The Role of Bank Capital and The Transmission Mechanism of Monetary Policy

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Abstract

This paper is a theoretical study of the transmission mechanism of monetary policy in the presence of an endogenous role of bank capital. The basic framework is a standard Dynamic New Keynesian model with price stickiness modified so as firms as well as banks face endogenous financial frictions in obtaining external funds from their respective debtors. This implies that an external financial premium exists, thereby motivating the endogenous role of entrepreneurial net worth and bank capital in the model. In the terminology of Van den Heuvel (2001), the model exhibits the unconventional '*bank capital*' channel of monetary policy. The simulation result highlights a financial accelerator effect in that endogenous evolution of bank capital, together with that of entrepreneurial net worth, operate to amplify and propagate the effect of a monetary shock in the macroeconomy.

Keywords: bank capital, monetary policy, financial accelerator effect
J*EL Classification*: E30, E44, E50, G21

1 Introduction

The goal of this paper is to understand the transmission mechanism of monetary policy in the context where bank capital together with entrepreneurial net worth interactively work to amplify and propagate the dynamics of key macroeconomic variables in addition to the otherwise conventional interest rate channel. This is motivated by a casual observation that most macroeconomic models for monetary policy evaluation, both with and without an explicit role of entrepreneurial net worth, abstract completely from the role of bank capital.[1] This consensus practice would be just a mere simplifying assumption only if one of the following conditions holds: 1) Unexpected monetary shock does not affect bank capital, or 2) if it does, change in the dynamics of bank capital must have no major effect on that of other important aggregate macroeconomic variables.

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[1] For macroeconomic models in which the transmission mechanism of monetary policy works only through the conventional interest rate channel, see, amongst others, Clarida, Gali and Gertler (1999). For those with an explicit role of entrepreneurial net worth, thus incorporating the balance sheet channel, see amongst other Bernanke, Gertler and Gilchrist (1999) and Carlstrom and Fuerst (2000).

One of the main functions that banks perform is the transformation of securities with short maturities, offered to depositors, into securities with long maturities that borrowers desire (Freixas and Rochet, 1997). This maturity mismatch on banks' balance sheets implies that lending rates are relatively stickier compared to deposit rates in response to unanticipated aggregate shocks. Consequently, as the central bank increases the interest rate unexpectedly, banks' interest rate cost will rise faster than their revenue counterpart thereby depleting their inside capital. This invalidates the aforementioned first condition; unexpected monetary shock can theoretically affect bank capital.

Concerning the second condition, there are many empirical findings which lend support to the importance of the role of bank capital in constraining bank lending and aggregate economic activities. Amongst others, Bernanke and Lown (1991), Furlong (1992) and more recently, Peek and Rosengren (1997) and Ito and Sasaki (1998) found that capital position of banks has positive and statistically significant effects on bank lending. Moreover, Hubbard, Kuttner and Palia (1999) found that higher bank capital lowers the rate charged on loans, even after controlling for borrower characteristics, other bank characteristics and loan contract terms. Given that the second condition is also violated, shunning bank capital from the model's dynamics can substantially distort our understanding of monetary policy transmission.

The framework undertaken in this paper is an extension to the well known financial accelerator model of Bernanke, Gertler and Gilchrist (1999). The key modification is that, in addition to firms, banks also face financial friction in obtaining external funds. This is achieved by embedding the double costly state verification approach into the otherwise standard dynamic general equilibrium model with price stickiness.[2] It is 'double' in the sense that both firms' debtors (banks) and banks' debtors (depositors) cannot verify their respective borrowers' investment return unless positive verification cost has been paid. On the one hand, because firms' project return is assumed to be private information, they have incentive to understate their liquidation return in the event of bankruptcy. In order to ensure truthful liquidation return, banks have to pay positive verification cost. As shown by Townsend (1979), the existence of verification cost drives a wedge between firms' internal and external cost of capital and provides them incentive to hold their inside capital (entrepreneurial net worth). On the other hand, as the riskiness of banks' loan portfolio cannot be fully diversified, the riskiness associated with firms' projects passes on to ultimate depositors. Therefore, similar to firms, as banks have incentive to understate their return in the event of bankruptcy, depositors are required to pay verification cost if they wish to observe the true liquidation return on banks' loan portfolio. This implies that external finance premium exists for banks in acquiring deposit from depositors which in turn motivates them to [illegible] capital works to amplify and propagate the effect of a monetary shock on real macroeconomic variables. This is primarily because lower bank capital and entrepreneurial net worth means that firms and banks have less inside capital to contribute to firms' investment projects, implying greater agency cost faced by ultimate lenders (depositors). A higher external finance premium in the form of higher non-default deposit rates is therefore required in order to compensate them for having to face greater perceived risk. As this directly imposes greater cost of borrowing on banks, non-default lending rates have to increase (though, as we shall see, with lag) which in turn implies that firms are exposed to greater cost of borrowing. Consequently, the extent of decrease in investment and aggregate output must be greater than it would have been had the firms' cost of external fund been equal to the risk free rate as implied by the conventional frictionless models. To the extent that entrepreneurial net worth and bank capital are procyclical, external finance premium will be countercyclical and thus operates as a propagation mechanism to the model's dynamics.

The organisation of the paper is structured as follows. Section 2 discusses related literature. Section 3 presents a partial equilibrium model of financial contracting. It discusses the source of credit market imperfection whereby firms and banks face financial friction in obtaining external funds. It then derives optimal demand for capital by firms as a function of entrepreneurial net worth, bank capital and external finance premium. Section 4 embeds the demand for capital derived in section 3 into the otherwise standard Dynamic New Keynesian (DNK) model of business cycle with price stickiness. Section 5 gives the definition of the equilibrium and describes the model in the completely log-linearised form. Section 6 elaborates the transmission mechanism of monetary policy implied by the model, highlighting the independent role that bank capital plays in enriching the dynamics of the transmission. Section 7 discusses the calibration and presents the simulation results. Section 8 concludes the paper.[3]

2 Related Literature

Bernanke et al. (1999) and Carlstrom and Fuerst (2001) analysed the role of credit market friction in business fluctuation. In their models, firms face financial friction in borrowing from banks thereby causing their net worth to become one of the key elements in determining their debt capacity. This allows monetary policy to have an independent effect on entrepreneurial net worth, the so-called balance sheet channel. However, there is no role for bank capital in these models. This is because they assume a perfectly diversifiable portfolio of bank loan, implying that any idiosyncratic risk associated with firms' investment return is completely diversified at the bank level and therefore does not pass on to ultimate depositors. Given that depositors are risk averse, they can therefore be guaranteed with an equivalently riskless rate of return.[4]

Another set of literature focuses on an explicit role of bank capital in the model of bank's asset and liability management. Van den Heuvel (2002) examines the role of bank lending in the transmission mechanism of monetary policy with the presence of capital adequacy regulations. In his model, the motive for banks to hold inside capital stems exogenously from the prevalence of the regulation. Given maturity mismatch on banks' balance sheets, this gives rise to a 'bank capital' channel in which monetary policy affects bank lending through its impact on bank capital. Schneider (1999) studied the relationship between bank's borrowing constraint and the observed heterogeneity in borrowing and

[3] Table of abbreviations is given in section 11.
[4] For a critical assessment of Bernanke et al. (1999) model, see Markovic (2002).

lending behaviour across banks. His model captures financial imperfection by assuming the moral hazard problem associated with strategic defaults by entrepreneurial bankers. Thus, banks have motive to hold inside capital as it can alleviate the moral hazard problem. In other words, similar to this paper, banks are subject to market based, rather than regulatory based, capital requirement. However, none of these models consider an independent role of entrepreneurial net worth. Thus their models do not exhibit dynamic interplay between bank capital and entrepreneurial net worth. Moreover, they are not fully general equilibrium models in the sense that they abstract from consumption, investment and aggregate demand effects relating to price stickiness.

The last set of literature endogenously incorporates both entrepreneurial net worth and bank capital. Bolton and Freixas (2000) analysed the transmission mechanism of monetary policy in the context where direct finance and indirect finance coexist. Bank capital matters in the model due to the presence of an exogenous capital adequacy regulation. However, asymmetric information on the value of bank capital implies the existence of endogenous cost in raising outside equity capital. The monetary policy transmission implied by their model exhibits an amplification effect on bank lending through its effect on bank capital. Cantillo (1997) adopted the double costly state verification approach to study the coexistence between direct and indirect finance. In his model, both firms and banks face financial friction in obtaining external funds thereby motivating explicit roles of entrepreneurial net worth and bank capital. In contrast to this paper, there are only two periods in these two models, thus the issue of dynamics cannot be disentangled. Moreover, similar to Van den Heuvel (2002) and Schneider (1999), they are not fully general equilibrium models.

Chen (2002) extended Holmstrom and Tirole's (1997) model into a dynamic general equilibrium setting in order to study the dynamic interaction amongst entrepreneurial net worth, bank capital and real economic activities. The moral hazard problem both at the firm and bank levels is assumed in order to motivate endogenous roles of entrepreneurial net worth and bank capital. However, the model has no money and price stickiness therefore cannot address the issue of the transmission mechanism of monetary policy.

3 The Partial Equilibrium Model of Financial Contracting: Double Costly State Verification (Double CSV)

3.1 Basic Assumptions and the Structure of the Model

There are five types of agents in the economy, namely entrepreneurs, banks, households (depositors), retailers and the central bank.[5] As this section discusses the financial contracting problem among entrepreneurs, banks and depositors, I shall only explain the basic structure of these sectors in the following subsections, addressing only what is relevant to the contract problem, leaving the rest to be discussed in section 4 where the results obtained from the partial equilibrium model is embedded into the general equilibrium framework.

[5] As the main focus of this paper is on the transmission of monetary policy, I shall abstract from the role of government and therefore fiscal policy in the model.

3.1.1 Entrepreneurial Sector

Entrepreneurs are assumed to be risk neutral and are the only type of agent in the economy who has access to investment technology which involves the transformation of capital together with hired labours (from household sector) into wholesale goods. A representative entrepreneur, say entrepreneur i, operates firm i.[6] At the end of period t, firm i purchases capital denoted by K_t^i. The unit price of capital is given by Q_t. All capitals are homogenous. However, it is assumed that capital purchased at the end of period t cannot be used in production until the end of period $t+1$. The return from investing $Q_t K_t^i$ is therefore denoted by GR_{t+1}^K.[7]

$$GR_{t+1}^K = \omega_{i,t+1} R_{t+1}^K \tag{1}$$

R_{t+1}^K and $\omega_{i,t+1}$ are the non-idiosyncratic and idiosyncratic components of firm i's return to capital, respectively. The random variable $\omega_{i,t+1}$ is assumed to be log normally distributed with a mean of unity, $E(\varpi_{i,t+1}) = 1$, and a variance of σ^2. It is also assumed to be independently and identically distributed ($i.i.d.$) across time and firms. [8]

Assumption 1: $\omega_{i,t+1} \overset{i.i.d.}{\sim}$ log normal$(1, \sigma^2)$

It is worth emphasising that, in addition to idiosyncratic risk, firm i also encounters aggregate risk. This arises because the non-idiosyncratic component of return invested at the end of period t, R_{t+1}^K, will not be realised until the end of period $t+1$. The timeline of the model will be discussed in detail in subsection 3.1.4.

I assume that firm i can borrow external fund from a representative bank, say bank j, to partially finance its capital investment. All financial contracts, including both loan and deposit contracts, are assumed to have one period maturity. Following the Costly State Verification literature (Townsend, 1979), the realisation of idiosyncratic component of return, $\omega_{i,t+1}$, is private information and bank j has to pay verification cost in order to observe the actual realisation of the return on firm i's investment. This, as mentioned earlier, motivates entrepreneur i to hold his inside capital as the existence of verification cost drives a wedge between internal and external cost of funds. Moreover, following Krasa and Villamil (1992), I assume that the realisation of $\omega_{i,t+1}$ is privately revealed only to the agent who requests CSV technology.[9]

Assumption 2: The realisation of $\omega_{i,t+1}$ is privately revealed only to the agent who requests CSV technology.

[6] Firm i and entrepreneur i will be used interchangeably throughout the paper.

[7] Thoughout the paper, the time subscript denotes the period in which the value of an underlying variable is realised.

[8] Denote $F(\varpi_{i,t+1})$ and $f(\varpi_{i,t+1})$ as $c.d.f.$ and $d.f.$ of $\varpi_{i,t+1}$, respectively. The assumption that $\varpi_{i,t+1}$ is log normally distributed implies that the following restriction on the hazard rate, $h(\varpi_{i,t+1}) \equiv \frac{f(\varpi_{i,t+1})}{1-F(\varpi_{i,t+1})}$, holds; $\frac{\partial(\varpi_{i,t+1} h(\varpi_{i,t+1}))}{\varpi_{i,t+1}} > 0$. This regularity condition is a relatively weak restriction as it is satisfied by most conventional distributions (Bernanke et al., 1999)

[9] This assumption differs from the Townsend's (1979) specification. In his model $\omega_{i,t+1}$ is *publicly* announced after CSV occurs, while in this model $\omega_{i,t+1}$ is privately revealed to the agent who requests CSV. This assumption is essential to the analysis since if all information could be made public *ex post* there would be no need for depositors to pay verification cost to observe banks' return on their portfolio of loan. This assumption is consistent with institutional features which characterise most lending arrangements. As Diamond (1984, p.395) illustrated, "Financial intermediaries in the world monitor much information about their borrowers in enforcing loan covenants, but typically do not directly announce this information or serve an auditor's function."

Denote firm i's inside capital (net worth) held at the end of period t by W_t^i, given that the total outlay of the investment is $Q_t K_t^i$, loan borrowed from bank j, L_t^i, is defined as follows;

$$L_t^i \equiv Q_t K_t^i - W_t^i \tag{2}$$

From equation (1), total return from investing $Q_t K_t^i$ is given by $\varpi_{i,t+1} R_{t+1}^K Q_t K_t^i$. Given this total return, I define the following relationship.

$$\overline{\varpi}_{i,t+1}^F Q_t R_{t+1}^K K_t^i = r_{i,t}^L L_t^i \tag{3}$$

where $r_{i,t}^L$ is defined as the non-default loan rate associated with the loan contract between firm i and bank j signed in period t.

$\overline{\varpi}_{i,t+1}^F$ is firm i's threshold value of $\varpi_{i,t+1}$. For the realisation of idiosyncratic component below the threshold level, $\omega_{i,t+1} < \overline{\varpi}_{i,t+1}^F$, firm i's total realised revenue from investing $Q_t K_t^i$ is strictly less than the amount required to fulfil its loan contract with bank j. Thus firm i declares bankrupt and faces liquidation. In contrast, for $\omega_{i,t+1} \geq \overline{\varpi}_{i,t+1}^F$, firm i's realised return in period $t+1$ is sufficient to cover its debt obligation and therefore does not go bankrupt. Lastly, it is important to note that, with the presence of aggregate risk, $\overline{\varpi}_{i,t+1}^F$ is realised in period $t+1$ which implies that its value is contingent on the ex-post realisation of R_{t+1}^K.

3.1.2 Banking Sector

Banks in this economy operate under a perfectly competitive environment. Similar to entrepreneurs, they are assumed to be risk neutral. They function as financial intermediary, i.e. they borrow from a representative depositor and lend to a representative entrepreneur.

As commonly shown in the conventional one-sided costly state verification literature[10] , given that $\omega_{i,t+1}$ is identically and independently distributed across firms, idiosyncratic risk associated with each particular investment project is fully diversified away in the infinitely large portfolio of bank loan by virtue of the law of large number. Thus, depositors can be guaranteed with an equivalently riskless rate of return and banks have no incentive to hold their inside capital.[11] However, as argued by Krasa and Villamil (1992), the diversification would not be completely in a *finite*-size portfolio of bank loan, in which case idiosyncratic risk associated with firms' investment projects remains at the bank level and therefore passes on to ultimate depositors. This gives an incentive for depositors to monitor banks and thus motivate an explicit role of bank capital.[12]

In general, given that the size of bank loan portfolio is finite, the distribution of return within each individual bank's loan portfolio becomes of crucial to the analysis thereby adding a lot of complication to the model. In order to simplify the analysis, I assume that each bank can only lend to one firm. This assumption is essentially tantamount to the case in which a bank can finance multiple firms

[10] Diamond (1984), Gale and Hellwig (1985) and Williamson (1986)), amongst others.

[11] This assumption is taken in the macro models which incorporate the balance sheet channel of monetary policy but completely ignore the role of bank capital, i.e. Bernanke et al. (1999).

[12] Thus the costly state verification problem becomes *two*-sided. On the one hand, banks act as delegated monitors on firms's investment projects. On the other hand, in the terminology of Krasa and Villamil (1992), depositors perform the role of 'monitoring the monitor'.

but the return on firms' investment projects is assumed to be perfectly correlated within a bank but *i.i.d.* across banks (Holstrom and Tirole (1997) and Chen (2001)).[13] [14] Although this assumption is obviously unrealistic, it greatly simplifies the analysis.[15] Moreover, as argued by Holmstrom and Tirole (1997), banks' loan portfolio has some degree of correlation due to the tendency to specialise in their expertise.

Assumption 3: A bank can only lend to one entrepreneur.

Bank j can borrow external fund from a representative depositor, say depositor m, to partially finance its lending to firm i. Given assumption 3, the idiosyncratic risk associated with firm i's investment passes on directly to bank j's return on its lending. As firm i's return on investment is private information, so is the return on bank j's loan. Assuming the presence of the CSV problem at the bank level, together with assumption 2, depositor m has to pay verification cost if he or she wishes to observe the return on bank j's lending. This creates an external finance premium for bank j in obtaining external funds from depositor m thereby motivating the bank to hold its inside capital. Thus the holding of bank capital in this model is market based, as opposed to regulatory-based, requirement.

Banks in this model are 'special' in the sense that they specialise in verifying the projects' return and thus pay the lowest verification cost as compared to other agents. In this economy, having banks as financial intermediaries dominates one-sided financial contracts between firms and depositors as the aggregate expected verification cost is lower. Given that the verification cost paid by bank j is assumed to equal a proportion 'θ^B' of the realised gross return to firm i's investment ($\theta^B \varpi_{i,t+1} R^K_{t+1} Q_t K^i_t$), the special role of banks as delegated verifiers is summarised in assumption 4.

