for the expected target level in equations (15) and (16), considering that $s_0 = w_1$ and $s_1 = m_1$, we find relations between the market order of the intervention operation, the spot rates in stages 0 and 1, and the employment level in stage 1. Plugging the new expressions for s_0, s_1, n_1 in the modified loss function (11), taking its expectation with respect to the forex authority's information and minimising with respect to I_0, we find that the optimal intervention operation is

$$\begin{aligned}I_0 \;&=\; \beta_{\mathcal{S}_e}(\bar{s}-\bar{s}^e) - \theta_{0|\mathcal{S}_e}\bar{n} - \beta_{\mathcal{S}_e}\lambda_{\mathcal{S}_e}h,\\ \text{where}\qquad \beta_{\mathcal{S}_e} \;&=\; \frac{1}{\lambda_{\mathcal{S}_e}}\frac{\lambda_{\mathcal{S}_e}+Q}{\lambda_{\mathcal{S}_e}+2Q},\qquad \theta_{0|\mathcal{S}_e} \;=\; \frac{2}{\lambda_{\mathcal{S}_e}+2Q}.\end{aligned}$$

The condition of consistency for the intervention operation of the forex authority and the expectations of the dealer are two. First, the constants h, $\theta_{0|\mathcal{S}_e}$ and $\beta_{\mathcal{S}_e}$ are such that $h = \theta_{0|\mathcal{S}_e}\bar{n} + \beta_{\mathcal{S}_e}\lambda_{\mathcal{S}_e}h$. This implies that $h = \theta_{\mathcal{S}_e}\bar{n}$ with $\theta_{\mathcal{S}_e} = 2/Q$. Second, $\beta_{\mathcal{S}_e}$ and $\lambda_{\mathcal{S}_e}$ solve simultaneously the system of equations

$$\beta_{\mathcal{S}_e} \;=\; \frac{1}{\lambda_{\mathcal{S}_e}}\frac{\lambda_{\mathcal{S}_e}+Q}{\lambda_{\mathcal{S}_e}+2Q},\qquad \lambda_{\mathcal{S}_e} \;=\; \frac{\beta_{\mathcal{S}_e}\Sigma}{\beta^2_{\mathcal{S}_e}\Sigma + \sigma_u^2}. \tag{21}$$

Substituting the expression for $\lambda_{\mathcal{S}_e}$ into that for $\beta_{\mathcal{S}_e}$ we obtain equation (17). This presents two solutions, one positive and one negative. Only the positive one satisfies the second order condition of the forex authority's optimisation problem, $\lambda_{\mathcal{S}_e}(\lambda_{\mathcal{S}_e}+2Q) > 0$. This completes the proof.

Proof of Proposition 4.
First, consider that from the expression for $\lambda_{\mathcal{S}_e}$ in (21) we see that this coefficient has maximum for $\beta_{\mathcal{S}_e} = \bar{\beta} \equiv (\sigma_u^2/\Sigma)^{1/2}$ equal to $\bar{\lambda} \equiv (\Sigma/\sigma_u^2)^{1/2}/2$. Second, we know that $n_1 = m_1 - w_1$. From equations (15) and (16) we find that $n_1 = (\bar{s}-\bar{s}^e_0)/2$. Inserting the expression for s^e_0 given in Proposition 3, we find that

$$n_1 \;=\; \frac{1}{2}\left((1-\lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})(\bar{s}-\bar{s}^e) - \lambda_{\mathcal{S}_e}u_0\right). \tag{22}$$

If we take the unconditional variance of n_1 when the forex authority intervenes in the foreign exchange market, we find the following

$$\mathrm{Var}_{\mathcal{S}_e}[n_1] \;=\; \frac{1}{4}\left((1-\lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})^2\,\Sigma + \lambda^2_{\mathcal{S}_e}\sigma_u^2\right). \tag{23}$$

Considering the expression for $\beta_{\mathcal{S}_e}$ in (21), we find that $1-\lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e} = Q/(\lambda_{\mathcal{S}_e}+2Q) < 1/2$. Finally, noticing that $\lambda_{\mathcal{S}_e}$ is smaller than $\bar{\lambda}$, we find that $\text{Var}_{\mathcal{S}_e}[n_1] < \Sigma/8$. This completes the proof of the first part of the statement, as the unconditional variances of n_1 when there is no foreign exchange intervention is $\Sigma/4$. As for the second part of the statement, consider that $s_1 = m_1$. Combining equation (16) with the expression for the expected target level, s_0^e, given in Proposition 3, we find that $s_1 = \bar{s}+\bar{n}+[(1-\lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})(\bar{s}-\bar{s}^e)-\lambda_{\mathcal{S}_e}u_0]/2$. Thus, given this expression and that for the employment level, one can easily check that $E_{\mathcal{S}_e}[\mathcal{L}_p] = 2\text{Var}_{\mathcal{S}_e}[n_1]+2\bar{n}^2$. The equivalent expected loss function without foreign exchange intervention is given in equation (10) Given the result for the variance of the employment level, we find that $E_{\mathcal{S}_e}[\mathcal{L}_p] < E_{\emptyset}\ [\mathcal{L}_p]$.

