

# Credit Card Debt and Default over the Life-Cycle

Paula Lopes<sup>1</sup>

September 2003

---

<sup>1</sup>Financial Markets Group, London School of Economics. E-mail: p.lopes@lse.ac.uk. I would like to thank Larry Kotlikoff for suggestions and guidance, and Jonathan Eaton, Simon Gilchrist, Sam Kortum, John Leahy, Claudia Olivetti and Alex Michaelides for the helpful comments. Financial support from Fundação para a Ciência e Tecnologia and Fundação Calouste Gulbenkian is gratefully acknowledged. I am also grateful for the helpful comments of Rui Albuquerque, Maria Carapeto, João Cocco, Francisco Gomes and Jan Mahrt-Smith.

## **Abstract**

This paper solves an empirically parameterised model of life-cycle consumption which extends the precautionary savings models of Carroll (1997), and Deaton (1991), to allow for uncollateralized borrowing and default. In case households choose to default: (i) their access to credit markets is restricted; (ii) lenders of funds may seize their financial assets above an exemption level, and up to the amount of outstanding debt; and (iii) there is a “stigma effect,” or a decrease in current utility caused by the social embarrassment of declaring bankruptcy.

The model shows that the decisions to borrow and default are closely related to the shape of the life-cycle labor income profile, and henceforth vary across household education levels. Moreover, the model explains two puzzling empirical facts: (a) why bankruptcy rates have been growing in periods of economic expansion and low unemployment; and, (b) why households hold simultaneously high cost debt and low return assets.

**Keywords:** consumer credit, life cycle, credit card, personal bankruptcy

**JEL classification codes:** D14, D91, E21

# 1 Introduction

Over the past 20 years personal bankruptcy rates grew from 0.35% to 1.43% per year. Unsurprisingly, politicians and economists are increasingly concerned with the magnitude of these numbers, and in particular, with their effects on the supply of credit. But to take the adequate measures for reverting this trend, policymakers first need a clear understanding of the factors that affect the bankruptcy decision.

The personal bankruptcy literature<sup>2</sup> has proposed three different types of explanations for the increase in bankruptcy rates: (i) macroeconomic factors, which affect households indebtedness and credit supply; (ii) current bankruptcy law, which may affect the incentives to file for bankruptcy; (iii) and, the decrease in the stigma, or social embarrassment associated with going bankrupt. However, no consensus has been reached.

In this paper, I try to assess the importance of each of these explanations by analyzing consumers' optimal response to each of the factors involved. In order to do so, I extend the classical precautionary saving model, developed in Deaton (1991) and Carroll (1997), to allow for unsecured borrowing and default. In this life cycle model, the representative consumer is endowed with a stochastic labor income stream, and may borrow from the credit card market, up to a certain credit limit.<sup>3</sup> In each period, after observing his current income, the agent chooses how much to consume (and save), how much to borrow, and in case of outstanding debt, whether he wants to default, where by default I mean to file for bankruptcy. However, if he chooses to default: -his assets (net of an exemption level) will be relinquished; -he will not be able to borrow in the future; -and, he will bear the social embarrassment ( ) associated with the bankruptcy filing.

The parameters of the model are carefully estimated and calibrated. The labor income

---

<sup>2</sup>See Kowalewski (2000) for a review of this literature.

<sup>3</sup>For a general equilibrium model with endogenous credit limits see Chatterjee, Corboe, Nakajima and Rios-Rull (2002).

process is estimated using the Panel Survey of Income Dynamics (PSID). The exemption level is estimated from the different state level asset exemption allowed by the Bankruptcy Reform Act of 1978. Interest rate premia charged by credit card companies and credit limits are calibrated to ensure zero expected profits in the credit card market, and the effect is calibrated in order to match U.S. annual bankruptcy rates with the rates from the simulated model.

The model makes the following important contributions. First, it delivers a natural explanation for the observed fact that most of the post war upward trend in personal bankruptcy has taken place in periods of economic expansion, with unemployment rates falling.<sup>4</sup> It is in periods of low uncertainty and low unemployment rates that buffer-stock consumers wish to consume and borrow more (see Carroll (1992)). In addition, if we allow these consumers to default, as I do here, they are more likely to do so when they expect low unemployment rates.<sup>5</sup>

Second, by introducing the option to default I find that the decision to borrow is not only driven by consumption smoothing but also by strategic behavior. Namely, consumers may choose to borrow with the intention of defaulting in the near future. The circumstances that trigger strategic borrowing are clearly drawn from the policy functions, and most depend on observable variables: age, education level, and credit limit. This will help lawmakers in the challenging task of making the distinction between consumers who intentionally cheat on their creditors and those who were unlucky and deserve a fresh start (see Sullivan, Warren and Westbrook (1989)).

Moreover, consumers' decision to default is determined by the relative importance of the benefits, i.e. waived liabilities, versus the costs of default, i.e., no access to credit, relinquished

---

<sup>4</sup>See Kowalewski (2000).

<sup>5</sup>The procyclicality of default is also found in general equilibrium models, where optimal contractual arrangements are made in the presence of commitment problems (see Kocherlakota (1996) and Kehoe and Levine (2001)). However, in these models default never happens because of the threat of autarky.

assets and . This balance is, in turn, heavily affected by level and shape of labor income profile. The level of current income affects the relative importance of the desutility caused by the social embarrassment of default, <sup>6</sup> On the other hand, the steepness of labor income profile affects the desirability of access to credit. Consequently, the decision to default varies over the life cycle and across individuals with different labor income profiles. This means that for consumers with flat and low labor income profiles (no high school education), neither the access to credit nor the stigma effect is sufficiently important to avoid default, making the probability to default very high. For households with an hump-shaped labor income profile (high school graduates), default is more likely to occur early and late in life. For consumers with very steep labor income profiles (college graduates) the probability of default is very low, at all ages. Supporting evidence for this result is found in the PSID.

Finally, allowing for a level of asset exemption, in the case of bankruptcy, generates the necessary edge that leads our consumers to hold simultaneously high cost debt and low return assets. Gross and Souleles (2002b), Bertraud and Haliassos (2001), among others, report empirical evidence for this portfolio puzzle. Lehnert and Maki (2002) combine state level bankruptcy laws with the Consumer Expenditure Survey, to find that households are more likely to hold simultaneously low return liquid assets and owe high cost unsecured debt in states where exemption levels are higher, a result predicted by my model.

The rest of the paper is organized as follows. Section 2 and 3 discusses the model's specification and solution method. The parameterization is presented in section 4. In sections 5 and 6 policy functions and the resulting simulation profiles are presented and discussed. In section 7 I match the model's predictions to available data. Finally, section 8 concludes.

---

<sup>6</sup>If current income is low, and consumption is also low, marginal utility of consumption is very high making the cost of stigma irrelevant. On the contrary if current income is high, marginal utility of consumption is low, and the relative importance of the cost of stigma is higher.

## 2 The Model

### 2.1 Time parameters and preferences

The representative consumer lives and works  $T$  periods. For simplicity,  $T$  is assumed to be exogenous and deterministic. I truncate the problem at retirement. Life-time preferences are described by the time-separable power utility function:

$$U = \sum_{t=1}^T \beta^{t-1} \frac{c_t^{1-\gamma}}{1-\gamma} - \beta^{T+1} V_{T+1}(A_{T+1}) \quad (1)$$

where  $c_t$  is the level of date  $t$  consumption,  $\gamma > 0$  is the coefficient of relative risk aversion, and  $\beta < 1$  is the discount factor.  $\beta$  is the stigma effect, or the social embarrassment caused by the decision to default, which affects current utility in the period when the consumer chooses to default (in which case  $c_t = 1$ , where the subscript  $t$  refers to the choice of defaulting).

$V_{T+1}$  represents the value to the consumer of assets  $A_{T+1}$  left at the time of retirement. The functional form assumed for the salvage function is the following:

$$V_{T+1}(A_{T+1}) = \frac{A_{T+1}^{1-\gamma}}{1-\gamma} \quad (2)$$

### 2.2 The labor income process

Consumer's age  $t$  labor income,  $y_t$  is exogenously given by:

$$\log(y_t) = (\alpha_t) + v_t \quad (3)$$

where  $(\alpha_t)$  is a deterministic function of age,  $\alpha$ , and a vector of other individual characteristics,  $\alpha$ . I assume that the consumer's labor income is subject to two different shocks: a family specific idiosyncratic shock,  $\varepsilon_t$ , and a persistent aggregate shock,  $\eta_t$ . Thus  $v_t$  can be described by:

$$v_t = y_t + \varepsilon_t \quad (4)$$

where  $\varepsilon_t$  is i.i.d and distributed as  $(0, \sigma_\varepsilon^2)$  and  $y_t$  is given by:

$$y_t = \phi y_{t-1} + \eta_t \quad (5)$$

where the parameter  $|\phi| < 1$  measures the degree of persistence of the aggregate income shock, and  $\eta_t$  is i.i.d. and  $(0, \sigma_\eta^2)$ .