Assumption 4: θ^B is sufficiently lower than θ^n, where n denotes other types of agent except banks, such that the aggregate expected verification cost in the economy with financial intermediaries is strictly lower compared to that of the economy without financial intermediaries.[16]

In period t, a representative bank who finances its lending to firm i (L^i_t) by its own inside capital (A^i_t) and deposit acquired from depositor m (D^i_t) has the following balance sheet identity;[17] [18]

[13] Technically, the assumption that each individual bank could only lend to *one* firm while $\omega_{i,t+1}$ is allowed to be *i.i.d.* across time and firms gives the same result as the case in which each bank can lend to multiple firms but $\omega_{i,t+1}$ is assumed to be perfectly correlated within a bank but is *i.i.d.* across banks.

[14] Therefore, idiosyncratic risk is fully diversified at the aggregate level, *but not at the bank level.*

[15] For example, the aggregation process would not depend on an individual bank's distribution of its risky loan portfolio.

[16] In the economy without financial intermediaries, firms and depositors have to engage in direct financial contracts. See footnote 33 for technical description of assumption 4.

[17] As mentioned, similar to loan contracts, I assume that all deposit contracts have only one period maturity.

[18] In reality, the simplest form of a typical bank's balance sheet can be written as;

Assets	Liabilities
Loans	Deposits
S-T government bonds	Bank Capital
Cash	
Equipments	

In comparision to the realistic version shown above, there are three important variables that the bank balance sheet assumed in this paper lacks, namely short term bonds, cash and equipments. Because banks have to maintain convertibility commitment with depositors at *any* point in time and that loan is a relatively illiquid kind of asset, banks have incentive to hold short-term government bonds. Thus, although short term government bonds give lower expected return compared to loans, they can reduce the degree of banks' exposure to liquidity risk. However, all financial contracts in the model, including loan contracts, have only one period maturity. Thus the model at hand is not rich enough to accommodate the prevalence of liquidity risk in bank balance sheets. In this light, incorporating liquidity risk into the model, i.e. by allowing loan contracts to have more than one period maturity, would be a fruitful extension of this paper.

$$L_t^i = D_t^i + A_t^i \tag{4}$$

In the event that firm i declares bankrupt, after paying for the verification cost, bank j would receive the net liquidation revenue from firm i equivalent to $(1-\theta^B)\varpi_{i,t+1}\ Q_t R_{t+1}^K K_t^i$. Given this net liquidation value, I define the following relationship.

$$(1-\theta^B)\overline{\varpi}_{i,t+1}^B\ Q_t R_{t+1}^K K_t^i = r_{i,t+1}^D D_t^i \tag{5}$$

where $r_{i,t+1}^D$ denotes the non-default deposit rate realised in period $t+1$ associated with the deposit contract between bank j and depositor m signed in period t.

$\overline{\varpi}_{i,t+1}^B$ is bank j's threshold value of $\varpi_{i,t+1}$. This is because, for $\varpi_{i,t+1} \geq \overline{\varpi}_{i,t+1}^B$, the bank's net liquidated revenue received from lending to firm i will be sufficient to repay its obligation to the depositor. In contrast, for $\varpi_{i,t+1} < \overline{\varpi}_{i,t+1}^B$, the bank will go bankrupt as its net liquidated revenue is insufficient to pay its debt obligation to the depositor.

Lastly, $\overline{\varpi}_{i,t+1}^B$ is realised in period $t+1$ which implies that its value is contingent on the ex-post realisation of R_{t+1}^K.

3.1.3 Depositor (Household)

Depositors invest their savings by depositing their money with banks. In contrast to entrepreneurs and banks, they are assumed to possess higher degree of risk aversion in the sense that they are neutral to idiosyncratic risk but are averse to aggregate risk. This implies that aggregate risk inherited in the firms' project has to be absorbed completely by entrepreneurs and banks. As can be seen from equations (3) and (5), unlike the non-default lending rate which is determined instantaneously once the loan contract is signed, the non-default deposit rate associated with the deposit contract signed in period t will not be realised until period $t+1$. Consequently, as period $t+1$ arrives and aggregate risk associated with period-t capital investment is uncovered, in response to a lower than expected realised return on non-idiosyncratic component of return to firm's i investment in period t ($R_{t+1}^K < E_t(R_{t+1}^K)$), depositors will be compensated by receiving a higher non-default deposit rate, thus are completely hedged against any plausible realisation of aggregate risk. Crucially, this assumption implies that non-default lending rates will adjust to aggregate shocks relatively slower compared to non-default deposit rates. In other words, the adjustment of lending rates will be relatively stickier in response to a monetary shock as compared to that of deposit rates. As discussed in the Introduction, this proxies realistically the effect of having maturity mismatch in the bank's balance sheet.[19]

Moreover, banks have to hold cash or reserve (the liabilities of central banks) owing to the inability of banks to perfectly forecast their payment flows and to arrange transactions in the interbank market throughout the day so as to maintain settlement balances constant at a balance greater than, or equal to, zero (Goodhart, 2000)). As central banks have monopoly power in issuing 'cash', this give them the power to set the short-term risk free rate. However, as emphasised by Woodford (2000), this power does *not* depend on the *size* of the banks' cash holding. To simplify the analysis, I therefore assume that the size of cash holding by banks is negligibly small, i.e. approaching zero, in the model.

Lastly, I assume for simplicity that the value of equipments required in conducting banking service is zero.

[19] As all the financial contracts in the model have only one period maturity, maturity mismatch in banks' balance sheet cannot be *explicitly* modelled. The assumption of the risk profile of depositors is meant to *implicitly* deliver the effect of having maturity mismatch in the model while, at the same time, maintaining the model's tractability.

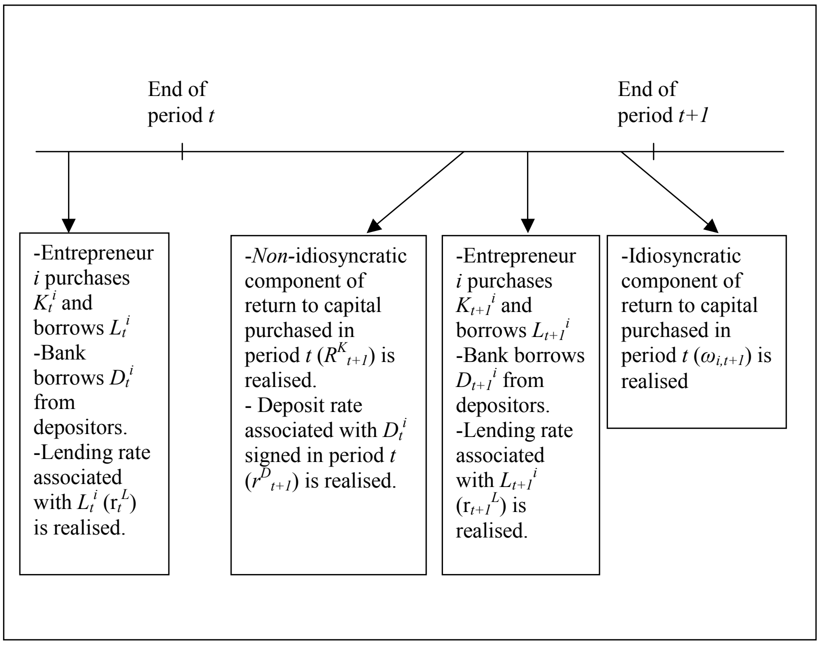


Figure 1:

As mentioned earlier, because the return on bank loan is private information, depositor m has to pay verification cost in the event that bank j announces bankruptcy in order to observe the realised return on the bank's portfolio of loan. The verification cost paid by the depositor is $\theta^D \varpi_{i,t+1} R^i_{t+1} Q_t K^i_t$.

3.1.4 The Timeline

To summarise the structure of the financial contract model, its timeline is shown in figure 1.

At the end of period t, entrepreneur i chooses his optimal demand for capital (K^i_t). To partially finance his investment, he engages in a loan contract with bank j and thus borrows L^i_t. The non-default lending rate associated with the loan contract $(r^L_{i,t})$ is simultaneously determined. In order to fund their lending to firm i, the bank also engages in a deposit contract with depositor m in which case it borrows D^i_t. However, the associated deposit rate is not simultaneously determined.

As time approaches the end of period $t+1$, non-idiosyncratic component of firm i's return on investment invested in period t (R^K_{t+1}) is realised. The deposit rate $(r^D_{i,t+1})$ associated with the deposit contract signed in period t is then realised, which, as emphasised before, implies that the depositor is perfectly hedged against any plausible aggregate risk.

After the aggregate risk (but not idiosyncratic risk) associated with period-t financial contract is uncovered, firm i decides on its optimal purchase of capital and its borrowing from the bank in period $t+1$, K^i_{t+1} and L^i_{t+1} respectively. The corresponding lending rate $(r^L_{i,t+1})$ is simultaneously determined. The bank then borrows D^i_{t+1} from the depositor.

Lastly, the idiosyncratic return to firm i's investment in period t ($\varpi_{i,t+1}$) is realised. In the event that the entrepreneur(the banker) bankrupts, he pays whatever is left to his debtor and departs from the scene.

3.2 The Contract Term

Given that all firms and banks are subject to limited liability clauses, as shown by Gale and Hellwig (1985), optimal loan and deposit contracts with the presence of CSV become those of risky debt contracts. In this section, I study an optimal financial contract amongst a representative firm, a representative bank and a representative depositor and derive a representative firm's optimal demand for capital.

In order to simplify the analysis, I impose the following equilibrium restriction on firm i's and bank j's threshold values of idiosyncratic component of return to firm i's investment.

Restriction 1: $\overline{\varpi}^F_{i,t+1} > \overline{\varpi}^B_{i,t+1}$

Descriptively, this restriction implies that, in the event of no bankruptcy, bank j's total revenue from its lending to firm i has to be sufficiently larger than its total repayment cost to depositor m. In other words, the bank's profit must be sufficiently large in the event that both firm i and bank j do not go bankrupt. Appendix A shows that this restriction holds under two assumptions, both of which are satisfied in the equilibrium under the parameterisation I used for calibration.

Given the above restriction, I find the expected profit functions for firm i, bank j and depositor m in subsection 3.2.1. subsection 3.2.2 then uses these expected profit functions to solve for the optimal demand for capital by firm i.

3.2.1 Finding Expected Profit Functions

A Representative Firm's Expected Profit Function From equation (3), when $\varpi_{i,t+1} \geq \overline{\varpi}^F_{i,t+1}$, the return to firm i's project is sufficient to repay its debt obligation with bank j, $r^L_{i,t}L^i_t$. So the firm does not default. It pays the contractual amount and retains the remaining profit, $[\omega_i Q_t R^K_{t+1} K^i_t - r^L_{i,t}L^i_t]$. However, when $\varpi_i < \overline{\varpi}^F_{i,t+1}$, firm i declares default and liquidates its asset. Bank j then pays verification cost and obtains the liquidation value netting off the verification cost as its revenue. Hence, in period $t+1$, firm i's expected profit function from investing in capital in period t, π^F_{t+1}, conditional solely on the realisation of idiosyncratic risk, is given by[20];

$$\pi^F_{i,t+1} = \int_{\overline{\varpi}^F_{i,t+1}}^{\infty} [\varpi_{i,t+1} Q_t R^K_{t+1} K^i_t - r^L_{i,t} L^i_t] f(\varpi_{i,t+1}) d\varpi_{i,t+1} - W^i_t r^f_{t+1} \tag{6}$$

where r^f_{t+1} is the real risk-free interest rate. The last term represents firm i's opportunity cost. Substituting equation (3) in equation (6), we obtain;

$$\pi^F_{i,t+1} = [\int_{\overline{\varpi}^F_{i,t+1}}^{\infty} \varpi_{i,t+1} f(\varpi_{i,t+1}) d\varpi_{i,t+1} - (1 - F(\overline{\varpi}^F_{i,t+1})\overline{\varpi}^F_{i,t+1}] Q_t R^K_{t+1} K^i_t - W^i_t r^f_{t+1} \tag{7}$$

[20] In period $t+1$, aggregate risk associated with period-t capital investment has been resolved. Therefore, expectation is taken solely over the remaining idiosyncratic risk.

A Representative Bank's Expected Profit Function Bank j's expected profit function, unlike that of firm i, depends in general on the relative value of the threshold $\overline{\varpi}^F_{i,t+1}$and $\overline{\varpi}^B_{i,t+1}$. However, given restriction 1, Appendix A shows that the expected profit function for bank j in period $t+1$, conditional on the realisation of idiosyncratic risk, is given by;

$$\begin{aligned} \pi^B_{i,t+1|\overline{\varpi}^F_{i,t+1} > \overline{\varpi}^B_{i,t+1}} &= \int_{\overline{\varpi}^B_{i,t+1}}^{\overline{\varpi}^F_{i,t+1}} [(1-\theta^B)\varpi_{i,t+1} Q_t R^K_{t+1} K^i_t - r^D_{i,t+1} D^i_t] f(\varpi_{i,t+1}) d\varpi_{i,t+1} \\ &\quad + [1 - F(\overline{\varpi}^F_{i,t+1})][r^L_{i,t} L^i_t - r^D_{i,t+1} D^i_t] - A^i_t r^f_{t+1} \end{aligned} \tag{8}$$

A Representative Depositor's Expected Profit Function Similar to that of bank j, depositor m's expected profit function depends, in general, on the relative value of the threshold $\overline{\varpi}^F_{i,t+1}$and $\overline{\varpi}^B_{i,t+1}$. However, given restriction 1, it can be seen from equations (3) and (5) that when $\varpi_{i,t+1} < \overline{\varpi}^B_{i,t+1}$, both firm i and bank j declare bankrupt. So after paying verification cost, depositor m retains $(1-\theta^D)(1-\theta^B)\varpi_{i,t+1} Q_t R^K_{t+1} K^i_t$. When $\varpi_{i,t+1} \geq \overline{\varpi}^B_{i,t+1}$, bank j does not go bankrupt and therefore does not default on its debt obligation with the depositor. In this case, the depositor gets $r^D_{i,t+1} D^i_t$. The depositor's expected profit function evaluated in period $t+1$, conditional on the realisation of idiosyncratic risk, is therefore given by;

$$\begin{aligned} \pi^D_{t+1|\overline{\varpi}^F_{i,t+1} > \overline{\varpi}^B_{i,t+1}} &= \int_0^{\overline{\varpi}^B_{i,t+1}} [(1-\theta^D)(1-\theta^B)\varpi_{i,t+1} f(\varpi_{i,t+1}) d\varpi_{i,t+1}] R^K_{t+1} Q_t K^i_t \\ &\quad + [1 - F(\overline{\varpi}^B_{i,t+1})] r^D_{i,t+1} D^i_t - D^i_t r^f_{t+1} \end{aligned} \tag{9}$$

where the last term represents the depositor's opportunity cost of fund from depositing his money with the bank.

For notational simplicity, I define the following notations;

$$\Gamma(\overline{\varpi}^F_{i,t+1}) \equiv \int_0^{\overline{\varpi}^F_{i,t+1}} \varpi_{i,t+1} f(\varpi_{i,t+1}) d\varpi_{i,t+1} + [1 - F(\overline{\varpi}^F_{i,t+1})] \overline{\varpi}^F_{i,t+1} \tag{10}$$

$$\Gamma(\overline{\varpi}^B_{i,t+1}) \equiv \int_0^{\overline{\varpi}^B_{i,t+1}} \varpi_{i,t+1} f(\varpi_{i,t+1}) d\varpi_{i,t+1} + [1 - F(\overline{\varpi}^B_{i,t+1})] \overline{\varpi}^B_{i,t+1} \tag{11}$$

$$G(\overline{\varpi}^F_{i,t+1}) \equiv \int_0^{\overline{\varpi}^F_{i,t+1}} \varpi_{i,t+1} f(\varpi_{i,t+1}) d\varpi_{i,t+1} \tag{12}$$

$$G(\overline{\varpi}^B_{i,t+1}) \equiv \int_0^{\overline{\varpi}^B_{i,t+1}} \varpi_{i,t+1} f(\varpi_{i,t+1}) d\varpi_{i,t+1} \tag{13}$$

Using the notations defined above together with equations (3)-(5), after some algebraic manipulations, the expected profit functions of firm i (equation 7), bank j(equation 8) and depositor m(equation 9) can be rewritten as;

$$\pi^F_{t+1} = [1 - \Gamma(\overline{\varpi}^F_{i,t+1})] R^K_{t+1} Q_t K^i_t - W^i_t r^f_{t+1} \tag{14}$$

$$\pi^B_{t+1|\overline{\varpi}^F_{i,t+1} > \overline{\varpi}^B_{i,t+1}} = \left[\Gamma(\overline{\varpi}^F_{i,t+1}) - (1-\theta^B)\Gamma(\overline{\varpi}^B_{i,t+1}) - \theta^B G(\overline{\varpi}^F_{i,t+1})\right] R^K_{t+1} Q_t K^i_t - A^i_t r^f_{t+1} \tag{15}$$

$$\pi^D_{t+1|\overline{\varpi}^F_{i,t+1} > \overline{\varpi}^B_{i,t+1}} = (1-\theta^B)\left[\Gamma(\overline{\varpi}^B_{i,t+1}) - \theta^D G(\overline{\varpi}^B_{i,t+1})\right] R^K_{t+1} Q_t K^i_t - D^i_t r^f_{t+1} \tag{16}$$

3.2.2 Optimal Demand for Capital

Thus far, I have derived the expected profit functions for firm i, bank j and depositor m, where expectation is conditional solely on the idiosyncratic risk, as given in equations (14)-(16) respectively. This section employs these equations to derive firm i's optimal demand for capital, K_t^i.

