Are Annuities Value for Money?
Who Can Afford Them?

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Abstract

This paper solves an empirically parameterized model of households' optimal demand for nominal and inflation indexed annuities. The model incorporates mortality, inflation, and real interest rate risk.

The model draws some interesting predictions. First, the welfare calculations on the access to annuities markets show that nominal annuities are welfare improving even when sold at the empirically parameterized cost, which is above fair value. Real annuities are welfare improving over nominal annuities when sold at a fair price, but when we incorporate the empirically parameterized annuity premium the gains become negative at all wealth levels.

Second, the simulation of the model for the British population wealth distribution shows that an important explanation for the little interest in annuities comes from the fact that annuities are extremely expensive, compared with the total accumulated assets held by households. In other words many households can not afford even to enter the annuity market.

The paper also compares the simulated annuity demand to the actual demands reported in the Family Resources Survey. For those individuals who buy annuities, the simulated demands are not very different from those observed in the survey.

Keywords: Annuities, Inflation Risk, Real Interest Rate Risk, Mortality Risk

JEL classification codes: E21, G11

1   Introduction

Currently, and in most countries, the government, through the first pillar of the social security system, insures retirees against inflation and longevity risk. At retirement, the state pension that each retiree is entitled to receive is computed based on his lifetime earnings. The retiree is entitled to receive this pension for as long as he lives, and in this way obtains insurance against longevity risk. In addition, and in most countries, nominal state pensions are indexed to a measure of consumer prices such as the consumer price index (CPI), so that the real purchasing power of retirees is relatively unaffected by inflation movements.[2]

In recent years, economists and politicians in most Western countries have become increasingly worried with the sustainability of the current social security system and its current level of benefits. The ageing of the population has led to a dramatic and unsustainable increase in liabilities.

Consequently the second and third pillar of the social security system, which are mostly funded systems, have become increasingly important in financing households' retirement. In a funded system, each person accumulates savings in a retirement account which will become liquid at retirement age.

However, accumulated financial wealth that is run down at retirement may not be a good alternative to state pensions, because it lacks some of the insurance features of the latter. A portfolio of financial assets does not necessarily provide insurance against inflation risk and longevity risk. Whereas in certain countries retirees may invest their financial portfolio in inflation-indexed bonds and in this way insure themselves against inflation risk, it may be harder to obtain insurance against longevity risk.

As demonstrated in the pioneering work of Yaari (1965), the optimal choice of a non-altruistic individual is to hold all his assets on actuarial notes, whenever the

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[2]We say relatively unaffected because state pensions are usually indexed to expected inflation which may differ from realized inflation, and because the basket of goods of a typical retiree may be different than the basket of goods used to compute the CPI.

probability of death is positive. This result is driven by the fact that the annuity system reduces the inefficiency created by the uncertainty of the time of death, as well as the fact that actuarial notes pay a higher interest (market interest rate plus a mortality premium) conditional of being alive.

Applying this result, one alternative is for individuals to use their accumulated financial wealth to purchase an annuity upon retirement. If the annuity is inflation-indexed it will insure the retiree against inflation movements in addition to longevity risk. Annuities are offered in the private marketplace in many countries, including the United Kingdom. Thus, it seems, the features of state pensions may be replicated in the marketplace, at some cost, for households who wish to do so.

One concern, though, is that the interest by retirees in annuities in the private marketplace has, to this date, been limited.[3] Literature on annuity demand has been trying to reconcile this fact with the theory by relaxing some of the Yaari's model restrictive assumptions. Namely, perfect information, no bequest motive, adjustable annuity payout and no uncertainty other than the time of death.

One line of reasoning is based on the imperfect information of the annuity market which results in self selection. This externality could drive households to insure themselves within the family units instead of relying on a private annuity market, as in Kotlikoff and Spivak (1981) and Brown and Poterba (2000). However, despite the self selection in annuities markets, empirical evidence shows that annuities are no longer unfairly priced. Mitchell, Poterba, Warshawsky and Brown (1999) find that the expected present discounted value of payoffs, relative to the initial cost of the annuity, has been increasing over time in the U.S.. Finkelstein and Poterba (2002) and Cannon and Tonks (2002) present evidence that annuity rates are not unreasonably low in the U.K..

Yagi and Nishigaki (1993) attribute the low demand for annuities to the fact that the annuity market is constrained to constant payouts (either nominal or real)

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[3]This empirical regularity has been well documented in previous studies. See Friedman and Warshawsky (1990), Poterba and Wise (1996) and Brown (1999), among others.

as opposed to the ones assumed in Yaari's model which are adjusted to optimal consumption during retirement.

Another type of explanations for the low demand of annuities is the bequest motive. Kotlikoff and Summers (1981) argue that intergenerational transfers account for the vast majority of savings in the U.S. and therefore households refrain from annuitizing their wealth. Recall that in Yaari's model there was no bequest motive for holding monetary wealth. However, Brown (2001) finds no evidence that bequest motives are important factor in making marginal annuity decisions.

