How Deep was the September 2001 Stock Market Crisis?

Putting Recent Events on the American and French Markets into Perspective

with an Index of Market Shocks *

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revised version

Abstract

Markets reacted strongly to the World Trade Center attacks both in Europe and in the United States. The extent of this crisis was difficult to assess at the time, underlining the need for a specific tool to measure the magnitude of financial crises. A first measure was recently proposed and applied to the foreign exchange market by Zumbach *et al* (2000-a and 2000-b). Their measure relies on an analogy with geophysics; the related Index of Market Shocks (IMS) that we propose here is also the counterpart of the Richter scale used for earthquakes. We apply this measure on the French and the American stock markets to put recent market events into perspective. The crisis triggered by the September attacks was actually the worst since 1987, and the 9th when compared to major historical ones.

Key words: Financial crises, Volatility, Risk Measurement, Heterogeneity of Economic Agents.

JEL Classification: G.10, G.14

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How Deep was the September 2001 Stock Market Crisis? Putting Recent Events on the American and French Markets into Perspective with an Index of Market Shocks

An Index of Market Shock (IMS) that is easily readable and computable is proposed by the authors to assess the gravity of a financial crisis and to put market events into perspectives.

Stock markets reacted strongly to the World Trade Center attacks both in Europe and in the United States. As information reached the market, indexes fluctuated widely, displaying large swings, often within the same day. The extent of this crisis was difficult to assess at the time, emphasizing once more the need for an accurate measure of turbulence on the financial markets. Widely-used scales exist in other fields to quantify event or shock strengths. Familiar examples are the Richter scale in geophysics (Richter, 1958) - which measures the intensity of earthquakes, the Beaufort scale in sailing - which measures the strength of winds and the condition of the sea, the International Nuclear Event Scale (INES) - that sums up the on- and off-site impact of a nuclear incident, the Saffir-Simpson Hurricane Scale – gauging a hurricane's current intensity… These examples emphasize the advantage of having a simple and intuitive measure in case of crisis for precautionary purposes. A comprehensive and accurate measure of market turbulence would likewise greatly benefit regulatory authorities, clearing firms and risk managers who are primarily concerned with the risk associated with extreme events and the use of capital to limit risk.

Accordingly, the main goal of this note is to provide a catastrophic scale to quantify the extent of financial crises following the approach first introduced by Zumbach *et al* (2000-a). An appealing measure should be easily interpretable, computable from market data and should encompass traditional measures of risk. Moreover, it should be theoretically sound and compatible with empirical stylized facts as well as economic features of financial markets - in particular the heterogeneity of economic agents' horizons, the strongly time-varying nature of volatility, known microstructure biases and inaccuracy of measures, and the usual distributional assumptions regarding return and volatility.

Traditional quantities such as daily historical volatility, implied volatility from options or tail indexes do not permit to accurately distinguish between normal and turbulent behaviour since we know that, in the latter case, they do not account for some of the properties of financial returns such as volatility clustering, leverage effect, increase in correlation coefficients. Moreover, those quantities do not offer a direct measure of the probability of occurrence of an event, and there is no universally accepted measure of the seriousness of crises, in order to assess them in real time, rank them, and put them into perspective.

The measurements of volatility traditionally used on the market are not well-suited to apprehend risk in all its dimensions, since they do not take into account the variability of the amplitude and the duration of crises - from a few hours to several months. Our measure is designed to deal with the heterogeneity of market participants by integrating the variability of their horizons of observation and decision. In fact, this indicator is none over than the analogue of the Richter scale used in geophysics. We propose an assessment of the September 2001 crisis compared to earlier crashes that marked the financial history. More precisely, we describe the method used to compute the proposed Index of Market Shock (IMS hereafter), highlighting how the differences in context with the original definition given by Zumbach *et al* (2000-a and 2000-b) lead to differences in design. After assessing the adequacy of the IMS, we measure the level of the recent crisis on the American and the French stock markets using samples with complementary characteristics. We then compare our diagnosis with the traditional measures of risk, focusing on the differences rather than on the common elements.