Given the assumption that depositors are completely averse to aggregate risk, depositors' optimisation requires that their expected profit functions conditional only on idiosyncratic risk must be equal to zero. Thus, from equation (16), the optimal zero expected profit condition for a representative depositor, depositor m, is given by;

$$(1-\theta^B)\left[\Gamma(\overline{\varpi}_{i,t+1}^B)-\theta^D G(\overline{\varpi}_{i,t+1}^B)\right]R_{t+1}^K Q_t K_t^i - D_t^i r_{t+1}^f = 0 \tag{17}$$

Unlike depositors, banks are risk neutral and therefore are willing to bear both aggregate and idiosyncratic sources of risk. Given that banks operate under a perfectly competitive environment, optimality condition for banks requires that their expected profit functions conditional on both aggregate and idiosyncratic sources of risk must be equal to zero. Thus from equation (15), the optimal zero profit condition for bank j is given by;

$$E_t\left[\{\left[\Gamma(\overline{\varpi}_{i,t+1}^F)-(1-\theta^B)\Gamma(\overline{\varpi}_{i,t+1}^B)-\theta^B G(\overline{\varpi}_{i,t+1}^F)\right]R_{t+1}^K\}Q_t K_t^i - A_t^i r_{t+1}^f\right]=0 \tag{18}$$

where $E_t(\cdot)$ denotes expectation taken as of time t

It is important to examine equations (17) and (18) carefully. The assumption that depositors will not bear any aggregate risk implies that as the risk free rate, r_{t+1}^f, rises unexpectedly, ceteris paribus, bank j's threshold $\overline{\varpi}_{i,t+1}^B$ will instantaneously increase via equation (17). Consequently, the non-default deposit rate associated with the deposit contract signed in period t, $r_{i,t+1}^D$, has to be increased correspondingly via equation (5) in order to compensate depositor m for an unexpected rise in his opportunity cost of fund. In contrast, banks are risk neutral and therefore are willing to bear aggregate risk. The non-default lending rate associated with the loan contract signed in period t, $r_{i,t}^L$, will be determined as of period t via equations (18) and (3). As a result, unlike the deposit rate, the lending rate associated with period-t loan contract has been predetermined as of period $t+1$ and therefore will not respond instantaneously to an unexpected monetary shock. Importantly, the result that the lending rate adjusts to an unexpected rise in the risk free rate relatively slower compared to the deposit rate implies that bank j's inside capital has to be depleting as its interest cost rises relatively faster compared to its revenue counterpart. This, as we shall see, underpins the operational mechanism of the bank capital channel of monetary policy transmission in the model.

Similar to banks, firms are risk neutral and therefore are willing to bear both idiosyncratic and aggregate sources of risk. Thus a representative firm, firm i, maximises its expected profit function, where expectation is conditional on both sources of risk, subject to bank j's balance sheet identity (equation (4)) and the zero expected profit conditions of depositor m and bank j (equations (17) and (18), respectively). The maximisation problem taken as given the values of $W_t^i, A_t^i, E_t(R_{t+1}^K), E_t(r_{t+1}^f)$ and Q_t, all of which are to be endogenised in the next section, can be written as follows;

$$\max_{K_t^i, r_{i,t}^D, r_{i,t}^L} E_t \sum_{j=0}^{\infty} \{[1 - \Gamma(\overline{\varpi}_{i,t+1+j}^F)] Q_{t+j} R_{t+1+j}^K K_{t+j}^i - W_{t+j}^i r_{t+1+j}^f\}$$

subject to

$$(1-\theta^B)\left[\Gamma(\overline{\varpi}_{i,t+1}^B) - \theta^D G(\overline{\varpi}_{i,t+1}^B)\right] R_{t+1}^K Q_t K_t^i - D_t^i r_{t+1}^f = 0 \quad \text{(a)}$$

$$E_t\{\left[\Gamma(\overline{\varpi}_{i,t+1}^F) - (1-\theta^B)\Gamma(\overline{\varpi}_{i,t+1}^B) - \theta^B G(\overline{\varpi}_{i,t+1}^F)\right] R_{t+1}^K\} Q_t K_t^i - A_t^i E_t(r_{t+1}^f) = 0 \quad \text{(b)}$$

$$Q_t K_t^i - W_t^i - A_t^i = D_t^i \quad \text{(c)}$$

Firm i chooses its optimal demand for capital by solving the above maximisation problem. Given its net worth, this directly gives rise to a schedule of demand for loan. Constraint (a), the expected zero profit condition for depositor m, then implies a schedule of supply of deposits. As loan and deposit markets are of perfectly competitive environment, given the schedule of demand for loan and the supply of deposit, the equilibrium non-default lending and deposit rates are determined so as the bank's expected zero profit condition given by constraint (b) and the bank's balance sheet identity given by constraint (c) are simultaneously satisfied.[21]

The solution to the maximisation problem is given in Appendix B. As shown in Appendix B, the first order necessary conditions from the maximisation problem yield the following approximated form of the optimal demand for capital[22];

$$\frac{Q_t K_t^i}{(W_t^i + A_t^i)} = \psi(E_t(\frac{R_{t+1}^K}{r_{t+1}^f})) \ , \psi'(\cdot) > 0 \quad (19)$$

[21] Descriptively, this implies that the *equilibrium spread* between the lending and deposit rates are determined so as banks yield zero expected profit, i.e. satisfying constraint (b). Then the *equilibrium level* of the two rates are chosen so as the bank's balance sheet identity holds, i.e. satisfying constraint (c). Thus the model crucially assumes that the lending and deposit rates always *perfectly and costlessly* adjust to their respective equilibrium rates. Although this assumption is widely adopted in the literature (i.e. Bernanke et al. (1999) and Carlstrom and Fuerst (2000), amongst others) as it greatly simplifies the analysis, it is worth mentioning that the assumption is rarely satisfied in reality owing the the prevalence of *information cost*. Banks *cannot perfectly* forecast the *ex post* demand for loans by firms and the supply of deposits from depositors when they *ex ante* announce their lending and deposit rates. Thus, the *ex post* demand for loans ($L_t^i(r_t^L)$) may not equal to the sum of *ex post* supply of deposit and bank capital ($D_t^i(r_t^D)$ + A_t^i). However, bank balance sheets have to be balanced at *any point in time*. This gives rise to the role of 'short term government bond' as a buffer stock, i.e. banks' actual holding of short term government bond may deviate from the optimal holding level, which in general depends on the size of deposit, the spread between loan rates and short term risk free rates, variance of deposit and loan streams.

[22] As shown in detail in Appendix B, the *exact* form of the firm's optimal demand for capital is given by $\frac{Q_t K_t^i}{(W_t^i + A_t^i)}$ = $\Psi_t(E_t(\frac{R_{t+1}^K}{r_{t+1}^f}), \frac{A_t^i}{W_t^i + A_t^i})$, where $\frac{d\Psi_t(\cdot)}{dE_t(R_{t+1}^K / r_{t+1}^f)} > 0$ and $\frac{d\Psi_t(\cdot)}{d[A_t^i/(W_t^i + A_t^i)]} < 0$. The rationale underlying a strictly negative sign of the latter derivative is as follows. When $\frac{A_t^i}{W_t^i + A_t^i}$ is higher (alternatively, $\frac{W_t^i}{A_t^i}$ is lower), ceteris paribus, the agency problem is relatively more severe at the firm-bank level, as compared to the bank-depositor level. This implies that bank j will optimally impose higher *interest rate margin* between non-default lending and deposit rates (via constraint (b)), which in turn would impose greater cost of borrowing on firm i. Consequently, firm i's optimal demand for capital must decline. However, under a reasonable parameterisation, the effect of changes in $\frac{A_t^i}{W_t^i + A_t^i}$ on the firm's optimal demand for capital will be so small that it can be simplified away from the analysis without affecting the result (see Appendix B for detail). More specifically, the firm's optimal demand for capital can be written as $\frac{Q_t K_t^i}{(W_t^i + A_t^i)}$ = $\Psi_t(E_t(\frac{R_{t+1}^K}{r_{t+1}^f}), \frac{A_t^i}{W_t^i + A_t^i}) \simeq \psi(E_t(\frac{R_{t+1}^K}{r_{t+1}^f}))$.

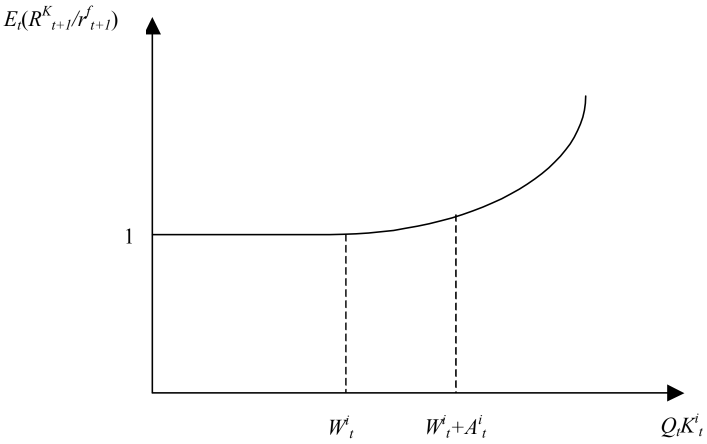


Figure 2:

Equation (19) shows the ratio of firm i's optimal demand for capital to the sum of the firm's and the bank's inside capital as a positive function of external finance premium, $E_t(\frac{R^K_{t+1}}{r^f_{t+1}})$.

To understand the intuition underlying this relationship, it is useful to consider figure 2. From figure 2, when the demand for capital is less than firm i's own net worth, K

capital is higher compared to available inside capital (the sum of entrepreneur i's net worth and bank j's capital), depositor m faces higher demand for his savings from the bank. Given that both entrepreneur i and bank j become more leveraging, depositor m has to request a higher non-default deposit rate in order to compensate for greater perceived risk, which in turn directly imposes higher cost to bank j. Eventually, bank j would have to raise its non-default loan rate in order to satisfy its zero expected profit condition.[24] Given a higher non-default lending rate, firm i's cost of borrowing is higher which implies that he has to require higher return from capital investment. Thus external finance premium, $E_t(\frac{R^K_{t+1}}{r^f_{t+1}})$, rises as more capital is demanded given the same total sum of entrepreneur i's and bank j's inside capital. In all, as firm i and bank j become more leveraging, external finance premium has to rise (through higher non-default lending and deposit rates) in order to induce depositor m to supply more of his savings to finance firm i's investment project.

3.3 Aggregation

Thus far, I have studied the optimal financial contract amongst a representative firm, a representative bank and a representative depositor and have derived a representative firm's optimal demand for capital. As discussed by Bernanke et al. (1999), and Carlstrom and Fuerst (2000) , in general, when demand for capital depends on financial position of agents, aggregation becomes difficult as it depends on the distribution of wealth among firms (similarly for banks). However, as shown by Bernanke et al. (1999), owing to the assumption of constant return to scale throughout the paper, a firm's demand for capital is proportional to its net worth with the factor of proportionality being the same for all firms. In other words, firms will have the same leverage ratio.

$$\frac{Q_t K^i_t}{W^i_t} = \frac{Q_t K^j_t}{W^j_t} = ... = \frac{Q_t K_t}{W_t} \tag{20}$$

where the variables without superscript denote aggregate variables.

Similarly, assuming that all competitive banks are the same in light of their lending and depositing services, their optimal leverage ratios have to be the same. In other words, each bank will optimally choose its lending in the same proportion to its inside capital.

$$\frac{L^i_t}{A^i_t} = \frac{L^j_t}{A^j_t} = ... = \frac{L_t}{A_t} \tag{21}$$

Given equations (20) and (21), the ratio $\frac{W_t + A_t}{Q_t K_t}$ is the same across firms.[25] This implies that the aggregation of entrepreneurs' demand for capital, equation (19), is straightforward.[26] Thus, the aggregate demand for capital as a positive function of external finance premium can be given as follows;

$$\frac{Q_t K_t}{W_t + A_t} = \psi(E_t \left[\frac{R^K_{t+1}}{r^f_{t+1}}\right]) \ , \psi'(E_t \left[\frac{R^K_{t+1}}{r^f_{t+1}}\right]) > 0 \tag{22}$$

[24] Though, as mentioned, the adjustment of lending rate will be relatively stickier compared to that of deposit rate. This, as I shall discuss more below, leads to a depletion of bank capital which in turn raises external finance premium in the subsequent periods, the bank capital channel.

[25] $\frac{W_t + A_t}{Q_t K_t} = \frac{W_t}{Q_t K_t} + (\frac{A_i}{L_t})(\frac{L_t}{Q_t K_t}) = \frac{W_t}{Q_t K_t} + (\frac{A_i}{L_t})(1 - \frac{W_t}{Q_t K_t})$

[26] Note that the aggregation would remain straightforward for the exact form of optimal demand for capital (equation B24 in Appendix B). This is because $\frac{A^i_t}{W^i_t + A^i_t}$ is the same for all i owing to the assumption of constant return to scale firms' and banks' production functions.

Because the ratios $\frac{W_t^i+A_t^i}{Q_tK_t^i}$ and $\frac{A_t^i}{Q_tK_t^i}$ are the same for all i, the zero expected profit conditions for depositors and banks, given in equation (17) and (18) respectively, hold in aggregate. As a result, via equations (3) and (5), the non-default lending rate (non-default deposit rate) charged to different firms (banks) will be the same. The intuition is as follows. Since all firms have the same leverage ratio (equation (20)), they possess the same degree of risk *ex ante*. This implies that banks will charge all firms by the same non-default lending rate. Similarly, as all banks have the same leverage ratio, depositors are exposed to the same degree of risk *ex ante*. As a compensation, they would thus universally charge the same non-default deposit rate to all banks.

Given the above argument, the *aggregate* zero expected profit conditions for depositors and banks as well as the *economy-wide* non-default lending and deposit rates can be given by;

$$(1-\theta^B)\left[\Gamma(\varpi_{t+1}^B)-\theta^D G(\varpi_{t+1}^B)\right]R_{t+1}^K Q_t K_t-(Q_tK_t-W_t-A_t)r_{t+1}^f=0 \tag{23}$$

$$E_t\{\left[\Gamma(\varpi_{t+1}^F)-(1-\theta^B)\Gamma(\varpi_{t+1}^B)-\theta^B G(\varpi_{t+1}^F)\right]R_{t+1}^K\}Q_tK_t-A_tE_t(r_{t+1}^f)=0 \tag{24}$$

$$\varpi_{t+1}^F Q_t R_{t+1}^K K_t = r_t^L L_t \tag{25}$$

$$(1-\theta^B)\varpi_{t+1}^B\ Q_t R_{t+1}^K K_t = r_{t+1}^D D_t \tag{26}$$

In the next section, equations (22)-(26) will be embedded into the general equilibrium setting. As we shall see, they add the source of financial imperfection both in the loan and deposit markets into the otherwise frictionless dynamic general equilibrium model and therefore underpin the operational mechanism of the balance sheet and bank capital channels of monetary policy transmission within the model.

4 General Equilibrium: Embedding financial contracting into the DNK model

In this section, I embed the partial equilibrium analysis developed in section 3 into the otherwise standard DNK model. This would allow the risk free interest rate, return to capital, price of capital, entrepreneurial net worth and bank capital, all of which were taken as given in the previous section, to be endogenised.

As mentioned earlier, there are five types of agent in this economy; entrepreneur, household, bank, retailer and the central bank. Section 3 explained the basic set-up of the first three sectors. However, it addressed only the issue relevant to the financial contract problem. This section completes the task by explaining the remaining, i.e. the production function and household's optimal consumption choice, and illustrating the basic set-up of the retailer and the central bank. However, as only entrepreneurial and banking sectors are non-standard in the DNK literature, the emphasis will be placed on them.

4.1 Entrepreneur: Endogenising the return to capital, price of capital, the evolution of capital and entrepreneurial net worth

In order to motivate the coexistence between aggregate and idiosyncratic sources of risk, as discussed in section 3, entrepreneurs cannot instantaneously use their purchased capital to produce wholesale goods. Thus, capital purchased in period $t-1$ will be used in production, together with labour hired in period t, to produce wholesale output in period t. Assuming constant returns to scale production technology, the aggregate production function is given by;

$$Y_t = T_t K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{27}$$

T_t is exogenous technology shock. K_{t-1} is the aggregate amount of capital purchased in period $t-1$. H_t is labour input hired in period t. Y_t is the aggregate wholesale output.[27]

Assume that entrepreneurs sell their wholesale output to retailers. Let $\frac{1}{X_t}$ be the relative price of wholesale goods. Equivalently, X_t is the gross mark-up of retail goods over wholesale goods. The gross return from holding capital from period $t-1$ to t is given by;

$$R_t^K = \left[\frac{\frac{1}{X_t}\frac{\alpha Y_t}{K_{t-1}} + (1-\varphi)Q_t}{Q_{t-1}} \right] \tag{28}$$

where $\frac{1}{X_t}\frac{\alpha Y_t}{K_{t-1}}$ is the rental payment paid to capital as implied by the Cobb-Douglas production technology, φ is the capital depreciation rate, Q_t is price of capital in period t. In sum, the non-idiosyncratic component of return from holding capital from period $t-1$ to t, R_t^K, is the sum of rental revenue and capital gain netting off depreciated capital stock.

Equation (28) represents the standard downward sloping demand for capital. The higher the level of capital demand is, ceteris paribus, owing to diminishing returns, the lower the return to capital will be. Essentially, this equation has endogenised the non-idiosyncratic component of return to capital, R_t^K, which was taken as given in the previous section.

In contrast to the conventional literature, entrepreneurs in this economy are no longer accessible to unlimited amount of fund at the sole opportunity cost equivalent to the risk free rate. Rather, equation (22) which expresses entrepreneurs' aggregate demand for capital as a function of external finance premium captures the source of financial imperfection in the model. In particular, firms as well as banks ca7(previ)-41quire more external fundnly at a higher extefinalnce premium.

Now I turn to findthe evolution of capital. I simply follow the standardliterature onthis; [28]

$$K_t = \Omega(\frac{I_t}{K_{t-1}})K_{t-1} + (1-\varphi)K_{t-1} \tag{29}$$

where I_t denotes aggregate investment expenditures and φ denotesdepreciation rate.

[27] Notice here that the idiosyncratic risk iscompletely diversi fiedinaggregateThis stemsfrom the assumptionthat $\varpi_{i,t+1}$ is identically andependently distributedross firmsandtime

[28] See,amongothers,KingandWolman(1996),ertler (2000) andBernanke et al (1999) for detail.