### 2.3 Borrowing constraints and credit history

In each period  $t$ , the consumer is allowed to borrow  $b_t$ , up to a certain limit  $\lambda$ , as long as he has a good credit history. In other words, I am assuming that there is a credit card market, willing to lend money against no collateral, whenever the consumer has not defaulted on his debt in the past. Thus, in the case of no previous default (in which case  $s_t^d = 0$ , where the subscript  $d$  refers to the state associated with previous default), the borrowing constraint is given by:

$$b_t \leq \lambda \quad s_t^d = 0 \quad (6)$$

If at any time the consumer decides to default, his assets will be relinquished up to an exemption level  $\lambda$ , and he will not be able to borrow in the future. That is, let  $a_t$  denote period  $t$  financial assets, then in periods where default occurs the credit card company is entitled to receive:

$$\pi_t = \min\{a_t, \lambda\} \quad c_t = 1 \quad (7)$$

and the borrowing constraint in subsequent periods is given by:

$$b_t = 0 \quad \text{if} \quad s_t = 1 \quad (8)$$

Finally, I assume that whenever a family files for bankruptcy, there is a stigma effect which lowers current utility by  $\sigma$ . Note that when  $b_t = 0$  is equal to zero borrowing is ruled out as in Deaton (1991).

## 2.4 Interest rate

In any given period  $t$ , the consumer earns a fixed real interest rate  $r_t$  on his assets.<sup>7</sup> The second interest rate that is relevant for consumers is the interest rate on debt, denoted by  $\tilde{r}_t$ . I assume that it is equal to the interest rate on financial savings plus a constant debt premium  $\theta$  such that:

$$\tilde{r}_t = r_t + \theta \quad (9)$$

In reality the debt premium may vary over time,<sup>8</sup> with the level of interest rates or the level of aggregate income. However, for the time being, I abstract from this.

## 2.5 The household's optimization problem

In our model the consumer may simultaneously hold positive savings ( $s_t$ ) and credit card debt ( $b_t$ ). In each period, after labor income  $y_t$  is realized, the individual chooses how much to consume,  $c_t$ , how much to save in financial assets,  $s_t$ , how much to borrow ( $b_t \geq 0$ ) or how much to repay ( $b_t \leq 0$ ) of the outstanding debt. In the case of positive outstanding debt, he also chooses whether he wishes to honor ( $\epsilon_t = 0$ ) or to default ( $\epsilon_t = 1$ ) on his liabilities.

---

<sup>7</sup>In Lopes (2001a) I analyze the effects of uncertainty in the real interest rate.

<sup>8</sup>For a model with endogenous debt premium see Lopes (2001b).



The equations describing the laws of motion for outstanding debt and financial assets are:

$$b_{t+1} = (b_t + d_t)(1 + r) - c_{t+1} \quad (10)$$

$$a_{t+1} = [(b_t + d_t - a_t)(1 + r)](1 - \delta) + (a_t - [a_t - 0] + d_t)(1 + r) \quad (11)$$

While the first equation is straightforward, the law of motion for financial assets deserves some explanation. The first part of this equation is the law of motion in case the household chooses not to default ( $\delta = 0$ ). In case he chooses to default current financial assets are decreased by the amount the credit card company is entitled to receive, which are current assets up to an exemption level, or the amount of current outstanding debt, whichever is smaller.

The consumer's optimization problem is to:

$$\max_{\{c_t, D_t^c, b_t\}} \sum_{t=1}^T \beta^{t-1} \frac{1-\gamma}{1-\gamma} - c_t + \beta^{T+1} a_{T+1} \quad (12)$$

subject to constraints (2) through (12) plus non-negativity constraints for financial assets,  $a_t \geq 0$ , and consumption,  $c_t \geq 0$ .

The control variables of the problem are  $\{c_t, D_t^c, b_t\}_{t=1}^T$ . The state variables are  $\{a_t, \varepsilon_t, d_t, s_t\}_{t=1}^T$ . The level of current income is a state variable since the decision of whether to default may depend on current income.

### 3 Solution Method

There is no analytical solution for the consumer's problem. Hence, the policy functions are derived numerically by discretizing the state-space and variables over which the choices are

made. In any period  $t$ , there are two cases, depending on whether the consumer has defaulted in the past or not. In the case of previous default, the Bellman equation is given by:

$$v_t(\mathbf{z}) = \max_C \{ u(\mathbf{z}) + \beta v_{t+1}(\mathbf{z}') \} \quad (13)$$

where the prime refers to date  $t+1$  variables. In the case of no previous default, we have the following equation:

$$v_t(\varepsilon, \mathbf{z}) = \max_{C, b, D^c} \{ u(\mathbf{z}^c) + \beta v_{t+1}(\varepsilon', \mathbf{z}') \} \quad (14)$$

The problem is solved using backward induction. At the time of death, the value function  $v_{T+1}$  is given by the power utility function on assets. This value function is fed in to the last period's problem. For each combination of the state variables, I then compute the utility associated with admissible values for the choice variables. This utility is equal to instantaneous utility plus the continuation value. I optimize over these different choices using grid search. This procedure is then iterated backwards.

In order to compute the value function corresponding to values of the state variables that do not lie on the grid I use cubic spline interpolation. Since full numerical integration is extremely slow, the distributions of the labor income shocks are approximated using Gaussian quadrature. To better capture the curvature in the value function at low levels of the state variables, the logarithmic function of the grid was used. The combination of the choice variables ruled out by the constraints of the problem, are given a very large negative utility such that they will never be optimal. Optimization is done over the different choices using grid search.<sup>9</sup>

---

<sup>9</sup>See Judd (1993) and Tauchen and R. Hussey (1991).

## 4 Parameterization

### 4.1 Labor Income Process

The labor income process described in section 2 was estimated using data from the Panel Study of Income Dynamics.<sup>10</sup> The following adjustments were made to the initial data set. In order to obtain a random sample, families that were part of the Survey of Economic Opportunities sample were dropped. Only male headed households are considered. This is due to the fact that age profiles of households headed by a female are potentially different and therefore require a sample separation. However, the later subsample is too small to be considered. I truncate the problem at retirement following Gourinchas and Parker (1996), so that households with the head aged over 65 and retirees are eliminated. Non-respondents, students, housewives and families reporting more than 6 children are also dropped from the subsample.

In order to estimate labor income variances which do not overstate the real income risk faced by the household, one has to include, in the definition of labor income, ways of self-insuring against this shocks. For this reason, labor income is composed by: total reported labor income plus unemployment compensation, workers compensation, social security, supplemental social security, other welfare, child support and total transfers (mainly help from relatives), all this for both head of household and his spouse, if present. Any observation reporting zero for this broad definition of income was dropped. Real labor income was calculated by deflating labor income using the Consumer Price Index, with 1992 as baseyear.

Age-profiles tend to differ in shape across education groups, a finding that has already been reported in several papers, including Attanasio (1995) and Hubbard, Skinner and Zeldes (1994). Following Cocco et al. (1998) I split the sample in three according to the education

---

<sup>10</sup>The PSID is a longitudinal study of a representative sample of U.S. individuals and family units. The study is conducted at the Survey Research Center, Institute for Social Research, University of Michigan, and has been running since 1968.

of the head of the household: no high school, high school and college graduates. In the few observations where education has changed over the life-cycle, I considered the household as a new entity.

One of the advantages of using the PSID is that the same household is followed over time and one can account for many sources of heterogeneity. For this reason, it is less likely to overestimate the family-specific variance of labor income shocks.<sup>11</sup> The sample runs from 1970 to 1992, therefore a household appears at most 23 times. Households with less observations were not removed from the sample, therefore an unbalanced panel is estimated.

I estimate equation (3) using fixed effects. The function  $\ln(y_{it})$  is assumed to be additively separable in  $\ln(w_{it})$  and  $\ln(h_{it})$ .  $\ln(w_{it})$  is composed by a set of age dummies, and the vector  $\ln(h_{it})$  includes individual characteristics, other than age and education, which potentially have an effect on labor income and therefore have to be controlled for. In this set I include: family size (number of children); marital status; and a family-specific fixed effect. The coefficients on the age dummies are all significant and the results match stylized facts.

In order to obtain smooth versions of the above estimates, third-order polynomials were fitted to the age-dummies. The resulting profiles are shown in Figure 1. The income profiles generated are similar to the ones reported in Gourinchas and Parker (1996), Attanasio (1995), and Carroll and Summers (1991). For consumers with no high school degree, earnings are almost flat during their life-cycle, whereas for families with a college graduate head, life-cycle earnings are hump-shaped. These differences in estimated profiles will allow us to study the effects of the shape of labor income profile on consumers' borrowing and default behavior.

Risk is another important element of the labor income process. I now proceed to estimate the variances of both individual specific and aggregate income shocks. In order to do so, I first sum over all households, for each year, in equation (4). Hence I obtain:

---

<sup>11</sup>Specially comparing with the synthetic-cohort approach.

$$\sum_{i=1}^N v_{it} = \eta_t + \sum_{i=1}^N \varepsilon_{it} \quad (15)$$

I assumed that the family specific shock is purely idiosyncratic. Therefore the second term on the right hand side of (17) cancels out, and thus we are able to estimate the time series of aggregate labor income shocks:

$$\hat{v}_t = \frac{\sum_{i=1}^N v_{it}}{N} \quad (16)$$

Using this series for  $\hat{v}_t$  I estimate equation (5). Given this it becomes straightforward to calculate an estimate for the variance of aggregate income shocks:

$$(\hat{v}_t) = \frac{\sigma_\eta^2}{1 - \phi^2} \quad (17)$$

This holds since by assumption  $\eta_t$  is i.i.d., distributed as  $(0, \sigma_\eta^2)$  and  $|\phi| < 1$ . Finally, I use equation (4) once more to obtain the variance of  $\varepsilon_{it}$ :

$$(\varepsilon_{it}) = (\hat{v}_t) - (v_{it}) \quad (18)$$

which holds since by assumption  $\varepsilon_{it}$  and  $\eta_t$  are uncorrelated and independent over time. These variance decomposition estimates are shown, for each education group, in Table 1.