The Index of Market Shocks (IMS) and Its Empirical Validation

Following Zumbach *et al* (2000-a and 2000-b), we propose a measure integrating the horizons of the various types of operators. Some are day traders, others observe the variations between two closing prices while those having a longer horizon focus on trends. The definition of a crisis should thus not depend on the periodicity of the observations, since significant movements are undetectable at low frequencies. The indicator proposed here is therefore based on a multidimensional approach of risk. It offers several advantages over commonly used single statistics. First, standard measures of risk, such as empirical volatility or maximum drawdown (see Dacorogna *et al*, 2001 and Johansen and Sornette, 2001), are nested in our indicator. By dealing with all return frequencies, the IMS indeed encompasses traditional (daily or close-to-close) volatility, the minimum return (which is at some frequency the maximum drawdown on the observation window), the negative returns as in the semi-volatility case and the VaR since the whole density is included. In the worst possible case where additional frequencies do not carry any information, the indicator is a rescaled standard measure. It should then be preferred to the traditional measures since by construction it incorporates all sources of risk. Second, as highlighted by Drost and Nijman (1993) in the case of weak GARCH processes, financial return properties are not identical when measured at different scales: returns become homoskedastic and Gaussian at a sufficiently low frequency. The choice of the optimal scheme of observations is then of importance when estimating models with market data (see Andersen and Bollerslev, 1998-b and Gerhard and Hautsch, 2002); dealing with all scales in the data is a possible answer. In fact, most single measures are known to be either biased or inaccurate because of microstructure effects (see Corsi *et al*, 2001). The multi-frequency approach extracts relevant information from each frequency - corresponding to different economic agents' horizons - and thus yields a denoised risk estimate. Third, the relations between scales have been shown to play an important role in the explanation of return distributions, notably in the volatility cascade hypothesis (Muzy *et al*, 2000). Fourth, the importance of path dependence in a definition of a coherent risk measure has been underlined by Artzner *et al* (1999); a multi-scale measure of risk incorporates all possible nodes of any decision tree since by definition it takes into account all historical paths.

The basis for the construction of the index are the recent studies showing that the empirical variance of returns is log-normally distributed (see Andersen *et al*, 2001) irrespective of the scale of observation. Accordingly, for each measurement of the IMS, we compute the variances of returns from the finest scale (high-frequency) to the coarsest, and interpret it as the realization of a multi-dimensional random variable[1]. For example, in the case of high-frequency data, we compute daily variances using returns ranging from ten minutes to close-to-close.

These empirical variances are naturally correlated depending on the sampling frequency selected. In order to avoid biases, we choose to extract the factors underlying

[1] On our samples, the findings of Anderson *et al* (2001) regarding log-normality of empirical volatilities are confirmed by the usual normality tests (Jarque-Bera and Goodness-of-fit tests) at a 5% threshold.

the variance time series by a Principal Component Analysis[2]. Since variances are lognormal, the decomposition of log-variances yields normal and independent factors. Combining probabilities of observing each factor (weighted by their relative importance) at each point in time yields the probability of observing the corresponding configuration of the set of variances.

The Richter scale is an increasing function of the total energy dissipated during an earthquake, and can be written as:

$$R_t \cong \frac{1}{\delta} \ln\left[\frac{1}{p(E)}\right]$$

where $p(E)$ stands for the probability of exceeding the total energy *E* and δ is a scaling factor.

To transpose this relation to financial markets, we need to define the arguments of this functional relation. The energy of a system is traditionally measured by the squared speed multiplied by mass. The speed, being the distance divided by time, corresponds in our framework to relative price changes *per* time unit (namely the returns). Thus the equivalent of energy will be the volatility (or squared price changes), and the Richter scale becomes:

$$R_t = -\frac{1}{\delta} \ln\left[p(v_t)\right]$$

where v_t is the aggregated volatility (or a volatility vector in our case) and *p(.)* its corresponding probability.

The measure of volatility at all scales can be written as a multinormal vector, thus yielding:

$$R_t = -\frac{1}{\delta} \ln\left[\frac{1}{1 - F\left[\ln\left[\hat{\sigma}_1(t)\right], ..., \ln\left[\hat{\sigma}_{\tau\max}(t)\right]\right]}\right]$$

where $\{\hat{\sigma}_1(t), ..., \hat{\sigma}_{\tau\max}(t)\}$ stands for the vector of volatilities at all scales. These volatilities are computed at different frequencies for each period with the classical definition of variance, that is the average of squared deviations from the mean.

We can now reduce the information carried in the volatility vector using PCA to recover the factors driving volatilities[3]. To ease interpretation and reading, we choose to use a base 2 logarithm, and then the IMS at time *t* reads:

$$IMS_t = -\sum_{k=1}^{K} \left\{\alpha_k \log_2\left[1 - F\left(fac_k\right)\right]\right\}$$

where α_k is the contribution of the ***k-th*** factor, denoted fac_k, to the total variance of volatilities and *F(.)* is the Gaussian cumulative distribution function.