Following the standard literature, I assume there are increasing marginal adjustment costs in the production of capital, which is captured by assuming that aggregate investment expenditures (I_t) yields a gross output of new capital goods $\Omega(\frac{I_t}{K_{t-1}})K_{t-1}$, where $\Omega(\cdot)$ is increasing and concave and $\Omega(0) = 0$. The introduction of adjustment cost is made in order to permit a variable price of capital (variable asset price) which in turn will enrich the model dynamics further through the asset price channel. In equilibrium, given the adjustment cost function, the price of a unit of capital in terms of the numeraire goods, Q_t, is given by;[29]

$$Q_t = \left[\Omega'(\frac{I_t}{K_{t-1}})\right]^{-1} \tag{30}$$

Next, I proceed to endogenise aggregate entrepreneurial net worth. As a technical matter, we are required to start entrepreneurs off with some net worth in order to allow them to begin operation. For simplicity, I assume that, in each period, each entrepreneur is endowed with a small endowment, e^F. Moreover, in order to prevent entrepreneurs from accumulating sufficient wealth to become self financed, I assume that each entrepreneur faces a constant probability of dying equal to γ^E. The dying entrepreneurs simply consume their remaining net worth $\left(C_t^E\right)$and die.

Thus we can write the evolution of aggregate entrepreneurial net worth and dying entrepreneurs' consumption respectively as;

$$W_t = (1-\gamma^E)[V_t^E + e^F] \tag{31}$$

$$C_t^E = \gamma^E[V_t^E + e^F] = (\frac{\gamma^E}{1-\gamma^E})W_t \tag{32}$$

where W_t is expected aggregate entrepreneurial net worth available in period t right before period-t capital decision is made. V_t^E is expected entrepreneurial gross profit from investing in capital excluding opportunity cost. The expectation is taken solely on the unrealised idiosyncratic risk associated with the last period capital investment as the aggregate risk component has been resolved. From equation (14), V_t^E can be written as;

$$V_t^E = [1-\Gamma(\varpi_t^F)]Q_{t-1}R_t^K K_{t-1} \tag{33}$$

Substituting equation (33) into equation (31), we obtain the evolution of entrepreneurial net worth equation which is given as follows;

$$W_t = (1-\gamma^E)\{[1-\Gamma(\varpi_t^F)]Q_{t-1}R_t^K K_{t-1} + e^F\} \tag{34}$$

Lastly, analogous to capital, the return to labour (real wage) is equal to the marginal product of labour. This demand for labour condition is given by;

$$\frac{N_t}{P_t} = \left[\frac{1}{X_t}\frac{(1-\alpha)Y_t}{H_t}\right] \tag{35}$$

[29] Following Gertler (2000), there are capital producing firms who use final goods (I_t) together with rented capital to produce new capital goods via the production function $\Omega(\frac{I_t}{K_{t-1}})K_{t-1}$. They then sell the newly produced capital to wholesale good producers at the price Q_t. Capital good firms therefore maximise their profit, $Q_t\Omega(\frac{I_t}{K_{t-1}})K_{t-1} - I_t - Z^k K_{t-1}$, where Z^k is the rental cost. FOC with respect to I_t is given by equation (32). Gertler (2000) showed that, via FOC with respect to K_{t-1}, the value of Z^k, and therefore the capital goods firms' profit will be approximately zero in the neighbourhood of the steady state.

where $\frac{N_t}{P_t}$ denotes real wage in period t.

4.2 Banking Sector: Endogenising the evolution of bank capital

As mentioned in the previous section, banks in this economy are competitive and operate by intermediating savings from depositors to finance borrowing from entrepreneurs. I have established equilibrium conditions for most of the key variables which are relevant to the banking sector treating as given only bank capital. In this section I endogenise bank capital.

Similar to the evolution of entrepreneurial net worth, we need to start off banks with some endowment so that they could begin their operation. I therefore assume that each bank is given a small endowment equal to e^B. Moreover, to prevent banks from accumulating sufficient capital to become self financed, I assume that they face a constant probability of dying, γ^B. The dying banks simply consume all of their remaining capital and depart from the scene.

We can write the evolution of aggregate bank capital and banks' consumption respectively as;[30]

$$A_t = (1-\gamma^B)[V_t^B + e^B] \quad (36)$$

$$C_t^B = \gamma^B[V_t^B + e^B] = \left(\frac{\gamma^B}{1-\gamma^B}\right) A_t \quad (37)$$

where A_t is expected aggregate bank capital in period t. V_t^B is expected banks' gross profit excluding their opportunity cost. Again, the expectation is taken conditional solely on the unrealised idiosyncratic risk associated with the capital investment in the previous period. From equation (24), V_t^B is given as follows;

$$V_t^B = \left[\Gamma(\overline{\varpi}_t^F) - (1-\theta^B)\Gamma(\overline{\varpi}_t^B) - \theta^B G(\overline{\varpi}_t^F)\right] R_t^K Q_{t-1} K_{t-1} \quad (38)$$

Substituting equation (38) into equation (36), I obtain the evolution of bank capital equation.

$$A_t = (1-\gamma^B)[\left[\Gamma(\overline{\varpi}_t^F) - (1-\theta^B)\Gamma(\overline{\varpi}_t^B) - \theta^B G(\overline{\varpi}_t^F)\right] R_t^K Q_{t-1} K_{t-1} + e^B] \quad (39)$$

4.3 Retail sector: Endogenising the evolution of aggregate price

This sector is introduced solely as a means to introduce some form of nominal price stickiness into the model without complicating the aggregation process.[31] This section directly follows Appendix B of Bernanke et al. (1999).

Monopolistic competition is assumed at the retail level. Retailers purchase wholesale output from entrepreneurs, slightly modify them and resell in the form of CES aggregate to households. Let $Y_t(z)$ be the quantity of output sold by retailer z, measured in units of wholesale goods, and let $P_t(z)$

[30] I assume that the cost of raising bank capital directly is prohibitively costly. Thus bank capital can only be accumulated via retain earnings.

[31] Had the retailers not been introduced, entrepreneurs would have to be price setters themselves thereby having to face a downward sloping demand curve. This would have led to non-linearity in an entrepreneur's demand for capital as a function of the sum of entrepreneurial net worth and bank capital which would have complicated the aggregation process.

be the nominal price of retail goods z. Total final usable goods, Y_t^f, are the following composite of individual retail goods;

$$Y_t^f = \left[\int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon} dz \right]^{\frac{\epsilon}{\epsilon-1}} \quad (40)$$

with the elasticity of substitution $\epsilon > 1$. The corresponding price index is given by;

$$P_t = \left[\int_0^1 P_t(z)^{(1-\epsilon)} \right]^{\frac{1}{1-\epsilon}} \quad (41)$$

Final output may be either transformed into a single type of consumption good, invested, or used up in verifying costs.[32] In particular, the economy wide resource constraint is given by;

$$Y_t^f = C_t + C_t^E + C_t^B + I_t + \left[\theta^B \int_0^{\overline{\varpi}_t^F} \varpi_t f(\varpi_t) d\varpi_t + (1-\theta^B)\theta^D \int_0^{\overline{\varpi}_t^B} \varpi_t f(\varpi_t) d\varpi_t \right] Q_{t-1} R_t^K K_{t-1} \quad (42)$$

where C_t^E is dying entrepreneurs' consumption, C_t^B is dying bankers' consumption, $\left[\theta^B \int_0^{\overline{\varpi}_t^F} \varpi_t f(\varpi_t) d\varpi_t + (1-\theta^B)\theta^D \int_0^{\overline{\varpi}_t^B} \varpi_t f(\varpi_t) d\varpi_t \right] Q_{t-1} R_t^K K_{t-1}$ is aggregate resource used up as verification cost.[33]

Given the index that aggregates individual retail goods into final goods, equation (41), the demand curve faced by each retailer is given by;

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t^f \quad (43)$$

To introduce price stickiness into the model, it is assumed that retailers can adjust their selling price only with probability $1-\rho$ in a given period (Calvo, 1983). Let P_t^* denote the price set by retailers who are able to change prices at t, and let $Y_t^*(z)$ denote the demand corresponding to this price. Retailer z chooses his price, $P_t(z)$, to maximise expected discounted profits taken as given the demand curve and the price of wholesale goods, P_t^w. The retailers' expected discounted profit is given by;

$$\sum_{k=0}^{\infty} \rho^k E_{t-1} \left[\Lambda_{t,k} \frac{P_t^* - P_t^w}{P_t} Y_{t+k}^*(z) \right] \quad (44)$$

where the discount rate $\Lambda_{t,k} \equiv \varkappa \frac{C_t}{C_{t+k}}$ is households' (i.e. shareholders) intertemporal marginal rate of substitution and $P_t^w \equiv \frac{P_t}{X_t}$ is the nominal price of wholesale goods.

The optimal price setting is obtained by differentiating the objective function, equation (44), with respect to P_t^*. This implies that the optimally set price satisfies;

[32] In general, aggregate final output, Y_t^f, differs from aggregate wholesale output, Y_t. However, as shown by Gertler (2000), they are approximately the same in the neighbourhood of the steady state. Hence, in the simulation analysis they will be treated as the same.

[33] Technically, assumption 4 implies the following;
$(\theta^D - \theta^B) \int_0^{\overline{\varpi}_t^F} \varpi_t f(\varpi_t) d\varpi_t > (1-\theta^B)\theta^D \int_0^{\overline{\varpi}_t^B} \varpi_t f(\varpi_t) d\varpi_t$
This states that the expected benefit from having financial intermediaries which arises from the fact that banks can verify the outcome of projects relatively cheaper compared to depositors (LHS) is greater than the expected cost of having intermediaries which arises from the fact that depositors have to pay extra cost of monitoring the monitors in certain states of the world (RHS).

$$\sum_{k=0}^{\infty}\rho^k E_{t-1}\left[\Lambda_{t,k}(\frac{P_t^*}{P_{t+k}})^{-\epsilon}Y_{t+k}^*(z)\left(P_t^* - \frac{\epsilon}{(\epsilon-1)}P_t^w\right)\right] = 0 \tag{45}$$

Intuitively, retailers set their prices so that expected discounted marginal revenue equals expected discounted marginal cost, given the constraint that the nominal price is fixed in period k with probability ρ^k. Given that the fraction ρ of retailers do not change their price in period t, the aggregate price evolves according to;

$$P_t = \left[\rho P_{t-1}^{1-\epsilon} + (1-\rho)\left(P_t^*\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{46}$$

where P_t^* satisfies equation (45)

Equations (45) and (46) form the evolution of aggregate price.

4.4 Household sector: Endogenising optimal consumption path and labour supply

Households in this model are standard. They consume CES aggregate of retail goods, save, and supply their labour. They save through depositing their money with banks, given that the opportunity cost of fund is equal to the risk free rate. In addition, because retailers are monopolistic competitors, they will earn positive profit in equilibrium. I assume that this positive profit is transferred to households. In other words, I assume that households own retail firms. A representative household's problem is given by;

$$\max_{C_t, H_t, \frac{M_t}{P_t}, D_t} E_t\{\sum_{j=0}^{\infty}\varkappa^j\left[\ln(C_{t+j}) + \varrho\ln(1-H_{t+j})\right]\}$$

subject to

$$C_{t+1} = \frac{N_{t+1}}{P_{t+1}}H_{t+1} + R_{t+1}^D D_t - D_{t+1} + \Pi_{t+1}$$

where $\varkappa$ is household's coefficient of relative impatience, C_t is household's consumption, D_t is interest-rate-earning deposit (in real term) held at the bank in period t, $\frac{N_t}{P_t}$ is real wage, H_t is household labour, Π_t is dividends received from owning retail firms. R_{t+1}^D is the actual rate of return on depositing money with banks which will not be realised until period $t+1$.[34]

Recall that depositors, although perfectly hedged against any realisation of aggregate risk, are still exposed to idiosyncratic risk. Importantly, as of period t, the expectation of the actual rate of return on deposit $(E_t(R_{t+1}^D))$ conditioning on the realisation of aggregate and idiosyncratic risk must be equal to the expected real risk free rate, $E_t(r_{t+1}^f)$, in order to satisfy the depositors' zero expected profit function as implied by equation (23). This implies that $E_t\left(R_{t+1}^D\right) = E_t(r_{t+1}^f)$. As a result, the solution to the above optimisation problem yields the following two standard first order conditions;

[34] Note here that the budget constraint is evaluated at the end of period $t+1$. Thus real consumption, new deposit contract are financed by realised return on deposit invested last period (period t), real labour income, and profit redistributed from retailers.

$$\frac{1}{C_t} = E_t \left[\varkappa \frac{r_{t+1}^f}{C_{t+1}} \right] \tag{47}$$

$$\frac{N_t}{P_t} \frac{1}{C_t} = \varrho \frac{1}{1 - H_t} \tag{48}$$

Equation (47) is a standard inter-temporal consumption Euler equation. Equation (48) is a standard intratemporal Euler equation between household's consumption and labour supply.

4.5 The Central Bank: Endogenising the risk free rate

The central bank sets the nominal risk free rate via a variant of Taylor rule.[35] Define nominal gross risk free rate, r_t^{nf}, as;

$$r_t^{nf} \equiv r_t^f E_t(\frac{P_{t+1}}{P_t}) \tag{49}$$

A form of Taylor's rule is given by;

$$r_t^{nf} = f(Y_t, \frac{P_{t+1}}{P_t}, ...) \tag{50}$$

5 Equilibrium and The Completely Log-linearised Version of the Model

Equilibrium is defined as an allocation $\{Y_t, C_t, C_t^E, C_t^B, I_t, K_t, W_t, A_t, H_t\}_{t=0}^{\infty}$ together with a vector of price variables $\{\overline{\varpi}_t^F, \overline{\varpi}_t^B, R_t^K, Q_t, r_t^f, r_t^{nf}, r_t^L, r_t^D, X_t, P_t, P_t^*, N_t\}_{t=0}^{\infty}$ satisfying equations (22)-(26), (27)-(30), (32), (34)-(35), (37),(39), (42), (45)-(50), given a sequence of the initial values of a vector of the model's state variables $\{Q_{-1}, K_{-1}, W_{-1}, A_{-1}, r_{-1}^L, r_{-1}^{nf}, P_{-1}\}$, a sequence of a vector of exogenous process $\{T_t\}_{t=0}^{\infty}$ and a sequence of interest rate shock $\{\varepsilon_t^r\}_{t=0}^{\infty}$.

In order to study the dynamic response of the model to a monetary shock, I log-linearise the model around the unique stationary steady state equilibrium. Define $\pi_t \equiv P_t - P_{t-1}$and let the variables with a tilda (˜) denote percentage deviations from the steady state and those without a time subscript denote the steady state values, the completely log-linearised version of the model around the steady state can be given by the following 20 equations in 20 variables. They are divided into 5 blocks of equations: 1) aggregate demand; 2) aggregate supply; 3) financial market; 4) evolution of state variables; and 5) monetary policy rule and exogenous process.

1) Aggregate demand

[35] The central bank has power to set the nominal risk free rate due to the assumption that banks have to hold cash, though the amount is assumed to be approaching zero (see footnote 18).

$$\widetilde{Y}_t = \frac{C}{Y}\widetilde{C}_t + \frac{C^E}{Y}\widetilde{C}^E_t + \frac{C^B}{Y}\widetilde{C}^B_t + \frac{I}{Y}\widetilde{I}_t + a_1[\widetilde{Q}_{t-1} + \widetilde{K}_{t-1} + \widetilde{R}^K_t] + a_2\widetilde{\overline{\varpi}}^F_t + a_3\widetilde{\overline{\varpi}}^B_t \tag{L1}$$

$$E_t(\widetilde{C}_{t+1}) = \widetilde{C}_t + E_t(\widetilde{r}^f_{t+1}) \tag{L2}$$

$$\widetilde{C}^E_t = \widetilde{W}_t \tag{L3}$$

$$\widetilde{C}^B_t = \widetilde{A}_t \tag{L4}$$

$$\widetilde{R}^K_t + \widetilde{Q}_{t-1} = (1-b_1)[\widetilde{Y}_t - \widetilde{X}_t - \widetilde{K}_{t-1}] + b_1\widetilde{Q}_t \tag{L5}$$

$$\xi[E_t(\widetilde{R}^K_{t+1}) - E_t(\widetilde{r}^f_{t+1})] = \{\widetilde{Q}_t + \widetilde{K}_t - \left(\frac{W}{W+A}\right)[\widetilde{W}_t + \widetilde{A}_t]\} \tag{L6}$$

$$\widetilde{Q}_t = \epsilon^Q[\widetilde{I}_t - \widetilde{K}_{t-1}] \tag{L7}$$

2) Aggregate Supply

$$\widetilde{Y}_t = \widetilde{T}_t + \alpha\widetilde{K}_{t-1} + (1-\alpha)\widetilde{H}_t \tag{L8}$$

$$\widetilde{Y}_t = \widetilde{C}_t + \widetilde{X}_t + (1+\frac{1}{\epsilon^H})\widetilde{H}_t \tag{L9}$$

$$\widetilde{\pi}_t = \varkappa E_t(\widetilde{\pi}_{t+1}) - u\widetilde{X}_t \tag{L10}$$

3) Financial Market

$$(\frac{K}{W+A} - 1)[\widetilde{r}^f_t - \widetilde{R}^K_t] + \widetilde{Q}_{t-1} + \widetilde{K}_{t-1} - \frac{W}{W+A}\widetilde{W}_{t-1} - \frac{A}{W+A}\widetilde{A}_{t-1} - l_1\widetilde{\overline{\varpi}}^B_t = 0 \tag{L11}$$

$$(\frac{K}{W+A} - 1)[\widetilde{R}^K_t + \widetilde{\overline{\varpi}}^B_t - \widetilde{r}^D_t] - (\widetilde{Q}_{t-1} + \widetilde{K}_{t-1}) + \frac{W}{W+A}\widetilde{W}_{t-1} + \frac{A}{W+A}\widetilde{A}_{t-1} = 0 \tag{L12}$$

$$j_1E_t(\widetilde{\overline{\varpi}}^F_{t+1}) - j_2E_t(\widetilde{\overline{\varpi}}^B_{t+1}) + E_t(\widetilde{R}^K_{t+1}) + \widetilde{Q}_t + \widetilde{K}_t - \widetilde{A}_t - E_t(\widetilde{r}^f_{t+1}) = 0 \tag{L13}$$

$$\widetilde{\overline{\varpi}}^F_t - (\frac{1}{\frac{K}{W}-1})[\widetilde{Q}_{t-1} + \widetilde{K}_{t-1}] + \widetilde{R}^K_t = \widetilde{r}^L_{t-1} - (\frac{1}{\frac{K}{W}-1})\widetilde{W}_{t-1} \tag{L14}$$