Since the data set is based on a survey of households, measurement error is a potential problem. More precisely it may lead to an overestimation of the variance values. Therefore, I have dropped observations on the top and bottom two percent of the income distribution and reestimated the variances of the labor income shocks.

Throughout, and otherwise stated the benchmark case is the income profile for high school graduates.

## 4.2 Interest Rate and Borrowing Contract

The real interest rate is set equal to the average of the three month Treasury bills rate from 1970 to 1992, the same time period which was used to estimate labor income profiles.

The parameters associated with the borrowing contract (borrowing rate and credit limit) are hard to parameterize. In recent years there has been a dramatic growth in credit card offers, both in terms of quantity and features of credit cards. Nowadays it is not uncommon for one household to own more than one credit card. There are credit cards with and without annual fee, with a low introductory rate, that give cashback, air miles, and so on. Obviously, the stylized model does not allow us to deal with this. Perhaps the simplest and most consistent way to parameterize the model is to assume that there is competition in the credit card industry, so that credit card firms on average earn zero abnormal profits. In particular, I set the interest rate premium  $\theta$  to 5% and let firms set the credit limit so that abnormal profits are on average zero. In that case the corresponding zero profit credit limit is 10,000 USD. Another zero profit combination would be  $\theta$  equal to 7% and a credit limit  $\lambda$  equal to 8,000 USD. Increasing the interest rate premium makes consumers more willing to default, and as a result credit card companies have to lower the credit limit in order to avoid big losses.

## 4.3 Bankruptcy Law and the Costs of Bankruptcy

In order to obtain some guidance for the value of the exemption level let us take a brief look at the bankruptcy law. Individuals who wish to file for bankruptcy under the U.S. Bankruptcy Code, have the right to choose between filing under Chapter 7 or Chapter 13. Under Chapter 7, debtors must turn over to the Bankruptcy court all their assets above a fixed exemption level, in turn for which many types of unsecured debt are discharged. Under Chapter 13, debtors do not give up any assets, but must propose a plan to repay a portion of their debts from future income. Given this, individuals have an incentive to choose Chapter 7 whenever their assets are less than the exemption level, since doing so allows them to completely avoid

the obligation to repay.<sup>12</sup> In practice 70% of the households filing for bankruptcy do so under Chapter 7. Therefore I assume that all filings for bankruptcy are done under Chapter 7. Although bankruptcy is a matter of federal law, individual states are allowed to adopt their own bankruptcy exemptions.<sup>13</sup> Most states have separate exemptions for equity in the debtor principal residence, equity in motor vehicles, personal property, the cash value of life insurance and IRA Keogh accounts, and a “wildcard” exemption that can be used for any type of property. Since in this model I abstract from housing, durable goods and savings which are illiquid until retirement, I focus on the exemption levels of personal property and “wildcard.” Although the exemption levels vary widely across states, going from zero in Oklahoma to 30,000 USD in Texas, the weighted average value for both these items is 2,000 USD, which is the value I set for  $\bar{w}$ .

There are three different types of costs, for the debtor, associated with the process of filing for bankruptcy: the out-of-the-pocket costs of lawyer’s and court’s fees, which are relatively small and therefore ignored in this model; the “stigma effect” ; and the effect on the debtors credit history records. As it may be expected, parameterizing the desutility caused by the decision to default is a difficult task. However, the recent study by Gross and Souleles (2002a) offers valuable guidance. They have found that the role of the “stigma effect” on default rates is far from trivial. Using a new and comprehensive data set, including all the information held by several credit card companies about its customers, the authors were able to control for risk-composition and other economic fundamentals, and investigate the role of the stigma effect in explaining bankruptcy and default. They found robust evidence that the decline in the social embarrassment from defaulting has an important role in explaining the recent

---

<sup>12</sup>See Domowitz and Sartain (1999) for a model which includes both the decision to file for bankruptcy and the choice between the two chapters.

<sup>13</sup>Gropp, Scholz and White (1997) show that differences in state-level bankruptcy exemptions affect the supply and demand for credit, in particular, higher exemption levels redistribute availability of credit from low asset to high asset households.

growth in personal bankruptcy. Given this I calibrate the stigma parameter in such a way that the average annual default rate predicted by the model<sup>14</sup> is the same as the one observed in the data. In particular, for the period after the Bankruptcy Reform act of 1978 and until 1992,<sup>15</sup> the percentage of households that filed for bankruptcy per year was 0.53%.<sup>16</sup> This results in a value of 0.001 for the stigma effect .

Another cost borne by the debtor as a consequence of filing for bankruptcy is the damage it causes on his credit history. Credit bureaus are allowed to report consumers' bankruptcy filings up to ten years. This will make access to new credit very difficult, if not impossible, for that period, as is well documented in Musto (1999). Given that this is a quite long period of time and in order to simplify the solution of the dynamic programming problem, I assume that credit is inaccessible at all after default.

#### 4.4 Other Parameters

Other parameters include preference and time parameters. In order to facilitate comparison with the existing consumption literature I set the discount factor  $\beta$  equal to 0.97 and the coefficient of relative risk aversion  $\gamma$  equal to 3 (these are also the benchmark parameters used by Deaton, 1991, and Carroll, 1997).

In our model the main source of uncertainty is labor income uncertainty. Broadly speaking, this is probably the main source of uncertainty faced by individuals during their working lives. However, during retirement uncertainty comes from other sources such as medical expenses and time of death. As a result consumption policy functions during retirement depend on variables not considered in the model. I think that studying consumption behavior during

---

<sup>14</sup>The simulations were done using a population consisting of 25% no high school, 50% high school and 25% college graduates.

<sup>15</sup>Same period for which the labor income process was estimated.

<sup>16</sup>Data taken from the Administrative Office of the U.S. Courts and the U.S.Census Bureau.



retirement lies beyond the scope of the present paper, therefore I truncate the problem at retirement/death time and consider that the salvage value function summarizes all retirement and bequest motives. Note that with this approach I am unable to disentangle the effect of retirement and bequest on life-cycle profiles, and refer to both interchangeably. Given this, I analyze individuals during their working lives, between age 20 and 65, which gives us a value for  $\lambda$  equal to 46. For college graduates, the working live starts at 22.

Table 2 summarizes the parameters used in the benchmark case.

## 5 Policy Functions

Before analyzing the simulated profiles over the life-cycle, let us study the underlying policy functions. Recall that my model extends those of Deaton (1991) and Carroll (1997) by introducing borrowing and default. To better understand the effects of each of these on consumption and saving, I first analyze the case where no borrowing is allowed, i.e.  $\lambda$  is set to zero. In subsection 5.1 I introduce each of the extensions in turn.

### 5.1 Effects of Borrowing and Default on Consumption

#### 5.1.1 First case: No Borrowing Allowed

The results are similar to the standard precautionary saving model with liquidity constraints. Following Deaton, I define cash on hand ( $c_t$ ) as the sum of labor income ( $y_t$ ) and financial assets ( $a_t$ ). Figure 2 shows the consumption rules, at various ages, as a function of cash on hand. Consumption is always a positive, increasing and concave function of cash on hand.<sup>17</sup>

Broadly speaking, the consumption function changes along two different dimensions over the life-cycle. One being shifts of the curve and the other changes in the slope.<sup>18</sup> Early in

---

<sup>17</sup>Analytical proof of these properties of the consumption function is given in Carroll and Kimball (1996).

<sup>18</sup>The movements in the consumption function over the life cycle are well explored in Gourinchas and Parker (2002).

life, households behave as standard buffer stock savers. For low levels of cash on hand, and in order to smooth consumption over life, they consume almost all their financial wealth. The marginal propensity to consume of near unity is the result of the liquidity constraint. For higher levels of cash on hand households are no longer liquidity constrained so that the marginal propensity to consume is strictly less than one.

Along the life-cycle consumption functions shift upwards and then downwards as a consequence of the hump shape in the permanent labor income profile. In addition, as agents approach the last period they face less labor income uncertainty and the precautionary savings motive disappears. As a result, marginal propensity to consume increases, being nearly equal to one in the last period. The interaction of these two movements of the consumption function results in the crossing of the curves.

### 5.1.2 Second case: Introducing Borrowing

I first analyze the effect of borrowing. I set the credit limit  $\lambda$ , to the benchmark value shown in Table 2 of 10,000 USD. In this particular case default is not an option, so that consumers are always forced to repay their debts. Figure 3 shows consumption functions for age twenty-five when borrowing is and is not allowed. The ability to borrow makes the consumption functions less concave.

For low levels of cash on hand there are important differences: when allowed, consumers borrow and consumption exceeds cash on hand. In addition, for low levels of cash on hand the marginal propensity to consume is now lower, and with a value closer to the one that we obtain for higher wealth levels. This is a direct consequence of relaxing the borrowing constraint. However, since there still is a limit on borrowing the policy function remains kinked. For higher values of cash on hand where liquidity constraints are not binding, consumption is slightly higher when borrowing is allowed. This is due to the fact that borrowing serves as an additional insurance mechanism against bad future income shocks, and therefore precautionary

savings are reduced.<sup>19</sup> However, as cash on hand increases, uncertainty becomes less relevant and the two consumption function converge. The borrowing policy functions for different ages are depicted in Figure 4. In accordance with the movements in the consumption policy functions over the life cycle so move the borrowing policy functions, expanding for the middle aged consumers in order to accommodate higher levels of consumption.