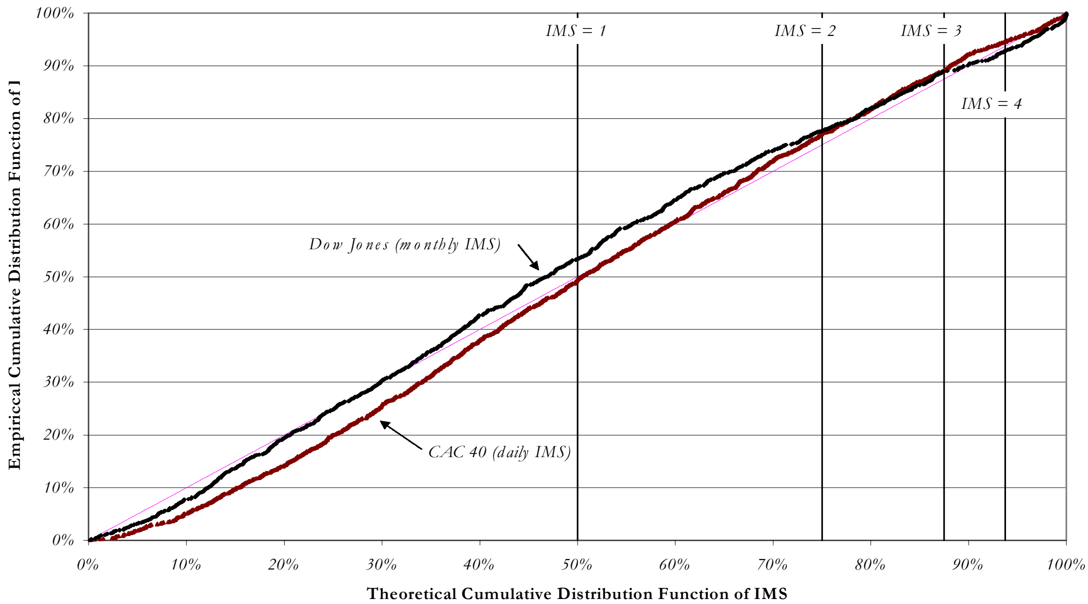
Thus, increasing the index by one point corresponds to a market condition twice less likely, and the value of one theoretically corresponds to the median state of the market such that 50% of the observations lie below.

[2] PCA of volatility time series has been already applied by Alexander (2001) on implied volatilities with different maturities.

[3] We do not use the kernel approach of Zumbach *et al* (2000-a and 2000-b) to compute an aggregate volatility. Unlike Zumbach *et al* (2000-a and 2000-b) measure, ours is not continuous and each value of the IMS uses only data that belongs to the period measured. The drawback of our approach is that the different scales contain redundant information pertaining to the same events and that the lowest frequency of analysis is limited to the frequency of IMS updating; the advantages are that there is no overlap between consecutive measures, no need for seasonal adjustments and that we do not have to specify an arbitrary smoothing (kernel) method in scale or time-space.

The *ex post* validation of the scale confirms that empirical frequencies of the various values of the index are close to their theoretical frequencies (see Figure 1). Although the fit between the theoretical distribution and the empirical distribution is not perfect, the largest discrepancies between both functions occur for small IMS values (which are out the scope of this analysis)[4]. For the highest quantiles (90% and more), corresponding to values of the index of crisis higher than 3, both distributions coincide.

Figure 1: Empirical Distribution of the Indexes of Market Shocks (IMS) Compared to their Theoretical Distributions



Source: Euronext and Economagic, computations from authors. Period: 1995-2002 with intra-day data for the CAC and 1896-2002 with daily data for the Dow Jones Index.

Putting Recent Events into Perspective

The analysis of the high-frequency data was carried out on the French CAC 40 Index, which comprises the forty leading companies on the Paris stock market weighted by capitalization, with a quote every thirty seconds. For practical considerations, we sample the data every ten minutes and compute nineteen daily variances from ten minute to close-to-close. Two factors are needed to take into account the full volatility structure of the high frequency CAC (together they explain more than 75 % of the variance).

Over the whole period 1995-2002[5], the most turbulent days were observed last September according to the IMS. In fact, the two most critical days were September the 11th and 12th; two days where the US market was suspended. This crisis is approximately 3 points above the crisis of October 1998 on our scale; in other words it is eight times less likely or so.

To better appraise the situation, it is necessary to use data spanning a long period, thus covering many crises. The Dow Jones Index is available at a daily frequency since

[4] The Kolmogorov-Smirnov goodness-of-fit test rejects (at the 10% level) the identity of theoretical and empirical distributions on the full sample. At the 10% level, the *maximum* discrepancy between the distributions tolerated by the test is 3% or so. This happens for an IMS under 1 on the CAC40 and 3 on the DJI. Thus, for at least the upper 10% of the samples (over 120 points in both cases), the distributions are close. This inadequacy for small turbulences is also found on the original application of the Richter scale in geophysics.