4) Evolution of state variables

$$\widetilde{K}_t = \varphi\widetilde{I}_t + (1-\varphi)\widetilde{K}_{t-1} \tag{L15}$$

$$\widetilde{W}_t = c_1(\widetilde{Q}_{t-1} + \widetilde{K}_{t-1} + \widetilde{R}^K_t) + c_2\widetilde{\overline{\varpi}}^F_t \tag{L16}$$

$$\widetilde{A}_t = d_1(\widetilde{Q}_{t-1} + \widetilde{K}_{t-1} + \widetilde{R}^K_t) + d_2\widetilde{\overline{\varpi}}^F_t + d_3\widetilde{\overline{\varpi}}^B_t \tag{L17}$$

5) Monetary policy rule and exogenous processes

$$\widetilde{r}^f_t = \widetilde{r}^{nf}_t - E_t(\widetilde{\pi}_{t+1}) \tag{L18}$$

$$\widetilde{r}^{nf}_t = g_1\widetilde{r}^{nf}_{t-1} + g_2\widetilde{\pi}_{t-1} + \varepsilon^r_t \tag{L19}$$

$$\widetilde{T}_t = v_2\widetilde{T}_{t-1} \tag{L20}$$

where
$a_1 \equiv [\theta^B G(\overline{\varpi}^F) + (1-\theta^B)\theta^D G(\overline{\varpi}^B)]R^K\frac{K}{Y}$

$a_2 \equiv \theta^B \overline{\varpi}^F G'(\overline{\varpi}^F) R^K \frac{K}{Y}$
$a_3 \equiv (1-\theta^B)\theta^D \overline{\varpi}^B G'(\overline{\varpi}^B) R^K \frac{K}{Y}$
$b_1 \equiv \frac{(1-\varphi)}{\frac{\alpha}{X}\frac{Y}{K}+(1-\varphi)}$
$c_1 \equiv (1-\gamma^E) R^K \frac{K}{W}[1-\Gamma(\overline{\varpi}^F)]$
$c_2 \equiv -(1-\gamma^E) R^K \frac{K}{W} \overline{\varpi}^F \Gamma'(\overline{\varpi}^F)$
$d_1 \equiv (1-\gamma^B) R^K \frac{K}{A}[\Gamma(\overline{\varpi}^F) - (1-\theta^B)\Gamma(\overline{\varpi}^B) - \theta^B G(\overline{\varpi}^F)]$
$d_2 \equiv (1-\gamma^B) R^K \frac{K}{A} \overline{\varpi}^F [\Gamma'(\overline{\varpi}^F) - \theta^B G'(\overline{\varpi}^F)]$
$d_3 \equiv -(1-\gamma^B)(1-\theta^B) R^K \frac{K}{A} \overline{\varpi}^B \Gamma'(\overline{\varpi}^B)$
$\xi \equiv \frac{\psi'(\frac{R^K}{r^f})}{\psi(\frac{R^K}{r^f})} \frac{R^K}{r^f}$
$u \equiv \frac{(1-\rho)}{\rho}(1-\varkappa\rho)$
$\epsilon^Q \equiv \frac{\partial Q}{\partial \frac{I}{K}} \frac{\frac{I}{K}}{Q}$
$\epsilon^H \equiv \frac{\partial H}{\partial \frac{N}{P}} \frac{\frac{N}{P}}{H}$
$j_1 \equiv \left[\frac{\Gamma'(\overline{\varpi}^F) - \theta^B G'(\overline{\varpi}^F)}{\Gamma(\overline{\varpi}^F) - (1-\theta^B)\Gamma(\overline{\varpi}^B) - \theta^B G(\overline{\varpi}^F)}\right] \overline{\varpi}^F$
$j_2 \equiv \left[\frac{(1-\theta^B)\Gamma'(\overline{\varpi}^B)}{\Gamma(\overline{\varpi}^F) - (1-\theta^B)\Gamma(\overline{\varpi}^B) - \theta^B G(\overline{\varpi}^F)}\right] \overline{\varpi}^B$
$l_1 \equiv \left[\frac{\Gamma'(\overline{\varpi}^B) - \theta^D G'(\overline{\varpi}^B)}{\Gamma(\overline{\varpi}^B) - \theta^D G(\overline{\varpi}^B)}\right] \left(\frac{K}{W+A} - 1\right) \overline{\varpi}^B$

Equation (L1) is the log-linearised version of equation (42), the economy wide resource constraint. The variation in aggregate output depends on the variation in consumption, investment, dying entrepreneur's and dying banker's consumption and the aggregate expected monitoring cost.[36] Equation (L2) is the log-linearised version of equation (47), the standard forward-looking consumption Euler equation. Equations (L3) and (L4) are the log-linearised version of equations (32) and (37), respectively. They imply that the variation in (dying) entrepreneur's and (dying) banker's consumption depends on the variation in the respective values of their inside capital.

Equations (L5)-(L7) characterise investment demand. They are the log-linearised version of equations (28), (22) and (30), respectively. Equations (L5) and (L7) are conventional in the DNK literature. While the former implies a standard downward sloping demand for capital, the latter relates investment demand to the price of capital. Equation (L6) is unconventional in the frictionless monetary model. It implies that the variation in external finance premium increases as the variation in aggregate demand for capital is higher compared to that of aggregate sum of entrepreneurial net worth and bank capital.

Equations (L8)-(L10), all of which are standard in the DNK literature, represent the aggregate supply block of the model. Equation (L8) is the log-linearised version of equation (27), the production function. Equation (L9) characterises the labour market equilibrium. It is obtained by equating the log-linearised aggregate labour demand (equation (

and the equation which characterises the equilibrium banks' threshold value of idiosyncratic component ($\tilde{\overline{\varpi}}_t^B$) (equation (26)), respectively. Equation (L11) implies that, ceteris paribus, an increase in the variation of aggregate capital demand relative to the sum of the aggregate entrepreneurial net worth and bank capital in the previous period (i.e. both firms and banks become more leveraging) will result in a higher variation of the current value of $\tilde{\overline{\varpi}}_t^B$. This implies, via equation (L12), that the variation in the non-default deposit rate in this period has to rise. Thus, equations (L11) and (L12) together imply that the variation in deposit rate will respond positively to an increase in the variation of the leverage ratio of firms and banks.

Equations (L13) and (L14) are the log-linearised version of the aggregate banks' zero profit condition (equations (24)), and the equation which characterises the equilibrium firms' threshold value $\tilde{\overline{\varpi}}_t^F$ (equation (25)), respectively. They together imply that, ceteris paribus, the variation in non-default lending rate is a positive function of the variation in non-default deposit rate. This is because a higher deposit rate imposes greater borrowing cost on banks. In order to maintain their optimal zero expected profit condition, they must increase the lending rate correspondingly. However, it is very crucial to notice from these two equations that the response of non-default lending rate to an unanticipated increase in the deposit rate will be subject to a one period lag. This implies that variation in the non-default lending rate will be relatively stickier compared to that of the non-default deposit rate in response to any unanticipated aggregate shock.

Equation (L15) is the log-linearised version of equation (29), the standard evolution of capital equation. Equations (L16) and (L17) are the log linearised version of equations (34) and (39), respectively. They are the transition equations for the aggregate entrepreneurial net worth and bank capital. Equation (L16) implies that the variation in the aggregate entrepreneurial net worth depends positively on the variation in the last-period price of capital, return to capital, demand for capital and negatively on the current value of firm's threshold value $\tilde{\overline{\varpi}}_t^F$. Equation (L17) implies that the aggregate bank capital is a positive function of the last period price of capital, return to capital, demand for capital, the current firm's threshold value $\tilde{\overline{\varpi}}_t^F$ and is a negative function of the current value of banks' threshold value $\tilde{\overline{\varpi}}_t^B$.

It should now become clear how adding financial imperfection at the firm and bank levels works to enrich the dynamic of the model. Equations (L11)-(L14), which represent the financial market block of the model, effectively link entrepreneurial net worth and bank capital to the equilibrium non-default lending and deposit rates. These links underpin the mechanism by which *financial* position of firms and banks works to augment the *real* investment decision of firms. This mechanism, which is completely absent in frictionless models, is captured by equation (L6).[37] Equations (L16) and (L17) then characterise the evolution of firm's and bank's financial positions, entrepreneurial net worth and bank capital, respectively.

[37] A rise in capital investment demand compared to the sum of aggregate entrepreneurial net worth and bank capital, i.e both firms and banks become more leveraging, implies, via equations (L11) and (L12), that the non-default deposit rate has to be higher in order to induce depositors to be willing to supply more of their savings. A higher non-default deposit rate implies a rise in the cost of borrowing for banks. In order to maintain their zero expected profit condition, they in turn have to raise their non-default lending rate (via equations (L13) and (L14)). Given higher opportunity cost of external fund, i.e. a higher lending rate, firms have to require higher return to capital in order to justify their investment. Thus external finance premium ($E_t(R_{t+1}^K) - E_t(r_{t+1}^f)$) rises as firms and banks become more leveraging. This is captured by equation (L6).

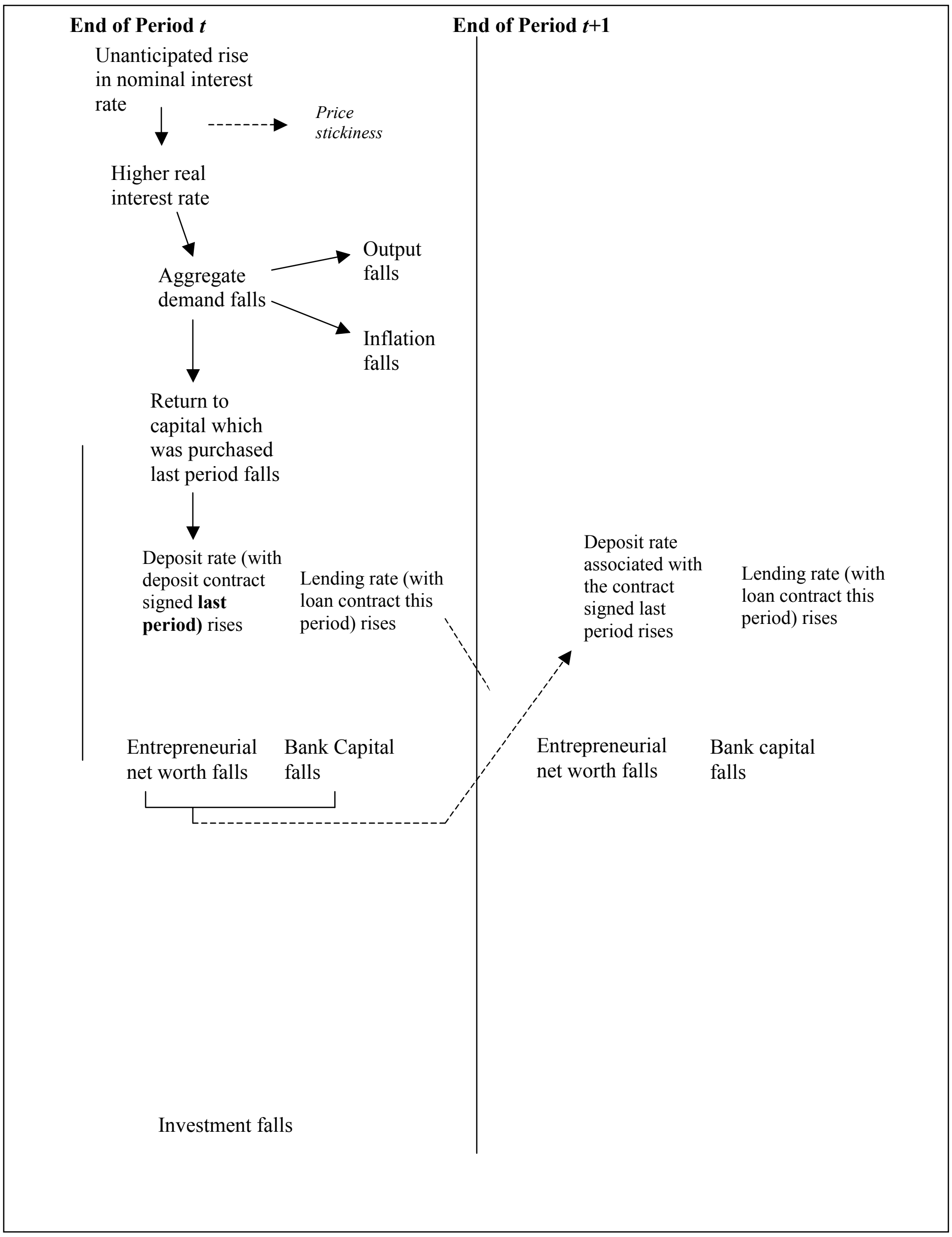
Equations (L18) and (L9) are the log linearised version of equations (49) and (50), respectively. The former relates the nominal risk free rate to the real risk free rate. The latter is a variant of Taylor-type interest rate rule. Following Bernanke et al. (1999), I consider the rule in which the central bank sets the current nominal risk-free interest rate as a function of lagged inflation and lagged nominal interest rate. Essentially this implies that the central bank puts zero weight on the output stabilisation objective.[38] This is intended to highlight the financial accelerator effect. As Bernanke et al. (1999) emphasised, the greater the extent to which monetary policy can stabilise output, the smaller is the role of any kind of propagation mechanism in amplifying and propagating business cycles.

Lastly, equation (L20) characterises the exogenous process of the variation in technology. As the main focus of this paper is on the transmission mechanism of monetary policy, I will not analyse the dynamic responses of the model to shocks in technology.

In the next section, I elaborate how the transmission of monetary policy works in the model, highlighting the explicit role that bank capital plays in the transmission.

6 The Transmission Mechanism of Monetary Policy: The Role of Bank Capital

Figure 3 summarises how bank capital works to enrich the transmission mechanism of monetary policy in the model. Owing to price stickiness, an unexpected rise in the nominal interest rate at the end of period t results in a higher real risk free rate. Consumption and thus aggregate demand fall due to the intertemporal substitution effect. Because capital stock has to be purchased one period in advance, an unexpected decline in aggregate demand causes the return to capital purchased last period to fall. As depositors are completely hedged against any realisation of aggregate risk (recall that they are completely risk averse to any aggregate risk by assumption), the non-default deposit rate associated with deposit contracts signed last period has to rise instantaneously in order to compensate them for the lower-than-expected realisation of return to capital, $R_t^K < E_{t-1}(R_t^K)$, as well as the higher-than-expected realisation of their opportunity cost of fund, $r_t^f > E_{t-1}(r_t^f)$. This directly imposes higher cost of borrowing on banks. In response, banks have to raise their non-default lending rate. However, they could only do so with loan contracts newly signed this period (period t) as a compensation to a higher expected deposit rate in period $t+1$. This is because the lending rate associated with loan contracts signed last period has already been determined in period $t-1$ (recall that banks are risk neutral and therefore are willing to bear aggregate risk). Given a higher lending rate associated with loan contracts in period t, firms face higher cost of borrowing which in turn implies that external finance premium, $E_t(\frac{R_{t+1}^K}{r_{t+1}^f}$


End of Period t
End of Period t+1
Unanticipated rise in nominal interest rate
Price stickiness
Higher real interest rate
Aggregate demand falls
Output falls
Inflation falls
Return to capital which was purchased last period falls
Deposit rate (with deposit contract signed last period) rises
Lending rate (with loan contract this period) rises
Entrepreneurial net worth falls
Bank Capital falls
Investment falls
Deposit rate associated with the contract signed last period rises
Lending rate (with loan contract this period) rises
Entrepreneurial net worth falls
Bank capital falls

rate, the interest rate cost of borrowing rises faster compared to its revenue counterpart causing bank capital in period t to decline. The decline of firms' and banks' inside capital means that both firms and banks have less to contribute to firms' investment projects which in turn implies that depositors are exposed to greater agency cost. As a compensation, the non-default deposit rate associated with deposit contracts signed in period t (which will not be realised until period $t+1$) has to rise. As the non-default deposit rate rises, banks will have to increase their lending rate, though with lag, which in turn implies greater cost of borrowing to firms. This higher borrowing cost faced by firms works to constraint demand for capital, investment and aggregate output in the next period $(t+1)$ in the form of higher external finance premium. A kind of multiplier effect arises because a higher non-default lending rate, together with a lower price of capital, decrease entrepreneurial net worth further. Moreover, after bank capital falls in the initial period, it then slowly accumulates back to trend at the rate equivalent to the risk free interest rate (banks' opportunity cost of fund). Given that the accumulation process is slow enough, bank capital will be persistently below trend. The negative effect of persistent decline in bank capital and entrepreneurial net worth then feeds into the subsequent periods by aggravating the deposit rate, the lending rate and therefore external finance premium which in turn works to depress demand for capital, investment and aggregate output further.

In sum, the transmission mechanism of monetary policy implied by the model exhibits the unconventional monetary policy transmission channel, the bank capital channel. This channel arises because a negative monetary shock causes bank capital to decline persistently. The dynamic interplay amongst declining bank capital, entrepreneurial net worth, and asset price exacerbates the extent of agency cost and increases external finance premium (through higher deposit and lending rates) which in turn works to amplify as well as propagate the negative effect of monetary shocks on investment and aggregate output. In the next section, I turn to the model simulation and shows quantitatively how the transmission of monetary policy operates in the model.

7 Model Simulation

7.1 Calibration

The model is calibrated at a quarterly frequency. The values assigned to most of the parameters relevant to preference, technology and price stickiness are standard in the DNK literature. The discount factor, $\varkappa$, is set to be equal to 0.99, which implies an annualised real interest rate of 4 percent. The depreciation rate, φ, is set to 2.5 percent. I select the steady state capital share, α, to be 0.35. I choose the labour supply elasticity, ϵ^H, to be 3 and, following Bernanke et al. (1999), the elasticity of the price of capital with respect to the investment capital ratio, ϵ^Q , to be 0.25. The elasticity of substitution, ϵ, is set so as the steady state mark-up price X is equal to 1.05. The probability that a retail firm does not change its price in a given period, ρ, is chosen to be 0.75, implying an average price duration of one year. The autoregressive parameters in the policy rule, g_1 and g_2, are set to 0.9 and 0.11, respectively.

The unconventional choices of parameterisation are those relevant to the financial contract problem. They are meant to be suggestive and therefore do not necessarily match with empirical evidence. Following Bernanke et al. (1999), I assume the idiosyncratic component of a firm's return on capital investment (ϖ) to be log-normally distributed with variance, σ^2, equal to 0.28. The endowment given to each firm and bank as a proportion of its inside capital, $\frac{e^E}{W}$ and $\frac{e^B}{A}$, is set to 0.01.[39] The proportional factors of monitoring cost paid by a firm (θ^F) and a bank (θ^B) are set to 0.12 and 0.16,

[39] Its magnitude is so small that it does not affect the dynamics of the model.