### 5.1.3 Third case: Introducing Default and Stigma

**Option value of default.** In order to understand the effects of introducing default, we have to keep in mind that the option of being able to discard liabilities at any time in the future in effect provides the consumer with an additional insurance policy.<sup>20</sup> Consumption will be higher for lower levels of wealth. However, for wealthier households the benefits of filing for bankruptcy are smaller; the value of the option of default decreases and consumption functions converge as cash on hand increases.

**Strategic borrowing.** Moreover with the option of default the optimal demand for credit is going to be higher. This is due to the fact that in our model rational consumers act strategically relative to bankruptcy law.<sup>21</sup> That is, for low realizations of cash on hand agents borrow all credit available with the intention of defaulting in the near future. This results in a discontinuity in the debt policy function, as it is illustrated in Figure 5. The size of the discontinuity increases for lower values of the stigma effect. In the limit, when the stigma effect is set equal to zero, debt policy becomes a step function. I will denote this feature of the model as the **strategic borrowing**.

Now that we have studied the effects of default on consumption and borrowing, we will go

---

<sup>19</sup>As mentioned in Carroll (1992), when liquidity constraints are relaxed impatient buffer-stock consumers will borrow more and will have a lower target wealth.

<sup>20</sup>White (1998) calculates the option value for a typical household and shows that it can be very high.

<sup>21</sup>Fay, Hurst and White (2000) find support for strategic bankruptcy behavior.

one step further and analyze what determines the decision to default.

**Asset level which triggers default.** In this model the default rule is always such that there is an asset level below which default is triggered. In other words, if in a given period the consumer has a low income realization, outstanding debt is at its limit and his assets are below this equilibrium trigger level, he chooses to default. In order to see this more clearly, Figure 6 plots the value of assets below which the agent chooses to default. In periods where the cut off level of assets is zero default will never occur.

Assets above the exemption level are seized by credit card firms in case of default. These assets are valuable for consumers who never choose to default when assets are very high. If we set the stigma effect to zero, the default trigger assets is around 3,000 USD, which means that in case of default consumers have to give up 1,000 USD in order to discharge their liabilities of 10,000 USD. This value is constant over the life cycle. However, introducing the stigma effect causes the trigger asset level to change over the life cycle, as illustrated in Figure 6. It is young and older households who choose to default up to higher asset values. That is, over life the default trigger asset value is U-shaped, a pattern inverse to that of labor income. To understand this shape it is important to realize that current income and consumption of young and old households are lower than those of middle-aged households, so that the utility function is evaluated at a point where the marginal utility of assets is larger. Because of this the dollar amount that young households are willing to forego to avoid the desutility associated with the stigma effect is smaller, making them more prone to default.

Lowering the levels of outstanding debt, results in parallel downward shifts of the asset curve, making default less likely to occur. This is due to the fact that, when the level of outstanding debt is below the maximum there is one additional consideration for the default decision: reneging on the borrowing obligations rules out the possibility of an increase in debt levels in future periods. However, default will still occur for some positive values of assets, for young and old households. This may be surprising as one may have expected that the

possibility of increasing debt levels in the future may be most attractive for young liquidity constrained households. However, for this age group borrowing constraints combined with a steep labor income profile effectively lower the discount factor, i.e. make them behave in a more myopic manner. The possibility of increasing debt levels in the future is less relevant for myopic households. When outstanding debt is sufficiently small (e.g. equal to eight thousand dollars), and individuals may increase it by more in the future, even the very young choose not to default for any value of assets.

## **5.2 Comparative Statics**

Now that we have some basic intuition of what determines credit card demand and default, let us go one step further and analyze the model's predictions for some policy oriented questions. Given the present debate on the adequacy of the current bankruptcy law it is important to determine how a change in the exemption level would affect consumers' behavior. Moreover, it is one of the main goals of the paper to clarify our understanding of how macroeconomic conditions and consumers' heterogeneity affects borrowing and default behavior. With this in mind I have solved the model for other parameter values.

### **5.2.1 No Asset Exemption**

I have solved the model not allowing for asset exemption in the case of default, i.e. credit card companies seize all of consumers' assets in the case of default. Consequently, the default trigger asset level is lower by roughly 2,000 USD, which is the exemption level in the baseline case. This also means that the probability to default in any given period is lower. Strategic borrowing is reduced and the corresponding consumption is also lower.

### 5.2.2 Positive Probability of Unemployment

One important issue which is not emphasized in the baseline parameterization of the model is the possibility, faced by consumers, of a sharp drop in current income. This could be due to becoming unemployed. In order to analyze the effect of such a possibility on the results, I have solved the case where there is a positive probability of the transitory income shock being such that current income is 10% of its permanent level. The probability of unemployment was calculated to insure that the mean and variance of the transitory income shock is the same as in the baseline case.

Since the shock to current income is temporary,<sup>22</sup> the optimal policy is to borrow and leave consumption unchanged.<sup>23</sup> In order to be able to do so, consumers need to have access to credit, and the way to insure this access is to minimize the endogenous probability to default. Agents will consume and borrow less in good states.

However in case the consumer does go unemployed the probability of default is higher. This is due to the fact that in that case default trigger asset level is positive and constant over the life cycle. In fact, the results are very similar to the case where no stigma effect is considered. This is understandable if we keep in mind that in the case of unemployment, sharp drops in income drives consumption down to levels where marginal utility is high, consequently the stigma effect becomes relatively less, for all ages, including middle aged consumers. Note also that the trigger asset level is not very high because, in this case, assets are very valuable to consumers.

---

<sup>22</sup>In this case I am assuming that unemployment is a temporary shock to income. However, this might not always be true.

<sup>23</sup>As opposed to the case of a permanent income shock where the optimal policy would to decrease consumption in order to accomodate the new permanent income level.

### 5.2.3 Higher Labor Income Uncertainty

Much of the labor income uncertainty comes from the transitory income shock. However it is worthwhile to analyze the case where aggregate income uncertainty is higher.

Due to the convexity of the marginal utility function, higher labor income uncertainty increases the valuation of future consumption inducing agents to consume less today and save more; this is the precautionary savings effect. One might expect that this drives down default, but in bad states consumers are more likely to default. The reason being the same as explained for the case above.

### 5.2.4 Different Labor Income Profiles

When we consider different income profiles, namely through the analysis of the consumer behavior of different education groups, the model draws some interesting implications.

**Flat Income Profiles.** First let us consider the group without high school education, in which case labor income earnings are almost flat over the life cycle. Due to the flatness of labor income consumers do not wish to borrow early in life; as a result the availability of credit is not very valuable. Moreover, since income and consumption are generally lower the relative importance of the stigma effect is reduced. These two factors make the cost of defaulting less severe, and as a result there is more strategic borrowing (Figure 7). In addition, the default trigger asset level is constant over all ages.

**Steep Income Profiles.** On the other hand, if we consider households with college education, the results described above are reversed. More precisely, for these households the labor income profile is extremely steep and peaks late in life, hence, credit availability is crucial for these consumers. Also, income is on average higher making the stigma effect dominate over any incentive to default. As a result the trigger value of assets for which consumers would choose to default is negative for almost all years, which means that default will almost never occur.

For the same reason there is no strategic borrowing.

## **6 Simulation Results**

Using the policy functions described above, I simulate the consumption, borrowing and saving profiles of ten thousand households over the life-cycle. The means of these simulated profiles are presented and discussed next.

### **6.1 Baseline Model**

First I will look at the simulated profiles for the baseline model. Figure 8 plots the average profi



Life cycle also changes consumer's attitude toward default decisions. As can be seen in Table 3, it is much more likely that default occurs due to a bad income shock when individuals are young than when they are old, suggesting that older individuals are more likely to behave strategically towards default.

## 6.2 Comparative Statics

I now discuss the simulation results from solving the model with different parameterizations. Table 4 presents default rates, amount of debt and savings for those who borrow. We are now able to answer the question of what factors affect default rates most and in which direction.

The average annual default rate for the baseline case, which represent households with high school degree, is 0.6%. However if the stigma cost is equal to 0 the default rate increases to 2.1%. This suggests the already documented fact that a decrease in the stigma effect could explain, to some extent, the observed increase in the actual rate of default. In the case where the credit limit is increased to 15,000 USD the rate of default also goes up to 1.5%.

The possibility of becoming unemployed creates the most interesting result: extremely low rate of default. Since in this context the availability of credit is precious, consumers refrain from borrowing and defaulting. Mean debt choice for those who borrow is 1,600 USD compared to 5,470 USD, in the baseline case. This is because people rely heavily on credit in case they do become unemployed. This result could also partially explain why default rates have been rising in a context of economic expansion, when unemployment rates are low.

For those without high school, earnings are flat and low reducing the cost of default. For these households, credit availability is not very important, specially early in life, and the stigma effect becomes less relevant. As a result default rates are very high, averaging 1.9%, and most of the borrowing is made strategically in order to default. On the other hand, households with college education, who have very steep income profiles and higher earnings, choose never to default.

Another interesting feature of the model is that it predicts that consumers simultaneously hold assets and debt. Bertraud and Halassios (2001) term this as “Puzzle of Debt Revolvers” and Lehnert and Maki (2002) call it “Borrowing to Save.” In my model, as in Lehnert and Maki’s model, this is due to the exemption level on assets, which households are allowed to keep in case they file for bankruptcy. The average amount of savings, for those individuals who are borrowing, is higher in cases where the probability of default is higher. For example, average savings for those who borrow is 600 USD for individuals with no high school, 300 USD for those with high school degree and 200 USD for those with college education.