Proof of Proposition 5.
Both under separation and coordination the conditional variance for the target level, $\bar{s}$, can be obtained from the projection theorem for normal distributions. We have that $\Sigma_{0|\Xi} = (1-\lambda_{\Xi}\beta_{\Xi})\Sigma$, where $\Xi = \mathcal{C}_o$ or $\mathcal{S}_e$. Since $\partial\lambda_{\Xi}\beta_{\Xi}/\partial\beta_{\Xi} = 2\sigma_u^2\lambda_{\Xi}/(\beta_{\Xi}^2\Sigma+\sigma_u^2) > 0$ and $\beta_{\mathcal{C}_o} < \beta_{\mathcal{S}_e}$, we find that $\Sigma_{0|\Xi}$ is larger for $\Xi = \mathcal{C}_o$.

From equation (19) we can see that under coordination the unconditional variance of the employment level is as follows

$$\text{Var}_{\mathcal{C}_o}[n_1] \;=\; \frac{1}{4}\left(1\,+\,Q\,\beta_{\mathcal{C}_o}\right)^2\left((1\,-\,\lambda_{\mathcal{C}_o}\beta_{\mathcal{C}_o})^2\,\Sigma\,+\,\lambda_{\mathcal{C}_o}^2\sigma_u^2\right). \tag{24}$$

In comparing equations (23) and (24) consider that: i) $(1-\lambda_{\Xi}\beta_{\Xi})$ is larger for $\Xi = \mathcal{C}_o$; ii) λ_{Ξ} is decreasing in β_{Ξ} for $\beta_{\Xi} > \bar{\beta}$; iii) $\beta_{\mathcal{S}_e} > \beta_{\mathcal{C}_o}$ and $\beta_{\mathcal{S}_e} \geq \bar{\beta}$; iv) for $Q < \bar{Q} \equiv (\Sigma/\sigma_u^2)^{1/2}\,\beta_{\mathcal{C}_o} > \bar{\beta}$. Then, it is immediate to see that $\text{Var}_{\Xi}[n_1]$ is larger for $\Xi = \mathcal{C}_o$ when $Q < \bar{Q}$. In the case in which $Q \geq \bar{Q}$, consider that $\forall Q$: i) $\beta_{\mathcal{C}_o} \geq (1/\sqrt[4]{2})\bar{\beta}$ and ii) $\lambda_{\mathcal{S}_e} \leq \bar{\lambda}$. It is not difficult to see that $(1+Q\beta_{\mathcal{C}_o})^2\lambda_{\mathcal{C}_o}^2\sigma_u^2 \geq ((\sqrt[4]{2}+1)^2/(\sqrt{2}+1)^2)\Sigma > \Sigma/4 \geq \lambda_{\mathcal{S}_e}^2\sigma_u^2$, so that even in this case $\text{Var}_{\Xi}[n_1]$ is larger for $\Xi = \mathcal{C}_o$.

Through the same argument proposed for the proof of Proposition 2 we can see that under separation the unconditional expected profits of the forex authority are

$$E_{\mathcal{S}_e}[\pi] \;=\; \frac{1}{2}\beta_{\mathcal{S}_e}(1\,-\,\lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})\,\Sigma \;=\; \frac{1}{2}\lambda_{\mathcal{S}_e}\sigma_u^2. \tag{25}$$