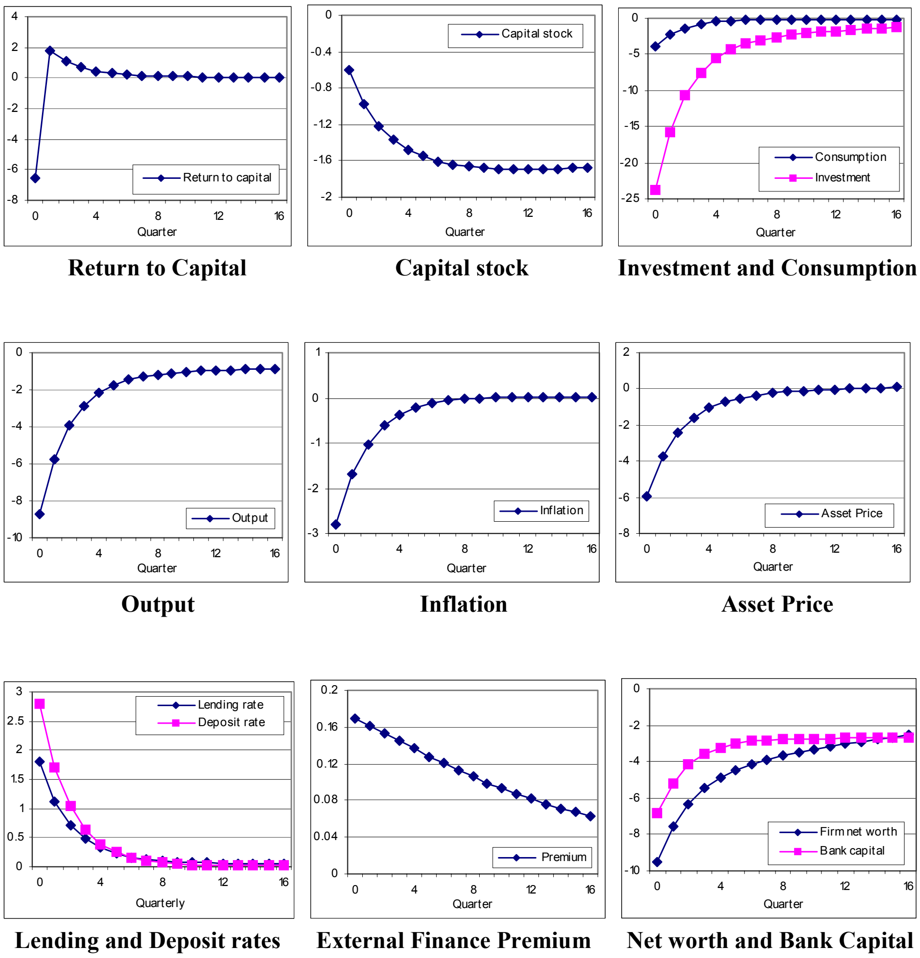


Figure 4: Response of the model to an interest rate shock

respectively.[40] I choose the bankruptcy rates for firms ($F(\varpi^F)$)and banks ($F(\varpi^B)$) to be 0.03 and 0.01, respectively. In order to obtain these steady state equilibrium parameter values, I set the death rate for a firm (γ^E) and bank (γ^B) to be 0.0269 and 0.0197, respectively.

7.2 Simulation result of the model to an interest rate shock

Figure 4 shows the response of the model to an unanticipated rise in the nominal interest rate by 1 percent from the steady state. The result lends support to the theoretical argument that I have illustrated in the previous section concerning how bank capital and entrepreneurial net worth work to enrich the transmission of monetary policy in the model.

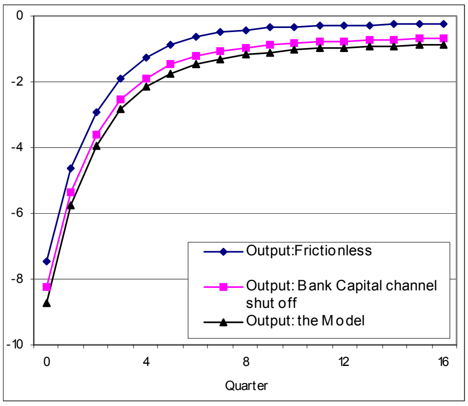
[40] The value of θ^F and θ^B are chosen so as the gap between the two values is large enough to ensure a gain from having banks in the economy (Assumption 4); i.e. banks perform a role in minimising expected aggregate verification cost.

In response to a negative monetary shock, investment and consumption fall. Evidently, investment falls relatively more dramatically and more persistently compared to consumption. This implies that the decline in investment is the main driving force in causing aggregate demand, and thus aggregate output, to decrease. The reason is because the amplification and propagation mechanisms via bank capital, balance sheet and asset price channels operate primarily through investment variable. As mentioned, because capital investment is made one period in advance, an unexpected decline in aggregate demand causes the return to capital to fall instantaneously (approximately seven percent from the steady state). Given a lower than expected realisation of return to capital as well as a higher than expected realisation of the risk free rate[41], depositors have to be compensated by being offered with a higher non-default deposit rate. Thus the deposit rate associated with last period deposit contracts increases instantaneously. However, banks cannot increase their lending rate associated with last period contracts as it has already been predetermined. As can be seen from figure 3, the deposit rate remains higher than the steady state value in the subsequent periods, thus, given rational expectation, banks will increase their non-default lending rate associated with loan contracts newly signed in this period to compensate for an expected higher cost of borrowing. As the lending rate is higher, external finance premium has to rise as firms face higher cost of borrowing from banks. A higher external finance premium works to decrease demand for capital, asset price and therefore investment. This amplification effect via investment explains why investment falls so dramatically in response to the shock. The propagation mechanism arises because a lower than expected realisation of return to capital decreases entrepreneurial net worth. Moreover, given that the lending rate adjusts to the shock relatively slower compared to the deposit rate, bank capital declines. The decrease in both entrepreneurial net worth and bank capital implies higher agency cost faced by depositors, which in turn increases the deposit rate, the lending rate (with lag) and external finance premium in the following periods. This causes further decline in demand for capital, asset price, investment, output, and therefore entrepreneurial net worth and bank capital in the subsequent periods. This kind of multiplier effect continues until entrepreneurial net worth and bank capital revert back to trend as dying firms and banks leave the market. As can be seen from figure 4, the reversion process is slow enough to make external finance premium persists above trend and therefore investment and aggregate output to persist below trend.

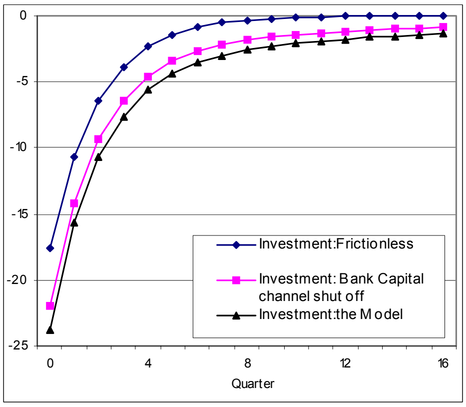
The importance of the role of bank capital in amplifying as well as propagating responses of aggregate economic activities to a monetary shock can be seen from figure 5. In the figure, I compare the dynamic responses of the model to a negative monetary shock with those of the frictionless model and the model with bank capital channel turned off. In the frictionless model, the role of entrepreneurial net worth and bank capital is completely shut off and firms can borrow external finance premium at the sole opportunity cost equivalent to the risk free rate. Thus the transmission mechanism of monetary policy in the frictionless model relies solely on the conventional interest rate channel. For the model in which bank capital channel is shut off, I ignore the assumption that depositors are averse to aggregate risk but assume instead that they are risk neutral. As a result, similar to banks, they are willing to bear aggregate risk. This eliminates the operational mechanism of bank capital channel that I discussed in the previous section as the adjustment of lending rate to aggregate shock is no longer stickier compared to that of deposit rate. This implies that, in response to a negative monetary shock, the interest rate cost may not necessarily rise faster compared to its revenue counterpart and thus bank capital may not decline. Put differently, turning off the bank capital channel is analogous to assuming away the effect of having maturity mismatch in banks' balance sheet.

As can be seen from the figure, adding bank capital channel amplifies as well as propagates the effect of a negative monetary shock on aggregate output and investment. Evidently, the decline of

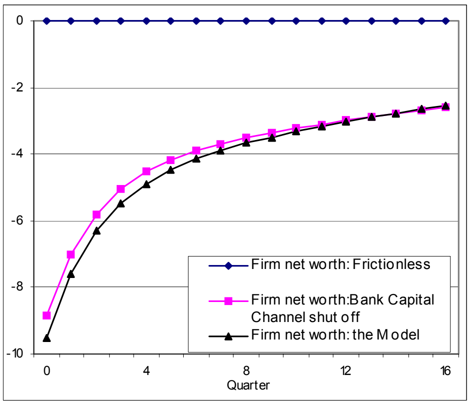
[41] Recall that the risk free rate is depositors' opportunity cost of fund.



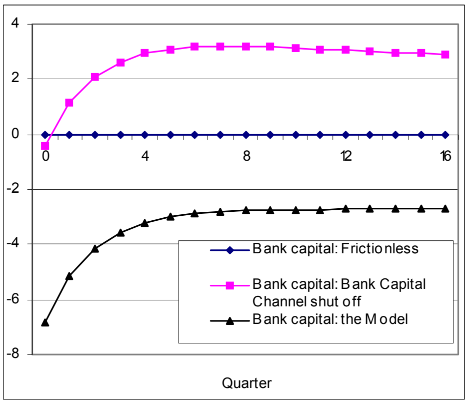
Output



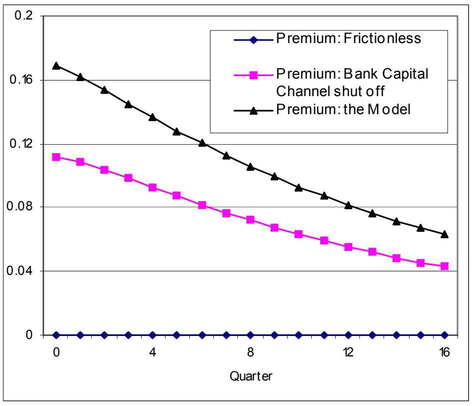
Investment



Entrepreneurial Net worth



Bank Capital



External Finance Premium

Figure 5: Response to an interest rate shock: frictionless model vs. the model with no bank capital channel vs. the full model

investment and output in response to a negative monetary shock in the frictionless model persists the least, i.e. investment in the frictionless model reverts back to trend only after nine quarters while that of the other two models remains below trend even after four years. This is because, as can be seen from the figure, the role of entrepreneurial net worth and bank capital are passive in the model. This implies that the role of external finance premium, which operates to constrain demand for capital and thus future investment and output, is nullified. More interestingly, when bank capital channel is turned on, in comparison to the model with no bank capital channel, not only that the magnitude of initial responses of investment and output is greater, but their persistence is also evidently longer. Although the responses of entrepreneurial net worth are pretty much the same in both models, the responses of bank capital implied by the two are substantially different. When bank capital channel is shut off, i.e. the deposit rate no longer adjusts to a monetary shock relatively faster compared to the lending rate[42], the immediate response of bank capital declines slightly and turns positive in the subsequent periods. Thus unlike the role of bank capital in the full model, the response of bank capital in the model when bank capital channel is turned off operates to lessen the agency problem arisen as a result of persistently declining entrepreneurial net worth. The relatively more active role of bank capital in magnifying the agency problem in the full model is mirrored by a substantially stronger response of external finance premium. Crucially, the strongest response of external finance premium in the full model compared to those in the other two models serves as the main amplifying and propagating mechanism in the model.

All in all, in the terminology of Bernanke, et al. (1999), the simulation result shows that the transmission mechanism of monetary policy implied by the model exhibits a *financial accelerator effect* in that endogenous evolution of bank capital, together with entrepreneurial net worth and asset price, work to amplify as well as propagate the effect of monetary shocks in the macroeconomy.

8 Conclusion

This paper is a theoretical study of the transmission mechanism of monetary policy in the presence of endogenous role of bank capital. The basic framework is a standard Dynamic New Keynesian model with the principal modification in that both firms and banks face financial friction in borrowing from their debtors. This implies that an external-finance premium exists for firms and banks in obtaining external funds. The existence of a wedge between internal and external cost of fund therefore motivates firms and banks to hold their inside capital endogenously.

The model's dynamics imply that the adjustment of the non-default lending rate to aggregate shocks is relatively stickier compared to that of the non-default deposit rate. This proxies realistically the effect of having maturity mismatch in banks' balance sheets and therefore underpins the operational mechanism of the bank capital channel in the model. The dynamic response of bank capital to a monetary shock, together with that of entrepreneurial net worth, operate to enrich the transmission mechanism of monetary policy primarily because lower bank capital and entrepreneurial net worth imply that firms and banks have less inside capital to contribute to the project. As agency problem faced by ultimate depositors is magnified, a higher external finance premium in the form of higher non-default deposit rate is therefore required in order to compensate depositors for having to face higher agency cost. In response to a higher external cost of borrowing, banks have to increase their non-default lending rate thereby directly imposing greater external cost of borrowing on firms. This affects the real investment decision of firms as capital investment becomes more expensive. Thus financial position of

[42] Thus, the interest rate cost (paid to depositors) does not rise relatively faster compared to the interest rate revenue (collected from firms).

firms and banks works to amplify as well as propagate the dynamics of demand for capital, investment and aggregate output compared to the benchmark frictionless type of model.

The principal simulation result shows that the model exhibits a financial accelerator effect in that endogenous evolution of bank capital, together with that of entrepreneurial net worth, operate to amplify and propagate the effect of a monetary shock in the macroeconomy. This signifies the quantitative importance of the role of bank capital in shaping the dynamics of the transmission mechanism of monetary policy.

9 Appendix A

This appendix shows that, under certain assumptions, firm i's threshold value of idiosyncratic component of return to firm i's investment ($\overline{\varpi}^F_{i,t+1}$) will be strictly greater than that of bank ($\overline{\varpi}^B_{i,t+1}$). In general, there are 3 plausible scenarios regarding the relative values of $\overline{\varpi}^F_{i,t+1}$ and $\overline{\varpi}^B_{i,t+1}$ and I shall discuss them sequentially. For notational simplicity, I ignore all of the time subscript in this appendix. To begin, I first re-state equations (3) and (5) in the text.

$$\overline{\varpi}^F_i QR^K K^i = r^L_i L^i \tag{A1}$$

$$(1-\theta^B)\overline{\varpi}^B_i\ QR^K K^i = r^D_i D^i \tag{A2}$$

Scenario 1: $\overline{\varpi}^F_i \leq (1-\theta^B)\overline{\varpi}^B_i$

This is an obvious case because when $\overline{\varpi}^F_i \leq (1-\theta^B)\overline{\varpi}^B_i$, equations (A1) and (A2) imply that $r^L_i L^i \leq r^D_i D^i$. Given a strictly positive opportunity cost for banks from holding inside capital, $A^i r^f$ (where r^f denotes the risk free rate), this implies that banks will always go bankrupt as their revenue from lending can never cover their total cost. Thus we can dismiss this scenario as a potential equilibrium solution.

Scenario 2: $(1-\theta^B)\overline{\varpi}^B_i < \overline{\varpi}^F_i \leq \overline{\varpi}^B_i$

When the realised ϖ_i is less than firm i's threshold $\overline{\varpi}^F_i$ ($\varpi_i < \overline{\varpi}^F_i$), the firm will go bankrupt and the remaining liquidation value amounts to $\omega_i QR^K K^i$. After paying for verification cost, bank j receives $(1-\theta^B)\omega_i QR^K K^i$ as its revenue. Since this revenue is less than the bank's obligation to repay depositor m, i.e. $(1-\theta^B)\omega_i QR^K K^i < (1-\theta^B)\overline{\varpi}^B_i QR^K K^i = r^D_i D^i$, the bank will go bankrupt and pass all its revenue to the depositor. This implies that the bank retains nothing and the depositor receives $(1-\theta^D)(1-\theta^B)\omega_i QR^K K^i$ after paying for the verification cost.

When $\varpi_i > \overline{\varpi}^F_i$, firm i does not go bankrupt and it pays the bank according to the contract, $r^L_i L^i$. Since the bank's revenue in this case is greater than the repaying amount specified in the deposit contract, $r^L_i L^i = \overline{\varpi}^F_i QR^K K^i > (1-\theta^B)\overline{\varpi}^B_i QR^K K^i = r^D_i D^i$, the bank does not default and pockets the profit equivalent to $(r^L_i L^i - r^D_i D^i)$. Given that the bank faces the opportunity cost of holding inside capital equivalent to risk-free rate, r^f, the bank's expected profit function conditional on the realisation of idiosyncratic risk is given by;

$$\pi^B_{|(1-\theta^B)\overline{\varpi}^B_i < \overline{\varpi}^F_i \leq \overline{\varpi}^B_i} = \int_{\overline{\varpi}^F_i}^{\infty} [r^L_i L^i - r^D_i D^i] f(\varpi_i) d\varpi_i - A^i r^f \tag{A3}$$

Using equations (A1) and (A2), equation (A3) can be rewritten as;

$$\pi^B_{|(1-\theta^B)\varpi^B_i<\varpi^F_i\leq\varpi^B_i} = (\overline{\varpi}^F_i - (1-\theta)\overline{\varpi}^B_i)[1-F(\overline{\varpi}^F_i)]QR^K K^i - A^i r^f \tag{A4}$$

Scenario 3: $\overline{\varpi}^F_i > \overline{\varpi}^B_i$

When $\varpi_i < \overline{\varpi}^B_i$, firm goes bankrupt. Since the bank's revenue after paying verification cost is insufficient to fulfil deposit contract, after declaring default, the bank passes all its revenue to depositor and retains nothing. This is because, $(1-\theta^B)\varpi_i R^K QK^i < (1-\theta^B)\overline{\varpi}^B_i R^K QK^i = r^D_i D^i$. When $\overline{\varpi}^F_i > \varpi_i \geq \overline{\varpi}^B_i$, the firm remains bankrupt. However, as for the bank, its revenue netting off verification cost is now enough to fulfil deposit contract, $(1-\theta^B)\varpi_i R^K QK^i \geq (1-\theta^B)\overline{\varpi}^B_i R^K QK^i = r^D_i D^i$. Hence the bank pockets the difference, i.e. $(1-\theta^B)\varpi_i R^K QK^i - r^D_i D^i$.