In order to gain some sense of what this default rate values mean, I have plotted the cumulative distribution of default in Figure 10. An annual default rate of 0.6% means that 27% of all households will default once over their life cycle. In the worst case scenario of a 1.9% annual default rate (when  $\beta$  is equal to zero), all households will choose to default at some point.

## 7 Validating the Model’s Predictions with Data

Before concluding, let us first compare quantitatively some of the model’s predictions to data. From the discussion above we learned that the model predicts the following: (i) default decreases with education level; (ii) default is negatively correlated with expectations about unemployment; and (iii) exemption levels drive consumers to simultaneously hold financial assets and credit card debt.

In order to assess the validity of the first result I have used 1996 PSID data, which was the only wave collecting data on personal bankruptcy filings. The survey asked households whether they had ever filed for bankruptcy and if yes, in which year. The number of filings was also asked, although there are very few observations with more than filing. Since in my model it is not possible to file for bankruptcy more than once, I have dropped observations with multiple filings. In order to be consistent with the model I also dropped households with

head aged less than 20 and over 65. Using this sample of 6400 observations, I have estimated a probit model of the bankruptcy decision on the education level,<sup>27</sup> controlling also for age<sup>28</sup> and income.<sup>29</sup> Results are reported in Table 5, together with the results of the same estimation but using a sample simulated by the model. The coefficient on education level is negative and statistically significant in both samples. However, the eff

card balances I regressed liquid financial wealth (which could be used to pay off credit card balances) on exemption level, controlling for age, education, income and credit card balance. The results are shown on Table 6. The coefficient on the exemption level is positive and significant, suggesting that consumers hold higher savings in regions where they are allowed to keep more of them in case they file for bankruptcy. This evidence is also confirmed by the estimation done by Lehnert and Maki (2002). Using the SCF in conjunction with other data sets they find supporting evidence that households are more likely to hold simultaneously low return assets and owe high-cost debt in states with higher exemption levels.

## 8 Concluding Remarks

The access to credit card debt has increased dramatically in recent years. However, the existing precautionary savings models rule out debt, either exogenously (Deaton (1991)), or by making assumptions so that it is never optimal for consumers to borrow (Carroll (1997), who assumes that there is a positive probability of zero labor income in every period). Instead, in this paper I make assumptions such that, as it happens in reality, consumers wish and are allowed to borrow against future labor income.

The model generates a number of interesting results. First, borrowing decision is driven not only by life cycle considerations, such as smoothing consumption early in life, but also by strategic behavior. Namely, some consumers choose to borrow with the intention of defaulting in the near future. Second, agents choose to default or not depending on how much they value credit availability and on how large is the relative importance of the stigma effect on the utility function. These two factors depend on the level and shape of consumers' labor income profile, and therefore their importance may vary over the life cycle. For consumers with flat and low labor income profiles (agents without high school education), neither the access to credit nor the stigma effect is sufficiently important to avoid default. Hence the model predicts strategic borrowing and large default rates, at all ages. For agents with an hump shaped labor

income profile (high school graduates), default is more likely to occur early and late in life. For consumers with very steep labor income profiles (college graduates) the model predicts little default.

The model also provides an explanation for the observed procyclicality of default. It is in periods of low uncertainty, which tend to coincide with those of low unemployment that buffer-stock consumers wish to consume and borrow more. Given the higher debt levels default becomes more likely.

Finally, the model incorporates an asset exemption level, which generates the necessary edge that leads consumers to hold simultaneously debt and savings.

## 9 References

Attanasio, O. (1995), "The Intertemporal Allocation of Consumption: Theory and Evidence," *Carnegie-Rochester Series on Public Policy*, 39-89.

Bertraud, C. C. and M. Haliassos (2001), "Debt Revolvers for Self Control," *HERMES Center Working Paper*, 01-11.

Carroll, C. D.(1992), "Buffer Stock Saving: Some Macroeconomic Evidence," *Brookings Papers on Economic Activity* 1992(2), 61-156.

Carroll, C. D. (1997), "Buffer Stock Saving and the Life Cycle/Permanent Income Hypotheses," *Quarterly Journal of Economics*, 112,1-55.

Carroll, C. D. and M. S. Kimball (1996), "On the concavity of the Consumption Function," *Econometrica*, 64, 981-92.

Carroll, C. D. and Summers (1991), "Consumption Growth Parallels Income Growth: Some New Evidence," in B.D. Bernheim and John B.Shoven, eds., *National Savings and Economic Performance*, Chicago University Press.

Cocco, J., F. Gomes, and P. Manheout (1999), "Consumption and the Portfolio Choice over the Life Cycle," *IFA Working Paper*, London Business School.

Deaton, A. 1991, "Savings and Liquidity Constraints," *Econometrica*, 59, 1221-1248.

Domowitz, Ian and Robert Sartain (1999), "Determinants of Consumer Bankruptcy Decision," *The Journal of Finance*, 54, 1241-77.

Fay, S., E. Hurst and M. White (2002), "The Household Bankruptcy Decision", *American Economic Review*, vol. 92:3, pp. 708-718.

Gourinchas, P.-O. and J. Parker (2002), "Consumption Over the Life Cycle," *Econometrica* 70(1), 47-89.

Gropp, R., J. Scholz and M. White (1997), "Personal Bankruptcy and Credit Supply and Demand," *Quarterly Journal of Economics*, vol. 112, pp. 217-251.

Gross, D. B. and N. S. Souleles (2002a), "An Empirical Analysis of Personal Bankruptcy

and Delinquency,” *The Review of Financial Studies*, 15(1), 319-347.

Gross, D. B. and N. S. Souleles (2002b), “Do Liquidity Constraints and Interest Rates Matter for Consumer Behavior? Evidence from Credit Card Data,” *Forthcoming Quarterly Journal of Economics*.

Hubbard, G., J. S. Skinner and S. Zeldes (1994), “The Importance of Precautionary Motives for Explaining Individual and Aggregate Savings,” in Allan H. Meltzer and Charles I. Plosser, eds., *Carnegie- Rochester Conference Series on Public Policy*, 40, 59-125.

Judd, K(1993), “Numerical Methods in Economics,” *Manuscript*, Hoover.

Kehoe, T. and D. Levine (2001), “Liquidity Constrained Markets versus Debt Constrained Markets,” *Econometrica*, 69 (3), 749-65.

Kocherlakota, N. (1996), “Implications of Efficient risk sharing without commitment,” *Review of Economic Studies*, 63 (4), 595-609.

Kowalewski, K. (2000), “Personal Bankruptcy: a Literature review,” *Congressional Budget Office*.

Laibson, D. A. Repetto and J. Tobacman (2000), “A Debt Puzzle,” *NBER Working Paper No. 7879*.

Lehnert, A. and D.Maki (2002), “Consumption, Debt and Portfolio Choice: Testing the Effect of Bankruptcy Law.” *Finance and Economics Discussion Series 14*. Board of Governors of the Federal Reserve System.

Lopes, P. (2001a), “Precautionary Savings with Borrowing and Default,” *Essays on Consumers’ Borrowing and Default Behavior*, Ph.D. Thesis, Boston University.

Lopes, P. (2001b), “On the Optimal Credit Card Debt Limit over the Life-Cycle,” *Essays on Consumers’ Borrowing and Default Behavior*, Ph.D. Thesis, Boston University.

Musto, D. (1999), “The Reacquisition of Credit Following Chapter 7 Personal Bankruptcy,” *Wharton Financial Institutions Center Working Paper No. 99-22*.

Sullivan, T. (2002), “Borrowing During Unemployment: Unsecured Debt as a Safety Net,”

Department of Economics, Northwestern University.

Sullivan, T. E. Warren and J. Westbrook (1989), "As We Forgive Our Debtors: Bankruptcy and Consumer Credit in America," New York: Oxford University Press.

Tauchen, G. and R. Hussey (1991), "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, 59, 371-396.

White M. "Why Don't More Households File for Bankruptcy?," *The Journal of Law, Economics & Organization*, V14 N2, 205-231.



Table 1:

$f_i$	$\varepsilon_{it}$	$\mu_t$
$f_i$	$\sigma_{\varepsilon_{it}}$	
	$\sigma_{\mu_t}$	

Table 2:

	$T$	
	$\beta$	
	$\gamma$	
$ff$	$s$	
		$\theta$
$\lambda$	$USD$	
	$e$	$USD$

Table 3

*USD.*

Panel A)

---

---

*B*

$\varepsilon$

---

$\mu$

---

Panel B)

---

---

*B*

$\varepsilon$

---

$\mu$

---

Table 4: This table shows annual default rates and average debt holdings (for those with positive debt), for different parameterizations.

Note: Values of debt are in thousand *USD*.