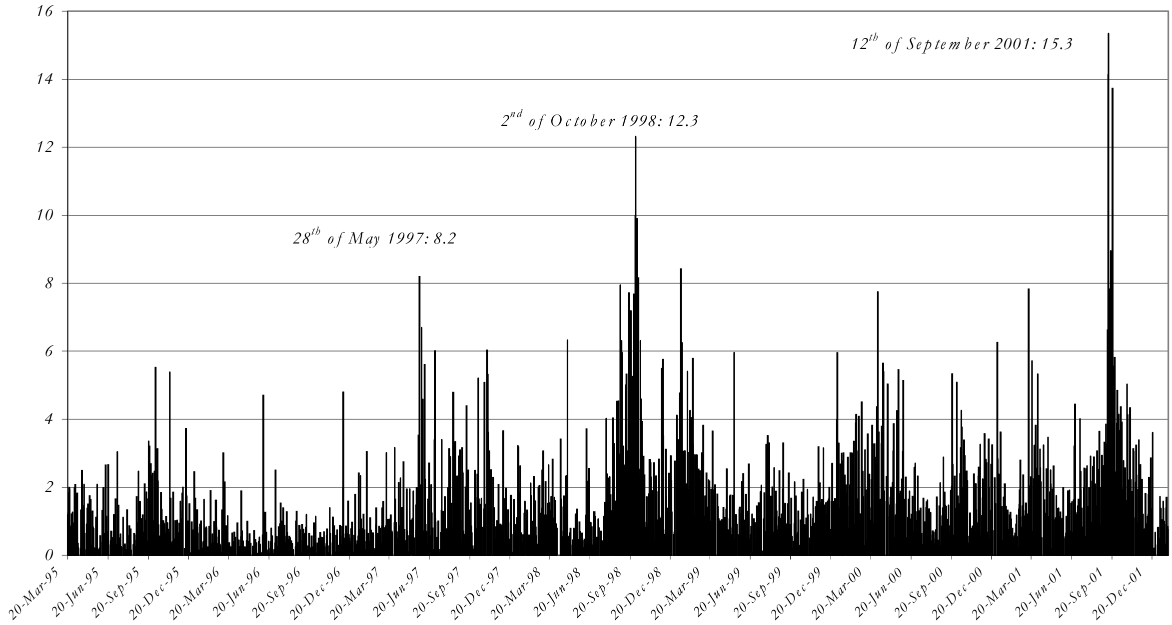
[5] The samples for the CAC 40 index and the DJI end on the 25th of January 2002.

1896. We then build a monthly IMS by sampling returns from a daily to a monthly basis by two-days increments until we reach the monthly squared return. In the case of the daily Dow Jones Index, we evaluate monthly IMS using nine frequencies.

In the monthly IMS one factor explains more than 80% of the total variance. In both cases, there is a common volatility component, with an added high-frequency volatility component on the CAC data[6].

For the Dow Jones Index series, the ranking of monthly market turbulence by the IMS and each of the volatilities indicates that the IMS incorporates mainly information present in the highest frequencies (daily to weekly for the Dow Jones Index), while the low-frequency volatilities appear noisy[7]. By contrast, using this time the *intra*-day data on the CAC40, the correlations between IMS and each volatility are lower and none of the volatilities contains exactly the same information as the IMS[8].

Figure 2: Evolution of the IMS on the CAC40 Index over 1995-2002 (daily measurements)



Source: Euronext, computation from authors. Period: 1995-2002, intra-day data.

The market events of September 2001 no longer correspond to the most significant perturbations of the sample, but only to the biggest since 1987. It ranks as the 28th greatest shocks since 1896 when comparing the monthly IMS values. However, many disturbances correspond to the same crisis, as they occur closely in time. To take this into account, we cluster together the consecutive periods with a high IMS value. The

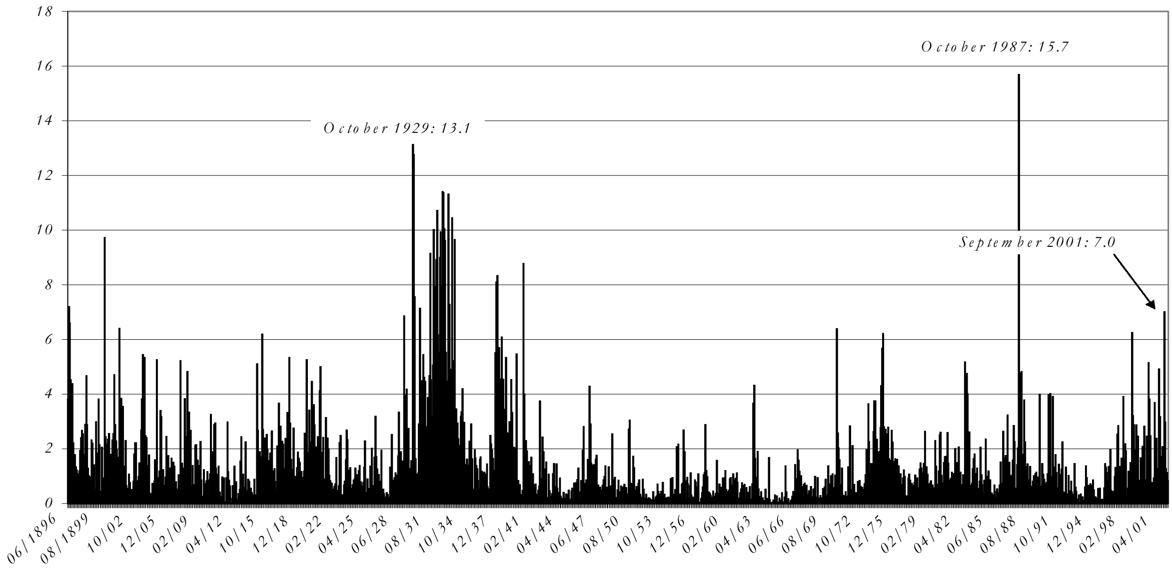
[6] The other possibility would be the computation the probability of the full volatility vector by a Monte-Carlo integration of the multi-normal cumulative density function. The ranking of crises, while very close (Spearman correlation coefficient of .95) is not strictly identical (significantly different at the 5% threshold). We choose to use the PCA because it is both faster and less arbitrary since it does not depend on the choice of the scaling grid.