Introducing a risky asset as means of investment can improve the budget constraint in the future and therefore create a real option value of delaying the decision to annuitize. This is the path taken in Moshe and Young (2002), in order to show that it might be optimal for the individual to delay annuitization.

Bernheim (1991) presents empirical evidence that strongly suggests that state pensions have provided most (or all) longevity risk that retirees required. Once that is not longer the case the demand for annuities will increase.

In order to investigate the plausibility of this explanation, the present paper proposes a dynamic model of household behavior, from retirement onwards, that is used to evaluate the benefits and predict the demand for nominal and inflation-indexed annuities. The model incorporates empirically parameterized mortality risk, both inflation and real interest rate risk, a bequest motive, and different annuity premia for nominal and real annuities. The distribution of wealth held by households is also empirically parameterized. The parameterization is done for two economies, namely the U.S. and the U.K.

The model draws some interesting predictions. First, the welfare calculations on the access to annuities markets show that nominal annuities are welfare improving even when sold at the empirically parameterized cost, which is above the fair value.[4] Real annuities are welfare improving over nominal annuities when sold at a fair price,

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[4]A simillar result is found in Mitchell et al (1999).

but when we incorporate the empirically parameterized annuity premium the gains become negative at all wealth levels. This means that when given the choice consumers would never choose to buy real instead of nominal annuities. This effect is stronger in the U.S. than in the U.K., mainly because the cost of inflation protection is higher in U.S.. When a scenario of lower state pension is considered welfare gains from both nominal and real annuities are increased.

The policy functions show that the demand for annuities is increasing in wealth, and how it varies across different parameterizations. Demand for annuities is higher when state pensions are lower and when current inflation is higher. On the other hand, a higher bequest motive decreases demand for annuities. The level of wealth above which individuals start buying annuities is also affected by different parameterizations. This has an important effect on simulated participating rates.

The simulation of the model for the British population wealth distribution shows that an important explanation for the little interest in annuities comes from the fact that annuities are extremely expensive, compared with the total accumulated assets hold by families. In other words many families can not afford even to enter the annuity market. These simulated demands are compared to the actual demands reported in the Family Resources Survey. For those individuals who buy annuities, the simulated demands are similar to those observed in the survey.

This paper also makes an important contribution for the literature debate on the presence of adverse selection in annuities markets. The evidence on the presence of adverse selection in annuities markets is usually based on the fact that the mortality rate of the annuitant population is lower than the mortality rate of the general population. However, mortality rate is also negatively correlated with wealth accumulation (see Attanasio and Hoynes, 2000). Therefore it becomes unclear whether the fact that annuitants are longer lived results from active selection or passive selection, as described in Finkelstein and Poterba (2002) and Murthi, Orzag and Orzag (1999). Distinguishing between the two types of selection is important because they have different policy implications. The present model provides a powerful example of

passive selection. Only wealthier individuals participate in the annuity market.

The remainder of the paper is organized as follows. In Section 2, model, solution technique and parameterization are presented. In Section 3, welfare gains and the policy functions for the demand for annuities are presented and discussed. Section 4 reports the simulation results, and Section 5 concludes.

2 The Model

2.1 Model specification

Time parameters and preferences

I model consumption and asset choices of an household from retirement onwards.[5] The household may either be an individual or a couple in which case the household only dies when both individuals die. The household lives for a maximum of T periods. I let each period in the model correspond to one year, and I allow for uncertainty in the age of death in the manner of Hubbard, Skinner and Zeldes (1995). Let p_t denote the probability that the household is alive at date t, conditional on being alive at date $t-1$. Of course, $p_1 \equiv 1$. Then, household i's preferences are described by the time-separable power utility function:

$$E_1 \sum_{t=1}^{T} \beta^{t-1} \prod_{j=1}^{t-1} p_j \left[p_t \frac{C_{it}^{1-\gamma}}{1-\gamma} + (1-p_t)B_i \frac{W_{i,t}^{1-\gamma}}{1-\gamma} \right], \quad (1)$$

where β is the time discount factor and γ is the coefficient of relative risk aversion. With probability $(1-p_t)$ the household dies and leaves his wealth W_t as a bequest.

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[5]The dynamic model is in the line of the precautionary savings literature models proposed by Carroll (1997) and Deaton (1991).

The parameter B measures the intensity of the bequest motive. In each period t, $t = 1, ..., T$, the household chooses consumption C_t.

The term structure of nominal and real interest rates

Nominal and inflation-indexed annuities differ because nominal interest rates are variable over time. This variability comes from movements in both the expected inflation rate and the ex ante real interest rate. Following Campbell and Cocco (2002) I use a model that captures variability in both these components of the short-term nominal interest rate, and allows for some predictability of interest rate movements. Thus in the model there will be periods when households can rationally anticipate declining or increasing short-term nominal interest rates.