Lastly, when $\varpi_i \geq \overline{\varpi}^F_i$, both the bank and the firm do not declare bankruptcy. The bank would then receive $r^L_i L^i - r^D_i D^i$ as its profit.

The bank's expected profit conditional on the realisation of idiosyncratic return ϖ_i in this case is given by;

$$\pi^B_{|\overline{\varpi}^F_i>\overline{\varpi}^B_i} = \int_{\overline{\varpi}^B_i}^{\overline{\varpi}^F_i} [(1-\theta^B)\varpi_i R^K QK^i - r^D_i D^i] f(\varpi_i) d\varpi_i + [1-F(\overline{\varpi}^F_i)][r^L_i L^i - r^D_i D^i] - A^i r^f \tag{A5}$$

Using the simplifying notations given in the text (equations (10)-(13)) together with equations (A1) and (A2), equation (A5) can be rewritten as;

$$\pi^B_{|\overline{\varpi}^F_i>\overline{\varpi}^B_i} = \left[\Gamma(\overline{\varpi}^F_i) - (1-\theta^B)\Gamma(\overline{\varpi}^B_i) - \theta^B G(\overline{\varpi}^F_i)\right] R^K QK^i - A^i r^f \tag{A6}$$

To sum, although scenario 1 can never be the equilibrium as the bank's expected profit is always negative, equilibrium $\overline{\varpi}^F_i$ could in general fall on to either scenario 2 or 3. However, I will show below that under certain assumptions, we can restrict the equilibrium $\overline{\varpi}^F_i$ to lie strictly in scenario 3. To do so, for notational simplicity, I first define R^B such that the following equation holds;

$$\pi^B = R^B R^K QK^i - A^i r^f \tag{A7}$$

From equations (A3) and (A6), using equation (A7), the value of R^B could be given as follows;

$$\begin{aligned} R^B &\leq 0 && \text{;scenario 1} \\ &= (\overline{\varpi}^F_i - (1-\theta^B)\overline{\varpi}^B_i)[1-F(\overline{\varpi}^F_i)] && \text{;scenario 2} \\ &= \left[\Gamma(\overline{\varpi}^F_i) - (1-\theta^B)\Gamma(\overline{\varpi}^B_i) - \theta^B G(\overline{\varpi}^F_i)\right] && \text{;scenario 3} \end{aligned}$$

As mentioned, scenario 1 can be immediately ruled out as $R^B \leq 0$, implying that $\pi^B \leq 0$.

Firstly, take limit of R^B at $\overline{\varpi}^F_i = \overline{\varpi}^B_i$;

$$\lim_{\overline{\varpi}^F_i \to \overline{\varpi}^{B+}_i} R^B = \lim_{\overline{\varpi}^F_i \to \overline{\varpi}^{B+}_i} \left[\Gamma(\overline{\varpi}^F_i) - (1-\theta^B)\Gamma(\overline{\varpi}^B_i) - \theta^B G(\overline{\varpi}^F_i)\right] = \theta^B \overline{\varpi}^B_i [1-F(\overline{\varpi}^B_i)]$$

$$\lim_{\overline{\varpi}^F_i \to \overline{\varpi}^{B-}_i} R^B = \lim_{\overline{\varpi}^F_i \to \overline{\varpi}^{B-}_i} (\overline{\varpi}^F_i - (1-\theta)\overline{\varpi}^B_i)[1-F(\overline{\varpi}^F_i)] = \theta^B \overline{\varpi}^B_i [1-F(\overline{\varpi}^B_i)]$$

So I have shown that;

$$\lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B+}} R^B = \lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B-}} R^B = \theta^B \overline{\varpi}_i^B [1 - F(\overline{\varpi}_i^B)] \geq 0 \tag{A8}$$

Next, I take partial derivative of R^B with respect to the threshold value $\overline{\varpi}_i^F$ for both scenarios (2) and (3);

$$\frac{\partial R^B}{\partial \overline{\varpi}_i^F}_{|(1-\theta^B)\overline{\varpi}_i^B < \overline{\varpi}_i^F \leq \overline{\varpi}_i^B} = [1 - F(\overline{\varpi}_i^F)][1 - \overline{\varpi}_i^F h(\overline{\varpi}_i^F)]$$

$$\frac{\partial R^B}{\partial \overline{\varpi}_i^F}_{|\overline{\varpi}_i^F > \overline{\varpi}_i^B} = \Gamma'(\overline{\varpi}_i^F) - \theta^B G'(\overline{\varpi}_i^F) = [1 - F(\overline{\varpi}_i^F)][1 - \theta^B \overline{\varpi}_i^F h(\overline{\varpi}_i^F)]$$

where, as defined in the paper, $h(\varpi_i) \equiv \frac{f(\varpi_i)}{1-F(\varpi_i)}$ is the hazard rate.

Then taking limit of the two derivatives at $\overline{\varpi}_i^F = \overline{\varpi}_i^B$, I obtain;

$$\lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B-}} \frac{\partial R^B}{\partial \overline{\varpi}_i^F}_{|(1-\theta^B)\overline{\varpi}_i^B < \overline{\varpi}_i^F \leq \overline{\varpi}_i^B} = [1 - F(\overline{\varpi}_i^B)][1 - \overline{\varpi}_i^B h(\overline{\varpi}_i^B)] \tag{A9}$$

$$\lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B+}} \frac{\partial R^B}{\partial \overline{\varpi}_i^F}_{|\overline{\varpi}_i^F > \overline{\varpi}_i^B} = [1 - F(\overline{\varpi}_i^B)][1 - \theta^B \overline{\varpi}_i^B h(\overline{\varpi}_i^B)] \tag{A10}$$

As ϖ_i is log normally distributed, it satisfies an increasing hazard rate restriction (see footnote 8). This implies that R^B reaches a global maximum at a unique $\overline{\varpi}_i^*$ and is an increasing function for $\overline{\varpi}_i^F < \overline{\varpi}_i^*$. In other words, we have;

$$\frac{\partial R^B}{\partial \overline{\varpi}_i^F} \begin{cases} = 0 & \text{for } \overline{\varpi}_i^F = \overline{\varpi}_i^* \\ > 0 & \text{for } \overline{\varpi}_i^F < \overline{\varpi}_i^* \\ < 0 & \text{for } \overline{\varpi}_i^F > \overline{\varpi}_i^* \end{cases}$$

Since $\overline{\varpi}_i^F > \overline{\varpi}_i^*$ can never be an equilibrium, in order to restrict equilibrium $\overline{\varpi}_i^F$ to be greater than $\overline{\varpi}_i^B$ as given by scenario 3, it must be the case that $\overline{\varpi}_i^B < \overline{\varpi}_i^*$. To achieve this, I must assume that $\frac{\partial R^B}{\partial \overline{\varpi}_i^F}$ evaluated at $\overline{\varpi}_i^F = \overline{\varpi}_i^B$ be greater than zero. From equations (A9) and (A10), this implies the following assumption A1.

Assumption A1: $[1 - F(\overline{\varpi}_i^B)][1 - \overline{\varpi}_i^B h(\overline{\varpi}_i^B)] > 0$

From equation (A8), another assumption to ensure that equilibrium $\overline{\varpi}_i^F$ will be greater than $\overline{\varpi}_i^B$ is given as follows;

Assumption A2: $\theta^B \overline{\varpi}_i^B [1 - F(\overline{\varpi}_i^B)] < \frac{A^i r^f}{R^K Q K^i}$

The intuition for these two assumptions will become clear as I explain figure 6. For notational simplicity, I define $VV_i \equiv [1 - F(\overline{\varpi}_i^B)][1 - \overline{\varpi}_i^B h(\overline{\varpi}_i^B)], SS^i \equiv \frac{A^i r^f}{R^K Q K^i}$.

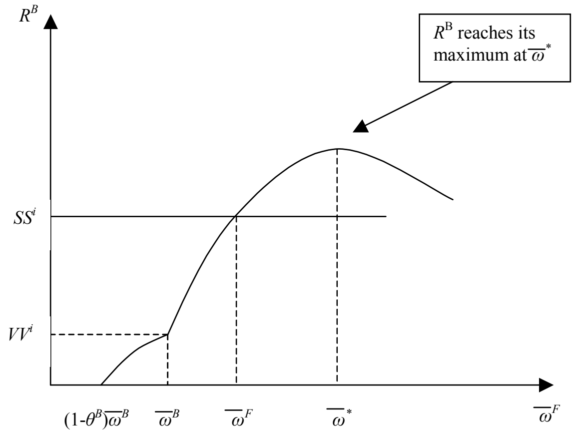


Figure 6:

From figure 6, SS^i is the normalised opportunity cost of fund for the bank. Assumption A2 implies that this opportunity cost of fund is greater than the bank's expected revenue when $\overline{\varpi}_i^F = \overline{\varpi}_i^B$. Thus, assumption A2 means that $\overline{\varpi}_i^F = \overline{\varpi}_i^B$ cannot be an equilibrium as the opportunity cost of fund outweighs the expected revenue. Assumption A1 means that at $\overline{\varpi}_i^F = \overline{\varpi}_i^B$, the slope of the R^B curve is positive. This together with the assumption of increasing hazard rate imply that equilibrium $\overline{\varpi}_i^F$ must lie within the range $(\overline{\varpi}_i^B, \overline{\varpi}_i^*)$ given that bank's opportunity cost is not too high, i.e. $SS^i \leq R^B{}_{|\overline{\varpi}_i^F = \overline{\varpi}_i^*}$. If bank's opportunity cost is too high, $SS^i > R^B{}_{|\overline{\varpi}_i^F = \overline{\varpi}_i^*}$, the firm is rationed. However, I will focus only on the non-rationing equilibrium.[43]

All in all, I have shown that, assumptions A1 and A2 are sufficient to ensure that equilibrium $\overline{\varpi}_i^F$ will be strictly greater than $\overline{\varpi}_i^B$. In other words, given assumptions A1 and A2, we can restrict our analysis on scenario 3 as required by the restriction 1 shown in the text.

10 Appendix B

In this appendix, I solve for the optimality conditions for the firm's demand for capital and show that the ratio of capital stock to the sum of entrepreneurial net worth and bank capital is a positive function of an external finance premium. For notational simplicity, I drop the i subscript.

First, I define the following variables;

$$k_t \equiv \frac{Q_t K_t}{W_t + A_t}, s_t \equiv E_t \left(\frac{R_{t+1}^K}{r_{t+1}^f} \right), u_{t+1} \equiv \frac{R_{t+1}^K}{E_t(R_{t+1}^K)} \frac{E_t(r_{t+1}^f)}{r_{t+1}^f}$$

[43] In order to ensure non-rationing equilibrium, it must be the case that $\frac{\partial R^B}{\partial \overline{\varpi}_i^F}_{|\overline{\varpi}_i^F > \overline{\varpi}_i^B} = \Gamma'(\overline{\varpi}_i^F) - \theta^B G'(\overline{\varpi}_i^F) > 0$.
This restriction holds under the parameterisation taken in this paper.

where u_{t+1} captures the source of aggregate risk in the model.

I restate firm i's optimisation problem given in the text as follows;[44]

$$\max_{k_t, r^D_{t+1}, r^L_t} E_t \sum_{j=0}^{\infty} \{[1 - \Gamma(\overline{\varpi}^F_{t+1+j})] u_{t+1+j} s_{t+j} k_{t+j}\}$$

subject to

$$(1-\theta^B)\left[\Gamma(\overline{\varpi}^B_{t+1}) - \theta^D G(\overline{\varpi}^B_{t+1})\right] u_{t+1} s_t k_t - (k_t - 1) = 0$$

$$E_t\{\left[\Gamma(\overline{\varpi}^F_{t+1}) - (1-\theta^B)\Gamma(\overline{\varpi}^B_{t+1}) - \theta^B G(\overline{\varpi}^F_{t+1})\right] u_{t+1}\} s_t k_t - \frac{A_t}{W_t + A_t} = 0$$

where

$$\overline{\varpi}^F_{t+1} = \frac{r^L_t (k_t - \frac{W_t}{W_t + A_t})}{r^f_{t+1} k_t s_t u_{t+1}} \tag{B1}$$

$$\overline{\varpi}^B_{t+1} = \frac{r^D_{t+1}(k_t - 1)}{\quad}$$

t

where $\wp_{t+1} \equiv [1-\Gamma(\overline{\varpi}^F_{t+1})]+\lambda^1_t[\Gamma(\overline{\varpi}^F_{t+1})-(1-\theta^B)\Gamma(\overline{\varpi}^B_{t+1})-\theta^B G(\overline{\varpi}^F_{t+1})]+\lambda^2_{t+1}(1-\theta^B)\left[\Gamma(\overline{\varpi}^B_{t+1})-\theta^D G(\overline{\varpi}^B_{t+1})\right]$

$$\lambda^2_{t+1}:\ (1-\theta^B)\left[\Gamma(\overline{\varpi}^B_{t+1})-\theta^D G(\overline{\varpi}^B_{t+1})\right]u_{t+1}s_t k_t-(k_t-1)=0 \tag{B6}$$

$$\lambda^1_t : E_t\{\left[\Gamma(\overline{\varpi}^F_{t+1})-(1-\theta^B)\Gamma(\overline{\varpi}^B_{t+1})-\theta^B G(\overline{\varpi}^F_{t+1})\right]u_{t+1}\}s_t k_t-\frac{A_t}{W_t+A_t}=0 \tag{B7}$$

From equations (B1)-(B7), there are 7 equations in 7 variables ($\overline{\varpi}^F_{t+1},\overline{\varpi}^B_{t+1},r^D_{t+1},r^L_t,k_t,\lambda^2_{t+1},\lambda^1_t$), taken as given the value of s_t and $\frac{A_t}{W_t+A_t}$. Thus, in equilibrium, we can write k_t solely as a function of s_t and $\frac{A_t}{W_t+A_t}$.

$$k_t=\Psi(s_t,\frac{A_t}{W_t+A_t})$$

In what follows, I will show that $\frac{d\Psi(s_t,\frac{A_t}{W_t+A_t})}{ds_t}>0$ and $\frac{d\Psi(s_t,\frac{A_t}{W_t+A_t})}{d(\frac{A_t}{W_t+A_t})}<0$.[45]

From equations (B1) and (B2), given that $k_t>1$[46], I could find the following derivatives;

$$\frac{\partial\overline{\varpi}^F_{t+1}}{\partial r^L_t}=\frac{(k_t-\frac{W_t}{W_t+A_t})}{r^f_{t+1}k_t s_t u_{t+1}}>0 \tag{B8}$$

$$\frac{\partial\overline{\varpi}^B_{t+1}}{\partial r^D_{t+1}}=\frac{(k_t-1)}{(1-\theta^B)r^f_{t+1}k_t s_t u_{t+1}}>0 \tag{B9}$$

Substituting equation (B8) into equation (B4), the first order condition with respect to r^L_t can be rewritten as;

$$\lambda^1_t=\frac{E_t[\frac{\Gamma'(\overline{\varpi}^F_{t+1})}{r^f_{t+1}}]}{E_t[\frac{(\Gamma'(\overline{\varpi}^F_{t+1})-G'(\overline{\varpi}^F_{t+1}))}{r^f_{t+1}}]} \tag{B10}$$

$$\frac{\partial \lambda_t^1}{\partial \varpi_{t+1}^F} = \frac{\theta^B [E_t(\frac{\Gamma'(\varpi_{t+1}^F)}{r_{t+1}^f})E_t(\frac{G''(\varpi_{t+1}^F)}{r_{t+1}^f}) - E_t(\frac{\Gamma''(\varpi_{t+1}^F)}{r_{t+1}^f})E_t(\frac{G'(\varpi_{t+1}^F)}{r_{t+1}^f})]}{\{E_t(\frac{\Gamma'(\varpi_{t+1}^F)}{r_{t+1}^f}) - \theta^B E_t(\frac{G'(\varpi_{t+1}^F)}{r_{t+1}^f})\}^2}$$

Using the fact that $\Gamma'(\varpi_{t+1}^F) = 1 - F\left(\varpi_{t+1}^F\right)$, and $G'(\varpi_{t+1}^F) = \varpi_{t+1}^F f(\varpi_{t+1}^F)$, where $F\left(\varpi_{t+1}^F\right)$ and $f(\varpi_{t+1}^F)$ are *cdf* and *df* of ϖ_{t+1}^F respectively, I can then write $\frac{\partial \lambda_t^1}{\partial \varpi_{t+1}^F}$ as follows,

$$\frac{\partial \lambda_t^1}{\partial \varpi_{t+1}^F} = \frac{\theta^B [E_t(\frac{1-F\left(\varpi_{t+1}^F\right)}{r_{t+1}^f})E_t(\frac{\partial(\varpi_{t+1}^F f(\varpi_{t+1}^F))}{\partial \varpi_{t+1}^F}\frac{1}{r_{t+1}^f}) + E_t(\frac{f(\varpi_{t+1}^F)}{r_{t+1}^f})E_t(\frac{\varpi_{t+1}^F f(\varpi_{t+1}^F)}{r_{t+1}^f})]}{\{E_t(\frac{1-F\left(\varpi_{t+1}^F\right)}{r_{t+1}^f}) - \theta^B E_t(\frac{\varpi_{t+1}^F f(\varpi_{t+1}^F)}{r_{t+1}^f})\}^2}$$

In general, the value of $\frac{\partial(\varpi_{t+1}^F f(\varpi_{t+1}^F))}{\partial \varpi_{t+1}^F}$ could be either positive or negative. However, under a reasonable parameterisation, including the one used for calibration in this paper, $\frac{\partial(\varpi_{t+1}^F f(\varpi_{t+1}^F))}{\partial \varpi_{t+1}^F}$ will be strictly positive in the neighbourhood of the steady state. Therefore, I have established that $\frac{\partial \lambda_t^1}{\partial \varpi_{t+1}^F} > 0$.