[7] Spearman rank correlations between the IMS and its volatility components range from 0.94 (P-stat for equal ranking is 2.5%) for the monthly volatility computed at the daily scale to 0.3 for the squared monthly return (P-stat for equal ranking is 0.0%).

[8] Spearman rank correlations range from 0.87 to 0.39, leading in all cases to the rejection of identity between IMS and volatility rankings.

threshold used to decide whether the IMS is high is 3, chosen roughly as the last decile of the empirical cumulative density function[9], as well as a point where the empirical and theoretical functions coincide (see Figure 1 above).
When grouping consecutive periods with high IMS, the current crisis is now ranked ninth in one century (see Table 1 below). Note that periods with high IMS initially correspond to market downturns even if subsequent recoveries can bring the index close to its starting level. This is in accordance with classical results about the tail asymmetry of return distribution.

Figure 3: Evolution of the IMS on the Dow Jones Index
over 1896-2002 (monthly measurements)



Source: Economagic, computation from authors. Period:1896-2002, daily data.

A close examination of the volatility vector for the month of September indicates that all frequencies equally contribute to the high level of the IMS, except for the monthly close volatility that is lower. The shock was spread over most market participants within our frame of reference, from very-short to medium term operators. Extending the horizons considered in the IMS to up to 6 months (thus computing only 2 values of the IMS *per* year) yields the same conclusion: most market participants strongly reacted to the shock.

On the French market (see Table 2 above), the largest crises over the period have a duration of 1 to 14 days. Amongst them, we can identify the Asian crisis (Summer 1997), the Russian turbulence (Fall 1998), the spread of the Brazilian crisis (early 1999), the burst of the NASDAQ bubble (Spring 2000), the turmoil following the collapse of Argentinean public debt (Spring 2001) and finally the consequences of the World Trade Center attack (Fall 2001). Regarding the last event, the crisis lasted 14 days (with an IMS higher than 3) starting from the 6th - the attacks on the 11th shocked indeed a market already strongly

9 Changing slightly the threshold does not change the ranking of the crises. The effect of a lowering of the threshold is to lengthen the crises by joining several into one. For instance, comparing the same crises with a threshold of 2 instead of 3 leads to a 40% increase in their length (a decrease of 1 of the threshold leads by construction to an overall twofold increase of the number of observations above the threshold).

volatile - with a quieter day on the 13th (where the IMS fell slightly below 3). The values of the IMS over October and the first days of November - if they still correspond to our *ad hoc* definition of a crisis - were no longer among the highest in the sample.

Table 1: Ranking the First 10 Crashes with the IMS
(US market since 1896)

Crisis #	Crisis starts in	Crisis ends in	*Maximum* IMS over the period	Crisis duration (in months)
1.	Oct-87	Feb-88	15.68	4
2.	Oct-29	Jan-30	13.12	3
3.	May-31	Jan-34	11.41	32
4.	Dec-1899	Jan-00	9.71	1
5.	May-40	Jul-40	8.77	2
6.	Sep-37	Feb-38	8.33	5
7.	Jun-1896	Dec-1896	7.19	6
8.	Jun-30	Jan-31	7.13	7
9.	Sep-01	Oct-01	7.00	1
10.	Dec-28	Jan-29	6.85	1

Source: Economagic, computations by the authors. Period 1896-2002, daily data.

Table 2: Ranking the First 10 Crashes with the IMS
(Paris Market since 1995)

Crisis #	Crisis starts in	Crisis ends in	*Maximum* IMS over the period	Crisis duration (in business days)
1.	09/06/01	09/27/01	15.34	14
2.	09/21/98	10/09/98	12.31	14
3.	01/13/99	01/18/99	8.41	3
4.	05/28/97	05/29/97	8.19	1
5.	08/28/98	09/02/98	7.94	3
6.	03/14/01	03/15/01	7.82	1
7.	04/05/00	04/6/00	7.75	1
8.	09/17/98	09/18/98	7.71	1
9.	06/2/97	06/03/97	6.70	1
10.	04/30/98	05/04/98	6.33	1

Source: Euronext, computations by the authors. Period 1995-2002, intra-day data.

An analysis of the principal components making up the IMS shows that the first factor explains the full extent of the last crisis; the level of the second factor remaining normal and carrying little information in this case. Thus there was no complementary high-frequency element added to the common trend during the peak of the crisis.