I assume that expected inflation follows a first-order autoregressive process. That is, log one-period expected inflation, $\pi_{1t} = \log(1 + \Pi_{1t})$, follows the process:

$$\pi_{1t} = \mu(1 - \phi) + \phi\pi_{1,t-1} + \epsilon_t, \tag{2}$$

where ϵ_t is a normally distributed white noise shock with mean zero and variance σ_ϵ^2. The expected log real return on a one-period bond, $r_{1t} = \log(1 + R_{1t})$, is given by:

$$r_{1t} = \bar{r} + \psi_t, \tag{3}$$

where $\bar{r}$ is the mean log real interest rate and ψ_t is a normally distributed white noise shock with mean zero and variance σ_ψ^2.

The log yield on a one-period nominal bond, $y_{1t} = \log(1 + Y_{1t})$, is equal to the log real return on a one-period bond plus expected inflation:

$$y_{1t} = r_{1t} + \pi_{1t}. \tag{4}$$

To model long-term nominal interest rates, I assume that the log pure expectations hypothesis holds for both nominal and real interest rates.[6] That is, we assume that the log yield on a long-term n-period nominal bond is equal to the expected sum of successive log yields on one-period nominal bonds which are rolled over for n periods:

$$y_{nt} = (1/n) \sum_{i=0}^{n-1} E_t[y_{1,t+i}]. \tag{5}$$

This model implies that excess returns on long-term bonds over short-term bonds are unpredictable, even though changes in nominal short rates are partially predictable.

Available nominal and inflation-indexed annuity contracts

I study both nominal and inflation-indexed annuity contracts. If at retirement age the interest rate on a nominal bond with maturity t is $Y_{t,1}$, and the household purchases a nominal annuity that makes an annual nominal payment of $A^{N,N}$ (where the double superscript refers to the nominal payout of a nominal annuity), the expected present discounted value (EPDV) of the annuity payouts is given by:

$$EPDV_1^N = \sum_{j=1}^{T} \frac{A^{N,N} \prod_{k=1}^{j} p_k}{(1+Y_{j,1})^j}. \tag{6}$$

This EPDV can be compared with the premium cost of the annuity to obtain a measure of the "money's worth" of the annuity. The difference between $EPDV_1^N$ and the premium cost of the annuity covers expenses and other administrative costs associated with the sales of annuities and normal profits of insurance companies. The annuity premium (P) and the money's worth are therefore defined as:

$$AnnuityCost = (1+P) \times EPDV$$

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[6]For a textbook exposition and summary of the empirical evidence on this model, see Campbell, Lo, and MacKinlay (1997), Chapter 10.

$$Money'sWorth = EPDV/AnnuityCost$$

In a nominal annuity payments are fixed at contract initiation, so that real annuity payments $A_t^{R,N}$ are inversely proportional to the price level P_t:

$$A_t^{R,N} = \frac{A^{N,N}}{P_t} \tag{7}$$

This implies that a nominal annuity contract is a risky contract because its real capital value is highly sensitive to inflation.

Similarly, if at retirement age the interest rate on a inflation-indexed bond with maturity t is $R_{t,1}$, and the household purchases an inflation-indexed annuity that makes an annual real payment of $A^{R,R}$ (where the double superscript refers to the real payout of a real annuity), the expected present discounted value (EPDV) of the annuity payouts is given by:

$$EPDV_1^R = \sum_{j=1}^{T} \frac{A^{R,R} \prod_{k=1}^{j} p_k}{(1 + R_{j,1})^j}. \tag{8}$$

Real annuity payments are fixed at contract initiation, and nominal payments increase in proportion to the price level P_t. Thus, unlike a nominal annuity, the real capital value of an inflation-indexed annuity is not sensitive to inflation.

Retirement Wealth and Pension Income

At retirement age the household has financial wealth W_1, which he can use to purchase an annuity. In addition, the household is endowed with gross real pension income in each period, L_t. As usual I use a lower case letter to denote the natural log of the variable, i.e., $l_t \equiv \log(L_t)$. Household j's age t real pension income is exogenous and is given by:

$$l_{jt} = f(t, Z_{jt}), \tag{9}$$

where $f(t, Z_{jt})$ is a deterministic function of age t and other individual characteristics Z_{jt}. Retirement or pension income is in most countries linked to lifetime earnings which are known as of retirement. In addition, the assumption that real pension income during retirement is deterministic implies that nominal pensions are inflation-indexed. This assumption may be relaxed. It is important because a nominal pension that increases at the rate of inflation is similar to an inflation-indexed annuity and therefore may decrease the benefits of inflation-indexed annuities.

Summary of the household's optimization problem

In summary, the household's control variables are $\{C_t\}_{t=1}^{T}$ and at the initial date whether to convert initial wealth into an annuity. The vector of state variables can be written as $X_t = \left\{ t, y_{1t}, W_t, P_t, \pi_{1t}, A_t^{R,i} \right\}_{t=1}^{T}$ where W_t is real liquid wealth or cash-on-hand, P_t is the date t price level and $A_t^{R,i}$, $i = N, R$ is the date t real annuity on the nominal/real annuity purchased at retirement.