	Baseline	$s = 0$	$\lambda = 15,000$	$e = 0$	$prob.unemp > 0$	$\sigma_{\mu}^2 = 0.01$	No Hi Sch	College
Default rate	0.006	0.021	0.015	0.004	0.0006	0.009	0.019	0.00004
Debt ( $B$ )	5.47	5.89	7.54	5.51	1.6	5.35	5.27	6.27

Table 5

ffi

---

ffi	ffi
-----	-----

---

Table 6

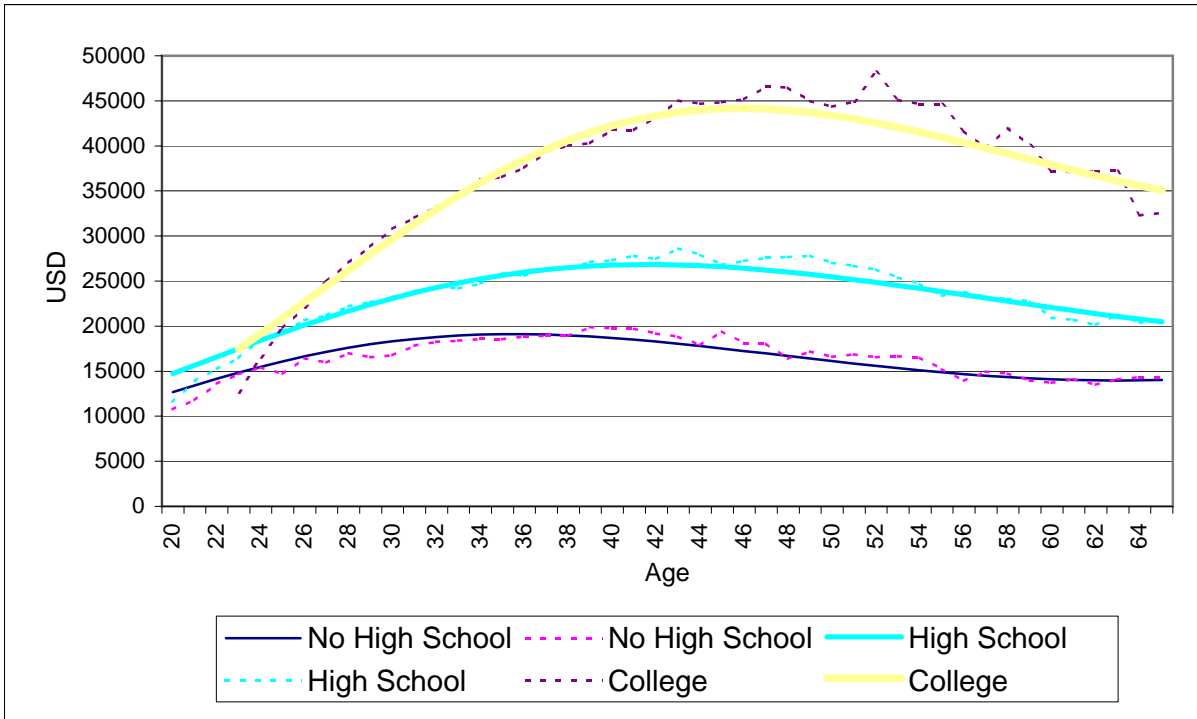
fi

---

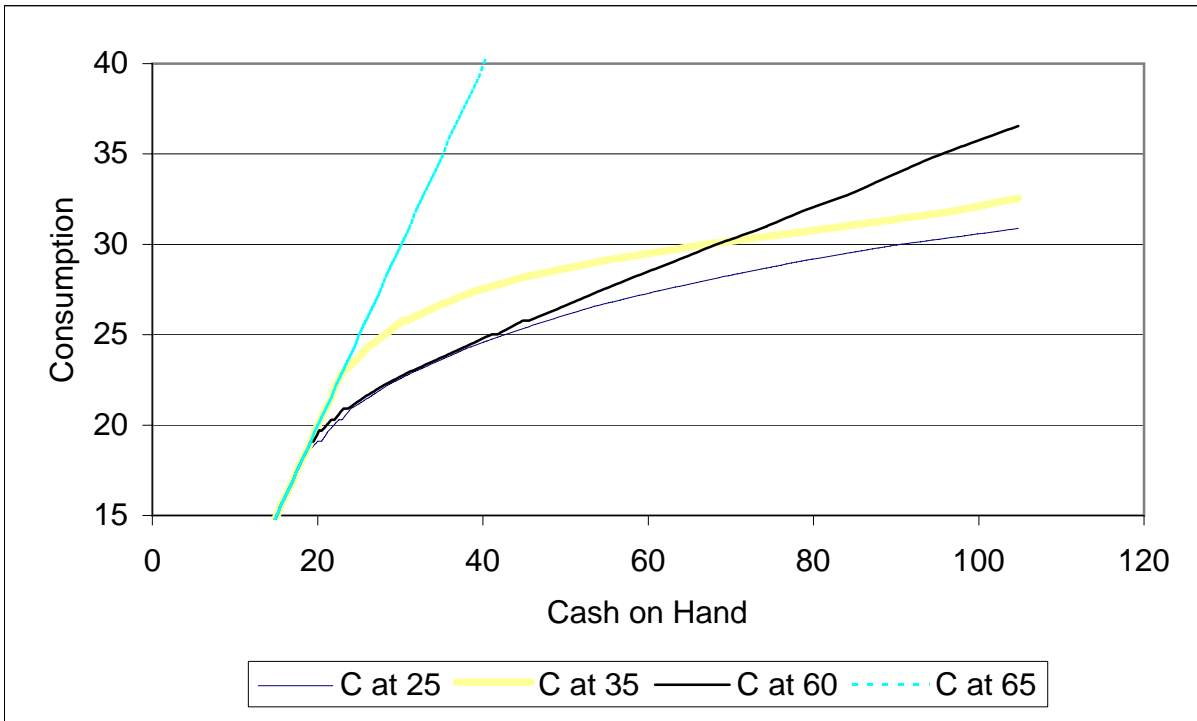
ffi
-----

---

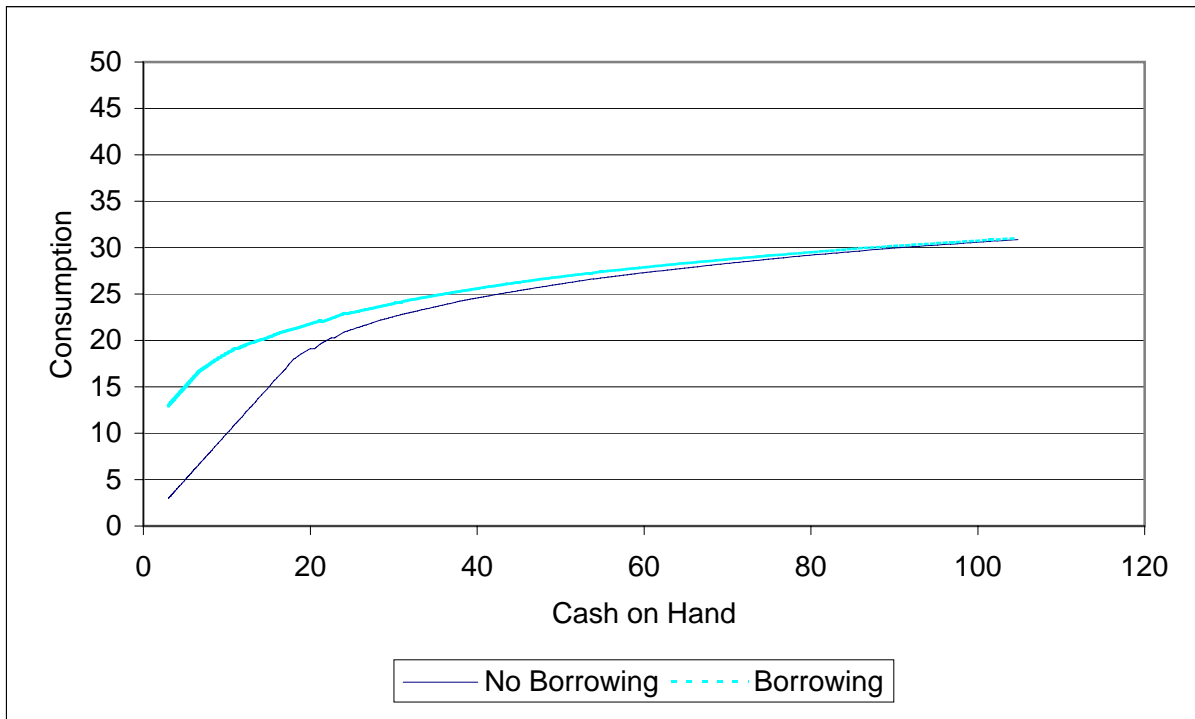
**Figure 1 - Labour Income Profiles**  
(Age Dummies and Fitted Polynomials)



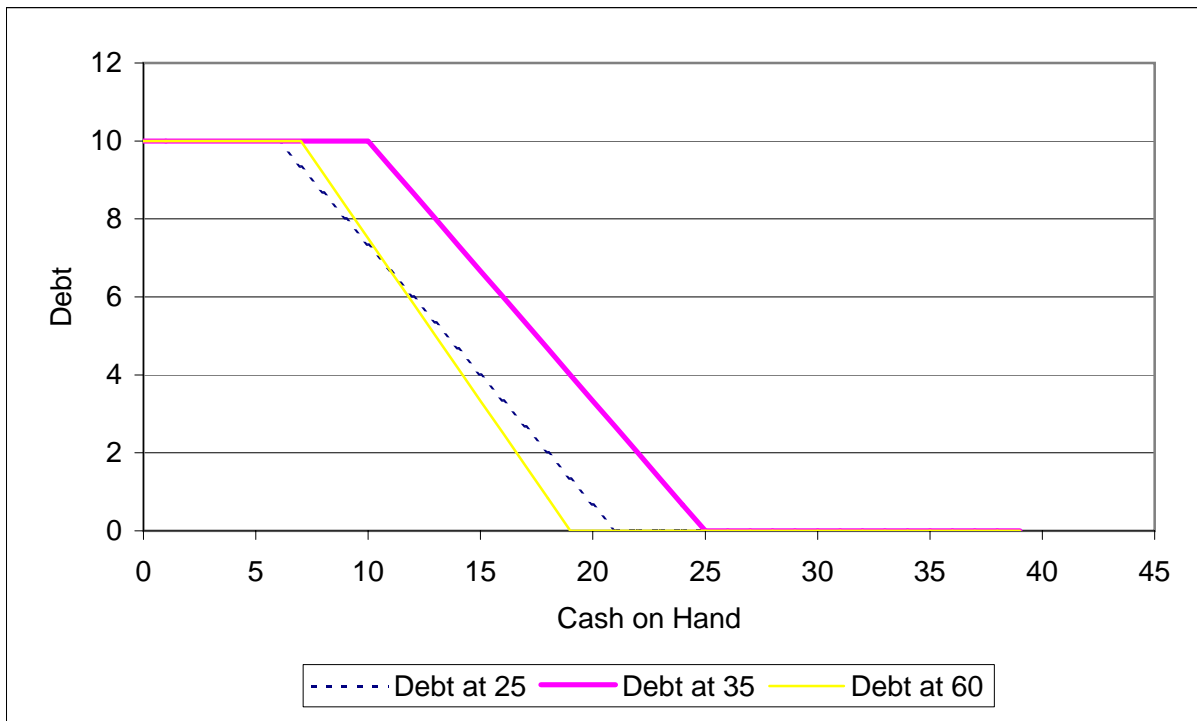
**Figure 2 - Consumption Policy Functions at Different Ages**  
(No Borrowing Allowed. Values in Thousand USD)