Finally, the September turbulence on the French and American markets share some characteristics: this crisis was the deepest since 1995, but the market recovery was relatively quick (the crisis did not go beyond October in both markets). The decision to close the US market might have limited the extent of the crisis, since on the French market the peak of the crisis occurred on the 12th of September while the US market was closed (during the days of market closure, the mean IMS on the French market was 10 corresponding to a probability of 0.1%).

Previous results have shown that the IMS was not reducible to one of its components. To complement the analysis, we focus in the next section on the comparison between the IMS assessments of risk and those of several traditional measures of risk.

Dissimilarities with Traditional Measures of Risk

The IMS should encompass traditional measures of risk, but the use of additional frequencies in a multivariate probabilistic framework allows for improvements in the assessment of risk.

Table 3 presents the Spearman and Pearson correlation coefficients corresponding to competing measures on the US market since 1896 based on daily data, whilst Table 4 gives theses coefficients for the French market since 1995 using intra-day data.

The four measures that we compare to the IMS are the maximum drawdown - that is by definition the maximum loss an investor could suffer over the period, the realized volatility corresponding to the empirical variance over the period of the highest frequency returns, the squared close-to-close return and the volatility forecast given by the RiskMetrics procedure (see Classic RiskMetrics, J-P Morgan/Reuters, 1996).

The IMS is highly correlated with realized volatility, negatively correlated with maximum drawdown, and weakly correlated with the two other volatility proxies (Squared Return and Canonical RiskMetrics estimate). At the 5% threshold, none of the measures is either independent or perfectly correlated, meaning that each responds to market turbulence similarly but does not capture the same component of a crisis at the same time.

The strong correlation between the monthly IMS and the realized volatility (the variance of the daily returns of that month) is explained by the fact that the PCA decomposition of the variance vector on the US market yields only one principal component that accounts for most of the daily variance, the rest being noise. With the daily Paris IMS, the strongest correlation is found with the maximum drawdown, meaning that the intra-day turbulence determine the value of the IMS.

In both cases, the RiskMetrics forecast is more correlated with the realized volatility than with the squared return, in accordance with the previous findings by Andersen and Bollerslev (1998-a) about GARCH forecasting power: dynamic volatility models yield poor forecasts of instantaneous volatility (squared returns), but relatively accurate ones of the realized volatility (integrated high-frequency squared returns).

Table 5 below illustrates the differences in the magnitude of the crises, as measured by the different criteria. Sorting observations on the French market by decreasing IMS, we compute here 5 different probabilities associated to each event. By construction, the IMS-based probability is simply 2^{-IMS}. The probability of observing a given maximum drawdown is estimated by fitting an extreme distribution (in this case the Frechet distribution[10]) to the series of maximum drawdowns. The other columns correspond to the value of the Gaussian distribution for the observed close-to-close returns, using three different variances: the high-frequency estimate of the realized volatility, the global variance of the close-to-close returns and the RiskMetrics forecast.

[10] The choice of Frechet as the limit distribution of extreme returns is documented for instance in Jansen and de Vries (1991) or Longin (1996); see Johansen and Sornette (2001) for an alternative.

Table 3: Correlation Coefficients between Measures of Risk (US market since 1896) Spearman [Pearson] Coefficients

	IMS	Maximum Drawdown	Realized Volatility	Squared Return	Canonical RiskMetrics
IMS	1.00 [1.00]	-0.71 [-0.79]	0.94 [0.96]	0.27 [0.51]	0.30 [0.50]
Maximum Drawdown	-0.71 [-0.79]	1.00 [1.00]	-0.71 [-0.8]	-0.68 [-0.83]	-0.21 [-0.36]
Realized Volatility	0.94 [0.96]	-0.71 [-0.8]	1.00 [1.00]	0.29 [0.54]	0.32 [0.50]
Squared Return	0.27 [0.51]	-0.68 [-0.83]	0.29 [0.54]	1.00 [1.00]	0.07 [0.22]
Canonical RiskMetrics	0.30 [0.50]	-0.21 [-0.36]	0.32 [0.5]	0.07 [0.22]	1.00 [1.00]

Source: Economagic, computations by the authors. Period 1896-2002, daily data.

Table 4: Correlation Coefficients between Measures of Risk (Paris Market since 1995) Spearman [Pearson] Coefficients

	IMS	Maximum Drawdown	Realized Volatility	Squared Return	Canonical RiskMetrics
IMS	1.00 [1.00]	-0.83 [-0.87]	0.69 [0.78]	0.39 [0.45]	0.49 [0.49]
Maximum Drawdown	-0.83 [-0.87]	1.00 [1.00]	-0.70 [-0.76]	-0.46 [-0.56]	-0.49 [0,49]
Realized Volatility	0.69 [0.78]	-0.70 [-0.76]	1.00 [1.00]	0.30 [0.38]	0.70 [0.68]
Squared Return	0.39 [0.45]	-0.46 [-0.56]	0.30 [0.38]	1.00 [1.00]	0.20 [0.27]
Canonical RiskMetrics	0.49 [0.49]	-0.49 [-0.48]	0.70 [0.68]	0.20 [0.27]	1.00 [1.00]

Source: Euronext, computations by the authors. Period 1995-2002, intra-day data.