The equation describing the evolution of real cash-on-hand can be written as:

$$W_{t+1} = (W_t - C_t)(1 + R_{1,t+1}) + L_{t+1} + A_{t+1}^{R,i}, \tag{10}$$

where $i = N, R$. This equation assumes that savings are invested in one-period inflation-indexed bonds.

Solution technique

This problem cannot be solved analytically. Given the finite nature of the problem a solution exists and can be obtained by backward induction. I discretize the state space and the choice variables using equally spaced grids in the log scale. The density

functions for the random variables will be approximated using Gaussian quadrature methods to perform numerical integration (Tauchen and Hussey 1991).

2.2 Parameterization

An important component of any quantitatively focused study is the parameterization of the model. With this respect, there are several issues that the reader should keep in mind. First, I take one period in the model to represent one year and use annual data throughout. Second, to the extent that inflation and interest rate processes or longevity risk differ across countries, the benefits of annuities will also differ. In order to explore these differences I calibrate the model for two countries, namely the US and the UK.

Inflation and Interest Rate Processes

In order to estimate the inflation rate process I have used data for the US Consumer Price Index from 1962 to 1999. The estimated parameters are shown in Table 1. The mean log inflation over this period was 4.6%, with a standard deviation of 3.9%, and a first-order autoregression coefficient equal to 0.754. For the inflation rate process for the UK I have used data from the Retail Price Index between 1985 and 2002. The mean and variances are considerably lower, this being partly due to the difference in the sample period.[7]

In order to estimate the parameters for the real interest rate process I constructed a measure of real interest rate by subtracting the logarithm of inflation to the log yield on one-year US Treasury bonds. The mean annual log real yield is equal to 2%, and it is fairly stable. For the UK I used the yield on Index-Linked British Government

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[7]Inflation indexed bonds where only available in the UK after 1985. In order to be consistent with the real interest rate estimation, the sample period for the inflation process also starts after 1985.

Securities from 1985 to 2002. Mean log yield was 4.6% and also fairly stable. Results are reported in Table 1.

Baseline Parameters

I study the problem of retirees, therefore, period one in my model is taken to be age 65, the most common retirement age. Conditional survival probabilities for male population is taken from the US census and the UK Government Actuary's Department, from ages 65 to 99. To simplify, I assume that the individual dies at age 100 with certainty, if he is still alive then. Several studies find supporting evidence for the fact that survival probabilities for the population that buys annuities is larger than for the population as a whole. However, it is not clear whether this is due to active selection, in the sense that annuitants have more information of their survival probabilities than insurance companies, or it results from passive selection, i.e., it results from the fact that wealth and survival probabilities are correlated.

In the baseline case I use a coefficient or relative risk aversion equal to 3 and a discount factor equal to 0.98. The bequest parameter is set to zero. I have estimated retirement income for the US by first calculating average income for the working population, at age 65, from the Panel Study of Income Dynamics.[8] Assuming a replacement ratio of 60% I obtain an estimate for the annual real pension at retirement L equal to ten thousand US dollars. I assume that it is constant throughout retirement. For the UK I have used the same methodology, but based on data from the 2002 Family Resources Survey.[9] This resulted in an estimate of 6.24 thousand British pounds. Table 2 summarizes these results.

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[8]The Panel Study of Income Dynamics is a longitudinal study of a representative sample of U.S. individuals and the family units in which they reside. the sample size has grown from 4,800 families in 1968 to more than 7,000 families in 2001. The study is conducted at the Survey Research Center, Institute for Social Research, University of Michigan.

[9]The Family Resources Survey collects information on the incomes and circumstances of approximately 25,000 private households in Great Britain. It is sponsored by the Department for Work and Pensions.

Annuity Premium

Different annuity markets show substantial pricing differences. In fact, as reported in Brown, Mitchell and Poterba (2001) there can be a difference as a high as 10 percent between the highest and the lowest industry payout, for a given annuity contract. Given this, I rely on previous studies of the U.S. and the U.K. annuity industries to calibrated the annuity premium parameter. More precisely, I obtain the value for the annuity premium P using reported $Money/sWorth$. These $Money/sWorth$ are calculated using average annuity payouts available to a 65 year old male, discounted using risk free interest rates and the population mortality tables described above.

For the U.S. market, and for nominal annuities, I set $Money/sWorth$ equal to 0.85, as reported by Poterba & Warshawsky (2000). The corresponding value for real annuities is 0.749 (Brown, Mitchell and Poterba (2000)).