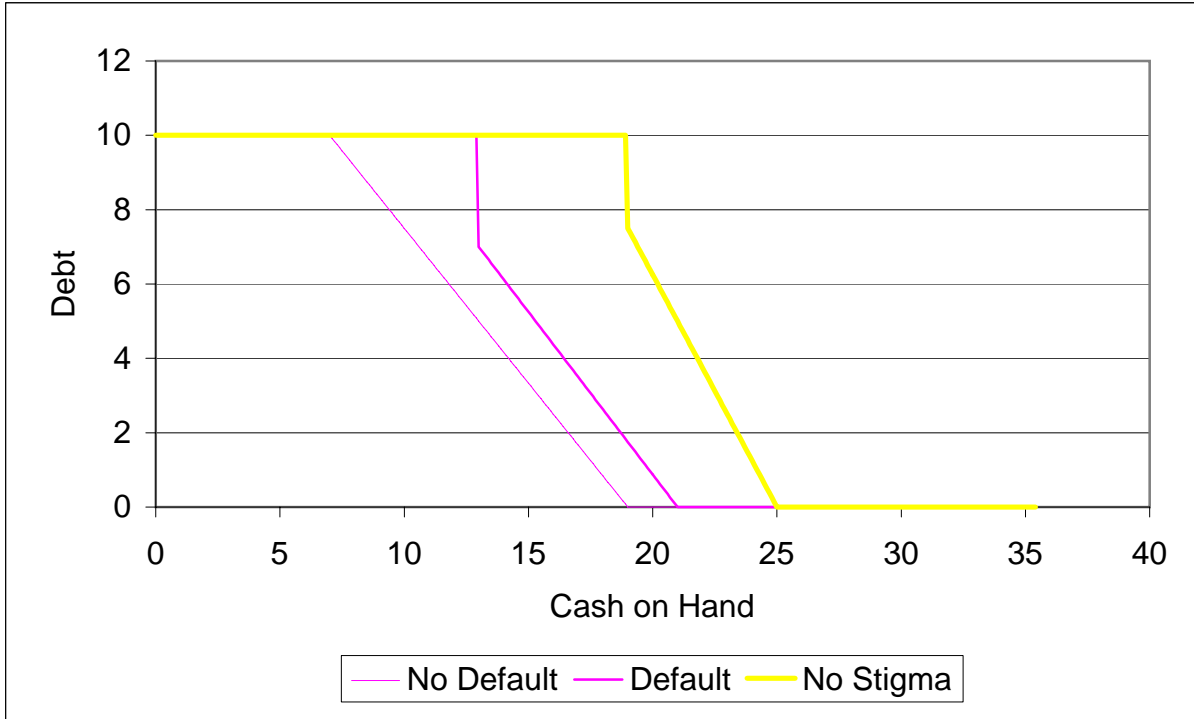
**Figure 3 - Consumption Policy With and Without Borrowing**  
(At Age 25. Values in thousand USD)



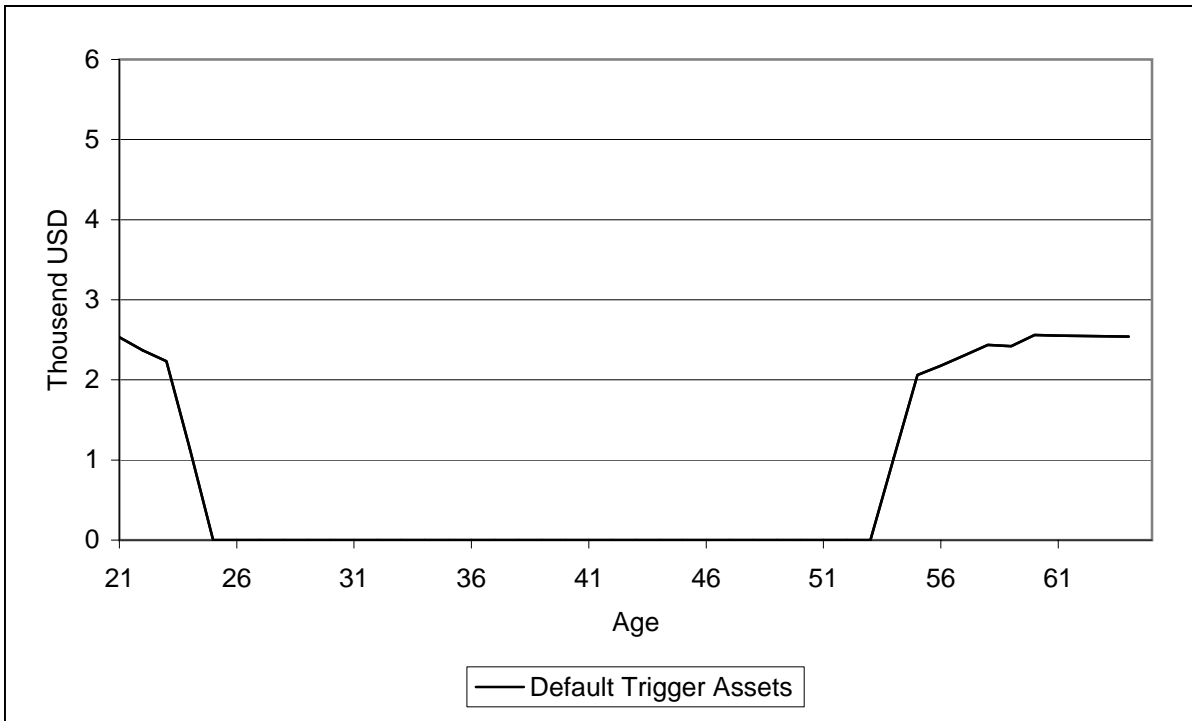
**Figure 4 - Borrowing Policy Functions at Different Ages**



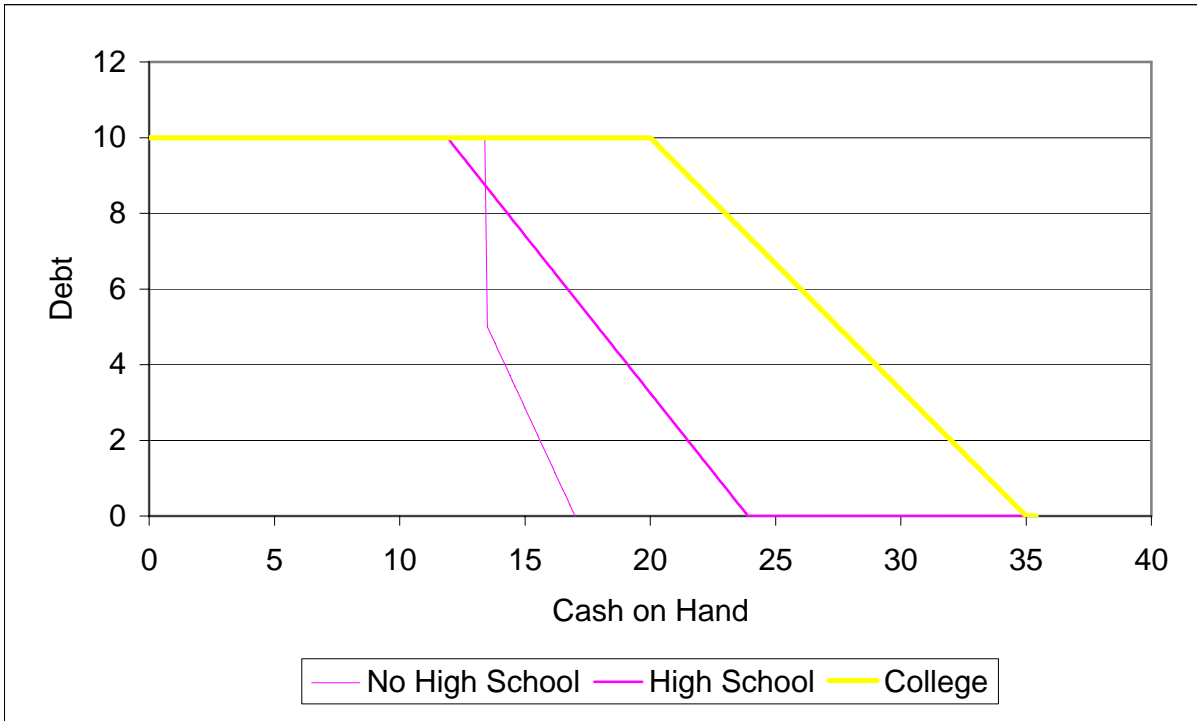
**Figure 5 - Borrowing Policy Functions**  
 (No Option to Default, Option to Default, and No Stigma)  
 (Values in Thousand USD. Age 60)