Table 5: Comparison of Probabilities assigned to the Most Turbulent Days by the IMS and Different Risk Measures (Paris Market since 1995)

	Dates	IMS	Maximum Drawdown	Realized Volatility	Global Variance	Canonical RiskMetrics
1.	09/12/2001	0.002%	0.208%	64.293%	82.728%	80.102%
2.	09/11/2001	0.006%	0.028%	3.005%	0.000%	0.000%
3.	09/21/2001	0.007%	0.101%	22.878%	4.085%	10.786%
4.	10/02/1998	0.020%	0.472%	49.801%	49.517%	50.724%
5.	10/01/1998	0.099%	1.903%	1.811%	0.007%	0.278%

Source: Euronext, computations by the authors. Period 1995-2002, intra-day data

The most turbulent day according to the IMS corresponds to a close-to-close change of +1.13 %, and thus is unremarkable by low-frequency measures. Moreover, the Gaussian with low-frequency variance, whether using traditional or RiskMetrics definitions, proves unable to cope with extreme events. The realized volatility measure may on the contrary overestimate the daily volatility in those cases, as seen from the example of the 11th of September 2001. Traditional measures based on close-to-close returns miss important intra-day turbulence, and either underestimate the frequency of extreme close-to-close events or overestimate it in the realized volatility case.

The maximum drawdown, by contrast, seems to yield reasonable estimates, whereas the IMS might slightly underestimate the extreme probabilities. The lowest probability according to the Frechet distribution of maximum drawdowns is 0.028% on September 11th, translating into a probability of 37% of observing such an event in a 1692 day sample (under the simplifying hypothesis of i.i.d. returns). For the same event, the IMS yields a 0.002% probability that only has a 4% chance of being observed for that sample size. Thus the accuracy of the IMS appears inferior in the most extreme cases.

Figure 4 represents the estimated quantiles of the IMS and the corresponding Frechet quantiles for each observation. Globally, the two measures, while close in many cases, are not identical. In the extreme cases (bottom left of Figure 4), the relation is slightly asymmetric: low probability IMS values correspond to low probability drawdowns, but low probability drawdowns can be associated with a relatively large range of IMS probabilities (up to 40 %). This likely corresponds to localized drops in prices within otherwise quiet days that do not match the traditional definition of turbulence. Such price patterns do not correspond to strong variability in agents' anticipations, have no lasting consequences and should not be singled out by the crash indicator.

Figure 5 compares the IMS with the implied volatility backed out from option market prices[11]. Unlike the previous measures, this one is not computed from the same market data and relies on different assumptions. Both indicators - corresponding to the spot and the option markets - are seemingly related - especially when strong turbulence happen - but do not lead to the same ranking in terms of crisis assessment. Large crises are detected in both cases but, like in the drawdown case, large ISD values can correspond to a range of IMS measures (an ISD higher than 35% can correspond to IMS values from 4 to 15).

All measures converge when market turbulence is generalized, but not all perturbations are detected by traditional measures. Specifically, the daily IMS is strongly linked to measures that incorporate intra-day information, that we believe important to assess the extent of crises. When these indicators signal crises, the IMS still displays a variety of values, meaning it can permit to draw finer distinctions between events. Unlike most measures, the IMS allows for plausible estimation of the probability of shocks, even if the accuracy of extreme events estimation can be refined by specific techniques.

[11] These statistics are provided since 01/2000 by Datastream™ and correspond to the inverted Black-Scholes formula for at-the-money options (on the French CAC 40 index for the shortest maturity) traded on the same business day.

Maximum Drawdown
ce 1995)