Now, in comparing equations (20) and (25) consider first of all that under separation the unconditional expected profits of the forex authority presents upper bound $E_{\mathcal{S}_e}[\pi] = (\sigma_u^2\Sigma)^{1/2}/4$. Then, consider that when $Q < \bar{Q}$, $\bar{\beta} < \beta_{\mathcal{C}_o}$. Moreover, notice that under coordination for all Q, $1-\lambda_{\mathcal{C}_o}\beta_{\mathcal{C}_o} > 1/2$. It is immediate to see that for $Q < \bar{Q}$, $E_{\Xi}[\pi]$ is larger for $\Xi = \mathcal{C}_o$. In the case in which $Q \geq \bar{Q}$, consider that $\forall Q$: i) $\beta_{\mathcal{C}_o} \geq (1/\sqrt[4]{2})\bar{\beta}$; and ii) $\beta_{\mathcal{S}_e} \geq \bar{\beta}$ and $\lambda_{\mathcal{S}_e} \leq \bar{\lambda}$. It is not difficult to see that $(1+Q\beta_{\mathcal{C}_o})\lambda_{\mathcal{C}_o}\sigma_u^2 \geq ((\sqrt[4]{2}+1)/(\sqrt{2}+1))(\sigma_u^2\Sigma)^2 > (\sigma_u^2)\Sigma/2 \geq \lambda_{\mathcal{S}_e}\sigma_u^2$, so that even in this case $E_{\Xi}[\pi]$ is larger for $\Xi = \mathcal{C}_o$.

Proof of Proposition 6.
Let us consider the separation case and define the implicit function $F(\beta_{\mathcal{S}_e}, Q, \sigma_u^2) \equiv \sigma_u^2(\sigma_u^2 Q + \Sigma\beta_{\mathcal{S}_e}) - Q\Sigma^2\beta_{\mathcal{S}_e}^4$. Using inequalities obtained from equation (17) and the fact that $\beta_{\mathcal{S}_e} > \bar{\beta}$, one can then check that $\partial F/\partial\beta_{\mathcal{S}_e} < 0$ and $\partial F/\partial Q < 0$. Thus, from the implicit function theorem one concludes that $\partial\beta_{\mathcal{S}_e}/\partial Q < 0$. Considering that $1-\lambda_{\Xi}\beta_{\Xi} = \sigma_u^2/(\beta_{\Xi}^2\Sigma+\sigma_u^2)$ one immediately sees that $\partial\Sigma_{0|\mathcal{S}_e}/\partial Q > 0$. Likewise, one should notice that $(1-\lambda_{\Xi}\beta_{\Xi})^2\Sigma + \lambda_{\Xi}^2\sigma_u^2 = \sigma_u^2\Sigma/(\beta_{\Xi}^2\Sigma+\sigma_u^2)$. It is not difficult to see that $\partial\text{Var}_{\mathcal{S}_e}[n_1]/\partial Q > 0$. For the profits notice that, since $\beta_{\mathcal{S}_e} > \bar{\beta}$, $\partial\lambda_{\mathcal{S}_e}/\partial Q > 0$. Since $E_{\mathcal{S}_e}[\pi] = \lambda_{\mathcal{S}_e}\sigma_u^2/2$, the result is immediate. For the loss of the policy makers, notice that $E_{\mathcal{S}_e}[\mathcal{L}_p] = 2\text{Var}_{\mathcal{S}_e}[n_1] + 2\bar{n}^2$, so the result is also immediate.

For the coordination case we obtain from equation (14) the new implicit function, $G(\beta_{\mathcal{S}_e}, Q, \sigma_u^2) \equiv$

$\sigma_u^2(\sigma_u^2 Q + \Sigma\beta_{\mathcal{S}_e}) - 2Q\Sigma^2\beta_{\mathcal{S}_e}^4$. With very similar steps we can show that $\partial\Sigma_{0|\mathcal{C}_o}/\partial Q > 0$ and $\partial\mathrm{Var}_{\mathcal{C}_o}[n_1]/\partial Q > 0$.

We now turn to the asymptotic properties. For $Q \uparrow \infty$, it is immediate to see from equations (17) and (14) that $\beta_{\mathcal{S}_e} \downarrow \bar{\beta}$, while $\beta_{\mathcal{C}_o} \downarrow \sqrt[4]{2}\bar{\beta}$. Then, all the limits follow immediately. On the contrary for $Q \downarrow 0$ in both cases $\beta_{\Xi} \uparrow \infty$ and once again the asymptotic results follow suit.