The U.K. annuity market is divided in two segments: -compulsory annuity market, where tax-qualified retirement funds are annuitized; - and a voluntary market, for all other annuitizations. Although some studies find that $Money/sWorth$ do not differ between the compulsory and the voluntary market (Murthy, Orzag and Orzag (1999)), others do find that prices differ in the two markets. For nominal annuities I have used the estimates of Finkelstein and Poterba (2002), who report values of 0.87 and 0.9 for the voluntary and compulsory markets, respectively. For real annuities, and for the compulsory market, I have set $Money/sWorth$ equal to 0.85, following Brown, Mitchell and Poterba (2001). $Money/sWorth$ for real annuities in the voluntary market is set equal to 0.8, following James and Vittas (1999). According to these parameters, inflation protection seems to be less expensive in the U.K. than in the U.S. This could be due to the fact that in the U.K. there is a developed inflation index bond market which can be used by insurance companies to hedge inflation risk.

Wealth Distribution

In my model the household takes the level of wealth, at age 65, as given. Therefore,

and in order to simulate the demand for annuities for the population as whole, we need to parameterize the wealth distribution. I have used the 1998 Survey of Consumer Finances, which offers a detailed description of U.S. families financial condition, to do so. In order to match the definition of wealth in my model to the one in the data I add two variables, namely: income (which represents households' gross income of the previous year) and financial wealth (which represents households' accumulated wealth in financial assets).

To parameterize the U.K. wealth distribution I have used data from the 2002 Family Resources Survey. Here again I have added total derived capital and labor income to obtain a measure of cash-in-hand. In both samples I have considered individuals aged between 55 and 65, and made the necessary adjustments to have the data for individual wealth instead of household wealth. Table 4 displays the distribution percentiles for the two countries.

2.3   Welfare Metric

In order to assess the benefits of purchasing annuities I perform welfare calculations. These calculations are done in the form of standard consumption-equivalent variations: for each annuity contract I compute the constant consumption stream that makes the household as well-off in expected utility terms as the consumption stream that can be financed by not acquiring an annuity. Relative utility losses are then obtained by measuring the change in this equivalent consumption stream.

3   Welfare Gains and Policy Functions

In this section I study the welfare gains resulting from having access to the annuity market and the optimal demand for annuities. At this stage I use the parameters calibrated for the U.S. economy. As mentioned in the previous section, estimated

inflation is higher for the U.S. than for the U.K. Therefore with the model calibrated to the American economy we are likely to gain better insights as to the effects of the inflation process on the demand for nominal and real annuities. In order to compute the welfare gains I consider different scenarios. First, I compute the gains associated from moving from a scenario with no access to annuities to one with access to nominal annuities. After this, gains are computed from moving from nominal annuities to real annuities. I will also consider different paprameterizations.

Currently, governments are planning to decrease the benefits that they pay retirees, so that households will have to rely more on accumulated financial wealth and less on state pensions to finance consumption when old. To assess the implications of such a scenario I consider a parameterization in which the annual real pension that the household is endowed with decreases by a half, to five thousand dollars.

Figure 1 shows the average welfare gains of nominal annuities relative to the scenario where households do not have access to annuity markets and do not purchase annuities. Welfare gains are increasing in financial wealth and vary according to different parameterizations. Obviously welfare gains are higher when annuities are available at a fair value, which means that their cost is equal to their EPDV, as compared to annuities that have a premium cost, i.e. are cost adjusted. However, when we consider the case where existing state pension in reduced to five thousand dollars instead of ten, the welfare gains are the highest. In order to obtain some insights on the magnitude of these welfare gains, let us consider the example of an individual with median financial wealth, namely thirty seven thousand dollars. He would have to be given an additional 3% increase in annual lifetime consumption in order to be as well off as in a scenario with access to nominal annuities at a fair value. However, if the individual only has access to nominal annuities subject to a premium cost, the welfare gain drops to 1%. On the other hand, if the individual only obtains five thousand dollars of state pension and has access to fair value nominal annuities the welfare gain is as high as 7%.

I now compare welfare gains from real annuities relative to nominal annuities

(Figure 2). Welfare gains are now much smaller. With nominal annuities the agent already obtains longevity insurance, so that the added value of real annuities comes from inflation risk protection. Inflation protection gains are positive and fairly relevant specially for wealthier households when annuities are sold at their fair value. Inflation protection becomes even more valuable when the state pension is decreased; this results from the fact that the state pension is in effect a real annuity. This is so in our model as it is in most of the economies, because retirement state pensions are indexed to the price level. However, if real annuities are sold at the price estimated for the American annuity market, i.e. 42% above the EPDV, welfare gains become negative. This means that consumers would never choose to buy these annuities if they have the choice to buy nominal annuities, even when these are also cost adjusted. This result explains why the market for real annuities almost does not exist in the U.S.

Figure 3 compares the welfare gains (in this case losses) in the real annuity markets in the U.S. and in the U.K.. Welfare losses are considerably smaller in the U.K., although still negative for higher wealth levels. According to this figure consumers with lower wealth are indifferent between real and nominal annuities, at the cost which they are sold in the marketplace.