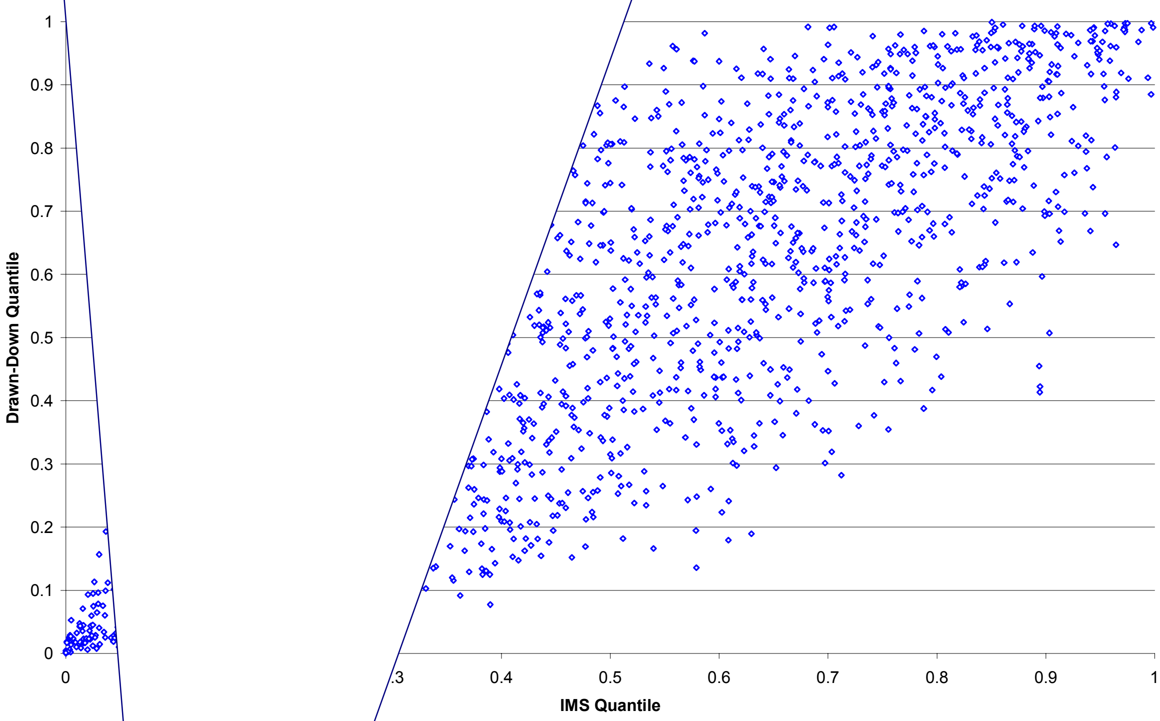


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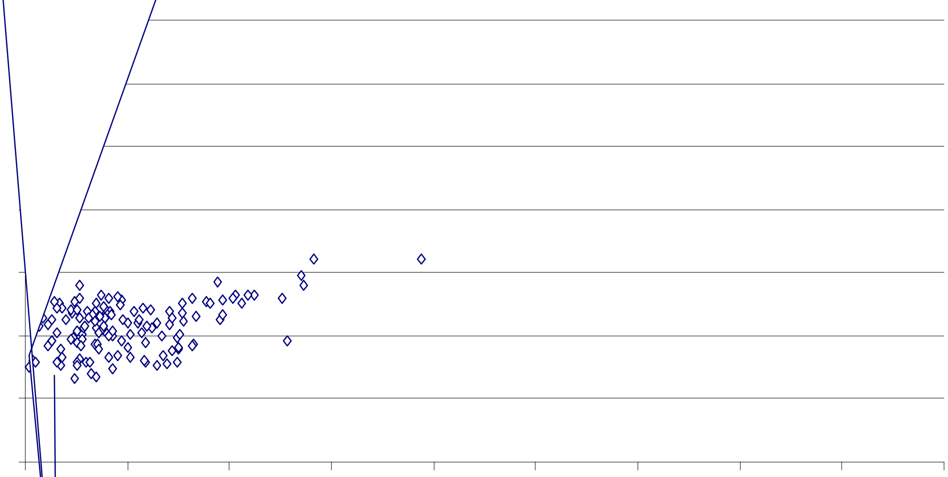
Figure 5: Plot of IMS *versus* ISD
(Paris Market since 2000)



Conclusion

The main goal of the paper was to develop and apply a simple measure of risk for the purpose of historical comparisons. The Index of Market Shocks presented here fulfills the requirements of a universal risk measure: it is easily computable and interpretable, provides a satisfactory quantitative assessment, encompasses traditional risk approaches and has an intuitive economic interpretation. The underlying method is a counterpart of the widely-used Richter Scale and the statistical assumptions are compatible with theoretical representations of return dynamics and well-known stylized facts. The IMS empirically scales close to the theoretical prediction for large events, it yields probabilities of observing risk conditions by filtering the noise and redundant information in each of the frequencies it includes and provides a better appraisal of the main factor(s) of risk.

The application shows that the September 2001 events in New York caused a major crisis - far more significant in France than the Asian and Russian crises were. We nevertheless see from the US market data that these turbulence are not of the same order of magnitude than the historical crises of October 1987 and October 1929. If we interpret the scale strictly, the crisis of 1987 would be 400 times less likely than the last one. More generally, while our index seems to indicate that the amplitude of turbulence increases since 1995, the long term American data does not confirm this observation since the IMS on the Dow Jones Index does not have a higher level now on average.

The applications of the IMS on two samples having different characteristics (daily data and long sample period, high-frequency data and short sample period) lead to similar interpretations, in particular with respect to the amplitude of the September 2001 crisis. The robustness of the indicator should allow for natural extensions as a risk scale on single stocks or stock portfolios. Thus, it should be of primary interest for investors, risk managers and regulators.

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