Proof of Proposition 7.
We can proceed as in the proof of Proposition 6, considering that $\partial F/\partial\sigma_u^2 > 0$. Thus, from the implicit function theorem one concludes that $\partial\lambda_{\mathcal{S}_e}/\partial\sigma_u^2 < 0$, and since $\beta_{\mathcal{S}_e} > \bar{\beta}$, one immediately finds $\partial\beta_{\mathcal{S}_e}/\partial\sigma_u^2 > 0$. Then, since $1 - \lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e} = Q/(\lambda_{\mathcal{S}_e} + 2Q)$, one sees that $\partial\Sigma_{0|\mathcal{S}_e}/\partial\sigma_u^2 > 0$. As for the derivatives of the conditional variance of n_1 and the expected value of $\mathcal{L}_p$ just notice that, after some tedious algebra, one can prove that $\partial[\lambda_{\mathcal{S}_e}^2\sigma_u^2]/\partial\sigma_u^2 = [(3\sigma_u^2\beta_{\mathcal{S}_e}\lambda_{\mathcal{S}_e}\Sigma)(\beta_{\mathcal{S}_e}^2\Sigma - \sigma_u^2)]/[(\beta_{\mathcal{S}_e}^2\Sigma + \sigma_u^2)(4Q\Sigma\beta_{\mathcal{S}_e}^3 - \sigma_u^2)]$. This is positive, as $\beta_{\mathcal{S}_e}$ respects (17) and is larger than $\bar{\beta}$. Since $(1 - \lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})^2$ is increasing with σ_u^2, this is sufficient to show that $\partial\mathrm{Var}_{\mathcal{S}_e}[n_1]/\partial\sigma_u^2 > 0$ and $\partial E_{\mathcal{S}_e}[\mathcal{L}_p]/\partial\sigma_u^2 > 0$. Finally, for the expected value of π notice that we have seen that $\partial\beta_{\mathcal{S}_e}/\partial\sigma_u^2 > 0$ and $\partial(1 - \lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})/\partial\sigma_u^2 > 0$. Then, it is immediate to see that $\partial E_{\mathcal{S}_e}[\pi]/\partial\sigma_u^2 > 0$.

The proof for the signs of the corresponding derivatives in the case of coordination is analogous. While for the asymptotic results consider that as for the proof of Proposition 6 we find that for $\sigma_u^2 \uparrow \infty$, $\beta_{\mathcal{S}_e} \to \bar{\beta} \uparrow \infty$, while $\beta_{\mathcal{C}_o} \to (1/\sqrt[4]{2})\bar{\beta} \uparrow \infty$. Then, all the limits follow immediately. On the contrary for $\sigma_u^2 \downarrow 0$ in both cases $\beta_{\Xi} \downarrow 0$ and once again the asymptotic results can be immediately shown.

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Figure 1: The time line representation of the model

Figure 2: Characteristics of the equilibrium as functions of Q.

Notes: $= 10$, $\sigma_u^2 = 1$. The continuous line represents the equilibrium value of the coefficient in the case of *coordination*. The long dash line represents the same coefficient in the case of *separation*, while the short dash line refers to the coefficient value in the absence of foreign exchange intervention. The efficiency coefficients, the unconditional variances of the employment level, alongside with the unconditional expected profits of foreign exchange intervention and the expected loss converge to limit values for $Q \uparrow \infty$, both in the case of coordination and separation. In particular, in the former case $_{0|\mathcal{C}_o} \uparrow (\sqrt{2}/(\sqrt{2}+1)) \quad = 5.8579$; while in the latter, $_{0|\mathcal{S}_e} \uparrow (1/2) \quad = 5$. Moreover, in the former case $\text{Var}_{\mathcal{C}_o}[n_1]$, $E_{\mathcal{C}_o}[\pi]$ and $E_{\mathcal{C}_o}[\mathcal{L}_a]$ all converge to infinity, while in the latter: $\text{Var}_{\mathcal{S}_e}[n_1] \uparrow (1/8) \quad = 1.25$, $E_{\mathcal{S}_e}[\pi] \uparrow (1/4)(\sigma_u^2 \quad)^{1/2} = 0.7906$ and $E_{\mathcal{S}_e}[\mathcal{L}_p] \uparrow (1/4) \quad + 2\bar{n}^2 = 4.5$. Likewise limits exist for $Q \downarrow 0$. In particular, $_{0|\mathcal{C}_o}$ and $_{0|\mathcal{S}_e}$ converge to 0 and so do $\text{Var}_{\mathcal{C}_o}[n_1]$, $\text{Var}_{\mathcal{S}_e}[n_1]$, $E_{\mathcal{C}_o}[\pi]$ and $E_{\mathcal{S}_e}[\pi]$. Finally, $E_{\mathcal{S}_e}[\mathcal{L}_p] \downarrow 2\bar{n}^2 = 2$ and so does $E_{\mathcal{C}_o}[\mathcal{L}_a]$.

Figure 3: Characteristics of the equilibrium as functions of σ_u^2.