In the cases considered so far, I have assumed away the bequest motive. However previous studies have shown that a bequest motive can be an important factor in households' consumption and saving decisions. I have solved the model with the bequest parameter equal to 7. Figure 4 compares the welfare gains from access to nominal annuities with and without a bequest motive. The presence of a strong desire to leave bequest significantly reduces the utility gains from annuitizing households' wealth. The bequest motive can significantly reduce the demand for annuities, and can be viewed as a possible explanation for the little demand observed in the U.S. annuity market.

Figures 5 and 6 plot the policy function for annuity demand as a function of financial wealth. Annuity demand is defined in terms of the annual payout offered

by each contract. In order to obtain some guidance for the correspondence between annual payouts and annuity cost, Table 5 shows the fair value cost for different annuity payouts. The demand function does not cross at the intercept for two reasons: (i) the consumer has to finance current consumption, out of his current financial wealth; (ii) there is a minimum amount of annuity the consumer has to buy, which I have set equal to 0.5 thousand dollars.[10] The wealth level above which individuals start buying annuities will determine the participation rate in annuity markets. As one would expect from the welfare analysis, annuity demand is an increasing function of financial wealth. The demand for nominal annuities is higher, for each level of financial wealth, than for real annuities. This is due to the higher cost of real annuities. Although not shown, the demand for nominal annuities is also higher the higher is inflation at the retirment age. This is due to the fact that this makes the cost of the annuity lower. In case the state pension is lower, i.e. five thousand dollars, annuity demand increases. The individual with median wealth would buy a nominal annuity contract that gave him a payout of 3.22 thousand dollars, in case the price was fair. If the annuity is sold with a cost premium, he would only buy 2.47. In case his pension income is reduced his demand for a nominal annuity at a fair value would increase to 3.78, which represents a 20% increase. This is a considerable value if we take into account that his lifetime resources are now lower. The demand for real annuity is the lowest, at 2.01, at a fair value price.

Figure 6 compares demand for nominal and real annuities in the U.S and the U.K.. The differences noticed are mainly due to differences in the inflation rate process and in the level of pension income. Higher current inflation makes nominal annuities cheaper in present value and therefore their demand is higher.

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[10]In the U.K. most of the insurance companies do not provide annuity contracts for values under 10,000 pounds. However, there are a few which would consider selling an annuity worth 5,000 pounds.

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4   Simulation Results

Using the policy functions that resulted from the U.K. parameterized model I have
simulated annuity demand over a sample of 1000 individuals. Individuals differ from
each other in their accumulated wealth at age 65. I have used the wealth distribution
estimated from the Family Resources Survey, which is representative of the British
population. The average demands for nominal and real annuities are shown in Table
6.

The different scenarios considered are mutually exclusive in the sense that con-
sumers only choose how much to buy of a particular annuity. They do not have the
option to choose between different annuity contracts. As we have seen before, this
would imply zero demand on some types of annuity contracts.

The first line of Table 6 shows average nominal annuity demand for the lower
and the upper half of the wealth distribution, when annuities are sold at a fair price.
Demand for nominal annuities is extremely low for the poorer half of the population,
with an average value of 150 pounds payout a year. When there is a positive annuity
premium cost, average demand is even lower. Given the higher cost of real annuities,
their demand is lower than that of nominal annuities. Moreover, the simulation results
show that only 65% of the population could afford to enter the annuity market. When
we adjust the price of the annuity contract to that observed in reality, participation
rates become even lower, reaching a value as low as 56% for the real annuity contract
sold in the voluntary market.

The valuable lesson we learn from these simulations is that, even without consid-
ering extreme deviations from the standard consumption model, such as a positive
bequest motive, extreme risk aversion or even myopic consumers, the model predicts
surprisingly low demand for annuities. This suggests that one important explanation
for the fact that one observes little interest in annuities in the real world is that many
individuals can not afford to buy them.

One of the components of income reported in the Family Resources Survey is an-

nuity income. Table 7 compares the summary statistics for the survey data and for the simulated data. The survey definition of annuity income excludes defined contribution occupational pension schemes. Therefore, and in order to be consistent, I compare it to the simulated demand for nominal annuities in the voluntary market. The average demands shown are calculated for individuals with positive annuity income. Simulated and survey average demands, for those participating in the annuity market, are reasonably close. The main difference being the fact that simulated demand is much more sensitive to wealth than survey demand.[11] However, the model over predicts participation rates. There are two possible ways of generating a smaller gap between the two rates. On one hand, by increasing the bequest parameter or reducing the discount factor one obtains lower predicted participation. On the other hand, there is strong evidence of underreporting in the survey annuity income data when compared with aggregate data (Banks and Emmerson (1999)).

5   Conclusion

This paper solves a dynamic model of households' demand for nominal and inflation-indexed annuities, in the context of uncertainty. Uncertainty comes from different sources: inflation, real interest rate and time of death. In order to draw realistic predictions the parameters of the model are empirically calibrated for two economies, namely U.S. and U.K.