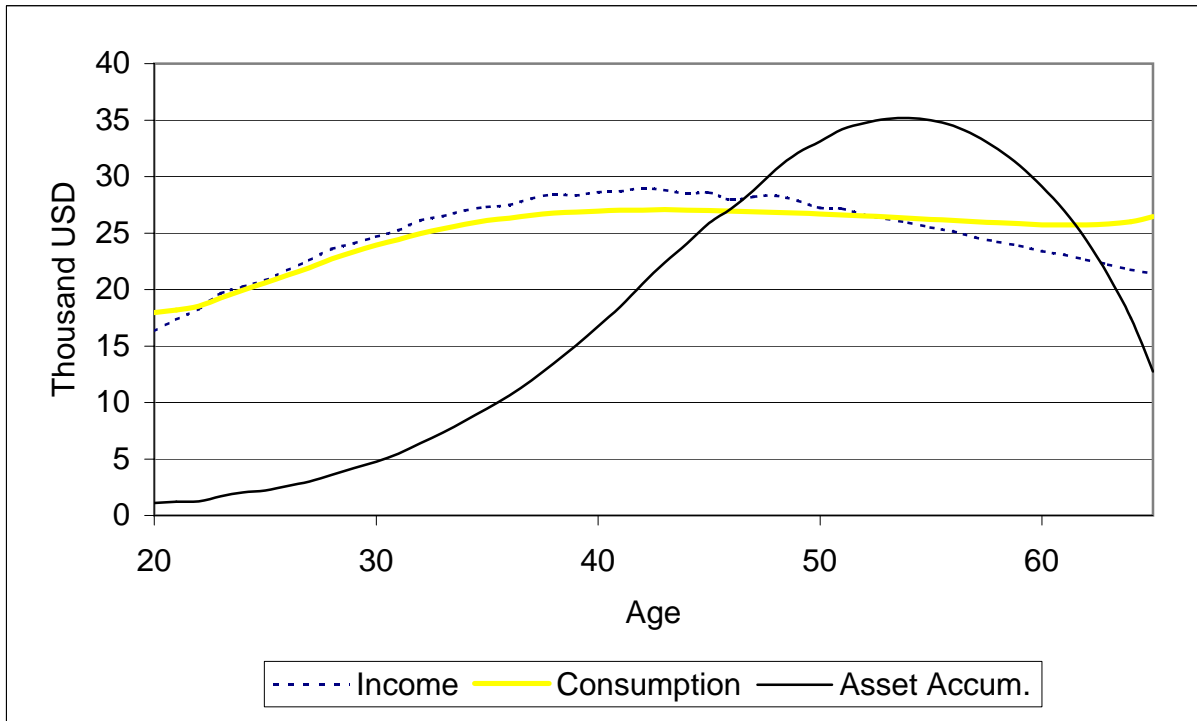
**Figure 6 - Asset Level that Triggers Default**  
 (Outstanding Debt is 10,000 USD)



**Figure 7 - Borrowing Policy Functions for Different Education Groups (Values in Thousand USD)**

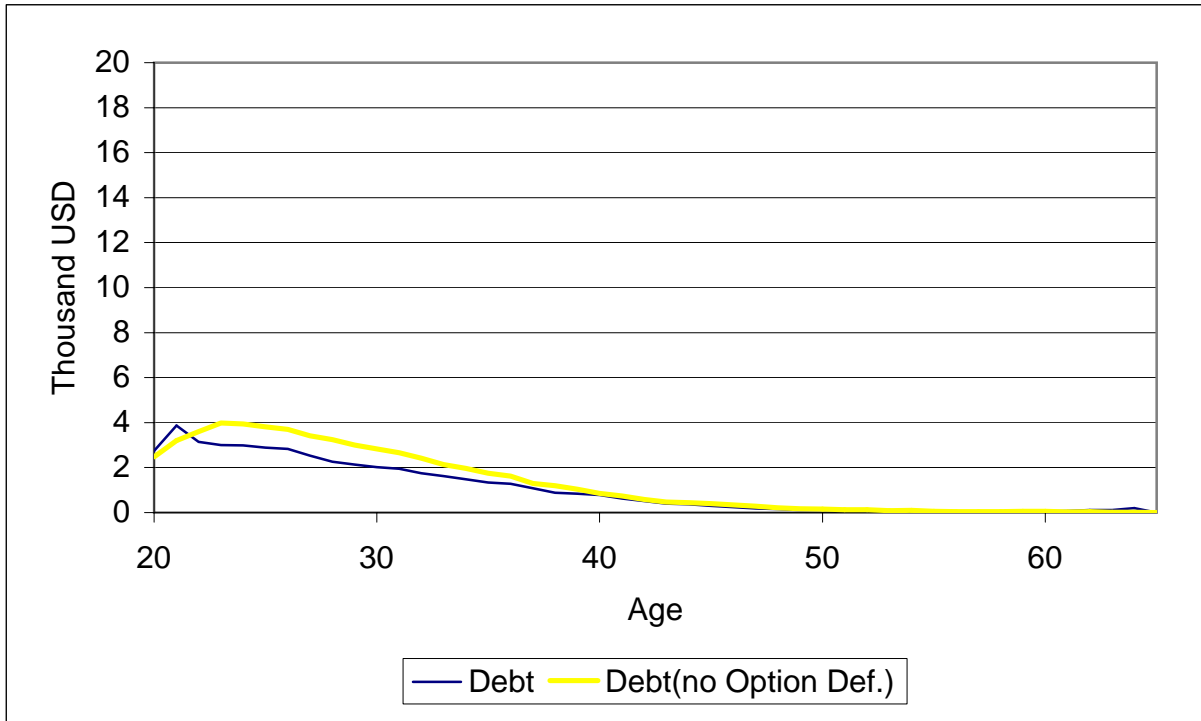


**Figure 8 - Average Simulated Profiles (Income, Consumption and Asset Accumulation)**

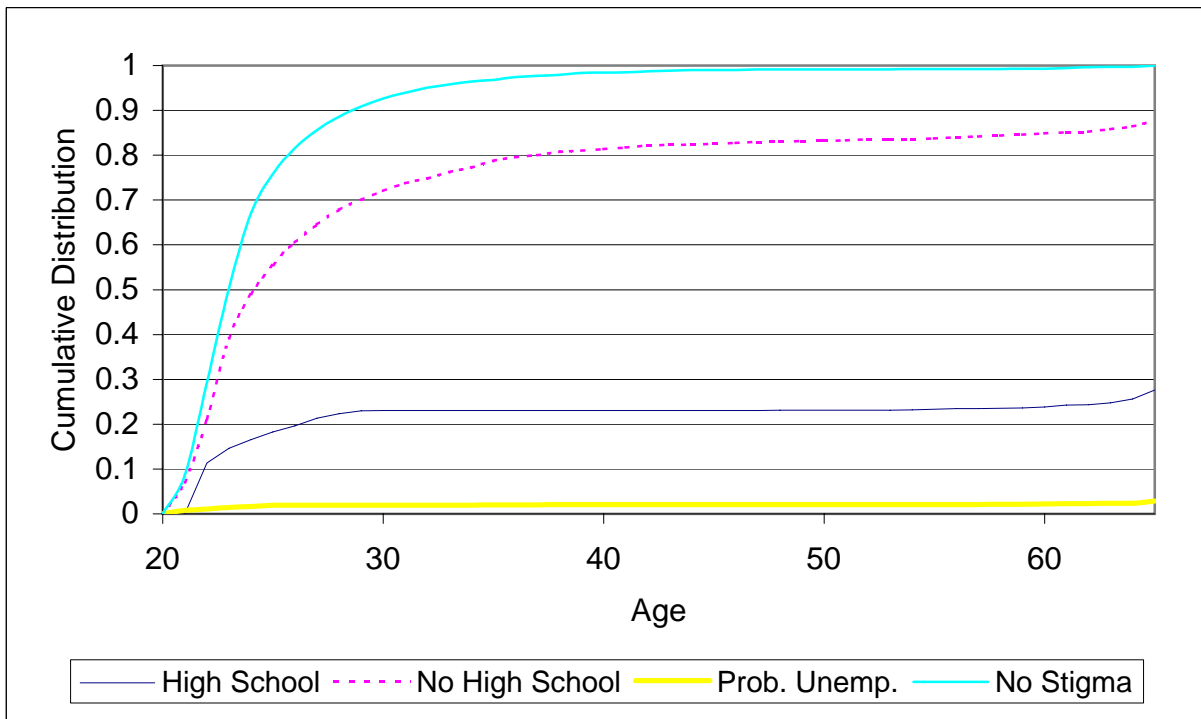




**Figure 9 - Average Debt**  
(With and Without Option to Default)



**Figure 10 - Cumulative Distribution of Probability to Default**  
(No High School, High School, Prob. Unemployment, No Stigma)



**Figure 11 - Unemployment Rate and Bankruptcy Rate**  
(US Data from 1980 to 2002)