Notes: $= 10$, $Q = 1$. The continuous line represents the equilibrium value of the coefficient in the case of *coordination*. The long dash line represents the same coefficient in the case of *separation*, while the short dash line refers to the coefficient value in the absence of foreign exchange intervention. The efficiency coefficients, the unconditional variances of the employment level, alongside with the unconditional expected profits of foreign exchange intervention and the expected loss converge to limit values for $\sigma_u^2 \uparrow \infty$, both in the case of coordination and separation. In particular, in the former case $_{0|\mathcal{C}_o} \uparrow (\sqrt{2}/(\sqrt{2}+1)) \quad = 5.8579$; while in the latter, $_{0|\mathcal{S}_e} \uparrow (1/2) \quad = 5$. Moreover, in the former case $\text{Var}_{\mathcal{C}_o}[n_1]$, $\text{Var}_{\mathcal{C}_o}[s_1]$ and $E_{\mathcal{C}_o}[\pi]$ all converge to infinity, while in the latter: $\text{Var}_{\mathcal{S}_e}[n_1] \uparrow (1/8) \quad = 1.25$, $E_{\mathcal{S}_e}[\pi] \uparrow \infty$ and $E_{\mathcal{S}_e}[\mathcal{L}_p] \uparrow (1/4) \quad + 2\bar{n}^2 = 4.5$. Likewise limits exist for $\sigma_u^2 \downarrow 0$. In particular, $_{0|\mathcal{C}_o}$ and $_{0|\mathcal{S}_e}$ converge to 0 and so do $\text{Var}_{\mathcal{C}_o}[n_1]$, $\text{Var}_{\mathcal{S}_e}[n_1]$, $E_{\mathcal{C}_o}[\pi]$ and $E_{\mathcal{S}_e}[\pi]$. Finally, $E_{\mathcal{S}_e}[\mathcal{L}_p] \downarrow 2\bar{n}^2 = 2$, while $E_{\mathcal{C}_o}[\mathcal{L}_a] \downarrow [1 + (1+Q)^2]\bar{n}^2 = 5$.

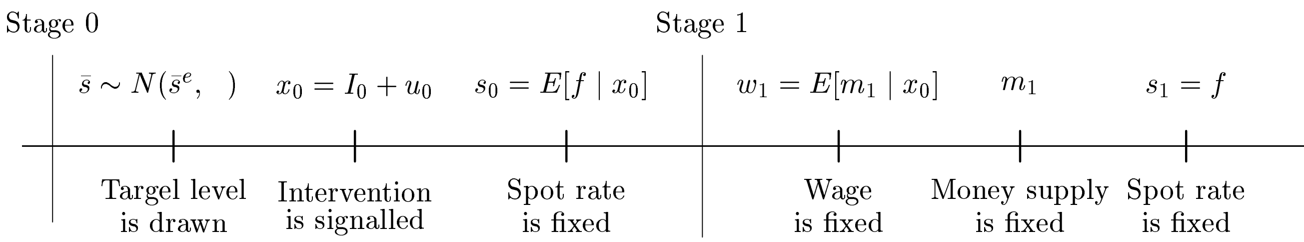
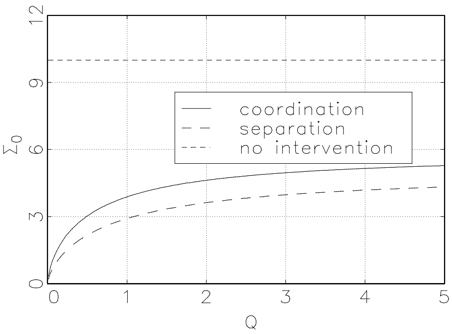
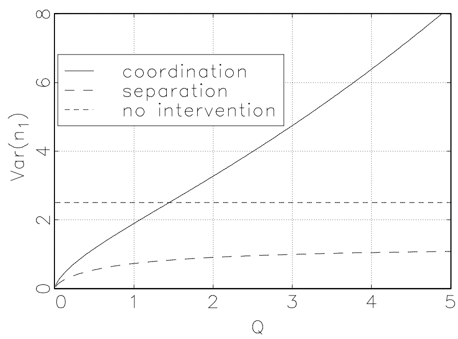


Figure 1: The time line representation of the model

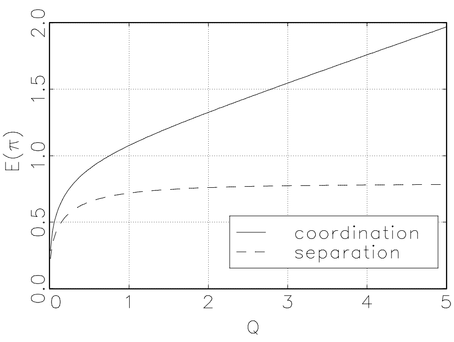
Figure 2: Characteristics of the equilibrium as functions of Q.