Welfare calculations over the access of annuities markets show that nominal annuities are welfare improving even when sold at the empirically parameterized cost.[12] Real annuities are welfare improving over nominal annuities when sold at a fair price. But when we incorporate the empirically parameterized annuity premium the gains become negative at all wealth levels. This means that when given the choice con-

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[11]In fact the regression slope of annuity income on wealth is 7 times higher in the simulated than the survey data.

[12]A simillar result is found in Mitchell et al (1999).

sumers would never choose to buy real instead of nominal annuities. This effect is stronger in the U.S. than in the U.K., mainly because the cost of inflation protection is higher in U.S.. When a scenario of lower state pension is considered welfare gains from both nominal and real annuities are higher.

The policy functions show that the demand for annuities is increasing in wealth, and how it varies across different parameterizations. Demand for annuities is higher when state pensions are lower and when current inflation is higher. On the other hand, a higher bequest motive decreases demand for annuities. The level of wealth above which individuals start buying annuities is also affected by different parameterizations. This has an important effect on simulated participation rates.

The simulation of the model for the British population wealth distribution shows that an important explanation for the little interest in annuities comes from the fact that annuities are extremely expensive, compared with the total accumulated assets held by families. In other words many families can not afford even to enter the annuity market.

Finally, the paper compares simulated demands to the actual demands reported in the Family Resources Survey. For those individuals who buy annuities, the simulated demands are not very different from those observed in the survey.

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Table 1: Estimated parameters for Inflation and Interest Rate Processes

Description	Parameter	Value US	Value UK
Mean log inflation	μ	0.046	0.016
S.d. of log inflation	$\sigma(\pi_{1t})$	0.039	0.010
Autoregression parameter	ϕ	0.754	0.690
Mean log real yield	$\bar{r}$	0.020	0.046
S.d. of real log yield	$\sigma(r_{1t})$	0.022	0.016

Note: Data Series used for estimating the US processes were the Consumer Price Index and Yield on one-year Treasury Bonds, from 1962 to 1999. Data Series used for estimating the UK processes were the Retail Price Index and annual Yield on Index-Linked British Government Securities, from 1985 to 2002.

Table 2: Baseline parameters

Description	Parameter	Value
Risk aversion	γ	3
Discount factor	β	0.98
Bequest motive	B	0
US Real Pension	L	$ 10,000
UK Real Pension	L	$ 6,250

Note: US and UK real pensions are in thousand US Dollars and thousand Pounds respectively. US Real Pension was estimated fromthe Panel Study of Income Dynamics, 1962-99. UK Real Pension was estimated using the Family Resources Survey year 2002.

Table 3: Annuity Premium above the Fair Value

| | Compulsory | | Voluntary | |
	Nominal	Real	Nominal	Real
US			17.64 (0.85)	42.85 (0.75)
UK	11.11 (0.90)	18.0 (0.85)	15.60 (0.86)	23.45 (0.81)

Note: Annuity premium above the fair price are in percentage points. Money's Worth of the annuity contract are in parenteses.

Table 4: Financial Wealth Distributions for the US and the UK

| | Financial Wealth | |
Percentile	US (Thousand US Dollars)	UK (Thousand Pounds)
10	3.81	3.53
20	8.29	6.29
30	16.60	8.64
40	25.78	11.50
50	36.75	15.08
60	54.97	19.25
70	82.45	25.51
80	129.95	36.58
90	246.18	61.63

Note: Financial Wealth is defined as liquid assets plus labor income held by individuals aged between 55-65. The US wealth distribution was estimated using the 1998 Survey of Consumer Finances, and the UK wealth distribution was estimated using the 2002 Family Resources Survey.

Table 5: Fair Value Annuity Cost, U.S. Parameterization.

Annual Payout	Nominal	Real
0.5	3.86	6.24
1.0	7.73	12.48
10.0	77.34	124.89

Note: Values in thousand U.S. Dollars.

Table 6: Demand for nominal and inflation-indexed annuities - U.K. calibration

		Avg Demand Low 50th pct	Avg Demand High 50th pct	Percentage with zero demand
Nominal	Fair Price	0.15	5.39	35
	Compulsory	0.11	4.96	39
	Voluntary	0.10	4.86	40
Real	Fair Price	0.12	4.87	37
	Compulsory	0.08	4.27	42
	Voluntary	0.06	4.11	44

Note: This table shows the average demand for nominal and real annuities, in thousand British Pounds, for the lower and the upper half of the wealth distribution. Different annuity premia are considered.