From equation (B3), taking partial derivative with respect to $\varpi_{t+1}^F$and ϖ_{t+1}^B, I obtain;

$$\frac{\partial \lambda_{t+1}^2}{\partial \varpi_{t+1}^F} = \frac{\Gamma'(\varpi_{t+1}^B)}{\Gamma'(\varpi_{t+1}^B) - \theta^D G'(\varpi_{t+1}^B)}\frac{\partial \lambda_t^1}{\partial \varpi_{t+1}^F} = \frac{\lambda_{t+1}^2}{\lambda_t^1}\frac{\partial \lambda_t^1}{\partial \varpi_{t+1}^F} > 0$$

$$\frac{\partial \lambda_{t+1}^2}{\partial \varpi_{t+1}^B} = \frac{\theta^D [\Gamma'(\varpi_{t+1}^B)G''(\varpi_{t+1}^B) - \Gamma''(\varpi_{t+1}^B)G'(\varpi_{t+1}^B)]}{\left[\Gamma'(\varpi_{t+1}^B) - \theta^D G'(\varpi_{t+1}^B)\right]^2}\lambda_t^1 = \frac{\theta^D [1 - F(\varpi_{t+1}^B)]^2 \frac{\partial \varpi_{t+1}^B h(\varpi_{t+1}^B)}{\partial \varpi_{t+1}^B}}{\left[\Gamma'(\varpi_{t+1}^B) - \theta^D G'(\varpi_{t+1}^B)\right]^2}\lambda_t^1$$

Due to the assumption of an increasing hazard rate, $\frac{\partial \varpi_{t+1}^B h(\varpi_{t+1}^B)}{\partial \varpi_{t+1}^B} > 0$, where $h(\varpi_{t+1}^B) \equiv \frac{f(\varpi_{t+1}^B)}{1-F(\varpi_{t+1}^B)}$ is the hazard rate evaluated at ϖ_{t+1}^B, it follows directly that $\frac{\partial \lambda_{t+1}^2}{\partial \varpi_{t+1}^B} > 0$.

Thus far, I have established that $\lambda_t^1, \lambda_{t+1}^2, \frac{\partial \lambda_t^1}{\partial \varpi_{t+1}^F}, \frac{\partial \lambda_{t+1}^2}{\partial \varpi_{t+1}^F}, \frac{\partial \lambda_{t+1}^2}{\partial \varpi_{t+1}^B}, \frac{\partial \varpi_{t+1}^F}{\partial r_t^L}, \frac{\partial \varpi_{t+1}^B}{\partial r_{t+1}^D}$ are all positive for $\varpi^F \in (\varpi^B, \varpi^*]$.

To show that $\frac{dk_t}{ds_t} > 0$, take derivative of $JJ_t(s_t, r_t^L, r_{t+1}^D)$ (the first order condition with respect to capital, equation (B5)) with respect to s_t.

$$\frac{\partial JJ_t}{\partial s_t} + \frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial s_t} + \frac{\partial JJ_t}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial s_t} + [\frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial k_t} + \frac{\partial JJ_t}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial k_t}]\frac{dk_t}{ds_t} = 0$$

rearranging to obtain;

$$\frac{dk_t}{ds_t} = -\frac{[\frac{\partial JJ_t}{\partial s_t} + \frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial s_t} + \frac{\partial JJ_t}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial s_t}]}{[\frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial k_t} + \frac{\partial JJ_t}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial k_t}]} \tag{B11}$$

From equation (B5),

$$\begin{aligned}
\frac{\partial JJ_t}{\partial s_t} &= E_t(\wp_{t+1}u_{t+1}) > 0 \\
\frac{\partial JJ_t}{\partial r_t^L} &= E_t[-\Gamma'(\overline{\varpi}^F_{t+1})\frac{\partial \overline{\varpi}^F_{t+1}}{\partial r_t^L}u_{t+1} + \lambda_t^1(\Gamma'(\overline{\varpi}^F_{t+1}) - G'(\overline{\varpi}^F_{t+1}))\frac{\partial \overline{\varpi}^F_{t+1}}{\partial r_t^L}u_{t+1}]s_t \\
&+ E_t[\frac{\partial \lambda_t^1}{\partial r_t^L}\left[\Gamma(\overline{\varpi}^F_{t+1}) - (1-\theta^B)\Gamma(\overline{\varpi}^B_{t+1}) - \theta^B G(\overline{\varpi}^F_{t+1})\right]u_{t+1}]s_t \\
&+ E_t\left[\frac{\partial \lambda^2_{t+1}}{\partial r_t^L}\left((1-\theta^B)\left[\Gamma(\overline{\varpi}^B_{t+1}) - \theta^D G(\overline{\varpi}^B_{t+1})\right]u_{t+1}s_t - 1\right)\right] \quad \text{(B12)}
\end{aligned}$$

From equation (B6), rearranging to obtain;

$$\left((1-\theta^B)\left[\Gamma(\overline{\varpi}^B_{t+1}) - \theta^D G(\overline{\varpi}^B_{t+1})\right]u_{t+1}s_t - 1\right) = -\frac{1}{k_t} \quad \text{(B13)}$$

From equation (B7), rearranging to obtain;

$$E_t\{\left[\Gamma(\overline{\varpi}^F_{t+1}) - (1-\theta^B)\Gamma(\overline{\varpi}^B_{t+1}) - \theta^B G(\overline{\varpi}^F_{t+1})\right]u_{t+1}\}s_t = \frac{A_t}{W_t + A_t} \quad \text{(B14)}$$

From equation (B3),

$$\frac{\partial \lambda^2_{t+1}}{\partial r_t^L} = \frac{\lambda^2_{t+1}}{\lambda_t^1}\frac{\partial \lambda_t^1}{\partial r_t^L} \quad \text{(B15)}$$

Substituting equations (B4), (B13), (B14) and (B15) into equation (B12), I obtain;

$$\frac{\partial JJ_t}{\partial r_t^L} = E_t\left(\frac{\partial \lambda^2_{t+1}}{\partial r_t^L}\frac{1}{k_t}\left[\frac{A_t}{W_t + A_t}\frac{\lambda_t^1}{\lambda^2_{t+1}} - 1\right]\right) \quad \text{(B16)}$$

Because $\frac{\partial \lambda^2_{t+1}}{\partial r_t^L} > 0$, $\frac{A_t}{W_t+A_t} \leq 1$, and $\frac{\lambda_t^1}{\lambda^2_{t+1}} < 1$, it must be that $\frac{\partial JJ_t}{\partial r_t^L} < 0$.

I turn now to solve for $\frac{\partial JJ_t}{\partial r^D_{t+1}}$.

From equation (B5);

$$\begin{aligned}
\frac{\partial JJ_t}{\partial r^D_{t+1}} &= E_t[-\lambda_t^1(1-\theta^B)\Gamma'(\overline{\varpi}^B_{t+1})\frac{\partial \overline{\varpi}^B_{t+1}}{\partial r^D_{t+1}}u_{t+1} + \lambda^2_{t+1}(1-\theta^B)[\Gamma'(\overline{\varpi}^B_{t+1}) - \theta^D G'(\overline{\varpi}^B_{t+1})]\frac{\partial \overline{\varpi}^B_{t+1}}{\partial r^D_{t+1}}u_{t+1}]s_t \\
&+ E_t[\frac{\partial \lambda^2_{t+1}}{\partial r^D_{t+1}}\{(1-\theta^B)\left[\Gamma(\overline{\varpi}^B_{t+1}) - \theta^D G(\overline{\varpi}^B_{t+1})\right]u_{t+1}s_t - 1\}]
\end{aligned}$$

Using equations (B3) and (B13), we can write;

$$\frac{\partial JJ_t}{\partial r^D_{t+1}} = -E_t[\frac{\partial \lambda^2_{t+1}}{\partial r^D_{t+1}}\frac{1}{k_t}] < 0 \quad \text{(B17)}$$

From equation (B6), the first order condition with respect to λ^2_{t+1}, I take partial derivative with respect to s_t and k_t respectively;

$$\frac{\partial r_{t+1}^D}{\partial s_t} = -\frac{\left[\Gamma(\overline{\varpi}_{t+1}^B) - \theta^D G(\overline{\varpi}_{t+1}^B)\right]}{\left[\Gamma'(\overline{\varpi}_{t+1}^B) - \theta^D G'(\overline{\varpi}_{t+1}^B)\right] \frac{\partial \overline{\varpi}_{t+1}^B}{\partial r_{t+1}^D} s_t} < 0 \tag{B18}$$

$$\frac{\partial r_{t+1}^D}{\partial k_t} = \frac{\left(1 - (1-\theta^B)\left[\Gamma(\overline{\varpi}_{t+1}^B) - \theta^D G(\overline{\varpi}_{t+1}^B)\right] u_{t+1} s_t\right)}{(1-\theta^B)\left[\Gamma'(\overline{\varpi}_{t+1}^B) - \theta^D G'(\overline{\varpi}_{t+1}^B)\right] \frac{\partial \overline{\varpi}_{t+1}^B}{\partial r_{t+1}^D} u_{t+1} s_t k_t}$$

Using equation (B13), $\frac{\partial r_{t+1}^D}{\partial k_t}$ can be rewritten as;

$$\frac{\partial r_{t+1}^D}{\partial k_t} = \frac{k_t^{-1}}{(1-\theta^B)\left[\Gamma'(\overline{\varpi}_{t+1}^B) - \theta^D G'(\overline{\varpi}_{t+1}^B)\right] \frac{\partial \overline{\varpi}_{t+1}^B}{\partial r_{t+1}^D} u_{t+1} s_t k_t} > 0 \tag{B19}$$

From equation (B7), the first order condition with respect to λ_t^1, I take partial derivative with respect to s_t and k_t respectively;

$$\frac{\partial r_t^L}{\partial s_t} = \frac{E_t\{(1-\theta^B)\Gamma'(\overline{\varpi}_{t+1}^B)\frac{\partial \overline{\varpi}_{t+1}^B}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial s_t} s_t k_t u_{t+1} - \left[\Gamma(\overline{\varpi}_{t+1}^F) - (1-\theta^B)\Gamma(\overline{\varpi}_{t+1}^B) - \theta^B G(\overline{\varpi}_{t+1}^F)\right] u_{t+1} k_t\}}{E_t\{\Gamma'(\overline{\varpi}_{t+1}^F) - \theta^B G'(\overline{\varpi}_{t+1}^F))\frac{\partial \overline{\varpi}_{t+1}^F}{\partial r_t^L} u_{t+1}\} s_t k_t}$$

Substituting equation (B18) into the above equation, I obtain;

$$\frac{\partial r_t^L}{\partial s_t} = -\frac{E_t\{(1-\theta^B)\frac{\lambda_{t+1}^2}{\lambda_t^1}\left[\Gamma(\overline{\varpi}_{t+1}^B) - \theta^D G(\overline{\varpi}_{t+1}^B)\right] k_t u_{t+1} + \left[\Gamma(\overline{\varpi}_{t+1}^F) - (1-\theta^B)\Gamma(\overline{\varpi}_{t+1}^B) - \theta^B G(\overline{\varpi}_{t+1}^F)\right] k_t u_{t+1}\}}{E_t\{\Gamma'(\overline{\varpi}_{t+1}^F) - \theta^B G'(\overline{\varpi}_{t+1}^F))\frac{\partial \overline{\varpi}_{t+1}^F}{\partial r_t^L} u_{t+1}\} s_t k_t} < 0 \tag{B20}$$

$$\frac{\partial r_t^L}{\partial k_t} = \frac{E_t\{(1-\theta^B)\Gamma'(\overline{\varpi}_{t+1}^B)\frac{\partial \overline{\varpi}_{t+1}^B}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial k_t} s_t k_t u_{t+1} - \left[\Gamma(\overline{\varpi}_{t+1}^F) - (1-\theta^B)\Gamma(\overline{\varpi}_{t+1}^B) - \theta^B G(\overline{\varpi}_{t+1}^F)\right] u_{t+1} s_t\}}{E_t\{\Gamma'(\overline{\varpi}_{t+1}^F) - \theta^B G'(\overline{\varpi}_{t+1}^F))\frac{\partial \overline{\varpi}_{t+1}^F}{\partial r_t^L} u_{t+1}\} s_t k_t}$$

Substituting equations (B19) and (B14) into the above equation, I obtain;

$$\frac{\partial r_t^L}{\partial k_t} = \frac{1}{k_t}\frac{E_t\{\frac{\lambda_{t+1}^2}{\lambda_t^1} - \frac{A_t}{W_t + A_t}\}}{E_t\{\Gamma'(\overline{\varpi}_{t+1}^F) - \theta^B G'(\overline{\varpi}_{t+1}^F))\frac{\partial \overline{\varpi}_{t+1}^F}{\partial r_t^L} u_{t+1} s_t k_t\}} \tag{B21}$$

Because, $\frac{\lambda_{t+1}^2}{\lambda_t^1} > 1$ and $\frac{A_t}{W_t+A_t} \leq 1$, it must be the case that $\frac{\partial r_t^L}{\partial k_t} > 0$.

Thus far I have shown that $\frac{\partial JJ_t}{\partial r_t^L}$, $\frac{\partial JJ_t}{\partial r_{t+1}^D}$, $\frac{\partial r_{t+1}^D}{\partial s_t}$, and $\frac{\partial r_t^L}{\partial s_t}$ are strictly negative and that $\frac{\partial JJ_t}{\partial s_t}$, $\frac{\partial r_{t+1}^D}{\partial k_t}$, and $\frac{\partial r_t^L}{\partial k_t}$ are strictly positive. Plugging these values into equation (B11) implies that $\frac{dk_t}{ds_t}$ is strictly positive as required.

I turn now to show that $\frac{dk_t}{d\frac{A_t}{W_t+A_t}} < 0$. From equation (B5), take derivative of JJ_t with respect to $\frac{A_t}{W_t+A_t}$;

$$\frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial \frac{A_t}{W_t+A_t}} + [\frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial k_t} + \frac{\partial JJ_t}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial k_t}]\frac{dk_t}{d\frac{A_t}{W_t+A_t}} = 0$$

$$\frac{dk_t}{d\frac{A_t}{W_t+A_t}} = -\frac{[\frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial \frac{A_t}{W_t+A_t}}]}{[\frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial k_t} + \frac{\partial JJ_t}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial k_t}]} \tag{B22}$$

In order to obtain $\frac{\partial r_t^L}{\partial \frac{A_t}{W_t+A_t}}$, take derivative of equation (B7) with respect to $\frac{A_t}{W_t+A_t}$ and rearrange to obtain;

$$\frac{\partial r_t^L}{\partial \frac{A_t}{W_t+A_t}} = \frac{1}{E_t\{\Gamma'(\overline{\varpi}_{t+1}^F) - \theta^B G'(\overline{\varpi}_{t+1}^F))\frac{\partial \overline{\varpi}_{t+1}^F}{\partial r_t^L}u_{t+1}\}s_t k_t} > 0 \tag{B23}$$

As $\frac{\partial JJ_t}{\partial r_t^L}$ and $\frac{\partial JJ_t}{\partial r_{t+1}^D}$ are strictly negative and $\frac{\partial r_t^L}{\partial \frac{A_t}{W_t+A_t}}$, $\frac{\partial r_t^L}{\partial k_t}$, $\frac{\partial r_{t+1}^D}{\partial k_t}$ are strictly positive, by substituting these values into equation (B22), it must be that $\frac{dk_t}{d\frac{A_t}{W_t+A_t}} < 0$.

In sum, I have shown that;

$$k_t = \Psi(s_t, \frac{A_t}{W_t + A_t}) \tag{B24}$$
$$\text{where } \frac{dk_t}{ds_t} > 0, \frac{dk_t}{d\frac{A_t}{W_t+A_t}} < 0$$

For any reasonable parameterisation, the magnitude of $\frac{dk_t}{d\frac{A_t}{W_t+A_t}}$ is very small compared to that of $\frac{dk_t}{ds_t}$ in the neighbourhood of the steady state. In other words, ignoring the effect of $\frac{A_t}{W_t+A_t}$ on k_t will not affect the dynamics of the model.[48] Thus equation (B24) can be approximately written as;

$$k_t = \Psi(s_t, \frac{A_t}{W_t + A_t}) \simeq \psi(s_t)$$
$$\text{where } \psi'(s_t) > 0$$

[48] By taking into account the effect of $\frac{A_t}{W_t+A_t}$ on k_t in the simulation analysis, the result in terms of the dynamic response of the key variables is virtually the same compared to the case when the effect is ignored.

11 Table of Abbreviations

K	capital
Q	price of capital
R^K	non-idiosyncratic component of return to capital
ϖ	idiosyncratic component of return to capital
$\overline{\varpi}^F$	a firm's threshold value of ϖ
$\overline{\varpi}^B$	a bank's threshold value of ϖ
W	entrepreneurial net worth
L	bank loan
D	bank deposit
A	bank capital
r^L	non-default lending rate
r^D	non-default deposit rate
π^F	expected profit for a firm, conditional on the realisation of idiosyncratic risk
π^B	expected profit for a bank, conditional on the realisation of idiosyncratic risk
π^D	expected profit for a depositor, conditional on the realisation of idiosyncratic risk
r^f	the real risk-free rate
r^{nf}	the nominal risk-free rate
X	markup
Y	wholesale output
Y^f	final goods (CES aggregate of retail output z)
$Y(z)$	retail good z
$Y^*(z)$	demand for retail good z by retailers who can choose a new price
P	price of final goods
$P(z)$	price of retail good z
π	inflation
P^w	price of wholesale goods
P^*	price of retail goods chosen by retailers who can choose the new price
H	household's labour
I	investment
$\frac{N}{P}$	real wage
C^E	dying entrepreneurs' consumption
C^B	dying bankers' consumption
e^F	a firm's endowment
e^B	a bank' s endowment
γ^E	entrepreneurs' probability of dying
γ^B	banks' probability of dying
θ^B	banks' proportional verification cost
θ^D	depositors' proportional verification cost
α	capital share in the production function

φ	depreciation rate
ρ	probability that a retailer cannot adjust his price
σ^2	variance of the log-normal distribution of ϖ
$\varkappa$	a household's coefficient of relative impatience
R^D	actual realisation of deposit rate
Π	profit of retail firms redistributed to households
C	households' consumption
ϵ^Q	elasticity of the price of capital w.r.t. the investment capital ratio
ϵ^H	labour supply elasticity
ξ	elasticity of $\frac{QK}{W+A}$ to external finance premium
T	technology

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