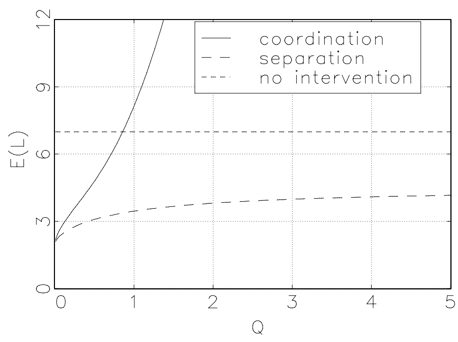
(a) Conditional Variance



( ) Employment Volatility

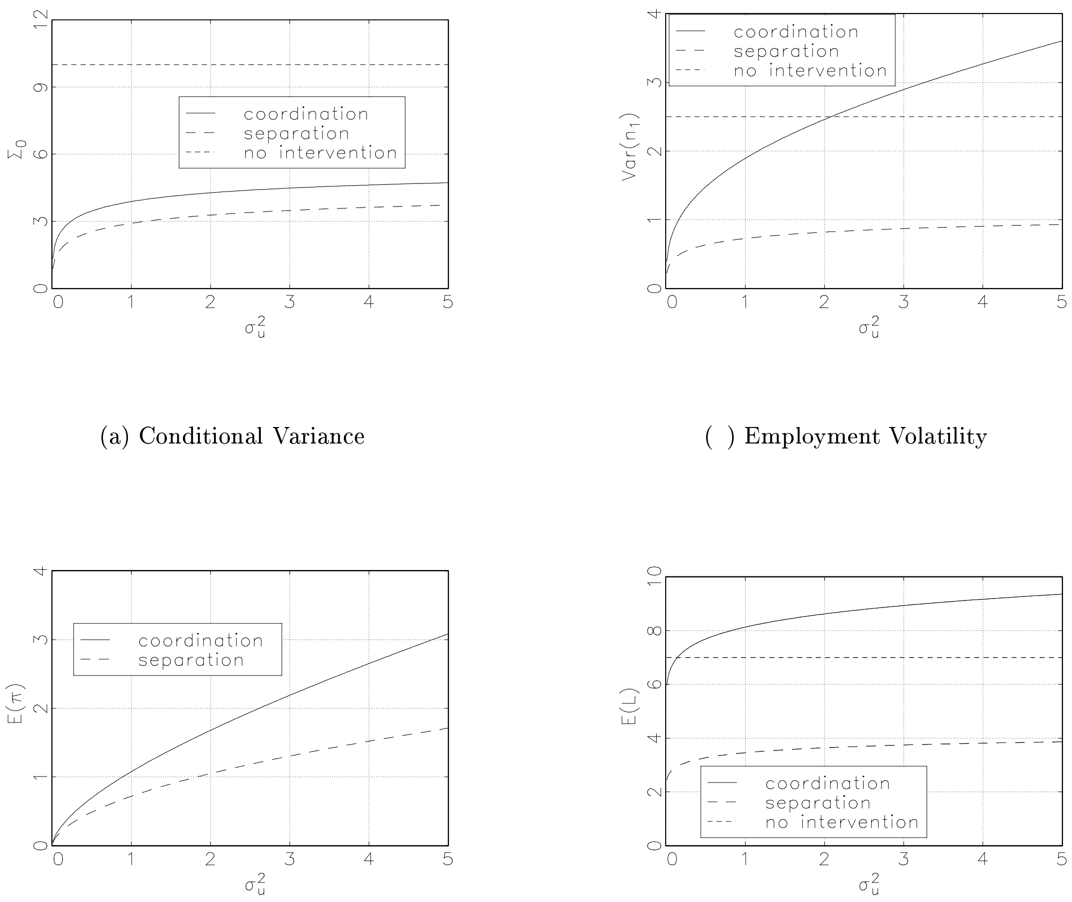


(c) Expected Profits



(d) Expected Loss

Figure 3: Characteristics of the equilibrium as functions of σ_u^2.



(a) Conditional Variance

( ) Employment Volatility

(c) Expected Profits

(d) Expected Loss