Table 7: Summary statistics for simulated and survey data

	Avg Demand Low 50th pct	Avg Demand High 50th pct	Whole Sample	Participation Rate
Simulated Data	0.52	4.86	4.13	60
Survey Data	1.57	3.18	2.71	3

Note: Average values are calculated only for individuals with positive annuity demand. Simulated data is for nominal annuity in the voluntary market. Values in thousand British Pounds

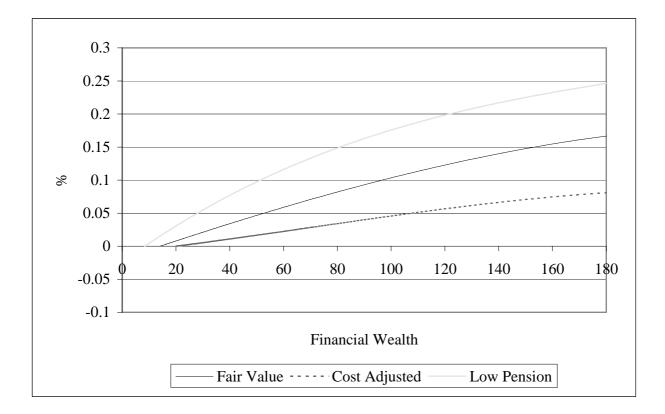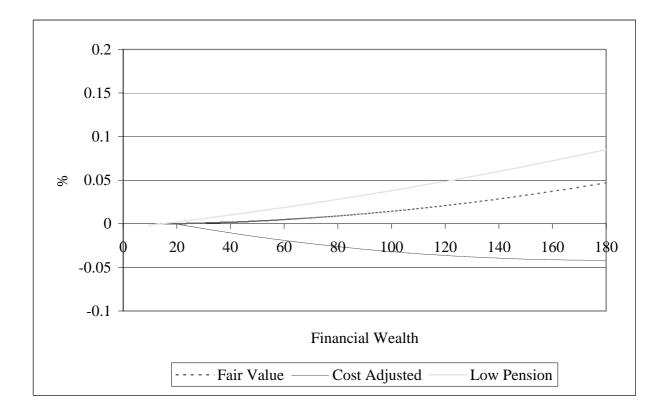**Figure 1**: Welfare Gains from access to Nominal Annuities.

Figure 2 : Welfare Gains from access to Real Annuities (Compared to Nominal Annuities).
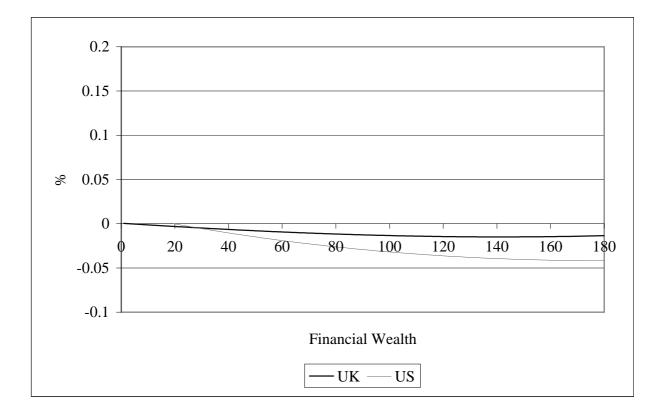
Figure 3 : Welfare Gains from cost adjusted Real Annuities in U.S. and U.K (Compared to Nominal Annuities).

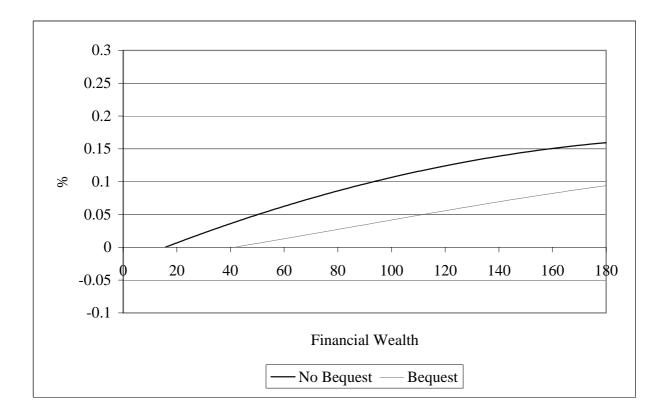**Figure 4** : Welfare Gains from access to Nominal Annuities with Bequest Motive
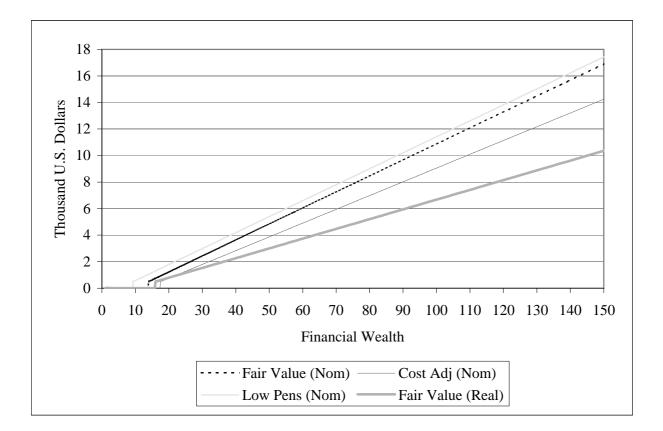
Figure 5 : Annuity Demand

Figure 6 : Annuity Demand in U.S. vs U.K. (Fair Value)